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A Novel Electric Fence Energizer: Design and Analysis

By

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A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy in Engineering



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Abstract

Continual advancements in technology have led to the development of reliable, efficient and economical farm management systems, many of which utilize electric fences for effective control of farm animals. An electric fence system constitutes a conducting fence structure that is energized by a high voltage signal generated from an electric fence energizer. Modern electric fence energizers employ a pulsed power supply together with an appropriate high voltage charging scheme to generate high voltage pulses that energize the fence structure. The high voltage pulse delivers a non-lethal electric shock to an animal that comes into contact with the fence, and the consequent psychological impact on the animal is such that it is less likely to come into contact with the fence again.

The complexity associated with modelling electric fence systems has hindered the development of proper mathematical tools that aid their design and optimization, and as a consequence, electric fence systems are currently designed using empirical rules together with a trial and error design approach. This Thesis therefore aims to fulfil this need by presenting new technologies and mathematical tools that can be used to design both intelligent and optimized electric fence systems. It presents a comprehensive study on electric fencing systems, which includes a detailed mathematical analysis on pulse propagation properties of electric fence networks and the development of high performance fence energizers that incorporates new pulses power supply technologies and high voltage charging schemes.

With regard to the pulsed power technologies, two novel topologies with the ability to adapt their output pulse shape according to the fence conditions are proposed. The performance of these technologies is analyzed mathematically, and verified experimentally. In comparison to

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the existing fence energizer technology, energizers that are based on the proposed pulsed power supply designs are superior in performance. Furthermore, a novel Buck-Boost pushpull parallel-resonant converter technique, which is suitable for charging high voltage storage capacitors in an energizer, is also presented. The proposed technique allows for the push-pull parallel-resonant converter to operate with a frequency dependent variable voltage gain over a wide load range while maintaining zero voltage switching (ZVS). The operation of the converter is analyzed mathematically and verified experimentally to validate the proposed technique.

In order to gain an insight into the propagation characteristics of electric fence networks, the Thesis presents a comprehensive mathematical model. The model uses the propagation properties of fence networks with frequency dependent distributed line parameters to obtain analytical solutions for the propagation function in the frequency-domain. As these analytical solutions are complex in nature, they are solved numerically to obtain time-domain solutions, the accuracy of which are verified through experiments and simulations.

The mathematical tools and new technologies proposed in the thesis can be used to design electric fence systems that are more efficient and effective than the existing systems. In addition, the tools proposed are also expected to aid the design of electric fence based communication channels for intelligent farm management systems.

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Nomenclature

Acronyms

- AC Alternating Current
- ADT Active Denial Technology
- DC Direct Current
- EMI Electro-Magnetic Interference
- GPS Global Positioning System
- GTO Gate Turn-Off Thyristor
- HV High Voltage
- HVPPS High Voltage Pulsed Power Generator
- IEC International Electrotechnical Commission
- IGBT Insulated Gate Bipolar Transistor
- LCD Inductor, Capacitor and Diode
- LCI Inductor, Capacitor and Current Source
- LHS Left Hand Side
- MOSFET Metal-Oxide Semiconductor Field Effect Transistor
- PPRC Push-Pull Parallel-Resonant Converter
- PSCAD Power Systems Computer Aided Design
- PWM Pulse Width Modulation
- RCD Resistor, Capacitor and Diode
- RF Radio Frequency
- RFI Radio Frequency Interference
- RHS Right Hand Side
- RMS Root Mean Square
- SC-PPRC Split-Capacitor Push-Pull Parallel-Resonant Converter
- SCR Silicon Controlled Rectifier
- TEM Transverse Electro-Magnetic
- VCO Voltage Controlled Oscillator
- ZCS Zero Current Switching
- ZVS Zero Voltage Switching

Symbols

A, B	(n x 1) vector for forward and backward travelling waves
ber, bei	Kelvin's functions
С	Capacitance (F)
C_l, C_m	Line, mutual capacitance (F/m)
D	Duty cycle
Е	Energy (J)
\mathbf{f}_{pr}	Pulse repetition rate (Hz)
$\mathbf{f}_{\mathbf{s}}$	Switching frequency (Hz)
$\mathbf{f}_{\mathbf{z}}$	Zero voltage switching frequency (Hz)
G _l	Shunt Conductance (S/m)
I, i	Current (A)
I_0	Modified Bessel functions of 1 st kind of order 0
I _{rr}	Reverse recovery current (A)
k	Coupling coefficient
L	Inductance (H)
L _{lk}	Leakage inductance (H)
L _m	Magnetizing inductance (H)
Ν	Number of stages
р	Complex depth
Р	Power (W)
R	Resistance (Ω)
r	Wire radius (m)
R _{dc}	Wire DC resistance (Ω)
R _r	Return path resistance (Ω/m)
S	Laplace transformation
T, t	Time (s)
Ton	Switch on-time (s)
T _p	Pulse duration (s)
T _s	Switching time period (s)
T_V, T_I	(n x n) similarity transformation matrices
Tz	Zero voltage switching time period (s)

U _(t)	Unit step function
V, I	(n x 1) vector for line voltage, current
V, v	Voltage (V)
V^m, I^m	Decoupled (n x 1) vector for line voltage, current
Ζ, Υ	(n x n) matrix for line impedance, admittance
Z_0	(n x n) matrix for characteristic impedance
Z_i, Z_e, Z_m	Internal, external, mutual impedance (Ω/m)
Z_T, Z_G	Terminal, generator external impedance (Ω)
Z_T, Z_G	Terminal, generator external impedances
$\Gamma_{\rm T}, \Gamma_{\rm G}$	Terminal, generator reflection coefficient
$\Gamma_{\mathrm{T}}, \Gamma_{\mathrm{G}}$	Terminal, generator reflection coefficients
μ_0	Free space permeability (H/m)
ϵ_0	Free space permittivity (F/m)
$ ho_e$	Earth resistivity (Ωm)
μ_r	relative permeability
α	Attenuation
γ	(n x 1) vector for propagation constants
η	Efficiency
σ	Wire conductance (S)
ω	Angular frequency (rad/s)
ω _r	Tank angular resonant frequency (rad/s)
ω _z	Damped angular resonant frequency (rad/s)

Subscript

max	Maximum
min	Minimum
out	Output
in	Input
eq	Equivalent
pk	Peak