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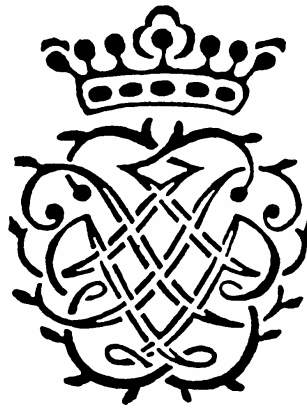
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ACCIDENT OR DESIGN?
*New Theories on the Unfinished
Contrapunctus 14 in J. S. Bach's
The Art of Fugue BWV 1080*

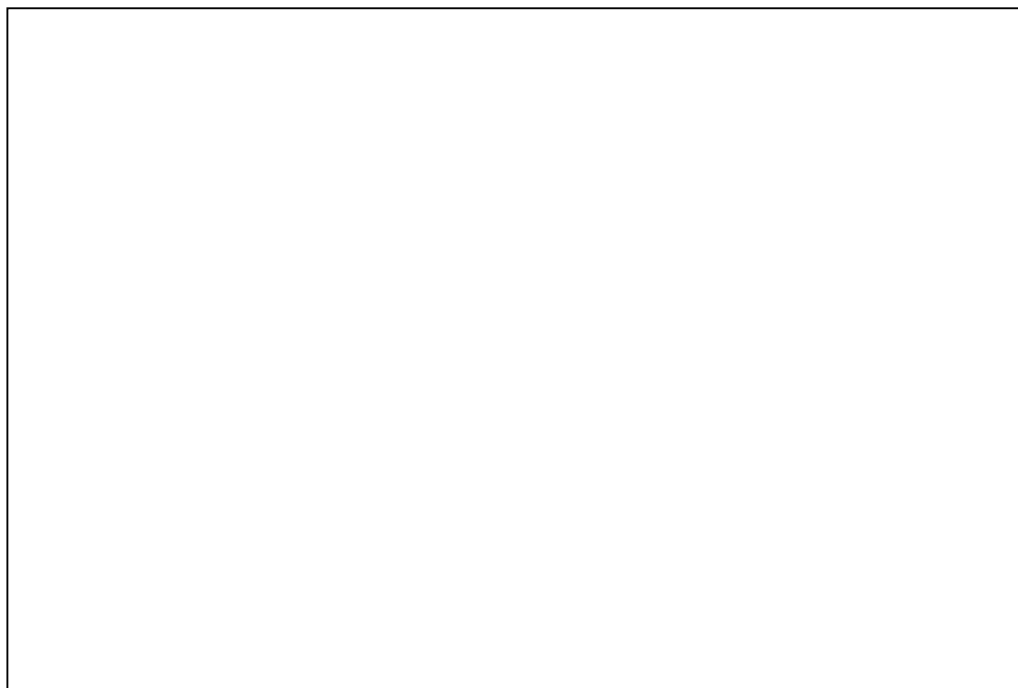


INDRA NICHOLAS MARTINDALE HUGHES

**A thesis submitted in partial fulfilment
of the requirements for the Degree of
DOCTOR OF MUSICAL ARTS**

University of Auckland, 2006

Frontispiece



Deutsche Staatsbibliothek, Berlin

Mus. ms. P200/Beilage 3 page 5

The last page of Bach's autograph manuscript of Contrapunctus 14

This reproduction is a colour postcard published by Carus, Stuttgart CV 40.331/90.
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last page.

ABSTRACT

The literature about the unfinished ending of J S Bach's *The Art of Fugue* (Die Kunst der Fuga) BWV 1080 is in universal agreement that the work remained unfinished at Bach's death; some texts go a step further to say that it is unfinished *because* he died. After giving a series of performances of the work, the author became convinced that this latter view must be incorrect, and that Bach left the work unfinished deliberately. This thesis explores this idea in detail and, by presenting a number of new theories, suggests not only that Bach left the work unfinished deliberately as an invitation to the reader, student or performer to work out his or her own completion, but also that he left a number of clues, hidden to a greater or lesser extent, to indicate that that was his intention and to supply vital information about the content of the missing bars.

Divided into two parts, the thesis first considers some of the evidence contained within the manuscript itself, up to and including the final written bar, and then in the second part goes on to consider two essential aspects of the completion.

By way of introduction, the first chapter surveys the controversial area of Bach's use of numbers in his music and draws attention to the number of the final bar, which can be interpreted as a clue to the fact that Bach expects the music to be continued.

Chapter Two invites a reconsideration of Christoph Wolff's famous "Fragment X" theory, which suggests that the continuation of the final fugue was written on a separate, now lost, piece of paper. Many inconsistencies and details in the manuscript suggest strongly that Wolff's theory is incorrect. As part of this theory, the author reports on his own examination of the original manuscript in Berlin.

Chapter Three, through a detailed study of the architecture of the final fugue, makes the bold claim that the author has definitively proved the exact number of bars required to complete the music in accordance with Bach's intentions: this theory develops and refines the work of Gregory Butler in this area, and, to corroborate the theory, presents a possible interpretation of the unusual markings at the end of Bach's score and of a significant correction made by Bach in his manuscript.

Finally, in Chapter Four, the question of the proposed inverted combination of all four fugue subjects is revisited – a combination that several writers have claimed to be impossible – and a new and convincing solution to this problem is presented and justified.

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I should like to thank my two Supervisors at the University of Auckland: Dr Fiona McAlpine, who was principally responsible for the supervision of this thesis; and Dr John Wells. I am very grateful for their expertise and wisdom.

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Dr Helmut Hell of the Deutsche Staatsbibliothek in Berlin graciously granted permission for me to view Bach's original manuscript of *The Art of Fugue*, enabling me to confirm my theories about the markings on Bach's last page, about the handwriting characteristics, and about the significant alteration made by Bach in one place in the score of Contrapunctus 14. The staff at the Bach-Archiv in Leipzig generously allowed me to take photocopies of some material that was not easily available elsewhere.

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