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Weibull distribution of brittle failures in the transition region

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Abstract

The Weibull stress is a well-known means of predicting the likelihood of weakest link brittle fracture that has been shown to accurately model the behaviour of ferritic steels in the lower transition region. Weibull stress is based on its use of a two parameter Weibull distribution, a commonly used distribution in probabilistic engineering. The distribution is defined by a shape parameter, the Weibull modulus, and a scaling parameter.

In the lower transition region, the Weibull modulus is relatively insensitive to temperature and the likelihood of failure can readily be defined by assuming it is constant and scaling the distribution with a 'measured' scaling parameter. This assumption, however, does not hold as the temperature increases into the upper transition zone and becomes less accurate as the upper shelf is approached.

This manifests itself as a broadening of the failure distribution that can be attributed to the increased size of the plastic zone ahead of a defect, which in turn 'samples' more potential failure sites, while simultaneously increasing the likelihood of blunting these sites and initiating ductile tearing. Thus, while more potential cleavage initiation sites are sampled, the likelihood of an individual defect causing failure is reduced.

This paper details the changes in the Weibull modulus and Weibull stress calculated from the 'Euro' fracture toughness data. The differences in the Weibull modulus and the mechanistic reason for these differences are explored to enable greater understanding of the factors that influence fracture toughness in the upper transition regime.

Keywords: Cleavage; Statistical analysis; Weibull stress; Fracture toughness; Ductile to brittle transition

1. Introduction

Fracture, by its nature, is a stochastic process and can be modelled using a probabilistic approach. This type of analysis assumes that the probability of fracture occurring follows a continuous distribution which can be represented by a standard statistical model. Typically for fracture of ferritic steels, the best representations of the likelihood of failure follow either a lognormal or a Weibull distribution.

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Nomenclature

β	Weibull modulus, a shape parameter in the Weibull distribution
γ	Location parameter in three parameter Weibull distribution
λ	The scaling parameter in the Weibull distribution
μ	Mean of a population
Φ	Cumulative normal distribution
ς	Standard deviation of a population
σ_y	Yield stress
b_0	Initial ligament length in a fracture toughness specimen
c	Ordinate-intercept of regression line
i	The true rank number in an ordered series of n measurements
i_{adj}	The adjusted rank number in a censored ordered series of n measurements
n	The number of measurements in a series of tests
J	The J-integral
K_J	Fracture toughness calculated from J-integral
P	Probability of failure
P_i	Estimated cumulative probability of failure at rank i
$P_{L,i}$	Linearised cumulative probability of failure at rank i
T	Temperature
T_0	Master Curve transition temperature

The lognormal distribution is defined by two parameters, μ and ς, which are the mean and standard deviation of the natural logarithm of the population of fracture toughness, K_J. The probability of failure at any given applied K_J is given by the cumulative distribution function, as shown in Equation 1.

$$P = \Phi\left(\frac{\ln K_J - \mu}{\varsigma}\right) \tag{1}$$

where Φ is the cumulative normal distribution.

The Weibull distribution is another commonly used probability distribution which is appropriate for data that follows a weakest link mechanism. This distribution is defined by two, or three, parameters. The two parameter Weibull distribution is defined by a shape parameter, β, which describes the general behaviour of the distribution, and a scale parameter, λ which shifts the peak of the distribution. In fracture parlance, the shape parameter is known as the Weibull modulus.

The probability of failure at a given level of applied K_J is given by Equation 2

$$P = 1 - e^{-(K_J/\lambda)^\beta} \tag{2}$$

The two parameter Weibull distribution can be modified through the addition of a third, location, parameter, γ. This parameter acts as an offset, guaranteeing a minimum K_J below which there is no probability of failure.

The Weibull distribution is of particular interest as it underpins the Master Curve distribution, as discussed in Wallin (2002), and will be used in the remainder of this paper. The Master Curve is defined by a three parameter Weibull distribution with a fixed Weibull modulus equal to 4.

For fracture toughness data in the lower transition region, the assumption that the Weibull modulus is equal to 4 holds reasonably well, but this assumption becomes less reliable in the upper transition region and there is, consequently, a limit imposed upon the Master Curve approach that reflects this. Wallin (2002) defines this limit to be:

$$-50^\circ C \leqslant (T - T_0) \leqslant +50^\circ C \tag{3}$$

where T_0 is the temperature where the median fracture toughness for a $25mm$ thick specimen is $100MPa\sqrt{m}$. For the 'Euro' dataset Wallin (2002) calculates T_0 to be between $-97^\circ C$ and $-87^\circ C$.

1.1. Weibull modulus in the transition region

With increasing temperature, ferritic steels show a transition from brittle to ductile behaviour, with a corresponding increase in fracture toughness. Within the upper transition region, there is an increase in the size of the plastic zone ahead of the crack tip and this in turn ‘samples’ a larger volume of material, thus increasing the likelihood that a potential cleavage initiation site will be affected. Simultaneously, however, this is compensated for by a lower stress as this stress is more readily redistributed through plasticity.

Additionally, with increasing plastic deformation, fracture as a result of ductile damage becomes more likely. This leads to a situation in which there are competing risks of failure that need to be considered, as discussed by Moskovic et. al. in Moskovic (1993), Moskovic (1995), Moskovic and Crowder (1995) and Moskovic (2006). Given the disparate natures of the failure processes, the distribution of K_J leading to failure becomes more spread to accommodate the different populations of failure.

2. Determining the Weibull parameters for the ‘Euro’ fracture dataset

The ‘Euro’ fracture dataset is a large body of fracture toughness tests carried out at different test temperatures with four different compact tension (CT) dimensions: 25mm, 50mm, 100mm and 200mm widths. Heerens and Hellmann (2002) discuss the development of this dataset in detail. The dataset has been widely analysed by numerous researchers and shows good applicability of the Master Curve approach in the lower transition region, but, as expected, deviation from the predicted behaviour in the upper transition region is observed.

In this paper, a graphical method was employed to estimate the Weibull parameters. In order to do this, an estimation of the cumulative probability of failure needs to be made. The measured fracture toughness was ranked in increasing order and the probability of failure estimated using Bernard’s approximation, after Wallin (2002) for median ranking as shown in Equation 4.

$$P_i = \frac{i - 0.3}{n + 0.4} \tag{4}$$

The resulting cumulative probability of failure was linearised through the use of Equation 5 and plotted on semi-log paper. A linear regression of the resulting plot was then used to estimate β and λ. β is the slope of the linear regression line and λ can be calculated from β and the ordinate-intercept, c, through Equation 6.

$$P_{L,i} = \ln\left[\ln\left(\frac{1}{1 - P_i}\right)\right] \tag{5}$$

$$\lambda = e^{-c/\beta} \tag{6}$$

2.1. Censoring ductile tearing failures

Two techniques were assessed as potential methods for accommodating the increasing importance of ductile tearing failures in an effort to more accurately predict the fracture toughness in the upper transition regime. The first of these was *censoring* data that can be definitively identified as ductile tearing, as defined by a lack of any cleavage. Strictly speaking a more rigorous scheme for censoring the data might be implemented, such as any data in which there is more than a set amount of ductile tearing (0.2mm for example), but for the purposes of this study, this was not explored. In the case where the failure is ductile tearing, an analogy can be drawn with fatigue testing, in which a run-out test is observed. Conceptually, if ductile tearing could be entirely suppressed, some level of applied K_J would lead to cleavage. Failure by ductile tearing, therefore, is similar to a suspended test and can be accounted as such.

To appropriately censor the ductile tearing failures, a modified rank was used to calculate the cumulative probability of failure. The adjusted rank described by Abernethy (2004) has been used to censor ductile tearing failures and can be calculated from the true rank through Equation 7.

$$i_{adj} = \frac{1 + i_{rev} \cdot j_{adj} + n}{i_{rev} + 1} \tag{7}$$

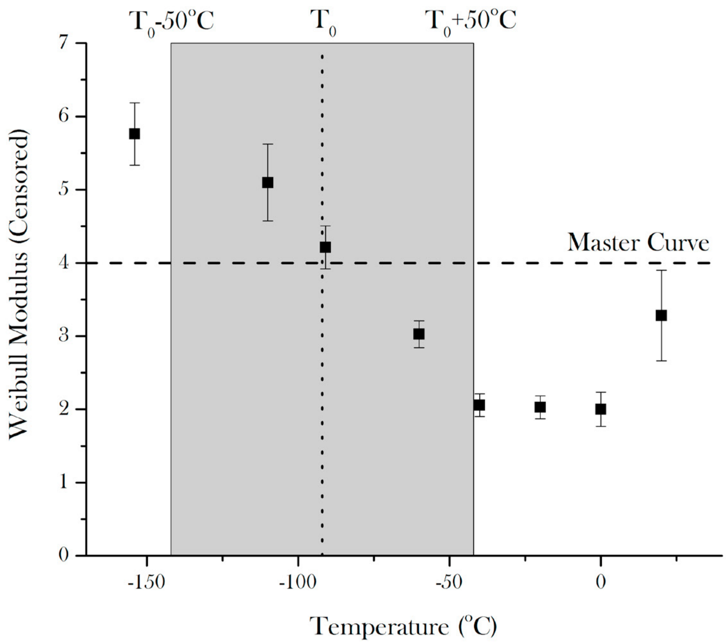


Fig. 1. Change in Weibull modulus with temperature

where i_{adj} is the adjusted rank, i_{rev} is the reversed rank, which equals $1 + (n - i)$, and j_{adj} is the previous adjusted rank for the uncensored data points. For censored data, the adjusted rank was not calculated and this altered the subsequent ranking, through varying j_{adj} from its expected value if no data was censored, which in turn altered the cumulative probability of failure.

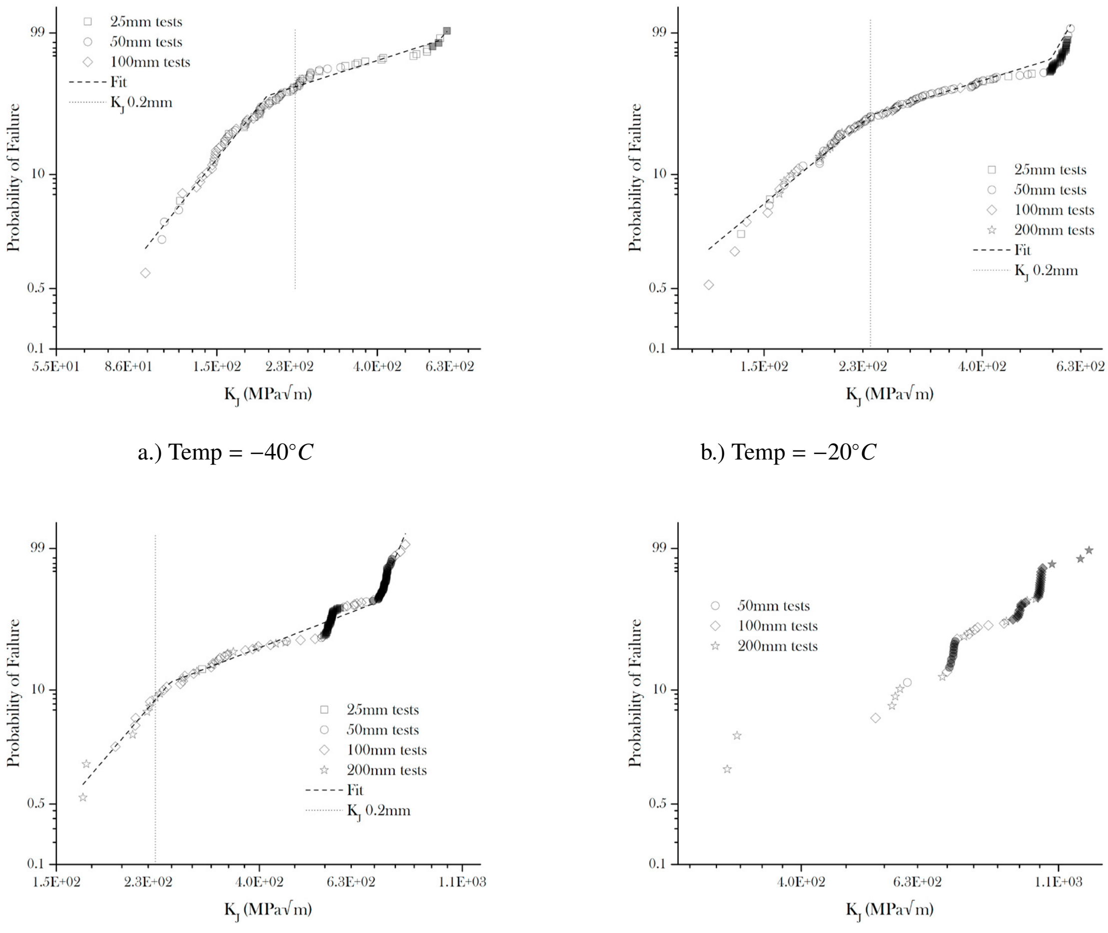
Censoring the data leads to a reduction in the Weibull modulus, particularly in the lower transition regime as can be seen in Figure 1. A reduction in the Weibull modulus leads to a less well defined peak, in other words the distribution is more spread. This supports the concept that there is a larger volume of material that is plastically deformed, which samples a larger number of defects, but at a lower average stress, so fewer defects are critical. It is also likely to be a reflection of the mixed mechanisms of failure, particularly where there is an observed cleavage failure preceded by some ductile tearing. These results are not censored and the amount of ductile tearing increases with increasing temperature, thus contributing to the increased width of the probability distribution.

It should be noted that the Master Curve value of 4 is a good average for the Weibull modulus across the validity range defined by Wallin (2002), as can be seen in Figure 1.

The censored data were compared to *uncensored* data, as well as to the best fit of the censored data, albeit with the Weibull modulus assumed to remain constant. In practice this was measured from the lower transition and held through the upper transition region. Essentially the slope of the linearised probability plot was held constant and the linear regression carried out on the basis of adjusting the ordinate-intercept only.

The uncensored data and the constant Weibull modulus fits were found to be similar, though the reason for why this should be the case is unclear. Wasiluk et. al. (2006) suggest that the Weibull modulus represents the distribution of defects in the material and is, thus, independent of the temperature, which might be argued is the reason for the seemingly constant value.

However, based on the probability plots generated during this study, such as those shown in Figure 2, this explanation for the Weibull modulus seems overly simplistic as it appears to be strongly influenced by the failure mechanism at work. Work by Afferrante et. al. (2006) also suggests that the Weibull modulus is more complex than this with the interaction between defects playing an important role in the observed modulus. The similarity between the uncensored fits and the constant modulus fits provides a possible explanation for the postulate that the modulus is a material con-



a.) Temp = $-40°C$

b.) Temp = $-20°C$

c.) Temp = $0°C$

d.) Temp = $20°C$

Fig. 2. Probability plot for samples tested at the indicated temperatures – filled symbols indicate ductile failure. Note: No fit was attempted for the $20°C$ data as there was no clear trend. An estimate was made for the purposes of determining the PDF.

stant, and in the case of multiple interacting influences the central limit theorem might result in an apparently constant modulus.

The median fracture toughness calculated from the *censored* probability distribution was compared with the median fracture toughness as determined using the constant Weibull modulus. Figure 3 shows the effect that measuring the Weibull modulus has on the predicted fracture toughness. Notable in this plot is the increase in the predicted median fracture toughness, particularly at higher temperatures. The fracture toughness for the 5% probability of failure is lower for the *censored* data due to the greater spread of this distribution.

This last observation shows that assuming the Weibull modulus is constant can be inherently conservative, depending on the exact means of prediction used, and is often excessively so.

2.2. Multimodal Weibull fitting

The second method of assessing the ductile tearing failures analysed in this work was to use the concept of competing risk and to fit the different distributions individually. The different distributions are readily apparent in the probability plots as a change in slope. A 2-part and a 3-part ‘broken stick’ fit was used to determine the Weibull parameters for the different distributions. In the first case the two distribution fit represents a distribution of cleavage

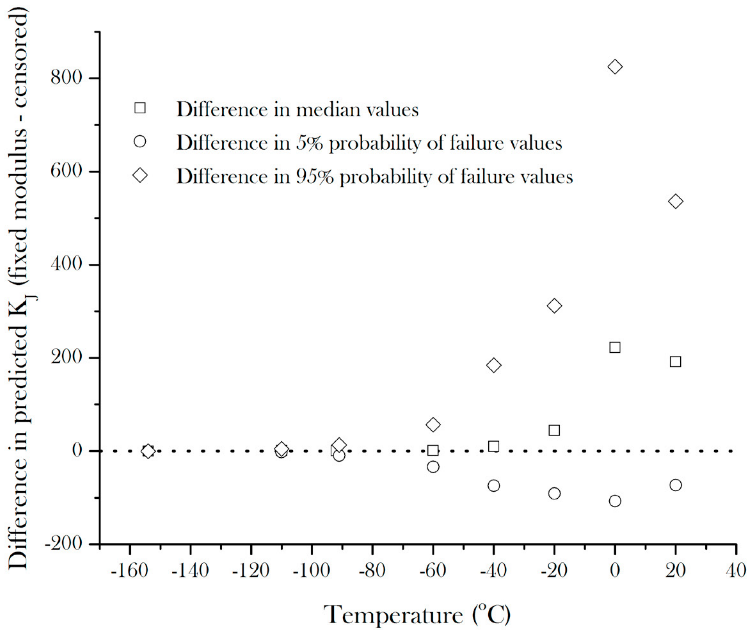


Fig. 3. Difference in the median fracture toughness when Weibull modulus is fit to data compared to an assumed constant modulus

failures and a distribution of ductile tearing failures. As can be seen in the probability plots in the upper transition region ($-40°C$ – $20°C$), Figure 2, there is a third distribution present. This represents cleavage failure following a degree of ductile tearing, and the observation led to the use of a 3-part fit.

When a 3-part fit was employed, the slope of the middle distribution was deliberately set to a lower Weibull modulus than that of the low and high distributions. This was done on the basis that the intermediate distribution represents cleavage failure following ductile tearing, which is expected to have a greater spread than either cleavage or ductile tearing failure. Fitting was done using MatLab and initial break points were estimated from the probability plots.

For the probability plots at $0°C$ and $20°C$, it appears that there are multiple probability distributions, i.e. in excess of the three distributions fit. On closer inspection, however, there appears to be a size effect, with larger specimens tending toward more brittle behaviour. This ties into the observations of Heerens and Hellmann (2002) and Neale (2002) who recognised that there was a size effect at higher temperatures that was not present on the lower shelf and lower transition regions.

To test the theory that the additional observed distributions were the result of a size effect, a probability plot of J, normalised by the initial ligament length, b_0 and the yield strength, σ_y was constructed, as shown in Figure 4.

First, the onset of ductile tearing failure occurs at an approximately constant normalised J. The additional distributions seen at high K_J in Figures 2c & d, for example, are not present in the normalised plot which suggests that these distributions are the result of the difference in the specimen size. Furthermore, this size effect dominates at $J > J_{Max}$, which Heerens et. al. (2005) report has been observed in previous studies.

The second trend that can be observed is the decreased probability of cleavage with increasing temperature, as reflected in the downward shift in the probability plots.

Figure 5 shows schematically how the multi-modal Weibull fit matches with the measured data. While the peak heights of the probability distribution functions are arbitrary, the location and spread of the distribution is accurate so that a valid comparison can be made. Subsequent development of this approach will follow the competing risk model discussed by Moskovic (1995) in order to fit the peak heights, that is the relative probability of a failure being part of the cleavage distribution, the cleavage following ductile tearing distribution or the ductile tearing distribution.

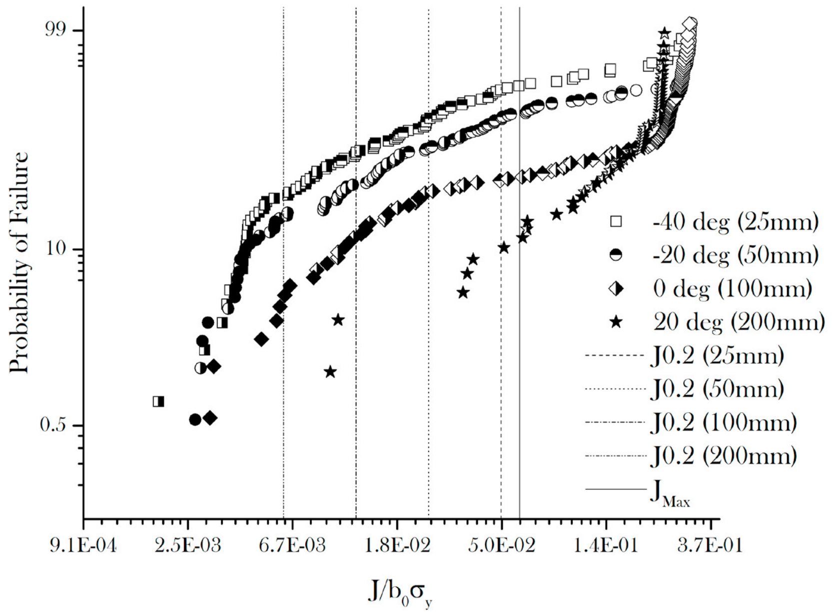


Fig. 4. Probability plot for *J*, normalised by the specimen size. Note: open symbols represent 25mm tests, top half shaded represent 50mm, right side shaded 100mm and fully shaded points are 200mm tests.

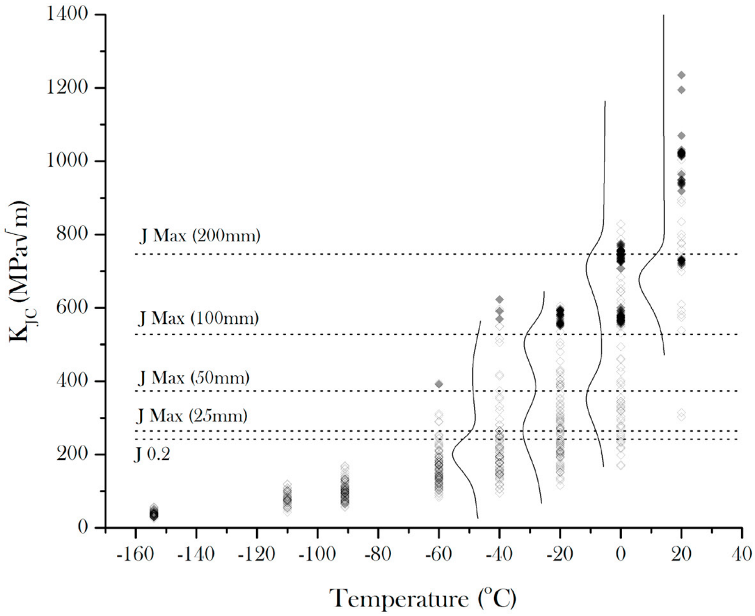


Fig. 5. Schematic showing the match between the multi-modal distribution fit and the measured data – filled data points indicate ductile failures.

3. Conclusions

The 'Euro' fracture toughness dataset is a rich dataset that offers a lot of insight into the fracture properties of ferritic steels. As many researchers have noted, the Master Curve method accurately represents the fracture properties in the lower shelf and lower transition regimes of the brittle-ductile transition curve. As the temperature increases, moving up the transition curve, however, this approach starts to become less reliable, resulting in overly conservative estimates for the fracture toughness with associated economic costs.

The Weibull modulus appears to be a more complex parameter than initially thought and may be the result of the different failure mechanisms, defect distribution and the interaction of defects. By using a linearised probability plot, some of this complexity can be interpreted.

In order to obtain an improved understanding of the Weibull modulus and how it changes with temperature, the presence of competing failure modes, such as ductile tearing and cleavage following ductile tearing need to be accounted for. Two methods have been trialled and reported in the paper, censoring of ductile failures and the use of 'broken stick' regression to identify multiple failure distributions. Both methods appear to more closely represent the measured dataset and a significant increase in the median predicted fracture toughness results within the upper transition region.

A note of caution needs to be raised, however, as much of the data lies outside of the validity limits for J. This issue was raised by Landes (2006), along with the usefulness of $J_{0.2}$ and it was concluded that the standard method of testing should be reviewed. With modern materials offering improved fracture properties, it appears that further work is needed to establish a robust Engineering approach to analyse upper transition fracture properties where cleavage failure follows some amount of ductile tearing.

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