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Communities of Practice in Mathematical E-Learning

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Communities of Practice in Mathematical E-Learning

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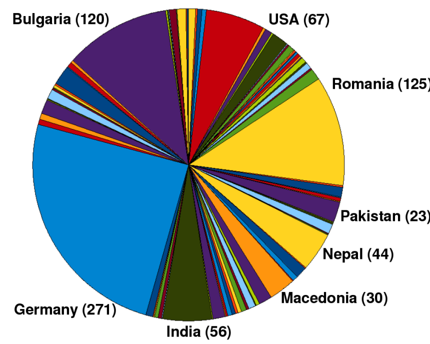
Abstract

With the globalization in education, bridging cultural differences by making course material more accessible and adaptable to individual user needs becomes an important goal. In this paper we attack this goal for the field of mathematics where knowledge is abstract, highly structured, and extraordinary interlinked. Modern representation formats like our OMDOC format allow us to capture, model, relate, and represent mathematical learning objects and thus make them *context-aware* and *machine-adaptable* to the respective learning contexts. But to make mathematical knowledge accessible to learners of diverse cultural backgrounds we also need to model mathematical practice.

In this paper, we show that many practices of mathematical communities can already be modeled in OMDOC and outline extensions to support further ones. We have implemented a collection of services that allow applications to interpret and manage OMDOC and its practice representations as well as to adapt OMDOC for users and communities. These services have been integrated into our prototype E-Learning platform *panta rhei* to demonstrate how systems can improve the accessibility of mathematical E-Learning materials.

Introduction

With the ever-increasing globalization of higher education, learning institutions have to cope with culturally induced differences in prerequisite knowledge and learning practices. This is especially pronounced at Jacobs University Bremen with an international student body: 1100 students from 88 countries on April 8^{th}, 2008 [University, 2008] (see figure on the right for the distribution). Surprisingly, this also affects subjects like Mathematics and Computer Science that are often considered culture-independent. Even though most of our students are well prepared and possess good mathematical knowledge[1], a recent study [AAS, 2008] shows mathematical discrepancies: Students of our one-year, introductory course on Computer Science (GenCS) reported that they had problems to get acquainted with the professor's notation systems, some had the feeling that the pace of the course was inappropriate and determined by the best students, some felt embarrassed to ask questions, while others did not face any problems and were able to balance out based on their previous education. Most students rate these discrepancies as problematic and believe that they can be associated with different educational and cultural backgrounds. In particular Romanian and Bulgarian students are very confident with their mathematical skills. Indian students are mostly well-educated in programming languages. Other nationalities struggle with the course. The grade distributions in Figure 1 show an unexpected peak in the failing grades and a concentration of nationalities.



We assume that these students are capable of passing the course, but eventually give up when they are not able to map their previous mathematical background and practices to our course. In this situation, we want to augment lectures with online material and E-Learning approaches that making it more accessible and adaptable to individual user needs.

We claim that the theory of *communities of practice* [Lave and Wenger, 1991] can help us understand different mathematical practices and backgrounds and to eventually counteract pre-existing differences. According to Lave and Wenger [Lave and Wenger, 1991], communities of practice

[1] Jacobs University is a private institutions which only accepts excellent students for its programs.

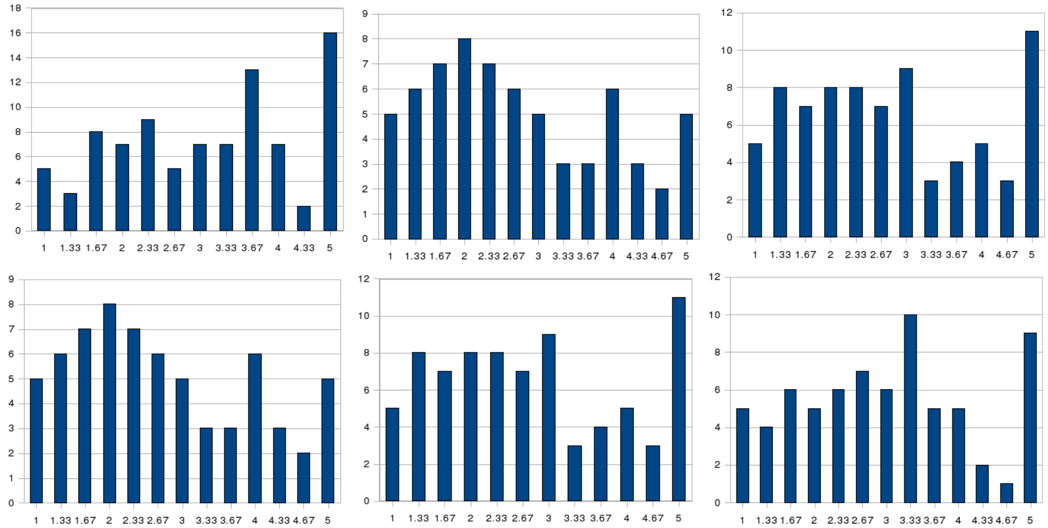


Figure 1: Grade Distributions Fall 05, Spring 06, Fall 06, Spring 07, Fall 07, and Spring 08

(CoPs) are groups of people who *share an interest* in a particular domain — in our case the interest in the GenCS course. By interacting and collaborating around problems, solutions, and insights they *develop shared practices*, i.e. a common repertoire of resources consisting of experiences, stories, tools, and ways of addressing recurring problems. Even though mathematical practitioners seem to form a homogeneous, unified community and share the same practices all over the world, they actually form various sub-communities that differ in their preferred notations, basic mathematical assumptions, and motivating examples. We can observe these sub-communities among our GenCS students and see that exactly these communities are valuable for deepening knowledge and learning.

To allow our students to *access mathematical knowledge efficiently* in the online materials mentioned above, we explicate their *knowledge structure* and the *mathematical practices* and use these to support the students in interacting with the course materials.

For determining the knowledge structure we make use of the fact that mathematical knowledge is *abstract*, *highly structured*, and *extraordinary interlinked* (cf. [Farmer, 2004]). This allows us to more easily capture, model, relate, and represent *mathematical learning objects*. For the practices we use that mathematical communities often *interact via their mathematical knowledge artifacts*, such as theories or learning objects (cf. [Müller and Kohlhase, 2008]). We claim that their practices are *inscribed* into these artifacts. For example, mathematical authors *choose notations*, *make assumptions*, build on *different foundations* as well as results, and *choose typical examples* to illustrate their mathematical concepts (cf. [Kohlhase and Kohlhase, 2006]).

Concretely we show in this paper, how to use our OMDOC format (Open Mathematical Documents, see [Kohlhase, 2006]) to represent mathematical learning objects as well as practices of mathematical communities. We illustrate which aspects of mathematical practices OMDOC supports and outline an extension of the format to further practices. We have implemented a collection of enabling technologies, which allow applications to interpret and manage OMDOC and its practice representations as well as to adapt OMDOC for users and communities. Our enabling technologies have been integrated into our prototype E-Learning platform *panta rhei* [panta rhei, 2008], to demonstrate how systems can improve the accessibility of mathematical E-Learning materials.

Knowledge Representation for Mathematics

Mathematical Objects are what we talk and write about when we do mathematics: Rather simple objects like numbers, functions, triangles, matrices, and more complex ones such as vector spaces and infinite series. In order to provide automated services such as search or computation, we need to represent these objects in a machine-processable format, such as MATHML [W3C, 2003] or OPENMATH [OpenMath, 2007]. The former is a W3C recommendation for high-quality presentation of mathematical formulae on the Web, whereas the latter concentrates on the meaning of objects[2].

OPENMATH Representation	MATHML Representation	Presentation
<om:OMOBJ> <om:OMA> <om:OMS cd="combinat1" name="binomial" /> <om:OMV name="n" /> <om:OMV name="k" /> </om:OMA> </om:OMOBJ>	<m:mrow> <m:mo>(</m:mo> <m:mfrac linethickness="0"> <m:mi>n</m:mi> <m:mi>k</m:mi> </m:mfrac> <m:mo>)</m:mo> </m:mrow>	$\binom{n}{k}$

Figure 2: OPENMATH and MATHML representation of the binomial coefficient.

Figure 2 provides the OPENMATH and MATHML representations of the number $\binom{n}{k} = \frac{k!}{(n-k)!}$ of k-element subsets of a n-element set. The `OMS` element represents the "binomial coefficient" function, which (via `cd` and `name` attributes) points to a definition in a **content dictionary** (CD) [OMCD-Core, 2008]. CDs specify *commonly agreed* definitions of basic mathematical objects

[2] In fact MATHML has a sub-language that is equivalent to OPENMATH, but we will concentrate on the presentational functionality of MATHML for simplicity.

and allow machines to distinguish the meaning of included mathematical objects. Consequently, OPENMATH expressions can be used by information retrieval or computation services while the MATHML expression is used for display: MATHML-aware browsers will present the middle expression in Figure 2 as $\binom{n}{k}$.

The OMDOC format serves as *semantics-oriented representation format* and *ontology language* for *mathematical knowledge*. The format extends OPENMATH and MATHML with markup primitives for the structure and interrelations of mathematical objects expressed as **mathematical statements**, i.e. definitions, theorems, and proofs. We have already seen above that content dictionaries serve as an explicitly represented context for mathematical symbols, formulae and thus learning objects. The OMDOC format allows to represent CDs as OMDOC documents containing mathematical statements, but extends this functionality with a very expressive infrastructure for inter-CD relations that facilitate concept inheritance, parametric reuse, and multiple views on mathematical objects and statements. We claim that this **theory level** makes OMDOC an ideal representation format for **mathematical learning objects** (MLO) i.e. reusable, granular, highly structured, and semantically marked up fragments of varying size:

The OMDOC approach negates the intuition of existing E-Learning approaches [Committee, 2005, Consortium, 2001, Learning, 2000] that learning objects (LO) should be context independent (see e.g. [LO, 2008]). We believe that this aim is not only impossible to achieve, but also misleading. Authors are biased when creating LOs and will always include subjective, context-dependent parts influenced by their didactic approaches or personal views. In mathematics, LOs also include the authors' individual and context-dependent practices such as their proving strategy, the choice of notations, or choice of typical examples. We believe that the context and practices of LOs should be represented explicitly so that machines can adapt them to the reader or learning goal. We will call LOs with explicitly represented context-dependence and practices **context-aware** to contrast them to the elusive "context-independent" ones. Thus, context-aware LOs allow to produce more accessible learning materials that are targeted to the individual needs and preference of the learner: For example, references to regional events or cultural aspects can motivate learners and allow them to more easily map new knowledge to prior experience.

OMDOC allows to represent context-aware MLOs since it preserves the *logical*, *narrative*, and *social* contexts of MLOs and provides an infrastructure that interprets contextual information allowing for sophisticated semantic services: For example, OMDoc supports a user-specific and context-aware selection and sequencing of LOs as well as their adaptive presentation that other non-semantic E-Learning approaches can not offer (cf. [Kohlhase and Kohlhase, 2008]).

Research on Mathematical Practices

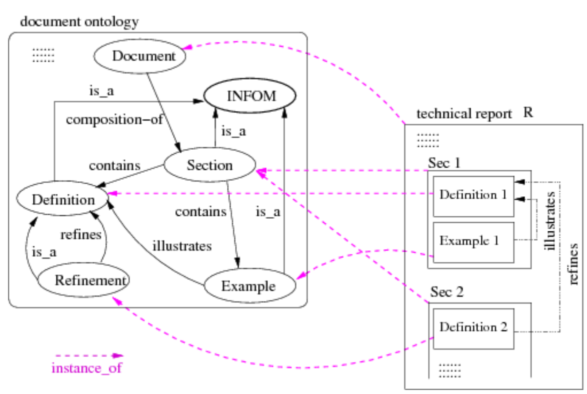
Before we come to the representation of mathematical practices in OMDoc let us recap the folklore: a first group of authors distinguish mathematical practices from those of other sciences. According to Philip Kitcher [Kitcher, 1988], the development of mathematics can be seen as a stepwise process from generalizations and observations to their symbolic substitutes. Kitcher underlines the dynamics in mathematics, i.e. the creating, revising, and dismissing of mathematical knowledge, as well as the process of abstracting experiences to gain symbolic substitutes. Sociologist Bettina Heintz observed the mathematical community [Heintz, 2000] and illustrates mathematical practices such as *proving*, *community assessment and acceptance of proofs*, and the spontaneous, problem-driven nature of *mathematical collaborations*. But there are also sub-communities inside Mathematics: Godfrey Hardy, a professional mathematician himself, reflects about mathematics in [Hardy, 1992]. He distinguishes *elementary* from *pure mathematics*, whereas the former include all school, most university and, in particular, applied mathematics and their different cultures. George Polyá distinguishes several mathematical strategies for problem solving in elementary mathematics [Polya, 1973] that differ among communities and their members.

Further research focuses on specific practices such as the choice of mathematical notations [Smirnova and Watt, 2006, Cajori, 1993], basic assumptions and logical foundations [Rabe, 2008], or the choice of typical examples [Kerber et al., 1992]. These practices depend on different context parameters such as nationality and language, the level of expertise, the area of application, the audience, the historical period, the output format, or the individual (author) style (cf. [Kohlhase et al., 2007]). Consider for instance the presentation of decimal numbers: while Germans use a comma for decimal numbers ($4,53$); English use a point (4.53). Vise verse, Germans use a point to structure large numbers ($1.000.000$), while English use a comma ($1,000,000$). If we think of the binomial coefficient, we can distinguish different presentation in various nationalities: In Germany $\binom{n}{k}$ is used, while Russians prefer C_k^n and French write C_n^k. The imaginary unit is presented differently according to the area of application: While mathematicians use an i, physicists use a j to not confuse the symbol with the presentation of electric current I.

Representing Practices in OMDoc

We will now detail how some of the practices above can be encoded in the OMDoc format. In this sense our paper can be seen as an instantiation of our earlier [Kohlhase and Kohlhase, 2008], where we state requirements for semantic representation formats for educational materials. We will concentrate on three practices on different levels of the OMDoc format: (1) The presentation of mathematical results (document level), (2) the structuring and contextualization of mathematical knowledge (theory level) and (3) the choice of mathematical notations (object level). The approach exemplified with these examples can be applied to any practice: we analyze the mathematical objects affected by the practice and try to find a represent a functional core that is independent of the practice. Then we try to reify — i.e. turn into represented objects — the other factors or parameters in the practice in question. For instance for notation practices, the functional core is already there in OpenMath representations of formulae, we only need to reify notation definitions into objects.

Representing Mathematical Documents



On the document level, OMDoc allows to separate narrative structure of mathematical documents from the content structure and thus makes it adaptable: Narrative elements such as section, subsections, definition, examples, or proofs, allow authors to explicate the didactic relations and sequencing of fragments in their documents. The figure to the right presents a technical report marked up using OMDoc's document ontology, which defines the *narrative concepts* such as section, example, or definition as well as their *interrelation*, e.g. an example *illustrates* a definition (cf. [Müller, 2006]). The narrative re-

```
<omdoc xmlns="http://omdoc.org/ns" ...>
  <notation> see Figure 4 </notation>
  <theory xml:id="MyTheory">
    <imports from="http :// omdoc.org/combinat1.omdoc#binomial"/>
    <omtext xml:id="id2">
      <CMP>
        The binomial coefficients is the number of ways of choosing m objects from a collection of n distinct
        objects without regard to the order. We denote it by see Figure 2
      </CMP>
    </omtext>
  </theory>
</omdoc>
```

Figure 3: OMDOC representation of an example document.

lations allow us to model the *didactic practice* of authors, i.e. their way of presenting mathematical results.

Representing Notation Practices in OMDOC

Figure 3 provides an example of a document represented in OMDOC. The OMDOC representation includes a `theory` element, which embeds a mathematical object represented in OPENMATH. The `import` element specifies the required prior mathematical knowledge and is used analogously to `include` operators in programming languages, which import required libraries and classes. In OMDOC, the `import` elements include all symbols from other theories that are used within the current theory, but have been defined and introduced in the imported theories: In the example, the `import` element of the theory `MyTheory` includes the symbol `binomial`, which is defined in the theory `combinat1`. Please note that mathematical objects in OPENMATH format can not be presented as OPENMATH represents the meaning, but can not be used for display. These objects have to be converted to MATHML. Consequently, we need to automatically process the author's notation practices, i.e. we need a mapping from OPENMATH to a respective MATHML representation.

In [Kohlhase et al., 2008] we presented the extension of OMDOC towards the representation of mathematical notation practices to provide a flexible and context-aware conversion from OPENMATH to MATHML: We reified *notation preferences* of scientists into artifacts, that is *notation specifications*, which are applied onto the meaning of mathematical objects (represented in OPENMATH) to generate their presentation (represented in MATHML).

Figure 4 presents the OMDOC representation of a notation specification. The `prototype` pattern matches the OPENMATH expression of the binomial coefficient in Figure 3. The `rendering` elements are applied to generate a concrete presentation for the symbol. The `context` attribute

```
<notation xmlns:m="http://www.w3.org/1998/Math/MathML"
          xmlns:om="http://www.openmath.org/OpenMath">
  <prototype>
    <om:OMA>
      <om:OMS cd="combinat1" name="binomial" />
      <expr name="arg1"/>
      <expr name="arg2"/>
    </om:OMA>
  </prototype>
  <rendering context="language:Russian,ru">
    <m:msubsup>
    <m:mi>C</m:mi>
    <render name="arg1"/>
    <render name="arg2"/>
    </m:msubsup>
  </rendering>
  ...
  <rendering context="language:German,de">
    <m:mrow>
      <m:mo>(</m:mo>
      <m:mfrac linethickness="0">
        <render name="arg1"/>
        <render name="arg2"/>
      </m:mfrac>
      <m:mo>)</m:mo>
    </m:mrow>
  </rendering>
  <rendering context="language:French, fr">
    <m:msubsup>
      <m:mi>C</m:mi>
      <render name="arg2"/>
      <render name="arg1"/>
    </m:msubsup>
  </rendering>
</notation>
```

Figure 4: An Example of a notation practice represented in OMDOC.

of the `rendering` element associates specific context parameters. In the example, the nationality of the respective notations are added. This allows to distinguish the German, Russian, and French notation of the binomial coefficient. Analogously, further context parameters such as the expertise level (novice, intermediate, expert) or area of application (mathematics, physics) can be added.

In order to select the appropriate presentation for a symbol, we proposed a context-aware conversion algorithm in [Kohlhase et al., 2008]: First we collect all notation specification for a mathematical object, then we collect the user's context parameters for the conversion, and finally we select an appropriate `rendering` element which best fits to the current context and apply it to generate a presentation for the mathematical object. To provide a flexible and context-aware conversion algorithm, we provide various options to collect notation specifications as well as concrete context parameters (cf. [Kohlhase et al., 2008]).

Given the notation specification in Figure 4 and a concrete context parameter, the mathematical object in the OMDOC document in Figure 3 can be presented differently: For example, depending on the nationality selected by the user, the binomial coefficient is presented with its German ($\binom{n}{k}$), Russian (C_k^n), or French notation (C_n^k).

Representing Structure and Context of Mathematical Knowledge

To structure collections of learning objects and provide them with context OMDOC groups them into **theories** and links them via **theory morphisms**. This mechanism reifies a practice that long been relatively overt in mathematical documents, e.g. the Bourbaki development of mathematics that starts with set theory [Bourbaki, 1968] and takes the mathematical practice of stating results with minimal preconditions to the extreme. OMDoc provides concrete markup for theory objects and extends the theoretically motivated accounts of inheritance and modularity in programming languages and mathematics to cover informal (but rigorous) mathematical practice (see [Rabe and Kohlhase, 2008] for the most recent theory, which will be incorporated into the upcoming version of OMDoc). Intuitively, a theory morphism is a mapping between theories that allow to "view" the source theory in terms of the target theory, if the mapping conserves truth. In the simplest case, theory morphisms model inheritance — the source theory can be viewed as an included part of the target theory — and thus allow to model the mathematical practice of modular/object-oriented development of knowledge in mathematics. For instance Figure 5 shows the inheritance graph of our GenCS course, and is used by students and the instructor for navigation and overview.

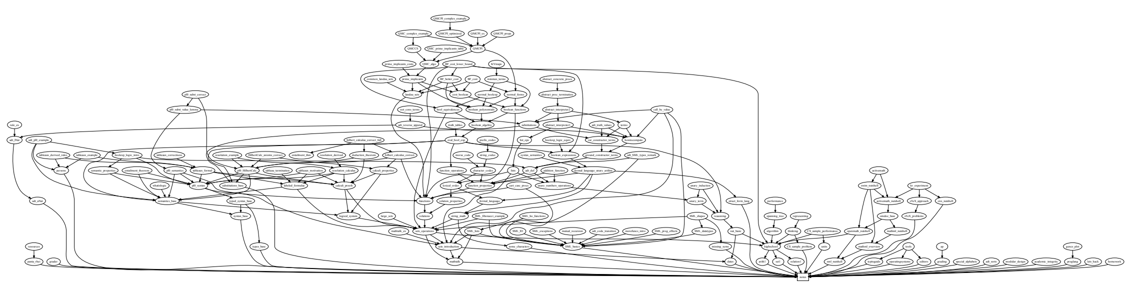


Figure 5: The inheritance Graph of the GenCS course

But theory morphisms can also be used to model intra-mathematical differences in practices, e.g. differing choices of basic concepts. To gain and intuition, let consider an elementary example, the choice measuring temperature with the Kelvin, Celsius, and Fahrenheit scales. This example is suitable, since these scales make different defining assumptions — we model these as OMDoc `axiom` elements. For instance the Fahrenheit scale *defines* zero degrees to be the temperature of the coldest winter night Mr. Daniel Gabriel Fahrenheit ever experienced whereas the Celsius scale puts zero degrees a the freezing point of water, while the Kelvin scale puts it at the at hypothetical point, where all atoms cease motion (cf. Figure 6). The crucial observation is that (after suitable rescaling) all arrive at compatible consequences — which we model as OMDoc theorems. This allows us to establish the rescaling mappings as theory morphisms, since they are truth-preserving.

The important implication for eLearning is that the elaborate theory structure that was theo-

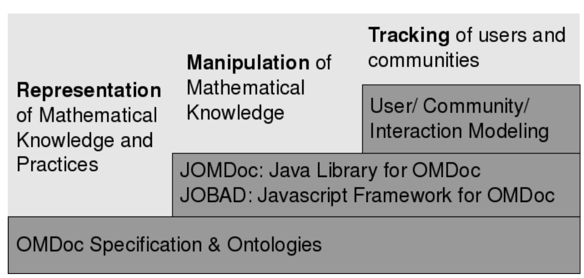
Theory	Temp. in Kelvin	Temp. in Celsius	Temp. in Fahrenheit
Signature	$^\circ K$	$^\circ C$	$^\circ F$
Axiom:	absolute zero at $0^\circ K$	Water freezes at $0^\circ C$	cold winter night: $0^\circ F$
Axiom:	$\delta(1^\circ K) = \delta(1^\circ C)$	Water boils at $100^\circ C$	domestic pig: $100^\circ F$
Theorem:	Water freezes at $271.3^\circ K$	domestic pig: $38^\circ C$	Water boils at $170^\circ C$
Theorem:	cold winter night: $240^\circ F$	absolute zero at $-271.3^\circ C$	absolute zero at $-460^\circ F$
Theory morphisms: $^\circ C \xrightarrow{+271.3^\circ} K$, $^\circ C \xrightarrow{-32/2^\circ} F$, and $^\circ F \xrightarrow{+240/2^\circ} F$			

Figure 6: Three equivalent theories of temperatures

retically motivated originally can be utilized for adaptation and bridging of context differences: We can automatically recontextualize learning objects. For instance we can move a LO from a Fahrenheit context to a Celsius context by translating it via the appropriate mapping above. This translation is safe, since we have established it to be a theory morphism earlier. Note that the recontextualization discussed here significantly surpasses the notation adaption discussed above as it is at the conceptual and content level. Another example of a service based on theory morphisms is to index *all* translations and thus make the virtual cloud of possible translations available to a formula search engine like our MATHWEBSEARCH service [Kohlhase and Şucan, 2006, MathWebSearchDemo, 2008]. Our experiments show that even for an introductory course like GenCS supports about three dozen non-trivial theory views, while the theory views from subsequent course into GenCS go in the hundreds, since these courses are given by other instructors.

Practice-aware adaptation: Implementation & Case Study

To support a practice-aware adaptation of the OMDOC-encoded knowledge, we also need to support *the manipulation of* OMDOC *representations* and *the tracking of users and communities*. The figure to the right presents a *framework for practice-aware adaptation*, which three components are described below:

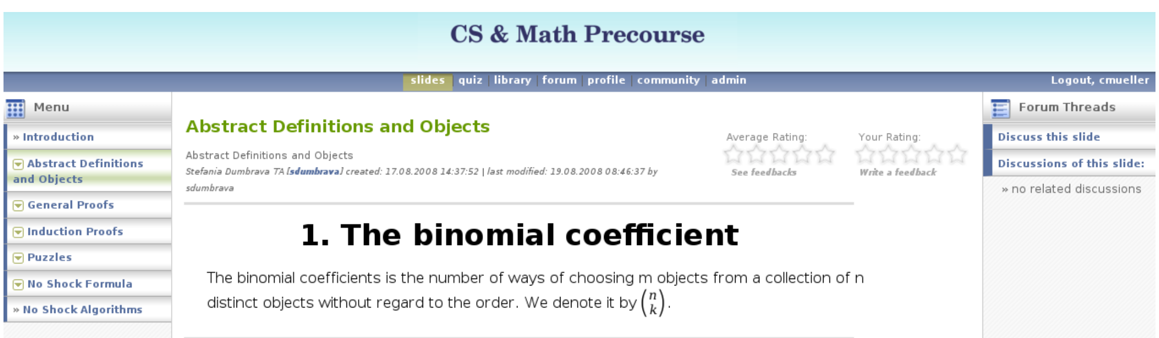


We provide tools for **representing mathematical knowledge and practices** in OMDOC: e.g. the OMDOC editor Sentido [González Palomo, 2006], and tools that facilitate a consistent maintenance and storage of OMDoc materials, e.g. the OMDoc repository [OMBase, 2008] and the change management system `locutor` [locutor, 2007].

For the **manipulation of the OMDoc representation** we provide enabling technologies such as the Java library JOMDoc [JOMDoc, 2008] which e.g. supports the context-aware conversion of mathematical notations and the Javascript framework JOBAD [JOBAD, 2008] which supports browser-based interactions with mathematical knowledge, e.g. the flexible display and hiding of brackets in mathematical formulae or the on-the-fly change of notations.

To **track user and community** behaviors, preferences, and practices, we build on related work on user and community modelling in mathematics, e.g. [Melis, 2001]. This allows us to adapt OMDoc materials to concrete contexts and preferences.

The panta rhei Case Study To demonstrate our approach, we are integrating and evaluating our services in our prototype E-Learning platform *panta rhei* [panta rhei, 2008] which uses JOMDoc to convert the lecture material (in OMDoc) to XHTML+MathML. During the import of the OMDoc materials, the lecturer can specify his notation preferences. In the figure below, the German notation has been chosen. In addition, JOMDoc considers user-specific contexts to adapt the presented material on-the-fly, i.e. users can change notations and indicate their preferences while reading the material.



However, dynamic adaptation of material is not always in the intention of the lecturer, as he might wish to introduce specific notations and allow students to learn new ones. Consequently, the panta rhei system does not simply overwrite the lecturer's notations, but can also add a hint for the user if his notation background differs with the presented symbols; see Figure 7 for the *panta-rhei*-generated variants of the material above.

The binomial coefficients is the number of ways of choosing m objects from a collection of n distinct objects without regard to the order. We denote it by $\binom{n}{k}$. But you know it as C_n^k .

The binomial coefficients is the number of ways of choosing m objects from a collection of n distinct objects without regard to the order. We denote it by $\binom{n}{k}$. But you know it as C_k^n .

Figure 7: User-specific Adaptation of Notations

Related Work

The ACTIVEMATH system [ActiveMath, 2008, Melis and Siekmann, 2004] is based on OMDOC and provides user-adaptivity in the the selection of examples [Goguadze et al., 2005]; the sequencing of learning objects into user-specific courses; as well as first attempts towards the adaptive presentation of course material based on user models. The focus of this work is on integrating knowledge representation issues with didactic and psychological criteria. However, the underlying representation and conversion does not yet consider the full power of OMDOC and our enabling technology: Consequently, practice-oriented adaptations are not yet specified or implemented.

The educational knowledge repository CONNEXIONS [CNX, 2008] is based on a corpus of semantic artifacts represented in CNXML [Hendricks and Galvan, 2007], a *lightweight* XML markup language for educational content. CNXML *embeds* MATHML as well as OPENMATH for the representation of mathematical objects. If provides markup for the document level, but lacks markup of theories and theory dependencies. However, CONNEXIONS provides “lenses” [Kelty et al., 2008] that allow to express the approval and authorship of organizations and individuals. Technically, lenses are selection of content *or tags* in the CONNEXIONS repository to help readers find content that is related to a particular topic or focus. Conceptually, lenses provide the assessment and relevance of content by a single user or group. The common assessment of mathematical knowledge by the mathematical community is an important process for approving or refusing mathematical results (cf. [Heintz, 2000]). Thus, CONNEXIONS implements a course-grained notion of practice-oriented adaptation, i.e. regarding the selection or sequencing of learning objects.

[Heeren et al., 2008] specifies strategies for interactive exercises. Strategies are procedures or procedural skills that help solving exercises and thus reflect mathematical practices, similar to the problem solving guidelines presented by Polyá [Polya, 1973]. Based on strategy specifications, [Heeren et al., 2008] implement a web service, which is used by several mathematical E-learning applications allowing them to consider alternative strategies to provide more adaptive feedback and guidance. The MathDox system [Cuypers et al., 2008] integrates this web service to provide interactive exercises. MathDox is based on still another XML-based format for interactive mathematical documents. These can be transformed to interactive mathematical web pages using the MathDox Player and, thus, implement more accessible and living documents. However, although the MathDox format embeds mathematical objects in OPENMATH and MATHML and provides a basic markup of the structure of documents, it is less suited for representing mathematical structures and practices as it is lacking OMDOC's theoretic foundations.

Conclusion & Outlook

In this paper we show how a modern, content-oriented document format (Open Mathematical Documents) can be used to represent *context-aware* mathematical learning objects as well as practices of mathematical communities. We have shown how the context awareness of MLO representations together reified practices can be used to recontextualize MLOs and adapt them to differing cultural backgrounds. Our enabling technologies have been integrated into our prototype E-Learning platform *panta rhei* [panta rhei, 2008], to demonstrate how systems can improve the accessibility of mathematical E-Learning materials

Future work will address the reification of further practices in OMDOC, e.g. to support the automatic selection of typical examples and exercises for a given set of theories and user-specific context parameters. Moreover, we want to extend OMDOC to represent the social context of mathematical knowledge, i.e. information and relations that are capture during the user's interaction with mathematical knowledge such as tags, annotations, or discussions.

We also want to elaborate more on further scenarios and contributions of our semantic document markup: [Bernareggi and Archambault, 2007, Archambault et al., 2007a, Archambault et al., 2007b] call for structured and semantically marked up documents to support the accessibility for visually impaired or blind people: In particular, representing mathematical expressions in MATHML can improve the accessibility and usability of online material for speech synthesis and Braille devices.

We also intend to discuss motivational issues that arise due to a common practices of mathematicians: In [Farmer, 2005, Farmer, 2004], William Farmer highlights that mathematicians do not fully articulate their mathematical knowledge and practices, although this can hamper the communication with others. Many mathematical details are never articulated; they reside only in the minds of mathematicians. The user of the knowledge is expected to be able to fill in what is missing as needed. This works well with human users who have strong mathematical skills. It does not work well with students and average human users. However, articulation and the semantic markup of mathematical knowledge is a precondition for enlarging the corpora of OMDoc contents. Consequently, motivating mathematicians to articulate and markup their documents is a key challenge for our further progresses and service evaluations (cf. [Kohlhase and Kohlhase, 2004]).

Acknowledgments

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