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A Novel Electric Fence Energizer: 
Design and Analysis

By

Duleepa J. Thrimawithana

A thesis submitted in partial fulfilment of
the requirements for the degree of
Doctor of Philosophy
in Engineering

Department of Electrical and Computer Engineering
The University of Auckland
New Zealand
June 2008
Abstract

Continual advancements in technology have led to the development of reliable, efficient and economical farm management systems, many of which utilize electric fences for effective control of farm animals. An electric fence system constitutes a conducting fence structure that is energized by a high voltage signal generated from an electric fence energizer. Modern electric fence energizers employ a pulsed power supply together with an appropriate high voltage charging scheme to generate high voltage pulses that energize the fence structure. The high voltage pulse delivers a non-lethal electric shock to an animal that comes into contact with the fence, and the consequent psychological impact on the animal is such that it is less likely to come into contact with the fence again.

The complexity associated with modelling electric fence systems has hindered the development of proper mathematical tools that aid their design and optimization, and as a consequence, electric fence systems are currently designed using empirical rules together with a trial and error design approach. This Thesis therefore aims to fulfil this need by presenting new technologies and mathematical tools that can be used to design both intelligent and optimized electric fence systems. It presents a comprehensive study on electric fencing systems, which includes a detailed mathematical analysis on pulse propagation properties of electric fence networks and the development of high performance fence energizers that incorporates new pulses power supply technologies and high voltage charging schemes.

With regard to the pulsed power technologies, two novel topologies with the ability to adapt their output pulse shape according to the fence conditions are proposed. The performance of these technologies is analyzed mathematically, and verified experimentally. In comparison to
the existing fence energizer technology, energizers that are based on the proposed pulsed power supply designs are superior in performance. Furthermore, a novel Buck-Boost push-pull parallel-resonant converter technique, which is suitable for charging high voltage storage capacitors in an energizer, is also presented. The proposed technique allows for the push-pull parallel-resonant converter to operate with a frequency dependent variable voltage gain over a wide load range while maintaining zero voltage switching (ZVS). The operation of the converter is analyzed mathematically and verified experimentally to validate the proposed technique.

In order to gain an insight into the propagation characteristics of electric fence networks, the Thesis presents a comprehensive mathematical model. The model uses the propagation properties of fence networks with frequency dependent distributed line parameters to obtain analytical solutions for the propagation function in the frequency-domain. As these analytical solutions are complex in nature, they are solved numerically to obtain time-domain solutions, the accuracy of which are verified through experiments and simulations.

The mathematical tools and new technologies proposed in the thesis can be used to design electric fence systems that are more efficient and effective than the existing systems. In addition, the tools proposed are also expected to aid the design of electric fence based communication channels for intelligent farm management systems.
To my loving

Father, Mother and Sisters
Acknowledgements

There are many people who have helped me in numerous ways to succeed in my PhD studies over the past three years. Although I wish to express my sincere gratitude to all of them, individually, I use this limited space to thank a few people who I should specifically mention in recognition of their invaluable support and guidance offered to me.

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Duleepa J Thrimawithana

Auckland

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Nomenclature

Acronyms

AC – Alternating Current
ADT – Active Denial Technology
DC – Direct Current
EMI – Electro-Magnetic Interference
GPS – Global Positioning System
GTO – Gate Turn-Off Thyristor
HV – High Voltage
HVPPS – High Voltage Pulsed Power Generator
IEC – International Electrotechnical Commission
IGBT – Insulated Gate Bipolar Transistor
LCD – Inductor, Capacitor and Diode
LCI – Inductor, Capacitor and Current Source
LHS – Left Hand Side
MOSFET – Metal-Oxide Semiconductor Field Effect Transistor
PPRC – Push-Pull Parallel-Resonant Converter
PSCAD – Power Systems Computer Aided Design
PWM – Pulse Width Modulation
RCD – Resistor, Capacitor and Diode
RF – Radio Frequency
RFI – Radio Frequency Interference
RHS – Right Hand Side
RMS – Root Mean Square
SC-PPRC – Split-Capacitor Push-Pull Parallel-Resonant Converter
SCR – Silicon Controlled Rectifier
TEM – Transverse Electro-Magnetic
VCO – Voltage Controlled Oscillator
ZCS – Zero Current Switching
ZVS – Zero Voltage Switching
### Symbols

<table>
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<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>A, B</td>
<td>(n x 1) vector for forward and backward travelling waves</td>
</tr>
<tr>
<td>ber, bei</td>
<td>Kelvin’s functions</td>
</tr>
<tr>
<td>C</td>
<td>Capacitance (F)</td>
</tr>
<tr>
<td>C_l, C_m</td>
<td>Line, mutual capacitance (F/m)</td>
</tr>
<tr>
<td>D</td>
<td>Duty cycle</td>
</tr>
<tr>
<td>E</td>
<td>Energy (J)</td>
</tr>
<tr>
<td>f_pr</td>
<td>Pulse repetition rate (Hz)</td>
</tr>
<tr>
<td>f_s</td>
<td>Switching frequency (Hz)</td>
</tr>
<tr>
<td>f_z</td>
<td>Zero voltage switching frequency (Hz)</td>
</tr>
<tr>
<td>G_l</td>
<td>Shunt Conductance (S/m)</td>
</tr>
<tr>
<td>I, i</td>
<td>Current (A)</td>
</tr>
<tr>
<td>I_0</td>
<td>Modified Bessel functions of 1st kind of order 0</td>
</tr>
<tr>
<td>I_rr</td>
<td>Reverse recovery current (A)</td>
</tr>
<tr>
<td>k</td>
<td>Coupling coefficient</td>
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<tr>
<td>L</td>
<td>Inductance (H)</td>
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<tr>
<td>Llk</td>
<td>Leakage inductance (H)</td>
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<tr>
<td>L_m</td>
<td>Magnetizing inductance (H)</td>
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<tr>
<td>N</td>
<td>Number of stages</td>
</tr>
<tr>
<td>p</td>
<td>Complex depth</td>
</tr>
<tr>
<td>P</td>
<td>Power (W)</td>
</tr>
<tr>
<td>R</td>
<td>Resistance (Ω)</td>
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<tr>
<td>r</td>
<td>Wire radius (m)</td>
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<tr>
<td>R_{dc}</td>
<td>Wire DC resistance (Ω)</td>
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<td>R_r</td>
<td>Return path resistance (Ω/m)</td>
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<tr>
<td>s</td>
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<td>T, t</td>
<td>Time (s)</td>
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<tr>
<td>T_{on}</td>
<td>Switch on-time (s)</td>
</tr>
<tr>
<td>T_p</td>
<td>Pulse duration (s)</td>
</tr>
<tr>
<td>T_s</td>
<td>Switching time period (s)</td>
</tr>
<tr>
<td>T_v, T_l</td>
<td>(n x n) similarity transformation matrices</td>
</tr>
<tr>
<td>T_z</td>
<td>Zero voltage switching time period (s)</td>
</tr>
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\( U(t) \)  Unit step function

\( V, I \)  (n x 1) vector for line voltage, current

\( V, v \)  Voltage (V)

\( V^m, I^m \)  Decoupled (n x 1) vector for line voltage, current

\( Z, Y \)  (n x n) matrix for line impedance, admittance

\( Z_0 \)  (n x n) matrix for characteristic impedance

\( Z_i, Z_e, Z_m \)  Internal, external, mutual impedance (\( \Omega/m \))

\( Z_T, Z_G \)  Terminal, generator external impedance (\( \Omega \))

\( Z_T, Z_G \)  Terminal, generator external impedances

\( \Gamma_T, \Gamma_G \)  Terminal, generator reflection coefficient

\( \Gamma_T, \Gamma_G \)  Terminal, generator reflection coefficients

\( \mu_0 \)  Free space permeability (H/m)

\( \varepsilon_0 \)  Free space permittivity (F/m)

\( \rho_e \)  Earth resistivity (\( \Omega m \))

\( \mu_r \)  relative permeability

\( \alpha \)  Attenuation

\( \gamma \)  (n x 1) vector for propagation constants

\( \eta \)  Efficiency

\( \sigma \)  Wire conductance (S)

\( \omega \)  Angular frequency (rad/s)

\( \omega_r \)  Tank angular resonant frequency (rad/s)

\( \omega_z \)  Damped angular resonant frequency (rad/s)

**Subscript**

<table>
<thead>
<tr>
<th>Subscript</th>
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<tr>
<td>max</td>
<td>Maximum</td>
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<tr>
<td>min</td>
<td>Minimum</td>
</tr>
<tr>
<td>out</td>
<td>Output</td>
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<tr>
<td>in</td>
<td>Input</td>
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<tr>
<td>eq</td>
<td>Equivalent</td>
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<tr>
<td>pk</td>
<td>Peak</td>
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1.1 An Introduction to Electric Fence Technology

Since the dawn of civilization until modern times, farmers have been using a variety of techniques to manage their livestock. Fuelled by continual advances in technology, these farm management tools have evolved into sophisticated systems to provide a reliable, economical, and efficient service. Electric fence technology, originated from New Zealand in the early 1930s, is widely used in farms all around the world as an integral part of contemporary farm management systems, allowing for efficient administration of farm animals. In addition, electric fences are used in security systems for preventing intrusion, large game reserves for managing wild life, and even along country borders to avoid the spread of deadly diseases among domestic animals. In comparison to physical barrier fence systems that were used in the early days, electric fence systems are reliable, cost effective and require less maintenance. Animal injuries, commonly associated with barbed-wire fences, are virtually eliminated by the electric fence technology. An electric fence does not require a high structural strength; therefore this technology can also be used to construct temporary fences to cater for seasonal demands and for advanced grazing techniques [1-4].

An electric fence system, illustrated in Fig. 1.1, is implemented by energizing one or more strands of bare conductors that are attached to fence posts through insulators. The high
voltage (HV) power converter that is used to energize the fence is termed the ‘electric fence energizer’. The earth serves as the return path for the signals produced by the energizers, although, in some situations, additional conductors are used to reduce the impedance of the return path. HV signals, generated by the energizer and transmitted along the fence, deliver a non-lethal shock to an animal that comes into contact with the fence. A physiological pain or discomfort is induced by the shock current that passes from the point of contact with the fence through the animal and into the return path. The consequent psychological impact in the animal is such that it is less likely to come into contact with the fence again. The shock voltage should be sufficiently high enough to effectively breakthrough the high resistance skin of the animal, allowing the current-flow through the body to increase above the animal’s pain threshold [1-5].

![An electric fence system](image)

**Fig. 1.1** An electric fence system

Although electric fence systems have been widely used throughout, to date a comprehensive study on electric fencing systems has neither been published nor the technology developed to deliver the optimum performance. The complexity associated with these systems has hindered the development of both analytical and numerical tools to characterize the performance of electric fence systems. As such, electric fence designers employ empirical
rules together with a trial and error design approach to develop electric fence equipment. Moreover, the layout of the electric fence, which is a vital part of the system, is not optimized due to the lack of proper optimization tools. In order to overcome the deficiencies found in the fence layout, electric fence energizers are designed to generate pulses with the highest voltage and the longest pulse duration allowed by the safety standards (IEC 60335-1 and IEC 60335-2-76). As a result, most electric fence systems are either overrated or not optimally designed, resulting in systems that are inefficient [1-4].

In addition, the increasing pulse energy levels produced by fence energizers, pose a danger of delivering a lethal shock to a human or an animal, which may come into contact with the fence. Although reports on lethal accidents are rare, there have been incidents that have resulted in loss of human life involving electric fence installations. As a consequence, the maximum output energy that can be generated by an energizer is limited to 5 J by international safety standards enforced on electric fencing equipment to assure safety of humans and animals. The low energy limit imposed by the safety standards has driven the development of new technologies to overcome the deficiencies found in the existing technology and also to increase the effective range of electric fence systems. The demand for an optimized fence system has led to the development of “intelligent electric fence systems”, which are autonomous and capable of sensing changes in the operating environment to adopt themselves to improve both efficiency and performance [1-12].

Furthermore, electric fences are intended to be used as a vital component in cutting-edge farm management systems. In such systems, the fence structure can be used as the communication channel, otherwise only possible through expensive long range radio frequency (RF) transmissions due to the limited coverage area of low cost, common RF solutions, such as ZigBee, WiFi or Bluetooth. The communication is realized in the form of modulated high
voltage pulse signals transmitted along the fence, through which all the components in the farm management system that are scattered across the farm land could communicate with each other and the central management system. This approach can reduce the cost of intelligent farm management systems to an affordable value, enabling wide spread use of such technology [1-5, 8].

Above mentioned technological advances in electric fence systems can only be achieved through the development of mathematical models to accurately characterize the behaviour of typical fence systems. Such mathematical models would serve as invaluable tools at the design stage of “intelligent electric fence systems”, and for electric fence based communication channels used for farm management systems. Furthermore, the existing electric fence energizer technologies are unable to meet the requirements imposed by these proposed technological advances, and as a consequence, manufacturers are continuously looking for new technologies to develop high performance energizers, which are capable of meeting these requirements. The main objectives of this thesis are therefore to,

- Analyze and develop technologies that can be used to enhance the performance of electric fence energizers
- Implement an electric fence energizer that can be used in an intelligent electric fence system and verify performance
- Propose a comprehensive mathematical model to characterize the propagation of signals along practical electric fence structures
- Experimentally verify the accuracy of the proposed mathematical model
1.2 Evolution of Electric Fence Technology

Both complexity and size of electric fence structures have grown immensely since the introduction of electric fence technology. A modern electric fence, shown in Fig. 1.2, takes the form of a complex network of meshed branches that extend over several hundred kilometres. As a consequence of the high demands set forth by these widely spread out fence structures, the fence energizer technology too has evolved considerably.

![Fig. 1.2 A typical electric fence layout](image)

The very first generation of energizers, introduced in the early 1930s, produced a continuous low power, high voltage signal to energize the fence. This early design utilized a high-impedance transformer to step-up the mains input, and produced a current limited output signal with an amplitude of several kilovolts. The limiting current value imposed on these designs was set to lower than the “let go” current limit, which fell in the range of 7-24 mA
RMS. However, the leakage current passing from the fence to the return path in wet conditions or at times of high vegetation growth over the fence was significantly higher than the low current limit that was enforced due to safety concerns. As a result, under the above mentioned conditions, the fence voltage was reduced to a point that it was too low to break though the animal’s high resistance skin barrier to stimulate an electric shock [1-4].

In an attempt to design a fence energizer that can perform effectively under the harshest farm conditions, designers developed a new breed of energizers that generated repetitive high voltage pulses. Since the duration of the output pulse was very short, a large pulse current could be applied to the fence while maintaining the RMS (effective) value of the current below safety limits. A special power converter, termed high voltage pulsed power supply (HVPPS), was utilized by these energizers to generate the high power pulses. The HVPPS stored energy into a reactive element, such as a capacitor or an inductor, and then rapidly discharged the stored energy into the fence wires as a short duration, high voltage and high current pulse. This new generation of energizers, termed pulsed output type energizers, was first introduced to the consumer in the late 1930s. At this stage semiconductors were not available, and therefore these energizers were implemented with mechanical switches and timing devices. Fig. 1.3 illustrates a few examples of such energizers, and a schematic diagram of a typical energizer constructed with a mechanical switch and inductive energy storage is shown in Fig. 1.4. The gradual degradation of the mechanical parts, especially the high voltage switch, was the major demise of this type of energizers. The sparks formed between the contacts of the mechanical switches gradually degraded the switch contacts causing irregular pulse voltages at the output [1-4, 6-19], thereby leading to unreliable and unsafe operation.
Fig. 1.3 Pulsed output energizers implemented with mechanical devices
As the technology evolved, the mechanical switches were replaced with vacuum/discharge type switches in an attempt to increase the reliability of pulsed output type energizers. In addition, energizers that utilized capacitive energy storage elements were preferred over the ones that utilized inductive energy storage elements due to higher efficiency and controllability of output pulse shape. A circuit diagram of such a system is shown in Fig. 1.5,
where a vacuum tube is used to generate a high voltage output pulse by discharging the storage capacitor through the load [1-4].

The use of vacuum/discharge type switches was short-lived as they were soon replaced by semiconductor switches such as semiconductor controlled rectifiers (SCR) and gate turn-off thyristors (GTO). In comparison to vacuum/discharge type switches, the semiconductor switches offered a longer operational life, higher reliability and higher efficiency. A pulse transformer based HVPPS, as shown in Fig. 1.6, is used by these converters. The step-up pulse transformer (T$_1$) boosts the output voltage thereby reducing the voltage stress on the switch (SCR$_1$) and capacitor (C$_1$), allowing for the use of inexpensive devices. Although the voltage stresses on the components are reduced, the transformer causes large currents to flow in the primary, causing high losses. In order to reduce the losses associated with the transformer and cater for the high winding current, a large transformer is required. In some commercial designs the transformer itself can weigh about 5-8 kg. Fig. 1.7 illustrates commercially available 75 J and 36 J energizer products. A summary of typical output parameters of a modern electric fence energizer is given Table 1.1 [1-3].

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![Fig. 1.6 A modern electric fence energizers](image)

---

Chapter 1  Introduction
Fig. 1.7 Commercial energizer products (a) 75 J unit (b) 36 J unit
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output Energy</td>
<td>0.05-50 J</td>
</tr>
<tr>
<td>Peak Output Voltage</td>
<td>5-15 kV</td>
</tr>
<tr>
<td>Peak Output Power</td>
<td>1-3000 kW</td>
</tr>
<tr>
<td>Peak Output Current</td>
<td>1-200 A</td>
</tr>
<tr>
<td>Pulse Repetition Rate</td>
<td>0.67 Hz</td>
</tr>
<tr>
<td>Pulse Width</td>
<td>10-500 µs</td>
</tr>
</tbody>
</table>

Table 1.1  Typical output parameters of a modern electric fence energizer

1.3  Scope of the Thesis

The thesis presents a comprehensive study on electric fencing systems with the intention of improving their overall performance. Initially, theoretical and experimental studies of HVPPS and HV charger technologies that can be used to enhance the performance of existing fence energizers are presented. The technologies investigated include a new energizer topology, a novel HVPPS and a Buck-Boost control technique for a push-pull parallel-resonant converter (PPRC). Finally, a detailed mathematical analysis on propagation characteristics of fence structures is presented and verified through experiments and simulations. The above study of this thesis has resulted in fourteen publications; four in IEEE and IET Transactions and ten in IEEE conference proceedings. The details of this study are presented in the chapters of this thesis as outlined below.

Chapter 2 – Pulsed transformer type electric fence energizers have a number of deficiencies, which make them unsuitable in intelligent electric fence systems. Therefore, in search of a new energizer technology, an investigation into properties of existing HVPPS is conducted.
This chapter presents an overview of four pulsed power technologies that are suitable for an electric fence energizer. These technologies are the direct discharge type, pulse transformer type, Marx generator type and vector inversion type. The advantages and disadvantages of each of these technologies in relation to their use in electric fence energizers are discussed with the aid of mathematical analysis and computer simulations. The chapter concludes with a comparison of properties between the investigated topologies, and recommendations for designing energizers with improved performance.

Chapter 3 – Vector inversion generators are the preferred type of HVPPS in a number of high power applications, as this type of converters only requires a single power switch to generate an output. However, a comprehensive theoretical analysis on a multi-stage transformer coupled vector inversion generator has not been reported in the literature. This chapter therefore presents a novel generalised circuit model for a multi-stage converter, through which explicit analytical solutions for the component stresses and output characteristics can be derived. In addition, a simplified circuit model that facilitates the analysis of lightly loaded converter is also discussed. A comparison between the theoretical solutions and SPICE simulations are presented to prove the validity of the proposed technique.

Chapter 4 – The use of very large electric fence installations are common in African countries and Australia. However, the existing energizers fail to meet the power requirements of these systems, and thus a number of energizers are used to power such large fence networks. As such, the design and implementation of a novel high power energizer that can deliver the power requirements of a large fence system is presented. The proposed design is based on the Marx generator concept and can generate output pulses with customizable wave shapes, facilitating the use of this technology in intelligent electric fence systems. In addition,
experimental results are presented to benchmark the performance of the prototype against existing electric fence energizers.

Chapter 5 – Although the Marx generator based implementation of a high power electric fence energizer has many advantages, the implementation of a low power energizer based on this technology proves to be uneconomical. As such, this chapter presents a novel high voltage pulse generation technique that facilitates a cost effective implementation of a low power electric fence energizer. In comparison to existing HVPPS, the proposed technology exhibits many advantages and thus will also have applications in other pulsed power fields. The operation of the converter is characterised through mathematical analysis and SPICE simulations to validate the proposed concept.

Chapter 6 – The hard-switched pulse width modulated (PWM) converters that are used to recharge the storage capacitor banks in conventional electric fence energizers exhibit high losses, switch stresses and electromagnetic interference (EMI). An analysis on soft-switching techniques that can be utilized to enhance the performance of these HV charger systems is presented in this chapter. Furthermore, a novel control technique that facilitates the operation of a push-pull parallel-resonant converter over a wide load range with output voltage control while maintaining zero-voltage switching (ZVS) is presented. The chapter presents a comprehensive mathematical model, through which the performance of the proposed PPRC topology in the Boost and Normal modes can be accurately predicted over the entire load range. In addition, effects of parasitic elements are modelled, and theoretical results are compared with simulations and experimental results of a 150 W prototype unit, used for a high voltage charging application, to verify the validity of the proposed control technique, and analytical solutions.
Chapter 7 - This chapter presents a detailed mathematical analysis on the operation of the proposed PPRC topology in the Buck mode. In contrast to both Normal and Boost modes of operation, theoretical analysis of Buck mode is complicated by the formation of two different resonant circuits in each switching cycle. As a result, the mathematical solutions presented are complex in nature and therefore a numerical approach is adopted to evaluate the solutions. Although the proposed model requires numerical evaluations of the mathematical expressions for circuit variables, the mathematical expressions themselves provide valuable information with regard to the expected behaviour of the converter, which would otherwise not be available through a complete numerical solution. Experimental and SPICE simulation results for a 150 W prototype converter are presented and compared with analytical solutions to verify the accuracy of the presented analysis.

Chapter 8 – The design of optimized fence structures, intelligent electric fence systems, and fence line communication channels require an insight into the propagation characteristics of electric fence structures. Therefore, to fulfil this requirement, a comprehensive semi-analytical model that facilitates the study of propagation characteristics of short HV pulses along typical multi-wire electric fence structures is presented. The technique presented utilizes the quasi-TEM approximation to model the propagation properties of the electric fence with frequency dependent distributed line parameters. The technique also models the discontinuities found along the fence line, caused by faults, loads, branches, etc. Analytical expressions for the line voltages and currents are derived in the frequency-domain and time-domain solutions are obtained through a MATLAB based numerical algorithm. The changes in line voltage and current under different fence conditions are investigated through experiments and PSCAD simulations. These results are compared with the theoretical solutions to verify the accuracy of the proposed analysis.
Chapter 9 – The generalized propagation model presented in Chapter 8 was derived in the Matrix-form and therefore complex matrix manipulations were required to obtain solutions, resulting in significantly long computation times. However, the majority of electric fences found all around the world are constructed with a single energized wire, and with multiple unenergized wires. Thus, a simplified scalar mathematical model that can be used to approximate the propagation properties of such fence structures is presented in this chapter. In comparison to the generalized model presented in Chapter 8, the proposed model results in a much simpler implementation and a faster computation time. In addition, theoretical results are compared with PSCAD simulations of various fence structures to validate the accuracy of the presented analysis.

Chapter 10 – This final chapter of the thesis summarizes the main findings of the study on electric fence systems.
1.4 References


2.1 Present State of Fence Energizer Technology

Over the past eight decades, electric fence energizers have evolved from low power units that were ineffective in harsh farm conditions, to high power, pulsed output type energizers that are usable in virtually any condition. These modern energizers utilize a pulse transformer based high power pulse generator to generate output pulses with a peak voltage of about 8 to 10 kV. The output pulses which are generated repetitively at a frequency lower than 0.7 Hz can contain as much as 60 J of energy. A generic energizer design that is implemented with an output pulse transformer is illustrated in Fig. 2.1. Each pulse cycle is initiated by charging a storage capacitor bank, $C_s$, to a voltage that is determined by the voltage feedback network which is normally in the range of 600 to 900 V. A typical charger consists of either a fly-back converter or a voltage quadrupler with adequate power throughput to charge the capacitors within a pulsing period. The energy stored in the capacitors is then discharged to the fence load which is coupled through a step-up pulse transformer $T_1$ by closing the switch, $SCR_1$. The passive filter network that follows the pulse-forming circuit determines the pulse width and the amplitude even though the primary purpose of the filter is to reduce radio frequency interference (RFI) generated by the output pulse [1-5].
The pulse transformer type energizer design is used extensively in modern fence energizers mainly due to its simplicity, cost effectiveness and robustness. As illustrated in Fig. 2.1, the output transformer isolates the high voltage stage of the design from the low voltage stage that includes all the control and switching circuitry. Thus the implementation of such a converter is simple, as it can be manufactured with standard industrial grade semiconductor devices [1-5].

Reliability and robustness are two of the most important features of electric fence energizers. The components in the energizers are regularly subjected to severe stresses in abnormal conditions that can occur in the electric fence line. These conditions include high voltage and current transients due to nearby lightning strikes, voltage transients propagating on nearby power transmission lines and faults on the electric fence lines. In these instances, the low-pass filter and the output transformer of the energizer protect the components in the energizer by attenuating the abnormal signals, and therefore improve the system reliability and robustness [1-5].

Even though the energizer designs based on the above topology have many useful properties, this topology has a few drawbacks. The step-up transformer causes the primary side current to increase considerably and in large energizers this primary current can be as high as several
kilo-Amperes. The high primary current increases the conduction losses, significantly reducing the system efficiency. Furthermore, the loss associated with the leakage inductance is greatly increased by the large currents in the transformer windings. Another drawback of having an output transformer with a large step-up ratio is the large reduction in the reflected load seen by the primary pulse circuit. For a heavily loaded fence, the reflected load is in the order of several ohms; and therefore the attenuation caused by the series resistance of primary side devices is significant. The attenuation of the output pulse caused by the conduction resistance could be as high as 10 dB in extreme cases. The fixed nature of the output pulse parameters which are mainly determined by the output filter, is another drawback of this topology. Hence the output of the energizer cannot be controlled to vary with the fence conditions to improve the performance and reduce the losses. In addition, the filter generates high circulating currents even in lightly loaded conditions, which further reduces the efficiency of the converter [1-4].

These deficiencies found in the aforementioned topology have hindered the development of high energy units; currently the largest unit is capable only of delivering less than 60 J at optimum load. At higher energy levels, the heat generated by the conduction and core losses of the converter is unacceptably high and degrade the performance of the components considerably. To reduce the losses, larger magnetic elements and semiconductors with larger die areas have to be incorporated into the design. The resulting products will then have a higher manufacturing cost and a larger physical size, making them uneconomical [1-6].

To overcome these associated problems, manufacturers are looking into new technologies that could be adapted to design pulsed output type energizers with improved performance. This chapter continues with the investigation into existing pulsed power converters, in search of suitable technologies for pulsed output type energizers. An overview of pulsed power technology is presented, followed by an analysis of suitable converters. The chapter
concludes with a comparison of the investigated topologies, and recommendations for energizer design with improved performance.

2.2 HV Pulsed Power Systems

A system that is capable of generating short duration, high voltage and high power pulses is termed a high voltage pulsed power supply (HVPPS). Such a system operates by storing energy in an energy compression element and discharging the stored energy through the target device as a high power pulse that lasts only for a short duration. A variety of HVPPS technologies can be found in the literature, utilising different switching and energy compression schemes to generate high power pulses. As indicated in Fig. 2.2, HVPPS technologies can be divided into three broad categories on the basis of their utilization of energy compression elements. Pulsed power supplies that utilize reactive elements, such as inductors and capacitors for energy compression, are used mainly in applications that require an output pulse, which lasts for hundreds of microseconds. In contrast, converters that make use of pulse forming transmission lines are suitable only for the purpose of generating output pulses shorter than a few microseconds. Converters that generate HV pulses by mechanical means constitute the third category. These are termed explosive-driven pulsed power supplies, where a subtle change in magnetic flux realized through mechanical displacement of components is utilized to generate a high voltage and high power pulse. Table 2.1 summarizes typical ranges of output parameters possible with HVPPS [1-3, 6-27].
Applications of pulsed power systems are found in a broad spectrum of disciplines. They include industrial applications such as food processing and electric fencing systems; medical instruments such as X-ray machines and defibrillators; military appliances such as radar systems and active denial technology (ADT) and also scientific applications such as particle accelerators and ion implantation systems [1-3, 6-12].

An electric fence energizer requires a pulsed power supply that can generate high voltage pulses with a pulse duration between 10 and 300 µs at a repetition rate of less than 0.7 Hz. Amongst the above mentioned power supply categories only the reactive element type converters have properties that suit these requirements. Moreover, capacitive energy
compression is preferred over inductive energy compression due to higher efficiency and controllability of output pulse shape [1-4].

Fig. 2.3 illustrates a conceptual circuit diagram of a pulsed power supply design that utilizes capacitors as the energy compression element. The typical waveforms of the input power, output power and load voltage of this converter are illustrated in Fig. 2.4. Essentially, the converter helps to establish a very low impedance source that is capable of supplying an instantaneous pulse of extremely high power. The power level transformation is achieved by storing energy in a very low impedance pulse grade capacitor bank and then discharging the capacitor bank rapidly through the target load in the form of a high voltage and high current pulse. As indicated in Fig. 2.3, the low-power HV charger charges the storage capacitor bank up to a predetermined voltage during the charging time interval (T₁) which is several magnitudes larger than the pulsing time interval (T₂). During the pulsing time period, the pulse-forming switch is closed and the energy that was stored in the capacitor is rapidly discharged into the load [6].

![Fig. 2.3 A conceptual circuit diagram of a direct discharge type converter](image_url)

In the literature, four pulsed power supply technologies can be found that utilize capacitors for energy compression. The technologies vary by the way they are designed to generate the high voltage pulses. Some technologies are centred entirely on semiconductor based solutions,
while others utilize magnetic elements for the purpose of generating high voltage pulses. These technologies, mentioned below, are discussed in detail in the following sections to investigate their suitability for electric fencing energisers [1-4, 6-27].

1. Direct discharge type pulse generator
2. Pulse transformer type pulse generator
3. Marx generator type pulse generator
4. Vector inversion type pulse generator

Fig. 2.4 Typical characteristics of an HVPPS (a) Input power (b) Output power (c) Output voltage (d) Output current
2.2.1 Direct Discharge Type Pulse Generator

The direct discharge type converter topology is a straightforward implementation of the HVPPS concept discussed in the previous section. Although the circuit topology appears to be simple, a reliable implementation of the converter is difficult to achieve. An implementation requires pulse-generating components, such as switches and diodes, to be able to withstand the high voltages and currents that are being generated by the converter. Thus these components require very high blocking voltages, fast switching capabilities and high surge current ratings. Unfortunately, commercially available common semiconductor devices cannot withstand such high voltages. Therefore these types of converters are designed with components that are connected in series to achieve the required voltage rating [1-4, 6, 8-12, 16-18].

A block diagram of a direct discharge type converter is illustrated in Fig. 2.5. The design utilizes an output RFI filter to reduce the conducted RFI caused by the output pulses. The charger that is used in this design must charge the storage capacitance (C) to a very high voltage (V_C), which is approximately equal to the required peak output voltage. Losses and ringing caused by the RFI filter would result in a deviation of the peak output voltage from the charged voltage of the storage capacitance. Under these conditions, the maximum amount of energy (E_{max}) that could be delivered to a load is given by (2.1). The maximum power throughput of the charger (P_{SMPS}) that is required to charge the capacitors in a pulse generator that has a pulse repetition rate of f_{pr}, pulse duration of t_p and an efficiency of \eta is given by (2.2). The maximum instantaneous output power (P_{out}) that can be delivered to a resistive load R_L is given by (2.3), where the insertion loss of the filter is represented by \alpha. A comparison between (2.2) and (2.3) illustrates the energy compression phenomenon since the output power relative to the input power can be very high for very short t_p and longer pulse repetition rates [1-3].
An implementation of a high voltage switch that is suitable for a direct discharge type converter is illustrated in Fig. 2.6. As discussed previously, the cascaded IGBT arrangement shown in Fig. 2.1 should be able to withstand a voltage of \( V_C \) and the current consumed by the load and the filter. Due to slight variations in the semiconductor fabrication technology, IGBTs in the HV switch will have slightly different switching properties. These differences will cause unequal sharing of voltage between them despite all the IGBTs are turned on at the exact time. Therefore, a successful implementation of the HV switch requires complex voltage balancing and driver circuitry to control the IGBTs as illustrated in Fig. 2.6.

Moreover, the high voltage stresses on the filter and pulse-forming components increase the system cost and the design complexity. The higher voltage requirement of the filter components also increases their physical size, resulting in a bulky system. Safety issues and the formation of corona due to high voltage across the storage capacitors are other drawbacks
of this system [1-3]. However, some useful properties can be found in this topology. Since it
does not require an output transformer, this topology can be used to generate pulses with a
wide range of duty cycles. The pulse duration can be controlled easily through the HV switch
according to the load requirements. In addition, the pulse generator can be designed to have a
very high efficiency as the transient current flowing through the components is smaller in
comparison to other topologies.

Fig. 2.6 An implementation of an HV switch with IGBTs [16-18]
2.2.2 *Pulse Transformer Type Pulse Generator*

The pulse transformer type HVPPS discussed in Section 2.1 is the most widely used pulse power topology in modern electric fence energizers. This topology has become popular among electric fence manufacturers due mainly to the simplicity of implementing an energizer based on this method. The output pulse transformer reduces the voltage stress on the components, enabling a simple, yet robust design that could be manufactured with common industrial grade devices. Hence such a system can be built at a very competitive cost [1-6].

The pulse transformer type generator operates similar to the direct discharge type generator. The two topologies differ from each other by the use of a step-up output transformer in the pulse transformer type topology. A block diagram of an energizer design based on the pulse transformer topology is illustrated in Fig. 2.7, and a generalised circuit diagram of the pulse-forming circuit is shown in Fig. 2.8 [1-6].

![Fig. 2.7 Pulse transformer type HVPPS](image)

![Fig. 2.8 Pulse generation stage of a traditional fence energizer](image)
The circuit shown in Fig. 2.8 can be analyzed to study the component stresses involved during the operation of this topology. The circuit is first simplified by referring all the parameters to the primary side and then introducing the T-equivalent transformation of the transformer. Based on the simplified equivalent circuit indicated in Fig. 2.9, it could be seen that the filter inductor \( L_f \), filter capacitor \( C_f \) and the leakage inductance \( L_{lk} = L_{lk1} + L_{lk2} \) of the transformer form a 3rd order low pass filter. The magnetizing inductance \( L_m \) is assumed to be very large and the ESR of the inductors and capacitors are assumed to be very small, so their effect on the filter response is negligible. The input-output transfer function of the filter for a fence load of \( R_f \) is given by (2.4). \( R_{fence} \) is the value of \( R_f \) referred to the primary of the pulse transformer. Fig. 2.10 illustrates the transfer function \( T(s) \) corresponding to a medium power energizer unit. The circuit exhibits resonance at about 1 kHz, resulting in a raised-cosine shaped output pulse. The pulse width of the output is governed entirely by the filter and in this case would be about 100 \( \mu s \) [1-3].

\[
T(s) = \frac{R_{fence}/C_f L_f}{s^3 + s^2 R_{fence}/L_{lk} + s \left( \frac{1}{C_f L_f} + \frac{1}{C_f L_{lk1}} \right) + \frac{R_{fence}/C_f L_{lk1}}{C_f L_{lk2} L_{lk1}}} 
\]

(2.4)

The maximum switch current occurs when a short circuit is applied at the output terminals. The peak value of this current can be approximated by (2.5). Thus, for a typical mid-range
electric fence energizer the current flowing through the switch and the inductor \((L_f)\) can be as high as 2000 A. This high current causes considerable (conduction and switching) losses in the components, thereby greatly reducing system efficiency. Moreover, the core losses and copper losses of the transformer are significantly elevated by the high currents. The diode that is fitted across the filter capacitor to prevent the negative voltage swing is the source of losses at light loads. Under this situation, the magnetizing inductance of the transformer and the filter cause a circulating current, leading to heat dissipation in the diode and the filter components [1-3].

\[
I_{\text{m}} = V_c \frac{C_v}{\sqrt{L_t + L_{ik}}} \tag{2.5}
\]

As discussed previously in Section 2.1, under heavy loading conditions the losses in the inductor \((L_f)\) and the transformer would have a significant impact on the output voltage, due to the large transformer turns-ratio. For example, a 50 Ω fence load is seen as 0.5 Ω on the
primary side of a pulse transformer with a turns-ratio of 1:10. Thus, if the combined winding resistance and filter inductor resistance is also 0.5 Ω, the output voltage is halved. In addition, under these conditions the leakage inductance of the output transformer leads to further reduction in output voltage. Due to the reasons discussed herein, the typical efficiency achieved by this topology is less than 75% [1-3].

2.2.3 Marx Generator Type Pulse Generator

In 1924, E. Marx proposed a pulsed power generator topology that utilizes a switching matrix to amplify the voltage to generate HV pulses from a low voltage supply without the aid of magnetic elements. The Marx topology can be implemented through readily available semiconductor devices since the voltage stress on the components can be reduced by increasing the number of stages. The technique facilitates the control of the pulse width and the amplitude of the output pulse after a pulse has been initiated. In comparison to the direct discharge type, the Marx generator does not require HV storage capacity, thus making the design safe and reliable. Hence, this topology has significant advantages over the other HVPPS technologies [1-3, 6, 19-23].

The Marx concept is realized by implementing the switching matrix, shown in Fig. 2.11, with active and passive switches such that the storage capacitors are charged in a parallel configuration and discharged in a series configuration. In the configuration illustrated, the storage capacitors $C_1$-$C_N$ are charged by closing switches $S_1$ while switches $S_2$ are left open. When the voltage across the capacitors reaches the desired value, charging is terminated by opening the switches $S_1$. To form a high voltage pulse at the output, switches $S_2$ are closed while switches $S_1$ are left open. During this time, the capacitors are arranged in a series configuration causing the peak output voltage to be $N$-times greater than the initially charged...
voltage of the individual capacitors. Thus an HV pulse can be generated at the output without the expense of high voltage stresses on the components [1-3, 6, 19-23].

The Marx generator concept discussed above can be implemented in a variety of ways, depending on the design requirements. Fig. 2.12 illustrates a schematic diagram of the pulse forming section of a prototype electric fence energizer implementation that is based on the Marx concept. The prototype has been implemented as a part of this research with the collaboration of Tru-Test research engineers. The design consists of cells that are made of a storage capacitor and the necessary switching devices. Each of these cells is charged to about 950 V through the isolation elements IE₁⁻IE₁₈. These elements isolate the individual cells from each other and from the charger during the pulsing time interval. During the pulsing interval the switches S₁⁻S₉ are closed, thereby connecting the storage capacitors in a series arrangement. A detailed description of this implementation together with experimental results is presented in Chapter 4 [1-3, 6, 19-23].

The major shortcoming of this topology is the high circuit complexity associated with implementation. Even though the topology can be implemented reliably through readily available devices, it requires a large number of discrete components. Similar to the direct discharge type, the Marx type also requires an output filter capable of handling high voltages. Thus implementation of an energizer based on the Marx concept is relatively expensive [1-3].
Fig. 2.12 An implementation of a 9-stage Marx type fence energizer

### 2.2.4 Vector Inversion Type Pulse Generator

The concept of a vector inversion generator is somewhat similar to the Marx concept. A vector inversion generator utilizes closely coupled transformers instead of switches to connect the storage capacitors in a series configuration to generate a high voltage pulse at the output. Fig. 2.13 illustrates a schematic diagram of an N-stage vector inversion generator. The storage capacitors are charged in a parallel configuration through the transformer magnetizing inductance. As indicated in the diagram, the charge stored in odd numbered capacitors is in opposite polarity to the charge stored in even-numbered capacitors. Thus the net voltage appearing across the load during the charging phase is zero. When the pulse-forming switch $S_2$ is turned-on, it causes the voltage across the odd numbered capacitors to invert, as a result of the resonance between the storage capacitance and the leakage inductance of the transformers. The voltage inversion process produces a high voltage pulse at the output of the generator.
which could be as high as 2N-times the initially charged voltage of a capacitor. In practice, the voltage amplification of the converter is less than 2N due to the losses and higher order resonances. However, the performance of this topology can be improved through proper design strategies, such as resonant frequency compensation. A detailed analysis of this converter together with simulation results is presented in Chapter 3 [1-3, 24-27].

![Fig. 2.13 A vector inversion type HVPPS](image)

Even though the vector inversion generator has a few attractive properties in comparison to the other methods, this technique also poses a few drawbacks. Since the unit does not employ a switch that directly controls the output, this technique is unsafe for use in electric fence type energizers. The output pulse shape and duration of this HVPPS are determined entirely by the load and the leakage inductances of the transformers. Moreover, the switch current during the
pulsing stage could be in excess of several kilo-Amperes because of the very high inversion current. This current also passes through the transformers contributing to high losses. Another drawback of the topology is the bulkiness of the large transformers that are required to handle the high inversion current [1-3, 24-27].

### 2.2.5 A Summary on Properties of HVPPS

Based on the preceding discussion, a comparison of the properties between these four HVPPS technologies is presented in Table 2.2.

<table>
<thead>
<tr>
<th>Property</th>
<th>Direct discharge</th>
<th>Pulse transformer</th>
<th>Marx generator</th>
<th>Vector inversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1  Output pulse width</td>
<td>Controllable</td>
<td>Filter dependent</td>
<td>Controllable</td>
<td>Load dependent</td>
</tr>
<tr>
<td>2  Output voltage</td>
<td>Restricted control</td>
<td>Restricted control</td>
<td>Controllable</td>
<td>Load dependent</td>
</tr>
<tr>
<td>3  Efficiency</td>
<td>High</td>
<td>Low</td>
<td>High</td>
<td>Lower</td>
</tr>
<tr>
<td>4  Reliability</td>
<td>Low</td>
<td>High</td>
<td>Medium</td>
<td>High</td>
</tr>
<tr>
<td>5  Design complexity</td>
<td>Medium</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td>6  Safety</td>
<td>Low</td>
<td>High</td>
<td>Medium</td>
<td>Lower</td>
</tr>
<tr>
<td>7  Voltage stress on components</td>
<td>High</td>
<td>Low</td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td>8  Current stress on components</td>
<td>Low</td>
<td>High</td>
<td>Low</td>
<td>High</td>
</tr>
<tr>
<td>9  Output filter design</td>
<td>Complex</td>
<td>Simple</td>
<td>Complex</td>
<td>Simple</td>
</tr>
<tr>
<td>10 Cost</td>
<td>Medium</td>
<td>Low</td>
<td>High</td>
<td>Medium</td>
</tr>
<tr>
<td>11 Size</td>
<td>Compact</td>
<td>Bulky</td>
<td>Compact</td>
<td>Bulky</td>
</tr>
</tbody>
</table>

Table 2.2 A comparison between HVPPS technologies
2.3 Conclusions

The performance of four pulsed power supply topologies has been reviewed in search of new technologies suitable to implement an efficient high power electric fence energizer that can produce customizable output pulses. The pulse transformer type HVPPS used in modern fence energizers is the most economical, and the simplest to implement. Due to high losses associated with the pulse transformer technique, this topology is recommended to be used only in small to medium size electric fence energizers. Alternatively, the Marx topology can be utilized to implement compact and efficient high power fence energizers. In addition, the ability of the Marx generator to produce output pulses with customizable wave shapes allows this technology to be used in intelligent electric fence systems. Although the direct discharge type has similar properties to the Marx type, the technique raises safety concerns, especially at elevated energy levels. The direct discharge type could be an attractive solution for low energy, solar-powered units where efficiency is important. The vector inversion type is not suitable for use in fence energizers as the output of the converter cannot be controlled.
2.4 References


[22] New Zealand patent application 535719 and foreign equivalents.


Chapter 3
Vector Inversion Generators

3.1 Principle of Operation

The vector inversion technique is one of the four high voltage pulsed power generation schemes that were discussed in the preceding chapter. Similar to other concepts, the vector inversion type too utilizes capacitive energy compression to generate high voltage and high power pulses. However, in comparison to other HVPPS technologies, the vector inversion technique requires only a single power switch to generate an output pulse. Thus vector inversion generators are the preferred type of HVPPS in a number of high power applications, even though these generators tend to be bulky in comparison to the counterparts [1-8].

Although this technique is extensively used in a number of high power applications, a comprehensive analytical model for a multi-stage transformer coupled vector inversion converter has not yet been reported in the literature. Hence a model that could predict the output characteristics of a vector inversion generator is an invaluable tool for engineers designing HVPPS. This could help designers to avoid time consuming computer simulations, and also to optimize the design parameters to improve efficiency of the overall system. For example such an analytical model can be used for resonant frequency compensation of a converter to achieve efficiencies over 90% [1-8]. As such, this chapter presents a simplified
circuit model for a multi-stage vector inversion generator, from which explicit analytical solutions for the component stresses and output characteristics can be derived.

A schematic diagram of a 2N-stage transformer coupled vector inversion generator is shown in Fig. 3.1(a), where N is the number of tightly coupled transformers in the converter. When switch \( S_1 \) is closed while \( S_2 \) is open, the charging system charges the energy storage capacitors \( C_1-C_{2N} \) to the required voltage through the magnetizing inductance of the transformers. During the charging phase, the magnetizing inductance of the transformers appears as a short circuit when a steady state is reached, resulting in the charging arrangement shown in Fig. 3.1(b).
This arrangement ensures that the odd numbered capacitors are charged with opposite polarity to the even numbered capacitors as indicated in Fig. 3.1(b). Hence the net voltage appearing across the output load is zero [1-8].

A pulse can now be generated at the output by closing the inversion switch $S_2$, while $S_1$ is left open. When $S_2$ is closed, the even numbered capacitors resonate with the leakage inductances of the transformers, inverting their voltages. In comparison to the fast voltage swing of even numbered capacitors, the voltages across odd numbered capacitors change slowly due to large magnetizing inductance that is in parallel with them. The result is an amplified ringing output voltage, which would ideally be equal to $2N$ times the initially charged voltage of a capacitor ($V_c$). Practically the peak output voltage is less than $2N V_c$ due to losses and higher order resonances. The ringing is mainly determined by the load impedance and could not be controlled without extra switches. The efficiency of the converter can be improved by increasing the transformer coupling and using resonant frequency compensation [1-8].

### 3.2 A Simplified Model

A direct mathematical analysis of the converter illustrated in Fig. 3.1 is complicated, and a closed form numerical solution cannot be derived for a converter that has more than two stages. Thus a simplified equivalent circuit is derived to facilitate a numerical solution for the output characteristics of a 2N-stage vector inversion generator. For simplicity, the solution assumes that all the transformers, which have a turns ratio of 1:1, are identical. However, the solution could easily be modified to account for variations in the turns ratio among the stages.

In order to derive a simplified model for the converter illustrated in Fig. 3.1, first the leakage inductances of the transformers are transformed into the secondary side. The resulting circuit
is illustrated in Fig. 3.2. The presence of transformers in Fig. 3.2 makes it rather difficult to obtain a simplified model for the converter. However, the transformers can be eliminated to derive a simplified model as described below.

As indicated in Fig. 3.2, the output of the converter is a function of the voltage variations across the individual storage capacitors. Thus the voltage variations across the capacitors are analyzed by simplifying the rest of the circuit to derive a solution. Consider the moment immediately after the inversion switch $S_2$ is closed. The nodes $V_1$ and $V_2$ are shorted together and $C_1$ is connected across the primary of transformer $T_1$. The transformer will induce a secondary voltage $V_{C1}$ across $V_2-V_4$, which is the voltage across capacitor $C_1$. Since $V_1$ and $V_2$...
are held at the earth potential by the switch, voltage that appears at \( V_3 \) and \( V_4 \) relative to the earth is \( V_{C1} \), even though \( V_{C1} \) might change as \( C_1 \) discharges through both the load and magnetizing inductance of the transformer. Hence the components above nodes \( V_3-V_4 \) will see two identical voltage sources that are in opposite polarity, cancelling out each other during the computation of \( C_2 \) voltage. This implies that the voltage variation across \( C_2 \) can be analyzed independent of the voltage variation across \( C_1 \).

Due to the large magnetizing inductance, the voltage variations across \( C_1 \) and the rest of the odd numbered capacitors would be small. Thus, for simplicity, it is assumed that the voltage across odd numbered capacitors remains constant, and only the voltages across the even numbered capacitors are analyzed. If required, the voltage variations across these capacitors can easily be incorporated into the model to derive a more accurate model.

As discussed above, \( T_1 \) and \( C_1 \) can be replaced by introducing a short circuit across nodes \( V_3-V_4 \) since they are held at the same potential and the resultant circuit is shown in Fig. 3.3. Similarly, the subsequent stages can be modified to derive a simplified model illustrated in Fig. 3.4, which represents the voltage variations across the even numbered capacitors. For example, consider \( T_2 \) and \( C_3 \) of the system in Fig. 3.3. As the transformer maintains the same voltage across both the primary and secondary,

\[
(V_7 - V_5) = (V_8 - V_6) \quad \text{(3.1)}
\]

The voltage at \( V_7 \) and \( V_8 \), relative to \( V_5 \) can be given by (3.2) and (3.3) respectively, where \( V_{C3} \) is the voltage across \( C_3 \). By combining (3.1) to (3.3), \( V_{8,5} \) is simplified to obtain (3.4).

\[
V_{7,5} = V_7 - V_5 = (V_6 - V_5) + V_{C3} \quad \text{(3.2)}
\]

\[
V_{8,5} = (V_6 - V_5) + (V_8 - V_6) \quad \text{(3.3)}
\]

\[
V_{8,5} = 2(V_6 - V_5) + V_{C3} \quad \text{(3.4)}
\]
Equations (3.2) and (3.4) express that the voltage across $V_8-V_7$ is equal to the voltage across $V_6-V_5$ during the pulsing period. Thus it could be treated as if the nodes $V_7-V_5$, and $V_8-V_6$ are separated from each other by two dependent, 1:1 voltage sources, generated by the transformer $T_2$. Since these two voltage sources are in opposite polarity to each other, they cancel out each other during the transient analysis of voltage across $C_4$. Therefore, $T_2$ and $C_3$ can be removed from the circuit by connecting nodes $V_7-V_5$ and $V_8-V_6$ together as shown by Fig. 3.4, to model the transient behaviour of the even numbered capacitors. This approach simplifies the complicated vector inversion topology, shown in Fig. 3.2, to a transient time equivalent model given in Fig. 3.4, facilitating a mathematical solution.

![A partially simplified equivalent circuit for a vector inversion generator](image-url)
3.3 The Derivation of Solutions

The validity of the simplified model presented in the preceding section is verified by analyzing a 4-stage vector inversion generator shown in Fig. 3.5. It is assumed that the converter is lightly loaded and the voltages across $C_1$ and $C_3$ are approximately constant at $V_C$ over the pulsing period. The circuit parameters are presented in Table 3.1.

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capacitance per stage</td>
<td>1</td>
<td>µF</td>
</tr>
<tr>
<td>Transformer magnetizing inductance</td>
<td>10</td>
<td>mH</td>
</tr>
<tr>
<td>Transformer coupling coefficient</td>
<td>0.999</td>
<td></td>
</tr>
<tr>
<td>$V_C$</td>
<td>1000</td>
<td>V</td>
</tr>
<tr>
<td>$R_L$ (heavy)</td>
<td>100</td>
<td>Ω</td>
</tr>
<tr>
<td>$R_L$ (medium)</td>
<td>1000</td>
<td>Ω</td>
</tr>
<tr>
<td>$R_L$ (light)</td>
<td>5000</td>
<td>Ω</td>
</tr>
</tbody>
</table>

Table 3.1 A 4-stage vector inversion generator
The concept derived in the previous section is applied to obtain the simplified equivalent circuit shown in Fig. 3.6. For simplicity it is assumed that the leakage inductances $L_{l1} = L_{l2} = L$. 

![Diagram of a 4-stage vector inversion generator](image)

**Fig. 3.5** A 4-stage vector inversion generator

![Diagram of a simplified equivalent circuit](image)

**Fig. 3.6** A simplified equivalent circuit
By considering the voltages across the capacitors and currents flowing through them as indicated in Fig. 3.6, (3.5)-(3.8) can be derived.

\[
v_{c2} = L \frac{di_3}{dt} \quad (3.5)
\]

\[
v_{c2} = -\frac{1}{C} \int i_1 dt \quad (3.6)
\]

\[
v_{c4} - v_{c2} = L \frac{di_2}{dt} \quad (3.7)
\]

\[
v_{c4} - v_{c2} = -\frac{1}{C} \int i_2 dt \quad (3.8)
\]

Substituting (3.5) in (3.7) and noting that \(i_3 = i_1 + i_2\),

\[
v_{c4} = L \frac{d(i_1 + 2i_2)}{dt} \quad (3.9)
\]

Manipulating (3.5) to (3.9) it can be shown that,

\[
2L \frac{d^2i_2}{dt^2} + \frac{i_2}{C} = -L \frac{d^2i_1}{dt^2} \quad (3.10)
\]

\[
L \frac{d^2i_1}{dt^2} + \frac{i_1}{C} = -L \frac{d^2i_2}{dt^2} \quad (3.11)
\]

Solving (3.10) and (3.11) in time-domain can be a tedious task. Therefore they are solved in s-domain, where the s-domain equivalents of (3.10) and (3.11) are given by,

\[
2L \left\{ s^2I_{2(s)} - \frac{di_{2(0)}}{dt} - si_{2(0)} \right\} + \frac{I_{2(s)}}{C} = L \left\{ s^2I_{1(s)} - \frac{di_{1(0)}}{dt} - si_{1(0)} \right\} \quad (3.12)
\]

\[
L \left\{ s^2I_{1(s)} - \frac{di_{1(0)}}{dt} - si_{1(0)} \right\} + \frac{I_{1(s)}}{C} = L \left\{ s^2I_{2(s)} - \frac{di_{2(0)}}{dt} - si_{2(0)} \right\} \quad (3.13)
\]
Since the initial currents in the leakage inductances are 0, and the voltages across the capacitors are initially \( V_C \), the initial conditions for the system are,

\[
i_{1(0)} = i_{2(0)} = 0
\]  
(3.14)

\[
L \frac{di_{2(0)}}{dt} = v_{C4(0)} - v_{C2(0)} = 0
\]  
(3.15)

\[
L \frac{di_{1(0)}}{dt} = \frac{v_{C2(0)}}{L} - \frac{di_{2(0)}}{dt} = \frac{V_C}{L}
\]  
(3.16)

Applying the initial conditions, (3.12) and (3.13) can be simplified and solved to obtain,

\[
I_{1(s)} = \frac{V_C s^2}{s^4 + 3s^2/LC + 1/(LC)^2} + \frac{V_c/L^2C}{s^4 + 3s^2/LC + 1/(LC)^2}
\]  
(3.17)

\[
I_{2(s)} = \frac{V_c/L^2C}{s^4 + 3s^2/LC + 1/(LC)^2}
\]  
(3.18)

Equations (3.17) and (3.18) can now be converted back to time-domain to obtain the time-domain analytical solutions given by,

\[
i_{1(t)} = A_1 \sin(\omega_1 t) - A_2 \sin(\omega_2 t)
\]  
(3.19)

\[
i_{2(t)} = B_1 \sin(\omega_1 t) - B_2 \sin(\omega_2 t)
\]  
(3.20)

where,

\[
A_1 = \frac{V_C}{\sqrt{5Lo_1}}, \quad A_2 = \frac{V_C}{\sqrt{5Lo_2}}(1-k_1)
\]

\[
B_1 = \frac{V_C}{\sqrt{5Lo_1}}, \quad B_2 = \frac{V_C}{\sqrt{5Lo_2}}
\]

\[
\omega_1 = \frac{1}{\sqrt{LC}}, \quad \omega_2 = \frac{1}{\sqrt{LC}}
\]
Substituting (3.20) in (3.7) the voltage across \( C_4 \) can be obtained as,

\[
v_{c4} = \frac{V_c}{\sqrt{5}k_1} \cos(\omega_1 t) - \frac{V_c}{\sqrt{5}k_2} \cos(\omega_2 t) \tag{3.21}
\]

From (3.7) and (3.20),

\[
v_{c2} = v_{c4} - \frac{V_c}{\sqrt{5}} \cos(\omega_1 t) + \frac{V_c}{\sqrt{5}} \cos(\omega_2 t) \tag{3.22}
\]

As indicated in Fig. 3.5, the output load voltage is equal to the addition of voltages across the capacitors. Therefore the open circuit output voltage of the converter can be determined employing (3.21) and (3.22), and is given by,

\[
v_{out} = 2V_c + v_{c2} + v_{c4} \tag{3.23}
\]

By substituting (3.21) and (3.22) in (3.23), the output voltage of the converter is obtained as,

\[
v_{out} = 2V_c + \Phi \cos(\omega_1 t) + \Psi \cos(\omega_2 t) \tag{3.24}
\]

where,

\[
\Phi = \frac{1 + \sqrt{5}}{\sqrt{5}(3 - \sqrt{5})}, \quad \Psi = \frac{\sqrt{5} - 1}{\sqrt{5}(3 + \sqrt{5})}
\]

The switch current is equal to the current flowing through the leakage inductance \( L_{01} \), which can be calculated from (3.19) and (3.20) and is given by,

\[
i_{\text{switch}(t)} = D_1 \sin(\omega_1 t) - D_2 \sin(\omega_2 t) \tag{3.25}
\]

where,

\[
D_1 = \frac{V_c}{\sqrt{5L_{01}}}(2 - k_1), \quad D_2 = \frac{V_c}{\sqrt{5L_{02}}}(2 - k_2)
\]
3.4 Verification of the Simplified Model

The 4-stage vector inversion generator, shown in Fig. 3.5, is analysed using PSPICE and the results are compared with the results derived through the analytical solution presented in previous sections to verify the validity of the theoretical model. A comparison between the voltages $v_{\text{out}}$, $v_{C2}$, and $v_{C4}$ obtained from the simulations and the theoretical model are presented in Fig. 3.7. The voltage gain of this converter is equal to the number of stages when operated with medium-light loads. As deduced from the simplified equivalent circuit, the output of the converter exhibits vigorous ringing at multiple resonant frequencies. The efficiency and the voltage gain of the converter can be improved by matching all the resonant frequencies to the lowest resonant frequency of the system. Since the resonant frequencies are controlled by the leakage inductances, resonant frequency compensation can be achieved by using transformers that have different coupling factors. The required coupling factor for each transformer can be derived from (3.19) and (3.20). Also it can be seen from the results that the output of the converter is oscillatory and load dependent. The theoretical results are in very good agreement with the simulation when the converter is lightly loaded validating the analysis presented in Sections 3.2 and 3.3.

![Graph showing voltages across the output and capacitors](image)

Fig. 3.7 Voltages across the output and capacitors
Chapter 3  Vector Inversion Generators

The proposed model does not account for the load current that flows through the storage capacitors and the transformers. Therefore, when the load is increased, a deviation between the simulated and the theoretical results can be observed as seen in Fig. 3.7. This deviation of the simulated values from the theoretical values increases with the pulse time. For example, the simulation and theoretical values for a medium load, shown in Fig. 3.7 correspond well with each other for within the first 60 µs of the pulse time and a significant discrepancy between the results can be observed after about 150 µs.

The simulated and theoretical switch current waveforms are shown in Fig. 3.8. It is noted that the switch current stress, which is determined by the leakage inductance, is very high. This is one of the disadvantages of this topology as large switch currents increase conduction losses considerably. In addition, the switch should be capable of handling AC currents. The theoretical solution closely follows the simulation when the converter is lightly loaded, but deviates for medium loads.

![Fig. 3.8 Current through the inversion switch](image)

Fig. 3.8  Current through the inversion switch
3.5 A Generalized Model

The model introduced in Section 3.2, and validated in Section 3.5, can only be used to model lightly loaded vector inversion generators. As both the load and pulse period increase, the theoretical results obtained from the aforementioned method become less accurate. However, the equivalent circuit shown in Fig. 3.4, can be modified to incorporate the loading effect on the circuit functionality. This could be achieved by introducing current sources at each LC node as indicated in Fig. 3.9. The magnitude of the current, sourced by each source, is related to the load current and is given in (3.26), where $V_{\text{out}}$ is the voltage across the load $R_L$.

$$I_L = \frac{V_{\text{out}}}{R_L} \quad (3.26)$$

Moreover, when the load is increased, the voltage variations across odd numbered capacitors become prominent. Therefore, the voltage variations across the odd numbered capacitors have to be included in the theoretical model to derive an accurate solution. However, the simplified model presented in Section 3.2 is still valid, and thus the odd numbered capacitors can be analysed independently from the even numbered ones. The voltage variations across the odd numbered capacitors ($V_{\text{C(odd)}}$) are similar to each other if the magnetizing inductances are equal. Therefore the total voltage resulting from odd numbered capacitors can be derived by analysing the behaviour of $C_1$. The voltage across $C_1$ is characterized by the load current and the resonance formed between the capacitor and magnetizing inductance as illustrated by an equivalent circuit in Fig. 3.10. Therefore, the voltage across $C_1$ can be calculated easily from this equivalent circuit. The output voltage of a 2N-stage vector inversion generator can thus be calculated using (3.27). Equations (3.26) and (3.27) can be combined to obtain an expression for $I_L$ given in (3.28).
The general model explained above is capable of accurately characterizing the operation of a multi-stage vector inversion generator under the entire load range. Moreover, it provides an insight into the operation of the converter and therefore provides an efficient way to optimize such designs. However, the introduction of the voltage dependent current sources complicates
the derivation of a complete analytical solution for output characteristics of a vector inversion generator. Therefore a semi-analytical technique is adopted to obtain solutions. The technique models the vector inversion generator in the s-domain through analytical formulae by further simplifying the equivalent model through Thevenin and Norton transformations. The analytical solution is then numerically transformed into time-domain to obtain the final solution.

3.6 Verification of the Generalised Model

For simplicity, a 2-stage vector inversion generator, shown in Fig. 3.11, is analysed to prove the validity of the proposed theoretical solutions. A complete list of circuit parameters is given in the Table 3.1.

![Fig. 3.11 A 2-stage vector inversion generator](image)

The equivalent circuit shown in Fig. 3.10 can be used to calculate the output voltage resulting from the odd numbered capacitors. This circuit can be further simplified by converting it into the s-domain and combining the two current sources through a Norton transformation to
derive the configuration indicated in Fig. 3.12. The value of the current source in Fig. 3.12 can be derived from (3.29) and the voltage across the capacitor by (3.30).

\[
i_{c1} = CV_c - \frac{v_{c1} + v_{c2}}{R_L} \quad (3.29)
\]

\[
v_{c1} = \frac{sL_m}{s^2L_mC_1 + 1} + i_{c1} = \frac{sL_m}{s^2L_mC + 1} \left( CV_c - \frac{v_{c1} + v_{c2}}{R_L} \right) \quad (3.30)
\]

Fig. 3.12 Equivalent circuit in s-domain illustrating \( V_{c1} \)

Similarly, the voltage across even numbered capacitor can be evaluated using the s-domain equivalent of the model illustrated in Fig. 3.9, which is shown in Fig. 3.13. Thus the equivalent current source and the voltage across \( C_2 \) can be obtained from (3.31) and (3.32).

\[
i_{c2} = -CV_c - \frac{v_{c1} + v_{c2}}{R_L} \quad (3.31)
\]

\[
v_{c2} = \frac{sL_{ik}}{s^2L_{ik}C_2 + 1} + i_{c2} = \frac{-sL_{ik}}{s^2L_{ik}C + 1} \left( CV_c + \frac{v_{c1} + v_{c2}}{R_L} \right) \quad (3.32)
\]

Let,

\[
X_{1,s} = \frac{sL_m}{s^2L_mC + 1} \quad (3.33)
\]

\[
X_{2,s} = \frac{sL_{ik}}{s^2L_{ik}C + 1} \quad (3.34)
\]
Then (3.31) and (3.32) can be solved simultaneously to obtain expressions for $v_{C_1}$ and $v_{C_2}$ as given below.

$$v_{C_1} = \frac{X_{1,s}R_{1}CV_{C} + 2X_{1,s}X_{2,s}CV_{C}}{R_{L} + X_{1,s} + X_{2,s}}$$  \hspace{1cm} (3.35)$$

$$v_{C_2} = -\left[ \frac{X_{2,s}R_{1}CV_{C} + 2X_{1,s}X_{2,s}CV_{C}}{R_{L} + X_{1,s} + X_{2,s}} \right]$$  \hspace{1cm} (3.36)$$

The output voltage of the converter is equal to the summation of voltages across individual capacitors as given by (3.27). Thus, the circuit parameters summarized in Table 3.1 can be substituted in (3.35) and (3.36) to derive the transfer function of the converter in the s-domain. A numerical Laplace inversion algorithm is utilized to convert this s-domain solution into time-domain and the results are presented in Fig. 3.14 and Fig. 3.15. From Fig. 3.14 it could be observed that the voltage across the even numbered capacitors resonate at a high frequency in comparison to the voltage across the odd numbered capacitors. And at light loads the high frequency resonance remains for a long duration due to low attenuation presented by the higher load resistance. This is the cause for the vigorous oscillations observed at light loads in the output voltage, as shown in Fig. 3.15. When the load is increased, the voltage oscillations across all capacitors are attenuated at a fast rate and reaches zero within 150 $\mu$s, as all the energy stored in the capacitors is consumed by the load. Therefore, as the load is increased the output pulse duration is significantly shortened. In both
situations, the theoretical results closely follow the simulation results, thus verifying the generalized model presented above.

![Graph](image1.png)

**Fig. 3.14** Capacitor voltages for a 2-stage converter

![Graph](image2.png)

**Fig. 3.15** Load voltage for a 2-stage converter

Similarly, a 6-stage vector inversion generator is analyzed and the results are presented in Fig. 3.16 and Fig. 3.17. Presence of several resonant frequencies can be observed in the illustrations as a result of the LC ladder arrangement formed between the capacitors and the
leakage inductances. The differences in resonant frequencies together with the high discharge rate cause a phase difference between the voltages across individual capacitors, thereby reducing the overall voltage gain of the converter. As a result, the gain of the converter has been reduced considerably from the ideal gain of 6. From Fig. 3.17, the gain of the converter can be calculated as 3.9. The resonant frequencies of the stages can be matched to increase the voltage gain of the converter as explained previously.

Fig. 3.16 Capacitor voltages for a 6-stage converter

Fig. 3.17 Load voltage for a 6-stage converter
3.7 Conclusions

The vector inversion generator is among the few broadly known high voltage pulsed power generation schemes. This scheme is extensively used in a number of high power applications, and therefore a mathematical model that would facilitate designers to gain an insight into the operation of the converter and to optimize such designs would be invaluable. As such, this chapter presented a semi-analytical model that can be used to analyze a multi-stage vector inversion generator across the entire load range.

The presented model, simplifies the complicated transformer coupled vector inversion generator by separating the analysis of even numbered capacitors from odd numbered capacitors into two independent circuits. The analysis of the even numbered capacitors is then accomplished by a further simplification to obtain an equivalent simplified circuit composed of a LCI ladder network. It has been shown that the voltage variation across the odd numbered capacitors is similar to each other, which is used to derive an equivalent LCI circuit to model the behaviour of odd numbered capacitors. These two simplified equivalent models were used to derive analytical solutions in the s-domain over the entire load range for the converter under investigation. The solutions were then numerically converted into the time-domain to obtain various circuit parameters. The results obtained from this technique for 2-stage and 6-stage converters have been compared with SPICE simulations to validate the accuracy of the proposed analysis. Also, an approximation to derive direct mathematical solutions for a lightly loaded converter has been presented. This method provides solutions within a reasonable margin of error when the converter is lightly loaded.
3.8 References


4.1 Introduction

The demand for high power electric fence energizers in African countries has been growing steadily over the past decade. Very large electric fence installations are widely used in these countries to enclose game reserves, national parks and even along the country borders. These systems provide an efficient means for managing wildlife, and also are used to control the spread of contagious diseases among livestock between neighbouring countries. Typically the length of such an electric fence installation is in excess of several hundred kilometres. Furthermore, large electric fence installations are also commonly found in Australia. However, a typical energizer cannot supply the energy that is required to energize such large fence structures, entirely by itself. Therefore these installations use a number of energizers to energize the entire fence, where it is divided into many isolated sections and each section is powered by a separate energizer. However in some situations, synchronized energizers that are connected at regular intervals along large non-isolated fence sections are also used. Construction costs and the complexity of these large fence systems can be significantly reduced by using very high power energizers, which effectively minimize the number of energizers required to energize these large fence systems [1-2].
However, as discussed in the previous chapters the efficiency of existing electric fence type energizers, which are based on the pulse-transformer type topology is very poor being less than 75% in most situations. Although expensive and bulky components can be used to improve the performance of such energizers the design of high power fence energizers based on the existing fence energizer technology is unfeasible. As a consequence manufactures are looking for new technologies to develop high performance and high power energizers to meet the requirements of consumers who have large fence installations [1-5].

In addition, some electric fence manufacturers are looking into a revolutionary fencing concept termed “intelligent electric fence systems”. An intelligent electric fence system adapts itself to suit conditions of a fence at any given time, and produces the most effective signals to energize the fence. Such a system senses the changes in the fence by monitoring the reflections. According to this data the energizer changes the pulse parameters of its output to enable efficient transmission of pulses. Furthermore, such a system contains control devices such as high voltage (HV) switches and monitoring devices, connected to the fence at various locations, to facilitate a dynamic fence layout. The communication between these devices and the energizer is achieved through modulated HV signals transmitted along the fence conductors. Thus, it would be advantageous to use an energizer that also has the capability of generating these HV communication signals; in addition to the ability of generating customizable high power pulses as per requirements of the intelligent fence system. This has also paved the way for the development of new energizer topologies, since the existing technology fails to fulfil these functional needs of intelligent fence systems [6-7].

Based on the analysis presented in Chapter 2, it is evident that the construction of an electric fence type energizer using the Marx generator principle will fulfil most of the requirements mentioned above. As previously discussed in Section 2.2.3, the losses associated with a Marx
generator are very low, therefore this topology facilitates the implementation of a highly efficient and high power energizer. The ability of generating output pulses with flexible voltage levels and pulse widths allows the use of this technology in intelligent electric fence systems. Furthermore, this topology facilitates a safe, compact and reliable implementation of an electric fence type energizer [1-14].

For the above reasons, a 120 J prototype energizer based on the Marx generator was developed in collaboration with Tru-Test engineers to investigate the performance of this technology. This novel implementation of an electric fence energizer is protected by international patents [12] and therefore this chapter only presents a generalized description of this technology. In addition, experimental results are presented to clarify the operation of the prototype [1].

4.2 Implementation of a Novel Fence Energizer

The design specifications for the prototype are summarized in Table 4.1. In addition to these basic specifications the prototype should also comply with the IEC 60335-2-76:2002 standards. A functional block diagram of the proposed implementation is shown in Fig. 4.1. As evident from this diagram, a high voltage capacitor charging unit is used to charge the capacitors, which constitute the pulse generation stages. The pulse generation stage utilizes a number of Marx stages to generate high voltage and high power pulses, required to energize the electric fence. A 3-stage low pass notch filter is used at the output of the Marx stages to limit the EMI produced by these pulses to an acceptable level. Furthermore, the proposed system incorporates two control subsystems to ensure proper operation of the unit [1].
### Table 4.1 Specifications for the prototype energizer

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input voltage</td>
<td>90 – 270 VAC</td>
</tr>
<tr>
<td>Maximum output energy</td>
<td>120 J</td>
</tr>
<tr>
<td>Maximum output voltage</td>
<td>10 kV</td>
</tr>
<tr>
<td>Maximum output current</td>
<td>300 A</td>
</tr>
<tr>
<td>Pulse rate</td>
<td>1.5 Hz</td>
</tr>
<tr>
<td>Pulse width</td>
<td>10-500 µs</td>
</tr>
</tbody>
</table>

Fig. 4.1 Functional block-diagram of the prototype
Chapter 4  A Novel Fence Energizer

The number of Marx stages required in the design is a function of the output voltage \( V_o \), and the acceptable voltage stress per stage \( V_c \) as given by,

\[
N = \frac{V_o}{V_c}
\]  

(4.1)

An evaluation of availability, performance and cost of suitable devices revealed that the optimum value for \( V_c \) is about 950 V. Substituting this in (4.1) results in,

\[
N = \frac{8.5 \text{kV}}{950 \text{ V}} \approx 9
\]  

(4.2)

The energy stored by the capacitors in the Marx stages are given by,

\[
E = \frac{1}{2} N C_{eq} V_c^2
\]  

(4.3)

The maximum pulse energy that can be generated by the converter is equal to the energy stored by the capacitors in the Marx stages. Therefore, to deliver 120 J of pulse energy, each Marx stage of the design should have a storage capacitance \( C_{eq} \) of approximately 30 \( \mu \text{F} \).

A schematic diagram of the pulse generation stage is shown in Fig. 4.2. At the start of each pulse cycle, the capacitors \( C_1-C_9 \) are charged to the desired voltage level by the HV charger indicated in Fig. 4.1. During this stage, the switches \( S_1-S_9 \) are in the off-state where as the isolation elements \( \text{IS-1 to IS-18} \) are in the on-state. Therefore, \( C_1-C_9 \) are connected together in a parallel arrangement and charged through the ‘Charging Bus’ to about 950 V. After the capacitors have reached this voltage level, charging is terminated and the isolation elements are switched to the off-state. The control system then turns-on the appropriate switches and isolation elements to generate an output pulse with the desired pulse shape. To reduce the complexity of the implementation, the control of switches was arranged into three groups, which can be controlled independently through ‘Drive In 1’ – ‘Drive In 3’. Therefore, the
Marx stages can be turned-on in three steps to generate an output voltage between 0 kV and 8.5 kV in increments of 2.8 kV.

Fig. 4.2 A schematic of the pulse generation stage
An example of the pulse shape that can be generated by this unit is illustrated in Fig. 4.3. During the time period $t_0$-$t_1$, only the first three stages are turned-on through ‘Drive In 1’, connecting $C_1$-$C_3$ in a series arrangement with the load. The states of IS-13 to IS-18 are set to be on, to allow for the pulse current to flow from the three capacitors to the load, generating a 2.8 kV pulse across the load. At $t_1$, the switches $S_4$-$S_6$ are turned-on together with isolation elements 15-18, while the isolation elements 1-14 are held at the off-state. Thus the output voltage is increased to about 5.7 kV, since the capacitors $C_1$-$C_6$ are now connected across the load. Finally, at $t_2$ all nine Marx stages are turned-on, while all the isolation elements are turned-off, to increase the output to about 8.5 kV. The output pulse is terminated by turning-off stages 9-7, 6-4, and 3-1 at $t_3$, $t_4$ and $t_5$, respectively.

Fig. 4.3 An output pulse shape that can be generated by the proposed topology
4.3 Results and Discussion

Based on the design presented above in Section 4.2, a prototype electric fence energizer was successfully implemented of which a photograph is shown in Fig. 4.4. A modular design strategy was used for the implementation to improve the serviceability and reliability, where each module consists of a self-contained Marx stage with its own storage, control and switching devices. During the implementation, a few difficulties were encountered due to the high dv/dt and di/dt stresses introduced by the operation of the pulse generation stage. However, they were successfully resolved to build a fully functional 120 J electric fence type energizer. The performance of the prototype has been investigated through laboratory experiments to validate the design. In addition, experiments were conducted at various test sites to benchmark the performance of this new technology against the existing energizers.
These tests showed promising results as the prototype unit outperformed all commercially available high-end energizers [1].

![Fig. 4.5](a)  Output pulses generated by the prototype energizer a.) 500Ω load b.) 50Ω load

The output of the prototype unit with the filter disconnected is illustrated in Fig. 4.5. A delay of about 30 µs is introduced between the switching of subsequent stages to produce the waveforms illustrated. Fig. 4.5(a) shows the output voltage of the energizer when a 500 Ω
load is connected at the output. As evident from this waveform, the discharge of the capacitor is very slow due to the low pulse current drawn by the load. Therefore, only a small portion of the stored energy is delivered to the load. The residual energy from this pulse cycle is carried over to the following pulse cycle, and the charger replenishes the storage capacitors to maintain the desired output voltage. Thus, in comparison to the existing schemes, this new method is significantly efficient, as it recycles the unused energy. However when the load is reduced to 50 $\Omega$, a significant discharge in the capacitor voltage is observed. As a result most of the energy stored by the storage capacitors ($C_1$-$C_9$) is delivered to the load. The experimental results verify the performance of the proposed design as a solution for a high power energizer with a reconfigurable output.

4.4 Conclusions

The design of a novel electric fence type energizer that utilizes the Marx principle to efficiently generate high power pulses has been presented. The operation of the proposed converter has been described with the aid of schematic diagrams. A 120 J energizer based on this design has been implemented and experimental results have been presented to validate the performance of the proposed design. Experimental results also demonstrated the ability of this technique to generate pulses with different amplitude levels and pulse widths, which are vital requirements for energizers used in intelligent electric fence systems. Furthermore, benchmark tests conducted at farms revealed that the performance of this technology is superior in comparison to existing high-end fence energizers.


4.5 References


12. New Zealand patent application 535719 and foreign equivalents.


5.1 Introduction

The novel implementation of a high power electric fence energizer presented in the previous chapter has many advantages over the existing fence energizers. This new energizer technology, which is based on the Marx concept, facilitates transformerless implementation of reliable, efficient, and high power energizers to cater for the power requirements of large fence installations found in African countries and Australia. Furthermore, the proposed technique has the unique ability of generating output pulses with flexible voltage levels and pulse widths, thus enabling the use of this technology in intelligent electric fence systems.

Although there is a high demand for high power electric fence energizers in African countries and Australia, the European energizer sales are dominated by low-medium power products. Moreover, the demand for low power energizers in European countries is expected to increase further with the introduction of new European safety standards for fence energizers that limit the maximum output energy of an energizer pulse to 5 J (and 20 A). The low energy level imposed by the standards constrains the effective range of the electric fence energizers significantly. This has prompted the manufacturers to investigate the use of intelligent electric fence systems to widen the coverage area of these low power conventional electric fence systems. In addition, some of the proposed intelligent electric fence designs utilize a number
of solar powered energizers that are connected at regular intervals along the fence network to cater for the high power demands of larger farms. The implementation of such systems demands for highly efficient electric fence energizers that can adapt the output pulse shapes to suit fence conditions, and therefore to facilitate effective transmission of energizer pulses.

Unfortunately, as discussed previously in Chapter 2, the existing energizer technology which is based on the pulse transformer type HVPPS is unable to fulfil the above mentioned design requirements. Although, the Marx generator based energizer implementation presented in Chapter 4 fulfils these requirements, such an implementation of a low power energizer is unfeasible due to high manufacturing costs arising from the high circuit complexity. In contrast, an implementation of a fence energizer based on the direct discharge topology which performs similar to a Marx generator is straightforward and therefore have lower manufacturing costs. Despite the low cost and simplicity, a prototype design of direct discharge topology revealed that the reliability of this technique is unacceptable for this particular application. This is essentially due to high voltage stresses experienced by the switches as a result of unequal voltage sharing between them.

As such, there is a need for a new HVPPS technology that can be used to implement an inexpensive, low power energizer with a similar performance to a Marx generator based design. A novel high voltage pulse generation technique that behaves identically to a Marx generator is presented in this chapter to address this need. In addition, the proposed technique also facilitates an economical and reliable implementation of a low power energizer due to the lower circuit complexity and its inherent ability to share voltage stresses equally among the components. The operation of the proposed converter is analysed mathematically and through simulations to verify the validity of the proposed concept [1-19].
5.2 The Proposed Topology

The proposed high voltage pulse generation scheme, shown schematically in Fig. 5.1, is derived from a multi-level converter topology and, hence, called a multi-level HVPPS. Although this technique appears to be similar to the direct discharge type, it has the unique ability of generating pulses with flexible amplitudes and durations similar to that of the Marx type, but with less circuit complexity. In addition, the special arrangement of switches and diodes reduce both voltage and current stresses on the switches, thereby facilitating a simple and reliable implementation [1-13].

Fig. 5.1  A multi-level type HVPPS

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¹ This technology is protected by New Zealand patent application 535719 and foreign equivalents
The operation of the multi-level HVPPS is initiated by charging a series connected storage capacitor bank \((C_1-C_N)\) to the desired voltage while the switches \((Q_1-Q_N)\) are turned-off. Although a charging circuit with a single high voltage output can be used for this purpose, it is preferable to use a multi-output charger to charge the stages separately. This will improve the efficiency of the design with better charge balancing between the capacitors. Alternatively, an active charge balancing circuit can be employed to charge all the capacitors to an equal voltage through a charger that has a single output. The maximum voltage, to which a single stage is charged, is limited by the breakdown voltage rating of the capacitor and the switch. Therefore, if these devices have a voltage rating of \(V_c\), then the required number of stages \((N)\) to generate a peak output voltage of \(V_o\) is given by,

\[
N = \frac{V_o}{V_c}
\]  

(5.1)

After the capacitors are charged to the specified voltage level, the switches \(Q_1\) to \(Q_N\) are turned-on sequentially to initiate an output pulse. During this state, the storage capacitors of the stages that are turned-on are in series with the load. They discharge through the load as the pulse current flows through the switches and the diode chain of the top-most stage that is turned-on. The pulse can be terminated by turning-off the switches sequentially from \(Q_N\) to \(Q_1\). Similar to a Marx generator, a delay between the switching of subsequent stages could be introduced easily to generate an output pulse with a variable amplitude and pulse width. During such an operation the diode links between the energy storage capacitors and corresponding switches ensure proper voltage sharing across the switches, eliminating the requirement for complicated snubber and driver circuitry.
An example of an output pulse shape that can be generated across a resistive load is shown in Fig. 5.2. During the time period $t_1$-$t_2$, only the first switch ($Q_1$) is turned-on connecting $C_1$ in a series arrangement with the load. The diode chain $D1_1$-$D1_{N-1}$ conducts the pulse current that flows from $C_1$ to the load, therefore discharging $C_1$ through the load and generating an output pulse with a peak amplitude of $V_c$. However, the maximum voltage stress experienced by the switches $Q_2$-$Q_N$ remains unchanged at $V_c$ since the diode chain $D1_1$-$D1_{N-1}$ clamps the emitter of $Q_2$ to $C_2$. At $t_2$, switch $S_2$ is also turned-on, which commutates $D1_1$-$D1_{N-1}$ to off-state and $D2_1$-$D2_{N-2}$ to on-state. As a result the output voltage is increased to $2V_c$ since the capacitors $C_1$-$C_2$ are now connected across the load. The $3^{rd}$ stage is turned-on at $t_3$ increasing the output voltage further to $3V_c$, since all three capacitors ($C_1$-$C_3$) are now connected in series with the load. Under this condition $D3_1$-$D3_{N-3}$ is forward-biased whereas $D1_1$-$D1_{N-1}$ and $D2_1$-$D2_{N-2}$ are reverse-biased. Therefore, the pulse current flows from the capacitors through $D3_1$-$D3_{N-3}$ and $Q_1$-$Q_3$ to the load. The output voltage can be increased further by turning-on the subsequent stages.
stages. However in this particular example the pulse is terminated by turning-off the switches $Q_3$, $Q_2$ and $Q_1$ at $t_4$, $t_5$ and $t_6$ respectively.

As evident from the above illustration, during all states of circuit operation the diode chains restrict the maximum voltage stress across each switch and capacitor to $V_c$, thus facilitating a reliable implementation without complex snubber and supervisory circuitry. However, the reverse voltage exerted across a particular diode chain can be much larger than $V_c$. The reverse voltage applied across a diode chain is determined by the location of the diode chain and the number of switches that are turned-on. Furthermore, the diode chains experience maximum voltage stress when all the stages are turned-on, and the voltage stress on the $n^{th}$ diode chain under this condition is given by (5.2). Hence, if the voltage rating of diodes are the same as for the switches, multiple diodes are required in each stage to meet the voltage rating given by (5.2). The converter illustrated in Fig. 5.1 thus utilizes diode chains that contain multiple diodes with individual voltage ratings of $V_c$.

$$V_{D(n)} = V_c \times (N - n)$$  \hspace{1cm} (5.2)

Based on the analysis presented above, it is evident that the proposed HVPPS technology has many advantages in comparison to the pulse generation technologies that were discussed in previous chapters. Mainly, it allows an implementation of a reliable solid-state HVPPS by utilizing the multi-level concept with a simple switching scheme as the component voltage and current stresses are low. Moreover, this topology also eliminates the need for expensive snubbing circuits, balancing circuits and driver circuits. The converter could also generate waveforms with variable amplitude and duration similar to the Marx generator, but with lower circuit complexity and cost. Owing to its superior performance, the technique has applications in many pulsed power disciplines that require reliable, compact and efficient solid-state HVPPS.
5.3 Theoretical Analysis

Prior knowledge of power requirements, voltage and current stresses on devices and output characteristics of a multi-level converter is essential to realize an optimized design. This can be achieved through a mathematical model that adequately represents the behavior of the converter. This section presents such a mathematical model that facilitates theoretical analysis related to output voltage, component stresses and power throughput on an N-stage converter.

It is assumed that the multi-level converter is supplying an output voltage of $V_o$ to a resistive load $R_L$ at a pulse repetition rate of $f_{rp}$ and a pulse width of $T_p$. The proposed converter stores energy in the storage capacitors $C_1$-$C_N$. Therefore, assuming that each stage of the converter has a capacitance of $C$, which is charged $V_c$, the total amount of energy stored in the converter is given by,

$$E_s = \frac{1}{2} NC V_c^2$$  \hspace{1cm} (5.3)

The amount of energy delivered to the load is a function of the output voltage and the load. Moreover, the output voltage of the converter is characterized by the switching times of each stage and RLC networks formed by the load and the corresponding storage capacitors. For the case of a purely resistive load of $R_L$, the output voltage can be derived as given by (5.4). The switching time delay between the $n^{th}$ and $(n+1)$ stages is indicated by $T_n$ and $U(t)$ denotes the unit step function.

$$V_{out} = V_c \sum_{n=1}^{N} X_n e^{-\tau_n(t^2)} \left[ U(t-T_n) - U(t-T_{n+1}) \right]$$
$$+ V_c \sum_{n=1}^{N-1} Z_n e^{-\tau_n(t^2-\tau)} \left[ U(t-T_{2N-n}) - U(t-T_{2N}) \right]$$  \hspace{1cm} (5.4)
where,

\[ X_1 = 1 \]

\[ X_2 = \left(1 + e^{-a_1(T_2-T_1)}\right) \]

\[ X_3 = 1 + \left(1 + e^{-a_1(T_2-T_1)}\right) e^{-a_2(T_3-T_2)} \]

\[ \vdots \]

\[ X_n = 1 + \left\{ 1 + \left[ 1 + \left(1 + e^{-a_1(T_2-T_1)}\right) \right] \times e^{-a_2(T_3-T_2)} \right\} \times e^{-a_{n-1}(T_n-T_{n-1})} \]

\[ Z_n = Z_{n+1} - C_{n+1} \]

\[ Z_N = X_{N+1} - 1 \]

\[ D_n^+ = \left[ X_n - (X_{n+1} - 1) \right] / n \]

\[ D_n^- = Z_n \left[ 1 - e^{-a_n(T_n-T_{n+1})} \right] / n \]

\[ C_m = 1 - \sum_{n=1}^{(N+1-m)} D_{m+n}^+ - \sum_{n=1}^{(N-m)} D_{m-n}^- \]

\[ \alpha_n = \frac{n}{RC} \]

The load current is given by,

\[ I_{\text{out}} = \frac{V_{\text{out}}}{R} \quad (5.5) \]
The energy delivered to the load is equal to the time integral of the output power and is given by,

$$E_{\text{out}} = \int_0^t \frac{V_{\text{out}}^2}{R} \, dt$$

(5.6)

where $E_{\text{out}} \leq E_s$.

The minimum power throughput of the charger that is required to charge the storage capacitors within one pulse period is a function of the stored energy as given by,

$$P_{\text{Charger}} = \frac{E_s f_{\text{rp}} p}{(1 - T_p f_{\text{rp}})}$$

(5.7)

The switch current of the proposed converter mainly constitutes the load current. However, in addition to the load current, during the turn-on of a stage, the reverse recovery current ($I_{rr}$) of the diode chain that is commutated-off, also flows through the switch. Therefore, the maximum current stress experienced by the switches during the turn-on period is given by,

$$I_{\text{sw(on)}} = I_{\text{out}} \bigg|_{t=T_s} + I_{rr}$$

(5.8)

### 5.4 Results and Discussion

A 3-stage multi-level type high voltage pulse generator, shown in Fig. 5.3, was designed and simulated to validate the proposed HVPPS concept. Moreover, simulations were compared with theoretical results to confirm the validity of the presented theoretical analysis. The prototype converter stores 1.5 J of energy in the storage capacitors when they are charged to 900 V, which is the maximum voltage rating of the capacitors. Therefore the converter is capable of generating output pulses with peak voltages ranging up to 2.7 kV. A complete list of circuit parameters is given in Table 5.1.
<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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</thead>
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<tr>
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<td>μF</td>
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<td>$T_p$</td>
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<td>μs</td>
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<td>Ω</td>
</tr>
<tr>
<td>$R_L$ (light)</td>
<td>1000</td>
<td>Ω</td>
</tr>
</tbody>
</table>

Table 5.1 Design parameters for a 3-stage multi-level HVPPS

All capacitors in this proposed design are charged together in a series arrangement through a single-output HV charger. Since this is a low power unit, passive balancing is used to balance voltages across the capacitors in order to minimize circuit complexity. A low power fly-back converter is used as the charger due to its ability of generating high voltages while utilizing a high frequency transformer with a lower turns-ratio.

As shown in Fig. 5.3, each IGBT switch in the converter is supplied with its own driver circuit. These driver circuits are controlled by a microcontroller that is powered with reference to the emitter of $Q_1$ through opto-isolators. The opto-isolators minimize the parasitic coupling between stages that could cause false triggering of switches. The power required by the driver circuits can be derived employing various methods. This implementation utilizes a diode chain ($D_1$-$D_3$) to supply power to the driver circuits. The diode chain replenishes the supply capacitor of the driver circuit of $Q_3$ when $Q_1$ to $Q_2$ are turned-on. This strategy minimizes circuit complexity in addition to providing protection against false triggering. The method ensures that the turning-on of a switch takes place only after the switches below it have been turned-on, thus ensuring proper operation of the converter.
The load voltage and current obtained from PSPICE simulations and the theoretical model for both heavy and light loads are shown in Fig. 5.4 and Fig. 5.5, respectively. The theoretical waveforms follow the simulations closely thus validating the theoretical analysis. As predicted by the theoretical model, the output voltage exhibits an exponential decay as the storage capacitors discharge through the load. As evident from these figures, the turn-on time delay between the switches can be varied to generate pulses with variable amplitudes and pulse widths, confirming the expected output performance of the proposed topology. It is also observed in Fig. 5.4 that the voltages across the capacitors in lower stages can reverse in polarity under heavy loads or when the switching delay is comparatively large. However, this could be easily prevented by connecting diodes across the storage capacitors to clamp the negative voltage.
The reliability of the design greatly depends on the ability of the topology to distribute voltage stresses among its switches equally even during abnormal operating conditions. An unequal voltage sharing between the switches could damage the switches due to high voltage stresses and lead to a catastrophic failure of the entire system. Therefore the voltage stresses across the switches and the diode chains are investigated through simulations and the results are illustrated in Fig. 5.6 and Fig. 5.7, respectively. From the waveforms it is evident that the maximum voltage stresses across switches are equal as predicted earlier. Moreover the voltage
stress across each switch is determined by the charged voltage of the corresponding storage capacitor \((V_c)\). A comparison between the switch and diode voltage stresses illustrates the importance of the diode chain to maintain voltage sharing across the switches. For example, consider the time interval from 28 µs to 50 µs where the first and second stages are turned-on and the third stage is in off-state. During this time period, diode chain D1₁-D1₂ is reverse-biased therefore connecting both \(C_1\) and \(C_2\) in a series arrangement with respect to the load. At the same time, diode chain D2₁ is conducting to ensure that the voltage stress across \(Q_3\) is maintained at \(V_c\). And if, for example, an abnormality turns-off \(Q_2\), causing the voltage stress across \(Q_2\) to rise above \(V_c\), then diode chain D1₁-D1₂ will initiate conduction, disconnecting 2\(^{nd}\) and 3\(^{rd}\) stages and therefore limiting the voltage stress across \(Q_2\) to \(V_c\). Thus, this confirms the capability of the proposed concept to share the voltage stress equally across the switches without requiring additional voltage balancing circuitry. The voltage stresses on the switches are significantly reduced after the termination of a pulse as the capacitors have discharged to a very low voltage by the heavy load. As indicated in Fig. 5.6, the maximum voltage on a switch is about 400 V after a pulse is terminated. It could also be observed that if the pulse length or the load is increased further, the voltage stress across \(C_1\) could reverse polarity. As explained earlier, this can be resolved by connecting diodes across the storage capacitors to clamp the negative voltage. It is evident from Fig. 5.7 that the voltage stress on any given diode chain is equal to the sum of voltages across the capacitors above this diode chain that are contributing to the output pulse. Hence this result suggests that there is a need for multiple diodes in lower stages to withstand the high voltage stresses and also to reduce the reverse recovery current.
The current stresses experienced by the switches for a heavy load are shown in Fig. 5.8. As predicted by (5.8), during the turn-on of a stage the current stress on the switch is higher than the load current due to the reverse recovery current of the diode chain prior to switch. After the diode is commutated-off the load current dominates the switch current and decreases as the capacitors discharge through the load causing the output voltage to decrease. Moreover, a sudden increase in the switch current is observed when a stage is turned-on, due to the increase in output voltage.


5.5 Conclusions

A novel solid-state, high voltage pulse generation technique termed multi-level HVPPS has been described. The proposed technique, which is suitable for a wide range of pulsed power applications, can be considered as a hybrid of the direct discharge type and the Marx generator but with considerably less complexity in both control and circuitry. As such, this technique has the ability for generating pulses with flexible amplitude and duration similar to that of a Marx generator; however unlike the direct discharge type, it does not require voltage balancing and snubbing circuitry. Since voltage balancing is inherent to the design, the timing between switching events is not critical to balance the voltage stresses and, as such, the implementation of the converter is relatively simple.

The operation of the proposed multi-level converter has been described in detail and theoretical solutions for the output characteristics and component stresses have been derived. The ability of the proposed concept to generate output pulses with flexible amplitudes and durations has been demonstrated through SPICE simulations and theoretical analysis performed on a 3-stage converter. Furthermore, the inherent protection and voltage stress balancing properties of the
converter, which would be useful to design simple yet more reliable pulse power generators, have been demonstrated through simulations.
5.6 References


6.1 High Voltage Charging Schemes

Conventional electric fence energizers utilize hard-switched PWM converters to charge high voltage (HV) storage capacitor banks in between each pulse interval. At the termination of each output pulse, the voltage across the storage system is approximately 0 V, and thus recharging the storage system back to a nominal level, determined by the controller, is necessary. Typically, this nominal charged voltage level is about 900 V, but it could range between 200-10,000 V depending on the pulse generating scheme. Therefore, this wide change in voltage (or load) forces these converters to operate in both continuous and discontinuous conduction modes within a single charging cycle. Moreover, in some situations the charging system is required to serve as a high voltage isolation barrier between the input and the output of an energizer (IEC 60335-2-76:2002, MOD). As a result of these stringent requirements, conventional high voltage charging schemes employed in electric fence type energizers exhibit high switching losses, switch stresses and electro magnetic interference (EMI), particularly at elevated switching frequencies and power levels [1].

However, modern soft-switching techniques are well-known for light weight, small size, high efficiency and low EMI; properties which are realized by controlling the voltage and current trajectories of the converter switches to be at zero or near zero at the moment of switching.
Such controlled operation of voltage and/or current is usually achieved by making use of either
the natural or forced resonance phenomenon that takes place between circuit components in the
converter itself. The resonance, which facilitates Zero-Voltage-Switching (ZVS) or/and Zero-
Current-Switching (ZCS), can be realized either by employing separate and dedicated
components or utilizing the parasitic elements in the converter circuit itself. Depending on
whether the resonance occurs during the whole switching cycle or part of the cycle, these
converters are called resonant or quasi-resonant converters [2-15].

As such, modern soft-switching schemes can be utilized to overcome the above mentioned
shortcomings of HV charger systems used in existing fence energizers. This chapter presents a
novel Buck-Boost push-pull parallel-resonant converter (PPRC) suitable for charging HV
capacitors. The proposed technique facilitates the operation of a PPRC over a wide load range
with output voltage control while maintaining ZVS. A comprehensive mathematical model is
presented, through which explicit analytical solutions are derived to accurately predict the
performance of the proposed converter. Furthermore, an analysis on quasi-resonant techniques
that can be utilized to enhance the performance of fly-back type HV charger systems is
presented.

6.2 Quasi-Resonant Techniques

Electric fence energizers frequently use high voltage charging schemes that are based on the
fly-back topology due to their ability to generate high voltages while utilizing a high-
frequency transformer with a relatively low turns-ratio. A schematic of a typical fly-back
converter and its equivalent model that can be used to analyze such a converter are shown in
Fig. 6.1 and Fig. 6.2 respectively [1, 8].
Fly-back converters perform very well in low power and low frequency charging systems, but encounter a few difficulties at elevated power levels and frequencies. The high leakage inductance of the transformer, due to large air-gap required for high power transfer, generates high voltage stresses across the switch as illustrated in Fig. 6.3. These high voltage stresses cause high switching losses and EMI that can be transmitted to the load through the interwinding capacitance of the transformer. The situation is exacerbated as the input-output
isolation requirements are increased. This is because under this condition the coupling co-
efficient between the primary and the secondary windings of the transformer becomes weaker
[1]. The power loss due to an imperfect coupling co-efficient (k) in a fly-back converter that
is supplying an output power of $P_o$ is given by,

$$P_{ik,loss} = kP_o$$  \hfill (6.1)

(a) Converter with an ideal transformer \hspace{1cm} (b) Converter with a non-ideal transformer

Fig. 6.3 Switch voltage of a fly-back converter in discontinuous mode

As explained earlier, the wide dynamic load range forces the fly-back converter to operate in
both continuous and discontinuous conduction modes during a single charging cycle. As
evident from Fig. 6.4, during continuous conduction mode of operation the switch current
stress is amplified due to the reverse recovery current of the secondary diode and the parasitic
winding capacitance. In addition, this effect increases the EMI produced by the converter [1].
Thus it is desirable to operate the converter in discontinuous conduction mode as the parasitic
effects become significant under continuous conduction mode of operation.
The minimum duty cycle of the switch (D) that is required to ensure discontinuous mode of operation is a function of the input voltage ($V_{in}$), output voltage ($V_{out}$) and the turns-ratio (n:1) of the transformer, as given by [1],

$$D \leq \frac{nV_{out} - V_{in}}{nV_{out}} \quad (6.2)$$

However, the operational duty cycle of the switch is determined by the switching frequency ($f_s$), magnetizing inductance ($L_m$), leakage inductance ($L_{lk}$) and peak primary current ($I_{pk}$) as given by [1],

$$D = \frac{(L_m + L_{lk})I_{pk}f_s}{V_{in}} \quad (6.3)$$

Furthermore, the power throughput ($P_o$) of the converter is related to the primary current as given by (6.4) [1]. Thus, it could be seen that the fly-back converter will operate in the continuous conduction mode for a longer duration if the switching frequency and/or the output power levels are increased.

$$P_o = 0.5L_mI_{pk}^2f_s \quad (6.4)$$
The above mentioned shortcomings of the hard-switched fly-back converter can be somewhat lessened by adopting quasi-resonant techniques along with a piecewise switching frequency controller. A piecewise switching frequency controller that monitors the current and voltage on the primary winding and controls the switching frequency to maintain the operation in discontinuous mode based on (6.2) to (6.4), can be implemented using a microcontroller [1, 19].

![Diagram of a fly-back converter with an active clamp](image)

**Fig. 6.5** A fly-back converter with an active clamp

An active-clamp, illustrated in Fig. 6.5, can be utilized to reduce the losses and EMI of a fly-back converter. The active-clamp recycles the energy that is stored in the leakage inductance and limits $dv/dt$ across the main switch ($Q_1$), facilitating zero-voltage turn-off. However the addition of an active-clamp complicates the control of the circuit and it is only effective within a narrow load range. Alternatively, a lossless snubber design indicated in Fig. 6.6 can be adopted to reduce losses and EMI with less circuit complexity. The LCD snubber recycles a portion of the energy stored in the leakage inductance and limits $dv/dt$ across the switch,
thereby increasing the system efficiency [1, 20-21]. Similar to many other quasi-resonant techniques, the effectiveness of the lossless snubber is also limited to a very narrow load range.

![Diagram of a fly-back converter with a lossless snubber](image)

**Fig. 6.6 A fly-back converter with a lossless snubber**

### 6.3 Resonant Converters

Most drawbacks of the hard-switched converter technology have been overcome through the development of a variety of resonant topologies such as half bridge, full bridge, and push-pull type resonant converters and their derivatives. These converters have a wide range of applications, and are usually controlled either by simple or sophisticated techniques on the basis of power level and type of application. The main advantage of these resonant technologies is that they minimize both the switching and capacitive losses of the converter, which can therefore be operated efficiently at an elevated frequency to facilitate a compact design. Moreover, the well defined waveforms across the components result in low dv/dt and di/dt, which in-turn reduce noise and simplify EMI filtering, assisting the development of a compact design [2-18].
However, the resonant technology has its own disadvantages. Mainly it suffers from higher peak voltage or/and current stresses on components than that imposed by the hard-switched converters. Thus, components with higher ratings are required at an additional cost. The limited controllability of the output is another disadvantage of the resonant technology as zero voltage or zero current switching is often sacrificed to achieve control over a wider load range. Converters that use a pre-regulating stage to control the output have also been reported but they are considered to be less attractive because of additional components, reduced efficiency and relatively high EMI generation [1-18].

Amongst the above mentioned resonant topologies, the current fed push-pull parallel-resonant converter is particularly popular. This could be attributed to its simplicity, low to medium power capability, high efficiency and low component cost. However, the conventional PPRC, which is operated with ZVS at a constant frequency that is just below its damped resonant frequency, also lacks the ability to operate over a wide load range. Usually the ZVS of the converter is sacrificed to tolerate a wider load range and to gain control over the output voltage [2-18].

Therefore, a technique that facilitates the operation of a PPRC over a wide load range with output voltage control, whilst maintaining ZVS would vastly improve the performance and the usability of a PPRC. Moreover, an implementation of a HV charger for an electric fence energizer based on such a technique will have significant advantages over the existing techniques discussed previously. To fulfil this requirement a novel control strategy that enables the operation of a PPRC as a Buck-Boost converter with ZVS over a wide load range is presented below.
6.4 A Novel Buck-Boost PPRC

The operation of the conventional PPRC, shown in Fig. 6.7, is well understood [2-19, 21-30]. Traditionally, the magnetizing inductances $L_1$ and $L_2$ of the split-winding transformer are equal and the converter is operated at a constant switching frequency ($f_s$) with ZVS. The switching frequency is either just below the tank resonant frequency ($f_r$) at light loads or approximately equal to the damped resonant (ZVS) frequency ($f_z$) at other loads. This form of operation will be referred to as Normal mode of operation in order to help identify other operating modes. Typical gate signals and drain-source voltages of $S_1$ and $S_2$ of a PPRC under the Normal mode of operation are shown in Fig. 6.8.

Fig. 6.7 A conventional current-fed PPRC

Under the Normal mode of operation, peak switch or load voltage (parallel coupled load) of the PPRC is independent of the loading at light load and is approximately ‘π’ times the input DC voltage when a 1:1 isolation transformer is used [2-15]. PPRC can be operated at a variable switching frequency to produce a higher (Boost) or lower (Buck) peak load voltage.
than that which would result in the Normal mode, through a slight modification to the resonant circuit. Fig. 6.9 illustrates the proposed circuitry that allows for the PPRC to be operated in any one of the following three modes.

1. Normal mode, where \( f_s = f_z \)
2. Boost mode, where \( f_s < f_z \)
3. Buck mode, where \( f_s > f_z \)

The variable peak capacitor voltage in Buck and Boost modes is facilitated by a modification to the conventional PPRC topology by splitting the resonant capacitor equally \((C_1 = C_2 = C)\) and connecting in parallel with the two switches. A conventional bifilar split-winding transformer is employed and the switches are operated in all three modes with equal duty cycles to avoid core saturation but at a frequency dictated by the mode. The conventional PPRC topology can be operated only in the Normal mode and has the advantage of having the resonant current confined to its resonant tank circuit. In contrast the proposed split-capacitor PPRC topology has the disadvantage of resonant current passing through the switch but has the advantage of being able to operate in any of the modes of Normal, Buck and Boost. The Normal mode of operation is defined by the waveforms in Fig. 6.8 while the Buck and Boost modes constitute operations, where peak capacitor (or load) voltage is below or above the nominal voltage that would result in the Normal mode, respectively.

It is evident from the literature that the Normal mode of operation of PPRC has been well researched through theoretical analysis, performed by using a variety of techniques [2-8, 14, 22-30]. Most of these theoretical analyses were largely based on numerical solutions due to the complexity associated with obtaining analytical solutions for voltage gain and load dependent ZVS frequency. Although this has been the case, a few closed-formed analytical
solutions have also been reported but over a limited load range and with some assumptions [22-25].

Fig. 6.8  Typical waveforms of Normal mode operation

Fig. 6.9  The proposed PPRC topology
According to literature, neither a comprehensive analysis of the proposed topology in different modes nor a mathematical solution that provides an insight into the behaviour of the converter in the Normal, Boost and Buck mode has ever been reported. Moreover an accurate insight into the behaviour of the converter under different operating conditions is vital at the design stage. Therefore, a comprehensive mathematical model is presented in this chapter through which the performance of the proposed PPRC topology in the Boost and Normal modes can be accurately predicted over the entire load range. Chapter 7 presents a detailed mathematical analysis on the operation of the converter in the Buck mode. Furthermore, effects of parasitic elements are modelled, and theoretical results are compared with simulations and experimental results of a 150 W prototype unit, which is used for a high voltage charging application, to verify the validity of the proposed control technique, and analytical solutions.

It is widely accepted that theoretical analysis of PPRC is particularly difficult when the entire load range is taken into account. Although this difficulty can somewhat be lessened with the assumption that the input DC current is equally divided between the two split-windings in situations where input DC inductance ($L_{DC}$) is sufficiently large [22], the analysis is still complicated by the presence of the two split-windings. However, this complication can be overcome by the model proposed in Fig. 6.10, which replaces the two split-windings with two equal current sources of magnitude $I_{DC}/2$ and a resonant inductor (L) equivalent to $2L_1+2kL_1$ for a split winding transformer with a coupling coefficient of $k$ between the two windings [1-3]. The proposed model is simple and allows determination of explicit analytical solutions for the entire load range as described below.
6.4.1 Boost Mode of Operation

As evident from Fig. 6.11, in the Boost mode the turn-on of a switch is similar to that in the Normal mode and takes place with ZVS. However the turn-on of a switch is not immediately followed by the turn-off of the second switch but instead it is delayed by incorporating a time delay (phase-shift) into the switching frequency. The delay at turn-off prolongs the on-time and, as a consequence, switches operate with above 50% duty cycle and at a frequency that is lower than the damped resonant frequency. As in the case of the Normal mode, the resonance takes place whenever a switch is tuned-off, and the load voltage corresponds to resonant voltage. But during the delay, both switches are turned-on and thus there is no resonance and the voltage \( (v_{c1}+v_{c2})/2 \) or the mid point voltage of the split-winding is zero. Since the volt-second product across \( L_{DC} \) is determined by the mid point voltage and it should essentially be zero at steady state, the peak resonant voltage increases beyond the nominal peak voltage of the Normal mode to compensate for the lost volt-second product over the delay period. The converter thus operates in the Boost mode and the peak resonant voltage (as given by (6.16) in

![Diagram](image-url)
Section 6.4.2) can be controlled by varying the switching frequency with an adjustable time delay (phase-shift). The steady state operation of the converter in boost mode, indicated in Fig. 6.11, can be described through the following 4 states.

State1: At $t=0$, switch $S_1$ is turned-off while $S_2$ is held turned-on by the control circuit. The inductor current has reached a maximum value and flows through $C_1$ and $S_2$, charging the capacitor.

State2: At $t=t_1$, the voltage across $C_1$ has reached a maximum and starts discharging through $S_2$.

State3: At $t=t_2$, the voltage across $C_1$ has reached zero. Due to the body diode of $S_1$, $C_1$ voltage cannot reverse as the body diode starts conducting clamping the voltage across $S_1$ to 0. The control circuit detects this and turns-on $S_1$ with ZVS.

State4: At $t=T_s/2$, half of the switching period has elapsed, and the control circuit turns-off $S_2$. The capacitor $C_2$ starts charging where as $C_1$ is held at 0 V repeating State1 to State4.

Fig. 6.11 Boost mode switch voltages of the proposed PPRC converter
6.4.2 Analysis on the Boost Mode

The Boost operation takes place when the converter is operated at a switching frequency \( (f_s) \) that is lower than the damped resonant or ZVS frequency \( (f_z) \) of the converter. The two switches are operated at a variable switching frequency with ZVS but at a duty cycle, which is always more than 50%. The following relationship thus holds.

\[
T_z < T_s
\]  \hspace{1cm} (6.5)

Only one-half of the switching period is considered in the presented analysis, due to the symmetrical nature of waveforms in each half cycle. Assume the switch \( S_1 \) is turned-off and \( S_2 \) is turned-on at time \( t=0 \), as discussed previously. The node at \( v_{c2} \) is shorted and applying KCL to the node at \( v_{c1} \) in Fig. 6.10 results in,

\[
i_{c1} + i_L + i_R = \frac{I_{DC}}{2}
\]  \hspace{1cm} (6.6)

The voltage \( v_{c1} \) and current \( i_{c1} \) of \( C_1 \) are related by,

\[
C \frac{dv_{c1}}{dt} = i_{c1}
\]  \hspace{1cm} (6.7)

And the voltages across the inductor \( L \) and the reflected load \( R \) are given by,

\[
v_L = L \frac{di_L}{dt}
\]  \hspace{1cm} (6.8)

\[
v_R = i_R R
\]  \hspace{1cm} (6.9)

where \( R = R_{load}/n^2 \) and ‘\( n \)’ is the turns-ratio of the isolation transformer. From Fig. 6.10,

\[
v_R = v_{c1} = v_L
\]  \hspace{1cm} (6.10)
By manipulating (6.6) to (6.10),

\[
\frac{d^2 v_{c1}}{dt^2} + \frac{1}{RC} \frac{dv_{c1}}{dt} + \frac{1}{LC} v_{c1} = 0
\]  

(6.11)

The solution for (6.11) is in the form given by,

\[
v_{c1} = A e^{\left(-\alpha t + \sqrt{\omega_r^2 - \alpha^2} t\right)} + B e^{\left(-\alpha + \sqrt{\omega_r^2 - \alpha^2} t\right)}
\]  

(6.12)

where \(A\) and \(B\) are constants defined by the circuit conditions while \(\alpha\) and \(\omega_r\) are given by,

\[
\alpha = \frac{1}{2RC} \quad \text{and} \quad \omega_r = \sqrt{\frac{1}{LC}} \quad \text{is the tank angular resonant frequency.}
\]

For loads that satisfy the condition \(R < \sqrt{L/4C}\) for a given \(L\) and \(C\), the resonant circuit is over-damped and, hence, voltage oscillations across capacitors cease to exist. Therefore the converter operation with ZVS is not possible. However, as load decreases and reaches a critical value \(R_{\text{critical}} = \sqrt{L/4C}\), the capacitor voltage decays down to zero within a finite time enabling the operation with ZVS. Thus for loads \(R > \sqrt{L/4C}\), the resonant circuit is under-damped and oscillatory and facilitates converter operation with ZVS. Under this condition the solution in (6.12) can be expressed by,

\[
v_{c1} = e^{-\alpha t} \left[A \cos(\omega_r t) + B \sin(\omega_r t)\right]
\]  

(6.13)

where \(\omega_r = \sqrt{\omega_r^2 - \alpha^2}\) is the angular damped resonant or ZVS frequency.

For the circuit in Fig. 6.10, the initial voltage across \(C_1\) is zero, and thus constant \(A\) becomes zero, reducing (6.13) to,

\[
v_{c1} = B e^{-\alpha t} \sin(\omega_r t) \quad \text{for} \quad 0 < t \leq \frac{T}{2}
\]  

(6.14)
Noting that the voltage $v_m$ at the centre point of the split winding transformer is $(v_{c1}+v_{c2})/2$, the constant $B$ can be evaluated by taking the steady-state volt-second product across $L_{\text{DC}}$ to be zero over a single period, and is given by,

$$B = \pi V_{\text{DC}} \left( \frac{T_s}{T_z} \right) \left[ \frac{2(\alpha^2 + \omega_z^2)}{\omega_z^2 \left( e^{-\alpha T_z/2} + 1 \right)} \right]$$

(6.15)

Substituting (6.15) in (6.14),

$$v_{c1} = \pi V_{\text{DC}} \left( \frac{T_s}{T_z} \right) \left[ \frac{2(\alpha^2 + \omega_z^2)}{\omega_z^2 \left( e^{-\alpha T_z/2} + 1 \right)} \right] e^{-\alpha t} \sin(\omega_z t)$$

(6.16)

For $(T_s/2) < t \leq (T_s/2)$, both $v_{s1}$ and $v_{s2}$ are held at zero as both switches are on during this period. Under light load conditions (6.16) reduces to,

$$v_{c1} \approx \pi V_{\text{DC}} \left( \frac{T_s}{T_z} \right) \sin(\omega_z t)$$

(6.17)

According to (6.16) and (6.17), the load voltage in the Boost mode is governed by both the switching and damped resonant frequencies; hence it could be controlled by varying the switching frequency.

Using (6.8), (6.10) and (6.16) the inductor current is determined as,

$$i_L = \frac{-2\pi V_{\text{DC}}}{L \omega_z^2 \left( e^{-\alpha T_z/2} + 1 \right)} \left( \frac{T_s}{T_z} \right) \left[ \omega_z e^{-\alpha t} \cos(\omega_z t) + \alpha e^{-\alpha t} \sin(\omega_z t) \right] + K$$

(6.18)

where $K$ is a constant.
During the time period \((T_d/2) < t \leq (T_s/2)\), where both switches are on, the two nodes at \(v_s1\) and \(v_s2\) are shorted to ground and thus the resonant and input DC currents in the split windings flow through the switches. The resonant current is expected to decay in accordance with the associated losses whereas the input DC current is expected to increase linearly with time. However, with a large DC inductor and with small losses, it is assumed that the split winding current remains constant during this small time period as justified by the experimental results. According to the proposed model, only resonant current flows through \(L\) and therefore the inductor currents at the end of each half cycle should be equal in magnitude but opposite in direction as given by,

\[
i_{L(at=0)} = -i_{L[at=(T_s/2)]} \quad (6.19)
\]

Imposing the condition given by (6.19) on (6.18),

\[
K = \frac{\pi V_{DC} \left(1 - e^{-\alpha T_s/2}\right) \left(T_s\right)}{L \omega_s \left(e^{-\alpha T_s/2} + 1\right) \left(T_s\right)} \quad (6.20)
\]

Substituting (6.20) in (6.18), the steady-state inductor current can be determined for \(t \leq \frac{T_s}{2}\) as,

\[
i_L = \frac{-2 \pi V_{DC}}{L \omega_s \left(e^{-\alpha T_s/2} + 1\right) \left(T_s\right)} \left[\omega_s e^{-\alpha t} \cos(\omega_s t) + \alpha e^{-\alpha t} \sin(\omega_s t)\right] + \frac{\pi V_{DC} \left(1 - e^{-\alpha T_s/2}\right) \left(T_s\right)}{L \omega_s \left(e^{-\alpha T_s/2} + 1\right) \left(T_s\right)} \quad (6.21)
\]

Under light loads, (6.21) reduces to,

\[
i_L \approx -\frac{\pi V_{DC}}{L \omega_s \left(T_s\right)} \left(T_s\right) \cos(\omega_s t) \quad (6.22)
\]
The actual currents through the split-windings can simply be obtained from,

\[ i_{L1} = i_{L2} = \frac{I_{DC}}{2} \pm i_L \quad (6.23) \]

As both load voltage and switching frequency are load dependent, it is useful to obtain an analytical expression for the output power of the converter. The power delivered to the load \((P_{out})\) can be calculated from (6.16) as given by,

\[
P_{out} = \frac{2B^2}{RT_s} \int_0^{T_s} e^{-2\alpha \sin^2 (\omega_z t)} dt
\]

\[
= \frac{2\pi^2 V_{DC}^2}{RT_s} \left( \frac{T_s}{T_z} \right)^2 \left[ \frac{(\omega^2 + \alpha^2)(1 - e^{-\alpha T_z})}{\omega^2 \alpha (1 + e^{-\alpha T_z}/2)^2} \right]
\]

At light loads,

\[
P_{out} \approx \frac{\pi^2 V_{DC}^2}{2R} \left( \frac{T_s}{T_z} \right)^2
\]

\[
(6.24)
\]

\[
(6.25)
\]

The main drawback of this proposed split-capacitor topology is the high conduction and diode loss of the switch due to the resonant current that flows through the switch and its anti-parallel diode. The switch current \(i_{s2}\) and diode current \(i_{D2}\) can be determined using,

\[
i_{s2} = \frac{I_{DC}}{2} - (i_R + i_L) \text{ for } i_L \leq \frac{I_{DC}}{2} - i_R \quad (6.26)
\]

and

\[
i_{s2} = \frac{I_{DC}}{2} - (i_R + i_L) \text{ for } i_L > \frac{I_{DC}}{2} - i_R \quad (6.27)
\]
The currents $i_R$ and $I_{DC}$, can be calculated from,

$$i_R = \pi V_{DC} \left( \frac{T_x}{T_z} \right) \frac{2\left(\alpha^2 + \omega_{z}^2\right)}{R \left( \omega_{z}^2 \left( e^{-\omega_{z}T_z} + 1 \right) \right)} e^{-\omega_{z}t} \sin(\omega_{z}t) \tag{6.28}$$

$$I_{DC} = \frac{P_{out}}{\eta V_{DC}} = \frac{2\pi^2 V_{DC}}{\eta RT_x} \left( \frac{T_x}{T_z} \right)^2 \left[ \frac{\left(\omega_{z}^2 + \alpha^2\right)\left(1 - e^{-\omega_{z}T_z}\right)}{\omega_{z}^2 \alpha \left(1 + e^{-\omega_{z}T_z}\right)^2} \right] \tag{6.29}$$

where $\eta$ is the efficiency of the converter. Under light loads, (6.28) and (6.29) reduce to,

$$i_R \approx \frac{\pi V_{DC}}{R} \left( \frac{T_x}{T_z} \right) \sin(\omega_{z}t) \tag{6.30}$$

$$I_{DC} \approx \frac{\pi^2 V_{DC}}{2R \eta} \left( \frac{T_x}{T_z} \right)^2 \tag{6.31}$$

### 6.4.3 Results on Boost Mode

In order to prove the viability of the proposed control technique and the validity of the presented theoretical analysis a 150 W split-capacitor push-pull resonant converter was built. The details of the converter, which has a measured tank resonant frequency of 74 kHz and an efficiency of about 92%, are given in Appendix A.

The performance of the proposed converter was verified by implementing the prototype with an open loop controller. However, the converter can also be operated with a closed-loop controller with either a microprocessor or Voltage Controlled Oscillator (VCO). The microprocessor based controller can be configured to maintain the desired load voltage by adjusting the switching frequency in accordance with the feedback of the output voltage.
With the VCO based controller, an error amplifier is required and the VCO will synthesise the switching frequency according to the error signal.

The waveforms of the converter during Boost mode of operation are compared in Fig. 6.12 for different loads. The converter was operated at a constant frequency of approximately 45 kHz with a variable phase-shift of 5-2.5 µs for the 2000-300 Ω loads, respectively. During the phase-shift period, where both switches are kept closed by the control circuitry, the DC input and inductor currents pass through the switches. A small increase in DC input current is expected as $V_{\text{DC}}$ appears across both the $L_{\text{DC}}$ and the split-winding. However, because of the small time period and large $L_{\text{DC}}$ this increase is minimal and not noticeable. Therefore, the winding current appears to be remained constant as evident from the simulated and experimental waveforms, justifying the assumption made in (6.19).

According to (6.17), at light loads the switch voltages (identical to the corresponding capacitor voltages) are sinusoidal and load independent because the ZVS frequency is close to the tank resonant frequency, and this effect is apparent from the waveforms for 2000 Ω. However, as load increases the switch voltage given by (6.16) becomes load dependent with a prolonged decay, resulting in a lower ZVS frequency. A slow decay of the switch voltage can be clearly evident from the 300 Ω load, which is close to the critical 220 Ω load, below which ZVS is not possible, for the prototype converter. The ZVS frequency decreased from approximately 74 kHz to 51 kHz when the load was increased from 2000 Ω to 300 Ω.

As predicted by (6.16) and (6.32), the peak switch voltage in this mode is relatively higher than that observed in the Normal mode with the same loads, and hence confirms the operation of the converter in the Boost mode. However, in contrary to the expected Boost operation, the peak amplitude of the voltage waveforms appears to decrease as the load increases even though it is still higher than what would result in the Normal mode of operation. This is
merely because the converter was operated at a constant frequency and could easily be avoided by changing the switching frequency with a constant phase-shift in accordance with (6.5). The comparisons indicate a close agreement with theoretical and experimental results, and therefore clearly demonstrate the validity of the analysis and the applicability of the proposed control technique.

Fig. 6.12 Comparison of waveforms in the Boost mode of operation
The variation of damped resonant (ZVS) frequency for different loads is presented in Fig. 6.13. The variation of measured ZVS frequency closely resembles with that predicted by simulations and theoretically. The small discrepancy is caused by the parasitic effects as discussed previously, and the converter can be operated in the Boost mode by choosing a switching frequency that is below the damped resonant frequency of any particular load. Based on the relationship \( \omega_z = \sqrt{\omega_r^2 - \alpha^2} \), the damped resonant frequency should increase and converge rapidly to the tank resonant frequency as load resistance increases. This is also clearly evident from Fig. 6.13, where the damped resonant frequency converges to the tank resonant frequency (74 kHz) of the prototype.

The variation of peak load voltage with load is shown in Fig. 6.14. For a given switching frequency, as predicted by (6.17), the peak load voltage is load independent for light loads in the Boost mode of operation. However, the peak load voltage converges rapidly to 120 Volts when the load is increased above 1.5 k\( \Omega \). This is because the peak voltage given by (6.16) at heavy loads is largely governed by the ratio \( (T_s/T_z) \) and not by the load dependent \( \omega \), which
decreases with the rapidly declining damped resonant frequency as the converter was operated at a constant switching frequency of 45 kHz with a variable phase-shift (time delay) during the measurements. Therefore, as the decreasing damped resonant frequency reaches the constant switching frequency, the ratio \( \frac{T_s}{T_z} \) becomes unity and the converter enters into the Normal mode of operation, which has a peak voltage of 120 Volts for the given circuit conditions. The small disagreement between the experimental and both the theoretical and simulated values can be attributed to the winding and conduction losses that have not been included during the analysis.

The measured efficiency of the converter is shown in Fig. 6.15 for different power levels. Losses have not been included in both theoretical analysis and simulations, and therefore no comparisons have been made. The low efficiency at light loads can be attributed to the standing losses, and as expected the efficiency improves at heavy loads and is approximately 92%. Thus, the efficiency of the proposed converter is comparable to the typical efficiencies of separate fly-back (60-90%) and forward (60-90%) converters reported in [16] for the same power level.
Fig. 6.15 Efficiency of the converter

Fig. 6.16 shows the variation of peak split-winding inductor ($L_1$) current with load. The measured variation in peak current is in good agreement with that predicted by simulations and theoretical analysis. The small deviation across the entire load range could be attributed to the losses as apparent from the slightly higher measured DC input current than that in both the simulations and theoretical analysis. Even though the change of resonant current flowing through the split-winding inductor is small as given by (6.21), the peak inductor current increases rapidly with increasing load due to the increase in $I_{DC}$ as predicted by (6.23).

Fig. 6.16 Peak inductor current
The load voltage of the proposed topology is expected to vary as the switching frequency is changed. This effect was verified by operating the prototype converter with a constant load resistance, equivalent to 25Watts in the Normal mode, and by changing the switching frequency from 53 kHz to 93 kHz. The experimental and theoretical peak voltages for different switching frequencies as a percentage of that, which would result in the Normal mode of operation, are shown in Fig. 6.17. The shapes of both theoretical and experimental variations are similar but shifted by about 4 kHz along the frequency axis. This is caused by the output capacitance of the switches, which reduces the damped resonant frequency \( f_z \) that depends on the load. As \( f_z \) varies with the load, both peak load voltage and power delivered to the load change with the switching frequency in accordance with (6.16) and (6.24), respectively. At light to medium loads (~2000-500 Ohms), \( f_z \) is relatively constant and thus the variations of peak load voltage would be approximately the same as shown in Fig. 6.17. However, \( f_z \) decreases rapidly as the load increases, resulting in a decline in the peak load voltage. This effect is shown in Fig. 6.17, where the theoretical variations of peak load voltage at high loads (~300 Ohms) tend to move away to the left as expected.

![Figure 6.17](image.png)

Fig. 6.17 Percentage change in peak load voltage with switching frequency
The conventional PPRC exhibits undesirable secondary oscillations due to parasitic components, arising mainly from the switch capacitance and the leakage inductance of the split-winding transformer [3-5]. However, in the split-capacitor PPRC the leakage inductance of the split-winding, being a part of the resonant tank circuit, does not cause any undesirable secondary oscillations [15]. It only effects the inductance that is associated with the resonance by $(2L_1+2kL_1)$, and as a result the damped resonant frequency for a given load increases as the coupling between the two windings of the split-winding is reduced. Fig. 6.18 shows the simulated and theoretical variation of damped resonant frequency as a function of the coupling co-efficient ($k$) for light, moderate and heavy load conditions. In practice, the coupling co-efficient between the two split-windings range from 0.97 to 0.99, and as shown in Fig. 6.18, only about 1.6% increase in the damped resonant frequency is expected for a 3% reduction in coupling co-efficient with 300 Ohms load.

![Graph](image-url)

Fig. 6.18 Effect of coupling on damped resonant frequency

In contrast to the effect of leakage inductance, the impact of parasitic capacitance of the switches, which are in parallel with the resonant capacitors, is to increase the effective resonant capacitance. The resonant capacitance influences the damping factor as well as the
tank resonant frequency of the system, which controls the damped resonant frequency. Hence the variation in damped resonant frequency due to switch capacitance is somewhat complicated. This could be seen from Fig. 6.19, where the switch capacitance has caused the damped resonant frequency to increase for light to medium loads, but not for the heavy loads. At heavy loads, the decrease in damping factor is comparable to the reduction in the tank resonant frequency. Therefore, the damped resonant frequency remains constant as given by (6.13). However, for light to medium loads, the damping factor is relatively low and hence the damped resonant frequency is mainly governed by the tank resonant frequency, which decreases with increasing resonant capacitance.

![Fig. 6.19 Effect of parasitic capacitance on damped resonant frequency](image)

The resonant circuit parameters used in both simulations and theoretical analysis were modified to account for a measured coupling co-efficient of 0.99 and an average switch output capacitance of 200 pF of the prototype. The results are compared in Fig. 6.20 with the measured results demonstrating a very close agreement. However as evident from both Fig. 6.13 and Fig. 6.20, the combined effect of parasitic elements in a split-capacitor PPRC is not
significant and, therefore, all theoretical analysis and simulations presented above were performed without the parasitic effects.

![Graph](image)

Fig. 6.20 The combined effect of the parasitic elements

### 6.4.4 Normal Mode of Operation

In the Normal mode, the switches are essentially operated at the load dependent ZVS (damped resonant) frequency with 50% duty cycle, and only one switch is turned-on at any given moment while the capacitor in parallel with the turned-off switch resonates with L. The resonance causes the capacitor voltage (the voltage seen by the load and the switch) to increase from zero to its peak value defined by the zero volt-second product across $L_{DC}$, and then decay down to zero. At zero volts, the resonance is terminated as the diode of the turned-off switch begins to conduct, short-circuiting the capacitance. Since the voltage across the turned-off switch is essentially zero while the diode is in conduction, the switch can now be turned-on with ZVS. Immediately after turn-on of the switch with ZVS, the switch that has been in conduction is turned-off to initiate another resonant cycle. The steady state operation of the
converter in the Normal mode, as illustrated in Fig. 6.8, can be described by dividing into the following three states.

State1: At \( t=0 \), switch \( S_1 \) is turned-off and \( S_2 \) is turned-on by the control circuit as the voltage across the capacitor \( C_2 \) has reached zero. The inductor current is a maximum and it flows through \( C_1 \) and \( S_2 \), charging the capacitor.

State2: At \( t=t_1 \), the voltage across \( C_1 \) has reached a maximum and starts discharging through \( S_2 \).

State3: At \( t=T_S/2 \), half of the switching period has elapsed. Since the switching period is equal to the damped resonant frequency, voltage across \( C_1 \) has also reached zero at this moment. Due to the body diode of \( S_1 \), \( C_1 \) voltage cannot reverse as the body diode starts conducting clamping the voltage across \( S_1 \) to 0. The control circuit detects this and toggles the states of \( S_1 \) and \( S_2 \) with ZVS, repeating State1 to State3.

### 6.4.5 Analysis on the Normal Mode

Since the resonant circuits formed during both Normal and Boost modes of operations are nearly identical, the analytical solutions that describe the operation of the converter in these two modes are similar. In fact, the only difference between these two modes of operation is the positive time delay introduced during the boost mode of operation of the converter. The normal mode of operation takes place when this time delay is reduced to zero, by operating the switches with a 50% duty cycle at a switching frequency that is equal to the damped resonant frequency. Therefore, analytical solutions that describe the operation of the converter in Normal mode can be deduced by equating the time delay to zero \( (T_s = T_z) \) in the solutions that were derived for the boost mode of operation. Thus, substituting \( T_s = T_z \) in (6.16), the voltage across the \( C_1 \) in the Normal mode can be obtained as,
\[ v_{c_1} = \pi V_{\text{dc}} \frac{2(\alpha^2 + \omega_z^2)}{\omega_z^2 \left( e^{-\frac{\alpha T_z}{2}} + 1 \right)} e^{-\alpha t} \sin(\omega_z t) \]  

(6.32)

Under light loads \( \alpha \) is small; therefore (6.32) can be reduced to,

\[ v_{c_1} \approx \pi V_{\text{dc}} \sin(\omega_z t) \]  

(6.33)

Equation (6.33) implies that the peak value of the output voltage is independent of the load under light load conditions. Using (6.21), the steady state inductor current is determined for \( t \leq \frac{T_z}{2} \) as,

\[ i_L = \frac{-2\pi V_{\text{dc}}}{L\omega_z^2 \left( e^{-\frac{\alpha T_z}{2}} + 1 \right)} \left[ \omega_z e^{-\alpha t} \cos(\omega_z t) + \alpha e^{-\alpha t} \sin(\omega_z t) \right] + \frac{\pi V_{\text{dc}} \left( 1 - e^{-\frac{\alpha T_z}{2}} \right)}{L\omega_z \left( e^{-\frac{\alpha T_z}{2}} + 1 \right)} \]  

(6.34)

Under light loads, (6.34) reduces to,

\[ i_L \approx \frac{-\pi V_{\text{dc}}}{L\omega_z} \cos(\omega_z t) \]  

(6.35)

Similarly the power delivered to the load (\( P_{\text{out}} \)), when the converter is operating in Normal mode can be calculated from (6.24) as given by,

\[ P_{\text{out}} = \frac{2\pi^2 V_{\text{dc}}^2}{RT_z} \left[ \frac{\left( \omega_z^2 + \alpha^2 \right) \left( 1 - e^{-\alpha T_z} \right)}{\omega_z^2 \alpha \left( 1 + e^{-\frac{\alpha T_z}{2}} \right)^2} \right] \]  

(6.36)

At light loads,

\[ P_{\text{out}} \approx \frac{\pi^2 V_{\text{dc}}^2}{2R} \]  

(6.37)
Chapter 6  A Novel Bock-Boost PPRC

The diode current and the switch current can be calculated from (6.26) and (6.27), where \( i_R \) and \( I_{DC} \) can be obtained by substituting \( T_s = T_z \) in (6.28) and (6.29) respectively.

\[
i_R = \frac{V_{Cl}}{R} = \pi V_{DC} \frac{2(\alpha_z^2 + \omega_z^2)}{R \omega_z^2} e^{-\alpha t} \sin(\omega_z t) \tag{6.38}
\]

\[
I_{DC} = \frac{P_{out}}{\eta V_{DC}} = \frac{2\pi^2 V_{DC}}{\eta RT_z} \left[ \frac{\omega_z^2 + \omega_z^2(1 - e^{-\alpha t})}{\omega_z^2 \alpha \left(1 + e^{-\alpha t}\right)^2} \right] \tag{6.39}
\]

where \( \eta \) is the efficiency of the converter.

Under light loads, (6.38) and (6.39) reduce to,

\[
i_R \approx \frac{\pi V_{DC}}{R} \sin(\omega_z t) \tag{6.40}
\]

\[
I_{DC} \approx \frac{\pi^2 V_{DC}}{2R \eta} \tag{6.41}
\]

### 6.4.6 Results on Normal Mode

Experimental results obtained from the prototype converter that was introduced in Section 6.3.3 together with simulation results were used to validate the analysis presented on the Normal mode. Fig. 6.21 compares the theoretical waveforms with simulated and experimental waveforms when the converter was operated in the Normal mode for three different loads. The switches were operated at a load dependent damped resonant (or ZVS) frequency, which was decreased from approximately 74 kHz to 51 kHz as the load was increased from 2000-300 Ohms, respectively. According to (6.33), the voltages across the switches are sinusoidal and load independent at light loads because the ZVS frequency is
close to the tank resonant frequency, and this effect is apparent from the waveforms for 2000 Ohms. However, as the load increases the switch voltage given by (6.32) becomes load dependent with a prolonged decay resulting in a lower ZVS frequency. A slow decay of the switch voltage is clearly evident from the 300 Ohms load, which is close to the critical 220 Ohms load, below which the ZVS is not possible for the prototype converter. The comparisons indicate a close agreement with theoretical and experimental results, and therefore clearly demonstrate the validity of theoretical analysis.

As predicted by (6.33), the peak load voltage in the Normal mode of operation is load independent for light loads and is equal to approximately ‘π’ times the input DC voltage. It is noted that for this prototype the load voltage is load independent for loads less than 500 Ω. However, when the load resistance decreases below 500 Ω, the peak load voltage becomes dependent on the load and rises rapidly due to significant increase in α, as given by (6.32). The variation of peak load voltage with load is shown in Fig. 6.22, where the small disagreement between the experimental and both the theoretical and simulated values can be attributed to the winding and conduction losses that have not been included in the analysis.
Fig. 6.21 Comparison of waveforms in the Normal mode of operation

(a) Theoretical (b) Simulated (c) Experimental
6.5 Conclusions

Present high voltage charging schemes that are being used to recharge the storage capacitors in electric fence energizers have many deficiencies, especially at elevated power levels and frequencies. As such, resonant and quasi-resonant techniques that can be utilised to improve the efficiency and reduce EMI emitted by these traditional, high-voltage charging schemes have been presented. An active-clamping technique and a LCD snubber design that can reduce undesirable parasitic effects of a typical HV charger have been presented and drawbacks of such quasi-resonant techniques have been discussed.

Moreover a novel control technique that facilitates the operation of PPRCs in Normal, Buck and Boost modes with ZVS has also been described. The capability of this technique to operate over a wide load range without compromising the ZVS makes it an ideal candidate for HV charger systems. The technique uses a split-capacitor topology and operates the converter essentially below the damped resonant frequency to realize the Boost mode with higher than nominal voltage gains. A voltage gain that is lower than nominal gains is realized by operating the converter above the damped resonant frequency. The converter has been represented by a
model, through which explicit analytical expressions have been derived for accurate prediction of its performance in the Normal and Boost modes. The validity of both the model and theoretical analysis has been verified by simulations and experimental results under various operating conditions. The impact of parasitic elements on the performance has also been analysed in detail with experimental verification to show that their effect is marginal and can be omitted during the design stage.
6.6 References


7.1 Buck Mode of Operation

A novel split-capacitor type push-pull parallel-resonant converter (SC-PPRC) topology, which can be operated with ZVS at a variable switching frequency, and regulate the output power with a higher (Boost) or lower (Buck) peak load voltage than that which would result in a conventional PPRC (Normal mode), was proposed in the previous chapter. Explicit analytical expressions were derived for accurate prediction of the performance of the Normal and Boost mode operations of the proposed topology. The validity of both the proposed control technique and theoretical analysis on the Normal and Boost mode operations were verified by simulations and experimental results and the impact of parasitic elements on the performance has been covered in Chapter 6 [1-5].

In contrast to both Normal and Boost modes of operation, theoretical analysis of Buck mode is complicated by the formation of two different resonant circuits in each switching cycle. Thus, in this chapter, a comprehensive mathematical model for the Buck mode of operation is presented separately from the Normal and Boost modes to provide an in-depth analysis of the Buck mode of operation. The solutions that are presented provide an insight into the behaviour of the converter under different operating conditions and will be vital at the design and optimization stages. Explicit mathematical solutions for all circuit variables are presented, but
as they are complex in nature, a numerical approach is adopted when evaluating the performance. Although the proposed model can be considered as semi-analytic as the solutions require numerical evaluations, the analytical solutions themselves provide valuable information with regard to the expected behaviour of the converter, which would otherwise not be available through a complete numerical solution [1, 3].

Fig. 7.1 shows the proposed SC-PPRC topology that allows the converter to operate in Buck, Boost or Normal mode with a lower or higher peak load voltage than that, which would result in the Normal mode of operation. The design incorporates a bifilar split-winding transformer, and the resonant capacitor is split equally \((C_1 = C_2 = C)\) and connected in parallel with the two switches, which are operated with equal duty cycles to avoid core saturation. As explained previously in Chapter 6, this topology not only allows for the converter to operate at a variable switching frequency, which can be higher or lower than the damped resonant frequency, but also negates any undesirable oscillations caused by the leakage inductance and switch capacitance. The equivalent circuit model shown in Fig. 7.2, which was derived in Chapter 6, is used to derive analytical solutions for the circuit variables when the converter is operated in the Buck mode [2-20].

![Fig. 7.1 The proposed SC-PPRC topology](image)
Chapter 7  
PPRC Operation in Buck Mode

Fig. 7.2 A model for the proposed split-capacitor PPRC

Fig. 7.3 Switch voltages of the proposed PPRC converter in the Buck Mode

The Buck mode of operation of the converter is similar to that of the Boost mode except now the switches are operated with a longer off-time, as can be seen from the typical waveforms shown in Fig. 7.3. The variable time delay \((0-t_2)\) that is introduced into the switching signal during the Buck mode of operation can be considered as being effectively negative with respect to the operation in the Normal mode. This is because the time delay (or phase-shift) shifts the voltage waveforms in such a way to cause an overlap and therefore produces a higher switching frequency than ZVS frequency in the Normal mode \(f_{z,\text{Normal}}\).
Unlike in the Normal and Boost modes of operation, the converter in the Buck mode can be represented by two different resonant circuit topologies during each half switching cycle. This is because during the time-delay (phase-shift) both switches are essentially off and therefore the converter operates in a state, where the two capacitors and the effective inductance L in Fig. 7.2 form a resonant circuit. In contrast, in the remaining period, during which only one switch is always on while the other is essentially off, the converter operates in a state where L resonates with one capacitor. Because both capacitors are resonating during the phase-shift, the average midpoint voltage of the split winding, given by \( \frac{v_{c1}+v_{c2}}{2} \) for any given \( V_{DC} \), would therefore be higher in comparison to the Normal mode. Consequently, in order to maintain zero ‘volt-second product’ across \( L_{DC} \) for the same \( V_{DC} \), the converter operates in the Buck mode with a lower peak resonant voltage than that, which would result in the Normal mode. The amount of reduction in peak voltage depends on the time-delay (phase-shift) and thus can be controlled by varying the switching frequency through the phase-shift to regulate the power output. Moreover, the resulting damped resonant frequency in the Buck mode is different to damped resonant or ZVS frequency \( f_{z,Normal} \) that is derived for the Normal mode of operation. The complete steady state operation of the converter in Buck mode over a half switching period, as shown in Fig. 7.3, can be described using the following five states.

State1: At \( t=0 \), switch \( S_1 \) is turned-off and \( S_2 \) is held-off by the control circuit as the voltage across the capacitor \( C_2 \) has not reached zero. The inductor current is reaching a maximum and it flows through the capacitors, charging \( C_1 \) and discharging \( C_2 \).

State2: At \( t=t_1 \), the charging capacitor \( C_1 \) and the discharging capacitor \( C_2 \) have reached equal voltages, and thus the load voltage given by \( v_{C1}-v_{C2} \) is zero and the inductor current is a maximum at this point.
State3: At \( t=t_2 \), the voltage across \( C_2 \) has reached zero and \( C_1 \) is still charging. Due to the body diode of \( S_2 \), \( C_2 \) voltage cannot reverse as the body diode starts conducting clamping the voltage across \( S_2 \) to 0. The control circuit detects this and turns-on \( S_2 \) with ZVS.

State4: At \( t=t_3 \), the voltage across \( C_1 \) has reached a maximum and starts discharging through \( S_2 \).

State5: At \( t=T_S/2 \), half of the switching period has elapsed and the control circuit turns-off \( S_2 \). The capacitor \( C_2 \) starts charging whereas \( C_1 \) is still being discharged repeating State1 to State5.

### 7.2 Analysis on the Buck Mode

As described above, the Buck operation for any given load can be guaranteed by operating the converter at a switching frequency (\( f_S \)) that is higher than \( f_{z,Normal} \). The higher \( f_S \) is achieved by operating the converter at less than 50% duty cycle with shorter ‘on’ time. If \( T_S \) and \( T_{z,Normal} \) are the time periods that correspond to \( f_S \) and \( f_{z,Normal} \), respectively, then the following relationship holds.

\[
T_S = 2T_{on} + 2t_2 < T_{z,Normal}
\]  \hspace{1cm} (7.1)

where \( t_2 \) is the variable time delay that corresponds to any given phase-shift, and \( T_{on} \) is the ‘on’ time of each switch, as illustrated in Fig. 7.3.

The voltage waveforms across \( C_1 \) and \( C_2 \) are identical in nature and shifted by half a switching period. Therefore, only one half of the switching period (0 to \( T_S/2 \)) is considered for analysis. During the Buck operation the switching cycle constitutes two different resonant topologies depending on the switch state as described below.
7.2.1 **Operation when Both Switches are Turned-off**

This is the time period of phase-shift \((t = 0\) to \(t_2)\), during which both switches are off.

Applying KCL to the nodes at \(v_{C1}\) and \(v_{C2}\) in Fig. 7.2 yields,

\[
i_{C1} + i_R + i_L = \frac{I_{DC}}{2} \quad (7.2)
\]

\[
i_{C2} - i_R - i_L = \frac{I_{DC}}{2} \quad (7.3)
\]

The currents \(i_{c1}\) and \(i_{c2}\) are given by,

\[
\frac{dv_{C1}}{dt} = \frac{1}{C_1} i_{C1} \quad (7.4)
\]

\[
\frac{dv_{C2}}{dt} = \frac{1}{C_2} i_{C2} \quad (7.5)
\]

The voltages across inductor \(L\) and the reflected load \(R\) are given by,

\[
v_L = L \frac{di_L}{dt} \quad (7.6)
\]

\[
v_R = i_R R \quad (7.7)
\]

where \(R = R_{load}/n^2\) and ‘n’ is the turns ratio of the isolation transformer. But,

\[
v_R = v_L = v_{C1} - v_{C2} \quad (7.8)
\]

From (7.2), (7.3) and (7.8),

\[
i_{C1} + i_{C2} = I_{DC} \quad (7.9)
\]

\[
i_{C1} - i_{C2} = C \frac{dv_R}{dt} = -2(i_R + i_L) \quad (7.10)
\]

where \(C = C_1 = C_2\)
Differentiating (7.10),

\[ C \frac{d^2 v_R}{dt^2} = -2 \frac{di_R}{dt} - 2 \frac{di_L}{dt} \]  \hspace{1cm} (7.11)

Substituting from (7.6)-(7.8),

\[ \frac{d^2 v_R}{dt^2} + \frac{2}{RC} \frac{dv_R}{dt} + \frac{2}{LC} v_R = 0 \]  \hspace{1cm} (7.12)

Equation (7.12) resembles (6.11) and therefore the solution will be of the form,

\[ v_R = e^{-\alpha_1 t} [A_1 \sin(\omega_{z1} t) + A_2 \cos(\omega_{z1} t)] \]  \hspace{1cm} (7.13)

where \( A_1 \) and \( A_2 \) are constants defined by the circuit conditions while \( \alpha_1 \) and the damped angular resonant frequency \( \omega_{z1} \) are given by,

\[ \alpha_1 = \frac{1}{RC}, \quad \omega_{z1} = \sqrt{\omega_{r1}^2 - \alpha_1^2} \quad \text{and} \quad \omega_{r1} = \sqrt{\frac{2}{LC}} \]

Using (7.4)-(7.5), (7.6)-(7.8) and (7.10),

\[ V_{C1} = \frac{I_{DC}}{2C} t + \frac{V_R}{2} + D \]  \hspace{1cm} (7.14)

\[ V_{C2} = \frac{I_{DC}}{2C} t - \frac{V_R}{2} + D \]  \hspace{1cm} (7.15)

where \( D \) is a constant. The inductor current \( i_L \), obtained using (7.7), (7.10) and (7.13), is given by,

\[ i_L = e^{-\alpha_1 t} \left\{ \left[ C(A_1 \omega_{z1} - A_2 \alpha_1)/2 + A_2/R \right] \cos(\omega_{z1} t) \right\} \right\} \\
- \left\{ \left[ C(A_1 \alpha_1 + A_2 \omega_{z1})/2 - A_1/R \right] \sin(\omega_{z1} t) \right\} \]  \hspace{1cm} (7.16)

The current in the split winding can thus be determined using the relationship \( I_{DC}/2 \pm i_L \).
7.2.2  Operation when One Switch is Turned-on

Consider the time period when switch $S_1$ is turned-off and $S_2$ is turned-on ($t = t_2$ to $T_{s/2}$). The node at $v_{C2}$ is shorted and applying KCL to the node at $v_{C1}$ in Fig. 7.2 results in,

$$i_{c1} + i_L + i_R = \frac{I_{DC}}{2} \quad (7.17)$$

The current through $C_1$ is expressed by,

$$C \frac{dv_{C1}}{dt} = i_{c1} \quad (7.18)$$

And the voltages across the inductor $L$ and the reflected load $R$ are given by,

$$v_L = L \frac{di}{dt} \quad (7.19)$$

$$v_R = i_R R \quad (7.20)$$

From Fig. 7.2,

$$v_R = v_{C1} = v_L \quad (7.21)$$

By manipulating (7.17) to (7.21),

$$\frac{d^2v_{C1}}{dt^2} + \frac{1}{RC} \frac{dv_{C1}}{dt} + \frac{1}{LC} v_{C1} = 0 \quad (7.22)$$

Similar to (7.12), the solution for (7.22) is given in the following form by,

$$v_{C1} = e^{-\alpha_2 t} [B_1 \sin(\omega_{22}t) + B_2 \cos(\omega_{22}t)] \quad (7.23)$$

where $B_1$ and $B_2$ are constants defined by the circuit conditions while $\alpha_2$ and the damped angular resonant frequency $\omega_{22}$ are given by,

$$\alpha_2 = \frac{1}{2RC}, \quad \omega_{22} = \sqrt{\frac{1}{LC} - \alpha_2^2} \quad \text{and} \quad \omega_{r2} = \sqrt{\frac{1}{LC}}$$
From (7.17), (7.18) and (7.22),

\[
i_L = \frac{I_{DC}}{2} + e^{-\alpha_2 t} \left\{ \frac{[C(B_1 \omega_2 + B_2 \omega_2) - B_1/R] \sin(\omega_2 t)}{[-B_2/R + C(B_1 \omega_2 - B_2 \alpha_2) \cos(\omega_2 t)]} \right\}
\]

(7.24)

The current in the split winding can be determined from \((I_{DC}/2) \pm i_L\). The five unknown constants of \(A_1\), \(A_2\), \(B_1\), \(B_2\) and \(D\) can now be determined by considering the following conditions.

### 7.2.3 Determination of constants

From the above analysis and Fig. 7.3, it is evident that the load voltage \(v_R\) is repetitive in each half cycle that can be represented by two different solutions, depending on the resonant topology.

As evident from Fig. 7.3, \(v_{C1} = 0\) at \(t = 0\) and \(v_{C2} = 0\) at \(t = t_2\). Thus, from (7.14) and (7.15),

\[
D = -\frac{A_2}{2}
\]

(7.25)

\[
\frac{I_{DC}}{C}t_2 = X_1A_1 + X_2(1 + A_2)
\]

(7.26)

where \(X_1 = e^{-\alpha_2 t_2 \sin(\omega_{z1}t_2)}\) and \(X_2 = e^{-\alpha_2 t_2 \cos(\omega_{z1}t_2)}\)

At \(t = t_2\) the load voltages given by (7.13) and (7.23) should be of the same value, and hence,

\[
X_1A_1 + X_2A_2 = X_3B_1 + X_4B_2
\]

(7.27)

where \(X_3 = e^{-\alpha_2 t_2 \sin(\omega_{z2}t_2)}\) and \(X_4 = e^{-\alpha_2 t_2 \cos(\omega_{z2}t_2)}\)

The magnitude of load voltage is equal and opposite in polarity during every half cycle, which implies,

\[
v_{R(t=0)} = -v_{R(t=T_2/2)}
\]

(7.28)
By substituting appropriate terms and simplifying (7.28),

$$A_2 = -X_5 B_1 - X_6 B_2$$  \hspace{1cm} (7.29)

where $X_5 = e^{-\alpha_2 (T_s/2)} \sin(\omega z_2 (T_s/2))$ and $X_6 = e^{-\alpha_2 (T_s/2)} \cos(\omega z_2 (T_s/2))$

Under steady state conditions, the ‘volt-second product’ across $L_{DC}$ over a half cycle is zero. Therefore,

$$\int_{0}^{T_s/2} (v_{mid} - V_{DC}) dt = 0$$  \hspace{1cm} (7.30)

where $v_{mid}$ is the midpoint voltage of the split-winding which is given by,

$$v_{mid} = \frac{1}{2} \left( v_{c1} + v_{c2} \right) = \frac{I_{DC}}{2C} t + D \text{ (for } 0 < t < t_2)$$

And

$$v_{mid} = \frac{1}{2} v_R \text{ (for } t_2 < t < T_s/2)$$

Substituting for $v_{mid}$ in (7.30) gives,

$$\frac{V_{DC} T_s}{2} = \frac{I_{DC} t_2^2}{4C} - \frac{A_2 t_2}{2} + X_7 B_1 + X_8 B_2$$  \hspace{1cm} (7.31)

where,

$$X_7 = \left[ \frac{e^{-\alpha_2 t}}{2(\alpha_2^2 + \omega^2 z_2^2)} \left[ -\alpha_2 \sin(\omega z_2 t) - \omega z_2 \cos(\omega z_2 t) \right] \right]_{t_2}^{T_s/2}$$

and

$$X_8 = \left[ \frac{e^{-\alpha_2 t}}{2(\alpha_2^2 + \omega^2 z_2^2)} \left[ -\alpha_2 \cos(\omega z_2 t) + \omega z_2 \sin(\omega z_2 t) \right] \right]_{t_2}^{T_s/2}$$

Now, using (7.25) to (7.30), it can be shown that,

$$B_2 = \frac{V_{DC} (T_s/2) - I_{DC} (t_2^2/4C + t_2 U_1/C/U_3)}{U_2 - U_1 U_4/U_3}$$  \hspace{1cm} (7.32)
where \( U_1 = X_5 t_2 / 2 + X_7 \), \( U_2 = X_6 t_2 / 2 + X_8 \), \( U_3 = X_3 - X_5 \), \( U_4 = X_4 - X_6 \) and

\[
B_1 = \frac{I_{DC} t_2}{C} - B_2 U_4 \quad (7.33)
\]

\[
A_2 = \frac{I_{DC}}{C} t_2 - X_3 B_1 - X_4 B_2 \quad (7.34)
\]

\[
A_1 = \left[ I_{DC} t_2 / C - (1 + X_2) A_2 \right] / X_1 \quad (7.35)
\]

Thus, for any given \( V_{DC} \), \( I_{DC} \), phase-shift and load, the constants \( A_1 \), \( A_2 \), \( B_1 \), \( B_2 \) and \( D \) can be calculated to determine the behaviour of the converter. However, both \( I_{DC} \) and the time that correspond to the phase-shift depend on the load, input voltage and associated losses and need to be estimated as they are not readily available. Following two conditions can be used to estimate \( I_{DC} \) and the time that correspond to the phase-shift.

If the efficiency of the converter is assumed to be \( \eta \), then from power balance,

\[
\eta V_{DC} I_{DC} = \frac{1}{(T_s/2)R} \int_0^{T_s/2} v_R^2 dt \quad (7.36)
\]

Which yields,

\[
\eta V_{DC} I_{DC} = \frac{1}{4(T_s/2)R} \left[ \frac{A_3^2}{\alpha_1^2 + \omega_{z1}^2} X_9 + \frac{B_3^2}{\alpha_2^2 + \omega_{z2}^2} X_{10} \right] \quad (7.37)
\]

where,

\[
X_9 = \left[ e^{-2\alpha_1 t} \left[ -2\alpha_1 \sin(\omega_{z1} t + \varphi_1) \right. \right. \\
\left. - 2\omega_{z1} \cos(\omega_{z1} t + \varphi_1) \sin(\omega_{z1} t + \varphi_1) - \frac{\omega_{z1}^2 e^{-2\alpha_1 t}}{\alpha_1} \right]_{0}^{T_s/2}
\]

\[
X_{10} = \left[ e^{-2\alpha_2 t} \left[ -2\alpha_2 \sin(\omega_{z2} t + \varphi_2) \right. \right. \\
\left. - 2\omega_{z2} \cos(\omega_{z2} t + \varphi_2) \sin(\omega_{z2} t + \varphi_2) - \frac{\omega_{z2}^2 e^{-2\alpha_2 t}}{\alpha_2} \right]_{0}^{T_s/2}
\]
\[ A_3 = \sqrt{A_1^2 + A_2^2}, \quad B_3 = \sqrt{B_1^2 + B_2^2} \]

\[ \tan(\varphi_2) = \frac{A_2}{A_1}, \quad \tan(\varphi_3) = \frac{B_2}{B_1} \]

The time \( t_2 \) that corresponds to the phase-shift can be estimated from the condition that \( i_L \) given by (7.16) and (7.24) at \( t = t_2 \), be of the same value. This implies,

\[
- e^{-a_1 t_2} \left[ \frac{[C(A_1 \omega z_1 - A_2 a_1)/2 + A_2/R] \cos(\omega z_1 t_2)}{[C(A_1 a_1 + A_2 \omega z_1)/2 - A_1/R] \sin(\omega z_1 t_2)} \right] = \frac{I_{DC}}{2} + e^{-a_2 t_2} \left[ \frac{[C(B_1 a_2 + B_2 \omega x_2) - B_1/R] \sin(\omega x_2 t_2)}{[B_2/R + C(B_1 \omega x_2 - B_2 a_2) \cos(\omega x_2 t_2)]} \right]
\]  

(7.38)

As evident from (7.37)-(7.38), it is extremely difficult to solve explicitly for both \( I_{DC} \) and \( t_2 \) and, therefore, the performance of the converter is evaluated by adopting a numerical method to solve for \( I_{DC} \) and \( t_2 \).

### 7.3 Results and Discussion

The validity of the proposed mathematical model is verified by comparing the predicted performance with simulations and measured results of the 150 W prototype SC-PPRC introduced in Chapter 6. The converter can be operated in the Buck mode using a closed-loop controller and a microprocessor. The microprocessor based controller initially operates the converter in the Normal mode and adjusts the switching frequency with a negative time-delay in accordance with the feed-back of the output voltage and relationship given by (7.1), to maintain the desired load voltage in the Buck mode. However, for simplicity and as proof of concept, the prototype converter was implemented with an open loop controller.

As illustrated in Fig. 6.13, the maximum value of \( f_{z,\text{Normal}} \) for the prototype is about 74 kHz, and therefore a switching frequency of approximately above 74 kHz guarantees the Buck mode...
of operation of the converter for all loads. The prototype was initially operated at a constant
switching frequency of 94 kHz, which was arbitrarily chosen with the view to achieve a wider
variable switching frequency over the full load range. Theoretical, simulated and measured
capacitor voltage and split-winding current of the converter in the Buck mode for a 5 kΩ load
resistance are shown in Fig. 7.4. According to (7.1), a switching frequency of 94 kHz results
in 1.4 µs phase-shift, which forces the converter into the Buck mode of operation. This is
evident from the overlap of voltages and the lower peak capacitor voltage (~ 400 V) than 471
V, which would be produced in the Normal mode for a 5 kΩ load resistance at 150 V DC input
voltage. As evident from Fig. 7.4, both simulated and measured waveforms are in very good
agreement with those predicted through the mathematical model.

![Waveforms for 5 kΩ at 94 kHz with 1.4µs phase-shift](image)

Fig. 7.4 Waveforms for 5 kΩ at 94 kHz with 1.4µs phase-shift

Fig. 7.5 shows the waveforms of Buck mode for a 1.7 kΩ load resistance. Despite the large
change in the load resistance from 5 kΩ to 1.7 kΩ, there is no apparent reduction in the peak
temperature, but a slight increase in the resonant period of capacitors can be noted. This is
expected because the peak capacitor voltage in the Normal mode is load independent for light
loads approximately above 1.3 kΩ as derived in Chapter 6. Therefore, according to (7.1) and
because of the very small reduction in $f_{z,\text{Normal}}$, the increase in phase-shift that is required to
maintain a constant switching frequency of 94 kHz is minimal, and as such the reduction in the peak voltage is barely noticeable. Although this has been the case, a change in load is evident from the offset of the split winding current in comparison to 5 kΩ load resistance. The offset is caused by the DC input current. Thus the off-set indicates an increase in input power that is required to cater for the increased load. A notable and important feature at the 1.7 kΩ load resistance is the slow decay of the capacitor voltage during the overlap. It is in the verge of not reaching the zero value to enable ZVS, as discussed in detail below.

![Waveforms for 1.7 kΩ at 94kHz with 1.6µs phase-shift](image)

Fig. 7.5 Waveforms for 1.7 kΩ at 94kHz with 1.6µs phase-shift

It was mentioned earlier that when the switching frequency was held constant at 94 kHz the peak voltage should remain approximately constant for load resistances above 1.3 kΩ because of approximately constant $f_{z,\text{Normal}}$ over this load range. However, for load resistances below 1.3 kΩ, a significant reduction in peak voltage and subsequent increase in phase-shift are expected because of the sharp drop (74 kHz to 54 kHz) in $f_{z,\text{Normal}}$ for load resistances below 1.3 kΩ. In contrary, the experimental results suggest that the operation of the converter in Buck mode at 94 kHz for load resistances below 1.7 kΩ is not possible. This is because for loads below 1.7 kΩ, the capacitor voltage does not resonate down to zero during the phase-
shift, during which both switches are on. Consequently, the converter cannot be operated in the Buck mode at 94 kHz with ZVS below 1.7 kΩ. Hence 1.7 kΩ can be regarded as the critical load resistance below which the Buck operation of the converter at 94 kHz is not possible.

![Graph](image)

(a) Peak load voltage vs load

![Graph](image)

(b) Phase-shift (time delay) vs \( f_s \)

**Fig. 7.6** Behaviour of the converter during Buck operation

The results in Fig. 7.4 and Fig. 7.5 imply that the operation of the converter in the Buck mode for variable loads can be realized with a constant switching frequency and a variable phase-shift. However a critical load, below which the Buck operation of the converter is not possible, exists for every switching frequency. Similarly, the converter can also be operated at a
variable switching frequency with either constant or variable phase-shift to regulate the power delivered to a variable or constant load, respectively. In this situation there is a critical switching frequency above which the Buck operation of the converter is not possible. The different options that are available to operate the converter in the Buck mode are illustrated in Fig. 7.6, which suggests that the converter in the Buck mode operates with a unique phase-shift for any given switching frequency and load. The behaviour of the converter under a variable load for any given switching frequency is shown in Fig. 7.6(a). As can be seen, for every switching frequency there exists a critical load resistance below which the converter cannot be operated in the Buck mode with ZVS. In contrast, Fig. 7.6(b) demonstrates the behaviour of the converter under a variable switching frequency for any given load. In this situation, the maximum or critical phase-shift of each load resistance is clearly visible. As the phase-shift is reduced, the switching frequency also reduces and approaches the corresponding \( f_{\text{z,Normal}} \) of the load given in Fig. 6.13, when the phase-shift is zero.

![Fig. 7.7 Boost, Buck and Normal modes of operation](image)

The operation of the converter in all three modes is shown in Fig. 7.7 for a moderate load. The percentage change in peak load voltage computed with respect to the peak load voltage in the Normal mode is used to help distinguish the transition from one mode of operation to another.
The converter was initially operated at 74 kHz in the Normal mode with ZVS and zero time delay (phase-shift). The switching frequency was then gradually increased up to 94 kHz by incorporating a negative and variable phase-shift, and as evident, the converter moved into the Buck mode of operation with a declining peak voltage. Similarly, the switching frequency was decreased down to 53 kHz by incorporating a positive and variable time delay (phase-shift), and as expected the converter operated in the Boost mode with a higher peak load voltage. The experimental and theoretical behaviours of the converter are in reasonably good agreement and show a similar trend despite the small discrepancy caused by the parasitic elements that have not been accounted for. The parasitic elements and their impact have been addressed in detail in Chapter 6.

Fig. 7.8 shows the measured efficiency of the converter, operated at 94 kHz with varying time delay (phase-shift) for different loads. The converter is efficient at heavy loads but at light loads the efficiency is poor. The poor efficiency at light loads is expected as standing losses become significant.

![Efficiency of the converter in Buck mode](image_url)
7.4 Conclusions

A Split-Capacitor PPRC and its Buck mode of operation with ZVS have been described. The converter, when operated at a variable switching frequency, which is higher than the ZVS frequency of the Normal mode, provides a lower peak output voltage than that, which results in the conventional Normal mode. The performance of the converter in the Buck mode has been analyzed using a mathematical model. The proposed mathematical model provides an insight into the behaviour of the converter in the Buck mode and thus can be regarded as a valuable tool for design and optimization of SC-PPRC. The validity of the mathematical model has been verified by simulations and measured performance of a 150 W prototype converter, and results indicate that the converter can be operated in the proposed Buck mode with ZVS but has a critical load below which its operation in the Buck mode is not possible.
7.5 References


8.1 Introduction

An electric fence structure facilitates the transmission of energizer pulses across farm land, and thus it is necessary to layout the electric fence in such a manner that it enhances the propagation of high voltage pulses. To facilitate a proper fence layout, an insight into the behaviour of various fence structures subject to high voltage pulses of different shapes is essential. As such, attempts have been made in the past to mathematically model the transient pulse behaviour of electric fence structures using the transverse electro-magnetic (TEM) propagation model. To reduce the complexity of such analysis, these techniques consider the electrical parameters of the fence to be linear and frequency independent. As a result of these simplifications, the solutions lacked the ability to fully characterize the transient propagation behaviour of an electric fence structure [1-12].

Therefore, due to complexity associated with modelling electric fence structures, to-date, a comprehensive study on electric fencing systems has neither been published nor technology developed to deliver optimum performance. Electric fence manufacturers currently employ empirical rules together with a trial and error design approach to develop electric fence equipment. The general manufacturing trend is to develop electric fence energizers that can generate pulses with the highest voltage and the longest pulse duration, allowed by the safety
standards (IEC 60335-1, IEC 60335-2-76 and UL 69), in order to overcome the deficiencies in the electric fence systems. Hence, most electric fence systems are either over rated or not optimally designed, resulting in systems that are less effective [1-12].

The increasing pulse energy levels produced by fence energizers pose a danger of delivering a lethal shock to a human or an animal. Although reports on such incidents are rare, there have been incidents that caused loss of human life involving electric fence installations. Moreover, it is essential that electric fences are installed assuring safety of humans and animals even in abnormal conditions. As a consequence, the maximum output energy and current that can be generated by an energizer are limited by international safety standards enforced on electric fencing equipment to assure safety of humans and animals. The low output limits imposed by these safety standards significantly reduce the effective range of electric fence installations narrowing down the room for inefficiently laid-out fence networks. Therefore, manufacturers are continuously searching for methods to optimize the design of electric fence systems to improve the effective range of their system over the competitors. The demand for highly optimized fence systems has driven the development of the concept of “intelligent electric fence system”, where the system is capable of sensing changes in operating environment and adapts accordingly to improve the efficacy [1-12].

In addition, the development of cutting-edge farm management systems demands that the electric fence system be an integral part of them. In such systems, the fence structure is intended to be used as a communication channel, otherwise only possible through expensive long range radio frequency (RF) transmissions due to the limited coverage area of low cost, common RF solutions, such as ZigBee, WiFi or Bluetooth. This approach can reduce the cost of intelligent farm management systems to an affordable value, promoting the wide spread use of such technology. The communication is realized in the form of modulated high voltage
pulse signals transmitted along the fence, through which all the components in the farm
management system that are scattered across the farm land communicate with each other and
the central management system [1-5, 8].

Above mentioned technological advances in electric fence systems are only possible through
detailed analysis of pulse propagation characteristics of electric fences. As such, to fulfil this
requirement a comprehensive semi-analytical model that facilitates a study on propagation
characteristics of short high voltage (HV) pulses along typical multi-wire electric fence
structures is presented. The proposed technique provides an insight into the behaviour of
typical electric fence lines under different operating and fence conditions, and thus serves as a
tool during optimization and design of electric fence systems for livestock control. The
technique can also be effectively used to determine the most appropriate fence layout and
pulse parameters for efficient pulse transmission for any given farm conditions, enabling the
design of “intelligent electric fence systems”. Moreover, this technique can also be used at
the design stage of electric fence based communication media in farm managements systems.

8.2 Modelling the Electric Fence

Electric fence structures range from simple single wire fences with earth return to complicated
multi-wire fences that run for hundreds of kilometres, depending on the application and the
budget allocated for construction costs [7-8]. Amongst the standard electric fence
installations illustrated in Fig. 8.1, the structures shown in Fig. 8.1(a) and Fig. 8.1(d) are the
most common constructions. An example layout of a fence is shown in Fig. 8.2. A
construction similar to Fig. 8.1(d) is typically used for the main branch of the fence that leads
to the furthest end of the farm and feeds all branches as indicated by the thick line in Fig. 8.2.
The number of live wires required for such a fence segment is determined to reduce the series
resistance of the line to an acceptable level. These live wires are shorted together at regular intervals to improve current and voltage sharing between them. Branches of the fence, shown by the thin lines, are usually single wire fences with an earth return, as indicated in Fig. 8.1(a). The dotted lines in the fence structure, shown in Fig. 8.1(a), represent conductors that are neither connected to the energizer nor earthed. In locations where the soil conduction is poor, a separate conductor is used to facilitate a low impedance return path for the pulse current, as indicated in Fig. 8.1(b). In some situations this return wire is earthed at multiple points to reduce the impedance of the return path further. Structures similar to the one shown in Fig. 8.1(c), are commonly used in security fence installations for line monitoring purposes.

![Fig. 8.1 Different electric fence structures](image)

The typical length of an electric fence can range from 1 – 10 km per segment and the combined length of all sub-sections can be in excess of several hundred kilometres. The initial experimental investigations carried-out on the pulse propagation properties of such typical electric fence structures at various test locations clearly indicated the presence of travelling waves on the fence wires, as illustrated in Fig. 8.3. Moreover, significant frequency components up to 1 MHz and pulse distortions due to frequency dependent propagation
characteristics of the fence structure are also observed. The results of these preliminary investigations thus suggest that electric fences exhibit wave propagation phenomenon and the simplified transmission line theory, which is based on the quasi-TEM approximation, can be applied to model signal propagation with acceptable accuracy [7-24].

![Diagram of a typical electric fence](image)

**Fig. 8.2** A layout of a typical electric fence

![Graph of measured voltage across the energizer output terminals](image)

**Fig. 8.3** Measured voltage across the energizer output terminals on a typical electric fence structure

The derivation of solutions for the propagation of fence pulses, based on the quasi-TEM approximation, requires the knowledge of distributed line impedance and admittance. Unfortunately, the line impedance is a complicated function that varies considerably with
frequency due to skin effect and proximity effect resulting from high frequency currents flowing in the conductors. Moreover, the earth return currents have a significant effect on the line impedances at low frequencies [12-14].

Modelling the influence of earth return currents is complicated, as the distribution of earth currents is difficult to determine. However similar challenges have been successfully dealt by researchers, who investigated the power transmission line transients in the past. The development of a method for determining the frequency dependent impedance of an overhead transmission line, caused by the earth return currents, by Carson in 1926 was a major leap in the transient analysis of transmission lines. This method, called the Carson’s method, has some approximations and validity limitations, and expresses the line impedance by integrals that can be evaluated numerically in a computer. With additional restrictions, such integrals can be evaluated through sums of “series” and may have convergence limitations. On the other hand, the complex depth method, proposed by Gary in 1976, that has additional approximations and validity limitations, provides closed-form solutions for the line impedance. Another milestone in transient analysis of power system research, which started in early 1900s, is the development of modal analysis technique to model propagation over multi-conductor transmission lines. This technique can be used to decouple the line voltage and current vectors in a multi-wire transmission line, thus facilitating a simpler solution [1-5, 12-29].

The transient analysis of transmission lines requires extensive numerical computations because of the complex nature of propagation models. Currently there are commercially available power systems analysis software packages, such as the Power Systems Computer Aided Design (PSCAD) package, that facilitate numerical analysis of transient propagation over power transmission lines at low-medium frequencies. Although these software packages are not optimized to simulate the pulse transmission over structures that are used for electric
fences, propagation properties of a limited number of simple fence structures can be simulated using current model libraries included in such software packages.

Although there are some similarities between a transmission line on a power system and an electric fence system, there are also a few dissimilarities between the two systems. In comparison to power distribution lines, electric fence lines exhibit strong coupling between lines. They present a higher leakage conductance to earth and have higher signal attenuation due to high series impedance. The currents in the lines are also unbalanced and some fences have complex structures. Moreover, the analysis should be done over a larger frequency range, typically from DC up to about 1 MHz, as the fence pulses are rich in high frequency harmonics. These dissimilarities make the direct use of power systems transient analysis techniques inappropriate for modelling of complex fence lines.

### 8.2.1 Derivation of Line Parameters

A cross-sectional view of an electric fence structure that has N-number of conductors is shown in Fig. 8.4(a). Since the conductors in most electric fence structures are vertically aligned, the horizontally displacement of fence conductors, shown in the diagram by x, is considered to be zero in this study. However, the line parameters of a fence with horizontally displaced conductors can be obtained, through a simple modification [28]. Fig. 8.4(a) also indicates the image conductors (n` and n``) that represent the return current flow through the earth. The image of conductor ‘n’ for a perfect earth is shown by n``. Whereas, when the earth is treated to be non-ideal, the resulting image conductor n` would appear at a greater depth than n``., which would cause the series impedance of the line to change considerably [1-5, 12-13].
As explained previously, pulse propagation along an electric fence can be approximated by the quasi-TEM mode. Therefore, the electrical model representation for an infinitesimally small segment of a two-conductor line, shown in Fig. 8.4(b) with frequency dependent distributed parameters, can be used to derive the propagation functions of the system. The series impedance of the N-wire system, which is composed of internal impedance \(Z_i\), external impedance \(Z_e\), mutual impedance \(Z_m\) and earth resistance \(R_e\), can be expressed by an \(n \times n\) impedance matrix as in (8.1).

\[
Z = \begin{bmatrix} Z_{nn} & Z_{nm} \\ Z_{mn} & Z_{mm} \end{bmatrix} = R + j\omega L
\]  

(8.1)

where \(Z_{nn} = Z_{i(n)} + Z_{e(n)} + R_{r(n)}\) and \(Z_{mn} = Z_{c(n,m)}\).

The internal impedance of the solid conductors can be obtained from a tabular conductor model [30] and is given by,
where $M_{(n)} = \sqrt{\frac{\mu_0 \mu_r \sigma_n}{4\pi}}$

Functions ber and bei are Kelvin’s functions and are defined by,

$$\text{ber}(M_{(n)} r_n) + j\text{bei}(M_{(n)} r_n) = I_0\left(M_{(n)} r_n \sqrt{j}\right)$$

(8.3)

As pointed out in [25], the internal impedance is not affected by the earth return currents, hence further simplification of (8.2) and conversion to $s$-domain results in,

$$Z_{i(n,s,n)} = \frac{kR_n}{2\sqrt{s}} \frac{I_0\left(k_{(n)} \sqrt{s}\right)}{I_1\left(k_{(n)} \sqrt{s}\right)}$$

(8.4)

where $s = j\omega$ and $k = r_n \sqrt{\mu_0 \mu_r \sigma_n}$

The external impedance of the $n^{th}$ wire, caused by the magnetic flux linkage outside the conductor, due to its return current, can be calculated from (8.5). The return current that flows through the earth is modelled by a conductor that is located at a distance $D_n$ away from the corresponding live conductor.

$$Z_{e(n)} = j\omega \frac{\mu_0}{2\pi} \ln\left(\frac{D_n}{r_n}\right)$$

(8.5)

The return currents flowing through the earth are significantly affected by the electrical and magnetic properties of the earth, which influence the external magnetic field. Thus $D_n$ is not equal to $2h_n$ for a non-ideal earth and is a function of resistivity, permeability and permittivity of the earth. Mathematical models, such as Carson’s method, weighting function method or complex depth method, can be used to accurately predict the effect on $D_n$ due to non-ideal properties of the earth. However, the complex depth method is usually preferred due to its improved accuracy and simplicity [1-5, 12-21].
The complex depth method is based on a few assumptions, and the limitations of the method are well reported. Moreover, the accuracy of the method for signals up to about 1 MHz has been proven to be acceptable under certain conditions as reported in [1-5, 14-30]. The complex depth method assumes that an imaginary return conductor \((n')\) could be placed at \(h_{n+2p}\) distance below the surface of the earth to model the earth return currents. The distance \(p\) is given by (8.6) and is a function of resistivity, permeability and permittivity of the earth \([12, 17, 30]\). Although earth resistivity and permittivity are frequency dependent, the original formulation of complex depth method assumes frequency independent soil conductivity and displacement currents are neglected. This is corrected in (8.6) by introducing the term \(1/(\sigma+j\omega\epsilon)\) replacing \(\sigma\) in the original formulation \([1, 30]\). However as discussed in Section 8.3.3 the effects of the frequency dependent earth conduction properties on the final solution are very small because the higher frequencies of the pulse are attenuated much faster due to significant skin effect and, therefore, a constant value of \(\sigma\) can be assumed during calculations.

\[
p = \sqrt{\frac{1}{(\sigma + j\omega\epsilon)j\omega\mu_0\mu_r}} \tag{8.6}
\]

Substituting (8.5) in (8.6) and then converting to s-domain results in,

\[
Z_e(s,n) = s\frac{\mu_0}{2\pi} \ln \left( \frac{2|h_n + p(s)|}{r_n} \right) \tag{8.7}
\]

\[
p(s) = \frac{1}{\sqrt{s}} \sqrt{\frac{1}{(\sigma + s\epsilon)\mu_0\mu_r}} \tag{8.8}
\]

Thus (8.7) can be used to obtain an accurate approximation for the external impedance of the multi-wire fence line. The mutual impedance, arising from magnetic coupling between the \(n^{th}\) and \(m^{th}\) wire, which is also affected by the earth return currents \([1, 12, 17]\), is given by,

\[
Z_{c(n,m)} = s\frac{\mu_0}{2\pi} \ln \left( \frac{h_n + h_m + 2P}{h_n - h_m} \right) = s\frac{\mu_0}{2\pi} \ln \left( \frac{D_{nm}}{D_{nm}} \right) \tag{8.9}
\]
The earth return resistance is considerably high for a typical fence system and is a function of the earthing mechanism, soil resistivity and environmental conditions. Thus the earth return impedance would be determined empirically in this study [1-5, 31].

Similar to series impedance, the shunt admittance, which is composed of line capacitance, mutual capacitance \((C)\) and shunt conductance \((G)\), can be expressed by an ‘\(n \times n\)’ matrix as given by,

\[
Y = \begin{bmatrix}
Y_{nn} & Y_{nm} \\
Y_{mn} & Y_{nn}
\end{bmatrix} = G + joC
\] (8.10)

The line and mutual capacitances between the conductors, arising from the external electric field, is unaffected by the earth return currents up to a few mega-hertz [1, 27]. Hence to calculate the capacitances, the earth return currents could be represented by imaginary conductors \(n``\) in Fig. 8.4(a), which are located at \(h_n\) distance below the surface of earth. The capacitance matrix \(C\), is obtained from the potential coefficient matrix as given by,

\[
C = P^{-1}
\] (8.11)

where ‘\(n \times n\)’ matrix \(P\) is composed of the elements given by,

\[
P_{(n)} = \ln \left( \frac{2h_n}{r_n} \right) \frac{1}{2 \pi \varepsilon_0}
\] (8.12)

\[
P_{(n,m)} = \ln \left( \frac{D_{nm}}{D_{mn}} \right) \frac{1}{2 \pi \varepsilon_0}
\] (8.13)

Generally, the shunt conductance matrix of the line is a very complicated parameter to evaluate mathematically because of faults in the line, voltage breakdowns and uneven leakage. Therefore it is determined experimentally and the unit length conductance of the
wire is calculated by assuming that the conductance is linear and homogeneous along the fence. Typically this is in the range of $10^{-11}$ to $10^{-7}$ S/m.

### 8.2.2 Frequency-Domain Analytical Solution

The matrix form of telegrapher’s equations, given in (8.14), could be used to model the propagation of pulses along the lines as they exhibit quasi-TEM propagation [1, 12, 21-30].

\[
\begin{align*}
-\frac{\partial V_{(z,s)}}{\partial z} &= Z_{(s)} I_{(z,s)} \\
-\frac{\partial I_{(z,s)}}{\partial z} &= Y_{(s)} V_{(z,s)}
\end{align*}
\]  

(8.14a)

(8.14b)

Due to the complexity of (8.1) and (8.10), (8.14) could not be solved directly in the time-domain to obtain an analytical solution. A solution to (8.14) could be obtained either by solving (8.14) in the frequency-domain through analytical techniques and converting back to the time-domain numerically or by solving (8.14) in the time-domain through numerical convolution. Both methods have their own advantages and disadvantages. The time-domain method lacks accuracy but has the flexibility of modelling non-linear, time-dependent systems compared to the frequency-domain method [1, 25]. Since all the line parameters are defined in frequency-domain, the problem could be solved most efficiently in frequency-domain. Otherwise the line parameters have to be converted numerically into the time-domain before applying to the time-domain telegrapher’s equations. Differentiation and substitution of (8.14), results in a set of equations given by (8.15), where the line current and voltage vectors are now decoupled from each other.

\[
-\frac{\partial^2 V_{(z,s)}}{\partial z^2} = Z_{(s)} Y_{(s)} V_{(z,s)}
\]  

(8.15a)
Obtaining a direct solution to (8.15) is somewhat complicated due to coupling that exists between the line to line voltages and currents. Thus the line to line voltage and current vectors in (8.15) can be decoupled by transforming the system into the modal-domain through the substitutions given by,

\[
V^m_{(z,s)} = T^{-1}_V V_{(z,s)} \tag{8.16a}
\]

\[
I^m_{(z,s)} = T^{-1}_I I_{(z,s)} \tag{8.16b}
\]

The transformation matrices \(T_V\) and \(T_I\) are chosen such that \(\gamma\), given by (8.17), is a diagonal matrix [21-29].

\[
T^{-1}_V Z_{(s)} Y_{(s)} T_V = \gamma^2 \tag{8.17a}
\]

\[
T^{-1}_I Y_{(s)} Z_{(s)} T_I = \gamma^2 \tag{8.17b}
\]

Therefore the decoupled system is given by,

\[
-\frac{\partial^2 V^m_{(z,s)}}{\partial z^2} = \gamma^2 V^m_{(z,s)} \tag{8.18a}
\]

\[
-\frac{\partial^2 I^m_{(z,s)}}{\partial z^2} = \gamma^2 I^m_{(z,s)} \tag{8.18b}
\]

The transient solution for the decoupled system given by (8.18), is a series of forward and backward travelling waves as given by,

\[
V_{(z,s)} = T_V \sum_{n=1}^{m} \left\{ A_{(n,s)} e^{-\gamma[2L(n-1)+z]} + B_{(n,s)} e^{\gamma[-2L+n+z]} \right\} \tag{8.19a}
\]

\[
I_{(z,s)} = T_I \sum_{n=1}^{m} \left\{ C_{(n,s)} e^{-\gamma[2L(n-1)+z]} + D_{(n,s)} e^{\gamma[-2L+n+z]} \right\} \tag{8.19b}
\]
Simplifying (8.19) further using (8.14) results in,

\[ I_{(z,s)} = Y_0 T_V \sum_{n=1}^{m} \left\{ A_{(n,s)} e^{-\gamma[2L(n-1)+z]} - B_{(n,s)} e^{\gamma[-2Ln+z]} \right\} \]  

(8.20)

where \( Z_0 = Y_0^{-1} = T_V \gamma T_V^{-1} Y_{(s)}^{-1} \)

The backward travelling waves (B) arise due to the reflections of forward travelling waves (A) from the terminal end and they are related by,

\[ B_{(n,s)} = \Gamma_T A_{(n,s)} \]  

(8.21)

However, considering the reflections formed at the generator end, B is related to A by,

\[ A_{(n,s)} = \Gamma_G B_{(n-1,s)} = \Gamma_T \Gamma_G A_{(n-1,s)} \]  

(8.22)

The reflection coefficient matrices at both ends are defined by,

\[ \Gamma_T = \left[ Z_{T(s)} - Z_{\theta(s)} \right] \left( Z_{T(s)} + Z_{\theta(s)} \right)^{-1} \]  

(8.23)

\[ \Gamma_G = \left[ Z_{\theta(s)} - Z_{G(s)} \right] \left( Z_{\theta(s)} + Z_{G(s)} \right)^{-1} \]  

(8.24)

The initial forward travelling waves, \( A_{(1,s)} \), which is related to the input voltage vector can be calculated by,

\[ A_{(1,s)} = \left[ Z_{G(s)} Z_{\theta(s)} T_V^{-1} + T_V^{-1} \right] V_{in} \]  

(8.25)

Thus combining (8.20) to (8.25), an analytical solution for line voltages and currents in the frequency-domain can be obtained as given by,

\[ V_{(z,s)} = T_V \sum_{n=1}^{m} \left\{ \Gamma_T^{(n-1)} \Gamma_G^{(n-1)} A_{(1,s)} e^{-\gamma[2L(n-1)+z]} \right\} \]  

(8.26a)

\[ I_{(z,s)} = Y_0 T_V \sum_{n=1}^{m} \left\{ \Gamma_T^{(n-1)} \Gamma_G^{(n-1)} A_{(1,s)} e^{-\gamma[2L(n-1)+z]} \right\} \]  

(8.26b)
8.2.3 Modelling Multiple Discontinuities

The solution presented by (8.26) can be extended further to account for discontinuities that could occur along the line due to branches, loads and faults. Modelling such discontinuities is complicated but can be achieved by dividing the fence into sections at the discontinuities and representing each section between two discontinuities as a 2-port network as shown in Fig. 8.5. The transfer function of each of these networks is (8.20), where $A$ and $B$ represent travelling waves resulting from the reflections at the two boundaries and waves introduced by the adjacent sections.

![Fig. 8.5 A multi-wire fence with discontinuities](image)

To solve this complicated fence network, the travelling waves need to be evaluated in terms of the boundary conditions of each section. Consider the $n^{th}$ section of the fence, shown in Fig. 8.6(a), where the input and output travelling wave vectors are as indicated in the diagram. The outgoing waves from the section are a result of the incoming waves and their reflections at the boundaries as indicated by the lattice diagram in Fig. 8.6(b).
As indicated in Fig. 8.6(b), each input creates a series of outputs, which are governed by the reflection coefficient at the boundary. Since this is a linear system, each input component can be analysed separately and superposition can be applied to obtain a final solution. Consider the waves $A^n_1$ entering the $n^{th}$ section that has a length of $L_n$. When $A^n_1$ reaches the right hand side (RHS) boundary, the discontinuity at the boundary causes part of the wave to reflect. This phenomenon creates a backward travelling wave within the section and a forward travelling wave that propagates to the next section. If the reflection coefficient at the RHS boundary is denoted as $\Gamma^n_e$, then the voltage and current at the boundary just after the reflection can be found from (8.19a) and (8.20) as given by,
\[ V^n_{(l_n,s)} = T_v^n \left( I_n^n + \Gamma^n_e A^n_1 e^{-\gamma_n l_n} \right) \]  
(8.27a)

\[ I^n_{(l_n,s)} = Y^n_0 T_v^n \left( I_n^n - \Gamma^n_e A^n_1 e^{-\gamma_n l_n} \right) \]  
(8.27b)

The voltage and current relationship at the discontinuity is given by,

\[ V^n_{(l_n,s)} = Z^n_e I^n_{(l_n,s)} \]  
(8.28)

where the equivalent impedance seen by the travelling wave at the boundary is given by,

\[ Z^n_e = \left( (Z^n_0)^{-1} + (Z^n)^{-1} \right)^{-1} \]  
(8.29)

Therefore, from (8.27) and (8.28), the reflection coefficient of the section at the RHS boundaries can be obtained as given by,

\[ \Gamma^n_e = \left[ Z^n_e - Z^n_0 \right] \left[ Z^n_e + Z^n_0 \right]^{-1} \]  
(8.30)

Similarly, the reflection coefficient at the left hand side (LHS) boundary can be obtained by,

\[ \Gamma^n_s = \left[ Z^n_s - Z^n_0 \right] \left[ Z^n_s + Z^n_0 \right]^{-1} \]  
(8.31)

The line voltages and currents at any given location along the section is made up of the reflections from \( A^n \) and \( B^n \). If \( B^n \) is assumed to be zero, then the line voltages and currents at a distance \( z \) from the LHS end can be obtained by summing all the reflections created by \( A^n \) as given by,

\[ V^n_{(z,s)} = T_v^n \sum_{\beta=1}^{m} \sum_{\alpha=1}^{m} \left\{ \left( \Gamma^n_s \Gamma^n_e \right)^{n-1} A^n_{\beta} e^{-\gamma_n [2l_n (\alpha-1)+z]} + \left( \Gamma^n_e \Gamma^n_s \right)^{n-1} \Gamma^n_e A^n_{\beta} e^{\gamma_n [-2l_n (\alpha+1)+z]} \right\} \]  
(8.32a)

\[ I^n_{(z,s)} = T_v^n Y^n_0 \sum_{\beta=1}^{m} \sum_{\alpha=1}^{m} \left\{ \left( \Gamma^n_s \Gamma^n_e \right)^{n-1} A^n_{\beta} e^{-\gamma_n [2l_n (\alpha-1)+z]} - \left( \Gamma^n_e \Gamma^n_s \right)^{n-1} \Gamma^n_e A^n_{\beta} e^{\gamma_n [-2l_n (\alpha+1)+z]} \right\} \]  
(8.32b)
A complete solution for the line voltages and currents in the section can be found by including the effects of \( B^n \) in to (8.32) and the solution is given by,

\[
V^n_{(z,n)} = T^n \sum_{\beta = 1}^{\Psi} \sum_{n=1}^{m} \left\{ \left( \Gamma^n_0 \Gamma^n_\infty \right)^{n-1} A^n_\beta e^{-\gamma_n[z_1 + (n-1) + z]} + \left( \Gamma^n_0 \Gamma^n_\infty \right)^{n-1} \Gamma^n_\infty A^n_\beta e^{-\gamma_n[z_1 - (n-1) + z]} \right\}
\]

(8.33a)

\[
I^n_{(z,n)} = T^n \sum_{\beta = 1}^{\Psi} \sum_{n=1}^{m} \left\{ \left( \Gamma^n_0 \Gamma^n_\infty \right)^{n-1} A^n_\beta e^{-\gamma_n[z_1 + (n-1) + z]} - \left( \Gamma^n_0 \Gamma^n_\infty \right)^{n-1} \Gamma^n_\infty A^n_\beta e^{-\gamma_n[z_1 - (n-1) + z]} \right\}
\]

(8.33b)

Implementation of a solution for (8.33) requires the knowledge of \( A \) and \( B \) for each section of the fence in terms of the initial conditions. As indicated by Fig. 8.6, \( A \) and \( B \) for a given section are generated due to the reflection and refraction of travelling waves reaching the section boundary from adjacent sections. Hence, vectors \( A \) and \( B \) for the neighbouring sections can be determined in relation to the line voltages and currents in the \( n^{th} \) section as given by,

\[
A^{(n+1)}_\beta = \sum_{a=1}^{m} \left\{ \left[ I^n_0 + \Gamma^n_0 \right] \left( \Gamma^n_0 \Gamma^n_\infty \right)^{n-1} A^n_\beta e^{-\gamma_n[z_1 + (n-1) + z]} \right\}
\]

(8.34a)

\[
B^{(n+1)}_\beta = \sum_{a=1}^{m} \left\{ \left[ I^n_0 + \Gamma^n_0 \right] \left( \Gamma^n_0 \Gamma^n_\infty \right)^{n-1} A^n_\beta e^{-\gamma_n[z_1 + (n-1) + z]} \right\}
\]

(8.34b)
This can be further simplified to obtain,

$$
\sum_{\beta=1}^{1} A^{(n+1)}(\beta) = V^{(n)}(z,x)|_{z=L_n} - \sum_{\beta=1}^{1} B^{(n)}(\beta)
$$

(8.35a)

$$
\sum_{\beta=1}^{1} B^{(n-1)}(\beta) = V^{(n)}(z,x)|_{z=0} - \sum_{\beta=1}^{1} A^{(n)}(\beta)
$$

(8.35b)

Assuming that initial charge stored on the fence conductors is zero, for an electric fence structure that has an N-number of sections, which is energized by the source $V_{in}$, the initial conditions can be calculated by,

$$
A^1_\beta = 0 \text{ for all } \beta \neq 1
$$

(8.36)

$$
A^1_1 = [Z_sZ_0^{-1}T_v^{-1} + T_v^{-1}]V_{in}
$$

(8.37)

$$
B^N_\beta = 0 \text{ for all } \beta
$$

(8.38)

**8.2.4 Time-Domain Numerical Solution**

The time-domain solution for line voltages and currents can only be obtained by numerically evaluating the inverse Laplace transformation of (8.33). A MATLAB algorithm that incorporates a standard Laplace inversion routine was developed to obtain time-domain solutions to (8.33) for a given period with an acceptable accuracy. As illustrated by the flow chart in Fig. 8.7, the frequency dependent line parameters are first calculated in the frequency-domain using (8.1) and (8.10). After the line parameters for all sections have been calculated, the initial conditions are calculated using (8.36)-(8.38). Then the components of the travelling waves resulting from wave reflection and refraction are calculated through (8.34). Finally, based on these values, the line voltages and currents at the point of interest are calculated from (8.33).
Fig. 8.7 The numerical algorithm to obtain time-domain solutions
8.2.5 Limitations of the Model

The proposed mathematical solution has a few limitations. Since the system is solved in frequency-domain and transformed into time domain through Laplace transform, the model cannot account for the non-linear properties of the fence. Also the model is based on fence structures that are constructed in a vertical plane as explained above. The solution assumes that the initial line voltages and currents are zero, which would be a valid assumption in most cases as the pulse interval is very long. Another limitation of the model is the use of experimental values for both earth return resistance and shunt conductance.

8.3 Verification of the Model

The validity of the mathematical model presented above was verified by comparing the results obtained from the MATLAB based theoretical implementation with PSCAD simulations and experimental data gathered from test fences. PSCAD is a proven software package that is capable of simulating transient propagation along power transmission lines. The simulation parameters of the relevant PSCAD models can be changed accordingly to facilitate the simulation of simple electric fence structures. Thus PSCAD was used to verify the theoretical analysis performed on simple fence systems to avoid costly and time consuming experimental analysis. Unfortunately some of the more complicated fence structures considered in this study cannot be simulated on PSCAD due to the limitations of model libraries found in PSCAD. In such situations, only a comparison between the theoretical values and experimental values has been performed.
8.3.1 Simple Fence Structures

The theoretical and simulated voltage and current waveforms, at a half-way point along an 8 km fence with an open terminal end, are shown in Fig. 8.8. A fence with very a low leakage conductance of about $10^{-11}$ S/m was considered for this study. The fence, of which the parameters are given by Appendix B, was energized by a 10 kV, 500 µs square pulse generator. The propagation delay of the line causes the pulse to appear about 18 µs after the input. Hence the speed of pulse propagation can be calculated to be $2.22 \times 10^8$ m/s. The initial voltage appears to be considerably lower than the input as the source impedance together with the characteristic impedance of the line form a voltage divider. Thus a low source impedance would generate a higher initial voltage, but at the same time the initial current would be higher. The first reflection from the terminal end reaches this point after about 42 µs and this is in phase with the initial pulse, causing the line voltage to increase significantly above its steady state value. This property could be used by energizers to create the initial high voltage that is required to effectively breakdown the high skin-resistance of the animal. The second reflection that reaches the midpoint from the source end is out of phase; hence it reduces the line voltage. The reflections are attenuated to a negligible level approximately at 250 µs, where the line voltage and current can be considered to have reached a steady state value. Although the wave dispersion is included in the analysis, it cannot be identified on its own from the reflections. However, the distorted nature of the reflections can be attributed to the combined effect of wave dispersion and frequency dependent attenuation. The waveforms, obtained from the semi-analytical method based on MATLAB algorithm, are in very good agreement with PSCAD simulation results and, hence, validate the accuracy of the analysis presented in this paper. However, a slight discrepancy between the line voltages and currents
of the bottom and top wires in Fig. 8.8(a) and Fig. 8.8(b), respectively, is observed in both simulated and theoretical waveforms.

Fig. 8.8 Voltage and current at the midpoint of an 8 km long low leakage fence

A typical electric fence has a reasonably high leakage current due to vegetation growth over the fence and damaged insulators. Moreover the leakage would be expected to increase during wet weather conditions. Fig. 8.9 shows the theoretical effect on the fence voltages and currents when the shunt conductance of the fence discussed above was increased up to $10^7$ S/m to represent the high leakage. Under the condition of increased shunt conductance,
PSCAD failed to simulate due to convergence problems, and therefore only the theoretical results are presented. In comparison to Fig. 8.8, the reflections are considerably attenuated in this case due to losses caused by leakage. The high leakage current also increases the voltage drop across the series impedance resulting in a considerably lower line voltage than in the earlier case.

Short circuits between the line and return path are common in electric fence applications. Therefore such a situation was introduced by simulating the electric-fence with a short circuit at the end of the fence, and the resulting waveforms at the same location are shown in Fig. 8.10. As expected, there is a significant increase and reduction in the steady state current and voltage, respectively, due to the shorted line. However, it is interesting to note that the fence voltage at the midpoint is still about 4 kV with a peak of 8 kV, which would be sufficient to deliver a shock to a bull [32].
Fig. 8.10  Voltage and current at the midpoint of an 8 km long fence with a fault


8.3.2 Complicated Fence Structures  

The aforementioned discussion and results confirm the validity of the presented theoretical analysis with respect to simulations performed on various electric fence structures. However, it is necessary to study the accuracy of the proposed analysis in comparison to experimental data gathered from test sites subjected to various conditions. Therefore, experimental results were gathered from an electric fence installation in a farm located near Karaka in New Zealand (GPS coordinates -37.1061583 and 174.8831783) and results obtained for the same structure through the theoretical analysis are compared and presented below.

The 660 m long test fence shown in Fig. 8.11, was constructed as a complicated 4-wire structure with multiple sections. Owing to the complicated nature of this structure arising from discontinuities and high leakage conductance, this system could not be simulated in PSCAD. A satellite map of the test fence, reconstructed through GPS data is illustrated in Fig. 8.12. The map indicates the locations of the discontinuities (C1-C3) found along the fence structure, where the 3-top wires of the fence were shorted together at these locations. Photographs that illustrate the construction of the fence are shown in Fig. 8.13. The bottom wire of the fence structure was not connected to the energizer, although in some experiments it was used as a part of the return path for the pulse current. Fig. 8.14(a) illustrates the use of the bottom wire to provide a low impedance return path for the current flowing through the load that is connected across the fence. The vegetation growth over the fence at point C3, shown in Fig. 8.14(b), significantly contributed to the leakage current. This effect was modelled by connecting a lumped impedance across the fence and the return path at C3. A complete list of fence parameters is given in Appendix C. The fence was energized by a 1 kV square pulse generator with a variable source impedance to simulate the output characteristics of different types of energizers.
Fig. 8.11  Construction of the test fence

Fig. 8.12  A GPS map of the test fence
Fig. 8.13 The test fence in Karaka (a) The 4-wire fence (b) Shorts between the top wires
Fig. 8.14 Faults on the fence
(a) External load connected to simulate a fault
(b) Vegetation growth over the fence at point C3
A comparison between the experimental and theoretical fence voltages at the energizer terminals, for two different source impedances, are shown in Fig. 8.15. As, evident from Fig. 8.15 the theoretical waveforms are in very good agreement with the experimental waveforms, therefore validating the accuracy of the theoretical analysis presented above. When the source impedance is increased, the large transient currents generate a higher voltage drop across it, thereby initially reducing the line voltage to a value, which is lower than the steady-state voltage. A few voltage reflections caused by the discontinuities that are present along the line can be seen in all the voltage waveforms. Among these reflections, the reflection caused by the discontinuity at 150 m and the reflections from the end of the line appear to be the most dominant. After about 1 µs, the reflection from the discontinuity at 150 m reaches the energizer terminals. This reflection is out of phase in relation to the initial pulse, and as a result it reduces the line voltage momentarily. However the reflection from the end of the line, reaching the energizer terminals after about 5 µs, is in phase with the initial pulse, thus it increases the line voltage considerably. The voltage transients caused by the reflections are attenuated to a negligible value after about 20 µs in both situations. In comparison to Fig. 8.9, the reflections have attenuated much faster due to the higher earth return resistance of this
fence system. Therefore, after about 20 µs voltage waveforms in Fig. 8.15 reach a steady-state value that is approximately equal to the source voltage. The slight discrepancy observed between the theoretical and experimental values can be attributed to the measurement errors as well as modelling uncertainties. Especially a slight error in line length can be observed, which has caused the reflections in the experimental results to appear earlier in comparison to the theoretical waveforms.

![Graph](image_url)  

**Fig. 8.16** Fence voltage with a 300 Ω load at TL1

Literature studies show that the body impedance of an animal that is in contact with a fence can be modelled by a load resistance between 100 and 500 Ω [32]. Therefore, to investigate the effects on fence voltage due to an animal in contact with the fence, loads were connected between the live wires of the fence and the earth at the point TL1, indicated in Fig. 8.12. Fig. 8.16 illustrates the voltages on the line when a 300 Ω load was connected to the fence at TL1, while the fence was energized by the same energizer with a 300 Ω source impedance. In comparison to Fig. 8.15, the effect on the steady-state line voltage caused by the extra load is negligible. This is mainly due to the poor earthing of the load that causes the effective impedance of the load seen by the energizer to increase significantly. In such situations, an
extra return wire is used to overcome the high return path impedance presented by poor earth conditions. However, the addition of the load has introduced more reflection into the system, which has lengthened the rise time of the line voltage in comparison to Fig. 8.15.

Fig. 8.17 Fence voltage when a return conductor is connected

(a) 300 Ω source impedance  (b) 50 Ω source impedance

The fourth wire (bottom wire) of the fence was connected to the earth at the energizer end, in order to reduce the return path resistance and the resulting line voltages for a source impedance of 300 Ω and 50 Ω, are shown in Fig. 8.17. In comparison to Fig. 8.15, a
significant change in the unloaded line voltages is not observed in Fig. 8.17. However, if a load is connected between the fence wires and the return wire, a significant change in the performance can be noticed. This is evident in the theoretical comparisons shown in Fig. 8.18. A greater voltage drop across the source impedance is observed when a load is connected between the return and energized conductors. This is caused by the significant increase in load current due to the low impedance current path introduced by the return conductor. The steady decline in line voltage, evident in Fig. 8.17(b), is caused by the termination of the energizer pulse after about 10 µs.

![Graph showing line voltage vs. time](image)

Fig. 8.18 The effect on fence voltage due to a load connected to the return conductor

The fence shown in Fig. 8.12 was modified by connecting a 500 m, 2-wire fence at the end of the existing fence to observe the changes in line voltage. The structure of the new electric fence is shown in Fig. 8.19. The resulting voltages at the energizer terminals for three different loads are shown in Fig. 8.20. The experimental results follow the theoretical values closely, confirming the validity of the theoretical analysis. In comparison to Fig. 8.15, the time taken for the reflection from the end to reach the measuring point is longer as a result of
the increased line length. Moreover, the discontinuity introduced by the two different fence sections has not produced large voltage reflections in the line.

Fig. 8.19 Structure of the new fence

Fig. 8.20 Line voltages for the longer fence

Line faults introduced by shorts, broken conductors, faulty cut-out switches, etc. are common in a typical electric fence system. Therefore to investigate the effect of a fault on the fence voltage, the live conductors of the fence were shorted to earth at TL1. The resulting voltage at the energizer terminals are shown in Fig. 8.21. The fault has reduced the steady-state line
voltage to about 0.6 kV from 1 kV as a result of the high current that is flowing through the short to the return path. A reflection produced by the short arrives at the energizer after about 2 µs and reduces the line voltage to about 0.4 kV until the reflection from the end arrives after 10 µs. The line voltage has reached a steady state value after about 18 µs.

![Graph showing line voltage after introducing a short at TL1](image)

**Fig. 8.21** Line voltage after introducing a short at TL1

### 8.3.3 Effects of Earth Return Currents

In Section 8.2.1, the frequency dependant nature of the line parameters due to the earth currents, skin effect and proximity effect were introduced. The complex depth method, which is an approximation to the Carson’s integral, was derived on the basis of a few assumptions on soil behaviour. The errors introduced by these assumptions in the theoretical solution have been investigated, in accordance with the method described in [28], to ascertain whether or not the proposed mathematical solution is of acceptable accuracy. The fence voltages were therefore computed using the proposed mathematical solution by varying the earth resistivity from 10 to 1000 Ωm. Fig. 8.22 compares the theoretically obtained line voltages of the 8 km fence introduced in Section 8.3.1, for resistivity values of 10, 100, 1000 and 100 + jωx8.85e-10 Ωm. During the analysis the permeability of soil was considered to be that of a vacuum
because of non-ferromagnetic nature of soil. The results indicated that the error introduced by these extreme variations of soil parameters is only less than 3% and, hence, considered to be acceptable for this work. The small error could however be attributed to the significant attenuation of high frequency components of the applied pulse caused by the presence of high skin effect in typical electric fence lines.

![Graph](image)

Fig. 8.22 Effect of earth properties on the solution

### 8.3.4 Safety Aspects

It is essential that electric fences are installed assuring safety conditions for humans and animals. Therefore, when designing an electric fence system it is necessary to evaluate the effects of current impulses passing through human body in unfavourable conditions. This should include contact conditions of the affected person with the fence and the ground, critical interval of cardiac period in what concerns higher probability for important cardiac effects, respiratory effects, muscular tetanization, and consequential effects.

However, the quantitative examples used in the above discussion do not account for the safety aspects of electrified fences. As a simple example, the quantitative analysis presented above with a pulse generator of square pulses of 10 kV, 500 µs, would lead to current impulses that will have amplitudes of the order of 5 A to 10 A, to flow through a person’s body that is in
contact with the fence. This current level exceeds the safety limit (of the order of 2 A in a specific set of assumptions, according IEC reports) for ventricular fibrillation, even for a healthy adult subjected to a single square impulse of 500 µs. So, it is essential to carefully evaluate the safety aspects of the electrified fence installations but such an evaluation is not presented in this Thesis as it is outside the scope of the study.

8.4 Conclusions

A generalized model and a semi-analytical technique that facilitate the analysis of HV pulse propagation along multi-wire electric fences with multiple discontinuities have been described. The technique models an electric fence as a multi-conductor transmission line with frequency dependent line parameters to account for the skin effect, proximity effect and earth current effects. The fence is divided into uniform sections at the discontinuities and each section is modelled as a 2-port network. Line parameters obtained from the complex depth method is applied to the telegrapher’s equation and simplified to obtain an analytical expression for the fence voltages and currents in the frequency-domain for each of these uniform sections. Afterwards, the boundary conditions for each section are derived in terms of adjacent sections and initial conditions. Finally, the frequency-domain solution was converted to time-domain by using a MATLAB based numerical algorithm, and its accuracy was verified by PSCAD simulations and experimental results for different operating conditions.

The proposed technique provides an insight into the behaviour of HV pulse propagation along multi-wire electric fence structures, and thus serves as a useful design tool for electric fence system engineers. The analysis conducted on the behaviour of experimental fence structures indicates that the discontinuities reflect a significant portion of the pulse. However, the effect
of such reflections on the final line voltage is small due to high level of attenuation caused by
the skin-effect. Moreover, an analysis on the errors introduced by the simplified soil
parameters in the theoretical solution has been presented to show that such errors are
negligibly small.
Chapter 8

8.5 References


9.1 Modelling a Single-Wire Fence

A detailed mathematical analysis on the pulse propagation characteristics of multi-wire electric fence structures was presented in Chapter 8. The distributed line parameters of the fence structure were derived in the frequency-domain and were applied in the Telegrapher’s equations to obtain an analytical expression for the propagation function of an electric fence. The discontinuities found along the fence structure were modelled by dividing the system into subsections that exhibit homogeneous propagation properties. These subsections were modelled as 2-port networks and a complete propagation model for the entire system was derived by applying boundary conditions and initial conditions. The frequency-domain analytical solution was then numerically converted into the time-domain to obtain solutions for line voltage and current at a specific location.

However, the analysis presented in Chapter 8 used voltage and current vectors to model the individual voltage and current waves that are propagating along each conductor in the multi-conductor fence. Furthermore, the interactions between line voltages and currents were modelled by incorporating mutual coupling terms into the impedance and admittance of the system. As a result, the system had to be modelled in a matrix-form and complicated matrix manipulations were required to obtain solutions. In addition, the implementation of the numerical algorithm that was required to solve such a system is somewhat complicated and
computation times are significantly long [1-5].

However, the majority of electric fences found all around the world, are constructed with a single energized wire as illustrated in Fig. 9.1. In addition to the energized conductor, most of these fences utilize a few unenergized conductors as indicated by the dotted lines in Fig. 9.1. Typically, these unenergized conductors are poorly isolated to reduce the construction cost. In addition, the vegetation growth over these unenergized conductors is high in comparison to the energized conductor. As a result, the shunt conductance of these conductors is significantly high, therefore reducing the induced voltages and currents on unenergized conductors to an insignificant level [6-10].

A comparison between the line voltages of a 4 km electric fence with a single energized conductor and multiple unenergized conductors, derived from the mathematical model presented in Chapter 8, are shown in Fig. 9.2. The voltage on the energized conductor, when a load is connected across the fence at a 2 km distance from the energizer, is also shown on the same diagram. As evident from these comparisons, the differences between the peak line voltages are less than 10% for four different fence structures. The slight difference between line voltages can be attributed to the mutual coupling that exists between the conductors as the induced voltages in the unenergized conductors have introduced extra reflections in the line voltage. However, due to the high attenuation introduced by the large leakage currents that are flowing in the unenergized conductors, these effects are very small. Therefore, such systems can be approximated by an equivalent single-wire electric fence structure to reduce the computation time.

This chapter introduces a simplified scalar model to analyze the propagation characteristics of single-wire electric fence structures. In comparison to the analysis presented in Chapter 8, such a scalar model will result in a much simpler implementation and a faster calculation time. The proposed model derives the frequency-dependent line parameters of the system in
the frequency-domain, which are then applied in the Telegrapher’s equations to obtain an analytical solution for the line voltage and current. The frequency-domain solutions can be converted to the time-domain through a simple numerical approach and the solutions can be used to optimize the design of fence structures that have a single energized conductor [1-34].

Fig. 9.1 A multi-wire fence with a single energized conductor

Fig. 9.2 Fence voltage for a multi-wire fence with a single energized conductor
9.1.1 Derivation of Line Parameters

A cross-sectional view of a single-wire electric fence structure that utilizes the earth as the return path is shown in Fig. 9.3(a). The return currents that flow through the earth can be modelled by an image conductor that is placed below the earth’s surface as indicated in the diagram by 1\` or 1``. The distribution of return currents that flows through the earth is greatly affected by properties of the earth. If the height of the energized conductor above the earth surface is $h_1$, then it could be shown that the location of the image conductor for an ideal earth is distance $h_1$ below the surface of the earth as indicated by 1``. However, when non-ideal properties of the earth is taken into consideration, the resulting image conductor 1`, will appear at a greater depth than at 1``. Moreover, this distance ($h_1+2p$) is a frequency dependent parameter which can be calculated through the complex depth method as explained in Section 8.2.1 [1-5, 10-34].

![Diagram of a single wire fence](image)

Fig. 9.3 A single wire fence (a) Cross-sectional view (b) Electrical model
The general transmission line theory, which is based on the quasi-TEM approximation, can be applied to model the signal propagation, if the separation between the conductors is smaller in comparison to the wavelength of the highest frequency component of the signal under investigation [12-28, 32]. The signals that are generated by a typical electric fence energiser can be accurately characterized by a frequency spectrum of less than 1 MHz, and the corresponding wavelengths are significantly greater than the height of a typical fence. Therefore the propagation properties of an electric fence line, similar to the one illustrated in Fig. 9.3(a), can be modelled by the general transmission line model containing distributed components. Fig. 9.3(b) shows a model of the distributed components, representing an infinitesimally small segment of line.

The frequency dependent distributed impedance and admittance of the single-wire fence can be calculated using the methods discussed in Section 8.2.1. However, the mutual impedance and admittance caused by the unenergized conductors are ignored. Thus, the solutions can be expressed in a scalar form as given below.

The series impedance of the line given in (9.1) is made up of three separate components: the internal impedance \( Z_i \), external impedance \( Z_e \) and return path impedance \( R_r \).

\[
Z = R + jω L = Z_i + Z_e + R_r
\]  

(9.1)

The internal impedance and the external impedance of the system can be calculated as explained in Chapter 8, and is given by,

\[
Z_{i(s)} = \frac{kR_{dc}}{2} \sqrt{s} \frac{I_0(k\sqrt{s})}{I_1(k\sqrt{s})}
\]  

(9.2)

\[
Z_{e(s)} = s \frac{\mu_0}{2\pi} \ln \left( \frac{2\sqrt{1 + p(s)}}{r} \right)
\]  

(9.3)
where \( s = j \omega, \ k = r \sqrt{\mu_0 \mu_r} \) and \( p_s = \frac{1}{\sqrt{s}} \sqrt{\frac{\rho_e}{\mu_0 \mu_r}} \)

The earth return resistance is considerably high for a typical fence system and is a function of earthing mechanism and soil resistivity. For example, one of the test fences used in this study has an earth return resistance of about \( 2 \Omega/m \). In addition, the earth resistance is highly dependent on the conditions of the farm and would vary with environmental conditions; hence it is determined empirically in this study [35-36]. Thus, the total series impedance of the line in s-domain, per unit length can be given by,

\[
Z(s) = \frac{k R_{dc}}{2} \sqrt{\frac{l_0 (k \sqrt{s})}{l_1 (k \sqrt{s})}} + s \frac{\mu_0}{2\pi} \ln \left( \frac{2 \left( h_{1} + p_s \right)}{r} \right) + R_f \tag{9.4}
\]

The shunt admittance of the system is made up from the line capacitance and the line to earth conductance as given by,

\[
Y = G + j \omega C = G_1 + j \omega C_1 \tag{9.5}
\]

The line capacitances can be calculated by considering the electric field generated between the energized conductor and the return conductor which is given by,

\[
C_1 = \frac{2 \pi \varepsilon_0}{\ln \left( \frac{2 h_{1}}{r} \right)} \tag{9.6}
\]

Generally the shunt conductance of the line is a very difficult parameter to evaluate mathematically because of faults in the line, voltage breakdowns, and uneven leakage. Therefore, similarly to the earth return resistance, the shunt conductance too is determined experimentally and the unit length conductance of the wire is calculated by assuming that the
conductance is linear and homogeneous along the fence [1-5, 12]. Substituting (9.6) in (9.5), the total unit length admittance of the system is obtained as,

\[
Y(s) = G + \frac{2\pi e_0}{\ln \left( \frac{2h_1}{r} \right)} s
\]

(9.7)

9.1.2 Frequency-Domain Analytical Solution

As explained in Section 8.2.2, the solutions for the line voltage and current are derived in the frequency-domain through analytical techniques. The frequency-domain expressions are then converted into the time-domain through a numerical algorithm to obtain a final solution. This method reduces the amount of numerical calculation required to obtain the final solution, thereby reducing the computation time significantly. The line parameters derived in (9.4) and (9.7) is substituted in the telegrapher’s equations to derive a relationship between the voltage and current that flow through the fence conductor as given below [1-5].

\[
-\frac{\partial V(z,s)}{\partial z} = Z(s) I(z,s)
\]

(9.8a)

\[
-\frac{\partial I(z,s)}{\partial z} = Y(s) V(z,s)
\]

(9.8b)

Differentiation and substitution of (9.8) results in,

\[
-\frac{\partial^2 V(z,s)}{\partial z^2} = Z(s) Y(s) V(z,s)
\]

(9.9a)

\[
-\frac{\partial^2 I(z,s)}{\partial z^2} = Z(s) Y(s) I(z,s)
\]

(9.9b)
Chapter 9  A Simplified Fence Model

The transient time solution to (9.9) is a series of forward and backward travelling waves as given by,

$$V_{(z,s)} = \sum_{n=1}^{m} \left\{ A_{(n,s)} e^{-\gamma [2L(n-1)+z]} + B_{(n,s)} e^{\gamma [-2L,n+z]} \right\} \quad (9.10a)$$

$$I_{(z,s)} = \sum_{n=1}^{m} \left\{ C_{(n,s)} e^{-\gamma [2L(n-1)+z]} + D_{(n,s)} e^{\gamma [-2L,n+z]} \right\} \quad (9.10b)$$

where $\gamma = \sqrt{Z_s Y_s}$

Substituting (9.10b) in (9.8a) results in,

$$I_{(z,s)} = \sum_{n=1}^{m} \left\{ \frac{A_{(n,s)}}{Z_0} e^{-\gamma [2L(n-1)+z]} - \frac{B_{(n,s)}}{Z_0} e^{\gamma [-2L,n+z]} \right\} \quad (9.11)$$

where $Z_0 = \sqrt{Z_s / Y_s}$

The backward travelling wave ($B_{(n,s)}$) arises due to the reflection of the forward travelling wave ($A_{(n,s)}$) from the terminal end, and are related by,

$$B_{(n,s)} = \Gamma_T A_{(n,s)} \quad (9.12)$$

Similarly, by considering the reflections formed at the generator end, $B_{(n,s)}$ is found to be related to $A_{(n,s)}$ by,

$$A_{(n,s)} = \Gamma_G B_{(n-1,s)} = \Gamma_G \Gamma_A (n-1,s) \quad (9.13)$$

where the reflection coefficients at both ends can be defined by,

$$\Gamma_T = \frac{Z_{Ti(s)} - Z_{(0,s)}}{Z_{Ti(s)} + Z_{0(s)}} \quad (9.14)$$

$$\Gamma_G = \frac{Z_{0(s)} - Z_{Gi(s)}}{Z_{0(s)} + Z_{Gi(s)}} \quad (9.15)$$
The initial forward travelling wave $A_{(1, s)}$ is related to the input by,

$$A_{(1, s)} = V_{in} \frac{Z_{0(s)}}{Z_{0(s)} + Z_{G(s)}}$$  \hspace{1cm} (9.16)

Thus an analytical solution for line voltage and current in a uniform fence section can be obtained from (9.10) to (9.16), and is given by,

$$V_{(z, s)} = \sum_{n=1}^{m} \left\{ \Gamma_T^{(n-1)} \Gamma_G^{(n-1)} A_{(1, s)} e^{-i[2L(n-1) + z]} + \Gamma_T^{(n)} \Gamma_G^{(n-1)} A_{(1, s)} e^{-i[2L_n + z]} \right\}$$  \hspace{1cm} (9.17a)

$$I_{(z, s)} = \sum_{n=1}^{m} \left\{ \Gamma_T^{n} \Gamma_G^{n} A_{(1, s)} e^{-i[2L(n-1) + z]} / Z_0 - \Gamma_T^{(n)} \Gamma_G^{(n-1)} A_{(1, s)} e^{-i[2L_n + z]} / Z_0 \right\}$$  \hspace{1cm} (9.17b)

### 9.1.3 Modelling Multiple Discontinuities

![Diagram of fence sections](image)

Fig. 9.4 Modelling discontinuities by dividing the fence into sections with uniform properties.

The 2-port model presented in Section 8.2.3 can be used to analyze the effects of discontinuities that could occur along the single-wire fence from branches, faults, loads, etc [1-5, 37-39]. Consider two adjacent sections of a fence that are divided by a discontinuity as illustrated in Fig. 9.4. The travelling waves entering the $n^{th}$ section from the preceding fence section are denoted by $A^n$ whereas the travelling waves entering from the following section are denoted by $B^n$. The wave components $A^n$ and $B^n$ are a result of all the reflections and
refraction that occur along the fence line and are related to the line voltages at the two boundaries by,

\[ \sum_{\beta = 1}^{i} A_{\beta}^{(n+1)} = V_{(z,z)}^{(n)} \bigg|_{z=L_n} - \sum_{\beta = 1}^{i} B_{\beta}^{(n)} \]  \hspace{1cm} (9.18a)

\[ \sum_{\beta = 1}^{i} B_{\beta}^{(n-1)} = V_{(z,z)}^{(n)} \bigg|_{z=0} - \sum_{\beta = 1}^{i} A_{\beta}^{(n)} \]  \hspace{1cm} (9.18b)

The voltage and the current in the \( n \)-th section can be calculated in relation to \( A^n \) and \( B^n \) from (9.17) as given by,

\[ V^n_{(z,z)} = T^n \sum_{\beta = 1}^{y} \sum_{a = 1}^{m} \left\{ \left( \Gamma^n_e \Gamma^n_s \right)^{n-1} A^a \left( e^{-\gamma_n [2L_n (a-1) + z]} + \Gamma^n_e \gamma_n [-2L_n a + z] \right) + \left( \Gamma^n_s \Gamma^n_s \right)^{n-1} B^a \left( e^{-\gamma_n [2L_n (a-1) + (z-L_n)]} + \Gamma^n_s \gamma_n [2L_n a + (z-L_n)] \right) \right\} \]  \hspace{1cm} (9.19a)

\[ I^n_{(z,z)} = T^n \sum_{\beta = 1}^{y} \sum_{a = 1}^{m} \left\{ \left( \Gamma^n_e \Gamma^n_s \right)^{n-1} A^a \left( e^{-\gamma_n [2L_n (a-1) + z]} - \Gamma^n_e \gamma_n [-2L_n a + z] \right) - \left( \Gamma^n_s \Gamma^n_s \right)^{n-1} B^a \left( e^{-\gamma_n [2L_n (a-1) + (z-L_n)]} - \Gamma^n_s \gamma_n [2L_n a + (z-L_n)] \right) \right\} \]  \hspace{1cm} (9.19b)

From (9.18) and (9.19),

\[ A^{(n+1)}_\beta = \sum_{a = 1}^{m} \left\{ \left[ I^n + \Gamma^n_e \right] \left[ \Gamma^n_e \Gamma^n_s \right]^{n-1} A^a \left( e^{-\gamma_n [2L_n (a-1) + L_n]} \right) \right\} \]  \hspace{1cm} (9.20a)

\[ B^{(n-1)}_\beta = \sum_{a = 1}^{m} \left\{ \left[ I^n + \Gamma^n_s \right] \left[ \Gamma^n_e \Gamma^n_s \right]^{n-1} B^a \left( e^{-\gamma_n [2L_n (a-1) + L_n]} \right) \right\} \]  \hspace{1cm} (9.20b)

where the reflection coefficient at the boundaries are given by,

\[ \Gamma^n_e = \frac{\left( Z^{n+1}_0 \right)^{-1} + \left( Z^n_1 \right)^{-1}}{\left( Z^{n+1}_0 \right)^{-1} + \left( Z^n_1 \right)^{-1}} - Z^n_0 \]  \hspace{1cm} (9.21)

\[ \Gamma^n_s = \frac{\left( Z^{n-1}_0 \right)^{-1} + \left( Z^{n-1}_1 \right)^{-1}}{\left( Z^{n-1}_0 \right)^{-1} + \left( Z^{n-1}_1 \right)^{-1}} - Z^n_0 \]  \hspace{1cm} (9.22)
The waves entering section-1 from the energizer end ($A^1$) is related to the supply as given in (9.16) whereas the waves entering the last section from the line termination ($B^N$) are zero if a passive network is used at this end. Thus, the voltage or current at any location along the fence can be calculated by evaluating (9.19) to (9.22) and converting the solutions to the time-domain.

### 9.1.4 Time-Domain Numerical Solution

Since it is impossible to obtain explicit time-domain solutions for both, line voltage and current from (9.19), a numerical approach similar to algorithm discussed in Section 8.2.4 is adopted. However, in comparison to (8.33), the solution given by (9.19) is in scalar form. Therefore, the implementation of the numerical solution is straightforward and the solution can be obtained with significantly less number of computations. The flowchart of the proposed algorithm, which numerically evaluates the inverse Laplace transformation of (9.19) to give a solution in time-domain for a given period and accuracy, is shown in Fig. 9.5. The numerical Laplace transformation uses an improved inversion algorithm with accelerated convergence as discussed in [12].
Fig. 9.5  MATLAB algorithm that calculates time-domain fence voltage and current
9.2 Results and Discussion

As discussed in Section 8.3, PSCAD is suitable for analysing simple fence structures, such as the single-wire fence considered in this chapter. Therefore PSCAD simulations were used as an efficient means of validating the presented theoretical analysis for a single-wire electric fence structure with an earth return. The voltage and current waveforms at a half-way point along an 8 km, single-wire fence with an open terminal end for various conditions are evaluated using the theoretical model, and the results are compared with simulations to validate the accuracy of the presented analysis. A list of fence parameters was given in Appendix B. Even though the proposed technique can be used to study the propagation of various pulse shapes, a 10 kV, 500 µs square pulse source with a low output impedance was considered for simplicity in the examples considered below.

9.2.1 Line Voltages and Currents

The line voltage and current for a fence with very low leakage conductance, which is about $10^{11}$ S/m is shown in Fig. 9.6. Since the measurements are done at a distance of 4 km from the energizer, the propagation delay of the line has caused the pulse to appear approximately 14 µs after the input. The speed of pulse propagation can be calculated from this data, and is approximately $2.67 \times 10^8$ m/s, which is considerably lower than the wave propagation speed of an ideal fence wire. Also the propagation velocity of the single-wire fence is about 20% faster than the propagation velocity of the 2-wire fence studied in Section 8.3.1. The initial voltage (approximately 6.5 kV) appears to be considerably lower than the input as the source impedance together with the characteristic impedance of the line forms a voltage divider. However, the first reflection from the terminal end, reaching the midpoint after about 32 µs is
in phase with the initial pulse, and therefore increases the line voltage above its steady state value. The second reflection reaching the midpoint (at ~ 75 µs) from the source end is out of phase, and reduces the line voltage. The reflections are attenuated to a point, where they are negligible after about 250 µs and the line voltage and current can be considered to have reached a steady state value. The variation of line parameters with frequency causes the pulses to propagate with different attenuation levels and phase velocities, resulting in a distorted pulse. As evident from Fig. 9.6, the waveforms predicted by the proposed semi-analytical technique are in very good agreement with PSCAD simulation results, hence confirm the accuracy of the model and subsequent theoretical analysis.

![Fig. 9.6 Voltage and current at the midpoint of an 8 km long low leakage fence](image)

Fig. 9.6 Voltage and current at the midpoint of an 8 km long low leakage fence

Fig. 9.7 shows the effect on the fence voltage and current when the shunt conductance of the fence was increased to $10^{-7}$ S/m to simulate a fence with high leakage current. In comparison to Fig. 9.6, the reflections are considerably attenuated in this case, due to losses through the leakage current. Also the high leakage current increases the voltage drop across the series impedance, resulting in a considerably lower line voltage than in the earlier case. In
comparison to Fig. 8.9, a faster propagation velocity is observed, although a significant
difference in the line voltages and currents are not evident.

The simulated and theoretical waveforms at the midpoint of the line, after introducing a short
circuit at the end of the line are shown in Fig. 9.8. As evident, due to the short circuit the
steady state current has increased considerably to 20 A whereas the voltage has reduced from
10 kV to about 4 kV with a peak of 6 kV, which would only be marginally sufficient to shock a
bull [40]. Furthermore, both simulated and theoretical waveforms follow each other very well, therefore validate the accuracy of the proposed single-wire fence model.

9.2.2 Frequency Dependence of Line Parameters

Fig. 9.9 Variation of characteristic impedance with frequency for different earth resistivities

In Sections 8.2.1 and 9.1.1 the frequency dependant nature of the line parameters due to earth currents and skin effect were discussed, showing how the changes in line parameters affect the characteristic impedance and the propagation function. These effects were investigated using the proposed model. Fig. 9.9 compares the variation in characteristic impedance of the line when the earth resistivity is changed, while the earth return resistance is kept constant. It is evident from Fig. 9.9, that the change in characteristic impedance is very small for earth resistivity up to 100 Ωm. Investigations also revealed that the effect of fence height on the characteristic impedance is small, as the characteristic impedance only increases by about 5% when the fence height is doubled. This is because the ratio of the impedance to admittance
cancels the height effect based on the definition of $Z_0 = \sqrt{Z_0 / Y_0}$. The characteristic impedance presented by this fence is very large for low frequencies, and converges to about 500 $\Omega$ for frequencies higher than 10 kHz. The variation of line parameters with frequency causes the frequency components of the input pulse to propagate with different attenuation levels and phase velocities, resulting in a distorted pulse.

### 9.2.3 Shock Voltage

![Figure 9.10 Variation of peak shock voltage](image)

Fig. 9.10 shows the peak voltage, which would be experienced by an animal in contact with the fence. The animal was modelled by a 300 $\Omega$ resistive load, which is the average muzzle to hoof impedance for a bull [40], and the same source that was introduced in the previous examples was used for this analysis. As expected the shock voltage is inversely proportional to the distance between the energizer and the point of contact. According to empirical measurement at least 5 kV is required to effectively shock a Bull [12, 40], hence the tested fence is only effective within the first 5 km.
9.3 Conclusions

A simplified mathematical model that can be used to analyze the propagation of transient pulses over a single-wire electric fence has been described. With a few limitations, the technique can also be used to analyze fence structures that have multiple unenergized conductors and a single energized conductor. The proposed technique models the electric fence as a transmission line with frequency dependent line parameters that account for the skin effect, proximity effect and earth current effects. The propagation properties of the fence are approximated by the quasi-TEM mode and the line parameters are substituted in the telegrapher’s equation to derive the voltage and current at a given location along the fence. The frequency-domain analytical expressions for the fence voltage and current were then converted to time-domain by using a MATLAB based numerical algorithm.

The accuracy of the proposed analysis was verified by PSCAD simulations for different operating conditions. The analysis shows that the propagation velocity of a single wire fence is about 20% higher than of a multi-wire fence. A comparison between the line voltages of multi-wire fences that have a single energized conductor and an equivalent single-wire fence was presented to show that the errors introduced by a single-wire approximation is less than 10%. In comparison to the generalized mathematical model presented in Chapter 8, a significant reduction in computation time can be achieved through the single-wire approximation due to the reduced number of mathematical manipulations required by the simplified model. Therefore, the proposed technique serves as a valuable tool at the design stage of electric fence structures that have a single energized conductor.
9.4 References


10.1 General Conclusions

This thesis has presented a comprehensive analysis on electric fence systems, focusing mainly on the following aspects.

- Analysis on HVPPS technologies suitable for electric fence energizers
- Analysis on HV charger schemes suitable for electric fence energizers
- Design and development of technologies for improving the performance of existing energizer technology
- Development and verification of comprehensive mathematical models to characterize the propagation properties of electric fence structures

A detailed review on the electric fencing technology, from its inception to the present state, has been presented in the first chapter. The functionality of typical fence systems and the key issues affecting the performance of existing technology have been discussed in detail. The need for analytical and numerical models to accurately characterize the behaviour of typical fence systems for advancements in electric fence technology has been justified. The benefits of a new energizer technology with improved efficiency and the ability to generate
customizable output pulse shapes for the development of intelligent fence systems and fence line communication systems have been demonstrated.

In Chapter 2, an investigation of the applicability of existing HVPPS technologies for implementing an efficient electric fence energizer that can produce customizable output pulses has been presented. Four HVPPS technologies, which fulfil these requirements, have been identified and analyzed, and a summary of these technologies is given below.

- **Pulse transformer type** – is the HVPPS technology used by most modern fence energizers, due to its simplicity, lower manufacturing costs and reliability. The pulses generated by this type of energizers have fixed parameters that are determined by the values of output filter components. In addition, due to high losses associated with the pulse transformer technique, this topology is recommended to be used only in small to medium size electric fence energizers.

- **Marx generator type** – facilitates the implementation of compact and efficient high power fence energizers since both the voltage and current stresses on components can be reduced. In addition, these generators have the ability to adapt the output pulse shapes to suit fence conditions, and therefore it is suitable for use in intelligent electric fence systems. However, the design of an energizer based on this technology incurs high manufacturing costs due to high circuit complexity.

- **Direct discharge type** - has similar properties to the Marx type, but the technique raises safety concerns, especially at elevated energy levels. However, in comparison to the Marx topology, the implementation of this topology is simple. As such, the direct discharge type could be an attractive solution for the implementation of low energy, solar-powered units, where high efficiency and low cost are vital design considerations.
• Vector inversion type – is commonly used in a number of very high power applications since this topology requires only a single power switch to initiate an output pulse. However, after it has been initiated, the pulse cannot be terminated without additional switches. Therefore, the vector inversion type is not suitable for use in fence energizers.

A comprehensive mathematical model that facilitates the analysis of output characteristics and the component stresses of a vector inversion generator has been presented in Chapter 3. Two equivalent circuits, which are independent from each other, have been derived to model the behaviour of the odd numbered stages and even numbered stages. The analysis of the even-numbered stages was then accomplished by a further simplification to obtain an equivalent simplified circuit composed of a LCI ladder network. It has been shown that the voltage variations across the odd-numbered stages are similar to each other, and an equivalent LCI circuit is derived to model the behaviour of these stages. These two simplified equivalent models provided valuable information with regard to the behaviour of the converter, which can be used at the design and optimization stages of HVPPS that are based on the vector inversion technique. The equivalent models were used to derive s-domain analytical functions of various circuit parameters, which were then numerically solved to obtain the time-domain solutions. The behaviour of a few vector inversion converters has been analysed through the proposed method and the accuracy of the solutions have been verified through SPICE simulations. In addition, a simplification has been proposed to facilitate the derivations of time-domain analytical solutions for a lightly loaded converter.

A novel design of a high power electric fence energizer has been presented in Chapter 4. The proposed design, which is based on the Marx principle, has the ability to generate output pulses with different amplitude levels and pulse widths. The operation of the proposed converter has been analyzed and a 120 J prototype energizer has been implemented to
investigate the performance. The ability of the prototype to generate high power output pulses with customizable pulse parameters has been demonstrated. Furthermore, benchmark tests have been conducted at various farms, and the results revealed that the performance of this technology is superior in comparison to existing high-end fence energizers.

In Chapter 5, a novel solid-state, high voltage pulse generation technique, termed multi-level HVPPS, has been described. The proposed technique, which is suitable for a wide range of pulsed power applications, is somewhat similar to the direct discharge type, but has the unique ability to share the voltage stresses equally among the components, therefore significantly improve the design reliability. The output characteristics and the component stresses of the multi-level converter have been analyzed mathematically and the results have been compared with SPICE simulations to verify the validity of the mathematical analysis and the proposed topology. Furthermore, the ability of the proposed concept to generate output pulses with flexible amplitudes and durations has been demonstrated through SPICE simulations and theoretical analysis. The inherent protection and voltage stress balancing properties of the converter, which would be useful to design simple yet more reliable high power pulse generators, have also been demonstrated through simulations.

Quasi-resonant techniques, which can be utilised to improve the efficiency and reduce EMI emitted by the high-voltage charging schemes used in existing energizers, have been presented in Chapter 6. The performance of a fly-back converter based HV charger, which is the most widely used technology among energizer, has been analysed, concluding that such a design has many deficiencies, especially at elevated power and frequency levels. An active-clamping technique and a LCD snubber design that can reduce parasitic losses found in a fly-back type HV charger, within a limited load range, have been presented.
A novel control technique that facilitates the operation of a PPRC as a Buck-Boost converter, with ZVS, over a wide load range, has been presented in Chapter 6 and 7. The technique requires a slight modification to the conventional PPRC circuit to enable operation above or below the damped resonant frequency to realize lower (Buck) or higher (Boost) than nominal voltage gains, respectively. The proposed split-capacitor type PPRC topology has a wide range of applications. Furthermore, this technique can be used to design an efficient, low EMI and compact charger for charging HV storage capacitors in electric fence energizers. The converter has been represented by a model, through which explicit analytical expressions have been derived for accurate prediction of its performance in the Boost, Normal and Buck modes. The analysis presented provides an insight into the behaviour of the converter in all three modes and thus can be regarded as a valuable tool for design and optimization of SC-PPRC. The validity of both the model and theoretical analysis have been verified by simulations and measured performance of a 150 W prototype converter under various operating conditions. In addition, it has been shown that the proposed topology is immune against undesirable parasitic oscillations. The critical operating conditions for each mode have been evaluated, and results indicated that the Buck mode has a critical load corresponding for each switching frequency, below which its operation in the Buck mode is not possible.

In Chapter 8, a generalized model and a semi-analytical technique that facilitate the analysis of HV pulse propagation along typical multi-wire electric fence networks have been described. The general transmission line theory, which is based on the quasi-TEM approximation, has been used to model the propagation of energizer pulses along the fence structures. The frequency dependent distributed line parameters of the fence have been calculated in the s-domain, accounting for skin effect, proximity effect and earth current effects. The discontinuities that can be found along the fence, caused by faults, branches and loads, have also been modelled to obtain a generalized mathematical model to predict the
propagation properties of typical fence networks. The fence has been divided into uniform sections at the discontinuities and each section has been modelled as a 2-port network. The telegrapher’s equations along with the frequency dependent line parameters have been used to obtain an analytical expression for the fence voltages and currents in the s-domain for each of these uniform sections. Afterwards, the boundary conditions for each section have been derived in terms of adjacent sections, and the analytical expressions have been numerically evaluated to obtain a time-domain solution. The accuracy of the proposed mathematical model and subsequent analysis have been verified through experiments and PSCAD simulations.

The technique that was proposed in Chapter 8, provides an insight into the behaviour of HV pulse propagation along multi-wire electric fence structures, and thus serves as a useful design tool for electric fence system designers. This technique has been used to investigate the performance of experimental fence structures and the results illustrated that although the discontinuities reflect a significant portion of the pulse, the effect of such reflections on the final line voltages are small due to high level of attenuation arising as a result of the skin effect. Furthermore, an analysis on the errors introduced by the simplified soil parameters in the theoretical solution has been presented to show that such errors are negligibly small.

In Chapter 9, a simplification to the generalized mathematical model has been proposed to enable the derivation of the propagation functions of a single-wire fence in the scalar-form. In comparison to the generalized mathematical model, a significant reduction in computation time can be achieved through the single-wire approximation due to reduced number of mathematical manipulations required by the simplified model. The technique can also be used to approximate propagation characteristics of fence structures that have multiple unenergized conductors or a single energized conductor. The accuracy of the simplified mathematical analysis has been verified by PSCAD simulations for different operating
conditions. A comparison between the line voltages of multi-wire fences that have a single energized conductor and an equivalent single-wire fence has been presented to show that the errors introduced by a single-wire approximation is less than 10%.

10.2 Contributions

The main contributions of this study are summarised below.

- Mathematical tools have been developed to characterise the propagation characteristics of electric fence networks. The accuracy of these tools has been verified through experimental and simulation results conducted on various electric fence structures.

- An analysis on HVPPS technologies suitable for electric fence energizers has been presented. A novel high power electric fence energizer design, which is based on the Marx concept, has been proposed. A 120 J energizer unit has been implemented to verify the functionality of the proposed design.

- A novel HVPPS technology, termed the multi-level HVPPS, which is suitable for a wide range of pulsed power applications, has been proposed. The proposed technology has been analysed mathematically and through simulations to verify its viability.

- A novel Buck-Boost control technique, which facilitates the operation of a PPRC over a wide load range with ZVS and output voltage control, has been proposed. A comprehensive mathematical analysis on the operation of the proposed converter in the Buck, Normal and Boost modes has been presented. The validity of both the proposed method and theoretical analysis have been verified by simulations and
measured performance of a prototype converter suitable for charging HV storage capacitors used in energizers.

The above outlined work have been published in four IEEE and IET Transactions and ten IEEE conference proceedings that are listed below. Three additional journal papers are in preparation.


**10.3 Recommendations**

The main objective of this study has been to propose mathematical tools and new technologies that will aid the design and development of optimized electric fence systems. Accordingly, mathematical tools have been developed to gain an insight into the propagation properties of electric fence energizers with the intention of aiding the design procedure. The accuracy of these tools has been verified experimentally and also through simulations. However, an optimized fence network has not been designed during this work as it was outside the scope of this study. Therefore, the author suggests the design of an optimized electric fence system using the developed tools to investigate the performance of such a system. In addition, the proposed mathematical tools can be grouped into a software package with a graphical user interface (GUI) to provide a platform, which designers can use to analyze and optimize electric fence networks.

The generalized mathematical model, developed to characterize pulse propagation over electric fence structures, requires empirical data for the earth return resistance. This can be avoided by incorporating mathematical models to predict the earth return resistance based on the soil parameters and earthing conditions. Moreover, these models can be used to design more effective earthing systems for fence energizers, thereby improving the return path resistance considerably. Therefore, future studies should investigate the use of mathematical models to characterize the earth return resistance of electric fence systems.

The proposed multi-level HVPPS technology enables the design of highly compact and efficient electric fence energizers at a competitive cost. This technology can be used to develop small solar-powered energizer units with a superior performance over existing products. The author, in collaboration with Tru-Test engineers, is currently investigating the design of an ultra compact energizer with 100% surface-mount components using this
technology. As this technology has potential uses in many fields, further investigations into such applications are necessary.

A novel Buck-Boost PPRC topology has been developed during this study. The converter has been operated with an open-loop controller and the design of a closed-loop controller has been proposed. The implementation of a robust closed-loop control strategy has to be investigated which requires a detailed study. Furthermore, a detailed transient analysis of the proposed topology has to be carried-out, and the performance of the converter under input and load transients has to be analyzed.
# Appendix A: Parameters of PPRC

<table>
<thead>
<tr>
<th>Parameter/Device</th>
<th>Value</th>
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<tbody>
<tr>
<td>$V_{DC}$</td>
<td>30-150 V</td>
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<tr>
<td>Load$_{critical}$</td>
<td>220 $\Omega$</td>
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<tr>
<td>Load</td>
<td>300-10000 $\Omega$</td>
</tr>
<tr>
<td>$f_r$ (measured)</td>
<td>74 kHz</td>
</tr>
<tr>
<td>$f_r$ (calculated)</td>
<td>77 kHz</td>
</tr>
<tr>
<td>$f_s$</td>
<td>45-74 kHz</td>
</tr>
<tr>
<td>$V_o$</td>
<td>variable</td>
</tr>
<tr>
<td>$P_{o,max}$</td>
<td>150 W</td>
</tr>
<tr>
<td>$C_1 = C_2 = C$</td>
<td>4.7 nF</td>
</tr>
<tr>
<td>$L_{DC}$</td>
<td>3 mH</td>
</tr>
<tr>
<td>$L_1 = L_2$</td>
<td>230 $\mu$H</td>
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<tr>
<td>Transformer core</td>
<td>ETD 39</td>
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<tr>
<td>Air-gap</td>
<td>1 mm</td>
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<tr>
<td>$k$</td>
<td>0.99</td>
</tr>
<tr>
<td>$N$</td>
<td>34</td>
</tr>
<tr>
<td>$S_1=S_2$</td>
<td>IRFB9N60 (600V/9A MOSFET)</td>
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</table>
## Appendix B: Fence Parameters Used in Simulations

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
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<tbody>
<tr>
<td>Length of the fence</td>
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<tr>
<td>Shunt conductivity</td>
<td>$10^{-7}$-$10^{-11}$ S/m</td>
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<tr>
<td>Ground resistivity</td>
<td>100 $\Omega$m</td>
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<tr>
<td>Wire resistivity</td>
<td>1.74e-7 $\Omega$m</td>
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<tr>
<td>Earth resistance</td>
<td>0.01 $\Omega$/m</td>
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<tr>
<td>Wire height - bottom</td>
<td>0.4 m</td>
</tr>
<tr>
<td>Wire height - top</td>
<td>0.8 m</td>
</tr>
<tr>
<td>Wire radius</td>
<td>1.25 mm</td>
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</table>
## Appendix C: Fence Parameters of Karaka Farm

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the fence</td>
<td>0.66-1.16 km</td>
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<tr>
<td>Shunt conductivity</td>
<td>$10^{-9}$-$10^{-11}$ S/m</td>
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<tr>
<td>Ground resistivity</td>
<td>100 $\Omega$m</td>
</tr>
<tr>
<td>Wire resistivity</td>
<td>1.74e-7 $\Omega$m</td>
</tr>
<tr>
<td>Earth resistance</td>
<td>0.3 $\Omega$/m</td>
</tr>
<tr>
<td>Wire radius</td>
<td>1.25 mm</td>
</tr>
<tr>
<td>Impedance at C3</td>
<td>$3000 + j\omega 1e^{-9}$ $\Omega$</td>
</tr>
<tr>
<td>Impedance at C3</td>
<td>$1000 + j\omega 2e^{-9}$ $\Omega$</td>
</tr>
</tbody>
</table>