A logical model of the dynamics of peer pressure

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Outline
A community of teenage girls is divided between two current fashions. Some strictly prefer $A$, others strictly prefer $B$, others are $I$ indifferent and yet others are conflicted $O$.

*Strong suggestion:* If all of their friends strictly prefer one of the styles then they will too.

*Weak suggestion:* Even if some of their friends are indifferent, if the rest strictly prefer one style, they will still be influenced, but not as strongly.
exhibit interesting dynamics
exhibit interesting dynamics

Loops:
exhibit interesting dynamics

Loops:

\[ A \rightarrow B \rightarrow \sim \rightarrow \]

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LAMAS 2011
exhibit interesting dynamics

Loops:

\[ A \rightarrow B \quad \mapsto \quad B \rightarrow A \quad \mapsto \]
Loops:

\[ A \rightarrow B \rightsquigarrow B \rightarrow A \rightsquigarrow A \rightarrow B \rightsquigarrow \]
exhibit interesting dynamics

Loops:

\[ A \rightarrow B \leftrightsquigarrow B \rightarrow A \leftrightsquigarrow A \rightarrow B \leftrightsquigarrow B \rightarrow A \leftrightsquigarrow \]
exhibit interesting dynamics

Loops:

\[ A \rightarrow B \rightsquigarrow B \rightarrow A \rightsquigarrow A \rightarrow B \rightsquigarrow B \rightarrow A \rightsquigarrow \cdots \]
An Example

exhibit interesting dynamics

Loops:

\[
A \rightarrow B \quad \sim \quad B \rightarrow A \quad \sim \quad A \rightarrow B \quad \sim \quad B \rightarrow A \quad \sim \cdots
\]

\[
\begin{tikzpicture}
\node (A) at (0,0) {$A$};
\node (B) at (-1,-1) {$B$};
\draw[->] (A) to (B);
\draw[->] (B) to (A);
\end{tikzpicture}
\]
exhibit interesting dynamics

Loops:

\[ A \rightarrow B \rightsquigarrow B \rightarrow A \rightsquigarrow A \rightarrow B \rightsquigarrow B \rightarrow A \rightsquigarrow \ldots \]
exhibit interesting dynamics

Loops:

\[ A \rightarrow B \leadsto B \rightarrow A \leadsto A \rightarrow B \leadsto B \rightarrow A \leadsto \ldots \]
An Example

exhibit interesting dynamics

Loops:

\[ A \rightarrow B \sim B \rightarrow A \sim A \rightarrow B \sim B \rightarrow A \sim \ldots \]

\[ B \leftarrow A \sim A \leftarrow B \sim B \leftarrow A \sim A \leftarrow B \sim B \leftarrow A \sim \ldots \]
exhibit interesting dynamics

Loops:

\[ A \rightarrow B \sim A \rightarrow B \sim B \rightarrow A \sim B \rightarrow A \sim \ldots \]

\[ B \leftarrow A \sim A \leftarrow B \sim B \leftarrow A \sim A \leftarrow B \sim \ldots \]
An Example

If one of the friends is indifferent
An Example

If one of the friends is indifferent

the pattern stabilizes:
If one of the friends is indifferent

the pattern stabilizes:

\[ A \sim B \sim A \sim B \sim I \]
If one of the friends is indifferent

the pattern stabilizes:

\[
\begin{array}{c}
A \rightarrow B \rightarrow A \rightarrow B \\
I \quad \sim \quad I
\end{array}
\]

\[
\begin{array}{c}
B \rightarrow A \rightarrow B \\
I \quad \sim \quad I
\end{array}
\]

\[
\begin{array}{c}
A \rightarrow B \\
I
\end{array}
\]

\[
\begin{array}{c}
B \rightarrow I \\
B
\end{array}
\]
An Example

If one of the friends is indifferent

the pattern stabilizes:

\[ A \rightarrow B \rightarrow A \rightarrow B \rightarrow I \rightarrow A \rightarrow B \rightarrow I \rightarrow B \rightarrow I \rightarrow B \rightarrow B \rightarrow I \rightarrow B \rightarrow I \rightarrow B \rightarrow\]
If one of the friends is indifferent
the pattern stabilizes:

\[
\begin{align*}
A & \rightarrow B \rightarrow A \rightarrow B \\
& \rightarrow I \\
B & \rightarrow I \\
& \rightarrow B \\
& \rightarrow B
\end{align*}
\]
If one of the friends is indifferent, the pattern stabilizes:
and with a mutual friend
and with a mutual friend

Indifference results in convergence to the preference of the mutual friend
and with a mutual friend

Indifference results in convergence to the preference of the mutual friend

$I \leftarrow A \leftarrow I \Rightarrow A \Rightarrow I \Rightarrow B \Rightarrow I \Rightarrow B$
and with a mutual friend

Indifference results in convergence to the preference of the mutual friend

$A \rightarrow I \leftarrow B$

$A \leftarrow B \rightarrow I$
An Example

and with a mutual friend

Indifference results in convergence to the preference of the mutual friend

\[
\begin{align*}
A & \leftarrow I \\
I & \leftarrow B
\end{align*}
\]

\[
\begin{align*}
B & \leftarrow A \\
I & \leftarrow I \\
I & \leftarrow B \\
B & \leftarrow B
\end{align*}
\]
and with a mutual friend

Indifference results in convergence to the preference of the mutual friend
when all three are friends
when all three are friends

indifference spreads
when all three are friends

indifference spreads

\[
\begin{array}{c}
I \\
A \\
B
\end{array}
\leadsto
\begin{array}{c}
I \\
A \\
B
\end{array}
\]
when all three are friends

indifference spreads

\[
\text{I} \xleftarrow{} A \quad \text{I} \xleftarrow{} \text{B} \quad \sim \quad \text{I} \xleftarrow{} I
\]
indifference is more stable than conflict
indifference is more stable than conflict
indifference is more stable than conflict
indifference is more stable than conflict
and soul-mates can resist any pressure!

\[ \text{Diagram:} \quad B \xrightarrow{I} I \xrightarrow{A} A \quad \text{and} \quad B \xrightarrow{O} O \xrightarrow{A} \]

Liang, Seligman (Chongqing, Auckland)
Outline
Facebook logic

Facebook logic


SELIGMAN, J.M; GIRARD, P.; LIU, F., ‘Facebook Logic’ Postponed pending generous grant from Facebook (please).
Community preference frames

\[ F = \langle W, A, \sim, \leq \rangle \]
Community preference frames

\[ F = \langle W, A, \sim, \leq \rangle \]

- \( A \) = set of agents
- \( W \) = set of possible states
- \textit{Friendship}: a symmetric and irreflexive relation \( \sim_w \) on \( A \) for each \( w \in W \)
- \textit{Preference}: a relation \( \leq_a \) on \( W \) for each \( a \in A \)
Community preference frames

\[ F = \langle W, A, \sim, \leq \rangle \]

- \( A \) = set of agents
- \( W \) = set of possible states
- Friendship: a symmetric and irreflexive relation \( \sim_w \) on \( A \) for each \( w \in W \)
- Preference: a relation \( \leq_a \) on \( W \) for each \( a \in A \)

\( \sim^* \) = community
  \( = \) relation between agents connected by a chain of friends
  \( = \) the transitive closure of \( \sim \).
Preference logic in the community

Language and semantics

$$\varphi ::= i \mid n \mid p \mid \neg \mid \land \mid F \mid F^* \mid P \mid U$$

Semantics

$$M, w, a \models p$$ \iff $$\langle w, a \rangle \in V(p)$$
$$M, w, a \models i$$ \iff $$\langle w, a \rangle \in V(i)$$ (iff $$w = i$$)
$$M, w, a \models \neg \varphi$$ \iff $$M, w, a \not\models \varphi$$
$$M, w, a \models (\varphi \land \psi)$$ \iff $$M, w, a \models \varphi$$ and $$M, w, a \models \psi$$
$$M, w, a \models F\varphi$$ \iff $$M, w, b \models \varphi$$ for each $$b \sim_w a$$
$$M, w, a \models F^*\varphi$$ \iff $$M, w, b \models \varphi$$ for each $$b \sim^*_w a$$
$$M, w, a \models P\varphi$$ \iff $$w \leq_a v$$ for each $$v \in W$$ such that $$M, v, a \models \varphi$$
$$M, w, a \models U\varphi$$ \iff $$M, v, a \models \varphi$$ for each $$v \in W$$
Language and semantics

\[ \varphi ::= i \mid n \mid p \mid \neg \mid \land \mid F \mid F^* \mid P \mid U \]

Semantics

\begin{align*}
M, w, a &\models p & \text{iff} & \langle w, a \rangle \in V(p) \\
M, w, a &\models i & \text{iff} & \langle w, a \rangle \in V(i) \text{ (iff } w = i) \\
M, w, a &\models \neg \varphi & \text{iff} & M, w, a \not\models \varphi \\
M, w, a &\models (\varphi \land \psi) & \text{iff} & M, w, a \models \varphi \text{ and } M, w, a \models \psi \\
M, w, a &\models F \varphi & \text{iff} & M, w, b \models \varphi \text{ for each } b \sim_w a \\
M, w, a &\models F^* \varphi & \text{iff} & M, w, b \models \varphi \text{ for each } b \sim^*_w a \\
M, w, a &\models P \varphi & \text{iff} & w \leq_a v \text{ for each } v \in W \text{ such that } M, v, a \models \varphi \\
M, w, a &\models U \varphi & \text{iff} & M, v, a \models \varphi \text{ for each } v \in W
\end{align*}
Language and semantics

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Semantics

\[
\begin{align*}
M, w, a & \models p & \text{iff} & \langle w, a \rangle \in V(p) \\
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M, w, a & \models \neg \varphi & \text{iff} & M, w, a \not\models \varphi \\
M, w, a & \models (\varphi \land \psi) & \text{iff} & M, w, a \models \varphi \text{ and } M, w, a \models \psi \\
M, w, a & \models F\varphi & \text{iff} & M, w, b \models \varphi \text{ for each } b \sim_w a \\
M, w, a & \models F^*\varphi & \text{iff} & M, w, b \models \varphi \text{ for each } b \sim^*_w a \\
M, w, a & \models P\varphi & \text{iff} & w \preceq_a v \text{ for each } v \in W \text{ such that } M, v, a \models \varphi \\
M, w, a & \models U\varphi & \text{iff} & M, v, a \models \varphi \text{ for each } v \in W \\
\end{align*}
\]

\textit{P} is not a normal modal operator.
Preference and preference change

Outline
Outline
Preference between propositions

\[
\begin{align*}
(\varphi \leq \psi) &\quad= U((\varphi \land \neg\psi) \rightarrow P(\psi \land \neg\varphi)) \\
(\varphi < \psi) &\quad= (\varphi \leq \psi) \land \neg(\psi \leq \varphi) \\
(\varphi = \psi) &\quad= (\varphi \leq \psi) \land (\psi \leq \varphi) \\
(\varphi \neq \psi) &\quad= \neg((\varphi \leq \psi) \lor (\psi \leq \varphi))
\end{align*}
\]

- weak preference for \(\psi\) over \(\varphi\)
- strict preference for \(\psi\) over \(\varphi\)
- indifference concerning \(\psi\) and \(\varphi\)
- conflicted about \(\psi\) and \(\varphi\)
Some further abbreviation are useful

\[ \langle F \rangle \varphi \; =_{df} \; \neg F \neg \varphi \]
\[ \langle P \rangle \varphi \; =_{df} \; \neg P \neg \varphi \]
\[ \langle U \rangle \varphi \; =_{df} \; \neg U \neg \varphi \]
\[ @_i \varphi \; =_{df} \; U(i \rightarrow \varphi) \]
\[ i = j \; =_{df} \; U(i \rightarrow j) \]

\[ M, w, a \models \langle F \rangle \varphi \iff M, w, b \models \varphi \text{ for some } b \sim_w a \]
\[ M, w, a \models \langle P \rangle \varphi \iff M, v, a \not\models \varphi \text{ for some } v \not\succeq_a w \]
\[ M, w, a \models \langle U \rangle \varphi \iff M, c, a \models \varphi \text{ for some } v \in W \]
\[ M, w, a \models @_i \varphi \iff M, i, a \models \varphi \]
\[ M, w, a \models i = j \iff i = j \]
Some further abbreviation are useful

⟨F⟩φ = df ¬F¬φ
⟨P⟩φ = df ¬P¬φ
⟨U⟩φ = df ¬U¬φ
@iφ = df U(i → φ)
i = j = df U(i → j)

M, w, a |= ⟨F⟩φ iff M, w, b |= φ for some b ∼_w a
M, w, a |= ⟨P⟩φ iff M, v, a ̸|= φ for some v ̸>_a w
M, w, a |= ⟨U⟩φ iff M, c, a |= φ for some v ∈ W
M, w, a |= @iφ iff M, i, a |= φ
M, w, a |= i = j iff i = j
Some further abbreviation are useful

\[\langle F \rangle \varphi =_{df} \neg F \neg \varphi\]
\[\langle P \rangle \varphi =_{df} \neg P \neg \varphi\]
\[\langle U \rangle \varphi =_{df} \neg U \neg \varphi\]
\[@i \varphi =_{df} U(i \rightarrow \varphi)\]
\[i = j =_{df} U(i \rightarrow j)\]
Some further abbreviation are useful

\(\langle F \rangle \varphi = \text{df} \neg \neg \varphi\)  
\(\langle P \rangle \varphi = \text{df} \neg \neg \varphi\)  
\(\langle U \rangle \varphi = \text{df} \neg \neg \varphi\)  
\(\@ i \varphi = \text{df} U(i \rightarrow \varphi)\)  
\(i = j = \text{df} U(i \rightarrow j)\)

\(M, w, a \models \langle F \rangle \varphi \iff M, w, b \models \varphi \) for some \(b \sim_w a\)

\(M, w, a \models \langle P \rangle \varphi \iff M, v, a \nvdash \varphi \) for some \(v \nvdash_a w\)

\(M, w, a \models \langle U \rangle \varphi \iff M, c, a \models \varphi \) for some \(v \in W\)

\(M, w, a \models \@ i \varphi \iff M, i, a \models \varphi\)

\(M, w, a \models i = j \iff i = j\)
Some dynamic operators

\[ X \leq Y \] \quad \text{change preference so that } Y \text{ is weakly preferred to } X

\[ X \preceq Y \] \quad \text{change preference so that } Y \text{ is strictly preferred to } X

where \( X \) and \( Y \) are subsets of \( W \).
Notation

\[(X \leq Y) = \text{the set of } \langle u, v \rangle \text{ such that } u \in X \text{ and } v \in Y.\]

\[M, w, a \models (\varphi \leq \psi) \iff ([\varphi] \leq [\psi]) \subseteq a\]
The definition

If $X$ and $Y$ are disjoint then

$$[X \leq Y](\leq) = \leq \cup (X \leq Y)$$

add links from $X$ to $Y$

$$[X < Y](\leq) = (\leq \cup (X \leq Y)) \setminus (Y \leq X)$$

also subtract links from $Y$ to $X$
The definition

If $X$ and $Y$ are disjoint then

\[
[X \leq Y](\leq) = \leq \cup (X \leq Y)
\]

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\[
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\]

also subtract links from $Y$ to $X$

If $X$ and $Y$ are not disjoint then

\[
[X \leq Y] = [(X \setminus Y) \leq (Y \setminus X)]
\]

\[
[X < Y] = [(X \setminus Y) < (Y \setminus X)]
\]
The definition

If $X$ and $Y$ are disjoint then

$$[X \leq Y](\leq) = \leq \cup (X \leq Y)$$

add links from $X$ to $Y$

$$[X < Y](\leq) = (\leq \cup (X \leq Y)) \setminus (Y \leq X)$$

also subtract links from $Y$ to $X$

If $X$ and $Y$ are not disjoint then

$$[X \leq Y] = [(X \setminus Y) \leq (Y \setminus X)]$$

$$[X < Y] = [(X \setminus Y) < (Y \setminus X)]$$

Note: add diagrams for LAMAS presentation!
These operations may not preserve the transitivity of $\leq$
These operations may not preserve the transitivity of $\leq$

There is a deterministic solution for $[X \leq Y]$ but not for $[X < Y]$. 
The result of upgrading a model $M$

\[ M = \langle W, A, \sim, \leq \rangle \]

\[ [X \leq Y]_a M = \langle W, A, \sim, \leq' \rangle \]

where $\leq'_b = \leq_b$ for all $b \neq a$ and $\leq'_a = [X \leq Y](\leq_a)$.
The result of upgrading a model $M$

$M = \langle W, A, \sim, \leq \rangle$

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Similarly for $[X < Y]_a M$. 

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Similarly for $[X < Y]_a M$.

Semantics

$$M, w, a \models [\varphi \leq \psi] \theta \quad \text{iff} \quad [[\varphi] \leq [\psi]]_a M, w, a \models \theta$$

$$M, w, a \models [\varphi < \psi] \theta \quad \text{iff} \quad [[\varphi] < [\psi]]_a M, w, a \models \theta$$
Outline
An agent is subject to *peer pressure* regarding the issue of \( \alpha \) vs. \( \beta \) when

If (*Strong Suggestion*) all of her friends strictly prefer \( \alpha \) to \( \beta \) (and she has at least one friend) then she will upgrade her preferences with \( \beta < \alpha \), otherwise

if (*Weak Suggestion*) all of her friends regard \( \beta \) as at least as good as \( \alpha \) and some of them strictly prefer \( \alpha \) to \( \beta \) then she will upgrade her preferences with \( \beta \leq \alpha \).
### Suggestion Dynamics

The suggestion operator in our language

| $S(\alpha, \beta)$ | $(F(\alpha < \beta) \land \langle F \rangle(\alpha < \beta))$
|-------------------|------------------------|
|                   | my friends strongly suggest that $\beta$ is better than $\alpha$

| $S(\beta, \alpha)$ | $(F(\beta < \alpha) \land \langle F \rangle(\beta < \alpha))$
|-------------------|------------------------|
|                   | my friends strongly suggest that $\alpha$ is better than $\beta$

| $W(\alpha, \beta)$ | $(F(\alpha \leq \beta) \land \langle F \rangle(\alpha < \beta) \land \langle F \rangle(\beta \leq \alpha))$
|-------------------|------------------------|
|                   | my friends only weakly suggest that $\beta$ is better than $\alpha$

| $W(\beta, \alpha)$ | $(F(\beta \leq \alpha) \land \langle F \rangle(\beta < \alpha) \land \langle F \rangle(\alpha \leq \beta))$
|-------------------|------------------------|
|                   | my friends only weakly suggest that $\alpha$ is better than $\beta$

| $N(\alpha, \beta)$ | The negation of the disjunction of the above.
|-------------------|------------------------|
|                   | my friends have no suggestion regarding $\alpha$ and $\beta$
Suggestion transforms a model

\[ M = \langle W, A, \sim, \leq, V \rangle \text{ goes to } \#_{\alpha, \beta} M = \langle W, A, \sim, \#_{\alpha, \beta}(\leq), V \rangle \text{ where} \]

\[
\#_{\alpha, \beta}(\leq a) = \begin{cases} 
[\alpha < \beta](\leq a) & \text{if } M, a, u \models S(\alpha, \beta) \\
[\beta < \alpha](\leq a) & \text{if } M, a, u \models S(\beta, \alpha) \\
[\alpha \leq \beta](\leq a) & \text{if } M, a, u \models W(\alpha, \beta) \\
[\beta \leq \alpha](\leq a) & \text{if } M, a, u \models W(\beta, \alpha) \\
\leq a & \text{if } M, a, u \models N(\alpha, \beta)
\end{cases}
\]
We add the suggestion operator to our language

Semantics

\[ M, a, u \models \#_{\alpha, \beta} \varphi \text{ iff } \#_{\alpha, \beta} M, a, u \models \varphi \]

\( \#_{\alpha, \beta} \varphi \) means that the agent is subject to peer pressure and after upgrading her preferences satisfies the description \( \varphi \), equivalent to the conjunction of

\[
\begin{align*}
S(\alpha, \beta) & \rightarrow [\alpha < \beta] \varphi \\
S(\beta, \alpha) & \rightarrow [\beta < \alpha] \varphi \\
W(\alpha, \beta) & \rightarrow [\alpha \leq \beta] \varphi \\
W(\beta, \alpha) & \rightarrow [\beta \leq \alpha] \varphi \\
N(\alpha, \beta) & \rightarrow \varphi
\end{align*}
\]
### Some abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y)</td>
<td>((\alpha &gt; \beta)) preference for (\alpha) over (\beta)</td>
</tr>
<tr>
<td>(N)</td>
<td>((\alpha &lt; \beta)) preference for (\beta) over (\alpha)</td>
</tr>
<tr>
<td>(I)</td>
<td>((\alpha = \beta)) indifference concerning (\beta) and (\alpha)</td>
</tr>
<tr>
<td>(O)</td>
<td>((\alpha \neq \beta)) conflicted preferences about (\beta) and (\alpha)</td>
</tr>
<tr>
<td>(SY)</td>
<td>(F_Y \land \langle F \rangle Y) my friends strongly suggest (Y)</td>
</tr>
<tr>
<td>(SN)</td>
<td>(F_N \land \langle F \rangle N) my friends strongly suggest (N)</td>
</tr>
<tr>
<td>(WY)</td>
<td>(F(I \lor Y) \land \langle F \rangle Y \land \langle F \rangle I) my friends only weakly suggest (Y)</td>
</tr>
<tr>
<td>(WN)</td>
<td>(F(I \lor N) \land \langle F \rangle N \land \langle F \rangle I) my friends only weakly suggest (N)</td>
</tr>
<tr>
<td>(Z)</td>
<td>(\neg(SY \lor SN \lor WY \lor WN)) my friends have no suggestion</td>
</tr>
</tbody>
</table>
We can think of these as states of a finite state machine.
Defining stability

An agent \( a \) in state \( u \) of model \( M \) has \textit{stable preferences} iff for some preference state \( \sigma \in \{Y, N, I, O\} \) for each \( n \geq 0 \),

\[
\#_{\alpha,\beta}^n M, u, a \models \sigma
\]

The model \( M \) is \textit{stable} at \( u \) iff every agent has stable preferences at \( u \).

A model \( M \) \textit{stabilizes} at \( u \) iff for some \( n \geq 0 \),
Invariants

φ is an invariant of \( \#_{\alpha,\beta} \) iff for any model \( M \), state \( u \) and agent \( a \),

\[
M, u, a \models (\varphi \rightarrow \#(\alpha, \beta)\varphi)
\]

Lemma

The following formulas are invariants of \( \#_{\alpha,\beta} \):

1. \( Y \land \langle F \rangle Y \)
2. \( I \land \langle F \rangle I \)
3. \( Y \land F(I \lor Y) \)
4. \( (Y \lor I) \land \langle F \rangle (Y \lor I) \)
5. \( N \land \langle F \rangle N \)
6. \( N \land F(I \lor N) \)
7. \( (N \lor I) \land \langle F \rangle (I \lor N) \)
8. \( F^*(NY \land FFN) \lor (FN \land FFY) \)
9. \( \langle F \rangle \top \)
An important sufficient condition for stabilization

Lemma

*If* $M, u, a |\models 1$ *then* $a$’s preferences in $M$ *stabilizes at* $u$ *for* $\#_{\alpha, \beta}$.
Our main result

Theorem

\( M \) stabilizes at \( u \) for \( \#_{\alpha, \beta} \) iff

\[
M, u \models \neg (\langle F \rangle^\top \land F^*( (F^Y \land FFN) \lor (FN \land FFY) ))
\]
Concluding remarks

- a toy model of the way in which preferences within a community can be affected by social relations
- non-trivial dynamics with an interesting structure (role of indifference, etc.)
- stability characterized by a simple formula
- methodological proof-of-concept (many changes possible)
- extend to knowledge or belief, asymmetric social relationships (subordination) and aggregation
- transitivity-preserving operators? (non-deterministic dynamics)
- relationship to other work...