Solution to the problem of time

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Despite the ultraviolet problems with canonical quantum gravity, as an effective field theory its infrared phenomena should enjoy fully quantum mechanical unitary time evolution. Currently this is not possible, the impediment being what is known as the problem of time. Here, we provide a solution by promoting the cosmological constant Λ to a Lagrange multiplier constraining the metric volume element to be manifestly a total derivative. Because Λ appears linearly in the Hamiltonian constraint, it unitarily generates time evolution, yielding a functional Schrödinger equation for gravity. Two pleasant side effects of this construction are that vacuum energy is dissociated from the cosmological constant problem, much like in unimodular gravity, and the natural foliation provided by the time variable defines a sensible solution to the measure problem of eternal inflation.

The problem of time [1–3], also known as the Hilbert space problem, is the absence of a positive definite probability current that is conserved under time evolution. It afflicts canonically quantized general relativity, and is unrelated to the ultraviolet problem of nonrenormalizability, because it also occurs in lower dimensions, where general relativity is renormalizable. It is often thought that the problem of time arises because of diffeomorphism invariance. This is not precisely the case, because diffeomorphism invariance is coordinate invariance, and any theory can be written in coordinate invariant language [4]. The origin of the problem of time is that in general relativity physical time is a *foliation*, essentially determined by the lapse function. Since no time derivatives are taken of the lapse function in the gravitational action, its canonical momentum vanishes. This is a so-called first-class primary constraint, and it ensures that physical-time translation must be arbitrarily chosen in different spatial regions as the spatial geometry is evolved. (Note that this is very different from the gauge freedom of coordinate invariance i.e., diffeomorphism invariance, which has no physical content.)

In classical general relativity, because the foliation is not determined by the equations of motion, it is a *gauge choice*, but this does not imply that it has no coordinate invariant reality. The foliation for geodesic slicing is not a coordinate transformation from the foliation corresponding to maximal slicing: They are different physical slicings, having different curvature invariants. They are only gauge equivalent from the point of view of the resulting 4-geometry that ensues. Quantum mechanically, all first-class constraints are gauge symmetries [5], and all states which depend on a gauge choice are projected out of the Hilbert space. Hence, no states in the physical Hilbert space evolve. Because classical theories should arise as an $\hbar \to 0$ limit of some quantum theory, this is a pathology of the Einstein-Hilbert action that is independent of its UV completion.

The most commonly attempted solutions to the problem of time involve the introduction of matter clocks, which are dynamical fields that (classically) evolve monotonically, and so might play the role of time [2]. However, for Hamiltonians bounded below, all momenta occur quadratically in the Hamiltonian, so backward propagating modes are inevitably produced [6]. This leads to negative probabilities. It is the first-order characteristic of the time-derivative in the Schrödinger equation that guarantees positivity of probabilities under time evolution, i.e., unitarity.

Remarkable progress has been achieved with Lagrange multipliers in Einstein aether theories [7] that use pressureless dust as a generator of unitary time evolution [8–11]. The primary difference between these models and our model is that here it is the cosmological constant that becomes a choice of initial conditions, rather than the matter density. A subtle advantage of this built-in landscape is that the natural foliation does not reward expansion, and so may provide a useful solution to the measure problem of eternal inflation [12].

A simple example of the problem of time involves a scalar field q in 0+1 dimensions (quantum mechanics), coupled to a dynamical metric, $\mathrm{d}t^2 = g_{\tau\tau}\,\mathrm{d}\tau^2$. Defining the lapse function $\eta(\tau)$ via $\eta^2 = g_{\tau\tau}$, the action is

$$S[\eta, q] = \int \eta\,\mathrm{d}\tau \left[\frac{\dot{q}^2}{2\eta^2} - V(q)\right]. \tag{1}$$

The momentum conjugate to q is $p = \dot{q}/\eta$, and that conjugate to η is $p_\eta \approx 0$. Here $\approx$ denotes a primary constraint, i.e., a momentum relation that does not determine a velocity. Instead, the classical velocity $\dot{\eta}(\tau)$ is an arbitrary gauge choice. The canonical Hamiltonian is

$$H = \eta\left[\frac{p^2}{2} + V(q)\right], \tag{2}$$

which generates τ-translations. Following Dirac [4], we ensure that the solution to the primary constraint, $p_\eta = 0$, is maintained under τ-evolution:

$$0 = \dot{p}_\eta = \{p_\eta, H\}_{\text{P.B.}} = -\partial H/\partial\eta, \tag{3}$$

and so we demand $H = 0$. This is known as a secondary constraint, because it involves the equations of motion. It is this constraint that preserves coordinate independence (diffeomorphism invariance). Canonical quantization is achieved by promoting the conjugate momenta to operators satisfying the usual commutation relations and then imposing both constraints, $p_\eta \approx 0$, $H = 0$, as the operator equations

$$\hat{p}_\eta \psi(q,\eta,\tau) = -i\frac{\partial}{\partial \eta}\psi(q,\eta,\tau) = 0, \tag{4}$$

$$\hat{H}\psi(q,\eta,\tau) = \eta\left[-\frac{1}{2}\frac{\partial^2}{\partial q^2} + V(q)\right]\psi(q,\eta,\tau) = 0, \tag{5}$$

on all physical states $\psi(q,\eta,\tau)$. Then the time dependent Schrödinger equation is

$$\mathrm{i}\frac{\partial}{\partial \tau}\psi(q) = \hat{H}\psi(q) = 0, \tag{6}$$

and so no time-evolution occurs. Because energy conservation is reduced to conservation of the number zero, τ-translation is a gauge (i.e. unphysical) symmetry of the theory.

Perhaps it is not surprising that τ-evolution is a gauge symmetry, since τ is just a coordinate. What is surprising is that, even though the classical theory exhibits *proper*-time evolution, the quantum theory does not. The reason is that the physical foliation is a gauge choice, so any quantum states that depend on it are projected out. This violates the correspondence principle — proper time exists classically but not quantum mechanically. Quantization removes so many states that no classical interpretation exists even as $\hbar \to 0$.

Let us reconsider the above quantum mechanical example, now *without* gravity, but still using a diffeomorphism invariant formalism. We will achieve this by rewriting the familiar non-relativistic action using the identity $\mathrm{d}t = \frac{\partial t(\tau)}{\partial \tau}\,\mathrm{d}\tau$. The new action is identical to Eq. (1) with the lapse substitution $\eta \to \dot{t}$,

$$S[t,q] = \int \dot{t}\,\mathrm{d}\tau\left[\frac{\dot{q}^2}{2\dot{t}^2} - V(q)\right]. \tag{7}$$

We will vary S with respect to $t(\tau)$ —not its endpoints of course — even though the action is certainly independent of it (Diff invariance). The momentum conjugate to q is $p = \dot{q}/\dot{t}$, and when computing the momentum conjugate to t we again find a primary constraint

$$p_t + \frac{p^2}{2} + V(q) \approx 0, \tag{8}$$

which is only associated with the Diff gauge symmetry (i.e., arbitrariness of $\dot{t}$). The canonical Hamiltonian is

$$H = \dot{t}\left(p_t + \frac{p^2}{2} + V(q)\right), \tag{9}$$

which vanishes when the primary constraint holds. Because

$$\left\{p_t + \frac{p^2}{2} + V(q), H\right\}_{\mathrm{P.B.}} = 0, \tag{10}$$

the primary constraint is preserved under τ-evolution, so no secondary constraint arises.

Canonical quantization imposes the operator version of the primary constraint Eq. (8),

$$\mathrm{i}\frac{\partial}{\partial t}\psi(q,t,\tau) = \left(\frac{\hat{p}^2}{2} + V(q)\right)\psi(q,t,\tau). \tag{11}$$

This implies $\hat{H}\psi = 0$ and so $\psi(q,t,\tau) = \psi(q,t)$. We have arrived at the familiar non-relativistic Schrödinger equation, and no problem of time exists. We could say that the "degree of freedom" t plays the role of the clock, and it is able to do so because its conjugate momentum p_t appears linearly in the constraint Eq. (8).

As expected, diffeomorphism invariance does not cause the problem of time. Only when the lapse function was a Lagrange multiplier did a secondary constraint arise. The derivative appearing in the lapse function $\dot{t}$ creates a natural physical foliation, defined by constant t surfaces. This hints at a strategy, used here as well as in references [8–11], to evade the problem, namely the use of a Lagrange multiplier to constrain the lapse function to be a time derivative. When the foliation is no longer totally arbitrary, it can appear in the Wheeler–DeWitt equation and generate physical time evolution.

Quantum mechanics is more fundamental than classical mechanics in the sense that it contains classical mechanics as a limit, but the reverse is not true. Each individual classical trajectory can be reproduced as an $\hbar \to 0$ limit of a minimum-uncertainty quantum state. For example, a classical non-relativistic point particle trajectory $q_{\mathrm{cl}}(t)$ with $t \in [t_{\mathrm{i}}, t_{\mathrm{f}}]$ is in correspondence with the distribution limit of the wavefunction squared:

$$q_{\mathrm{cl}}(t) \quad \longleftrightarrow \quad \lim_{\hbar\to 0}|\Psi_{\mathrm{cl}}(q,t)|^2 = \delta(q_{\mathrm{cl}}(t) - q), \tag{12}$$

where Ψ_{cl} is a wavepacket of minimum time-averaged position uncertainty. Because quantum mechanics is more fundamental, we should expect that some states (e.g., those with small quantum numbers) do not have classical interpretations, whereas all classical states should arise in the $\hbar \to 0$ limit of some quantum theory.

The story becomes more complicated in the presence of gauge symmetries. At the classical level, gauge symmetries correspond to the appearance of arbitrary functions of time, i.e., the gauge choice [4, 5]. In the language of Dirac's mechanics, first-class primary constraints always correspond to gauge symmetries. Passage to quantum mechanics then requires that the wavefunction be annihilated by the gauge generators, because a quantum gauge symmetry means that the physical Hilbert space is orthogonal to all gauge generators.

Because the foliation is not determined by the equations of motion, it is a gauge choice.[1] This leads to the problem of time in canonical quantum gravity, where the wavefunctional $\Psi[\bar{g}_{ij}, t)$ for the spatial geometry $\bar{g}_{ij}(x)$, does not evolve in time, even though the classical configurations $(\bar{g}_{ij}(x,t), \Sigma_t)$ do evolve in (an arbitrarily chosen but coordinate invariant) time. Here Σ_t is a foliation by space-like hypersurfaces. Much like unimodular gravity [13], we will modify the Einstein-Hilbert action by constraining the metric volume element using a Lagrange multiplier. However, we do not use a non-dynamical background volume form. Instead we use a dynamical scalar field χ whose velocity will provide a local clock. We propose the full gravitational action (suppressing boundary terms) is

$$S[g, \Lambda, \chi] = \int \sqrt{-|g|}\, \mathrm{d}^4 x \left[R - 2\Lambda(1 - \nabla^2 \chi) \right]. \quad (13)$$

The equations of motion are

$$\nabla^2 \chi = 1, \quad (14)$$
$$\nabla^2 \Lambda = 0, \quad (15)$$
$$R_{\mu\nu} - \tfrac{1}{2} R g_{\mu\nu} + (\Lambda + \partial_\rho \Lambda \partial^\rho \chi) g_{\mu\nu} - \partial_\mu \Lambda \partial_\nu \chi - \partial_\nu \Lambda \partial_\mu \chi = 0. \quad (16)$$

The equation of motion for χ coincides with that of a scalar field with an exactly linear potential. The global symmetry corresponding to χ-translation invariance gives rise to a conserved current $j_\mu = \partial_\mu \Lambda$.

If we choose constant Λ, the χ-field does not contribute to the Einstein equations, much like the harmonic coordinate model [14]. Thus this theory contains as a subset all solutions of general relativity for any value of cosmological constant. More generally, solutions resemble quintom [15] dark energy. Although the bilinear kinetic term would appear to necessarily suffer from a ghost instability, this is not the case, because the Hamiltonian constraint[2] ensures Λ is not an independent field [14].

The identity of the gravitational degrees of freedom can be elucidated by writing the metric in the ADM form,

$$\mathrm{d}s^2 = -N^2\, \mathrm{d}t^2 + \bar{g}_{ij}(\mathrm{d}x^i + N^i\, \mathrm{d}t)(\mathrm{d}x^j + N^j\, \mathrm{d}t), \quad (17)$$

because then the lapse function N and shift vector N^i appear without time derivatives in the action and so lead to primary constraints $\pi_N \approx 0$, $\pi_{N^i} \approx 0$. The Hamiltonian density of our theory is

$$\mathcal{H} = N \mathcal{H}_0 + N^i \mathcal{H}_i, \quad (18)$$

where

$$\mathcal{H}_0 = \sqrt{|\bar{g}|} \Big[\frac{\pi_{ij}\pi^{ij}}{|\bar{g}|} - \frac{(\pi^i{}_i)^2}{2|\bar{g}|} - \bar{R} + \frac{\pi_\Lambda \pi_\chi}{2|\bar{g}|} + 2\partial_i \Lambda \partial^i \chi + 2\Lambda \Big], \quad (19)$$
$$\mathcal{H}_i = -2\bar{\nabla}_j \pi^j{}_i + \pi_\Lambda \partial_i \Lambda + \pi_\chi \partial_i \chi, \quad (20)$$

where $\bar{R}$ and $\bar{\nabla}_i$ are computed from the spatial metric $\bar{g}_{ij}$, and π^{ij} is conjugate to it. The primary constraints are preserved under time evolution when the Hamiltonian and momentum (secondary) constraints are satisfied,

$$0 = \frac{\partial \mathcal{H}}{\partial N} = \mathcal{H}_0, \qquad 0 = \frac{\partial \mathcal{H}}{\partial N^i} = \mathcal{H}_i. \quad (21)$$

We can use the momentum constraints to eliminate $\partial_i \Lambda$ from the Hamiltonian constraint, which, upon canonical quantization ($\Lambda(x) \to \mathrm{i}\delta/\delta\pi_\Lambda(x)$), becomes the Tomonaga–Schwinger equation (a local version of the Schrödinger equation),

$$-2\mathrm{i} \frac{\delta}{\delta \pi_\Lambda(x)} \Psi[\pi_\Lambda, \chi, \bar{g}] = \hat{h}_\Lambda(x) \Psi[\pi_\Lambda, \chi, \bar{g}], \quad (22)$$

with

$$\hat{h}_\Lambda(x) = \frac{\hat{\pi}_{ij}\hat{\pi}^{ij}}{|\bar{g}|} - \frac{(\hat{\pi}^i{}_i)^2}{2|\bar{g}|} - \bar{R} + \frac{\pi_\Lambda \hat{\pi}_\chi}{2|\bar{g}|} - \frac{2\hat{\pi}_\chi}{\pi_\Lambda} \partial_i \chi \partial^i \chi + \frac{4}{\pi_\Lambda} \partial_i \chi \bar{\nabla}_j \hat{\pi}^{ij}, \quad (23)$$
$$\hat{\pi}^{ij} = -\mathrm{i} \frac{\delta}{\delta \bar{g}_{ij}(x)}, \quad (24)$$
$$\hat{\pi}_\chi = -\mathrm{i} \frac{\delta}{\delta \chi(x)}. \quad (25)$$

To turn this into an ordinary functional Schrödinger equation, we need to choose a foliation $\pi^t_\Lambda(x)$. This is a choice for the value of π_Λ at each spatial point x that is monotonic in the parameter t. Notice that like all conjugate momenta here, π_Λ is a tensor *density*, and so makes an unusual time parameter [2]. We cannot use $\sqrt{|\bar{g}|}$ to factor out the volume form, because the spatial metric is an operator, not a c-number. The natural choice is just the product of t and a fiducial spatial volume element $\bar{\mu}$:

$$\pi^t_\Lambda(x) = -2t\bar{\mu}. \quad (26)$$

Classically, this corresponds to a lapse function

$$N = \frac{\bar{\mu}}{\sqrt{|\bar{g}|}(1 - \bar{\nabla}^2 \chi)}. \quad (27)$$

It is far from clear that different choices of foliation are equivalent [16], but because there is a natural choice, $\bar{\mu} = 1$, this may not matter. Then, because

$$\frac{\partial}{\partial t} \Psi[\pi^t_\Lambda] = \int \mathrm{d}^3 x \frac{\partial \pi^t_\Lambda(x)}{\partial t} \frac{\delta}{\delta \pi_\Lambda(x)} \Psi[\pi_\Lambda] \bigg|_{\pi_\Lambda = \pi^t_\Lambda}, \quad (28)$$

[1] It is only unphysical if *spacetime* is the only observable.

[2] Similarly, bosonic string theory has no world-sheet ghost instability, despite the presence of a negative kinetic term.

we can write the time dependent functional Schrödinger equation for gravity

$$\mathrm{i}\frac{\partial}{\partial t}\Psi(t,\bar{g},\chi] = \int \mathrm{d}^3x\,\bar{\mu}\hat{h}_\Lambda(x)\Big|_{\pi_\Lambda=-2t\bar{\mu}}\Psi(t,\bar{g},\chi]. \quad (29)$$

In $d = 4$ spacetime dimensions the spatial metric has $d(d-1)/2 = 6$ components, but the $d-1 = 3$ momentum constraints reduce this to $(d-1)(d-2)/2 = 3$ degrees of freedom, the correct number for gravity plus a scalar field Λ. Hence, the dynamical content is simply the spatial geometry, as well as the χ field. Note that $\pi_\chi \geq 0$, (i.e., $\dot{\Lambda} \geq 0$) in order for the gradient energy in the χ-field to be nonnegative. Given appropriate initial conditions for Λ, this ghost-free condition can be maintained for positive times, thanks to the conservation of the current $\partial_\mu\Lambda$. Note that constraining $\pi_\chi \geq 0$ does not restrict the geometrical phase space.

We have neglected to resolve operator ordering ambiguities, nor have we attempted to regularize UV divergences, since this task is likely impossible given the nonrenormalizability of gravity in 3+1 and higher dimensions [17]. Nevertheless, this equation may prove to be a useful tool for understanding infra-red phenomena such as dark energy, eternal inflation, the measure problem, and the black-hole information paradox.

The solutions to the equations of motion are straightforward in the mini-superspace ansatz

$$\mathrm{d}s^2 \;=\; -N^2(t)\,\mathrm{d}t^2 + a^2(t)\,\mathrm{d}\Omega_3^2, \quad (30)$$

and we assume Λ and χ depend only on t. We will choose the gauge $N = 1$. The scalar equations of motion are

$$\ddot{\Lambda} = -3H\dot{\Lambda}, \qquad \ddot{\chi} = -3H\dot{\chi} - 1. \quad (31)$$

There is a conserved quantity associated with global χ-translation invariance, namely

$$p_\chi = 4\pi^2 a^3\dot{\Lambda}. \quad (32)$$

Extremizing the action with respect to the lapse N gives the Friedmann equation

$$H^2 = \frac{\Lambda}{3} - \frac{1}{a^2} + \frac{\dot{\Lambda}\dot{\chi}}{3} \quad (33)$$

$$= \frac{\Lambda}{3} - \frac{1}{a^2} + \frac{p_\chi\dot{\chi}}{4\pi^2 a^3}, \quad (34)$$

where $H = \dot{a}/a$. Notice that if we choose initial conditions $\Lambda > 0$, $\dot{\Lambda} = 0$, we find global de Sitter space is a solution.

If we perturb away from an expanding de Sitter space by choosing a positive initial $\dot{\Lambda}$, the solution rapidly approaches de Sitter space, because the energy density in the homogeneous $\dot{\Lambda}$ perturbation decays like $1/a^3$, although it grows relative to the critical density during the radiation and matter eras, raising the possibility of detection. The canonical Hamiltonian

$$H_\mathrm{c} = N\left(\frac{p_\Lambda p_\chi}{4\pi^2 a^3} - \frac{p_a^2}{48\pi^2 a} - 12\pi^2 a + 4\pi^2 a^3\Lambda\right), \quad (35)$$

vanishes according to to the single secondary constraint

$$0 = -\frac{p_\Lambda p_\chi}{48\pi^4 a^6} + \frac{p_a^2}{576\pi^4 a^4} + \frac{1}{a^2} - \frac{\Lambda}{3}. \quad (36)$$

The generator of time translations should appear linearly in the constraint equation, and so Λ is the only sensible choice. Time is therefore played by p_Λ, obeying

$$p_\Lambda = 4\pi^2 a^3\dot{\chi}\ , \qquad \dot{p}_\Lambda = -4\pi^2 a^3. \quad (37)$$

Because the 4-volume increases like $\mathrm{d}V = 2\pi^2 a^3\,\mathrm{d}t$, the clock measures 4-volume:

$$\frac{\mathrm{d}p_\Lambda}{\mathrm{d}V} = -2. \quad (38)$$

This theory has some similarities with unimodular gravity [6, 13, 18], namely the arbitrariness of the cosmological constant. Indeed, if in the action we replaced $\partial_\mu\chi$ with a (Hodge dual) three-form potential $\Lambda(1-\nabla^2\chi) \mapsto \Lambda(1-\epsilon^{\mu\nu\rho\sigma}\partial_\mu A_{\nu\rho\sigma})$, we would precisely recover the background independent form of unimodular gravity [19]. However, because the dynamical Λ is then constrained to be constant, the problem of time remains [20], since the wavefunctional $\Psi[\bar{g}_{ij}, t)$ would be invariant under all 4-volume preserving deformations of the Cauchy surface Σ_t. Thus only one of the infinitely-many fingers of time is successfully ungauged by unimodular gravity.

The model we have presented is unusual for a theory of gravity in that it has a natural arrow of time and foliation, although this foliation need not be imprinted on the Einstein tensor. Because the lapse function is inversely proportional to the spatial volume element, expansion is not rewarded in the sense that the total four-volume of the universe is not dominated by the region with the largest expansion rate. This fact allows conditional probabilities of what observers should measure to be compatible with our observation of an old universe with a small cosmological constant, evading what is known as the youngness paradox [12] .

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