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# Robust and Integrated Airline Scheduling

A thesis submitted in partial fulfilment of the requirements  
for the Degree of Doctor of Philosophy

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2009



# Abstract

In airline scheduling a variety of planning and operational decision problems have to be solved. In this thesis we consider the problems aircraft routing and crew pairing: aircraft and crew must be allocated to flights of a schedule in a minimal cost way.

Although these problems are not independent, they are usually formulated as independent mathematical optimisation models and solved sequentially. This approach might lead to a suboptimal allocation of aircraft and crew, since a solution of one of the problems may restrict the set of feasible solutions of the problem solved subsequently.

Also, in minimal cost solutions, aircraft and crew are highly utilised and short turn around times are usually used for aircraft and crew. If such a solution is used in operations, a short delay of one flight can cause very severe disruptions of the schedule later in the day due to the lack of buffer times. We formulate an integrated aircraft routing and crew pairing model that can generate solutions that incur small costs and are also robust to typical stochastic variability in airline operations.

We propose two new solution methods to solve the integrated model. The first approach is an optimisation based heuristic approach that is capable of generating good quality solutions quickly, the second approach can solve the integrated model to optimality.

In an extension of the integrated model we allow the departure times of some flights in the schedule to vary in some time window. This creates additional flexibility that leads to aircraft routing and crew pairing solutions with improved cost and robustness compared to the integrated model without time windows.

Using data from domestic Air New Zealand schedules, we evaluate the benefits of the approaches on real world problem instances. Our solutions satisfy all rules imposed for these problems and are ready to be implemented in practice. We generate solutions that dramatically improve the cost and robustness of solutions obtained by existing methods.

# Acknowledgements

First of all I would like to thank my supervisors Professor David Ryan and Associated Professor Matthias Ehrgott. Without their great support and enthusiasm for the project this thesis would not have been possible.

I also would like to thank Air New Zealand for providing the crew pairing solver, data, and valuable feedback. I particularly would like to thank Chris Cullinan, Graeme Andrews, and Jeremy Hutson for providing the required data and for their time to discuss results and provide important feedback that led to improvements of the algorithms.

I would also like to thank all colleagues at The Optima Corporation, particularly Amanda, Jody, Paul, and Jeff. Their suggestions during my studies and help with the crew pairing optimiser is very much appreciated. Special thanks go to Amanda for taking the time to proofread an earlier draft of this thesis.

I am grateful for all help I received from many different colleagues and friends from the Department of Engineering Science, especially Lizhen, Andrea, Richard, Hamish, and Andrew.

I would also like to thank my family, in particular my loving parents Monika and Manfred, and my sister Carolin for their great support to help me reach my goals. Thanks also to all friends in New Zealand, Germany, and wherever else for making my life as enjoyable as it is. Most importantly, very special thanks to Andrea, for simply everything. With you, writing a PhD is a sweet dream rather than a nightmare.



# Contents

<b>Introduction</b>	<b>1</b>
<b>1 Mathematical Background</b>	<b>9</b>
1.1 Set Partitioning Problem . . . . .	10
1.2 Multi-Commodity Flow Problem . . . . .	12
1.3 Column Generation . . . . .	13
1.4 Branch-and-Price . . . . .	15
1.5 Linear Program Decomposition Principles . . . . .	16
1.5.1 Dantzig-Wolfe Decomposition . . . . .	16
1.5.2 Benders Decomposition . . . . .	19
1.5.3 Lagrangian Relaxation . . . . .	21
1.5.4 Comparison of Decomposition Methods . . . . .	22
1.6 Multiobjective Optimisation . . . . .	24
<b>2 Airline Scheduling Background and Literature</b>	<b>27</b>
2.1 Airline Scheduling Problems . . . . .	29
2.1.1 Problem Characteristics . . . . .	29
2.1.2 Schedule Design . . . . .	32
2.1.3 Fleet Assignment . . . . .	33
2.1.4 Aircraft Routing . . . . .	36



2.1.5	Crew Pairing . . . . .	39
2.1.6	Crew Rostering . . . . .	45
2.2	Integration of Airline Scheduling Problems . . . . .	48
2.3	Robustness . . . . .	55
2.4	Overview of Solution Approaches for Airline Scheduling Problems	60
<b>3</b>	<b>Aircraft Routing Problem</b>	<b>65</b>
3.1	Model . . . . .	65
3.2	Rules . . . . .	67
3.3	Solution Methods . . . . .	72
3.3.1	Preprocessing . . . . .	72
3.3.2	LP-Relaxation . . . . .	74
3.3.3	Column Generation . . . . .	74
3.3.4	Branch-and-Price . . . . .	77
3.3.5	Alternative Set Partitioning Formulation . . . . .	80
3.3.6	Decomposition Methods . . . . .	80
3.4	Computational Experiments . . . . .	85
<b>4</b>	<b>Crew Pairing Problem</b>	<b>91</b>
4.1	Model . . . . .	91
4.2	Rules . . . . .	94
4.3	Operational Robustness . . . . .	97
4.4	Solution Methods . . . . .	102
4.4.1	LP-Relaxation . . . . .	102
4.4.2	Column Generation . . . . .	103
4.4.3	Branch-and-Price . . . . .	104

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4.4.4	Cost Constraint Approach . . . . .	104
4.5	Computational Experiments . . . . .	106
4.5.1	Cost Constraint Approach . . . . .	118
<b>5</b>	<b>Robust and Integrated Aircraft Routing and Crew Pairing</b>	<b>121</b>
5.1	Model . . . . .	122
5.2	Solution Methods . . . . .	124
5.2.1	Iterative Approach . . . . .	126
5.2.2	Dantzig-Wolfe Decomposition Approach . . . . .	132
5.2.3	Benders Decomposition Approach . . . . .	135
5.2.4	Discussion of Approaches . . . . .	139
5.3	Computational Experiments . . . . .	141
5.3.1	Iterative Approach for a Single Crew Group . . . . .	141
5.3.2	Comparison of Iterative Approach and Optimisation Ap- proaches . . . . .	157
5.3.3	Iterative Approach for Multiple Crew Groups . . . . .	164
5.4	Simulation . . . . .	166
5.5	Visualisation . . . . .	168
<b>6</b>	<b>Robust and Integrated Aircraft Routing and Crew Pairing with Time Windows</b>	<b>173</b>
6.1	Model . . . . .	174
6.2	Rules . . . . .	177
6.3	Solution Methods . . . . .	179
6.3.1	Time Window Branching Approach . . . . .	180
6.3.2	Re-timing of Flights for Fixed Aircraft Routings and Crew Pairings . . . . .	185

6.4 Computational Experiments . . . . .	188
<b>Conclusion</b>	<b>197</b>
<b>References</b>	<b>201</b>

# List of Figures

1.1	Block-diagonal matrix structure. . . . .	22
1.2	Supported and non-supported non-dominated points. . . . .	26
2.1	Connection network with 5 flight arcs. . . . .	31
2.2	Time-line network for a single airport. . . . .	31
4.1	Comparison of a non-robust and a robust solution . . . . .	100
4.2	Solutions for variation of AIRCRAFTCHANGECOST penalty for first officer scenario, summer 2005, 7 days. . . . .	117
4.3	Solutions for variation of DPACLIM rule for first officer sce- nario, summer 2005, 7 days. . . . .	117
5.1	Schematic view iterative approach. . . . .	129
5.2	Schematic view iterative approach with two crew groups. . . . .	131
5.3	Iterative approach solutions for first officer scenario, summer 2005, 7 days. . . . .	142
5.4	Variation of DPACLIM for first officer scenario, summer 2005, 7 days. . . . .	156
5.5	Results for first officer scenario, summer 2006, 7 days, with cabin crew solved simultaneously. . . . .	165
5.6	Results for cabin crew scenario, summer 2006, 7 days, with first officers solved simultaneously. . . . .	165

5.7	Screen-shot of aircraft routing solution for first officer scenario, summer 2006. . . . .	170
5.8	Screen-shot of traditional approach crew pairing solution for first officer scenario, summer 2006. . . . .	171
5.9	Screen-shot of iterative approach (iteration 5) crew pairing solution for first officer scenario, summer 2006. . . . .	172
6.1	Re-timed solutions of iterative approach, AKLWLG flights fixed, $\pm 10$ minute windows, first officer scenario, summer 2006, 7 days.	189
6.2	Re-timed solutions of iterative approach, AKLWLG flights flexible, $\pm 10$ minute windows, first officer scenario, summer 2006, 7 days. . . . .	190
6.3	Time window branch-and-bound solutions of iterative approach solutions (iterations 3 and 5), AKLWLG flights fixed, $\pm 10$ minute windows, first officer scenario, summer 2006, 7 days. . . . .	192
6.4	Screen-shot of time window branch-and-bound solution (iteration 3), AKLWLG flights fixed, $\pm 10$ minute windows, for first officer scenario, summer 2006. . . . .	194
6.5	Screen-shot of iterative approach solution (iteration 3) without time windows for first officer scenario, summer 2006. . . . .	195

# List of Tables

2.1	Overview of solution approaches for airline scheduling problems.	64
3.1	Characteristics of scenarios.	86
3.2	Computational results for summer 2005.	89
3.3	Computational results for winter 2005.	89
3.4	Computational results for summer 2006.	90
3.5	Computational results for winter 2007.	90
4.1	Computational results for captain, first officer, and cabin crew scenarios, summer 2005, winter 2005, summer 2006, and winter 2007.	109
4.2	Variation of DPAC <sub>LIM</sub> rule and AIRCRAFTCHANGE <sub>COST</sub> penalty $p$ for first officer scenario, summer 2005, 7 days.	114
4.3	Improvements of solutions for variation of AIRCRAFTCHANGE <sub>COST</sub> penalty $p$ for first officer scenario, summer 2005, 7 days.	115
4.4	Improvements of solutions for variation of DPAC <sub>LIM</sub> for first officer scenario, summer 2005, 7 days.	116
4.5	Variation of $o$ and $t$ of cost constraint approach for first officer scenario, summer 2005.	120
5.1	Results of iterative approach for first officer scenario, summer 2005.	146
5.2	Results of iterative approach for first officer scenario, winter 2005.	148

5.3	Results of iterative approach for first officer scenario, summer 2006. . . . .	150
5.4	Results of iterative approach for captains scenario, summer 2005.	152
5.5	Results of iterative approach for cabin crew scenario, summer 2005. . . . .	154
5.6	Variation of DPAC <sub>CLIM</sub> for first officer scenario, summer 2005, 7 days. . . . .	156
5.7	Comparison of iterative approach, Dantzig-Wolfe decomposition approach, and Benders decomposition approach. . . . .	163
5.8	On-time performance for iterative approach solutions, first officer and cabin crew scenarios, summer 2006, 7 days. . . . .	167
5.9	Minutes of delay listed by reason for iterative approach solutions, first officer and cabin crew scenarios, summer 2006, 7 days.	168
6.1	Results for re-timed solutions of iterative approach for first officer scenario, summer 2006, 7 days. . . . .	191
6.2	Results for re-timed solutions of iterative approach for first officer scenario, winter 2005, 7 days. . . . .	191