Reasoning in and about situations and agents: a hybrid logic story

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Monday 19 May, 2014
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Abstract

We reason, for the most part, not about the universe as a whole but about small parts of it: the situations we encounters and navigate within on a daily basis. This thought inspire the development of situation theory and situation semantics in the 1980s and 90s. My own work in this area turned out to coincide with a parallel development of hybrid logic. In this talk I will try to chart the motivations for a logic of situated reasoning, explain the relevance of hybrid logic to this project and make connections with more recent work on social reasoning.
1. Situations

2. Internalisation

3. Social Epistemic Logic
Outline

1. Situations
2. Internalisation
3. Social Epistemic Logic
Why situations are relevant to logic

- Formal logic was developed for application to mathematics, with its ontology of abstract, timeless, unchanging objects.
- How well is the logical structure of our thoughts about ordinary objects and situations captured by formal logic?
- Applied logic: develop a language and logic for reasoning about specific scenarios, e.g. the epistemic states of agents while playing cards.
- Says little about the general structure of our thoughts or how we can reason across different scenarios.
Why situations are relevant to logic

Suppose you have developed a language, for reasoning about a certain kind of scenario, and a semantic theory whose outputs are statements like this:

\[ s \models \varphi \]

where \( s \) is a model of a specific scenario of the relevant kind and \( \varphi \) is a formula of your language. Then do the same for another application to get

\[ t \models \psi \]

So what is the relationship between the two?
Prospective for Situation Theory

To provide a general framework for reasoning in and about different situations, allowing for

- partiality: $s \not\models \psi$ and $s \not\models \neg \psi$.
- parameters: $s \models Rab$ if $s \models R'f(s)ab$.
- local constraints: $\varphi_1$ implies $\varphi_2$ (but not in all situations)
- situations: explicit reasoning about situations: $s, t, \ldots$

1980s and 90s: Barwise, Perry, Etchemendy, Israel, Aczel, etc.
Reasoning *in a situation*

Plenty of examples of this in applied logic. We assume the situation is fixed and study reasoning within a situation

\[ K_b \neg K_a p, \ K_b (q \rightarrow p) \therefore K_b \neg K_a q \]

Note: no mention of the (model of) the situation.
Reasoning *about* a situation

Need reference to situations: $s$, $t$, etc. Then:

- $\vdash$ elimination: $s$ and $s \models \varphi$ imply $\varphi$
- $\vdash$ introduction: $s$ and $\varphi$ imply $s \models \varphi$

$s$ acts as a *nominal proposition*, true in only one situation, the one to which it refers.

Alternative: $\downarrow x \ (x = s)$
Say that a formula is guarded if it is of the form $t \models \psi$ (or Boolean combination thereof)

Name If $s$ and $\Gamma$ imply $\varphi$ then $\varphi$ \hspace{0.5cm} (s not in $\varphi$ or $\Gamma$)

Guard If $s$ and $\Gamma$ imply $\varphi$ then $\varphi$ \hspace{0.5cm} ($\varphi$ and $\Gamma$ are guarded)

E.g. $s \models s$, if $s \models p$ and $t \models s$ then $t \models p$
But what about partiality?

1. Either $s \models \varphi$ or $s \models \neg \varphi$
2. If $s \models \varphi$ then $t \models s \models \varphi$ for all $t$

For (1) we can move to a partial logic; (2) is a bit trickier...
Whether or not $s \vDash \varphi$ depends on the situation within which you are reasoning about $s$. Call it $t$. So we are really asking whether $t \vDash s \vDash \varphi$. But that depends on the situation within which you are reasoning about $t$, ...

Perspective: $\ldots s_3, s_2, s_1, s_0 \vDash \varphi$

*But what does this mean?*
Simple model

\[ \langle S, +, - \rangle \text{ where } +_S, -_S : \text{pow } S \rightarrow \text{pow } S \] Given perspective \( \alpha \), an infinite sequence of situations, define:

\[
\begin{align*}
\llbracket \varphi \rrbracket_0^\alpha &= \llbracket \varphi \rrbracket \\
\llbracket \varphi \rrbracket_{n+1}^\alpha &= +_\alpha [\llbracket \varphi \rrbracket]_n^\alpha
\end{align*}
\]

Then \( \alpha \models \varphi \) iff \( \alpha_n \in [\llbracket \varphi \rrbracket]_n^\alpha \) for all \( n \).
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To provide a language and system of natural deduction which captures the general features of applied modal logic, within which new operators can be defined by their semantics and from which inference rules can be obtained ‘automatically’.
Situated Predicate Logic (sic)

\[ \varphi ::= t \mid rt_1 \ldots t_n \mid \neg \varphi \mid (\varphi \land \varphi) \mid \exists \varphi \mid \Diamond_x \varphi \mid \downarrow x \varphi \]

\[
\begin{align*}
M, a \models t & \quad \text{iff} \quad M(t) = a \\
M, a \models rt_1 \ldots t_n & \quad \text{iff} \quad \langle a, M(t_1), \ldots, M(t_n) \rangle \in r^M \\
M, a \models \Diamond_x \varphi & \quad \text{iff} \quad M, M(x) \models \varphi \\
M, a \models \downarrow x \varphi & \quad \text{iff} \quad M[^{\overline{x}}_{\overline{a}}], a \models \varphi \\
M, a \models \exists \varphi & \quad \text{iff} \quad M, b \models \varphi \text{ for some } b
\end{align*}
\]

Define \( \exists x \varphi \) to be \( \downarrow y \exists \downarrow x \Diamond_y \varphi \) (new y)
Rigidity Assumption

The denotation of singular terms does not depend on the evaluation point.

This can be relaxed but it gets more complicated.
Natural Deduction

Fitch-style system with the usual rules for Booleans but with an extra structural feature: horizontal bars which divide the deduction into segments which restrict these rules only to apply within a segment. In addition, the following rules which are not so-restricted:

@/
Interpretations

Define $\langle r \rangle \varphi$ to be $\exists x (rx \land \Diamond_x \varphi)$ and derive rules.
Outline

1 Situations

2 Internalisation

3 Social Epistemic Logic
Two Dimensional Framework

Agents $A$ and states $W$
Two Dimensional Framework

Agents $A$ and states $W$

Cognitive structure $F_a = \langle W, R_a \rangle$ for each $a \in A$
Two Dimensional Framework

Agents $A$ and states $W$

Cognitive structure $F_a = \langle W, R_a \rangle$ for each $a \in A$
Social structure $G_w = \langle A, S_w \rangle$ for each $w \in W$

E.g. Epistemic Logic of Friendship: $R_a = \{\sim_a\}$ and $S_w = \{\sim_w\}$
Social Epistemic Logic of Observation

$\rho \mid \eta \mid \neg \varphi \mid (\varphi \land \varphi) \mid [K] \varphi \mid [S] \varphi \mid [A] \varphi \mid @_{\eta} \varphi \mid \downarrow \eta \varphi$
Social Epistemic Logic of Observation

\[ \rho \mid \eta \mid \neg \varphi \mid (\varphi \land \varphi) \mid [K] \varphi \mid [S] \varphi \mid [A] \varphi \mid \odot_\eta \varphi \mid \downarrow \eta \varphi \]

I need help

N

I can see someone who needs help

\[ [S](N \rightarrow [K] N) \]

I know that a person needs help if I can see them (and they need help)

\[ (K @ d N \land \neg S d) \]

I know that Dennis needs help but I cannot see him

\[ \downarrow [A](\neg S n \rightarrow [K] @ n N) \]

everyone who can see me knows I need help
Social Epistemic Logic of Observation

\[ \rho | \eta | \neg \varphi | (\varphi \land \varphi) | [K]\varphi | [S]\varphi | [A]\varphi | \Box_\eta \varphi | \downarrow \eta \varphi \]

\(N\) I need help
\(\langle S \rangle N\) I can see someone who needs help
Social Epistemic Logic of Observation

\[\rho \mid \eta \mid \neg \varphi \mid (\varphi \land \varphi) \mid [K] \varphi \mid [S] \varphi \mid [A] \varphi \mid \Box \eta \varphi \mid \downarrow \eta \varphi\]

\begin{align*}
N & \quad \text{I need help} \\
\langle S \rangle N & \quad \text{I can see someone who needs help} \\
[S](N \rightarrow [K]N) & \quad \text{I know that a person needs help if I can see them (and they need help)}
\end{align*}
Social Epistemic Logic of Observation

\[ \rho \mid \eta \mid \neg \varphi \mid (\varphi \land \varphi) \mid [K] \varphi \mid [S] \varphi \mid [A] \varphi \mid \Box_{\eta} \varphi \mid \downarrow \eta \varphi \]

\begin{align*}
N & \quad \text{I need help} \\
\langle S \rangle N & \quad \text{I can see someone who needs help} \\
[S](N \rightarrow [K]N) & \quad \text{I know that a person needs help if I can see them (and they need help)} \\
([K] \Box_{d} N \land \neg \langle S \rangle d) & \quad \text{I know that Dennis needs help but I cannot see him}
\end{align*}
Social Epistemic Logic of Observation

\[
\rho \mid \eta \mid \neg \varphi \mid (\varphi \land \varphi) \mid [K] \varphi \mid [S] \varphi \mid [A] \varphi \mid @_\eta \varphi \mid \downarrow \eta \varphi
\]

\begin{align*}
\text{\textit{N}} & \quad \text{I need help} \\
\langle S \rangle \text{\textit{N}} & \quad \text{I can see someone who needs help} \\
[S](N \rightarrow [K]N) & \quad \text{I know that a person needs help if I can see them (and they need help)} \\
([K]@_d N \land \neg \langle S \rangle d) & \quad \text{I know that Dennis needs help but I cannot see him} \\
\downarrow n [A](\langle S \rangle n \rightarrow [K]@_n N) & \quad \text{everyone who can see me knows I need help}
\end{align*}
Social Epistemic Logic of Observation -
semantics

$R_a = \{ \approx_a \}$ and $S_w = \{ \prec_w \}$ and $g$ maps agent nominals to agents.
Social Epistemic Logic of Observation - semantics

$R_a = \{\approx_a\}$ and $S_w = \{\prec_w\}$ and $g$ maps agent nominals to agents.

\[
\begin{align*}
M, w, a & \models \rho \quad \text{iff} \quad \langle w, a \rangle \in V(\rho) \\
M, w, a & \models \eta \quad \text{iff} \quad a = g(\eta) \\
M, w, a & \models \neg \varphi \quad \text{iff} \quad M, w, a \not\models \varphi \\
M, w, a & \models (\varphi \land \psi) \quad \text{iff} \quad M, w, a \models \varphi \text{ and } M, w, a \models \psi \\
M, w, a & \models [K] \varphi \quad \text{iff} \quad M, v, a \models \varphi \text{ for all } v \text{ such that } \approx_a (w, v) \\
M, w, a & \models [A] \varphi \quad \text{iff} \quad M, w, b \models \varphi \text{ for all } b \in A \\
M, w, a & \models [S] \varphi \quad \text{iff} \quad M, w, b \models \varphi \text{ for all } b \text{ such that } \prec_w (a, b) \\
M, w, a & \models \Theta \eta \varphi \quad \text{iff} \quad M, w, g(\eta) \models \varphi \\
M, w, a & \models \downarrow n \varphi \quad \text{iff} \quad M[a^n], w, a \models \varphi
\end{align*}
\]
Interpret $s \models \varphi$ as $\Box_s K \varphi$.

If $\Box_s K \varphi$ and $s$ then $\varphi$. (cf. $\models$ elimination)
If $s$ and $\varphi$ then $\Box_s K \varphi$. (cf. $\models$ introduction)

The latter only works for those $\varphi$ which imply $K \varphi$. 
Thank you!