Knowledge, Friendship and Social Announcements

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October 15, 2013

Logic Meets the University
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Joint work with

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Reasoning about and within social networks

- communication channels
  - proximity
  - “friendship”
  - trust
  - …
- authority
  - political representation
  - group organisation
  - …

Social Sciences (psychology, economics, etc) use models to describe/explain/predict human activity.

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Highlights

• logical dynamics of networks (stability/flux, fragility)
  • peer pressure
  • belief influence
  • pluralistic influence

• indexical aspect of communication
  • “You are my friend"
  • sender/receiver indexicality

• dimensions of group knowledge
  • common knowledge among whom about what
  • explicit/indexical
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Outline

What this paper is about

Epistemic logic: a quick summary

Epistemic logic of friendship

Social announcements

Knowing your friends

Common knowledge reconsidered

Conclusion
Reasoning about Knowledge and Friendship

- knowledge of social relationships
- knowledge on the basis of social relationships
- communication via social relationships
- (changes to social relationships)
Reasoning about Knowledge and Friendship

- knowledge of social relationships
  Which friends of mine does she know about?
- knowledge on the basis of social relationships
- communication via social relationships
- (changes to social relationships)
Reasoning about Knowledge and Friendship

• knowledge of social relationships
  Which friends of mine does she know about?

• knowledge on the basis of social relationships
  What do her friends know?

• communication via social relationships

• (changes to social relationships)
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• communication via social relationships
  If she told her friends, would I know?

• (changes to social relationships)
Reasoning about Knowledge and Friendship

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  Which friends of mine does she know about?

• knowledge on the basis of social relationships
  What do her friends know?

• communication via social relationships
  If she told her friends, would I know?

• (changes to social relationships)
  If he became her friend, what would he know?
Berlin, 1978. A spy network has recently been uncovered by the Stasi, who are rounding up the spies and their associates. Bella (b) is friends with Charlie (c) and Erik (e), neither of whom are friends with each other. Unknown to the others is that Erik is a spy (s). The others are not spies, and Erik knows that because all spies know who else is a spy (we suppose). Bella knows that Charlie is not a spy, but Charlie does not know about her. After the network is exposed, all the spies and their friends will be interrogated by the police. But just before this happens a message is relayed to all agents revealing whether or not they are in danger, that is, whether they are a spy (which they would know in any case) or a friend of a spy. Who now knows that Erik is a spy?
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- Epistemic logic (S5): reasoning about knowledge
  - If I don’t know that I’m in danger then I know that I don’t know it.

- Multiagent epistemic logic (multi S5): reasoning about other people’s knowledge
  - Bella knows that Erik knows that Bella knows that Erik is in danger.

- Logic of common knowledge
  - It is common knowledge that Erik is in danger.

- Public announcement logic (PAL)
  - Were it to be announced that Bella is in danger, Charlie and Erik would know that.

- Dynamic epistemic logic (DEL)
  - If Bella does not know that he is in danger, and Charlie were told this in private, Bella would still not know it.
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Epistemic logic: the symbols

\( K_e(p \rightarrow q) \)  Erik (e) knows that if (p) the network has been exposed then (q) there is a mole.

\( K_b \neg K_c p \) Bella knows that Charlie (c) does not know the network has been exposed.

\( C_{bc} p \) It is common knowledge among Bella and Charlie that the network has been exposed.

\( [p!] K_e q \) If it were publicly announced that the network has been exposed, Erik would know there is a mole.

\( (\neg K_e q \land [p]) \rightarrow \neg K_e q \) Erik doesn’t know that there is a mole, and were it to be announced privately to the others that the network has been exposed, he would still not know.
Epistemic logic: the models
Epistemic logic: the models

\[ K_e(p \rightarrow q) \]
\[ K_b \neg K_c p \]
\[ C_{\{bc\}} p \]
\[ [p!] K_e q \]
Epistemic logic: the models

\[ K_e(p \rightarrow q) \]

\[ K_b \neg K_c p \]

\[ C\{bc\} p \]

\[ [p!] K_e q \]
Epistemic logic: the models

\[ K_e(p \rightarrow q) \checkmark \]

\[ K_b \neg K_c p \]

\[ C_{\{bc\}} p \]

\[ [p!] K_e q \]
Epistemic logic: the models

- $K_e(p \rightarrow q)$
- $K_b \models K_c p$
- $C_{\{bc\}} p$
- $[p!] K_e q$
Epistemic logic: the models

\[ K_e(p \rightarrow q) \quad \checkmark \]

\[ K_b \neg K_c p \quad \checkmark \]

\[ C_{\{bc\}} p \quad \times \]

\[ [p!] K_e q \]

Diagram:
- Points labeled pq, c, p, bc, bce, q, bce
- Lines connecting the points
- Logical expressions and their truth values marked
Epistemic logic: the models

\[ K_e(p \rightarrow q) \checkmark \]
\[ K_b \neg K_c p \checkmark \]
\[ C_{\{bc\}} p \times \]
\[ [p!]K_e q \]
Epistemic logic: the models

$K_e(p → q)$

$K_b ¬ K_c p$

$C_{\{bc\}} p$

$[p!] K_e q$
Epistemic logic: the models

\[ K_e(p \rightarrow q) \checkmark \]
\[ K_b \neg K_c p \checkmark \]
\[ C\{bc\} p \times \]
\[ [p!]K_e q \checkmark \]
\[ [p!]C\{bce\} p \checkmark \]
Epistemic logic: the models

\((\neg K_e q \land [p]^{e} \neg K_e q)\)
Epistemic logic: the models

\( \neg K_e q \land (p \land e \rightarrow \neg K_e q) \)
Epistemic logic: the models

\((\neg K_e q \land [p!!!]^e_i \neg K_e q)\)
Epistemic logic: the models

\((-K_e q \wedge p) \Rightarrow \neg K_e q\)
Epistemic logic: the models

\((-K_e q \land \neg K_e q\)\)
Epistemic logic: the models

\[
(\neg K_e q \land \neg K_e q)
\]

\[
\begin{array}{c}
pq \quad c \\
  \quad p \\
  \quad c \\
  pq \quad bc \\
\end{array}
\]

\[
\begin{array}{c}
pq \\
  \quad p \\
  \quad c \\
  pq \\
\end{array}
\]

\[
\begin{array}{c}
pq \quad bc \\
  \quad p \\
  \quad bc \\
  \quad bce \\
\end{array}
\]
Epistemic logic: the models

\((\neg K_e q \land \neg K_e q)\)
Epistemic logic: the models

\( (\neg K_e q \land \neg K_e q) \)
Epistemic logic: the models

\((\neg Keq \land [p]_e Keq) \checkmark\)
Epistemic logic: the models

\[ C_{bc}p \]

\[ C_{be}p \]
Epistemic logic: the models

\[ C_{bc}p \quad C_{be}p \]
Epistemic logic: the models

\[ C_{bc}p \quad \checkmark \quad C_{be}p \]
Epistemic logic: the models

$C_{bc}p$ ✓ $C_{be}p$
Epistemic logic: the models

\[ C_{bc}p \quad \checkmark \quad C_{be}p \quad \times \]
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Indexical propositions
Naming agents
Friendship modality
Anaphora

¬d  I am not in danger (d)
¬Kₐₑ s  I don’t know that Erik (e) is a spy (s)
F d  All my friends are in danger
KF d  I know that all my friends are in danger
@ₑKF d  Erik knows that all his friends are in danger
FK d  All my friends know they are in danger
⟨F⟩ d  Some of my friends are in danger
⟨F⟩ c  Charlie (c) is my friend
⟨F⟩Kₐₑ d  Some of my friends know that Erik is in danger
↓ n ⟨F⟩Kₐₙ d  I have a friend who knows that I am in danger
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\( \neg d \) \hspace{1cm} \text{I am not in danger (d)}
\( \neg K \@ e s \) \hspace{1cm} \text{I don’t know that Erik (e) is a spy (s)}
\( F d \) \hspace{1cm} \text{All my friends are in danger}
\( K F d \) \hspace{1cm} \text{I know that all my friends are in danger}
\( \@ e K F d \) \hspace{1cm} \text{Erik knows that all his friends are in danger}
\( F K d \) \hspace{1cm} \text{All my friends know they are in danger}
\( \langle F \rangle d \) \hspace{1cm} \text{Some of my friends are in danger}
\( \langle F \rangle c \) \hspace{1cm} \text{Charlie (c) is my friend}
\( \langle F \rangle K \@ e d \) \hspace{1cm} \text{Some of my friends know that Erik is in danger}
\( \downarrow n \langle F \rangle K \@ n d \) \hspace{1cm} \text{I have a friend who knows that I am in danger}
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⟨F⟩ d Some of my friends are in danger
⟨F⟩c Charlie (c) is my friend
⟨F⟩K@e d Some of my friends know that Erik is in danger
↓n ⟨F⟩K@n d I have a friend who knows that I am in danger
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↓n ⟨F⟩K@nd I have a friend who knows that I am in danger
Epistemic logic of friendship: quick summary

Indexical propositions

Naming agents

Friendship modality

Anaphora

\(\neg d\) \quad I am not in danger (\textit{d})
\(\neg K @_e s\) \quad I don’t know that Erik (\textit{e}) is a spy (\textit{s})
\(F d\) \quad All my friends are in danger
\(KF d\) \quad I know that all my friends are in danger
\(@_e K F d\) \quad Erik knows that all his friends are in danger
\(FK d\) \quad All my friends know they are in danger
\(\langle F \rangle d\) \quad Some of my friends are in danger
\(\langle F \rangle c\) \quad Charlie (\textit{c}) is my friend
\(\langle F \rangle K @_e d\) \quad Some of my friends know that Erik is in danger
\(\downarrow n \langle F \rangle K @_n d\) \quad I have a friend who knows that I am in danger
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⟨F⟩K@e)d Some of my friends know that Erik is in danger
↓n ⟨F⟩K@n)d I have a friend who knows that I am in danger
Epistemic logic of friendship: models

knowledge: $k$ friendship: $f$

I have a friend who does not know whether I am a spy.
Epistemic logic of friendship: models

knowledge: $k$

friendship: $f$

$\neg d$
$\neg K@e s$
$Fd$
$@e KF d$
$FKd$
$\langle F \rangle d$
$\langle F \rangle c$
$\langle F \rangle K@e d$
$\downarrow n \langle F \rangle K@n d$
Epistemic logic of friendship: models

knowledge: \( k \)

friendship: \( f \)

\(-d\)
\(-K@s\)
\(Fd\)
\(eKFd\)
\(FKd\)
\(\langle F \rangle d\)
\(\langle F \rangle c\)
\(\langle F \rangle K@e d\)
\(\downarrow n \langle F \rangle K@n d\)
Epistemic logic of friendship: models

knowledge: $k$

friendship: $f$

$d$

$s, d$

$s, d$

$d$

$d$

$s, d$

$s, d$

$c$

$b$

$e$

$\neg d$

$\neg K@e s$

$Fd$

$@e KFd$

$FKd$

$\langle F \rangle d$

$\langle F \rangle c$

$\langle F \rangle K@e d$

$\downarrow n \langle F \rangle K@n d$
Epistemic logic of friendship: models

knowledge: $k$

friendship: $f$

$d$

$s, d$

$s, d$

$d$

$d$

$s, d$

$d$

$s, d$

$c$

$b$

$e$

$¬d$

$¬K\@_e s$

$Fd$

$\@_e KFd$

$FKd$

$\langle F \rangle d$

$\langle F \rangle c$

$\langle F \rangle K\@_e d$

$\downarrow n \langle F \rangle K\@_n d$
Epistemic logic of friendship: models

I have a friend who does not know whether I am a spy.

\[ \neg d \]
\[ \neg K @ e s \]
\[ F d \]
\[ @ e K F d \]
\[ F K d \]
\[ \langle F \rangle d \]
\[ \langle F \rangle c \]
\[ \langle F \rangle K @ e d \]
\[ n \langle F \rangle K @ n d \]
Epistemic logic of friendship: models

<table>
<thead>
<tr>
<th>Knowledge:</th>
<th>Friendship:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d$</td>
<td>$\neg d$</td>
</tr>
<tr>
<td>$s, d$</td>
<td>$\neg K \odot_e s$</td>
</tr>
<tr>
<td>$s, d$</td>
<td>$F d$</td>
</tr>
<tr>
<td>$\neg K \odot_e K F d$</td>
<td></td>
</tr>
<tr>
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<td>$\langle F \rangle d$</td>
</tr>
<tr>
<td>$\langle F \rangle c$</td>
<td></td>
</tr>
<tr>
<td>$\langle F \rangle K \odot_e d$</td>
<td></td>
</tr>
<tr>
<td>$\downarrow n \langle F \rangle K \odot_n d$</td>
<td></td>
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</tbody>
</table>
Epistemic logic of friendship: models

I have a friend who does not know whether I am a spy
Epistemic logic of friendship: models

knowledge: \[ k \]

friendship: \[ f \]

\[-d \] \( \times \)
\[-K@e{s} \] \( \checkmark \)
\(Fd\) \( \times \)
\(eKFd\) \( \checkmark \)
\(FKd\) \( \times \)
\(\langle F\rangle d\) 
\(\langle F\rangle c\) 
\(\langle F\rangle K@e d\)  
\(\downarrow n \langle F\rangle K@n d\)
Epistemic logic of friendship: models

Knowledge:

-\(d\)  
-\(\neg K@es\)  
\(Fd\)  
\(\Diamond eKFd\)  
\(\langle F\rangle d\)  
\(\langle F\rangle c\)  
\(\langle F\rangle K@ed\)  
\(\downarrow n \langle F\rangle K@nd\)

Friendship:

-d  
\(\neg K@es\)  
\(Fd\)  
\(\Diamond eKFd\)  
\(\langle F\rangle d\)  
\(\langle F\rangle c\)  
\(\langle F\rangle K@ed\)  
\(\downarrow n \langle F\rangle K@nd\)
Epistemic logic of friendship: models

knowledge: \( k \)

friendship: \( f \)

\[ \neg d \times \]

\[ \neg \mathcal{K} @ e s \checkmark \]

\[ F d \times \]

\[ @ e K F d \checkmark \]

\[ F K d \times \]

\[ \langle F \rangle d \checkmark \]

\[ \langle F \rangle c \checkmark \]

\[ \langle F \rangle K @ e d \]

\[ \downarrow n \langle F \rangle K @ n d \]
Epistemic logic of friendship: models

knowledge: $k$

friendship: $f$

$d$

$s, d$

$s, d$

$d$

$d$

$s, d$

$s, d$

$c$

$b$

$e$

$\neg d$ ×

$\neg K @_e s$ ✓

$Fd$ ×

$@_e KFd$ ✓

$FKd$ ×

$\langle F \rangle d$ ✓

$\langle F \rangle c$ ✓

$\langle F \rangle K @_e d$ ✓

$\downarrow n \langle F \rangle K @_n d$
Epistemic logic of friendship: models

I have a friend who does not know whether I am a spy
I have a friend who does not know whether I am a spy

\[ \downarrow n \langle F \rangle \neg (K@n s \lor K@n \neg s) \]
Transforming the spy network model

\[
\text{cut}_K(d) = (d?; K; d?) \cup (\neg d?; K; \neg d?)
\]
Transforming the spy network model

\[ \text{cut}_K(d) = (d?; K; d?) \cup (\neg d?; K; \neg d?) \]
Transforming the spy network model

\[ \text{cut}_K(d) = (d?; K; d?) \cup (\neg d?; K; \neg d?) \]

Diagram:

- Nodes: \(d\), \(s, d\), \(s, d\), \(d\), \(d\), \(d\), \(d\), \(s, d\), \(d\), \(s, d\), \(d\), \(b\), \(c\), \(e\)
- Edges: connecting each node as per the cut set description.
Transforming the spy network model

\[ \text{cut}_K(d) = (d?; K; d?) \cup (\neg d?; K; \neg d?) \]
Transforming the spy network model

\[ \text{cut}_K(d) = (d?; K; d?) \cup (\neg d?; K; \neg d?) \]

\[ [K := \text{cut}_K(d)] \]

\[ \Rightarrow \]
Transforming the spy network model

Who then knows that Erik is a spy (and didn’t know before)?
Who then knows that Erik is a spy (and didn’t know before)?

For which $n$,

$\forall_n (\neg K@es \land [K := \text{cut}_K(d)] K@es)$?
Transforming the spy network model

Who then knows that Erik is a spy (and didn’t know before)?

For which $n$,

$$\hat{n}(\neg K \oplus e s \land [K \equiv \text{cut}_K(d)] K \oplus e s)$$?

Answer: $b$ (Bella)
Restricting to Bella’s friends

\[
\text{send}_{\langle F \rangle b}(d) := [K := (\langle F \rangle b?; \text{cut}_K(d)) \cup (\neg \langle F \rangle b?; K)]
\]
Restricting to Bella’s friends

\[ \text{send}_{\langle F \rangle b}(d) := [K := (\langle F \rangle b?; \text{cut}_K(d)) \cup (\neg \langle F \rangle b?; K)] \]
Restricting to Bella’s friends

\[
\text{send}_{\langle F \rangle b}(d) := [K := (\langle F \rangle b?; \text{cut}_K(d)) \cup (\neg \langle F \rangle b?; K)]
\]
Restricting to Bella’s friends

\[
\text{send}_{\langle F \rangle b}(d) \; := \; [K \; := \; (\langle F \rangle b?; \text{cut}_K(d)) \cup (\lnot \langle F \rangle b?; K)]
\]
Restricting to Bella’s friends

\[
\text{send}_{\langle F \rangle b}(d) := [K := (\langle F \rangle b?; \text{cut}_K(d)) \cup (\neg \langle F \rangle b?; K)]
\]
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Sending messages

Revealing whether $\psi$ to agents $\theta$:

\[ \text{send}_\theta(\psi) = [K := (\theta?; \text{cut}_K(\psi)) \cup (\neg \theta?; K)] \]

Were $\eta$ to send the message $\psi$ to agents $\theta$ then $\phi$.

\[(\eta \text{ has the information } \psi \quad \rightarrow \quad [\text{send}_\theta(\psi)]\phi)\]

But the details depend on the kind of message being sent…
Announcements about the sender

Were \( \eta \) to announce to \( \theta \) that \( \psi \) (about \( \eta \)), then \( \varphi \).

\[
[\eta \triangleleft \psi!: \theta] \varphi = (@_\eta \mathcal{K} \psi \rightarrow [\text{send}_\theta (@_\eta \psi)] \varphi)
\]
Announcements about the sender

Were $\eta$ to announce to $\theta$ that $\psi$ (about $\eta$), then $\varphi$.

$$[\eta \triangleleft \psi! : \theta] \varphi \; = \; (\@_\eta K \psi \rightarrow [\text{send}\_\theta (@_\eta \psi)] \varphi)$$

Public announcements

$$[e \triangleleft s! : \top] AK @_e s$$

Were Erik to announce to everyone that he is a spy, then everyone would know he is.

$$[b \triangleleft s! : \top] AK @_b s$$

Were Bella to announce to everyone that she is a spy, then everyone would know she is.
Announcements about the sender

Were $n$ to announce to $\theta$ that $\psi$ (about $n$), then $\varphi$.

$$[n \triangleleft \psi! : \theta] \varphi = (\oplus_n K \psi \rightarrow [\text{send}_\theta(\oplus_n \psi)] \varphi)$$

Personal announcements

$$[e \triangleleft s! : b] \ominus_b K \ominus_e s$$

Were Erik to announce to Bella that he is a spy, then she would know he is.

$$(- (b \lor K \ominus_e s) \rightarrow [e \triangleleft s! : b] \neg K \ominus_e s)$$

An agent who is neither Bella nor (already) knows that Erik is a spy, still would not know this after he announces it to Bella.
Announcements about the sender

Were \( n \) to announce to \( \theta \) that \( \psi \) (about \( \eta \)), then \( \varphi \).

\[
[\eta ◁ \psi! : \theta] \varphi = (@_\eta K \psi \rightarrow [\text{send}_\theta (@_\eta \psi)] \varphi)
\]

Announcements to friends

\[
[b ◁ \neg s! : \langle F \rangle b] @_b FK @_b \neg s
\]

If Bella were to tell her friends that she is not a spy then they would all know that she isn’t a spy.
Announcements about the receiver

Were \( \eta \) to announce to \( \theta \) that \( \psi \) (about each of them), then \( \varphi \).

\[
[\eta : \psi ! \triangleright \theta] \varphi = (\circlearrowright \eta A(\theta \rightarrow \psi) \rightarrow [\text{send}_\theta(\psi)] \varphi)
\]
Announcements about the receiver

Were η to announce to θ that ψ (about each of them), then ϕ.

\[[η:ψ!▷θ]ϕ = (⊕_η A(θ → ψ) → [send_θ(ψ)]ϕ)\]

Public announcements

\[↓ n [n: d!▷ ⊤]AKd\]

Were I to announce to everyone ‘you are in danger’, everyone would know they are.
Announcements about the receiver

Were $\eta$ to announce to $\theta$ that $\psi$ (about each of them), then $\varphi$.

$$[\eta : \psi! \triangleright \theta] \varphi = (\Box_\eta A(\theta \rightarrow \psi) \rightarrow [send_\theta(\psi)] \varphi)$$

Personal announcements

$[e : d! \triangleright b] \varphi$  Were Erik to announce to Bella, ‘you are in danger’, then $\varphi$.

$[e \triangleleft \Box_b d! : b] \varphi$  Were Erik to announce to Bella, ‘Bella is in danger’, then $\varphi$. 
Announcements about the receiver

Were η to announce to θ that ψ (about each of them), then ϕ.

\[ [\eta: \psi! > \theta] \varphi = (\ominus_{\eta} A(\theta \rightarrow \psi) \rightarrow [\text{send}_{\theta}(\psi)] \varphi) \]

Announcements to friends

\[ \downarrow n [n: \langle F \rangle n! > \langle F \rangle n] FK \langle F \rangle n \]

Were I to announce to my friends, ‘you are my friend’, then all my friends would know they are my friends.
Private social announcements

\[ [\eta \triangleleft \circ_{\eta} \psi! : \theta] \]

\((\circ_{\eta} K \psi) \rightarrow \text{send}_{\theta}(\circ_{\eta} \psi) \quad K' \quad I \quad \varphi)\)

\(K := (K \cup (-\theta?; K'))^*\)

Were \(\eta\) to send a private message that \(\psi\) (about herself) to agents \(\theta\), then \(\varphi\).
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Peggy ($p$) knows that Roger ($r$) is cheating ($c$) on his wife, Mona ($m$). What’s more, Roger knows that Peggy knows, because they met accidentally while he was with his mistress. Mona does not know about the affair, and both Peggy and Roger know this. The situation (for Roger) deteriorates when he discovers that Peggy is a terrible gossip. She is bound to have told all her friends about his affair. What Roger does not know is whether Mona is a friend of Peggy (she is). Who knows what, exactly?
Roger’s Quandry

\[ c \]
\[ \downarrow \ n \ K(\odot_p K \odot_n c \land \odot_m \neg K \odot_n c) \]
\[ \downarrow \ n \ \odot_p K \odot_n K \odot_p K \odot_n c \]
\[ (\neg K \odot_m \langle F \rangle p \land \neg K \odot_m \langle F \rangle p) \]
\[ \downarrow \ n \ \odot_p K \odot_n \neg K \odot_m \langle F \rangle p \]
I wouldn’t know that Mona would know about my cheating, were Peggy to tell her friends about it.
I wouldn’t know that Mona would know about my cheating, were Peggy to tell her friends about it.
Roger’s Quandry

\[ \downarrow n \ [p \triangleleft @_n c!] : \langle F \rangle p \circ_m K \circ_n c \]

I wouldn’t know that Mona would know about my cheating, were Peggy to tell her friends about it.
Before returning home to face Mona, Roger is uneasy. He would really like to know whether or not she knows about his affair. He already knows that she knows if and only if she is friends with Peggy. So if Peggy told him that they are friends, he would be prepared for Mona’s fury. But for his planned excuses to be convincing, Mona must not know that he knows she knows (about the affair). It is therefore very important that Peggy tells him in private.
Roger needs a little more privacy

\[ [p \land (F)\Diamond m! : r] (\Diamond_{r}K\Diamond_{m}K\Diamond_{r}c \land \neg\Diamond_{m}K\Diamond_{r}K\Diamond_{m}K\Diamond_{r}c) \]

Were Peggy to tell Roger privately that she is friends with Mona, Roger would know Mona knows he has been cheating but Mona wouldn’t know that he knows.
Were Peggy to tell Roger, privately

\[
[p \triangleright (F) m! : r]_\phi
\]

\[
(\otimes_p K \langle F \rangle m \rightarrow \text{send}_r(\otimes_p \langle F \rangle m) \xrightarrow{K'} I)
\]

\[
K := (K \cup (\neg r ?; K'))^*
\]
After Peggy tells Roger, privately

\[send_r(\langle p, F \rangle m) \xrightarrow{K'} I\]

\[K := (K \cup (\neg r?; K'))^*\]

\[(\Diamond_r K \Diamond_m K \Diamond_r c \land \neg \Diamond_m K \Diamond_r K \Diamond_m K \Diamond_r c)\]
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Common knowledge reconsidered

It is common knowledge that $\varphi$
It is common knowledge that $\varphi$

**Group identification**

Charlie, Bella, and Erik
Bella’s friends
Friends of mine
Common knowledge reconsidered

It is common knowledge that $\varphi$

Group identification

Charlie, Bella, and Erik
Bella’s friends
Friends of mine

Perspective

It is common knowledge that Charlie is not a spy
It is common knowledge among Charlie’s friends that I am in danger
It is common knowledge among my friends that they are in danger
For common knowledge, we have:

\[
\overline{K}_n = (A; n?; K)
\]

\([\overline{K}_n] \varphi\) (a PDL formula) is equivalent to \(\Diamond_n K \varphi\) (an EFL formula).
Common knowledge, reconsidered

\[ K_n = (A; n?; K) \]

\([K_n]\varphi\) (a PDL formula) is equivalent to \(\circledast_n K\varphi\) (an EFL formula)

Now, for common knowledge,

\[ c_\theta = (A; \theta?; K)^*; A; \theta? \]

\([c_\theta]\varphi\) means that there is common knowledge among \(\theta\)-agents that \(\varphi\).
Common knowledge among enumerated groups

There is common knowledge among Bella and Erik that Bella is not a spy.

\[ [c(b \lor e)] @ b \neg s \]

\[ [(A; (b \lor e)更重要; K)^*; A; (b \lor e)] @ b \neg s \]
There is common knowledge among Bella and Erik that Bella is not a spy.

\[
[c_{(b \vee e)}]_{b \neg s}
\]

\[
[(A; (b \vee e)?; K)^*; A; (b \vee e)?]_{b \neg s}
\]
Common knowledge among enumerated groups

There is common knowledge among Bella and Erik that Bella is not a spy.

\[ [c_{(b \lor e)}] @ b \neg s \]

\[ [(A; (b \lor e)\otimes K)^*; A; (b \lor e)] @ b \neg s \]
There is common knowledge among Bella and Erik that Bella is not a spy.

\[ [c_{(b \lor e)}]_{b \not\leftrightarrow s} \]

\[ [(A; (b \lor e)^*; K)^*; A; (b \lor e)]_{b \not\leftrightarrow s} \]
There is common knowledge among Bella’s friends that Bella is not a spy.

\[ [c_{\langle F \rangle b}@_b \neg s] \]

\[ [(A; \langle F \rangle b?; K)^*; A; \langle F \rangle b?]@_b \neg s] \]
Common knowledge among non-indexically described groups

There is common knowledge among Bella’s friends that Bella is not a spy.

\[[c_{⟨F⟩b}]@_b \neg s\]

\[[(A; ⟨F⟩b?; K)^*; A; ⟨F⟩b?]@_b \neg s\]
Common knowledge among non-indexically described groups

There is common knowledge among Bella’s friends that Bella is not a spy.

\[ [c_{\langle F \rangle b}]_{b} \neg s \]

\[ [(A; \langle F \rangle b?; K)^*; A; \langle F \rangle b?]_{b} \neg s \]

send\(\top(d)\)
Common knowledge among non-indexically described groups

There is common knowledge among Bella's friends that Bella is not a spy.

\[ [c_{\langle F \rangle b}]@b \neg s \]

\[ [(A; \langle F \rangle b?; K)^*; A; \langle F \rangle b?]@b \neg s \]

\[ \Rightarrow \text{\text{send}_{\top}(d)} \]
Common knowledge among non-indexically described groups

There is common knowledge among Bella’s friends that Bella is not a spy.

\[
[c\langle F \rangle b]_{b} \neg s
\]

\[
[(A; \langle F \rangle b?; K)^{*}; A; \langle F \rangle b?]_{b} \neg s
\]

\[
\text{send}_{c}(d)
\]
Common knowledge among non-indexically described groups

There is common knowledge among Bella’s friends that Bella is not a spy.

\[ [c_{\langle F \rangle b}]_{b} \models s \]

\[ [(A; \langle F \rangle b?; K)^*; A; \langle F \rangle b?]_{b} \models s \]

send\(_{c}(d)\)  \(\Rightarrow\)
Common knowledge among an indexically described group of agents about an indexical proposition.

There is common knowledge among my friends that they are not friends of a spy.

\[ \downarrow n \ [c\langle F\rangle n] \rightarrow \langle F\rangle s \]

\[ \downarrow n \ [(A; \langle F\rangle n?; K)^*; A; \langle F\rangle n?] \rightarrow \langle F\rangle s \]

send_{\top}(d) \Rightarrow
The interaction between indexical descriptions of groups, and indexical contents of propositional attitudes give rise to numerous logical distinctions regarding common knowledge, and its relationship to communication.
Highlights

- logical dynamics of networks
- indexical aspect of communication
- dimensions of group knowledge