Varieties of Group Knowledge

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Varieties of Group Knowledge

While the concepts of common knowledge (we know it and we all know that we all know it) and distributed knowledge (were we to talk to each other, we would know it) are well-known to epistemologists and epistemic logicians, both ignore the role played by social relationships within our community. I will consider the effect of such relationships in structuring both the content and mode of access that we have to group knowledge, and show that consideration of this structure reveal many distinctions in the way in which knowledge can be shared, and how we reason about this.

In a nutshell...

My friends know they are my friends.

Outline

1. Reasoning about what a group knows
2. Reasoning from within a group
3. Reasoning about social relations
4. Social Cognitive Logic

Based on joint work with

$\star$ Liu Fenrong (Tsinghua) $\star$ Patrick Girard (Auckland) $\star$ Liang Zhen (SWU/Auckland) $\star$

Outline

Group knowledge

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Bella's friends know that she is a spy.

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Bella's only friends are Andre and Charlie, neither of whom are friends with each other.

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Who knows what?

Independent access

Andre, when alone with Bella, sees her secretly sliding a knife into her boot. Charlie, jealously rifling though Bella's handbag find three passports in different names all with Bella's photograph.

Joint acquisition

Bella has a rendezvous with Erik, a known enemy agent, in a location she supposes to be secret. Unknown to her, Andre and Charlie have been following her and observe her with Erik. They also see each other.

Collusion

Andre discovers a code pad engraved into a tube of Avon lipstick, discarded in his bathroom. It could only have been left by Bella or Danielle. Charlie knows that Danielle never wears Avon.

Three kinds of group knowledge

General/Mutual knowledge: Andre and Charlie both knows that Bella is a spy

$$E_{a,c}\ s(b)$$

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Distributed knowledge: Andre and Charlie would know that Bella is a spy if they shared their information about her.

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c.f. Halpern and Moses,1990

Some Principles of Epistemic Logic

$Kp \supset p$	knowledge implies truth
$K(p \supset q) \supset (Kp \supset Kq)$	epistemic closure
$Kp \supset KKp$	positive introspection
$\sim Kp \supset K \sim Kp$	negative introspection
$C_G p \equiv E_G(p \,\&\, C_G p)$	fixed point
$D_G \supset D_H$ if $H \subseteq G$	inclusion

Induction rule: from $\varphi \supset E_G(\varphi \,\&\, \psi)$ infer $\varphi \supset C_G q\psi$

Outline

Subject-indexical propositions

s I am a spy

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s is true of Bella but is false of Danielle.

Subject-indexical propositions

s I am a spy

s is true of Bella but is false of Danielle.

From Bella's perspective, s is true.
From Danielle's perspective, s is false.

Shifting perspectives

$@_b\, s$ [Shift to Bella's perspective] I am a spy

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$@_b\, s$ is true of Andre, who is not a spy, because Bella is a spy.

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$@_b\, s$ is true of Andre, who is not a spy, because Bella is a spy.

From Andre's perspective, $\sim s \ \& \ @_b s$ is true.

Shifting perspectives about knowledge

$@_b\, Ks$ Bella knows that she is a spy

Shifting perspectives about knowledge

$@_b\, Ks$	Bella knows that she is a spy
$@_a{\sim}K@_b s$	Andre does not know that Bella is a spy.

The Essential Indexical

Bella suffers a traumatic injury, loses her memory and undergoes extensive reconstructive surgery. While recovering she read the detailed file on secret agent Isabella “Bella” Donna. She knows that the beautiful and deadly Bella is a spy but she does not know that she is Bella.

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$$\sim Ks \ \& \ K@_b s$$

The Essential Indexical

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$$\sim Ks \ \& \ K@_b s$$

Also $\sim Kb$.

Group knowledge revisited

Andre and Charlie know that Bella is a spy.

General knowledge	$@_a K @_b s \ \& \ @_c K @_b s$
Common knowledge	$C_{a,c} @_b s$
Distributed knowledge	$D_{a,c} @_b s$

Andre and Charlie know that they are not spies.

General knowledge	$@_a K s \ \& \ @_c K s$
Common knowledge	$C_{a,c} s$?
Distributed knowledge	$D_{a,c} s$?

Common and distributed knowledge of indexical propositions

Andre and Charlie know that they are not spies.

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$$@_a K{\sim}s \ \& \ @_c K{\sim}s$$

Common and distributed knowledge of indexical propositions

Andre and Charlie know that they are not spies.

$$@_a K{\sim}s \;\&\; @_c K{\sim}s \;\&$$
$$@_a K @_c K{\sim}s \;\&\; @_c K @_a K{\sim}s$$

Common and distributed knowledge of indexical propositions

Andre and Charlie know that they are not spies.

$$@_a K \sim s \;\&\; @_c K \sim s \;\&$$
$$@_a K @_c K \sim s \;\&\; @_c K @_a K \sim s \;\&$$
$$@_a K @_c K @_a K \sim s \;\&\; \ldots$$

Common and distributed knowledge of indexical propositions

Andre and Charlie know that they are not spies.

$$@_a K \sim s \ \& \ @_c K \sim s \ \&$$
$$@_a K @_c K \sim s \ \& \ @_c K @_a K \sim s \ \&$$
$$@_a K @_c K @_a K \sim s \ \& \ \ldots$$

Equivalent to $C_{a,c}(@_a \sim s \ \& \ @_c \sim s)$

Common and distributed knowledge of indexical propositions

Andre and Charlie know that they are not spies.

$$@_a K \sim s \ \& \ @_c K \sim s \ \&$$
$$@_a K @_c K \sim s \ \& \ @_c K @_a K \sim s \ \&$$
$$@_a K @_c K @_a K \sim s \ \& \ \ldots$$

Equivalent to $C_{a,c}(@_a \sim s \ \& \ @_c \sim s)$

Similarly $D_{a,c}(@_a \sim s \ \& \ @_c \sim s)$

Self-identification

Bella knows that Andre does not know she is a spy.

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What does Bella know (from her perspective)?

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$$K@_a{\sim}Ks$$

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$\downarrow x\ K@_a{\sim}@_xKs$	

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What does Bella know (from her perspective)?

$K@_a{\sim}Ks$	I know that Andre does not know that he is a spy.
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$\downarrow x\ K@_a{\sim}@_xKs$	I know that Andre does not know that I am a spy.

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What does Bella know (from her perspective)?

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$\downarrow x\ K@_a{\sim}@_xKs$	I know that Andre does not know that I am a spy.
$@_b \downarrow x\ K@_a{\sim}@_xKs$	

Self-identification

Bella knows that Andre does not know she is a spy.

What does Bella know (from her perspective)?

$K@_a{\sim}Ks$	I know that Andre does not know that he is a spy.
$K@_a{\sim}K@_bs$	I know that Andre does not know that Bella is a spy.
$\downarrow x\ K@_a{\sim}@_xKs$	I know that Andre does not know that I am a spy.
$@_b \downarrow x\ K@_a{\sim}@_xKs$	Bella knows that Andre does not know that she is a spy.

Some Principles of Subject-Indexical Logic

$@_n(p \supset q) \supset (@_n p \supset @_n q)$	perspective closure
$@_n K p \equiv @_n K @_n p$	transparency
$@_n p \supset \sim @_n \sim p$	self-dual
$n \supset (p \equiv @_n p)$	intro
$@_n n$	ref
$@_n @_m p \equiv @_m p$	agree

Name: if n is not in φ then from $@_n \varphi$ infer φ

Outline

Knowing who your friends are

Charlie is my friend.

Knowing who your friends are

Charlie is my friend. $\langle F \rangle c$

Knowing who your friends are

Charlie is my friend. $\langle F \rangle c$

I don't know whether Erik is my friend.

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K \sim \langle F \rangle e)$

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K \sim \langle F \rangle e)$
None of my friends is a spy.	

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K \sim \langle F \rangle e)$
None of my friends is a spy.	$\sim \langle F \rangle s$

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K \sim \langle F \rangle e)$
None of my friends is a spy.	$\sim \langle F \rangle s$
I know that none my friends is a spy.	

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K\sim\langle F \rangle e)$
None of my friends is a spy.	$\sim\langle F \rangle s$
I know that none my friends is a spy.	$K\sim\langle F \rangle s$
Each of my friends knows s/he is not a spy.	

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K\sim\langle F \rangle e)$
None of my friends is a spy.	$\sim\langle F \rangle s$
I know that none my friends is a spy.	$K\sim\langle F \rangle s$
Each of my friends knows s/he is not a spy.	$FK\sim s$
I am a friend of all my friends.	

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K\sim\langle F \rangle e)$
None of my friends is a spy.	$\sim\langle F \rangle s$
I know that none my friends is a spy.	$K\sim\langle F \rangle s$
Each of my friends knows s/he is not a spy.	$FK\sim s$
I am a friend of all my friends.	$\downarrow n\ F\langle F \rangle n$
Each of my friends knows I'm a friend.	

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K\sim\langle F \rangle e)$
None of my friends is a spy.	$\sim\langle F \rangle s$
I know that none my friends is a spy.	$K\sim\langle F \rangle s$
Each of my friends knows s/he is not a spy.	$FK\sim s$
I am a friend of all my friends.	$\downarrow n\ F\langle F \rangle n$
Each of my friends knows I'm a friend.	$\downarrow n\ FK\langle F \rangle n$
I know I'm a friend of each of my friends.	

Knowing who your friends are

Charlie is my friend.	$\langle F \rangle c$
I don't know whether Erik is my friend.	$\sim(K\langle F \rangle e \vee K\sim\langle F \rangle e)$
None of my friends is a spy.	$\sim\langle F \rangle s$
I know that none my friends is a spy.	$K\sim\langle F \rangle s$
Each of my friends knows s/he is not a spy.	$FK\sim s$
I am a friend of all my friends.	$\downarrow n\ F\langle F \rangle n$
Each of my friends knows I'm a friend.	$\downarrow n\ FK\langle F \rangle n$
I know I'm a friend of each of my friends.	$\downarrow n\ KF\langle F \rangle n$

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Charlie is my friend.	$\langle F\rangle c$
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None of my friends is a spy.	$\sim\langle F\rangle s$
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Each of my friends knows s/he is not a spy.	$FK\sim s$
I am a friend of all my friends.	$\downarrow n\ F\langle F\rangle n$
Each of my friends knows I'm a friend.	$\downarrow n\ FK\langle F\rangle n$
I know I'm a friend of each of my friends.	$\downarrow n\ KF\langle F\rangle n$
	$\downarrow n\ F\downarrow m\ @_n K@_m\langle F\rangle n$

Indexical groups

Bella's friends	$@_n @_b \langle F \rangle n$
Those of whom Bella is a friend	$\langle F \rangle b$

(Assume symmetry of friendship for the moment.)

Knowledge among friends

My friends know I am spy.

General knowledge	$\downarrow n\ FK@_n s$
Common knowledge	$\downarrow n\ C_{\langle F \rangle n}@_n s$?
Distributed knowledge	$\downarrow n\ D_{\langle F \rangle n}@_n s$?

Common non-indexical knowledge among friends

My friends know I am spy.

It is common knowledge among my friends that I am spy.

Common non-indexical knowledge among friends

My friends know I am spy.
It is common knowledge among my friends that I am spy.

Andre is my friend, so Andre knows I am a spy.
Charlie is my friend, so Charlie knows I am a spy.

Common non-indexical knowledge among friends

My friends know I am spy.
It is common knowledge among my friends that I am spy.

Andre is my friend, so Andre knows I am a spy.
Charlie is my friend, so Charlie knows I am a spy.

$$@_n FK @_n s$$

So Andre knows Charlie knows I am a spy.
And Charlie knows Andre knows I am a spy.

Common non-indexical knowledge among friends

My friends know I am spy.
It is common knowledge among my friends that I am spy.

Andre is my friend, so Andre knows I am a spy.
Charlie is my friend, so Charlie knows I am a spy.

$$@_n FK @_n s$$

So Andre knows Charlie knows I am a spy.
And Charlie knows Andre knows I am a spy.

$$@_n FKFK @_n s$$

Common non-indexical knowledge among friends

My friends know I am spy.
It is common knowledge among my friends that I am spy.

Andre is my friend, so Andre knows I am a spy.
Charlie is my friend, so Charlie knows I am a spy.

$$@_n FK @_n s$$

So Andre knows Charlie knows I am a spy.
And Charlie knows Andre knows I am a spy.

$$@_n FKFK @_n s$$

But even if my only friends are Andre and Charlie, this leads to a notion of common knowledge that is *not* equivalent to $@_n C_{\langle F \rangle n} @_n s$ unless it is also common knowledge that my only friends are Andre and Charlie.

Indexical knowledge among friends

My friends know they are my friends.

General knowledge	$\downarrow n\ FK\langle F\rangle n$
Common knowledge	$\downarrow n\ C_{\langle F\rangle n}\langle F\rangle n$?
Distributed knowledge	$\downarrow n\ D_{\langle F\rangle n}\langle F\rangle n$?

Distributed indexical knowledge among friends

My friends know they are my friends.
It is distributed knowledge among my friends that I am their friend.

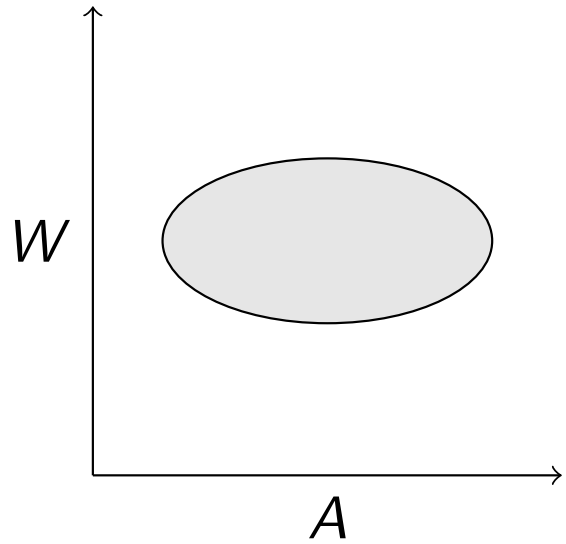
Distributed indexical knowledge among friends

My friends know they are my friends.
It is distributed knowledge among my friends that I am their friend.

By communicating, each of my friends can infer that s/he is my friend.
But this can be done with or without each friend knowing that each friend knows that s/he is my friend.

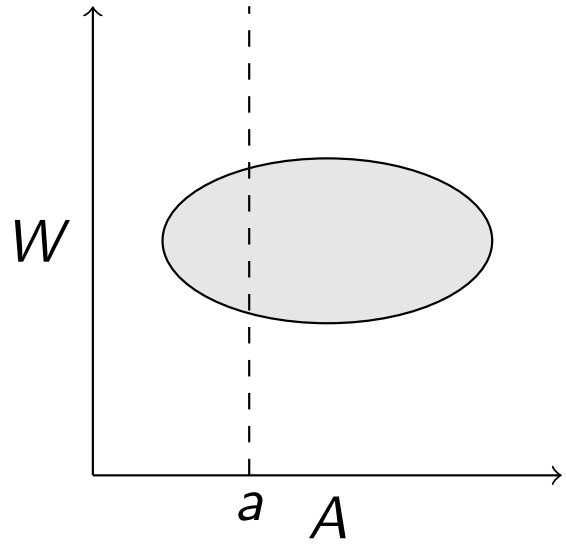
Outline

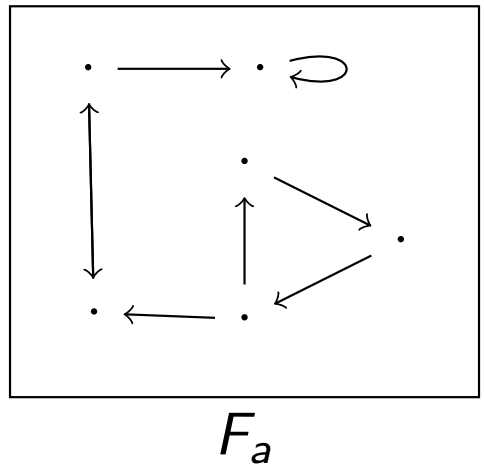
Two Dimensional Framework



Agents A and states W

Two Dimensional Framework



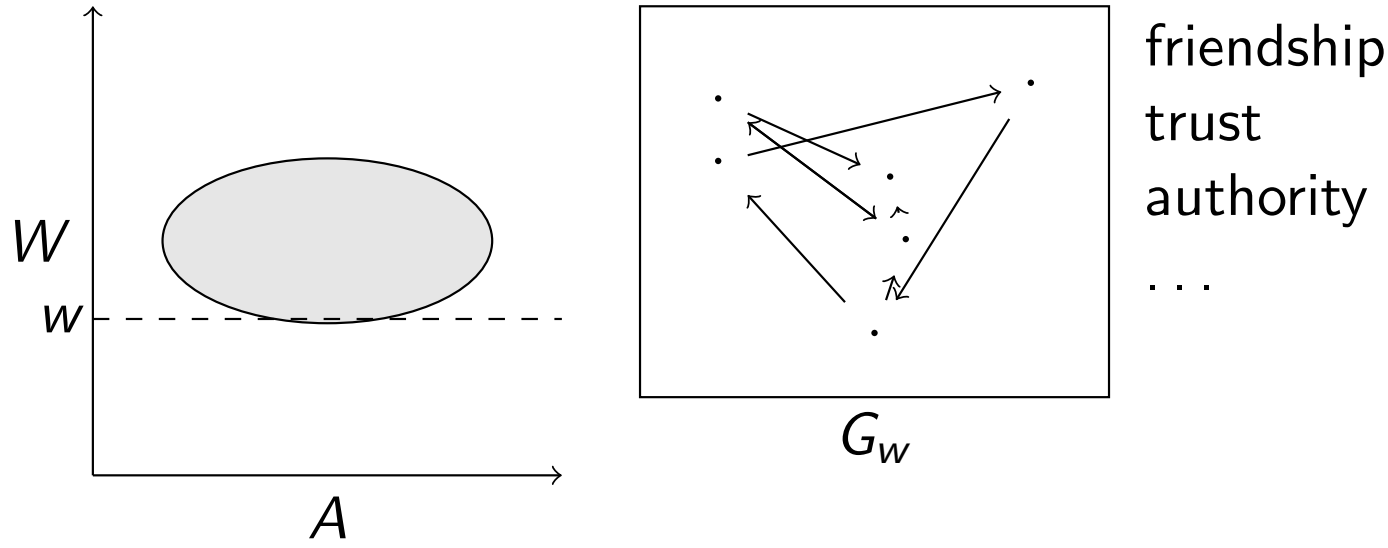


belief
knowledge
preference
...

Agents A and states W

Cognitive structure $\quad F_a = \langle W, R_a \rangle$ for each $a \in A$

Two Dimensional Framework



Agents A and states W

Cognitive structure	$F_a = \langle W, R_a \rangle$ for each $a \in A$
Social structure	$G_w = \langle A, S_w \rangle$ for each $w \in W$

E.g. Epistemic Logic of Friendship: $R_a = \{\approx_a\}$ and $S_a = \{\sim_a\}$

Epistemic Logic of Friendship

$$\rho \mid \eta \mid \sim\varphi \mid (\varphi \,\&\, \varphi) \mid K\varphi \mid F\varphi \mid A\varphi \mid \downarrow\eta\ \varphi \mid C_{\varphi}\psi \mid D_{\varphi}\psi$$

$$@_{\eta}\varphi \quad = \quad A(\eta \supset \varphi)$$

Further questions

- distributive knowledge as potential knowledge of a group
- communication via social relations

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