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Raman scattering and four-wave mixing

From fundamentals to fibre lasers

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12th May 2009

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Stéphane Coen

Abstract

Raman scattering and four-wave mixing are two fundamental nonlinear phenomena present in optical fibres with important implications for applications in fields ranging from modern telecommunications networks to biophotonics.

This thesis investigates three situations when these two phenomena interact: Firstly we investigate the interplay of multiple four-wave mixing processes using coherent and incoherent pump waves in the presence of Raman scattering. We experimentally demonstrate that despite the requirements of phase-matching conditions it is possible to observe multiple phasematched and non-phasematched four-wave mixing processes. Furthermore we show that an incoherent light wave provided by amplified spontaneous emission noise can act as an effective pump wave for degenerate four-wave mixing.

The main part of the thesis is occupied by the investigation of a mode-locked Raman fibre laser. The use of dissipative four-wave mixing for the passive mode-locking technique in combination with Raman scattering as the gain mechanism offers the possibility of achieving ultra-high repetition rates at very high average output powers. We experimentally demonstrate the mode-locked operation of the laser at 500 GHz and achieve an average output power of almost 1 W. Additionally we examine the key limitation of the laser which is supermode noise caused by mode-locking the laser at very high harmonics of the cavity resonance frequency. In order to gain qualitative insight into the influence of supermode noise on the laser dynamics we create a laser model which takes account of supermode noise. Furthermore we design a scheme to reduce supermode noise using additional subcavities, and evaluate the scheme using a lower repetition rate laser. We show that by including the subcavities into the setup the amount of supermode noise can be reduced by at least a factor 100.

Lastly we introduce a novel method to measure the noise fluctuations of continuous wave lasers at timescales prohibiting the use of traditional noise measurement techniques. The noise is measured using a technique which transfers the fast noise from the continuous wave laser to a low repetition rate mode-locked laser which can be measured with traditional methods. We demonstrate that a continuous wave Raman fibre laser exhibits ultrafast, high contrast intensity fluctuations at timescales of tens of Gigahertz.

This work has led to three publications and six conference presentations.
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Since the dawn of mankind have humans been fascinated by the properties of light. In ancient India light was one of the five fundamental elements and philosophers developed theories of its nature as early as the 5th to 6th century BC [1]. Philosophers in ancient Greece were likewise fascinated with light and about 300 BC the philosopher and mathematician Euclid wrote *Optica* also known by its latin translation *De visu* in which he developed a theory of vision and light based on geometrical concepts [1]. In the islamic world Ibn al-Haytham or Alhazen, who is often considered the father of modern optics, developed a theory on the properties of light in the *Book of Optics* (1012). He was also the first person to carry out experiments on the dispersion of white light into its composing colours [2, 1]. The philosophers and scientist of the enlightenment later elaborated and developed these theories and optics was one of their main interest of research as is evident for example by Sir Isaac Newton who devoted an entire book to optics (*Opticks* (1704)). After Newton light remained a focus of interest for many scientists and in 1864 James Clerk Maxwell wrote in *A Dynamical Theory of the Electromagnetic Field*:

*The agreement of the results seems to show that light and magnetism are affections of the same substance, and that light is an electromagnetic disturbance*
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propagated through the field according to electromagnetic laws. (A Dynamical Theory of the Electromagnetic Field, 1864).

He was therefore the first person to describe light as an electromagnetic wave. By using the equations which became later known as Maxwell’s equations to derive the electromagnetic wave equation, Maxwell laid the foundation for the modern mathematical description of optics. Today Maxwell’s equations form the basis of almost all theoretical treatments in optics.

The field of nonlinear optics is a branch of optics describing the behaviour of light propagating in media with a nonlinear response of the polarization ($\vec{P}$) to the electric field ($\vec{E}$). Although nonlinear effects such as the Kerr- or the Pockels-effect have been known since the late 19th century, the required high intensities could only be produced by DC-fields and thus it was only possible to observe the nonlinear optical response to applied external fields. The field of nonlinear optics therefore remained largely unexplored for many years. This changed with the advent of the ruby laser in 1960 [3, 4] which suddenly provided the means to reach the optical intensities necessary to observe the polarization to respond nonlinearly to the electromagnetic radiation. In 1961 Franken and co-workers published a paper titled “Generation of optical harmonics”, in it they reported the “unambiguous indication of second harmonic” [5]. This paper is generally considered the dawn of the field of nonlinear optics. The availability of a source of coherent optical radiation with nearly monochromatic properties spurred the research into nonlinear optics and soon let to the discovery of various novel phenomena such as stimulated Raman scattering [6], Brillouin scattering [7], two photon absorption [8] and more [9, 10]. Today nonlinear optics is a vast field encompassing different research areas such as pattern formation [11, 12], solitary waves like spatial [13, 14] and temporal solitons [15, 16] or holographic data storage [17].

Although optical fibres which guide light by total internal reflection had been developed as early the 1920’s it was only with the development of fibres with losses below 20 dB/km [18] that nonlinear optics in optical fibres became feasible. Soon after this development stimulated Raman and Brillouin-scattering were studied in optical fibres [19, 20]. Since then the field of nonlinear fibre optics has grown rapidly and enormous progress has been made.
in the recent decades. Fibre optics and nonlinear fibre optics in particular have revolutionized the nature of telecommunications. Today a world without fibre optics is unimaginable. The backbone of the world wide web is based on fibre technology and optical fibres have now superseded copper cables for almost all telecommunications applications. Currently there is a push to replace the last domain of RF-cable technology with the development of fibre to the home technologies [21]. The key advantage of telecommunications using light-wave technology is the very high carrier-wave frequency of light which amounts to several hundred THz in the near-infrared region. Because the bandwidth of a modulated carrier depends on the carrier frequency, lightwave communications systems offer several orders of magnitude increase in data transfer rates compared to RF transmission systems with carrier-wave frequencies in the GHz region. The transmission rates of optical fibre networks have therefore been increasing continuously in the last decade. In 2001 a record transmission rate of over 10 Tbit/s was demonstrated [22, 23]. Despite the tremendous advantages in the fibre-based telecommunications technology, current systems still rely on time and wavelength division multiplexing of channels with 40 Gbit/s to reach bit rates of several 100 Gbit/s. However the telecommunications requirements are constantly increasing and the development in bit rate per wavelength is heading past 40 Gbit/s to 160 Gbit/s and beyond [24]. One of the main factors impeding the increase of the bandwidth inside a single telecommunications channel is the limitation imposed by the processing speed of the involved electronic equipment, i.e. the electronic bandwidth. Traditional methods of actively modulating and altering the light using electronic equipment fail at timescales above 40 GHz. Researchers have therefore started investigating all-optical means of ultra-fast signal processing, thus circumventing the restrictions imposed by the use of electronic equipment. Significant effort is currently being invested in developing ultra-high repetition rate fibre lasers [25, 26, 27], all-optical switching [28, 29] and signal-processing [30, 24] capabilities. More recently the advent of photonic crystals and silicon photonics has created a push to miniaturize optical components in order to perform signal-processing with “light on a chip” [31, 32, 33] promising to revolutionize signal-processing.

On the background of its application to all-optical telecommunications the present dis-
Chapter 1 Introduction

The dissertation is part of the research effort to broaden the knowledge of phenomena in nonlinear optics and nonlinear fibre optics in particular. We concentrate on three aspects on the boundary between parametric and non-parametric processes and their interaction, notably four-wave mixing and Raman scattering. The three key areas we will address in this thesis are: Multiple four-wave mixing processes, which will be discussed in chapter 2, mode-locking by dissipative four-wave mixing of a Raman fibre laser and its noise properties (chapter 3 and chapter 4 respectively) and the measurement of fast, high-contrast intensity fluctuations of a continuous wave Raman fibre laser in chapter 5. The aim of this work is to give an overview and understanding of the research carried out during the course of my PhD thesis. This encompasses providing the necessary background of the research as well as giving an overview of the state of current knowledge on the specific topics. In the following we will give a brief overview about the topics in the chapters and place them into perspective in their relation to the current research.

Four-wave mixing and Raman scattering are two of the key effects in nonlinear optics [34]. Four-wave mixing is the interaction between four waves mediated by the instantaneous optical Kerr effect, or the $\chi^{(3)}$ nonlinearity. Similarly, Raman gain can also be attributed to the third-order nonlinearity, however it is induced by the non-instantaneous nonlinearity associated with the vibrational contribution to $\chi^{(3)}$. The Raman effect is caused by inelastic scattering of photons from an atom or molecule, with a fraction of the photon energy dissipated through a phonon. The combination of Ramam scattering with stimulated emission often denoted stimulated Raman scattering (SRS) is the underlying effect of Raman amplification and thus the basis of many applications such as Raman fibre lasers or amplifiers. Although Raman scattering and four-wave mixing have long been known to affect each other [35] (effects ranging from Raman amplification of non-phasematched waves [36] to parametric Raman gain suppression [37] have been reported), applications taking advantage of their combination are scarce.

In general four-wave mixing has to adhere to strict phase-matching conditions for the process to be efficient. The existence of multiple independent four-wave mixing processes presented in chapter 2 is therefore remarkable. We show that several four-wave mixing pro-
cesses of different nature can exist simultaneously. In particular we show that spontaneous and seeded, phasematched and non-phasematched, Raman assisted four-wave mixing can co-exist and that incoherent amplified spontaneous emission (ASE) noise can function as a pump to the four-wave mixing process. This has important implications for applications such as wavelength conversion.

One of the key advantages of both Raman scattering and four-wave mixing is that they are wavelength versatile, i.e. we are not restricted to specific spectral regions for their use. This is particularly advantageous for the creation of wavelength versatile lasers, and mode-locked lasers in particular. By combining Raman gain with a mode-locking technique based on four-wave mixing, we are able to create a mode-locked laser which in principle operates at every desired wavelength. Combining Raman gain with a passive mode-locking technique offers the ability to create a fast repetition rate mode-locked laser at virtually any desired spectral region, overcoming the limitations of rare-earth-doped lasers. Surprisingly it was not until 2005 when Chestnut et al. published their article on a Raman-pumped figure-of-eight laser that someone created such a laser [38]. However their setup did not achieve the ultra-high repetition rates possible with passive mode-locking techniques. In contrast to Chestnut et al. we combine the large bandwidth of the Raman gain with a passive-mode locking technique allowing us to achieve repetition rates of up to 500 GHz. We will introduce the passively mode-locked Raman fibre laser and explain the mode-locking principle which was first conceived by Quiroga-Teixeiro [39] in chapter 3. We then present a detailed theoretical, numerical and experimental investigation of the laser and demonstrate that the laser does not only achieve ultra-high repetition rates but also offers exceptionally high average output powers, the highest that have been achieved for a laser with comparable repetition rates. The following chapter (chapter 4) encompasses a discussion of one of the key limitations of the laser, i.e. the existence of supermode noise. Although supermode-noise is a well-known concept in the investigation of mode-locked lasers [40, 41, 42] it is often ignored when discussing passively mode-locked lasers. This is particularly the case at repetition rates above 100 GHz, due to the difficulties in assessing the noise properties of such high repetition rate lasers. The discussion in chapter 4 of the noise properties of
our laser is to the best of our knowledge the first treatment on the noise properties of such a high repetition rate mode-locked laser. Furthermore we will propose and demonstrate a technique to reduce the impact of the noise on the passively mode-locked laser.

In chapter 5 we then will discuss noise properties of lasers at the opposite end of the timescale, i.e. continuous wave cascaded Raman fibre lasers. There exist several studies on the noise properties of continuous wave Raman fibre lasers [43, 44, 45], however these studies were limited to the discussion of relatively slow fluctuations of the laser intensity based on supermode beating. Simultaneously numerical studies into the influence of pump coherence on supercontinuum generation have resulted in contradictory results [46,47,48] about the nature of the noise. Some of the studies showed evidence that cascaded Raman fibre lasers exhibit high-contrast fluctuations at timescales in the range of 100 GHz or more [48]. We present a novel method to measure these ultra-fast fluctuations. The results do not only have implications for continuous wave supercontinuum generation, but also for telecommunications transmission systems where Raman fibre lasers function as pump lasers for signal amplification. With the increase in repetition rates, these fast fluctuations which could previously be ignored will gain increasing significance.

1.1 Assumptions

Before we proceed to the individual chapters we need to clarify some key assumptions used throughout this work. Firstly we need to note, that although some of the results may be applicable to other areas of nonlinear optics, we restrict our treatment to the propagation of light inside single-mode fibres (SMF) made from silica. This has significant implications on the theoretical description of the phenomena we encounter.

In general electro-magnetic wave propagation inside a nonlinear dielectric medium is described by the so-called wave equation, which can be derived directly from Maxwell’s equations [34,49]. However under a set of assumptions which we discuss in more detail later the wave equation can be significantly simplified to yield the nonlinear Schrödinger equation.
(NLSE) [34]:
\[
\frac{\partial A}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} = i\gamma |A|^2 A
\] (1.1)

Here $\beta_2$ is the group-velocity dispersion and $\gamma$ is a parameter describing the strength of the nonlinearity. The time $T = t - \beta_1 z$ is defined in a reference frame travelling at the group velocity of the light inside the fibre. The NLSE can be considered the swiss-army knife of the fibre-optic physicist. It describes a large part of all the phenomena encountered when light propagates along a fibre, among them self-phase modulation [50, 51] and soliton propagation [52], just to name two. The NLSE also forms the basis for the majority of mathematical descriptions in this thesis. However before we proceed we need to discuss the major assumptions which validate the NLSE and verify them in the context of our experiments.

The key hypothesis leading to the NLSE are [34]:

- The optical frequency of the electric field is far away from any medium resonances.
- The medium response is instantaneous.
- Transverse effects such as diffraction can be ignored.
- The polarization state of the electric field upon propagation is maintained.
- The *slowly varying envelope approximation* is valid.

In the following we will shortly discuss the implications and limitations of each of these assumptions and how they apply to our experiments.

In general to obtain the polarization density $\vec{P}$, i.e. the medium response to an electric field, a quantum-mechanical approach is necessary. However under the conditions that the frequency of the field is far away from any resonance frequencies of the medium and that the nonlinear response of the medium is instantaneous, we can use a phenomenological approach based on the Lorentz oscillator model. Under this approach we treat the nonlinear part of the polarization density as a small perturbation to the linear part and thus describe $\vec{P}$ in terms of a power series of the optical field strength [49, 34]:

\[
\vec{P} = \varepsilon_0 \left( \chi^{(1)} \vec{E} + \chi^{(2)} \vec{E} \vec{E} + \chi^{(3)} \vec{E} \vec{E} \vec{E} + \cdots \right)
\] (1.2)
Chapter 1 Introduction

Where $\chi^{(1)}$ through $\chi^{(3)}$ denote the orders of the optical susceptibility which describe the optical properties of the medium and $\varepsilon_0$ is the electric permittivity of free space. As there are no resonances in silica for optical wavelengths between 500 and 2000 nm and the wavelengths in our experiments fall well within this range, this assumption is valid. However by ignoring the non-instantaneous parts of the polarization density we neglect the vibrational contribution to $\chi^{(3)}$ which is responsible for the Raman effect. Although Raman scattering plays a significant role in our work it can often be treated by an abstract gain parameter in our calculations. We therefore continue with the discussion and return later to discuss how to modify the resulting equations to account for Raman scattering. Equation 1.2 reduces further because $\chi^{(2)}$ vanishes due to the inversion symmetry of silica glasses. The nonlinear response of the medium can therefore be described by a linear dependency of the refractive index to the local intensity of the light $I$, i.e. $n = n_0 + n_2 I$. This is often denoted an instantaneous Kerr nonlinearity.

Because we are exclusively dealing with single mode fibres light does not diffract and we can ignore any transverse effects. The electric field thus becomes a function of the time and only one spatial dimension, the longitudinal coordinate $z$ [$\vec{E} = \vec{E}(z, t)$]. By also assuming that the polarization state of the electric field upon propagation is maintained, the description of the light propagation reduces to a scalar differential equation of $t$ and $z$. However optical fibres in general are not polarization maintaining due to random core shape fluctuations and stress-induced anisotropy. We also did not utilize special polarization maintaining equipment in the experiments. However in practice the polarization instabilities cancel because of averaging of the nonlinear polarization effects due to the randomness of the perturbations to the polarization state. We can accompany for this averaging effect by multiplying the nonlinear refractive index $n_2$ by $\frac{8}{9}$ [53]. This is further validated by the fact that most of the waves in our experiments are randomly polarized.

The final hypothesis is the validity of the slowly varying amplitude approximation (SVEA) [34]. The SVEA states that the electric field envelope varies slowly with respect to the frequency of the electric field. In other words this means that the optical field is assumed to be quasi-monochromatic or the spectral width ($\Delta\omega$) of the field is small with respect to the
Assumptions

centre frequency of the spectrum \((\omega_0)\), i.e. \(\Delta \omega / \omega_0 \ll 1\). In mathematical terms we can express the SVEA by the inequality \(\left| \frac{\partial^2}{\partial t^2} \right| \ll \left| \omega_0 \frac{\partial}{\partial t} \right|\). Because the frequency of optical fields is on the order of several hundred terahertz, this approximation is valid for light pulses which are longer than 100 fs. The use of the SVEA is therefore justified in our experiments, as the shortest phenomena we encounter is longer than 500 fs. Under the SVEA we can separate the electric field into its envelope and a fast carrier wave \(E(z, t) = A(z, t) \exp i(\beta_0 z - \omega_0 t)\), where \(\omega_0\) and \(\beta_0\) are the angular frequency and the propagation constant of the carrier wave respectively. The SVEA significantly simplifies the mathematical description of the propagation of the electric field inside the fibre as we can expand the propagation constant \(\beta(\omega)\) in a Taylor series around the carrier frequency:

\[
\beta(\omega) = \beta_0 + \beta_1 (\omega - \omega_0) + \frac{\beta_2}{2} (\omega - \omega_0)^2 + \cdots \tag{1.3}
\]

The parameter \(\beta_1 \equiv 1/ \nu_g\) is the inverse of the group velocity of light in silica fibres \((\nu_g \approx 2 \cdot 10^8 \text{ m/s})\) and \(\beta_2\) is the so-called group-velocity dispersion (GVD). In single-mode silica fibres the GVD coefficient vanishes at \(\lambda_0\) which is called the zero-dispersion wavelength. Light propagation is often categorised by the sign of the GVD parameter; for light propagating in normal dispersion \((\beta_2 > 0)\) low frequencies travel faster than high frequencies, while in the anomalous dispersion regime \((\beta_2 < 0)\) the opposite occurs. In silica fibres the normal dispersion regime is at wavelengths below the zero-dispersion wavelength and the anomalous dispersion regime occurs for wavelengths higher than the zero-dispersion wavelength. However with the recent advent in photonic crystal fibres it is possible to engineer quite different dispersion properties and even create fibres with two zero-dispersion wavelengths [54]. We have not explicitly written the higher-order dispersion terms in equation 1.3. However if the carrier wavelength approaches \(\lambda_0\), cubic and quartic terms need to be included in the above equation. We will add the necessary terms to the NLSE when appropriate. Because the higher-order terms are non-zero, propagation is not dispersion-free at the zero-dispersion wavelength. Although the zero-dispersion wavelength in normal single-mode silica fibres is at around \(\lambda_0 \approx 1310\, \text{nm}\) it is possible to shift \(\lambda_0\) towards longer
wavelengths using either special index profiles or some dopants. These so-called dispersion shifted fibres (DSF) are often used in experiments to take advantage of low GVD values at wavelengths around 1.55 μm.

![Figure 1.1](image)

**Figure 1.1** – Measured imaginary part of the Raman susceptibility in silica fibres as a function of frequency detuning.

Finally we would like to return to the treatment of the vibrational contribution to the third-order susceptibility, in particular the Raman effect. In silica fibres the vibrational or Raman contribution to the polarization density occurs over a timescale of 60 – 70 fs and its contribution to the nonlinear propagation equations can thus be neglected for pulse widths above one picosecond. As we experimentally observe pulses with widths below 1 ps we need to include the Raman effect into our treatment. After some mathematical discussion (see reference [34] for more details) we arrive at an additional integration term to the NLSE
to take account of the delayed Raman response and the propagation equation becomes:

\[
\frac{\partial A}{\partial z} + \frac{i\beta_2}{2} \frac{\partial^2 A}{\partial T^2} = i \gamma \left[ (1 - f_R)|A|^2 + f_R \int_{-\infty}^{T} \chi_R^{(3)}(T - T')|A(T')|^2 dT' \right] A,
\]  

(1.4)

where \( f_R \) is the fractional contribution of the Raman susceptibility to the instantaneous Kerr effect which has been measured in the past to be \( f_R = 0.18 \) [55]. \( \chi_R^{(3)} \) is the Raman response function, it is the Fourier transform of the Raman gain spectrum \( \tilde{\chi}_R^{(3)}(\omega) \) of silica fibres which can be measured experimentally [55]. Such a measurement of the raman gain spectrum is depicted in figure 1.1. We see that the spectrum is relatively wide at about 30 THz and it exhibits a broad peak at around 13 THz. The nonlinearity coefficient \( \gamma \) can also be expressed in terms of fibre parameters: \( \gamma = (n_2 \omega_0)/(c A_{eff}) \) where \( n_2 \approx 3 \cdot 10^{-20} \text{m}^2/\text{W} \) is the nonlinear refractive index of silica, \( c \) is the speed of light in vacuum, and \( A_{eff} \) is the effective core area of the fibre. \( A_{eff} \) is typically about 50 – 100\( \mu \text{m}^2 \) at \( \lambda = 1550 \text{ nm} \) however in highly nonlinear fibres (HNLF) the effective area can be smaller than 20 \( \mu \text{m}^2 \). This results in values of the nonlinearity \( \gamma \) between 1 – 15 \( \text{W}^{-1}\text{km}^{-1} \). We should note that we have neglected fibre attenuation in this discussion, however the attenuation can easily be accounted for by an additional parameter \( \alpha \). In single-mode silica fibres this coefficient is generally very low with a minimum value of around 0.2 dB/km at 1550 nm.

In general both equation 1.1 and equation 1.4 can not be solved analytically. However there exist a number of numerical methods for solving nonlinear Schrödinger-type equations. Throughout this thesis we make extensive use of the the so-called “split-step Fourier method” for numerical simulations of the light propagation inside a fibre. Appendix B discusses some of the subtleties involved in the numerical techniques used.

### 1.2 Publications

The research in this dissertation has been presented at a number of international conferences and been subject to publication in several scientific journals:


Before we come to the main part of this thesis; the mode-locked Raman fibre laser, we introduce some work on multiple simultaneous four-wave mixing processes. Although this work has only played a minor role in this dissertation, it does nevertheless offer some valuable insights into the interplay of four-wave mixing and Raman scattering and is thus the ideal topic to begin this thesis.

2.1 Introduction

The nonlinear phenomena of four-wave mixing (FWM) and parametric interactions in optical fibres have attracted considerable research interest for several decades. The first investigations started as early as 1974 when Stolen et al. first observed FWM in a glass fibre [65]
Chapter 2  Four-wave mixing processes

and initially called the phenomenon three-wave mixing. Since then FWM has proved to be immensely popular in fibre optics research due to its efficiency in generating waves at new optical frequencies [36, 66–76]. Today FWM is the underlying process of a large number of applications ranging from parametric amplification [76–79] to wavelength conversion [80–83] or the generation of high repetition rate pulsed light sources [84].

In this chapter we concentrate on two lesser known phenomena of FWM. In section 2.3 we demonstrate the interplay of multiple four-wave mixing processes. When FWM was discovered the theoretical and experimental investigation initially concentrated on treating the various FWM processes separately [72, 75, 85, 86]. However in 1991 Thompson et al. demonstrated [87], and later theoretically described [88] the interaction between two pump waves including all involved FWM processes. The authors show that the two pumps create two sidebands which in turn produce additional sidebands. Since then, several authors investigated four-wave mixing with two pump waves, predicting various instabilities such as sideband oscillations along the fibre propagation [70, 89] as well as reporting a self-stabilization effect [90]. The concept of FWM between two pump waves was recently expanded to very broadband cascaded FWM products [67, 68]. In contrast to the cascaded creation of sidebands through multiple FWM processes, we demonstrate the interaction of largely independent FWM processes. Two processes stand out in particular; a non-phase-matched interaction between three wavelengths and FWM with an incoherent pump beam comprised of an amplified spontaneous emission (ASE) source. Parametric processes at a large phase-mismatch have mainly been ignored in the discussions on FWM as they are significantly less efficient. However recent research has shown that non-phase-matched FWM can be assisted by gain processes such as Raman amplification [36, 91, 92]. The efficiency of the FWM is significantly improved in the presence of gain leading to strong FWM effects even for relatively large phase-mismatches. Section 2.4 provides an in depth investigation of FWM with an incoherent pump. FWM of incoherent waves has only received little attention in the past. Although some authors have presented FWM mixing with partially coherent waves [93–95], it was only recently that FWM with an ASE pump field was shown [96]. Our results are remarkable in the fact that they demonstrate that the bandwidth of the FWM
product can be narrower than the bandwidth of the signal wave. Finally I would like to point out that this work has been carried out in collaboration with Anne Boucon and Thibaut Sylvestre at the Institut FEMTO-ST at the Université de Franche-Comté. Therefore part of these experiments were performed by Anne Boucon and I am grateful to her for providing me with the relevant data.

2.2 Background

In section 1.1 we already discussed the fact that if we ignore non-instantaneous effects, nonlinear light propagation in silica optical fibres is to a large degree dominated by the Kerr-nonlinearity. The Kerr-nonlinearity originates in the third-order susceptibility which is responsible for effects such as self-phase modulation (SPM), cross-phase modulation (XPM), four-wave mixing (FWM) and third harmonic generation [34]. In general these parametric processes involve the nonlinear interaction of four optical waves. In this chapter we concentrate on the processes of FWM and ignore third-harmonic generation which does not occur in these experiments. Furthermore we only discuss SPM and XPM with respect to their role in the phasematching. We also neglect all polarization effects according to the reasoning given in section 1.1. As stated, FWM is the interaction between four optical waves mediated by the nonlinear response of the propagation medium, in our case the optical fibre. In the quantum-mechanical picture FWM can be understood as the annihilation of photons from one or more waves and the subsequent creation of photons of the other waves. Generally FWM processes are subject to energy- and momentum conservation, i.e. the frequency and wave-vector of the waves have to be conserved. In almost all cases the name FWM is restricted to mixing between one or two pump waves with angular frequencies \( \omega_1 \) and \( \omega_2 \), which create a Stokes and anti-Stokes component from noise at frequencies \( \omega_3 \) and \( \omega_4 \), or if seeded by a small signal wave at \( \omega_3 \) amplify this wave while simultaneously creating a wave at \( \omega_4 \). We can classify FWM into two main categories, degenerate and non-degenerate FWM. In degenerate FWM two of the photons originate from the same wave, i.e. \( \omega_1 = \omega_2 \) thus \( 2\omega_1 = \omega_2 + \omega_3 \). This process is commonly known as modulation instability.
(MI) and plays a crucial role in soliton formation and supercontinuum generation [97–99]. The case when $\omega_1 \neq \omega_2$ is non-degenerate FWM, which some authors have recently begun to classify by the relative positions of the pump and signal waves [100]. Names such as Bragg-scattering and phase-conjugation are used to indicate the similarities to known optical processes. In this work we do not distinguish between different types of non-degenerate FWM processes in this way, however we use the term modulation instability to denote degenerate FWM.

In addition to energy conservation, momentum needs to be conserved for the mixing to be efficient. This requires the phase mismatch between the waves to be zero, $\kappa = 0$. The overall phase mismatch is a combination of linear mismatch ($\Delta k_L$) due to material dispersion and waveguide dispersion, and nonlinear mismatch ($\Delta k_{NL}$). We can thus write the phase-matching condition in the form [34]

$$\kappa = \Delta k_L + \Delta k_{NL} = 0.$$  \hspace{1cm} (2.1)

We can write the linear phase mismatch in terms of the fibre wave number and get [88]

$$\Delta k_L = \beta^3 + \beta^4 - \beta^1 - \beta^2$$ \hspace{1cm} (2.2)

where $\beta^i = \beta(\omega_i)$ is the wavenumber at the frequencies of the mixing waves. If the power of the pump waves is much stronger than the signal and idler power, the contribution of the idler and signal to the nonlinear phase mismatch is negligible and the nonlinear phase mismatch can be written as [34]:

$$\Delta k_{NL} = \gamma (P_1 + P_2),$$ \hspace{1cm} (2.3)

where $P_i$ are the powers of the two pump waves. Equation 2.2 can be further simplified by using the Taylor expansion of the dispersion around a frequency $\omega_0$ (section 1.1):

$$\beta(\omega) = \beta_0 + \beta_1(\omega - \omega_0) + \frac{1}{2}(\omega - \omega_0)^2 \beta_2 + \cdots$$ \hspace{1cm} (2.4)
Multiple independent FWM processes

In the special case of degenerate FWM the two pump photons are provided by the same wave, i.e. $\omega_p = \omega_1 = \omega_2$ and two new waves are created symmetrically around the pump frequency. These two waves are commonly denoted the Stokes and anti-Stokes waves and are separated from the pump by the frequency $\Omega = \omega_p - \omega_3 = \omega_4 - \omega_p$ where $\omega_3$ is the frequency of the Stokes wave and $\omega_4$ is the frequency of the anti-Stokes wave. Retaining dispersion terms up to fourth order the expression for the linear phase mismatch for degenerate four-wave mixing simplifies to

$$\Delta k_L = 2\beta_2 \Omega^2 + \frac{\beta_4}{12} \Omega^4. \quad (2.5)$$

Furthermore as $P_1 = P_2 = P_p$, the nonlinear contribution to the phase mismatch becomes $\Delta k_{NL} = 2\gamma P_p$.

Although FWM does not strictly require the phase mismatch to be zero, the efficiency significantly decreases for larger phase mismatches. However some recent work has shown, that Raman scattering or a gain process can assist FWM [36,91,92]. The relevant details will be discussed when we present the results below.

2.3 Multiple independent FWM processes

Generally it is rather difficult to observe multiple FWM processes in optical fibre systems due to the efficiency of the FWM decreasing significantly for large phase mismatches, and the difficulties of satisfying the phase-matching conditions of the different processes simultaneously [101]. In this section we demonstrate the observation of a number of independent FWM processes inside a dispersion-shifted fibre. The results nicely demonstrate the interaction and dynamics of the individual FWM processes.

2.3.1 Experimental setup

The experimental setup of the four-wave mixing experiment is very simple. We use a cw Raman fibre laser (RFL) with a fixed wavelength of 1455 nm and a continuous wave (cw) Erbium-doped fibre laser (EDFL) with a wavelength tunable from 1535 to 1565 nm. The
Chapter 2 Four-wave mixing processes

EDFL is amplified by an 33 – dBm Erbium-doped fibre amplifier (EDFA) which also creates some additional broadband high-power amplified spontaneous emission (ASE). Both light sources are launched into 3.1 km of dispersion-shifted fibre (DSF) using a 1450/1550 nm wavelength-division multiplexer. The fibre has a zero-dispersion wavelength of 1549.2 nm and a dispersion slope of 0.071 ps nm$^{-2}$km$^{-1}$ (see appendix A.1 for further parameters of the fibre). The output spectrum is recorded using an optical spectrum analyser (OSA). A schematic of the setup is depicted in figure 2.1. As the RFL exhibits an essentially random polarization the assumptions discussed in chapter 1 apply and no polarization control is necessary.

2.3.2 Experimental observations

In this section we discuss the experimental observations as we vary the wavelength of the EDFL laser. Because the EDFL can be tuned from the normal to the anomalous dispersion region, we can observe very, qualitatively different FWM effects. In the following set of experiments we kept the power of the Raman fibre laser constant at 1.3 W. The EDFA was adjusted to always yield approximately 500 mW of output power. We did not ensure that the power ratio of amplified EDFL and spontaneous emission was constant. However we kept the power of the EDFA at a constant level and it is therefore safe to assume that the fluctuations of the EDFL to ASE power ratio are small. A graph of the output spectra for values of the EDFL wavelength between 1536 and 1554 nm is shown in figure 2.2(a) and as
a colour plot in figure 2.2(b). The spectra reveal a number of new optical waves generated by independent FWM processes. The different processes are easily distinguishable and this is to the best of our knowledge the first observation of such a large number of independent simultaneous FWM processes. In the following we describe the individual waves created by the different FWM processes.

The most obvious feature of the spectra is the large sidebands around the EDFL for larger EDFL wavelengths (see figure 2.3(a) for an enlargement of the relevant part of the spectrum). The sidebands appear when the EDFL laser reaches a wavelength of around 1549 nm. Initially they are detuned from the EDFL by approximately 50 nm, however they quickly converge towards the EDFL wavelength. The FWM effect here is the well-known, so-called modulation-instability (MI). MI is a degenerate FWM process where two pump photons are converted to a Stokes- and anti-Stokes photon. In spontaneous MI which is the effect observed here, the two sidebands grow spontaneously from noise. Harvey et al. showed [102], that the phase-matching condition is defined by two inequalities.

\[
\beta_2 + \frac{\beta_4}{12} \Omega^2 < 0 \quad (2.6)
\]
\[
\left| \beta_2 + \frac{\beta_4}{12} \Omega^2 \right| \Omega^2 < 4\gamma P \quad (2.7)
\]
where \( \Omega \) is the detuning of the sidebands from the pump wave. The two inequalities provide an upper and lower limit for the MI gain band, i.e. the detuning of the sidebands. The frequency region defined by the two equations is very narrow for large normal dispersion however it gets significantly wider for values around the zero-dispersion wavelength [102]. Figure 2.3(b) shows a colour plot of the MI spectrum depicting the spectral power as a func-

![Figure 2.3](image)

**Figure 2.3** – (a) Enlargement of the figure 2.2(a) centred around the EDFL laser demonstrating MI. (b) Corresponding colour plot of the output spectra with phase matching conditions. The \( y \)-axis represents the detuning from the EDFL-frequency and the \( x \)-axis the detuning of the EDFL from the zero-dispersion wavelength.

...portion of detuning of the EDFL from the zero-dispersion wavelength (\( \Omega_{\text{EDFL}} - \Omega_0 \), with \( \Omega_0 \) the angular frequency of the zero-dispersion point). The solid and dashed line indicate the solutions for the two phase-matching conditions (equation 2.6 and equation 2.7) respectively. We can see that the theoretical phase-matching condition and the experimental results agree well, and that the generated sidebands lie within the region defined by the phase-matching condition. Remark the structure of the gain bands: when the pump is in the anomalous dispersion regime (\( \Omega_{\text{EDFL}} - \Omega_0 < 0 \)) we observe a single wide gain region to both sides of the pump. However once the EDFL-detuning approaches zero and when the EDFL experiences normal dispersion, the gain structure changes significantly into two separate sidebands on either side of the pump. This behaviour is in fact a feature of the presence of fourth-order dispersion [103] and it is essential that the contribution of fourth-order dispersion is included in equations 2.6 and 2.7.
More interesting than the modulation instability are the two waves at the long wavelength side of the spectrum in figure 2.2. Figure 2.4 shows an enlargement of the relevant part of the spectrum as a line [figure 2.4(a)] and colour [figure 2.4(b)] plot. Superimposed on the colour plot is the wavelength calculated from the energy conservation condition $\omega = 2\omega_{EDFL} - \omega_{RFL}$. When the wavelength of the EDFL is varied, we see that the wavelength of one of the waves changes from around 1627 nm to over 1668 nm. The second wave remains stationary in wavelength at around 1651 nm. In the further course of this discussion we will call the moving Stokes wave S1 and the stationary Stokes wave S2. Examining figure 2.4(b) we see that the maxima of S1 coincides exactly with the wavelength calculated from energy conservation between the RFL and the EDFL, i.e from $\omega_{S1} = 2\omega_{EDFL} - \omega_{RFL}$. It is therefore clear that S1 is generated by degenerate FWM between the EDFL and the RFL with the EDFL functioning as the pump wave. It can be easily shown that the phase-mismatch cannot be zero for most of the EDFL wavelengths, the underlying FWM process is thus non-phasematched. The origin of the stationary wave S2 is not immediately clear. As the wavelength of S2 does not change when varying the EDFL wavelength, we can safely assume, that the EDFL is not involved in it’s creation. Assuming a would-be degenerate FWM process involving S2 and the RFL we can calculate the position of the pump using energy
conservation $\omega_{\text{pump}} = (\omega_{\text{RFL}} + \omega_{S2})/2$. The wavelength calculated in this manner is constant at approximately 1547 nm. Under these assumptions $S2$ is created by FWM between the RFL and the broadband ASE noise of the EDFA present around 1547 nm. To further validate the findings we calculated the phase-mismatch of the two processes. However due to the power of the RFL, the nonlinear contribution of the EDFL and the RFL to the phase-matching have to be taken into account. Assuming a lossless fibre the phase-mismatch can be calculated as [104]:

$$\Delta k = \beta_2 \Omega^2 + \frac{\beta_4}{12} \Omega^4 + \gamma (2P_{\text{pump}} - P_{\text{RFL}}) \quad (2.8)$$

The phase-mismatch and the power of $S1$ and $S2$ as a function of the frequency detuning between the EDFL and the RFL $\Delta f = f_{\text{EDFL}} - f_{\text{RFL}}$ are shown in figure 2.5(a) and figure 2.5(b) respectively. In the calculations we have used the combined average power of the EDFL and the ASE of the EDFA for the pump power, because their power cannot easily be separated. As we already stated above the FWM process responsible for $S1$ exhibits a large phase-mismatch for most EDFL wavelengths, supporting the notion that $S1$ is created by a non-phasematched FWM process. However the power of wave $S1$ exhibits a peak at a

![Figure 2.5](image-url)
Multiple independent FWM processes

frequency detuning \( \Delta f = -12.3 \) THz. The detuning corresponds to an EDFL wavelength of 1547 nm, i.e. the position of the stationary wave. For larger values of the detuning the power is significantly lower at around \(-45\) dBm. The phase-mismatch changes almost linearly with the EDFL detuning, and passes through the origin at around \(-12.1\) THz detuning. The maximum power is thus not observed exactly at zero phase-mismatch. However the error in the mismatch calculation can be explained by the error in the values of \( \beta_2 \) and \( \beta_4 \) which are calculated from the zero-dispersion wavelength and the zero-dispersion slope. At more than 50 nm away from the zero-dispersion wavelength this approximation is not very accurate. Remarkably the wave still contains a considerable amount of power even for relatively large phase-mismatches, even though the efficiency of FWM processes decreases significantly if the phase-mismatch is large. However we have not accounted for non-parametric processes in the discussion thus far. The frequency detuning between the EDFL and S1 increases from around 11 THz to about 13 THz when the EDFL wavelength is increased. S1 is therefore well within the Raman gain bandwidth of the EDFL. A similar effect has been observed by Sylvestre et al. in 1999 [92]. The non-phasematched FWM processes that creates S1 is seeded by spontaneous Raman scattering and the subsequently created Stokes wave is amplified by stimulated Raman scattering (SRS) from the EDFL. The Raman scattering breaks the symmetry of the non-phasematched, degenerated FWM process which without SRS transfers equal energy to both the RFL and S1 thus amplifying S1. The effect is stronger when S1 is closer to the maximum of the Raman gain. Indeed in figure 2.5(a) the power of S1 is slightly larger on the left hand side at detuning values of around 13 THz.

Examining the phase-matching and power of wave S2, we see that the power exhibits a peak at the same detuning as S1. In fact, figure 2.4 demonstrates that S1 and S2 merge to form one wave once they get close. However when S1 is far away from S2, there is significant power in S2, thus there is energy transferred from the ASE pump wave to S2. Examining the phase-matching condition we see that apart from a linear offset due to errors in the dispersion measurement, the phase-mismatch is zero. Note that between \(-12.5\) and \(-12\) THz detuning S2 merges with S1 and thus the phase-mismatch of S1 is shown in the graph. We can thus summarize that the S1 wave is caused by non-phasematched Raman assisted de-
generate FWM between the RFL and the EDFL. Because the EDFL acts as the pump of the process changing the EDFL wavelength also changes the wavelength of S1. The power is at a maximum when the EDFL wavelength is such that the phase-mismatch for the process is zero. The second wave S2 is caused to be phase-matched FWM between the RFL and the ASE pump. Its wavelength is therefore stationary when the EDFL wavelength is varied. This process is intriguing because it possibly demonstrates degenerate FWM with an incoherent pump. Although FWM with partially incoherent waves have been observed previously, it was either limited to non-degenerate FWM [93] or the incoherent light acted as the signal not the pump [95]. However it is not entirely clear if S2 is caused by the stipulated FWM process. It could also be caused by non-degenerate FWM involving two pump waves. We will therefore examine this effect in more detail to determine its nature in the following section.

### 2.4 FWM with an incoherent pump

FWM in optical fibres has been investigated for several decades. However most investigations treated the interacting waves as perfectly temporally coherent. Nevertheless the interaction between incoherent waves as caused by a spectrum-sliced amplifier does exhibit interesting properties. Jang et al. predicted that the FWM interaction between two incoherent pump waves has a 6 dB higher efficiency than the same process with coherent waves [93]. Other authors demonstrated the wavelength conversion of an ASE signal by degenerate FWM with a strong coherent pump beam [95]. The work by Cavalcanti et al. and Sauters et al. is particularly interesting because they investigate MI of a partially incoherent wave [94, 105]. Some very recent work demonstrated how FWM with an ASE noise wave can be utilized to measure the ratio between third and fourth-order dispersion [96]. However with the exception of [95] all these investigations were either about non-degenerate FWM or the involved waves were only slightly incoherent. In contrast to [95] we will discuss the FWM with a pump consisting of ASE noise.
2.4.1 Experimental setup

The experimental setup for most of the experiments in this section is very similar to the setup described in section 2.3.1. However as we are interested in the FWM between the RFL signal and the ASE noise spectrum, we remove the EDFL and the pump is only provided by the ASE created from the EDFA. In some of the experiments the fixed wavelength RFL was exchanged for a tunable RFL with a wavelength between 1430 nm and 1490 nm. As is the case in the experiments in section 2.3 the FWM is documented by measuring the output spectrum using an OSA.

2.4.2 Experimental results

We recorded a number of spectra for various powers of the EDFA and the RFL to observe the influence of pump and signal power on the generated Stokes wave. The resulting spectra for mixing the RFL laser with the broadband ASE noise from the EDFA are shown in figure 2.6. Figure 2.6(a) demonstrates the influence of varying the RFL power while the power of the EDFA is held constant at 30 dBm. In figure 2.6(b) we present the results when the RFL power is held constant and the power of the ASE source is varied. The broadband ASE noise spectrum is centred around 1550 nm. The ASE spectrum is around 45 nm wide, with the exact value depending on the power of the EDFA. The spectra show that a Stokes wave is created at a wavelength of about 1651 nm. The graphs reveal that the power in the Stokes waves increases with increasing ASE power and increasing RFL power. Moreover figure 2.6(b) shows that the RFL output power decreases when we increase the ASE power. This is an initially surprising result because in a parametric process both the signal and the idler are amplified. We should therefore observe a higher spectral power of the RFL when the power of the EDFA is higher. The discrepancy can be explained by the asymmetry introduced by stimulated Raman scattering (SRS). Because SRS causes an energy transfer from the lower wavelength to higher ones, it will cause a larger gain for the Stokes than the anti-Stokes wave, i.e. an effective energy transfer from the Stokes to the anti-Stokes wave [92]. In other words, the Stokes wave experiences gain due to Raman amplification by the ASE pump in
addition to the FWM gain. On the other hand the anti-Stokes wave is transferring energy to the ASE pump by Raman amplification, while simultaneously experiencing gain due to FWM.

The dependence of the spectral power of the Stokes wave on the power of the RFL and the EDFA is depicted in logarithmic units in figure 2.7(a) and 2.7(b) respectively. The inset in figure 2.7(b) additionally depicts the power in linear units. The linear fit to figure 2.7(a) has a slope of approximately 1.1, i.e. the Stokes power depends linearly on the RFL power. This linear relationship is consistent with the constant Stokes to anti-Stokes power ratio predicted for combined Raman scattering and four-wave mixing processes [35, 36, 107]. The slope of the linear fit in figure 2.6(b) is approximately 3.2 which would correspond to the Stokes power being proportional to $P_{\text{ASE}}^3$, however a fit of an exponential function to the linear data reveals an equally good agreement. The exponential dependence of the Stokes power on the pump power is consistent with theoretical predictions e.g. in [35]. We have to point out that this agreement is quite remarkable. The theoretical predictions are based on a model consisting of single frequency waves, i.e. the pump, Stokes and anti-Stokes wave.

Figure 2.6 – Spectra of FWM between RFL and ASE noise for (a) a constant ASE power of 30 dBm and an RFL power of 50 mW (solid) and 950 mW (dotted) and (b) a constant RFL power of 1 W and an ASE power of 28 dBm (solid) and 33 dBm (dotted) [106].
are considered to be coherent cw waves at distinct wavelengths. In contrast the pump wave in this experiment is provided by an incoherent noise source, consisting of a broad band of wavelengths. Furthermore we plot the Stokes power as a function of the average power of the ASE, however the shape and bandwidth of the ASE spectrum varies significantly with average power. It is therefore by no means clear that the exponential dependence of the Stokes power on the pump power should be reproduced in this experiment. The experimental results therefore give a good indication that FWM with an incoherent pump possesses very similar characteristics to FWM with coherent light sources.

The efficiency of the FWM process is revealed by the fact that there is significant FWM even for powers of the RFL as low as 50 mW. In fact we have been able to further reduce the power of the RFL to 14 mW if we tune the EDFA power to 32 dBm and by using 500 m of highly non-linear fibre (HNLF) instead of the DSF [figure 2.8]. If we assume a degenerate FWM process, we can calculate the pump wave from energy conservation, i.e. $\omega_{\text{pump}} = (\omega_{\text{RFL}} + \omega_{\text{Stokes}})/2$. We can then use equation 2.8 to calculate the phase-matching condition for different wavelengths around the Stokes wave and their corresponding pump waves within the ASE spec-
Figure 2.8 – Optical spectrum (solid line) and phase matching curve (dotted) for FWM between RFL and ASE noise inside 500 m of HNLF. The RFL power is 14 mW and the average power of the EDFA is 32 dBm. The dashed line indicates the zero phase-mismatch line.

Comparing the wavelength of the Stokes wave with the phase-matching curve in figure 2.8 we see that its wavelength is very close to the zero-phase-mismatch, providing strong evidence that the observed effect is indeed the postulated degenerate FWM.

Another property of the incoherent FWM process reveals itself when we examine the width of the Stokes wave. For a constant width and power of the ASE source, the width of the Stokes wave depends linearly on the bandwidth of the RFL. Remarkably the Stokes wave is narrower than the RFL laser [figure 2.9]. The narrow Stokes wave is a direct consequence of the phase-matching due to fourth-order dispersion. Let us ignore the contribution of the RFL power to the phase matching and recall equations 2.6 and 2.7. These inequalities define the upper and lower boundaries of the FWM gain region. In the normal dispersion regime the width of the gain region decreases rapidly with pump-signal detuning [103,108], which can also be seen in figure 2.3(b). In the present case the separation between pump and signal is approximately 105 nm and the gain bandwidth is thus narrower than the width.
FWM with an incoherent pump

Figure 2.9 – Width of the Stokes wave as a function of RFL spectral width; dots: experimental data, dashed line: linear fit. Note that this the full-width-half-maximum (FWHM) of the wave measured in dBm. The power of the EDFA was constant at 24.2 dBm [106].

of the anti-Stokes wave provided by the RFL. In fact calculating the width of the gain-band from equations 2.6 and 2.7 results in a width of 0.5 nm smaller than the smallest width of the Stokes wave seen in figure 2.9. However the uncertainties in $\beta_2$ and $\beta_4$ as well as the fact that the equations do not take account of the contribution of the RFL power to the phase matching explain the discrepancy. Furthermore fluctuations of zero-dispersion wavelength which are present in all fibres also cause the gain spectrum to broaden [103, 108].

Finally we would like to offer an interpretation of the FWM process with an incoherent pump. When a relatively strong signal wave is launched together with the broadband pump wave such as the present ASE source the frequency component of the pump that mixes with the signal to create the idler is determined by the phase matching condition (e.g. equation 2.8), thus determining the position of the idler. In other words the wavelength inside the ASE band that fulfils the phase matching condition functions as the pump for a degenerate FWM process. If the ASE band is close to the zero-dispersion wavelength it can sustain FWM for a large wavelength range of the signal. Figure 2.10 depicts the FWM spectra from

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an experiment using a tunable RFL for RFL wavelengths between 1455 nm and 1489 nm. The fibre used in this case was HNLF 1 in the appendix A. We see that although its power varies we can observe a Stokes wave for almost all RFL wavelengths. While the position of the RFL and Stokes wave changes over a large wavelength range, the calculated wavelength of the pump varies only minimally by approximately 1.5 nm between the maximal and minimal values. This is consistent with the theoretical predictions by Marhic et al. [103], which predict that a detuning of the pump wavelength by only a few nanometres can result in a change of the pump-to-signal separation of 100 nm.

We have provided strong evidence that the Stokes wave is created by degenerate FWM, i.e. $2\omega_{ASE} - \omega_{RFL} = \omega_{Stokes}$. However we should note, that recent research reported on a similar FWM process, but the authors stipulated the wave to be generated by multiple non-degenerate FWM processes $\omega_l + \omega_{l'} = \omega_{RFL} + \omega_{Stokes}$. Where $\omega_l$ and $\omega_{l'}$ are frequencies symmetrically around the centre of the ASE spectrum $\omega_A$ [96]. It is difficult to determine
the exact nature of the FWM mixing, as both a superposition of non-degenerate processes such as proposed by the authors of [96], or degenerate FWM yield a very similar output spectrum, this however is beyond the scope of this thesis.

2.5 Summary and discussion

In this chapter we provided a brief introduction into the fascinating field of FWM and parametric processes. We demonstrated that contrary to common belief it is possible to observe multiple distinct independent FWM processes simultaneously. The result is remarkable because the efficiency of FWM is determined by the phase-matching condition and it is difficult to achieve low values of the phase-mismatch for multiple processes. The individual FWM processes could be identified as the spontaneous MI of the EDFL plus two FWM mixing processes between the RFL and the EDFL and the ASE background generated by the EDFA. The two generated MI sidebands display the typical dependency on the phase-matching condition and the wavelength of the pump laser provided by the EDFL. The agreement between the experimentally observed sideband positions and the theoretically calculated wavelengths is very good and well within the uncertainties mainly determined by the values for the fibre dispersion. The FWM between the RFL and the EDFL and ASE noise prove to be even more intriguing. Two Stokes waves at wavelengths between 1600 nm and 1700 nm are created. One is created by non-phasematched FWM between the RFL and the strong EDFL pump. The second wave is created by FWM between the RFL and part of the ASE noise from the EDFA. The non-phasematched FWM process demonstrates how the strict limitations imposed by the phase-matching condition are mitigated by non-parametric processes such as stimulated Raman scattering. The effectiveness of the FWM is remarkably high even for relatively large values of the phase-mismatch. This is particularly obvious when comparing the wave generated by the non-phasematched FWM to the wave generated by the phasematched FWM with the ASE noise. We investigated the phase-matched FWM process in more detail because FWM with a largely incoherent pump has thus far been mainly ignored in the research of parametric processes. We could show that
Chapter 2 Four-wave mixing processes

by mixing a small signal with a strong incoherent pump wave we create an idler wave. The position of the idler wave and the pump wave within the ASE spectrum is determined by phase-matching and the theoretically calculated wavelength of the Stokes wave agrees well with the experiment. Further investigation revealed that the power of the generated wave depends linearly on the power of the signal wave but exponentially on the average power of the ASE spectrum, which is consistent with theoretical predictions for FWM with a Raman scattering contribution. The Raman scattering also causes an energy transfer from the low wavelength signal wave to the high wavelength idler wave, i.e. the two waves are not as in the case of pure FWM amplified equally. The conversion is so efficient, that we observe that the generated wave has a larger power than the signal if the ASE power is sufficiently large. Furthermore we could observe that for low powers of the ASE spectrum the width of the generated idler wave is smaller than the width of the signal, due to the influence of the phase-mismatch on the spectral width of the idler. FWM with an incoherent pump is an intriguing process and could be promising for wavelength conversion processes, in particular because it eliminates the necessity to tune a laser to the correct wavelength for optimal phase matching. The exact nature of the mixing is still unclear, Chavez-Boggio et al. hypothesise that the idler wave is generated by the superposition of many non-degenerate FWM processes [96], however our results can be accurately described by a degenerate FWM process with a single pump wave within the ASE band. Future investigations into FWM with incoherent pump waves might clarify this discrepancy, however such an investigation is not part of this work. All of the individual processes in these experiments have been observed before. Nevertheless, this is the to the best of our knowledge the first observation of the interaction between such a large number of parametric processes. This study is an educational example of the interplay and simultaneous existence of many individual parametric processes. It can provide valuable information for understanding systems with simultaneous FWM processes, in particular supercontinuum generation, where the spectral broadening is caused by the interaction between parametric and non-parametric processes.
In recent years extensive research has gone into the feat of increasing the repetition rate of pulsed lasers. As the demand for higher and higher telecommunications bandwidths has been increasing so has the demand for laser sources which can provide the required repetition rates. Simultaneously the number of applications requiring ultra-fast repetition rates has been growing for example in emerging fields such as biophotonics or sensing. High
Chapter 3  Mode-locking by dissipative four-wave mixing

repetition rate lasers with large average output powers can be advantageous for probing the coherent accumulation effect [109] of fast decaying transitions in molecular high resolution spectroscopy [110] for example. The current commonly used mode-locking methods have arrived at a limiting value where the repetition rate cannot be increased significantly without a major change in the mode-locking technique. This can be attributed to the fact that currently most high repetition rate lasers rely on active mode-locking where a cavity parameter such as gain or loss is varied electronically. Because these techniques require an electronic signal to achieve mode-locking, they are limited to electronic bandwidths and it is therefore not possible to increase the repetition rate significantly higher than 40 GHz. Instead telecommunications systems have to rely on time [111, 112] or wavelength [22, 23, 113] multiplexing techniques to achieve bandwidths above 160 GBit/s. In order to directly reach repetition rates above 100 GHz new mode-locking methods have to be explored. In particular passive mode-locking techniques promise to overcome the limitations of the traditional active mode-locking techniques [25, 27, 114]. Although the concept of passive mode-locking is not new, the first passively mode-locked laser was demonstrated in 1972 [115], research into different passive mode-locking methods has skyrocketed in the last decade [25–27, 38, 39, 114–148] (also see [149] for a general review of mode-locking techniques). As passive mode-locking usually attains the pulsed laser operation by all-optical means, there is no in-principle limitation to the maximum possible repetition rates and rates in excess of several hundred GHz have been realised [26, 27, 114, 131, 145]. A great number of passive mode-locking schemes have been proposed and demonstrated, ranging from techniques using saturable absorber such as carbon nanotubes [132, 139, 140, 146], or quantum dots [122, 125, 130, 136] or saturable absorber-like methods using loop mirrors [38, 150, 151] or nonlinear polarization rotation [121, 129, 137, 152, 153] to modulation instability lasers [25–27, 118, 154, 155]. Of these techniques the saturable absorber based ones are the most commonly used and recognized. Apart from the possibility of reaching ultra-high repetition rates, passively mode-locked lasers have the advantage of realising mode-locking with comparatively simple experimental setups. Where actively mode-locked lasers require phase or amplitude modulators plus the electronic equipment to con-
trol them, passively mode-locked lasers work with all-optical setups requiring only spectral filters or only an additional fibre loop to achieve mode-locked operation. Furthermore, while actively mode-locked lasers are mainly limited to pulse widths of picoseconds, passively mode-locked lasers can achieve femtosecond pulses. This is not so interesting for telecommunications applications but for applications which require very fast, high peak-power pulses for example in biophotonics. Despite the advantages, passive mode-locking techniques have mainly been relegated to research only and have not made their way into applications in a large way, in part due to their susceptibility to noise fluctuations. While noise is well controlled for actively mode-locked lasers [156], efficient techniques to reduce noise have yet to be established for passively mode-locked lasers. Therefore actively mode-locked lasers generally exhibit significantly more stable pulses than passively mode-locked ones. With the desired repetition rates within the bandwidth of active mode-locking techniques, the incentive to investigate noise reduction of passively mode-locked lasers was relatively small. However now that the telecommunications industry is demanding stable, low noise lasers with repetition rates beyond electronic bandwidths, the increased interest hopefully overcomes the current limitations of passively mode-locked lasers.

With a few exceptions (e.g. [38]) most of the demonstrations of passive mode-locking methods in fibres were performed taking advantage of rare-earth doped fibres such as Yb- or Er-doped fibres as the gain medium [26, 133, 148, 157]. The use of rare-earth elements to attain the gain has some drawbacks however, due to their operation-principle they are limited to the gain band of the dopant, e.g. Er-doped fibres can provide amplification in the C- and L-band, while Yb-doped amplifiers work in the one micron region of the spectrum. Rare-earth doped amplification schemes also have an inherent saturation mechanism, linked to the finite number of dopant ions.

Raman scattering as a gain mechanism on the other hand, is not limited in such a way. It is wavelength versatile and virtually the same at every wavelength. Thus a Raman laser is only restrained by the availability of pump sources in reaching a wavelength region. Apart from pump depletion Raman gain does also not contain an inherent gain saturation, which can be highly advantageous for many applications. Raman lasers in the continuous wave
(cw) regime have been commercially available for a long time. They are well-known to yield very high average output powers up to tens of Watts. Thus Raman scattering lends itself for creating applications which aim to achieve high average output powers especially at wavelengths not covered by the rare-earth doped amplifiers. As the bandwidth of Raman gain is sufficiently large to support repetition rates in the range of possibly several hundred of GHz to even a THz, it is rather surprising that work of combining Raman amplification with mode-locking techniques has been very limited. There have only been few examples of mode-locked Raman fibre lasers, and until recently nobody had combined Raman gain with a passive mode-locking technique [38]. Even the given example did not take advantage of the possibilities of Raman gain and passive mode-locking to achieve ultra-high repetition rate. In the course of this dissertation we combined Raman amplification with a passive mode-locking technique to create a laser capable of reaching average output powers of up to one Watt, at repetition rates of more than 100 GHz [61].

In this chapter we introduce a mode-locking technique based on multiple FWM processes in combination with gain and dissipation. The method was first theoretically conceived by Quiroga-Teixeiro and co-workers and denoted mode-locking by dissipative four-wave mixing (DFWM). In the following we present the original theoretical model by Quiroga-Teixeiro et al. [39] describing the mode-locking technique. Consecutively we develop and discuss an analytic model to describe the laser. Next we describe the experimental setup of the passively mode-locked Raman fibre laser and discuss the results obtained from the setup. In particular we will look at the pulse shape and width of the output pulses and the dependence of the output power on the pump power. Thereafter we will investigate the behaviour of the laser with respect to the propagation direction of the pump wave and finally discuss the influence of dispersion on the laser output. To obtain further insight into the laser dynamics we introduce a novel numerical model of the laser including the effects of higher-order dispersion and intrapulse Raman scattering. We then represent the results of numerical simulations reproducing the experimentally observed phenomena and discuss limitations of the laser and the numerical model in the summary.

The work presented in this chapter was part of an ongoing collaboration between Stéphane
Coen and myself at the Department of Physics in Auckland and Thibaut Sylvestre at the Institut FEMTO-ST in Besançon in France. Some of the experiments were performed during my two visits to Besançon as part of this collaboration.

3.1 Principle

Passive mode-locking by dissipative four-wave mixing (DFWM) is a mode-locking technique which was first proposed in 1998 in a theoretical paper by Quiroga-Teixeiro, Balslev Clausen, Sorensen, Christiansen and Andrekson [39] and the first experimental implementation was demonstrated by Thibaut Sylvestre in an Er-doped fibre ring laser in 2001 [141]. We combine the DFWM mode-locking technique with Raman amplification as the gain mechanism to create a high average output power, ultra-high repetition rate laser.

The method of DFWM is as simple as it is elegant. In the temporal domain the underlying process is best interpreted as the nonlinear conversion of a high-frequency beat signal into a soliton pulse train. Under the influence of amplification and nonlinearity a dual-frequency beating signal develops into a clean soliton pulse train because upon propagation through a dispersive medium, such as an optical fibre, the two frequency components create new spectral components while simultaneously experiencing amplification. The phase of the new components self-adjusts so that a soliton pulse train is created. The mechanism has been demonstrated as early as 1991 [158] and has recently been shown to yield very high repetition rate pulse trains [84]. The DFWM mode-locking technique extends this concept considerably by transferring it into a cavity. In contrast to the work in the above references the DFWM technique creates a mode-locked laser and is not simply a mechanism for pulse-train generation. While most mode-locking techniques can be better understood by a consideration of the process in the temporal domain, DFWM is better understood by regarding the mode-locking process in the spectral or frequency domain. Let us look at the description of mode-locking in the frequency domain: Consider a simple ring type laser cavity, the light propagating inside the cavity is subject to interference, only light that regenerates every round trip of the cavity interferes constructively, leading to standing
waves inside the cavity. The standing waves are associated with discrete frequencies related to the cavity round trip time $t_{rt} = nL/c$ which are called the longitudinal or eigenmodes of the cavity. An eigenmode is therefore light which reaches the same state after an integer number of cavity round trips. Only the light at the cavity eigenmodes oscillates inside the cavity, while all other frequencies are suppressed by destructive interference and the cavity eigenmodes are separated by the cavity resonance frequency $f_{res} = 1/t_{rt} = c/(nL)$. A laser consisting of only one longitudinal mode is usually denoted single-mode, while a laser with several longitudinal modes is called multi-mode. Theoretically the number of eigenmodes is infinite, however in practice it is limited by the bandwidth of the gain medium and the
cavity losses, and the laser spectrum is bandwidth limited. Figure 3.1 depicts the longitudinal mode spectrum of a cavity, the gain structure and the resulting laser spectrum. In

![Figure 3.1](image)

**Figure 3.1** – (a) Hypothetical spectrum of a mode-locked laser with a Gaussian envelope, (b) intensity of the electric field in the time domain when all modes have the same phase, (c) intensity of the electric field in the time domain when the modes have a random phase

general the modes of the laser can oscillate independently, i.e. there is no fixed relationship between the modes. The individual modes therefore act like independent lasers at slightly different wavelengths, a concept which has been taken advantage of to create multiwavelength lasers [160]. The phase of the individual modes fluctuates due to environmental influences such as thermal expansion and contraction of the fibre. The temporal distribution therefore displays interference or beating between the modes, resulting in random power fluctuations. However for lasers with a large number of modes these fluctuations
average to create an essentially continuous wave output. Mode-locking of the laser can be achieved by fixing the phase relationship between the different modes, when the phases of the longitudinal modes are constant with respect to each other, we see a periodic constructive interference of the modes, and the output of the laser becomes a continuous train of pulses. The repetition rate of the pulses $f_{rep}$ is equal to the inverse of $t_{rt}$ the round trip time of the cavity. The envelope of all the involved modes determines the pulse shape and width due to the properties of the Fourier transform. Figure 3.2 illustrates the influence of the phase relationship of the modes on the pulse train. Figure 3.2(a) shows a hypothetical spectrum of individual modes with a Gaussian envelope. If the modes have a constant phase relationship the field in the time domain is a regular pulse train [figure 3.2(b)], while a random phase distribution between the modes results in an irregular pattern [figure 3.2(c)].

The process of fixing (“locking”) the phase-relationship of the different modes is commonly denoted mode-locking.

Figure 3.3 – Principle of mode-locking by dissipative four-wave mixing. (a) A two humped gain structure causes (b) the growth of two modes at the gain maxima. (c) These two modes then distribute their energy to higher order modes via four-wave mixing. The repetition rate $f_{rep}$ of the laser is determined by the frequency separation of the two gain peaks, i.e. the two central modes.

The elegance of the mode-locking by dissipative four-wave mixing (DFWM) method is, that it achieves the phase-locking of the modes automatically, simply through the process by which the modes of the laser are created. The central element of the DFWM process is
a bandwidth-limited two-humped shaped gain curve similar to the one depicted in figure 3.3(a), with the two humps separated by a frequency $\Delta f_{\text{rep}}$. The two-humped gain will result in the growth of two modes centred at the two peaks of the gain structure, i.e. at $-f_{\text{rep}}/2$ and $+f_{\text{rep}}/2$ [figure 3.3(b)]. Once the modes reach a threshold power, they create new modes through four-wave mixing. In general four-wave mixing does not exhibit a threshold power. However here the modes create their own phase-matching condition, in combination with the lossy cavity round trip this results in a threshold power at which four-wave mixing starts to occur. We will therefore denote this threshold four-wave mixing or mixing threshold due to the close ties with the four-wave mixing. The two newly created modes are located at $\pm 3/2 f_{\text{rep}}$ and they again mix with their neighbouring modes to create two further modes located at frequencies $\Delta f_{\text{rep}}$ to their left and right. This process continues until the newly created modes do not reach the FWM threshold. The elegance of this method is, that the FWM automatically fixes the phase relationship between the modes and therefore all modes are phase-locked to each other and a mode-locked laser is created. The key requirement of this method is that the gain is bandwidth-limited, so that only the two central modes experience a net positive gain and the energy transfer is unidirectional from the lower to the higher order modes. This is important to ensure a constant phase relationship between the modes, because a randomly varying direction of energy transfer, i.e. when the higher-order modes transfer their energy back to the lower-order modes, destroys the constant phase relationship. For a laser to reach a stationary state the cavity gain needs to saturate, otherwise the power inside the cavity grows ad infinitum and the laser never becomes stationary. The saturation mechanism of the DFWM laser is inherent to the finite bandwidth of the gain and can be understood as follows: The energy inside the laser is transferred from the lower order modes to higher orders. The energy is then dissipated by the higher-order modes due to the bandwidth-limited nature of the gain, which causes the higher-order modes to experience a net-loss. If the power of the central modes is increased, more energy is transferred to the harmonics, thus more energy is dissipated and the cavity gain of the laser saturates.

Mode-locking by dissipative four-wave mixing is a close relative of the modulation-instability
Chapter 3  Mode-locking by dissipative four-wave mixing

laser [154,161], and the DFWM laser has been mistakenly identified as a modulation-instability laser [142]. However the mode-locking mechanism is distinctly different from modulation instability (MI). Modulation instability is characterised by a degenerate four-wave mixing process where two pump photons are annihilated to create two photons at the signal and idler frequencies under energy and momentum conservation conditions. Although MI is also present in the DFWM it only accounts for part of the process. Apart from two MI processes there is an additional non-degenerate FWM process that is essential for the mode-locked operation of the system (see section 3.2.2 for a more detailed explanation). Additionally, in contrast to the DFWM laser which mode-locks at both normal and anomalous dispersion, MI lasers were known to only operate in the anomalous dispersion regime. Although novel research has shown that modulation-instability laser can operate in the normal dispersion regime when taking advantage of fourth-order dispersion [162], the conditions remain very specific while no fourth-order dispersion is necessary for DFWM mode-locking.

3.2 Theory

3.2.1 QT model

When they first proposed the technique of mode-locking by dissipative four-wave mixing in 1998, Quiroga-Teixeiro et al. developed a model based on the Ginzburg-Landau equation [163] to describe the laser [39]. For brevity the model will be denoted QT model in the course of this thesis. The model considers the dissipative four-wave mixing process in a ring laser configuration. Although there is no principal reason why it should not work in different cavity settings, the ring laser simplifies the treatment considerably. The cavity includes the two central elements, i.e. the active medium, and the filter. For short fibre lengths a distribution of the actions along the cavity length is justified and one can consider the propagation of a quasi-monochromatic light packet along a homogeneous nonlinear fibre with frequency-dependent gain. We start from the Nonlinear Schrödinger equation
Theory

(equation 1.1) and ignore fibre loss, third-order dispersion and stimulated Raman scattering. After adding a gain term which is a polynomial function of the frequency, we get:

$$\frac{\partial}{\partial z} A + \frac{i\beta_2}{2} \frac{\partial^2}{\partial T^2} - i|A|^2 A = g_0 A - g_2 \frac{\partial^2}{\partial T^2} A - g_4 \frac{\partial}{\partial T^4} A$$

(3.1)

Normalizing by the cavity length, the cavity dispersion and the nonlinear power, i.e. $u = \sqrt{\gamma L} A$, $\tau = T/\sqrt{\beta_2 L}$ and $\zeta = z/L$ leads to the following equation governing light propagation inside the cavity:

$$iu_{\zeta} - \eta u_{\tau\tau} + |u|^2 u = i\left(\gamma_0 u - \alpha_g u_{\tau\tau} - \beta_g u_{\tau\tau\tau\tau}\right)$$

(3.2)

with $\eta = \text{sgn}(\beta_2)/2$ determining the nature of the dispersion; if $\eta > 0$ it is normal, and if $\eta < 0$ the dispersion is anomalous. $\gamma_0 = g_0 L$, $\alpha_g = g_2 L/|\beta_2|$ and $\beta_g = g_4/\beta_2^2 L$ are the terms for the dispersive gain over the laser cavity. The subscripts $\zeta$ and $\tau$ denote derivatives with respect to the corresponding variable. Note that the normalization used here is slightly different to the normalization in [39].

---

**Figure 3.4** – The spectral gain profile for parameters $\gamma_0 = 0.05$, $\alpha_g = 0.1$ and $\beta_g = 0.1$ (after [39])
In the spectral domain, the dispersive gain takes the form of,

$$\gamma(\omega) = \gamma_0 + \alpha_g \omega^2 - \beta_g \omega^4.$$

(3.3)

We are only interested in the case where \(\gamma_0, \alpha_g\) and \(\beta_g\) are positive. The gain \(\gamma(\omega)\) then has two frequency maxima \(\omega_m = \pm \sqrt{\alpha_g/(2\beta_g)}\) and a cutoff frequency \(\omega_c\) where the gain becomes zero can be calculated as,

$$\omega_c = \left[\frac{\alpha_g + \sqrt{\alpha_g^2 + 4\beta_g \gamma_0}}{2\beta_g}\right]^{\frac{1}{2}}.$$

(3.4)

Figure 3.4 depicts the spectral gain, for typical parameters of \(\gamma_0, \alpha_g\) and \(\beta_g\). The condition \(\omega_c < 2\omega_m\) enforces the requirement of only the two fundamental modes having positive gain, i.e. all higher harmonics will have a net loss under this condition.

The QT model produces trains of bright and dark pulses in the anomalous and normal dispersion regime respectively. However the width of the simulation time window \(T_{max}\) has a significant influence on the output of the laser [39], because changing the simulation window effectively changes the discretization of the gain structure in the spectral domain. For large values of \(T_{max}\), i.e. sampling the gain structure by a large number of points, we can interpret the discrete numerical points as being equivalent to the cavity modes of a ring laser. However for a smaller discretization this interpretation becomes physically unrealistic. As typical fibre lasers have a length of several tens or hundreds of meters, the cavity mode spacing is in the range of kilo- or megahertz. The sampling of the gain by only a small number of points therefore translates to a physically unrealistic gain bandwidth of only a few megahertz. In this case the spectral discretization is better interpreted as the ring modes of an additional Fabry-Perot filter inside the cavity.

When \(T_{max}\) is small, i.e. the spectral gain is sampled by only a few points, three different states can be observed for slightly different time windows. Figure 3.5 depicts the evolution of the temporal distribution for three different values of \(T_{max}\), with parameters \(\gamma_0 = \alpha_g = \beta_g = 0.1\) and \(\eta = -1\). A value of \(T_{max} = 8\) results in two pulses propagating inside the cavity.
Figure 3.5 – Evolution of the temporal distribution $|u(t)|^2$ for parameters: $\gamma_0 = \alpha_g = \beta_g = 0.1$ in the anomalous dispersion regime. The subfigures depict the (a) SYM state ($T_{max} = 8$), (b) irregular evolution ($T_{max} = 10$), (c) ASYM state with pulses moving to the left ($T_{max} = 20$) and (d) ASYM state with pulses moving to the right ($T_{max} = 20$). After [39]
Increasing the time window to $T_{\text{max}} = 10$ changes the propagation to become unstable and no steady state solution is reached. A further increase in the time width changes the result to a train of moving pulses, moving in the left or right direction depending on the initial noise conditions. When we examine the spectrum of the two interesting regimes, i.e. motionless bright pulses and moving bright pulses, we see that they are easily distinguishable. The spectrum of the motionless pulses [figure 3.6(a)] is symmetric around the origin with two modes inside the gain band and two modes in the dissipative frequency region. The two central modes transfer their energy to the higher order modes by four-wave mixing. In the case of the moving pulses [figure 3.6(c) and (d)] the spectrum is asymmetric around the
origin with again two central modes within the gain bandwidth. The two modes again create two sets of higher-order modes, which are evenly spaced to the left and right of the two central modes. Quiroga-Teixeiro et al denote the two different states the SYM and ASYM states for obvious reasons. It should be noted that the moving pulses are of no relevance in almost all experimental settings. The delay or acceleration can be interpreted as a difference in the group velocity to the velocity of the reference frame. However an experimental observer outside this reference frame simply observes a continuous pulse train, and can not distinguish between accelerated or delayed pulses. Note that the spectrum of the unstable regime [figure 3.6(b)] does not provide any further insight, as it varies from roundtrip to roundtrip.

3.2.2 Analytic model

The numerical simulations in the previous section indicate that the autowaves formed by the laser consist of only four components. The two central frequency waves seem to transfer their energy to the odd harmonics spaced at the frequency separation between the two central waves. Due to the limited number of frequencies the derivation of an analytic model for the description of the laser should be possible promising to provide a more indepth understanding of the involved processes. Of particular interest is the development of the phase relationship between the different modes because the phase-mismatch between the modes has to be constant for the laser to be mode-locked. It is not clear from the numerical simulations how this condition is achieved.

Similar to the QT model we start from the Nonlinear Schrödinger equation (equation 1.1), by adding a gain term $g$ and a fibre loss $\frac{\alpha}{2}$ we obtain the following equation:

$$\frac{\partial}{\partial z} A + \frac{i \beta_2}{2} \frac{\partial^2}{\partial T^2} A - \frac{\beta_3}{6} \frac{\partial^3}{\partial T^3} A = i \gamma |A|^2 A + g A - \frac{\alpha}{2} A.$$  \hspace{1cm} (3.5)

Further normalizing by the cavity length $L$, the frequency separation between the modes $f_{rep}$ and the nonlinearity coefficient $\gamma$. 

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we obtain a Ginzburg-Landau type equation that reads

\[ \frac{\partial}{\partial \zeta} U + i\kappa \frac{\partial^2}{\partial \tau^2} U - \sigma \frac{\partial^3}{\partial \tau^3} U = i|U|^2 U + GU, \] (3.10)

Here \( \kappa = \frac{b_2 f^2 L}{2} \) and \( \sigma = \frac{b_3 f^3 L}{6} \) are the normalized second and third-order dispersion terms respectively. The gain and loss have been combined into the normalized term \( G = (g - \frac{\alpha}{2})L \). If we only consider autowaves consisting of two central frequencies and their odd harmonics, i.e. four modes located at normalized angular frequencies \( -\frac{3}{2}, \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \) we can make the ansatz:

\[ U(\zeta, \tau) = U_{\frac{1}{2}} e^{i\frac{\pi}{2}} + U_{-\frac{1}{2}} e^{-i\frac{\pi}{2}} + U_{\frac{3}{2}} e^{3i\frac{\pi}{2}} + U_{-\frac{3}{2}} e^{-3i\frac{\pi}{2}} \] (3.11)

By substituting equation 3.11 into the Ginzburg-Landau type equation 3.10, and dropping all new frequency terms and third-order terms in the harmonics, we arrive at four coupled-mode equations for the field amplitudes of the individual waves:

\[ \frac{\partial}{\partial \zeta} U_{\frac{1}{2}} = \begin{cases} \frac{\kappa \Omega^2}{4} + |U_{\frac{1}{2}}|^2 + 2|U_{-\frac{1}{2}}|^2 + 2|U_{\frac{3}{2}}|^2 + 2|U_{-\frac{3}{2}}|^2 & U_{\frac{1}{2}} \\ +i \left[ 2U_{-\frac{3}{2}} U_{\frac{1}{2}} U_{\frac{3}{2}} + 2U_{-\frac{1}{2}} U_{\frac{3}{2}}^* U_{\frac{3}{2}} + U_{\frac{3}{2}}^* U_{-\frac{3}{2}} \right] \\ +G_{\frac{1}{2}} U_{\frac{1}{2}} \end{cases} \] (3.12)

\[ \frac{\partial}{\partial \zeta} U_{-\frac{1}{2}} = \begin{cases} \frac{\kappa \Omega^2}{4} + |U_{-\frac{1}{2}}|^2 + 2|U_{\frac{1}{2}}|^2 + 2|U_{\frac{3}{2}}|^2 + 2|U_{-\frac{3}{2}}|^2 & U_{-\frac{1}{2}} \\ +i \left[ 2U_{-\frac{3}{2}} U_{-\frac{1}{2}} U_{\frac{3}{2}} + 2U_{-\frac{1}{2}} U_{\frac{3}{2}}^* U_{\frac{3}{2}} + U_{\frac{3}{2}}^* U_{-\frac{3}{2}} \right] \\ +G_{-\frac{1}{2}} U_{-\frac{1}{2}} \end{cases} \] (3.13)
The subscript denotes the frequency position of the field and $G_i$ denotes the gain at that particular position. Note that in the derivation the contribution of third-order dispersion has been neglected. Apart from the cross-phase (XPM) and self-phase (SPM) modulation terms, the above equations reveal three FWM processes that can transfer energy from the central modes to the harmonics, two degenerate FWM processes and one non-degenerate FWM process, which are depicted in figure 3.7. The fields can be decomposed into their power and phase using $U_{±1/2} = \sqrt{A_{±}} e^{iφ_{±}}$ for the central waves and $U_{±3/2} = \sqrt{B_{±}} e^{iφ_{±}}$ for the harmonics. The equations for the evolution of the power can then be written as:

\[
\frac{∂}{∂ζ} A_+ = 2G_1 A_+ - 4T_1 - 4T_2 + 2T_3 \quad (3.17)
\]
\[
\frac{∂}{∂ζ} A_- = 2G_{-2} A_- - 4T_1 - 4T_3 + 2T_2 \quad (3.18)
\]
\[
\frac{∂}{∂ζ} B_+ = 2G_2 B_+ + 4T_1 + 2T_2 \quad (3.19)
\]
\[
\frac{∂}{∂ζ} B_- = 2G_{-2} B_- + 4T_1 + 2T_3 \quad (3.20)
\]

with the amplitude and phase terms for the three four-wave mixing processes:
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Figure 3.7 – The three FWM processes present in the DFWM laser model.

\[
T_1 = \sqrt{A_+ A_- B_+ B_-} \sin \Delta \phi_1 \quad (3.21) \quad \Delta \phi_1 = \phi_{B_+} + \phi_{B_-} - \phi_{A_+} - \phi_{A_-} \quad (3.24)
\]
\[
T_2 = \sqrt{A_- B_+ A_+ B_-} \sin \Delta \phi_2 \quad (3.22) \quad \Delta \phi_2 = \phi_{A_+} + \phi_{B_-} - 2\phi_{A_-} \quad (3.25)
\]
\[
T_3 = A_- \sqrt{B_- A_+ B_+ A_-} \sin \Delta \phi_3 \quad (3.23) \quad \Delta \phi_3 = \phi_{A_-} + \phi_{B_+} - 2\phi_{A_-} \quad (3.26)
\]

In accordance with the numerical model in section 3.2.1 we assume a symmetric dissipative gain distribution where the energy is dissipated through the harmonics, therefore \( G_{\frac{1}{2}} = G_{-\frac{1}{2}} = g \) and \( G_{\frac{3}{2}} = G_{-\frac{3}{2}} = -p \). By confining the discussion to only symmetric solutions, i.e. \( A_+ = A_- = A \) and \( B_+ = B_- = B \) the equation for the phase evolution simplifies to:

\[
\frac{\partial \Delta \phi_2}{\partial \zeta} = 2\kappa \Omega^2 + 2(A - B)(1 + 2 \cos \Delta \phi_1) + A \sqrt{\frac{A}{B}} \left(1 - \frac{3B}{A}\right) \cos \Delta \phi_2 \quad (3.27)
\]

From further symmetry considerations it is possible to deduce a relation for the phases \( \Delta \phi_2 = \Delta \phi_3 \). The equation for the evolution of \( \Delta \phi_3 \) and \( \Delta \phi_1 \) can then easily be obtained by substituting \( \Delta \phi_2 \) with \( \Delta \phi_3 \) and utilizing the relation \( \Delta \phi_1 = \Delta \phi_2 + \Delta \phi_3 \) respectively. Using the relationship \( \Delta \phi_2 = \Delta \phi_3 \) the coupled equations can be reduced further to a set of only
three equations:

\[
\frac{\partial}{\partial \zeta} A = 2gA - 4T_1 - 2T_2 \quad (3.28)
\]

\[
\frac{\partial}{\partial \zeta} B = -2pB + 4T_1 + 2T_2 \quad (3.29)
\]

\[
\frac{\partial \Delta \phi_2}{\partial \zeta} = 2\kappa + 8(A - B) \left( \cos^2 \Delta \phi_2 - \frac{1}{4} \right) + A \sqrt{\frac{A}{B}} \left( 1 - \frac{3B}{A} \right) \cos \Delta \phi_2 \quad (3.30)
\]

Equation 3.28 and 3.29 nicely demonstrate the energy transfer between the pump wave and the sideband. The first term on the right hand side of the equations describe the gain experienced by the pump and the power dissipated through the sideband. We can immediately see that in the stationary solution when energy is conserved, the amount of energy gained by the system needs to be equal to the energy dissipated from the system and therefore \( gA = pB \). The second and third terms are responsible for the energy transfer due to the degenerate and non-degenerate four-wave mixing respectively. Both processes transfer energy from the pump to the harmonic wave, with the degenerate process transferring twice as much power as the non-degenerate process. Interpreting equation 3.30 is not as straightforward as the equations for the powers. Initially the \( \Delta \phi_2 \) is dominated by the linear phase-mismatch \( 2\kappa \), however as the central frequency power grows, the second and third terms have an accretive impact on the phase evolution. Once a significant amount of power has been transferred to \( B \) the influence of these two terms again diminishes and the system possibly reaches a steady state. Taking advantage of energy conservation we can write a closed-form expression of the stationary solution \( \frac{\partial B}{\partial \zeta} = 0 \) for the sideband power.

\[
B = \frac{g}{4 \sin \Delta \phi_2 \left[ \cos \Delta \phi_2 + \frac{1}{4} \sqrt{\frac{p}{g}} \right]} \quad (3.31)
\]

Because the ratio of the central wave power to the sideband power is determined by the
structure of the dissipative gain, we can use the relation $gA = \rho B$ to calculate the power of the central frequency component. Unfortunately we cannot easily write an analytic expression which is equivalent to equation 3.31 for the phase of the stationary solution. However the roots to equation 3.30 can be found numerically and exists for a large range of experimentally relevant parameters. The analytic results therefore confirm the numerical findings of section 3.2.1. The existence of fixpoints for this system of equations shows that the laser model possesses a stationary autowave solution.

![Graphs](image) 

**Figure 3.8** – (a) Phase $\Delta \phi_2$ and (b) ratio of sideband to central wave power $B/A$ for an anomalous cavity ($\kappa = -0.2$) and the gain structure parameters $g = 0.1$ and $p = 0.51$. (c) Evolution of the $B/A$, $\Delta \phi_2$, $A$ and $B$ which have been normalized to their final values respectively. (d) phase plane of the transient laser evolution. The dashed line in (b) indicates the ratio $\frac{\rho}{g}$. 

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Numerically solving the coupled equations 3.28-3.30 for a anomalous dispersion cavity with gain ratio $p/g = 5.1$ shows that the solution quickly converges to a stationary state where both the power ratio and phase remain constant upon propagation. Figure 3.8(a) depicts the phase evolution. Initially we observe a quick decrease of the phase from its initial zero value. At $\zeta \approx 10$ the slope changes significantly and the phase varies much less rapidly. After around 50 cavity lengths of propagation the phase reaches the steady state and does not change any further. The ratio $\frac{B}{A}$ undergoes a similar evolution as can be seen in figure 3.8(b). From the initial random value it quickly decreases to zero, as the sideband power experiences dissipative loss. However between 10 and 20 cavity lengths the ratio starts to grow exponentially because four-wave mixing starts to convert energy from the central mode to the sideband. These observations provide interesting insights into the nature of the four-wave mixing process and in particular the phase-matching. The earlier discussion of the equation of the phase evolution (equation 3.30) is nicely reflected in figure 3.8(c) which depicts the evolution of the power ratio, the central wave power, the sideband power and the phase, normalized for an easier comparison. The initial phase evolution is determined by the linear phase-mismatch $\kappa$ and the normalized phase increases strongly in the first 10 cavity lengths. Note that the non-normalized value decreases in agreement with the negative dispersion parameter $\kappa$. Simultaneously the central wave power slowly grows while the power of the sideband remains essentially zero. Once the central wave power has reached a threshold value, the impact of power dependent terms in the phase evolution equation becomes more significant, which is reflected by the large change in the phase gradient. The sideband power and power ratio follow the central wave power curve, however with a delay and a steeper gradient. We should note that because the simulations start from random initial conditions, the starting ratio of $B/A$ can artificially high. However at the start of the simulation the value of $B$ initially decreases to zero and a physically meaningful state is restored.

To illustrate the transient dynamics of the system it is convenient to graph the system using trajectories in the phase plane [142, 163], as shown in figure 3.8(d). Initially the system undergoes a strong change from the random initial conditions to the origin of the graph.
This occurs during the propagation along the first five cavity lengths and can be seen by the initial decrease in $B/A$ in graph 3.8(c). From the origin the ratio grows monotonically before the trajectory spirals into a fixed point, constituting a stable attractor of the dynamical system. The spiralling indicates a damped oscillation, i.e. the power ratio changes between growth and loss. The small oscillation can also be observed in figures 3.8(a)-(c) where we can identify an overshoot combined with a short oscillatory tail for $A$, $B$ and the ratio $B/A$. Once the power and phase $A$, $B$, $\Delta \phi_2$ of the stationary solution of the system have been found, it is possible to construct the electric field using

$$U(\tau, \zeta) = 2\sqrt{A} (e^{i\tau} + e^{-i\tau}) + 2\sqrt{B} (e^{i3\tau} + e^{-i3\tau}) e^{i\Delta \phi_2}. \quad (3.32)$$

Figure 3.9(a) depicts the calculated field intensity of the stationary solution for the same parameters as used in figure 3.8, showing a pulse train of bright pulses. The phase profile of the generated pulses is plotted with a dotted curve. The phase difference between adjacent pulses is $\pi$ with a sudden jump in between the pulses. This behaviour is consistent with the fact that in-phase soliton trains in a loop cavity are intrinsically unstable [164]. In contrast to the simulation results in [142] however, the phase is not constant over the individual pulses but exhibits a shape similar to the pulse intensity. It is impor-
tant to stress again the fact that the mode-locking mechanism is not modulation instability, i.e. the laser is not a modulation instability laser. Modulation instability is characterised by degenerate four-wave mixing with a pump and two sidebands, however DFWM mode-locking involves three FWM processes, two of which are degenerate, however the third non-degenerate FWM process is crucial to achieve mode-locking. Additionally the DFWM laser exhibits mode-locking in the normal dispersion regime but in the absence of higher-order dispersion modulation-instability lasers do not exist for normal dispersion. Figure 3.9(b) depicts the stationary pulse train for a set of parameters in the normal dispersion regime. The dark pulses exhibit an abrupt $\pi$ phase jump over its profile indicating the solitonic nature of the pulses. It should be noted that the pulse shape information that can be gathered from the analytic model is limited because the resulting pulse train is always described by a beating between two sinusoidal waves due to the nature of the model.

![Figure 3.10](image.png)

**Figure 3.10** – (a) Power ratio of the sideband to the central mode and (b) power of the central mode for $\kappa = -0.1$ (solid), $\kappa = -0.2$ (dashed) and $\kappa = -0.3$ (dotted), $g = 0.1$ and $p = 0.51$.

To investigate the effect of small changes of the dispersion parameter to the dynamics of the stable system, we have solved the model for fixed values of $g$ and $p$ and three different values of the dispersion $\kappa$. Figure 3.10(a) and figure 3.10(b) depicts the ratio of the sideband power over the power of the central mode and the power of the central mode for the three
dispersion values. Although all dispersion parameters exhibit a stable solution of the system, the dynamics are subtly different. For larger absolute values of the dispersion $A$ and $B/A$ increases at larger values of $\zeta$, i.e. after more cavity round trips, furthermore we see that the final value of $A$ is larger for larger values of dispersion. This is related to what we alluded to as the FWM threshold in section 3.1. In an experimental setup with cavity losses this results in less or no power being transferred to the sidebands for higher dispersion values because the central modes do not reach the power necessary to transfer their energy to higher-order modes.

For a given dispersion parameter the stability of the laser depends on the ratio of $p/g$. If the ratio is too small, the power in the sideband becomes too large because it is not dissipated quickly enough. However the dynamics are quite different in the different dispersion regions. In the normal dispersion regime, we observe a reversal of the energy transfer and power is transferred from the sideband to the central wavelength component. This is illustrated in figure 3.11(a) where the phase space trajectory of the system approaches a closed curve. A closed curve in phase space is denoted a limit-cycle in nonlinear dynamics and is usually associated with self-sustained oscillations of the system. Here the oscillations stem from the fact that when the dissipation from the sideband is too small the sign of the en-
energy transfer reverses. Thus the sideband transfers energy to the central wave component until the transfer reverses again at a different power ratio and the system keeps oscillating between these two states. When the dispersion is anomalous, the phase space trajectory is significantly different [figure 3.11(b)]. Initially the phase space trajectory spirals inwards towards a stable fix-point, however the direction of the spiral changes, i.e. the fix-point moves. The system approaches a state with zero phase difference between the modes and increasing power ratio. It is not quite clear if the result correspond to a stable or unstable fix point, as the numerical algorithm for solving the coupled equations becomes unstable and does not converge upon further propagation.

The simplified analytic model confirms the conclusions from the numerical simulations of the QT model that the laser laser exhibits stationary autowave solutions. It should be highlighted that phase-matching of the four-wave mixing is reached automatically, specific engineering of the dispersion is therefore not necessary. Let us further mention that Hontzatko et al. [123] did perform a similar analysis for modulation instability lasers. However they fail to realise the fact that one of the models they describe is a DFWM mode-locked laser instead of a modulation instability laser.

### 3.3 Experimental implementation

It is non-trivial to find a gain medium which provides the two-humped gain structure required for the DFWM process to work, however this proves not to be necessary. Instead of matching the exact gain structure it is possible to utilise a broadband gain and introduce a filter into the cavity which modifies the gain to the desired shape. A number of requirements have to be met by both the gain and the filtering mechanism however. First, the gain has to be sufficiently broadband to support the desired repetition rates, i.e. its spectral width should be larger than the repetition rate and additionally it should be sufficiently flat over this width, so both central modes experience approximately the same gain. The requirements on the filter are relatively simple: It should be low loss and be able to withstand the optical powers inside the cavity.
The gain mechanism which stands out as an ideal candidate is amplification through stimulated Raman scattering. The Raman gain bandwidth is more than 20 THz, sufficiently wide to support repetition rates well in excess of what is currently desired for optical systems. If the repetition rates is limited to below 1 THz and the maximum of the Raman gain curve is used, the gain will be reasonably symmetric so that if both central modes are set up symmetrically to the centre of the gain they experience similar gain. This holds especially true when using a tunable pump laser, as that allows to tune the maximum of the Raman gain curve so that this criteria is met. Raman amplification also offers some benefits beyond the ones necessary for DFWM. Compared to fibre amplifiers doped with rare-earth elements, e.g. Erbium, Raman amplification has two key advantages; it does not exhibit an inherent saturation mechanism and Raman scattering is virtually the same for all wavelengths in the optical spectrum, thus the only limitation on the possible laser wavelength is the availabl-
ity of suitable pump lasers. Rare-earth based amplification in contrast exhibits a saturation based on the concentration of the dopant. Furthermore their output wavelength is limited to the gain band of the dopant. Therefore Raman lasers are well known to provide very high average powers in the cw regime, and commercial cw Raman laser are readily available with powers in excess of 5 W. Surprisingly however there have been only limited examples [38] of combining the possible high average output powers with mode-locking techniques to form pulsed Raman fibre lasers.

The optical filtering can be achieved through the combination of two filters, a Fabry-Perot filter combined with a bandpass filter. The Fabry-Perot filter will provide the differentiation into modes, the equivalent of the numerical discretization and the two hump structure of the gain described in section 3.2.1, as only modes at the transmission peaks of the Fabry-Perot will grow. The dissipative element can be provided by a second filter with bandpass characteristics. Provided the correct filter is selected and is tuned correctly, only two central modes of the Fabry-Perot filter experience a net gain, fulfilling the requirement for DFWM.

As the average powers inside the cavity of the laser can be high (well over 1 W), choosing filters which supports these high optical powers becomes a challenge. Although it is possible to incorporate free-space optical components into the cavity, it has the significant drawback of introducing tremendous losses. We therefore choose to use custom made fibre Bragg gratings (FBG) as the filtering element. There are some disadvantages to using a FBG, in particular is there no straightforward way of tuning the centre-wavelength of the bandpass filter or the FSR of the Fabry-Perot filter. However the ability of withstanding high average optical powers at significantly lower losses than offered by bulk components far outweighs the drawbacks. On a further note, it can be assumed that for many applications it is not necessary to be able to tune the centre-wavelength or the repetition rate of the laser. Furthermore there has recently been some research into tunable Fabry-Perot type FBG filters [165], so this limitation of FBG filters might be overcome in the near future.

Figure 3.12 depicts the experimental implementation of the laser. The pump laser seeds the laser through spontaneous Raman emission inside the signal band. The seed is amplified subsequently through stimulated Raman scattering. However the wavelength of light
inside the signal band is limited by the modes of the Fabry-Perot filter. The bandpass characteristics of the FBG filter limit the net-gain to the two central modes only. The central modes then distribute their energy to the higher harmonics, exactly as described in 3.1.

### 3.3.1 Setup

The basic experimental setup of the passive mode-locked Raman fibre laser is depicted in figure 3.13. We use a regular ring cavity configuration with two 1450 nm/1550 nm wavelength division multiplexers (WDM) that couple the pump into the cavity and reject the remaining pump from the cavity. The pump does therefore not contaminate the signal and the circulator is protected from the high average power of the pump. The two WDMs also offer the additional advantage of allowing a setup reconfiguration with respect to pumping direction, i.e. the laser can be changed from forward to backward pumping to measure its effect on the output of the laser. As the experiments were partly carried out in two different locations, different pump lasers were used. In both cases commercial cw Raman fibre lasers (RFL) were chosen for their high average output powers. One pump laser was a tunable RFL.
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with a wavelength range between 1428 nm and 1489 nm and a maximum average output power of approximately 5.5 W, the second laser was a fixed wavelength RFL with a centre wavelength of 1445 nm and approximately 6 W of average output power. Unless noted otherwise the pump laser used was the tunable RFL. The output of the cavity is monitored behind a broadband output coupler (OC) with varying coupling ratios (see section 3.4.3 for a further discussion about the influence of the coupling ratio on the output). The nonlinearity and the gain are provided by a length of fibre in between the two WDMs. In most experiments this was either a highly nonlinear fibre (HNLF) or dispersion shifted fibre (DSF). An HNLF or DSF considerably lowers the lasing threshold and increases the average output power of the laser, due to the high nonlinear and Raman gain coefficients. See appendix A for the parameters of the different fibres. The FBG filter, the central element of the DFWM

Figure 3.14 – Reflection spectrum of the fibre Bragg grating filter.

mode-locking is incorporated into the cavity with a polarization insensitive circulator. The circulator acting as an isolator in the backward direction also ensures the unidirectional operation of the laser. Figure 3.14 shows the reflection spectrum of one of the employed FBG
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grating filters. The reflection spectrum was designed as the transmission of a Fabry-Perot filter multiplied with a Gaussian-shaped bandpass filter, acting as the dissipative element. The full-width-half-maximum (FWHM) of the Gaussian envelope is 250 GHz with the width of the individual reflection peaks being between 2 and 3 GHz. The free spectral range (FSR) of the Fabry-Pérot is 100 GHz determining the repetition rate of the laser. A number of these FBG filters with varying parameters were used. Appendix A contains a list of the different FBG filters and their parameters. To minimize losses, the connections inside the cavity were spliced. However the joints from the WDMs to the central fibre are connectorized to allow easy change of the fibre without the need to resplice. The laser output was monitored using an optical spectrum analyser (OSA) and a second harmonic frequency resolved optical gating (SHG-FROG) instrument from Southern Photonics.

3.4 Experimental results

The mode-locking is self starting; once the pump is switched on, it creates spontaneous Raman emission in the signal band. When the pump power is increased sufficiently, amplification surpasses the cavity losses and lasing starts. Just above the threshold we initially observe only light from one of the central modes of the FBG, due to imbalances in the Raman gain. However when the pump power is increased a few milliwatts above the lasing threshold, both central modes assume approximately the same power and sidebands are created by four-wave mixing causing the laser to mode-lock. The lasing threshold for a laser with a 90/10 output coupler and 1 km of HNLF is at 303mW pump power. The parameters of the FBG and the HNLF used in this experiment can be found in the appendix A (FBG 1 and HNLF 1). Figure 3.15(a) shows the spectrum at the lasing threshold when only one mode dominates and (b) when the pump is increased further so that the two central modes create higher harmonics through four-wave mixing. We should point out that the modes we are talking about here correspond to the reflection modes of the Fabry-Perot filter, these are different from the cavity modes of the laser. In fact these modes can contain a large number of cavity modes and we will discuss the implications of this in chapter 4. For
Experimental results

![Graphs showing spectral power vs. wavelength for different pump powers.](image)

(a) $P_{pump} = 303$ mW  
(b) $P_{pump} = 700$ mW

**Figure 3.15** – Spectra of the laser with pump power (a) just above the threshold and (b) after increasing it so that significant four-wave mixing occurs.

In the remainder of this chapter, we will use the term modes in this broader sense unless we explicitly talk about cavity- or eigenmodes of the laser. Investigating figure 3.15(b) we see that the number of modes is significantly larger than the reflection bands of the FBG filter. The spectrum therefore does not simply reproduce the transfer function of the filter, a good indication for the effectiveness of the four-wave mixing process. However, the large number of modes contradicts the findings of the numerical QT model and the assumptions of the analytic model, because in both models only two central components and their first odd harmonics exist. Additional measurements by frequency resolved optical gating (FROG), using a SHG-FROG from Southern Photonics, confirm the mode-locked operation of the laser. An SHG-FROG in its basic form is an optical autocorrelator followed by a spectrum analyser. It thus measures a spectrally resolved autocorrelation, usually denoted a spectrogram. Theoretically it is possible to recover the pulse shape and phase from the measured spectrogram (for an overview on the topic of pulse shape recovery see e.g. [166]). However, for reasons discussed in the section 3.4.2, a pulse recovery was not possible. Nevertheless, the autocorrelation can be calculated from the FROG spectrogram. Figure 3.16(a) and (b) show a measured FROG spectrogram and the corresponding autocorrelation respectively.
This measurement was taken with 2.22 W of pump power, with one kilometre of highly non-linear fibre as the active element and a 10% output coupling ratio. It can be seen that the laser indeed produces a train of pulses. The pulse separation is almost exactly 10 ps as expected from the 100 GHz free spectral range of the FBG filter and the width of the central autocorrelation pulse is 870 fs. Examining the autocorrelation further we can distinguish three features: The autocorrelation exhibits a background of 20 – 30%, we see an envelope over the autocorrelation; the peak power of the crosscorrelation pulses decreases with increasing delay, i.e. higher order crosscorrelations have lower power, and the width of the cross- and autocorrelation pulses is approximately the same. A decreasing crosscorrelation power is the signature of a limited coherence of the pulse train, i.e. the pulses are not fully coherent to each other, often due to noise. However the similar width of the cross- and autocorrelation pulses testifies for a low timing jitter and the decrease in coherence must therefore be attributed to intensity noise of the pulse train. The noise also partly explains the existence of the background in the traces. Finally we want to draw attention to the subpulse-like structure in between the main pulses present in the autocorrelation traces, which is particularly evident in the FROG spectrogram. Comparing the autocorrelations
Experimental results

![Autocorrelation plots](image)

(a) $P_{\text{pump}} = 1.5 \text{ W}$

(b) $P_{\text{pump}} = 2.6 \text{ W}$

Figure 3.17 – Autocorrelation of the laser operating at (a) $P = 1.5 \text{ W}$ and (b) $P = 2.6 \text{ W}$ pump power respectively.

in figure 3.16(b) and figure 3.17(a) and (b), all taken at different pump powers, indicates that the subpulse structure varies with pump power, i.e. gain. Additionally the background of the autocorrelation traces decreases with pump power. The source of the subpulses is not evident, however a similar structure did appear under some conditions when solving the analytic model numerically. The question of the subpulses will be addressed in more detailed simulations in section 3.6.

### 3.4.1 Pulse width

Some of the output characteristics of a laser that are of key interest are the pulse shape, chirp and pulse width. The pulse shape will be discussed in section 3.4.2, while we discuss the width and chirp of the output pulses in this section. Several other important laser characteristics are directly linked to the pulse width. For a fixed repetition rate of the laser the pulse width essentially determines the duty cycle of the laser. For the same average output power shorter pulses will also result in higher peak power, a characteristic desirable for applications requiring high nonlinear responses. A simple examination of the mode-locking by dissipative four-wave mixing process shows that the pulse width of the output should
be pump power dependant. Considering the laser setup, we easily see that an increase in pump power yields a stronger amplification of the signal band. The two central modes of the laser spectrum therefore contain more power, which they then transfer to the next set of higher-order modes through the four-wave mixing process. These subsequently create new modes and so forth, until a set of modes does not reach the mixing threshold and thus does not create a new set of modes. When there is more power available, a larger number of modes reach the mixing threshold and create new sets of modes, resulting in more modes being created overall. Therefore the width of the laser spectrum should increase. The notion is confirmed by the experimental results. Figure 3.18 shows a comparison between the spectrum at two different pump power levels. We see a significant increase in the number of modes. In the case of low pump power (\(P = 370\) mW), the number of distinguishable modes is only slightly higher than the reflections bands of the FBG filter. However at high pump power (\(P = 2.22\) W) the number of modes has increased dramatically and can be estimated to significantly above 50. As a side effect the mode contrast, i.e. the ratio of the mode peak to the minimum between modes, decreases. This is caused by an increase in spontaneous Raman emission, that can be observed as the background underneath the modes in figure 3.18. Additionally we observe a widening of the spectral tails in particular on the red side of the spectrum. The asymmetry of the spectrum can already be observed in fig-

![Figure 3.18](image-url)

(a) \(P_{\text{pump}} = 370\) mW  
(b) \(P_{\text{pump}} = 2.22\) W

Figure 3.18 – Optical spectrum of the laser for (a) \(P = 370\) mW and (b) \(P = 2.22\) W pump power
Experimental results

Figure 3.15(b), and is consistent with third-order dispersion and intrapulse Raman scattering effects which will be discussed in more detail in section 3.6.3.

![Autocorrelation at pump powers $P_{\text{pump}} = 1.85$ W (solid line) and $P_{\text{pump}} = 2.6$ W (dashed line).](image)

**Figure 3.19** – Autocorrelation at pump powers $P_{\text{pump}} = 1.85$ W (solid line) and $P_{\text{pump}} = 2.6$ W (dashed line).

Provided that there is no additional chirp an increase of a pulses spectral width results in a shorter pulse width in the time domain due to the properties of the Fourier theorem. This is indeed the case, as can be seen in the autocorrelations of the laser for pump powers of $P_{\text{pump}} = 1.85$ W and $P_{\text{pump}} = 2.6$ W, shown in figure 3.19. We can clearly see the difference in pulse width; at lower pump power the FWHM of the central pulse is approximately 1 ps while it is 840 fs at the higher pump power. Graph (a) in Figure 3.20 depicts the FWHM of the central pulse of the autocorrelation as a function of the pump power, while Figure 3.20(b) shows the dependence of the time-bandwidth product on pump power. The pulse width appears to be inversely proportional to the pump power, and decreases asymptotically, while the time-bandwidth is almost constant apart from the first point. The minimum autocorrelation pulse width is about 840 fs at 2.6 W pump power. This translates to approximately 530 fs or 590 fs FWHM of the pulses assuming a sech$^2$ or Gaussian respec-
Chapter 3  Mode-locking by dissipative four-wave mixing

**Figure 3.20** – (a) Autocorrelation width and (b) time-bandwidth product dependence on pump power. (c) Autocorrelation width as a function of pump power (2nd experiment with 20% output coupling ratio), the grey line represents an exponential fit.

tively. The time-bandwidth product is 0.54 for the autocorrelation pulses, corresponding to a value of 0.34 for a sech\(^2\) or 0.38 for Gaussian. Comparing these values to the values of a transform limited Gaussian (0.44) and sech\(^2\) (0.315) pulse, we see that it is below the value of a transform limited Gaussian pulse indicating that the pulse trains cannot be Gaussian shaped. We can conclude that the laser produces a train of nearly transform-limited pulses assuming a sech\(^2\)-shape and there is only limited chirp on the pulses. A second measure-
ment taken with a slightly different setup (a 20% output coupling ratio was used) clearly shows that the pulse width decreases exponentially with pump power [figure 3.20(c)]. However the width asymptotically approaches a value of approximately 1 ps slightly higher than the 800 fs observed in figure 3.20(a). This slightly larger pulse width can be explained by the difference in coupling ratio. A smaller output coupling ratio causes more power to recirculate inside the cavity, i.e. a higher intracavity power. Thus more modes reach the FWM threshold and more modes are generated, i.e. the spectrum is broader and the pulse width is shorter.

### 3.4.2 Pulse shape

The QT model (section 3.2.1) and the theoretical analysis (section 3.2.2) predict that the DFWM laser generates a train of pulses and that the nature of the pulses depends on the sign of the cavity dispersion. In the anomalous dispersion regime the models predict the laser to generate bright pulses with a $\pi$-phase shift in between the pulses. For normal dispersion the laser emits a train of dark soliton-like $\tanh^2$ pulses. However in contrast to the QT model and the analytic analysis the laser can be affected by other factors that influence the pulse shape, in particular the shape of the transfer function envelope of the filter might affect the pulse shape. All the filters used in the experiments had a Gaussian shaped envelope. Furthermore the pulse shape could be influenced by averaging over pulse fluctuations. An investigation of the influence of the filter shape will be carried out numerically in a later section (section 3.6.1), while the influence of noise on the laser is discussed in chapter 4. Experimentally determining the pulse shape and phase is a non-trivial task and pulse characterisation techniques are the topic of extensive research [166]. In this experiment we used a SHG-FROG from Southern Photonics to measure the pulse train. The common numerical algorithms to retrieve the pulse shape and phase from the FROG-spectrogram are only valid for single pulse recoveries, however it is possible to modify the recovery algorithms to work for the recovery of pulse trains (see reference [167] for a detailed discussion). Unfortunately it was not possible to recover a sensible pulse shape from the spectrograms with two factors contributing to the failure of recovering a pulse shape.
One reason is of technical nature, the FROG instrument from Southern Photonics uses a CCD chip in the spectrometer and the resolution of the CCD chip is to small to properly resolve the mode structure of the laser, i.e. every mode is only sampled by three pixels. However there is a second more severe limitation on the pulse recovery. Similar to a SHG-autocorrelation the FROG takes an average over many pulses, however the laser pulse train is noisy, mainly due to supermode noise. Because the FROG averages over varying pulses there is no one pulse train to be recovered, as the spectrogram is an average of many slightly different pulse trains (see reference [168] for a description of a similar phenomenon when recovering FROG traces from a supercontinuum). The effects and causes of the supermode noise will be discussed in more detail in chapter 4.

Despite the failure to recover a sensible pulse shape from the FROG-spectrogram, it is possible to get an indication of the shape of the generated pulses. Figure 3.21 depicts the fit of a sech\(^2\), a Gaussian function and an exponential to the envelope of the laser spectrum at 0.7 W of pump power. Although the Gaussian and sech\(^2\) function fit exhibit the same error, we can see in figure 3.21(a) that the tails of the spectrum are significantly better reproduced by a hyperbolic secant. Even though the exponential fit displays similar behaviour in the
Experimental results

As for the sech² fit, the fitting error is significantly higher due to the strong mismatch for the central modes of the spectrum, which can be clearly seen on the linear scale plot (figure 3.21(b)). Both the sech and the Gaussian are self-reciprocal functions, i.e. both the function and its Fourier transform have the same shape [169]. The Fourier transform of an exponential is a Lorentzian function. We can therefore gather information from the shape of the spectral envelope about the pulse shape in the time domain. The central autocorrelation pulse with a best-fitted sech², a Gaussian and a Lorentzian function is shown in figure 3.22. The plot is in a logarithmic scale to better demonstrate the quality of the fits. Similar to the spectrum the sech² resembles the autocorrelation trace more closely than the Gaussian function. However, the Lorentzian function is clearly the best of the three curve fits. The mechanism causing the pulses to resemble a Lorentzian function is not clear. A Lorentzian pulse envelope is usually associated with an exponential decay of the frequency.

Figure 3.22 – Central pulse of autocorrelation with a sech (dashed), Gaussian (dotted) and Lorentzian (dashed-dotted) fit, logarithmic plot.
components in the spectral domain. However the spectrum does not fit an exponential decay, although the spectral tails seem to decay almost linearly [see the logarithmic plot in figure 3.21(a)] the central modes do not match the exponential very well [figure 3.21(b)]. A possible explanation for the Lorentzian shape is that it is caused by averaging over a number of different pulse trains due to noise fluctuations. The pulse shape will be investigated numerically in section 3.6.1 and the influence of noise will be discussed in more detail in chapter 4.

3.4.3 Output power

One of the design goals of the DFWM mode-locked Raman fibre laser was to achieve high average output powers. Raman scattering was specifically chosen as the amplification process because it promises these high output powers. In this section we will investigate the output power behaviour of the laser. Figure 3.23 depicts the output power of the laser with the parameters given in section 3.3.1 and a 20/80 output coupler, i.e. 20% output coupling ratio. The data shows that once the pump power is increased above the lasing threshold, i.e. the unsaturated gain exceeds the cavity losses, the output power increases linearly with pump power. The maximum average output power is 426 mW at 4.32 W pump power, a further increase on the previously reported value for a laser of similar repetition rate [25]. The slope of the linear increase, or the slope efficiency, is approximately 10%. The linear relationship between pump and output power is typical for a laser without any saturation of the gain. Indeed the graph does not indicate that it is approaching an asymptote, evidence of gain saturation. Therefore the gain is not saturated for high pump powers. As the only possible saturation effect for Raman amplification is pump depletion, we can assume that the laser operates in the non-depleted pump regime. Observing the rejected pump at the second WDM, confirms this assumption, about 30% of the input pump power can be observed at the pump rejection port. Thus only about 50 – 60% of the pump energy is converted into energy in the signal band. Generally it is desirable to convert more of the pump power into the signal band. The low utilization of the available power can be improved in several ways. As a significant proportion of the pump light is rejected from the cavity, it
Experimental results

Figure 3.23 – Output power as a function of pump power. The dots indicate measured data, the curve is a linear fit.

should be possible to reinject the rejected pump into the cavity, to fully utilize the pump energy. In order to test the effect of pump reinjection, a FBG was spliced to the pump re-jection port of the laser. However the pump reinjection yielded only a slight improvement of the the lasing threshold and the slope efficiency of the laser by around 10%. The benefit could be more significant, as the FBG is not exactly centered at the pump wavelength and only about 50% of the rejected pump is reflected back into the cavity. It was not possible to adjust the wavelength of the pump laser, because the Raman laser used for this part of the experiment was not tunable. A FBG properly tuned to the pump wavelength should give additional 5 – 10% improvement.
Chapter 3  Mode-locking by dissipative four-wave mixing

Variation of the output coupler

The output coupling ratio determines the percentage of power that is coupled out of the laser cavity. It therefore directly influences the cavity loss, and thus the lasing threshold, as well as the slope efficiency of the laser, because an increase in cavity power means a larger increase in output power for a higher coupling ratio. To investigate the influence of different coupling ratios on the output power of the laser, we carried out experiments with output couplers with 30%, 50% and 70% output coupling percentage and measured the output powers of the laser. Figure 3.24, shows the output power as a function of pump power for the three different couplers. The lasing threshold increases with the amount of light coupled out of the cavity. The lasing threshold is 1.12 W, 1.2 W and 1.4 W for the 30/70, 50/50 and 70/30 coupler respectively. As the cavity loss increases with larger output coupling ratio, it is non-surprising that the lasing threshold increases. Simultaneously we observe a higher

![Graph showing output power as a function of pump power for different coupling ratios. Squares (data) and dotted line (linear fit): 30% output coupling ratio, circles (data) and solid line (linear fit): 50% coupling ratio, diamonds (data) and dashed line (linear fit): 70% coupling ratio.](image-url)
slope efficiency for the larger output couplers. The slope efficiency of the 70/30 coupler is above 30% with 0.33. Using the 70/30 coupler the laser reaches a maximum output power of 926 W at 4 W of pump power. This is a further twofold increase on the previous maximum power and to the best of our knowledge the highest average power achieved for a laser in this repetition rate region. Furthermore extrapolating the results of the pump reinjection setup to the 70% output coupling ratio should yield an increased output power of over one Watt.

3.4.4 Pump direction

So far all experimental results have been attained by employing a backward pumping configuration. However in principle there should be no significant difference between pumping the laser in the forward or backward direction. Before we proceed further, let us define what is meant by forward or backward pumping: Here forward pumping refers to the setup where the pump light from the RFL propagates inside the cavity in the same direction as the signal light. Backward pumping, or pumping in the backward direction refers to the case when the pump light propagates in the opposite direction as the signal. This section will present a comparison of the laser output when it is pumped in forward and backward direction. The results will make evident why the backward pumping scheme is favoured compared to the forward pumping scheme in all the experiments. As the numerical simulations only utilize an abstract gain parameter instead of the more realistic Raman gain curve, they do not account for pumping direction and we have to rely on experimental results to investigate the effect of the pump propagation direction. The experimental setup is the same as the one described in section 3.3.1, however we use an output coupler with a larger coupling ratio (20% versus 10% in the previous experiments). The configuration can easily be changed from backward to forward pumping by moving the pump input from the input WDM to the second WDM, i.e. the pump rejection port. For the following experiments all parameters of the cavity except for the pump direction have been kept constant to facilitate the comparison between pumping directions. Figure 3.25 depicts the optical spectrum of the laser pumped in the forward direction, at the pump power
Chapter 3  Mode-locking by dissipative four-wave mixing

Figure 3.25 – Spectrum of the laser pumped in the forward direction.

of 1.1 W. Comparing the spectrum of the forward pumped laser with the spectrum of the laser pumped in a backward configuration shown in figure 3.15, we see that, while the spectrum of backward pumped laser contains a large number of modes, the forward pump laser only has few modes on top of a large background. Additionally there is significant residual pump power left in the output of the forward pumped laser, further reducing its quality. For a better comparison of the autocorrelation traces we took a measurement of the laser in forward and backward pumping configuration at the same pump power of 1.1 W. The results are shown in figure 3.26(a) and (b) respectively. In contrast to the laser pumped in the backward configuration, we see a strong autocorrelation peak, but the crosscorrelation peaks are very weak. The first order crosscorrelation peak has about a 25% of the power of the autocorrelation peak, disregarding the background. In the backward pumping configuration the difference in peak power is negligible. A low crosscorrelation peak power is the signature of a very low coherence time. Because subsequent pulses are only partly coherent to each other they only cause a weak crosscorrelation signal. One possible reason
Experimental results

![Autocorrelation forward pumping](image1)

![Autocorrelation backward pumping](image2)

Figure 3.26 – Autocorrelation trace of the DFWM laser pumped in the (a) forward and (b) backward direction.

for this might be the stronger generation of spontaneous emission causing a larger noise variation. The autocorrelation peak is also significantly shorter in the case of the forward pumped laser, being 0.6 ps wide compared to 2 ps width for the backward pumped laser. The decreased width can also be interpreted in terms of a lower coherence time. A light signal composed to a large proportion of noise will yield a very narrow autocorrelation peak at zero delay. However as the delay increases, the degree of coherence will rapidly decrease and thus the autocorrelation signal vanishes. We can therefore conclude that the forward pumped laser is significantly noisier and yields a much less coherent pulse train than the laser pumped in the backward direction. It should also be noted that the output power of the laser pumped in the forward direction increases more quickly, i.e. the slope efficiency is higher. However this is only the case up to approximately 50 mW output power (at around 1.5 W pump power) and a further increase in power of the RFL pump does not yield any more growth in output power. The reason for the asymptotic behaviour of the output power is not clear, possibly there is a significant amount of amplified spontaneous emission (ASE) noise, which is filtered and lost every time the light passes through the FBG filter. There have been extensive studies about the influence of pump direction on signal noise for Raman amplifiers [170–173], these studies mainly dealt with transfer of amplitude or phase noise from the pump to the signal. There is no study about the dependence of spontaneous
Raman emission generation on pump direction. Although such a study would provide further insight into the Raman amplification process, it is beyond the scope of this thesis.

3.4.5 Filters

One factor in the assessment of a laser system is the supported repetition rates and how easy it is to design the system to a desired repetition rate. The maximum repetition rate of a laser depends on a number of factors. In passively mode-locked systems these are mainly the response time of mode-locking element, such as a saturable absorber and the gain bandwidth, which has to be sufficiently wide to provide amplification over the laser bandwidth. The only time-dependent process in DFWM is the Kerr-nonlinearity and it can be regarded as instantaneous for almost all use cases. Therefore the only limiting factor for the repetition rate is the gain bandwidth. Raman gain with a width of \( \approx 13 \text{ THz} \) supports repetition rates in excess of several THz. As the repetition rate of the laser is determined by the FSR of the filter, changing the repetition rate becomes simply a matter of changing the filter making it very easy to build lasers with the desired repetition rate.

![Optical spectrum](image1)

![Autocorrelation](image2)

Figure 3.27 – (a) Optical spectrum and (b) autocorrelation of the 160 GHz laser with 2.96 W pump power.

Figure 3.27(a) and (b) show the optical spectrum of the laser with a 160 GHz FSR FBG (FBG 2 in appendix A). The setup for this experiment was the same as in section 3.4.3. The nonlinear element is 500 m of HNLF (HNLF 2 in appendix A) and the laser pump laser is the fixed
Experimental results

wavelength 5 W Raman fibre laser already described in section 3.3.1. The output coupler has a 30% output coupling ratio. The spectrum shows the mode separation to be approximately 1.3 nm in agreement with the 160 GHz repetition rate set by the FSR of the filter. The pulses in the autocorrelation traces are separated by ≈ 6.3 ps also consistent with the FSR of the filter. The width of the autocorrelation pulses is approximately 1 ps. Note that the width of the spectrum is significantly smaller than the width of the 100 GHz laser spectrum. There are several reasons for this, for one, the losses in this setup are slightly higher, as some of the fibre connections which were spliced in the 100 GHz setup, are connectorised in this setup. The larger mode-spacing also causes less modes to overlap with the Raman gain curve and thus to experience less gain that partially upsets the loss due to the filter. As the filter is designed to be centred around the maximum of the Raman gain curve, a larger separation of the modes results in the modes being further away from the maximum of the Raman gain, translating to less gain. Lastly and most significantly, due to the larger mode separation, the dispersion difference, i.e. the linear phase-mismatch between the modes is larger. This is related to what we called the FWM threshold in section 3.1 and we also observe in the analytic model (see section 3.2.2), for a larger linear phase-mismatch the FWM threshold is higher and therefore less laser modes are created.

We achieved a further increase in the repetition rate of the laser by exchanging the FBG of the 100 GHz laser with a grating with an 500 GHz FSR (FBG 3 in appendix A). The Raman gain curve is only approximately symmetric, and the flat top of the curve is quite narrow. Due to the asymmetry of the Raman gain, two modes equally spaced around the gain maximum possibly experience different gain. The probability for this to happen increases with the separation between the modes. With the mode separation of approximately 4 nm in the case of the 500 GHz laser, the two central modes possibly experience significantly different gain depending on their respective position to the Raman gain curve, effectively preventing a proper mode-locked operation. However this can be compensated by adjusting the pump laser wavelength, i.e. shifting the centre of the Raman gain curve. It is possible to shift the curve to a position where both modes experience approximately the same gain, however especially at larger separations they will be significantly away from the peak of the gain.
curve. The gain will therefore be smaller which increases the lasing threshold and decreases the slope efficiency. In practice the wavelength where both central modes experience the same gain can easily be found, by simply monitoring the output of the laser on an optical spectrum analyser. To allow the tuning of the Raman gain curve, the 500 GHz experiment was carried out with the tunable RFL pump. The autocorrelation [figure 3.28(a)] taken at 2.96 W of pump power reveals pulses separated by approximately 2 ps, in agreement with the 500 GHz repetition rate. The pulse train resembles a sinusoidal beating, and there is no observable narrowing of the pulses. The strong autocorrelation background is another indication for the sinusoidal nature of the pulse train. Observing the FROG spectrogram [figure 3.28(b)], we notice the small number of spectral modes. Although there are more than just the two central modes, i.e. the pulse-train is not merely a sinusoidal beating, the energy of the higher-order modes is small and they do not have a strong effect. This can be explained by the same effects that were relevant for the 160 GHz laser. The smaller gain due to the mode spacing and the larger linear phase-mismatch have a detrimental effect on the spectral width of the laser, the power in the central modes is not high enough to overcome the FWM threshold for the creation of more higher-order modes. However this can be compensated by either using a stronger pump laser, reducing the cavity loses, or by using a fibre...
Numerical model

It should further be noted that other methods of modifying the FSR of the filter are possible. Zhang et al demonstrated tuning to different harmonics of the FSR of their FBG filter by taking advantage of polarization rotation in an Erbium-doped fibre laser, achieving a repetition rate of almost 700 GHz [147]. Another option is to use a tunable Fabry-Perot filter, which becomes more viable as tunable FBG FP-filters like the ones recently demonstrated [165] become more mature.

3.5 Numerical model

The QT model (section 3.2.1) assumes the length of the cavity to be short in order to justify the distribution of the actions along the cavity round trip, however at fibre lengths of several hundred meters this assumption becomes questionable. Furthermore some of the experimentally observed phenomena are not reflected by the numerical results of the QT model or the analytic analysis. In particular the background and subpulse structure in the autocorrelation traces and the large number of modes in the optical output spectra is not observed in the numerical simulations of the QT model. A novel model should therefore separate the different actions, such as the filter, nonlinearity and gain, and be able to account for the background of the autocorrelation traces. Additionally we would like to acquire information about the pulse shape of the laser, because the QT model and the analytic model only provide very limited information.

Starting from the Ginzburg-Landau type equation (equation 3.10) that was introduced in section 3.2.2 and introducing gain saturation yields the following equation:

\[
\frac{\partial}{\partial \zeta} U + i \kappa \frac{\partial^2}{\partial \tau^2} U - \sigma \frac{\partial^3}{\partial \tau^3} U = i |U|^2 U + \frac{G}{1 + \frac{Q}{I_S}} U - \frac{\alpha}{2} U
\] (3.33)

Where Q is the average power of the pulse train, I_S is the gain saturation parameter and \( \alpha \) is the cavity loss. The gain parameter saturation is not strictly necessary because with the dissipative bandwidth filter there already exist a cavity gain saturation mechanism, i.e. a
mechanism which limits the total gain per cavity roundtrip, however introducing the gain saturation into the equation provides a means to model the effects of pump depletion. In order to distinguish between the two saturation mechanisms we denote the saturation of the gain parameter gain saturation, while the saturation of the overall cavity gain is called cavity gain saturation. The cavity gain saturation is the combination of all gain and loss mechanisms over the cavity round trip, in which the saturation is mainly achieved by the transfer of energy to the dissipative higher-order modes. The transfer and thus the loss increases with cavity power, therefore saturating the unsaturated gain. The gain parameter saturation is a separate mechanism which saturates the propagation gain only if the average power increases too high. Experimentally this corresponds to a case where the fibre amplifier gain saturates. In case of a Raman amplifier the only saturation is caused by pump depletion, the saturation parameter thus provides a means of simulating the effect of pump depletion for the present laser. It should be pointed out that due to the normalization of the equation to the repetition rate $f_{\text{rep}}$ pulses will be separated by the normalized time 1. The parameter $f_{\text{rep}}$ thus corresponds to the FSR of the experimental Fabry-Perot filter. For sufficiently short pulse widths (< 1 ps) the spectral becomes so wide that the high frequency components of the spectrum amplify the low frequency components via stimulated Raman scattering (SRS) [34]. Because the experiments show pulse widths of around 700 fs and less (see section 3.4.1), a full treatment of the laser needs to include a term for intrapulse Raman scattering. Note that in the experiment there are two distinct Raman scattering effects; The Raman amplification of the laser that is caused by Raman scattering between the strong cw pump laser and the signal band, is modelled by the generic gain parameter in equation 3.33. The model does not reproduce the Raman nature of the amplification process. An inclusion of the pump to signal Raman scattering complicates the model considerably due to the necessity of modelling the pump band without adding any significant detail. In contrast intrapulse Raman scattering takes place within the signal band and thus does not complicate the numerical treatment. The above normalization applied to the nonlinear Schrödinger equation with Raman scattering (equation 1.4) results in the following equa-
tion:
\[
\frac{\partial}{\partial \zeta} U + i k \frac{\partial^2}{\partial \tau^2} U - \sigma \frac{\partial^3}{\partial \tau^3} U - i \left( (1 - f_R) |U|^2 U + f_R \int_{-\infty}^{\tau} \chi_R(\tau - \tau') |U(\tau')|^2 d\tau' \right) = \frac{G}{1 + \frac{Q}{2}} U - \alpha U,
\]

(3.34)

The same abbreviations as in equation 3.10 have been used. \(\chi_R(\tau)\) is the normalised delayed Raman response function. Its Fourier transform \(\tilde{\chi}_R(\nu)\) is the Raman susceptibility, normalised to one. Note that the Raman susceptibility curve is frequency dependent, we therefore need to assume an actual value for the repetition rate of the laser due to the normalization of the numerical frequency.

### 3.6 Numerical results

Equation 3.10 and 3.34 are typical propagation equations. There exists a well-known algorithm for solving these equations, the split-step Fourier method \([34]\), which was also used for the present simulations. The simulations are carried out with two loops, an inner loop simulating the propagation along the cavity, and a second outer loop over a set number of cavity round trips, in order to let the laser converge to a stable solution. Equation 3.33 does not account for the experimental filter, to complete the model, once every round trip the intracavity electric field is multiplied by a function that mimics the transfer function of the experimental FBG filter, thus resembling the experimental implementation more accurately than the QT model. Unless noted otherwise the filter function is the transmission of a Fabry-Perot filter multiplied with a Gaussian bandpass filter. The field was saved before multiplying with the filter, to reflect the position of the output coupler in front of the filter. After the filter the field is multiplied by a one off loss factor \(\sqrt{1 - l_{OC}}\) to simulate the effect of the output coupler that removes energy from the cavity. The number of simulation points was typically 4096. The discretisation was chosen in such a way, so that the width of the spectral filter is significantly smaller than the width of the simulation window, eliminating boundary effects. The convergence depends on the gain and loss of the laser. However a
stable solution was typically reached after approximately 200 to 300 cavity round trips if it existed. Appendix B contains a more detailed discussion of the numerical method employed in the simulations. Figure 3.29 depicts the numerically obtained spectrum and tem-

Figure 3.29 – (a) Spectrum and (b) temporal distribution and (c) autocorrelation of a typical simulation of the laser with parameters, $\kappa = -0.001$, $G = 0.9$, $I_S = 4.0$, $\alpha = 0.4$.

poral distribution and corresponding autocorrelation for a numerical simulation with parameters closely matching the experiments. Experimental parameters of $\beta_2 = -0.2$ ps$^2$/km, $L = 1000$ m, and a repetition rate of $f_{rep} = 100$ GHz calculate to a dispersion parameter $\kappa = -0.001$. Translating the gain parameter from the Raman amplification used in the experiment is not straight forward, we therefore have chosen a value of $G = 0.9$ which yields similar results to the experiment. Because we observe significant pump rejection in the
Numerical results

experiment we chose a high saturation parameter of $I_S = 4.0$ to simulate the undepleted pump regime. The FBG filter is modelled by a Fabry-Perot filter with a finesse of approximately 50 multiplied by a Gaussian with a FWHM of 2.5, closely matching the width of the individual reflection peaks and the envelope of the experimental filter. The coupler loss parameter is $l_{OC} = 0.25$ equivalent to a 20% output coupler plus some additional loss due to the laser components and connections which are concentrated around the filter. The numerical model accurately reproduces the large number of harmonics which are observed in the experiment. This is in stark contrast to the QT model which only results in one set of harmonics being created. The autocorrelation [figure 3.29(c)] also reveals a background similar to the background in the experimental autocorrelation traces, however the autocorrelation pulse width is significantly larger than in the experiment. By further examining the pulse train in the time domain, we can attribute the autocorrelation background to the relatively low duty-cycle of the pulses. The experimental results in contrast contradict such an interpretation due to the pulse width being significantly shorter. The origin of the experimental autocorrelation background will be investigated further in the later sections.

3.6.1 Pulse shape

Generally speaking the pulse shape of the laser is determined by pulse creation mechanisms. That includes cavity parameters such as the characteristics of the nonlinearity, the gain and propagation parameters such as dispersion. The mode-locking mechanism also plays a significant role, is the mode-locking active or passive, mode-locking by polarization rotation, using saturable absorption etc.. The QT model and the analytic model offer only limited insight into the output pulse shape. As the analytic model decomposes the image into four frequencies spaced symmetrically around the zero, the resulting pulse train will be described by the superposition of two sinusoidal waves. The QT model does not offer significantly more insight, because the number of frequency components is the same as in the analytic model due to the shape of the gain structure. The experimental autocorrelation trace closely resembles a Lorentzian shape, however the reason for this is not clear. One possible explanation is that it is caused by averaging over
noisy pulse trains, which will be discussed in more detail in chapter 4. Another possibility is that it is due to the influence of the spectral envelope of the spectral filter. Although this is unlikely, an investigation of the influence of the filter envelope on the pulse shape is warranted. The possible role of the spectral filter envelope can be understood by considering the Fourier relationship between the temporal and spectral domain. If the spectral filter determines or significantly influences the spectral field envelope it will also determine the pulse shape in the temporal domain. In particular in the case of a Gaussian-shaped filter, we expect Gaussian-shaped pulses, due to the fact that the Gaussian is a self-reciprocal function. In this section we numerically investigate the pulse shape of the laser and concentrate on three main factors; the reproduction of the experimental pulse shape, the influence of the filter envelope on the output pulses and the differences to the QT and analytic model due to the existence of higher harmonics in the spectrum. This allows us to establish the influence of the filter shape on the pulse shape, by determining the shape of pulses for a simulation with a Gaussian envelope of the Fabry-Perot filter. If the emitted pulses are non-Gaussian shaped, we can conclude that the envelope of the filter is not the determining factor.

Figure 3.30(a) depicts the same temporal distribution as shown in figure 3.29(a) but with the pulse phase added to the plot. We see a π phase jump in between the pulses in good agreement with the analytic considerations in section 3.2.2 and consistent with the results found by the QT model. To get an indication of the pulse shape, one pulse from the temporal distribution was fitted with four different functions; a squared Gaussian function, a \( \text{sech}^2 \) function, a squared Lorentzian function and a beating between four harmonics of a sinusoidal wave. The single pulse and the graphs of the four fits are shown in figure 3.30(b) on a logarithmic scale to ease comparison. The fitting error \( \epsilon \) was calculated by the normalized Euclidean distance

\[
\epsilon = \frac{1}{N} \left( \sum_{i=1}^{N} (|O_i|^2 - |F_i|^2)^2 \right)^{1/2},
\]

where \( O \) denotes the measured pulse \( F \) the fitted function and \( N \) the number of discretization points. The calculated errors are shown in table 3.1. The beating provides the best fit
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Figure 3.30 – (a) Temporal distribution (solid) and phase (dotted) after 300 round trips for anomalous dispersion and a Gaussian-shaped filter envelope. (b) Logarithmic plot of a single pulse of the distribution (solid) with a fit of a squared Gaussian function (dotted), a sech² function (dashed), a squared Lorentzian function (dashed-dotted) and the beating between four harmonics of a sinusoidal wave (grey).

<table>
<thead>
<tr>
<th>Filter</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon$</td>
<td>$A \exp \left[ -\frac{(x-x_0)^2}{2w_0^2} \right]$, $\frac{A}{\cosh \left[ \frac{v-x_0}{w_0} \right]}$, $\frac{A}{\pi \gamma \left[ 1 + \left( \frac{x-x_0}{\gamma} \right)^2 \right]}$, $\sum_{i=1}^{4} A_i \sin \left[ \omega_i \frac{x}{T} + \phi_i \right] \right] \right</td>
</tr>
<tr>
<td>Gaussian</td>
<td>$2.3 \cdot 10^{-3}$, $1.7 \cdot 10^{-3}$, $3.4 \cdot 10^{-3}$, $3.8 \cdot 10^{-4}$</td>
</tr>
<tr>
<td>hyperbolic secant</td>
<td>$1.9 \cdot 10^{-3}$, $1.8 \cdot 10^{-3}$, $3.4 \cdot 10^{-3}$, $4.0 \cdot 10^{-4}$</td>
</tr>
</tbody>
</table>

Table 3.1 – Error of the pulse fit functions for simulations with a Gaussian and hyperbolic secant filter envelope.

To the pulse envelope. This is not surprising, due to the fact that the temporal distribution is composed by a discrete number of harmonics in the spectral domain, thus it consists of a superposition of sinusoidal waves. The error of the three other functions is relatively close, however we can see from the graph, that all three functions differ mainly in the pulse tails from the original pulse shape. The sech² function is the closest match for the original pulse shape. However the relative small difference between the fits is not decisive enough to determine the impact of the filter envelope. We therefore performed a second simulation with the same parameters, except for a hyperbolic secant shaped filter. The corresponding errors are in table 3.1. We can see that they are very similar to the errors for the Gaussian-
Chapter 3  Mode-locking by dissipative four-wave mixing

shaped filter, in fact the difference between the sech$^2$ function and the Gaussian function fit did decrease. This gives substantial evidence that the shape of the laser pulses is mainly determined by the interplay between the propagation parameters, i.e. nonlinearity and dispersion and the filter shape does not have a significant impact on the pulse shape. The fact that the pulse shape is accurately described by only four harmonic waves demonstrates that the analytic model is a very good approximation of the laser.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.31}
\caption{(a) Temporal distribution (solid) and phase (dotted) after 300 round trips for normal dispersion and a Gaussian-shaped filter envelope. (b) A single pulse of the distribution (solid) fitted with a tanh$^2$ (dashed) function and a beating between four sinusoidal harmonics (grey). Note that the grey line from the fit of the beating almost fully overlaps the solid line of numerical distribution.}
\end{figure}

The analytic model and the QT model predict a train of dark soliton-like pulses if the cavity dispersion is normal, this was confirmed by Sylvestre et al. [141] who observed a train of dark solitonic waves with a laser operating in the normal dispersion regime. Similar to the above systems, the laser indeed emits a dark pulse train when the dispersion parameter is normal. Figure 3.31 clearly shows a train of dark pulses when the dispersion is $\kappa = 0.019$. The pulse shape is very well approximated by a tanh$^2$ function with an error of $\epsilon = 6.9 \cdot 10^{-4}$. Comparing the phase of the dark pulse train to the bright pulses, we see that the dark pulses exhibit the $\pi$ phase jump at the centre of the dark pulse, opposed to the bright pulses, where the phase jump was located in between pulses. The $\pi$ phase jump is a defining character-
Numerical results

istic of dark solitons propagating in normal dispersion fibre, supporting the evidence that the present output can be approximated by a dark soliton train.

To measure the dependence of the pulse shape on the cavity dispersion and in particular the transition from bright to dark pulses a number of simulations scanning the dispersion from the anomalous ($\kappa = -0.05$) to the normal ($\kappa = 0.05$) dispersion, were conducted. The pulses are distinguished by measuring the duty cycle of the pulse train, $D = \tau_{\text{HIGH}} / T$, where $T$ is the period of the pulse train and $\tau_{\text{HIGH}}$ is the on time of the pulses. Due to the shape of the pulses it is more convenient to redefine the duty cycle using the FWHM of the pulses, i.e. $D = \tau_{\text{FWHM}} / T$. The result is depicted in figure 3.32. Examining the region around the zero dispersion point the duty cycle exhibits a steep linear increase from around 20% to just under 80%. This suggests that the pulses undergo smooth transition from bright pulses to dark pulses with a stage of sinusoidal beating in between. The graph is point symmetric with respect to the origin of the graph. Observing the graph over the whole range of $\kappa$ we see that the duty cycle exhibits a minimum and maximum at $\kappa = -0.013$ and $\kappa = 0.013$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.32}
\caption{Duty cycle $D$ of the generated pulse train as a function of dispersion parameter $\kappa$.}
\end{figure}
respectively. For larger absolute values of the dispersion the duty cycle increases in the anomalous, and decreases in the normal dispersion. The two maxima correspond to the points where the dark and bright pulses possess a minimum pulse width.

A scan over a larger range of dispersion parameters [figure 3.33] reveals the behaviour of the pulse train for larger absolute values of the dispersion. We can see, that the transition observed in figure 3.32 is periodic, with a period of approximately $\Delta \kappa = 0.8$. The duty cycle undergoes a sudden change at values $N\Delta \kappa$ where $N$ is an integer number and a slow transition in between these values. Note that the transition appears more abrupt in figure 3.33.

![Figure 3.33](image)

**Figure 3.33** – Duty cycle of the generated pulse train as a function of dispersion parameter $\kappa$. The dashed square indicates the region shown in figure 3.32.

due to the limited resolution compared to figure 3.32. As we observe both maxima and minima for negative and positive values of the cavity dispersion means that we are encountering bright and dark solitonic waves in either normal and anomalous dispersion regions. The fact that one encounters bright soliton-like pulses in the normal dispersion regime, and
dark pulses in the anomalous dispersion regime is rather surprising, because the nonlinear Schrödinger equation which describes light propagation in optical fibres does not support these solutions. However the presence of a laser cavity with dissipation and gain fundamentally changes the system. Instead of the NLSE the laser is governed by a Ginzburg-Landau equation with gain and dissipation, which is well known to exhibit various kinds of soliton solutions, that are unstable in the NLSE [121,174–176]. Observing the periodic transitions of the pulse shape is experimentally challenging. \( \kappa \) depends on the repetition rate the length of the fibre and the group-velocity dispersion (GVD) a change in either of these parameters therefore corresponds to a change in \( \kappa \). However presuming the parameters of the experiment, i.e. \( L = 1000 \text{ m}, f_{\text{rep}} = 100 \text{ GHz} \) and \( \beta_2 \approx 0.1 \text{ps}^2/\text{km} \), means one would need to change the dispersion to a value of \( 160 \text{ ps}^2/\text{km} \) or the length to more than \( 15000 \text{ km} \) to achieve a variation in dispersion by \( \kappa \approx \pm 0.8 \), provided that the other parameters are kept constant. This is experimentally very difficult. We have performed experimental measurements using a fibre cut-back method to change the dispersion in order to observe the transition at \( \kappa = 0 \). Although we did observe a change in the pulse shape in the experiments, the results are inconclusive due to large errors and we were not able to see the transition from bright to dark pulses. We have therefore chosen to omit these results. However it remains very desirable to demonstrate this transition, especially at non-zero dispersion.

### 3.6.2 Gain variation

It is easy to understand that the gain has a significant impact on the laser system. If the gain parameter is too small, every round trip more energy will be lost than gained, and the laser will not reach the lasing threshold. If the round trip gain is large enough that the laser surpasses the threshold, the power will increase every cavity round trip. However the cavity losses are power dependent due to the fact that more energy is transferred to the dissipative higher-order modes. The laser thus attains a steady state where round trip loss and gain are exactly balanced. Yet the analytic model predicts that if the ratio of loss to gain parameter is too small the system becomes unstable as can be seen from the phase trajectory in figure 3.11. Two factors exacerbate the understanding of the effect of increased
gain on the full laser system. Firstly, it is difficult to quantify the loss mechanism of the laser. An increased gain leads to larger four-wave mixing and thus to stronger dissipation, however the limitations to this effect are not known. Secondly the separation of gain and loss. Unlike in the QT and the analytic model the gain and loss are not evenly distributed along the cavity. The gain is achieved along the fibre propagation, thus it is almost evenly distributed over the cavity. However the main loss occurs at the filter, thus at a specific point once every cavity round trip. It is not quite clear if this introduces additional instabilities into the cavity.

In this section we will investigate the dynamics of the laser system with respect to the gain parameter and compare the numerical results with the analytic predictions and the experimental data. In the experiment the equivalent action to increasing the gain parameter is to increase the pump power, because the small signal gain parameter in Raman amplification is proportional to the pump power. We will discuss simulations of the numerical model for varying values of the gain parameter. All other parameters have been kept constant at the same values described in the previous sections, except for the dispersion which was changed slightly to $\kappa = -0.002$ and the finesse of the Fabry-Perot filter which was increased so that only a single mode inside each filter band experiences any gain. This eliminates possible supermode noise effects which will be discussed in detail later in chapter 4. For an easier comparison with the experimental power inside the cavity and the involved losses, the output coupler loss parameters $l_{OC}$ was split into two parameters $l_{OC}$ as a parameter for the output coupler only plus an additional parameter to model the component loss $l_C$. In all the following simulations $l_{OC} = 0.2$ and $l_C = 0.05$.

Figure 3.34 depicts the average power and the width of the autocorrelation pulse as a function of the gain parameter. The normalized values for both width and power have been translated to non-normalized variables using the experimental parameters for easier comparison with the experiment. Experimentally we observed a linear dependency of the average output power on the pump power. The simulations [figure 3.34(a)] nicely reproduce this linear relationship between average output power and gain. However there is a significant discrepancy between the experimentally and the numerically observed powers, the
values from the simulations are smaller by about a factor of 10. Similarly the slope efficiency differs between experiment and the numerical results, however this difference is not very surprising because the proportionality factor between the gain parameter and the pump power is not known. The difference between the output powers has a number of possible explanations. Apart from the uncertainty of the fibre parameters which possibly accounts for a factor of two, there is a cw wave contribution due to Raman ASE which is unaccounted for in the simulations. Furthermore there are additional factors like Rayleigh backscattering which contribute to the differences as well. Nevertheless the difference cannot be explained by a higher maximum gain in the experiment, as the pulses train becomes very irregular for very high gains as we will show later. This is not seen in the experimental autocorrelation traces. The width of the autocorrelation also does not agree with the experimentally observed behaviour, although initially the width decreases we observe large fluctuations for higher gain values. We will return to the pulse width in chapter 4, where we will discuss a possible explanation for the fluctuations and the discrepancy to the experimentally observed behaviour.

Figure 3.34 – (a) Average power and (b) central pulse width of the autocorrelation as a function of gain parameter. The normalized power and width have been translated back non-normalized values for easier comparison to the experiment using the experimental parameters $\gamma = 14 \text{ W}^{-1}\text{km}^{-1}$, $L = 1000 \text{ m}$ and $f_{rep} = 100 \text{ GHz}$. 
In the following we discuss the simulations at the different gain parameters in more detail, to gain a better understanding of the laser dynamics. To visualise the dynamics, we utilize surface plots of the evolution of the temporal distribution and phase space trajectories as described in section 3.2.2. Let us first investigate the pulse formation for the laser in the steady state condition, i.e. for relatively low gain parameters. Figure 3.35(a)-(d) illustrate the dynamics and pulse evolution of the laser system over 250 round trips for two gain parameters. The system is in steady state after 250 round trips for both simulations, although the evolution to the state is significantly different. When the gain parameter $G = 0.7$ [figure 3.35(a) and (c)] is just above the lasing threshold we see the laser undergoing very limited formation dynamics; the phase space trajectory displays only a short motion before it reaches a fix point. The graph of the pulse evolution displays an exponential growth of the pulses starting at about 170 round trips before the laser reaches the steady state regime after approximately 220 round trips. When the gain is increased to $G = 0.92$ the dynamics change. The phase trajectory [figure 3.35(c)] reveals a long spiral towards the stable fixed point. A circular motion in phase space is the signature of oscillations in time space, and indeed the pulse evolution [figure 3.35(d)] shows that the pulses undergo relaxation oscillations. Although this graph only displays the pulse evolution up to 250 round trips, no variation in the pulses has been observed up to 500 round trips. It is therefore safe to conclude that the laser is in steady state.

If the gain is increased further a second regime of pulse dynamics can be observed. Figure 3.36 depicts the results for simulations with gain parameters $G = 1.0$ and $G = 1.15$. For $G = 1.0$ the system initially approaches a fixed point, but the trajectory changes and converges into a limit cycle. A limit cycle implies sustained oscillations, which can be observed in the evolution of the temporal distribution in figure 3.36(b). The figure reveals oscillations of both the pulse peak power as well as the peak position of the pulses. The oscillations have been checked to persist for at least 500 round trips. The dynamics complicate further for $G = 1.15$ [figure 3.36(c)]. Although the system possesses a limit set, the nature of the attractor cannot easily be determined. Observing the temporal distribution we see that it displays a pseudo-periodic behaviour, i.e. the pulse train appears to be periodic but never
Figure 3.35 – Phase space trajectory [(a) and (c)] and pulse evolution [(b) and (d)] for simulations with gain parameters $G = 0.7$ and $G = 0.92$ respectively.

exactly repeats itself. Calculating the autocorrelation of the pulse train after 250 round trips for the simulation with gain $G = 1.15$ [figure 3.37(a)] we observe a subpulse structure similar to the one observed in experiments. The temporal distribution of the field intensity [figure 3.37(b)] reveals that the subpulse structure can be attributed to the formation of subpulses in between the main pulse train. From these results we can conclude that the experimental results obtained at above 1.5 W pump power correspond to gain parameters above $G = 1.0$. This means that for part of the experimental results the laser does not emit a single pulse train, but the train contains subpulses. However the system can be easily changed back to the steady state regime by increasing the cavity losses, i.e. by using a larger
output coupling ratio. The additional benefit is, that the maximum output power is increased. The phase space trajectory and temporal evolution for a simulation with $G = 1.1$ but a larger output coupling ratio ($\ell_{OC} = 0.5$) are displayed in figure 3.38. The pulse train exhibits similar behaviour to the simulation with a gain of $G = 0.92$ and converges to the steady state regime.

At very high gains [$G = 2.0$, figure 3.39], the system behaviour becomes completely erratic. The temporal distribution appears random, i.e. there is not distinguishable relationship between the outputs for subsequent round trips. The phase space does not have an attractor and the system appears to be in a chaotic state. Note that we are using the term chaotic in
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![Numerical results](image)

**Figure 3.37** – (a) Autocorrelation and (b) temporal distribution of the laser after 250 round trips for a gain parameter $G = 1.15$.

![Numerical results](image)

(a) Phase space trajectory high output coupling ratio  
(b) Pulse evolution high output coupling ratio

**Figure 3.38** – (a) Phase space trajectory and Pulse evolution for a simulation with gain $G = 1.15$ and higher output coupling ratio $\ell_{OC} = 0.5$.

In a loose sense, no complete analysis of the system dynamics were carried out to determine if the system is chaotic in the strict mathematical sense.

Ignoring the case where the gain is below the lasing threshold we can distinguish three different propagation regimes: The steady state where the system converges into a stable fix point solution, the limit set regime where the system appears to possess an attractor and the chaotic regime where no attractor exists. The above raises the question about the transition
between the different propagation regimes. Considering the gain as our governing system parameter we see a significant change in the phase space from one regime to the next. In the theory of dynamical systems such an abrupt change in the phase space is denoted a bifurcation [163], with the gain G being the bifurcation parameter. A bifurcation can thus be described as a sudden appearance of a qualitative change of the system dynamics. It is usually accompanied by the disappearance of the systems attractors and the appearance of new attractors. Bifurcations can be identified into different families according to their patterns, in particular the different dynamical regimes they are connecting (see [163] for an introduction into bifurcation theory).

The first bifurcation occurs between the steady state regime and the periodic or quasi-periodic regime. The steady state is characterised by a fix-point in phase space, while the periodic and to a limited degree the quasi-periodic regime exhibit a limit cycle in phase space. A transition where a limit cycle develops from an equilibrium fix-point is denoted a Hopf-bifurcation [163]. If the resulting limit cycle is stable, i.e. the phase space trajectories converge to the limit cycle, the bifurcation is a supercritical Hopf-bifurcation. If the limit cycle is unstable, i.e. the trajectories diverge from away from the limit cycle it is denoted a subcritical Hopf-bifurcation. Examining the phase space trajectory of figure 3.36(a) we see that the trajectory in fact converges to the limit cycle. The bifurcation is therefore a super-

![Phase trajectory G = 2.0](image1)

![Pulse evolution G = 2.0](image2)

**Figure 3.39** – (a) Phase space trajectory and (b) pulse evolution for a simulation with gain G = 2.0
critical Hopf-bifurcation. It should be noted that the quasi-periodic state can possibly be also considered a new different state of the dynamic system.

The remaining dynamics are not easily categorised. The system undergoes one or two further bifurcations into a chaotic state. The appearance of the subpulses indicates a further bifurcation similar to a period doubling bifurcation [163]. The transition into the chaotic regime is almost certainly accompanied by an additional bifurcation as well. However the categorisation of these bifurcations is beyond the scope of this thesis, as they lie outside the region of interest for a stable laser operation.

The numerical investigation of the variation of the laser dynamics with changing gain parameter shows good qualitative agreement with the experimentally obtained results. The linear relationship between the pump power and the output power observed in section 3.4.3 is reproduced by the numerical simulations. However the simulations fail to quantitatively reproduce the results and the experimentally observed exponentially decreasing pulse width could not be observed in the simulations either. The detailed discussion of the numerical simulations of the system with different pulse parameters reveals the laser operating in different states. The states differ significantly in the dynamical characteristics and undergo an abrupt change in stability when the gain parameter is increased beyond a threshold value. The transitions conform to the definitions of a bifurcation and the first bifurcation from the stable equilibrium state to a oscillatory limit set, was identified as a Hopf-bifurcation. Additionally the numerical model predicts the subpulse structure observed in the experimental autocorrelation traces. They can be attributed to an appearance of subpulses in the temporal distribution, however the numerical simulations show that an increased output coupling ratio changes the state of the laser back to the steady state, while offering the additional benefit of an increased average output power. It should be noted that using a larger output coupling ratio also increases the lasing threshold, however this is of limited importance, as the change is only necessary for large pump powers where the laser is above threshold even for the increased output coupling.
### 3.6.3 Third-order dispersion and Raman

Up until now, we have neglected any effects of third-order dispersion and stimulated intrapulse Raman scattering in the treatment of the laser. The theoretical model presented by Quiroga-Teixeiro et al [39] also does not include these effects. However, if the laser is operating very close to the zero-dispersion wavelength, higher-order dispersion terms have an increasing influence and the inclusion of third-order dispersion into the model becomes essential. Similarly if the pulse width approaches a picosecond, the corresponding optical spectrum becomes so wide that intrapulse Raman scattering can have a significant effect on the pulse propagation. It is important to note that we are discussing the effect of intrapulse Raman scattering, i.e. Raman scattering within the signal band that transfers energy from the blue to the red side of the pulse spectrum. The Raman nature of the amplification process, i.e. the Raman scattering from the pump to the signal band that is present in the experiment is still being modelled by the gain parameter.

#### Third-order dispersion

Once the central wavelength of a pulse spectrum approaches the zero-dispersion wavelength, i.e. when the magnitude of second- and third-order dispersion become comparable, a description has to include the third-order dispersion. One of the best known consequences of third-order dispersion is a red-shift similar to the so-called soliton self-frequency shift [177], caused by SRS. A soliton pulse in the anomalous dispersion regime close to the zero-dispersion wavelength slowly shifts to higher wavelengths upon propagation, and at the same time part of its energy is transferred to a so-called dispersive wave located in the normal dispersion regime, with the position of the dispersive wave determined by phase-matching (see e.g. [178, 179] for a detailed discussion). The effect is similar to Cherenkov radiation in nonlinear optics [178], and has therefore also been denoted soliton Cherenkov radiation. The direction of the soliton shift depends on the sign of $\beta_3$, however except for some exotic photonic crystal fibres, $\beta_3$ is always positive causing the soliton to shift to higher wavelengths. As the present experiment operates within 5 nm of the zero-dispersion
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wavelength, the numerical treatment needs to include an investigation of the effects of third-order dispersion on the laser operation.

\[ \kappa = 0.01 \]

\[ \kappa = -0.01 \]

**Figure 3.40** – Color plots of the pulse evolution with third-order dispersion in the (a) normal dispersion regime \((\kappa = 0.01, \sigma = 8 \cdot 10^{-6})\) and (b) anomalous dispersion regime \((\kappa = -0.01, \sigma = 8 \cdot 10^{-6})\).

Figure 3.40(a) and (b) displays the evolution of the temporal distribution for a train of dark and bright solitons for slightly normal \((\kappa = 0.01)\) and anomalous \((\kappa = -0.01)\) dispersion. The third-order dispersion parameter \(\sigma = 8 \cdot 10^{-6}\) is still some orders of magnitude smaller than the second order dispersion, however it is close to the parameter calculated from the experimental setup. Although the general behaviour of both pulse trains is similar, there are some significant differences. After the initial formation of the pulses, both trains experience a shift with respect to the propagation time frame, however the two pulse trains are shifted in opposite directions. The bright pulses are delayed with respect to the reference frame, while the dark pulses are accelerated. The delay of the bright pulses is well known and directly related to the red-shift due to the Cherenkov radiation, in fact this phenomenon is often characterised by the decrease in the soliton group velocity [178]. The acceleration of the dark soliton pulse train, although not quite as well known, as also been the subject of some research [180–183]. However it should be stressed that the present system is modelled by a Ginzburg-Landau equation with gain, in contrast to the propagation regime modelled.
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by a NLSE where these effects have first been discovered and which does not include any dissipation or gain. Examining the spectra for normal [figure 3.41(a)] and anomalous [figure 3.41(b)] dispersion cavities, there are a number of observations to make. Note that the third-order dispersion in these simulations with $\sigma = 8 \cdot 10^{-5}$ is an order of magnitude larger than in figure 3.40, enhancing the effects so that they are easier to distinguish in the spectra. The spectrum of the bright pulses is significantly wider than the dark pulse spectrum, probably because the system is in a different state due to the different dispersion parameter. However we would like to draw attention to the two central modes around the zero frequency point seen in the inset of figure 3.41(a) and (b). In the case of bright pulse, i.e. an anomalous dispersion inside the laser cavity, the low frequency central mode has a higher power than the high frequency mode. This shift in energy to low frequencies, for bright pulses can be interpreted as a manifestation of the self-frequency shift, or soliton red-shift described above. Additionally we observe the tail of the spectrum to be wider on the high frequency side than the low frequency side, suggesting a transfer of energy from the low to high frequency components. This is an effect similar to the dispersive wave generation, i.e. the so-called Cherenkov radiation [178]. It should be noted that the frequency of the dispersive wave can not be calculated by the method stipulated in [178]. The phase match-
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ing condition is significantly changed by the presence of gain and dissipation, which are not accounted for in the theory of Akhmediev et al.. The exact opposite is true for dark pulses in the normal dispersion regime. Comparing the two central spectral modes, we note that the higher frequency mode is larger than the lower frequency mode. We also see a broadening of the spectral tails, however again the broadening is occurring into the opposite direction as in the case of bright pulses, to low frequencies. These results show that the dark pulses experience a blue-shift while shedding energy through radiation on the red side of the spectrum. In the literature the blue-shift in dark solitons is always attributed to Raman scattering, and although there has been some research on dark solitons perturbed by third-order dispersion and the results show radiation, the authors do not mention a blue-shift of the dark solitons.

Intrapulse Raman scattering

Considering a transform-limited sech$^2$ pulse with a width of around 1 ps in the time domain, the width of the spectrum in the frequency domain is about 0.5 THz. For shorter pulses or if the pulse is not transform-limited the spectral width is even higher. Once the spectral width of a pulse or pulse train reaches approximately one THz, intrapulse SRS becomes important. Intrapulse SRS occurs when the low frequency side of a pulse spectrum overlaps with the Raman spectrum of the high frequency side of the spectrum. In such a case SRS takes place between the low and high frequencies of the spectrum, i.e. the low frequency side of a pulse spectrum is amplified by the high frequencies via Raman scattering. The aforementioned soliton self-frequency shift, is often incorrectly attributed to only the effect of self-Raman scattering. However in the case of bright solitons both the frequency shift due to Raman scattering and the shift due to third-order dispersion act in the same direction if $\beta_3 > 0$ which is the case for almost all experimental conditions. Thus the combination of both enhances the soltion self-frequency shift.

Raman scattering delays the pulses with respect to the propagation timeframe, in both the normal and anomalous dispersion regime. The magnitude of the delay is also very similar for dark and bright pulses, as can be seen in figure 3.42. If the frequency shift due to Ra-
man scattering and frequency shift due to third-order dispersion can be engineered to act in different directions the shifts can cancel each other. Skryabin et al showed that it is possible to design a photonic crystal fibre such that $\beta_3 < 0$ [184]. In the anomalous dispersion regime, a negative third-order dispersion causes a blue-shift for bright solitons, opposed to the red-shift caused by Raman scattering. The authors demonstrated that the two effects can compensate each other stabilizing the soliton position. In the time domain, we observe the Raman process delaying the pulses and the third-order dispersion shift accelerating them, i.e. Raman scattering causing a continuous decrease of the group velocity and the third-order dispersion an increase. The resulting pulse train exhibits a constant group velocity as the decrease and increase cancel each other. Here we have a similar situation. In a normal dispersion cavity, Raman scattering causes a decrease of the dark pulses group velocity, while third-order dispersion accelerates the pulses. By adjusting the relative magnitude it is possible for both processes to cancel each other. Figure 3.43 shows a colour plot for the pulse evolution with the same parameters as figure 3.42(a) including a third-order dispersion parameter of $\sigma = 8 \cdot 10^{-6}$. Indeed the delay observed in figure 3.42(a) is almost fully compensated by the acceleration caused by third-order dispersion. It should be noted

\begin{figure}
\centering
\includegraphics[width=\textwidth]{Fig3_42.png}
\caption{Color plots of the pulse evolution with Raman scattering in the (a) normal dispersion regime ($\kappa = 0.01$) and (b) anomalous dispersion regime ($\kappa = -0.014$).}
\end{figure}
however, that the result is only of qualitative nature. A calculation of the exact parameters, necessary for the two effects to compensate each other, requires a detailed theoretical investigation of dark and bright pulse propagation inside the laser. However this is beyond the scope of this thesis.

Although the moving pulse train solutions both in the case of third-order dispersion and self-Raman are similar to the ASYM states in the Quiroga-Teixeiro model, the origins are fundamentally different. The ASYM moving pulse solution in the Quiroga-Teixeiro model are caused by a stable solution in which the modes are asymmetrically positioned around the frequency origin. Thus the $+\text{ and } -$ mode have slightly different frequencies, causing moving pulses in the time domain. In the numerical model this solution is highly unlikely, as the Fabry-Perot part of the filter is aligned with the discretization, i.e. there is always a discretization point at the maxima of the Fabry-Perot transmission function, which is symmetrical around the origin. Notwithstanding self-Raman and third-order dispersion cause
Chapter 3  Mode-locking by dissipative four-wave mixing

a power imbalance between the central modes of the laser, by either effecting an energy-transfer from high to low frequencies or vice versa. So the central modes of the laser are asymmetric in power in contrast to the ASYM solution of the QT model, which is asymmetric in frequency.

The experimental implication of the moving pulses is small. The laser emits a continuous train of pulses, the change in group velocity can therefore not be observed directly. However, the effect of third-order dispersion and intrapulse Raman scattering is observable in the output spectrum, as the spectra displays a broadening to higher wavelengths. The numerical investigations in this section show that this broadening is caused by intrapulse Raman scattering or a combination of intrapulse Raman scattering and third-order dispersion.

3.7  Summary and discussion

Mode-locking by dissipative four-wave mixing is an intriguing technique to overcome the limitations of more traditional mode-locking techniques. We have shown that in combination with Raman amplification as the gain mechanism, it is possible to accomplish average output powers close to one Watt at repetition rates over 100 GHz. The QT model and the analytic treatment provide accurate qualitative descriptions of the laser dynamics. The analytic model predicts the laser to operate in the steady state if the dissipation from the system is large enough so that no instabilities develop. It also gives valuable insight into the phase matching conditions for the laser. Although four-wave mixing is the principle mechanism for creating the higher-order modes which create the mode-locked condition, the laser is not reliant on a specific phase-matching condition. Indeed in the stable parameter region, the laser converges into a state where the phase-matching condition is met automatically. The analytic model also tells us that there are three different mixing mechanism involved in the mode-locking, two degenerate four-wave mixing processes where two pump photons are annihilated and one photon is created at the harmonic wave and the neighbouring pump wave respectively, and a non-degenerate four-wave mixing process annihilating a photon of each pump wave and creating a photon at each of the harmonics.
However the two models fail to account for a number of experimentally observed phenomena. A novel numerical model was thus developed to be able to achieve a more accurate, quantitative and qualitative description of the laser and the involved processes. The new model takes into account the non-distributed nature of the different actions along the cavity, i.e. the fact that the gain and nonlinearity act over most of the cavity length while the filter or dissipative element only acts at a discrete point inside the cavity. It offers the ability to take higher-order dispersion and intrapulse SRS into account and thus reproduces the asymmetric widening of the experimental optical spectrum to longer wavelengths. It also reproduces the experimentally observed linear relationship between the pump power and the average optical output power. It does not however give accurate quantitative predictions of the output power of the laser. Although the discrepancy could be explained by a significantly larger gain in the experiment, this interpretation is falsified by the numerical simulations which predict chaotic behaviour of the laser for very large gain parameters, which is not observed in the experiment. Nevertheless there is some evidence for the onset of instabilities in the experiments, as a small subpulse-like structure appears in the autocorrelation traces for larger pump powers. The numerical investigation of the laser dynamics with varying gain indicates that the structure appears in a periodic or quasi-periodic regime of the laser. In this regime the laser output still consists of a train of pulses, however small subpulses form in between the pulses and pulses and subpulses move independently upon propagation. Although it appears to be similar to the supermode noise effects that will be discussed in the following chapter, it is important to stress that the effect is distinctively different and care has been taken that the results were not effected by supermode noise. Although the numerical simulations suggest that the laser operates in a semi stable regime for larger pump powers, this can easily be overcome by introducing an additional loss into the cavity such as increasing the output coupling ratio. The additional advantage of an increased output coupling ratio is an increased slope efficiency of the laser. Although the lasing threshold increases simultaneously, the increase in slope efficiency results in a larger maximum output power, as shown experimentally. Using a 70% output coupling ratio, we could achieve a maximum output power of almost 1 W, the highest average output power
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for an ultrahigh repetition rate laser experimentally reported so far.

The numerical investigation of the effect of gain variation provides some interesting insights into the laser dynamics. The examination of the phase space trajectories of the laser at different values of the gain parameter reveals at least three different stability regimes of the laser. The steady state regime where the laser emits a continuous train of pulses is characterised by a stable fix point attractor in phase space. When the gain parameter is increased the system undergoes a supercritical Hopf-bifurcation into a periodic regime identified by a stable limit-cycle in the phase plane. This changes into a quasi-periodic regime, which manifests itself by a “varying” limit cycle in phase space. It is not quite clear if the system undergoes a bifurcation when transitioning between the limit cycle and the “changing” limit cycle. When the gain parameter is increased further, the system becomes chaotic and no attractors can be identified in phase space trajectories. Evidence of these instabilities are already partially present in the analytic treatment of the laser, where a bifurcation from a fix point to a limit cycle attractor can be observed. This examination shows the laser system to provide interesting dynamics which are experimentally challenging but rewarding to investigate.

Similar to both the analytic and the QT model, the numerical model predicts the existence of bright pulses in the anomalous and dark pulses in the normal dispersion regime. In contrast to the above models, dark pulses also exist for anomalous and bright pulses for normal dispersion. The pulses undergo a periodic variation from bright to dark and back to bright pulses when the dispersion parameter is changed. This result is rather surprising as it seems to contradict the known solutions to the NLSE. However it is important to remember that the DFWM laser is not described by the NLSE but by a Ginzburg-Landau type equation which is known to exhibit various types of solitons in both the normal and anomalous dispersion regime. The exact pulse shape was also found to vary when changing the dispersion parameter as demonstrated by the dependence of the duty cycle on the dispersion parameter. The experimental variation of the dispersion with a fibre cut-back method, did not demonstrate the existence of dark pulses in neither the normal nor anomalous dispersion regime and could not reproduce the transition between bright and dark pulses observed
numerically, due to various possible reasons. We have therefore omitted these results, however observing this effect is still very interesting and should be part of any future investigation of this laser.

Although the numerical model accurately reproduces a number of experimentally observed effects, it fails to reproduce some of the experimental findings. The experimental autocorrelation trace shows the pulses to exhibit a Lorentzian shaped envelope, opposed to the numerical results which never show the pulse shape to be best approximated by a Lorentzian shape. The Lorentzian function is well-known to be the probability density function of the Cauchy or Lorentzian distribution. A possible interpretation of the Lorentzian shape is that it is caused by averaging over the correlated supermode noise which is present in experiment but not accounted for in the simulations. This hypothesis is further investigated in chapter 4. The numerical simulations not only differ from the experiment with respect to the pulse shape, but also with respect to pulse width. Although the numerically predicted values agree quite well with the experimentally measured pulse width, the behaviour with respect to gain variation is significantly different. In the experimental results we see an exponential decrease of the autocorrelation pulse width approaching a minimum of 1 ps. The plot of the numerically obtained pulse width over the gain parameter, does not show the exponential relationship. For higher gain values the pulse widths vary considerably and seemingly randomly. This again can be attributed to the presence of supermode noise in the experiment not taken into account for the numerical simulations. The influence of supermode noise on the laser dynamics will be discussed in the following chapter and the effect on the pulse width and shape is examined in detail.

There are some aspects of the experimental laser which can not be investigated numerically due to the limitations of the model. The numerical model does not specifically account for the Raman nature of the amplification, because a full treatment of pump to signal SRS would need to include the pump band thus increasing computation time significantly. However the main effects; pump depletion and non-symmetry of the gain curve can be accounted for without specifically simulating the pump band, and simulations did not provide any novel insights into the laser. Furthermore, because of the chosen normalization to
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the repetition rate of the laser, the model cannot give information over the possible bandwidth, i.e. the achievable repetition rate. We experimentally demonstrated mode-locked operation at repetition rates of 100, 160 and 500 GHz. The maximum obtainable repetition rate is generally determined by the bandwidth of the gain medium. As the width of the Raman gain is significantly larger than 500 GHz, it should in principle be possible to achieve even higher repetition rates, if a filter with a larger FSR is used.

One of the more astounding results is the considerable difference in the laser output between pumping the laser in a backward or forward configuration. When the pump light is travelling in the same direction as the signal light, we observe a large increase in the ASE noise generated by Raman scattering. The spectrum of the forward pump laser only reveals a very small number of laser modes, compared to the very large amount which is observable when the laser is pumped in the opposite direction. The autocorrelation also reveals the pulse quality to be significantly lower in the forward pumping configuration. While the difference between the peak power of the autocorrelation pulse compared to the crosscorrelation pulses is quite small in the case of backward pumping, the crosscorrelation peak power for the forward pumped autocorrelation is only half of the peak power of the autocorrelation, indicating the pulses to be relatively incoherent with respect to each other. Additionally the forward pumped laser experiences an asymptotic limit to the average output power. After the output power reaches a certain level, a further increase in pump power does not yield an increase in output power. This is in stark contrast to the case of backward pumping, where the output power depends linearly on the pump with no observable saturation effects over the whole range of available pump powers. The exact mechanisms causing the significant differences in generated ASE noise for forward or backward Raman amplification are not clear. Although there are some studies about the effect of the direction of Raman amplification on signal quality, these studies did not cover the topic of ASE. The topic warrants further investigation, it is however beyond the scope of this thesis.

Finally the relatively large background in the autocorrelation traces of around 30% as well as the envelope over the autocorrelation needs to be addressed. The background can be explained by the onset of laser instabilities and the presence of subpulses within the pulse.
train for high pump powers, or by the overlap between pulses when the pump power is lower. Part of the background can also be attributed to supermode noise which is also the cause for the autocorrelation envelope. Supermode noise, i.e. the presence of many cavity-modes within one reflection peak of the FBG filter will be discussed in the next chapter. Supermode noise is the main limiting factor to the quality of the train of pulses generated by the laser. It causes both frequency and intensity fluctuations. In the following chapter we will present a numerical technique to simulate the impact of many supermodes within the laser, as well as experimental measurements for a more quantitative insight into the noise of the laser. We also demonstrate a technique to reduce the amount of supermodes in the laser.
In the previous chapter we have introduced dissipative four-wave mixing as a mode-locking technique. The experimental and numerical results show that this technique in combination with Raman amplification can produce ultra-high repetition rate lasers at very large average output powers. In this chapter we will discuss the main difficulties of the laser, the existence of supermode noise.

4.1 Introduction and background

For a typical fibre laser to attain sufficient gain to overcome the cavity losses, the length of the active medium usually has to be several meters long. Ytterbium-doped fibre lasers which currently provide the highest gain per metre of cavity length, are still rarely shorter than a metre, although some fibre lasers with cavity lengths of centimetres or even millime-
tres have been reported [134,140]. Even a laser with a cavity length of one centimetre has an approximate resonance frequency of 2 GHz (recall that the resonance frequency is the inverse of the cavity round trip time and therefore inversely proportional to the cavity length, \( f_{\text{res}} = \frac{c}{nL} \) for a ring cavity or \( f_{\text{res}} = \frac{c}{2nL} \) for a linear cavity). Thus, to reach repetition rates in the order of several tens or hundreds of GHz and higher, fibre lasers have to be mode-locked at harmonics of the cavity resonance frequency. Harmonically mode-locked lasers [185], i.e. lasers operating at a multiple of their natural repetition rate or cavity resonance frequency, are known to suffer from a phenomenon called supermode noise. Becker et al. first discovered this effect in a bulk harmonically mode-locked Nd:YAG laser in 1972 [40]. To understand the effect of supermode noise, it is first necessary to understand what is meant by a supermode.

Consider a typical spectrum of a pulsed laser, i.e. a number of modes separated by the cavity resonance frequency with for example a Gaussian type envelope. When harmonically mode-locking the laser, we force the laser to operate at an integer multiple \( N \) of the cavity repetition rate \( \frac{c}{nL} \), for example by using an amplitude or phase modulator. For simplicity’s sake let us confine the discussion to the case of \( N = 2 \), i.e. mode-locking a laser at twice the natural frequency. The principle of harmonic mode-locking is depicted in figure 4.1. When the laser is mode-locked at the second harmonic of its natural repetition rate, the new spectrum is composed of every second mode of the original spectrum. However both all the odd, or all the even modes of the spectrum can form the desired spectrum with a mode separation of \( 2 \frac{c}{nL} \), i.e. the solution is degenerate. Becker et al. named the possible combinations the hypermodes of the laser, however since then the term supermodes has been adopted to denote the phenomenon [41, 153, 186].

When mode-locking at higher harmonics the number of supermodes increases accordingly, thus a harmonically mode-locked laser operating at the 100th harmonic has 100 possible supermodes. Because all supermodes satisfy the mode-locking condition equivalently, they possibly all oscillate simultaneously. This ambiguity between the supermodes results in competition and mode-jumping. In the case of harmonically, actively mode-locked lasers the presence of a number of supermodes usually manifests itself in additional peaks at mul-
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Figure 4.1 – Principle of harmonic mode-locking, if the laser is mode-locked at twice the natural frequency (2nd harmonic) we see an ambiguity between the sets of modes, i.e. the laser can be compromised of either the odd or even natural modes. The two sets are denoted supermodes.

tiples of the cavity resonance frequency in the experimentally observed power spectrum caused by the beating between the supermodes [41, 187]. The effect is often called supermode noise [41, 153, 188] and has a detrimental effect on the laser operation by causing pulse amplitude fluctuations and timing jitter. Its impact becomes increasingly important when raising the repetition rate of the laser by working at very high harmonics. For these reasons, the effects of supermode noise have been extensively studied in high harmonically, actively mode-locked lasers [41, 186, 187, 189]. Various schemes have been proposed for reducing the effect of supermode noise, such as taking advantage of self-phase modulation [41], composite cavity setups [42], dithering the cavity length [190]. Other methods include adding filters to the cavity such as linear [191] and nonlinear [192, 193] Fabry-Perot filters or saturable absorbers [186, 194, 195] or including saturable absorber techniques based on nonlinear polarization rotation [153] or negative impulse modulation [196].

In the past supermode noise has mostly been ignored for passively mode-locked lasers. Although there are studies on the noise characteristics of passively mode-locked lasers, the investigated noise was usually not ascribed to the presence of supermodes. In particular for ultra-high repetition rates passively mode-locked lasers the effect of supermode noise as been completely ignored, due to a lack of techniques for quantifying the noise of the lasers operating at repetition rates above 100 GHz.

The application of the definition of supermode noise from actively mode-locked lasers to
the passive mode-locking technique presented in this thesis is not straight forward. Dissipative four-wave mixing (DFWM) does not “force” the laser into operating at a higher harmonic like active mode-locking does. The ambiguity between different sets of supermodes does not apply to the DFWM mode-locked laser, the supermodes of the DFWM laser are not degenerate solutions to the harmonic repetition rate. However the laser can exhibit noise characteristics very similar to the supermode noise in actively mode-locked lasers.

Consider a spectral filter similar to the one introduced in section 3.3.1: A Fabry-Perot filter combined with a Gaussian-shaped bandpass filter. The individual reflection peaks of the filter have a finite width $\nu_{fwhm}$, that is defined by the finesse of the filter. As discussed above, the cavity resonance frequency $f_{res} = c/nL$ determines the frequency spacing of the cavity eigenmodes of the laser. If the cavity resonance frequency is of similar magnitude or larger than the width of the reflection peaks, only one cavity mode coincides with every reflection peak, thus only these modes mix with each other to create the mode-locking. The resulting laser is mode-locked at the Nth-harmonic of the eigenfrequency, where $N$ is given by the ratio of the free spectral range and the cavity resonance frequency $N = \frac{f_{FSR}}{f_{res}}$ with the repetition rate determined by the free spectral range (FSR) of the filter. This situation is depicted in figure 4.2(a). However the picture is significantly different if the width of the reflection peaks is much larger than the cavity resonance frequency, $\nu_{fwhm} \gg f_{res}$, because each reflection peak contains a (possibly large) number of cavity eigenmodes, as depicted in figure 4.2(b). New modes are created by four-wave mixing between cavity modes inside the two central reflection modes. Due to the fact that several modes are present inside each reflection peak it is possible that not only the modes spaced exactly at the FSR of the filter mix with each other. Instead of the central mode in one of the central reflection peaks mixing with the mode at the centre of the other reflection peak, it can also mix with the next mode to the right or left of this mode. The set of modes created through this mixing process has a slightly different frequency spacing ($f_{FSR} + f_{res}$), and the resulting pulse train thus has a slightly different repetition rate. The effect of the mixing of modes at different frequency separations is indicated in the inset in figure 4.2(b). Depending on the number of cavity modes inside the reflection peaks, there are possibly multiple sets of modes at different fre-
(a) One cavity mode per reflection band

(b) Multiple cavity modes per reflection band

Figure 4.2 – Reflection spectrum of the filter with cavity modes of the laser, when (a) $\nu_{\text{FWHM}} \sim f_{\text{res}}$, only one cavity mode inside each reflection band, and (b) when $\nu_{\text{FWHM}} \gg f_{\text{res}}$, i.e. multiple cavity modes inside each reflection band. The inset demonstrates how modes with different frequency separation can mix.
Chapter 4 Supermode noise

frequency spacings. It has to be pointed out, that although we denote the effect supermode noise, it is distinctively different to supermode noise in actively mode-locked lasers as the traditional supermode noise does not contain sets of supermodes at different frequency spacings.

If we recall the relevant experimental parameters of the laser, a cavity length of 1000 m corresponding to a cavity-mode spacing of 200 kHz and a full-width-half-maximum (FWHM) of the filter reflection peaks of around 2 GHz, we calculate up to 10 000 eigenmodes inside the cavity. Due to the large number of possible supermodes, it is necessary to examine the noise characteristics of the laser. In particular we want to investigate if supermode noise can explain the envelope of the autocorrelation seen in the experimental results in chapter 3, that does not show up in the simulations. The noise characteristics of the laser are examined in two ways. Firstly to gain a better understanding of how the laser dynamics are effected by supermode noise a method of including the noise into the numerical simulations is introduced in section 4.2 and the results are presented in section 4.3. Secondly experiments are carried out with a laser mode-locked at 10 GHz to obtain quantitative measurements of the laser using traditional RF-equipment. Finally we explore a strategy for suppressing the supermode noise in the DFWM laser. Generally we can reduce the amount of supermode noise by decreasing the number of supermodes oscillating inside the cavity. The reduction can either be achieved by increasing the cavity resonance frequency, i.e. mode-locking the laser at a lower harmonic, or by filtering the cavity modes more effectively using a higher finesse filter or additional filters. In section 4.5 we demonstrate the implementation of a supermode noise reduction technique by adding additional fibre-based filters into the cavity. We should note that other means of supermode noise reduction are possible, for example an approach by filtering the fluctuations in the “temporal domain”, using a saturable absorber type filter. We have numerically and experimentally investigated such an approach, by integrating a nonlinear loop mirror (NOLM) into the cavity, however as we were not able to obtain mode-locked operation either in experiments or simulations we have omitted these results.
4.2 Numerical method

The laser is described by the model introduced in section 3.5; A Ginzburg-Landau equation with gain, multiplied by a filtering function once every cavity round trip. However to simulate the supermode noise behaviour of the laser, we need a way to take into account the effect of a number of discrete cavity modes inside each mode of the filter. Cavity eigenmodes represent a reduction of the continuous spectrum of the free-propagation regime to the discrete solution spectrum of the cavity. The cavity induces a restraint to only allow solutions at discrete frequencies at multiples of the cavity resonance frequency. Therefore in order to simulate the effect of the cavity eigenmodes it is necessary to induce a discretization of the spectrum on a scale which is smaller than the free spectral range of the main filter. One strategy of introducing this discretization is by applying a second Fabry-Perot filter with a smaller FSR than the main filter. The modes of the second Fabry-Perot filter then act as the cavity modes of the laser. However, the disadvantage of this approach is that a large number of numerical discretization points is required, especially if one wants to simulate a large amount of cavity-modes thus increasing the simulation time significantly. Fortunately a discretization of the spectrum is already present in the numerical simulation. We can simply take advantage of the computational discretization of the field by regarding the numerical discretization points in the spectral domain as the cavity modes of the laser, i.e. each discrete point in frequency space corresponds to a cavity mode of the laser. In this way, we can vary the amount of supermode noise by changing the numerical discretization, i.e. the numerical sampling of the main Fabry-Perot filter. Additionally a change in discretization corresponds to a change of the harmonic of the mode-locking process, due to the normalization of the model. The effect of different discretization on the number of cavity modes and the harmonic of the mode-locking is depicted in figure 4.3. The figure shows the two central peaks of the filter in figure 4.3(a). The discretization has been chosen such that the FSR of the filter is sampled by 100 points while the FSR is sampled by 10 points in figure 4.3(b). The repetition rate of the laser is determined by the FSR of the filter. Because of the equivalence between a discretization point and a cavity eigenmode,
the repetition rate of the laser is 100 and 10 times the cavity resonance frequency in both cases respectively, i.e. the laser is mode-locked at the 100th and 10th harmonic. Although it seems counterintuitive, a change in the harmonic of the laser does not correspond to a change of the repetition rate due to the normalization of the model. The closest analogy to the change of the sampling is changing the cavity length in experiment. However in contrast to altering the discretization in the simulations, changing the experimental cavity length also affects the cavity gain and dispersion. Similar to the simulations in section 3.5,

![Image](a) FSR sampling 100  
(b) FSR sampling 10

**Figure 4.3** – Demonstration of the effect of discretization on the number of cavity modes and harmonic of the mode-locking. (a) Fabry-Perot filter numerically sampled by 100 points per FSR. (b) Fabry-Perot filter numerically sampled with 10 points per FSR.

the numerical algorithm is the well-known split-step Fourier method [34].

## 4.3 Simulations

Figure 4.4(a) depicts the experimental autocorrelation trace of the laser mode-locked at 160GHz. The main characteristic of the experimental autocorrelation that cannot be explained by noise-less simulations is the envelope. The crosscorrelation peaks have less peak power than the central autocorrelation pulse and the peak power of the crosscorrelation pulses gets smaller for larger delays. A decrease in crosscorrelation power with increasing delay is an indication of a limited temporal coherence due to noise on the pulse.
Figure 4.4 – (a) Experimental autocorrelation trace (see figure 3.27 for parameters) (b) autocorrelation from numerical simulation with the following parameters: $\kappa = -0.0050$, $I_{sat} = 4$ and $G = 0.8$. There were $2^{17}$ simulation points and the FSR of the filter was sampled by 400 points, i.e. mode-locking at the 400th harmonic.

train. In particular power fluctuations and timing jitter cause the crosscorrelation intensity to decrease. The autocorrelation of a simulation including supermode noise is shown in figure 4.4(b). Comparing the numerical autocorrelation to the experimental autocorrelation in figure 4.4(a), we see that the numerical simulations with supermode noise reproduces the experimentally observed autocorrelation envelope. It should be noted that the number of cavity modes within one filter reflection band in the numerical simulations is significantly lower than the experimental number of supermodes. The effect of varying the filter sampling will be discussed in detail in section 4.3.1. Suffice to say that the sampling does not have to be the same for qualitative agreement between simulation and experiment. The field intensity of the simulations gives insight into the influence of supermode noise on the pulse dynamics of the laser. As the field could not be retrieved from the experimental FROG spectrograms we have to rely on the numerical results as the strongest tool for understanding the effects of supermode noise on the laser output. Figure 4.5(a) shows the field intensity resulting in the autocorrelation in figure 4.4(b). The graph nicely demonstrates the effect of the supermode noise. Instead of a continuous pulse train the field consists of bunches of pulses with varying envelope width and peak power. Figure 4.5(b) displays only part of the full simulation window. It depicts the overlapping region between two bunches.
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Figure 4.5 – Field intensity resulting in the autocorrelation given in figure 4.4(b). (a) shows the full simulation time window, while (b) shows only a partial cut-out to illustrate the beating between different bunches of pulses.

of pulses, demonstrating a beating between the underlying pulse trains of the two bunches. The bunches therefore consist of pulse trains of slightly different frequency. This observation agrees nicely with the earlier discussion about supermode noise inside the DFWM laser; As each of the two central reflection bands of the filter contains several cavity modes there is not only mixing between the modes precisely at the centre of the two bands that are spaced at exactly the FSR of the filter, but also between modes spaced at slightly different frequencies. Because the four-wave mixing is the process that is responsible for creating the higher-order modes and induces the mode-locking, the modes spaced at a different frequency create a pulse train at that frequency, slightly different to the FSR of the filter. The different sets of cavity modes share some modes with each other, causing the pulse trains to be interdependent. Therefore the pulses form bunches not several independent pulse trains at different frequencies, in contrast to a simple regular beating which would be the result of totally independent supermodes.
4.3.1 Variation of supermode noise

With the experimental laser mode-locked at the 500 000th harmonic and a finesse of approximately 50 the number of cavity modes in one reflection band of the filter could exceed 10 000. Compared to the preceding simulations that simulated mode-locking at the 400th harmonic, the experiment contains several orders of magnitude more cavity modes than the simulations. Nevertheless the numerical and experimental results agree well qualitatively. In this section we investigate the effect of varying the number of cavity modes inside a filter reflection band on the output dynamics of the laser, by varying the filter sampling parameter in the numerical simulations. Changing the sampling of the filter is equivalent to changing the cavity-mode spacing in the experiment, thus effectively changing the harmonic of the mode-locking. If the finesse of the filter (i.e. the ratio of FSR to the FWHM of the reflection bands) is kept constant, decreasing the mode spacing results in more cavity modes inside each reflection band. Figure 4.6(a)-(d) depicts the autocorrelation traces for four different values of the filter sampling, i.e. four different harmonics of the mode-locking. The finesse was kept constant at 37. The number of simulation points was varied so that the ratio of number of points to sampling was always larger than 150, ensuring that the frequency window remains approximately the same for all simulations, to prevent boundary effects. Note that the window still changes slightly, as the number of discretization points was restricted to powers of two to keep simulations times low. However this slight variation does not have a significant effect on the results, which was verified by comparing the results obtained in this fashion with results obtained with a fixed number of points. All other simulation parameters were kept constant at the same values as in the previous section ($\kappa = -0.005$, $\sigma = 0$, $I_{sat} = 4$ and $G = 0.7$).

Below a filter sampling value of 350, corresponding to around 115 cavity modes inside one reflection band, the autocorrelation has no envelope. Once the harmonic is increased slightly further to 355, the autocorrelation exhibits a strong background and a narrow envelope. A further increase in filter sampling does not result in a narrower autocorrelation envelope, instead the envelope width fluctuates randomly around a mean value when the sampling is changed as illustrated by figure 4.6(c) and (d). Figure 4.6(c) with a harmonic of
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Figure 4.6 – Autocorrelation for simulations with different filter sampling, i.e. different mode-locking harmonic. (a) 350th harmonic, (b) 355th harmonic, (c) 450th harmonic and (d) 600th harmonic.

450 shows a autocorrelation trace with a narrower envelope than figure 4.6(d) with a sampling value of 600. The results show that there is a critical threshold value below which there is no significant fluctuations on the pulse train. This value is surprisingly high, even at more than 100 cavity modes inside each reflection band there is no indication of significant variations on the pulse train. This is due to the fact that modes which are detuned from the centres of the filter reflection bands experience a higher loss, and therefore do not reach the lasing threshold. Furthermore the higher-order modes created by mixing between the off-centre modes are even further detuned from the filter band centres causing the fluc-
Simulations

Simulations to be damped. Once the number of supermodes is increased past the threshold value pulse train fluctuations start to manifest themselves in an observable envelope over the autocorrelation trace. Surprisingly the variations which are indicated by the width of the autocorrelation envelope do not increase proportionally to the number of supermodes. Instead the envelope varies randomly, indicating that the strength of the fluctuations is not directly proportional to the number of supermodes. For a more direct observation of the effect of the supermode noise on the pulse train figure 4.7(a)-(d) shows the field intensities corresponding to the autocorrelations given in figure 4.6. At a filter sampling of 350 no

![Figure 4.7](image-url)
variation of the peak intensities is observable. A qualitative comparison of the fields at the higher sampling values of (b) 355, (c) 450 and (d) 600 does not reveal a strong difference in the pulse variation. However figure (b) which corresponds to the field with the narrow-est autocorrelation has slightly stronger fluctuations than the other two fields. The results support the presumption taken from the autocorrelation traces, that once a certain amount of variation is reached, a further increase in the number of supermodes does not result in stronger fluctuations.

Obtaining a more quantitative measurement of the noise of the laser is not straightforward. Usual pulse noise analysing techniques require the pulses to be well separated [197]. However in the present case the bunches of pulses are overlapping, thus individual pulses cannot easily be distinguished. Nevertheless we can attain an approximate measurement of the pulse energy fluctuations. As the average pulse separation $\Delta \bar{t}$ is known, it is possible to calculate the energy $E_{\Delta t}$ inside each pulse separation interval by integration. The variance of the interval energy over the average energy $\delta E_{\Delta t}/\bar{E}_{\Delta t}$ gives an estimate of the pulse energy fluctuations. Note that the measurement is very similar to the relative intensity noise (RIN) measured in experiments, for simplicities sake we therefore denote the measurement RIN. We should point out, that the measurement does not take into account frequency or timing jitter fluctuations, it only gives an indication of the pulse energy fluctuations. However it is a reasonable assumption that timing jitter and frequency fluctuations change similarly to the pulse energy fluctuations and we therefore get valid information about the overall pulse fluctuations. Figure 4.8 shows the RIN as a function of the filter sampling for the simulations shown in figures 4.6 and 4.7 plus some additional filter sampling values. The measured values are in good agreement with the qualitative observations from the autocorrelation envelopes above. While there are no pulse fluctuations at a sampling of 350 at a slightly higher value of 351 there are significant pulse fluctuations. The measured fluctuations are maximal at a sampling of 355 where we also observed the narrowest autocorrelation envelope. Similarly to the earlier observations the measured RIN is smaller for 600 filter sampling points than 450. We can conclude that autocorrelation envelope is correlated to the pulse fluctuations measured by the RIN and that the conclusions
drawn from the autocorrelations are indeed valid. We did not increase the sampling above 600 points because the computation time increases significantly due to the need to increase the number of computation points. There is however no indication that the observed behaviour changes for higher sampling values. Additional measurements of the pulse width as a function of the filter sampling show no obvious correlation between the supermode noise and the autocorrelation pulse width.

### 4.3.2 Gain variation

For a fixed filter sampling value, only a few modes in the centre of the reflection bands will experience an overall cavity gain. If the gain parameter is increased the number of modes experiencing overall gain will increase. Additionally we have seen in section 3.6.2 that the laser experiences instabilities when the gain is increased. However we do not know how a larger gain affects the supermode noise and what effect the presence of supermodes has on the laser stability (Note that we talk about the stability as discussed in section 3.6.2 not the
stability with respect to supermode noise). In this section we present a numerical investigation of the effect of an increase of the gain parameter on the output pulses of the laser. Figure 4.9(a) to (d) shows the autocorrelation for simulations with four different values of the gain parameter. The filter sampling was held constant at 300 and all other parameters are the same as in the simulations presented in the previous section. For the lowest gain 

\( G = 0.7 \) there is no envelope to the autocorrelation function, indicating that there are no pulse fluctuations. However once the gain is increased the autocorrelation develops an envelope. This envelope narrows with increasing gain parameter, indicating an increase in the pulse fluctuations. This is not entirely surprising; The cavity modes at the edges of the reflection bands of the filter which previously experienced a net loss, experience a net gain

\( G = 0.8 \) \( G = 1.0 \) \( G = 1.5 \)

**Figure 4.9** – Numerical autocorrelation traces for gain values (a) \( G = 0.7 \), (b) \( G = 0.8 \), (c) \( G = 1.0 \) and \( G = 1.5 \).
with the increased overall gain. They are not damped, but instead grow every cavity round trip. Therefore more cavity modes become involved in the mode-locking, thus supermode noise increases. Additionally the phase matching of the four-wave mixing is affected by the gain, a larger value causes a higher effectiveness of the four-wave mixing. This results in modes which are spaced at larger frequency separations to mix, thus creating more sets of supermodes at frequencies which would not be present otherwise. In contrast when we only increase the filter sampling the gain is not large enough to provide phase matching at more frequencies, thus the absence of a correlation between filter sampling and pulse fluctuations. It should be pointed out, that the envelope to the autocorrelation is not observed in the laser stability simulations in section 3.6.2. Those simulations used a very narrow filter with only one discretization point within each reflection band of the filter, eliminating the influence of supermode noise. The present results therefore provide evidence to the fact, that the autocorrelation envelope is a feature of the supermode noise, not of laser instabilities due to the high gain.

The graph in figure 4.10 shows the RIN calculated as described in the previous section as a function of the gain parameter for a filter sampling of 300. In agreement with the observations from the autocorrelations the RIN is zero at a gain of $G = 0.7$ and it increases with increasing gain. Although the results are not conclusive due to the number of points, it appears that the fluctuations depend linearly on the gain parameter. The deviation from the linear graph can be attributed to the random nature of the fluctuations which depend on the initial noise conditions.

The experimental equivalent of the gain parameter is the pump power. We noted earlier (section 3.4.1) that the width of the autocorrelation pulses experiences an exponential decay when the pump power is increased. However the numerical simulations without supermode noise did not reproduce this behaviour, but showed no observable dependence of the autocorrelation width on the gain parameter. Repeating a similar numerical experiment with a filter-sampling of 300 displays a entirely different picture. Figure 4.11 depicts the width of the central autocorrelation pulse as a function of the gain parameter. The simulation accurately reproduces the experimentally observed exponential decay, as displayed.
by the fitted curve. The numerically observed pulse width is also very similar to the experiment. In section 3.4.1 we measured a minimum autocorrelation pulse width of 840 fs the numerical curve approaches a minimum width of about 600 fs. Considering the numerical assumptions and experimental uncertainties this a very good quantitative agreement. The numerical results also indicate that the width of the actual pulses does not fluctuate significantly in the experiment as can be seen from the numerical field intensities which do not exhibit large pulse width fluctuations.

Finally we would like to come back to the discrepancy between the experimental pulse shape and the numerical observations. The experimental autocorrelation trace resembles most closely a Lorentzian function, however the numerical results did not show the resemblance to a Lorentzian. Figure 4.12 depicts a graph of the error when fitting a Gaussian, sech$^2$ and Lorentzian to the autocorrelation as a function of the gain parameter. We can see that for low values of the gain, the sech$^2$ function provides the best fit to the autocorrelation.

Figure 4.10 – RIN as a function of the gain parameter. The line indicates a linear fit to the data.
However at gain values above $G = 1.0$ the error of the Lorentzian fit is smaller. Incidentally this is also the threshold value where the supermode noise starts to appear (see figure 4.10). The supermode noise therefore causes a change of the autocorrelation pulse shape to more closely resemble a Lorentzian distribution. The exact mechanism that causes the change is not entirely clear, however it can be assumed that the supermode noise causes a broadening of the autocorrelation tails due to the pulse bunches with slightly different frequencies. The numerical model presented in section 3.5 had a number of shortcomings when the numerical simulations failed to reproduce the experimental observations. The above results show that once supermode noise is included into the model most of the shortcomings are overcome. The simulations with supermode noise accurately reproduce the envelope, which could be seen in the experimental autocorrelation traces. The width of the autocorrelation pulses decreases exponentially in agreement with the experimental behaviour.
Chapter 4 Supermode noise

Figure 4.12 – Error $\epsilon$ of a sech$^2$ (dashed), Gaussian (dotted) and Lorentzian (dashed-dotted) function fitted to the central autocorrelation peak. The FSR sampling parameter was set to 300.

We also see that the autocorrelation pulse shape closely resembles a Lorentzian function once supermode noise is included in the simulations, reproducing the pulse shape of the experimental autocorrelation pulses. The conclusion to be drawn from the above results is, that supermode noise has indeed a significant impact on the experimental DFWM laser, and it is necessary to find ways to more accurately measure and reduce the amount of supermode noise of the laser. The measurement and suppression of supermode noise will be investigated in the following sections.

4.4 Experiment

The high repetition rate of more than 100 GHz effectively prevents acquiring quantitative measurements of the laser noise, as the timescale exceeds the capabilities of the electronic
Supermode filtering with subcavities equipment. Although the numerical simulations in conjunction with the autocorrelation give some indication about the influence of the supermode noise on the laser dynamics, they are no substitute for measurements using RF-equipment. In order to obtain such measurements a laser with a repetition rate of 10 GHz repetition rate was constructed using a different FBG filter. As 10 GHz is well within electronic bandwidths we can use conventional RF-electronics to measure the noise characteristics of the laser. The quantitative measurements enable us to explore and evaluate noise reduction techniques which would not be possible without the means to effectively compare the noise for different setups.

The experimental setup of the laser is the same as the one introduced in section 3.3.1. A fibre ring cavity with 1000 m of HNLF (fibre 1 in appendix A) pumped by a tunable cw Raman fibre laser. The filter is a FBG filter similar to the one described in chapter 3 however with a FSR of 10 GHz (FBG 4 in the appendix A). The width of the individual reflection peaks is about 2 GHz similar to the 100 GHz filter. Note the very low finesse of the filter of around five, compared to 50 for the 100 GHz filter. The FWHM of the Gaussian envelope to the filter is 25 GHz, i.e. 2.5 times the FSR of the filter. The laser output is measured using a fast photodiode with a bandwidth of 40 GHz and analysed using a RF-spectrum analyser with 25 GHz bandwidth. Figure 4.13 depicts the spectral density function of the 10 GHz laser taken with a resolution bandwidth (RBW) and video bandwidth (VBW) of 1 MHz. The pump power is 870 mW resulting in 10 mW average output power. As expected, we see a spectral peak centred around the 10 GHz repetition rate. The peak is however very broad, indicating a large number of cavity modes resonating simultaneously. Examining the inset we can see, that the full width of the 10 GHz RF peak is slightly over 2 GHz, which agrees well with the FWHM of the FBG reflection bands.

4.5 Supermode filtering with subcavities

Clearly, a large amount of supermode noise as seen in the power density function in figure 4.13 is undesirable in a laser. Ideally the RF-spectrum consists of only one mode with a width less than the cavity resonance frequency centred at 10 GHz. Therefore we need to
Figure 4.13 – RF-Spectral density of the 10 GHz repetition rate laser. The main window shows the full spectrum with a resolution bandwidth (RBW) and video bandwidth (VBW) of 1 MHz. The inset is a cut-out with the same RBW and VBW.

find strategies to reduce the supermode noise of the laser. Numerous techniques for reducing supermode noise have been suggested in the literature [41, 153, 188, 192]. However, most of these techniques are impractical in the present case, either because of the high optical powers involved, or due to the high repetition rate at which we want to apply them (i.e. above 100 GHz).

The most obvious approach for decreasing the supermode noise is to reduce the number of cavity modes inside the reflection bands of the filter. This reduction can either be accomplished by increasing the cavity mode spacing, i.e. a larger cavity resonance frequency, or by filtering a larger number of modes by increasing the finesse of the filter or introducing additional filters. In the case of our experiment, the length of the cavity and thus the cavity resonance frequency is determined by the length of the highly nonlinear fibre which cannot easily be changed for two reasons. Firstly a technical reason: because the fibres were used in other experiments we could not cut them. Secondly a physical reason: a shorten-
Supermode filtering with subcavities

...ing of the fibre also increases the threshold and decreases the slope efficiency of the laser as both the cavity nonlinearity and gain become smaller. The possible change in fibre length is therefore limited, as too short a length prevents mode-locking. Increasing the finesse of the filter is not possible either, because the filters are already manufactured with the smallest possible FWHM of the reflection bands. The remaining option is therefore to add additional filters to reduce the number of cavity modes in the laser. The requirements for the additional filters are very similar to the requirements on the main laser filter; they need to be low loss, thus excluding any free space coupled components and they should be able to handle the high average optical powers inside the cavity. In this section we investigate the addition of a subcavity to the main laser cavity, which is equivalent to an additional filter inside the cavity. Numerous subcavity setups for supermode noise reduction have been proposed in the past. In particular the so-called composite cavity setup, which splits the laser cavity into two slightly detuned cavities [42]. Although the technique looks very advantageous, it has been criticised to not achieve the desired noise reduction [187, 198].

![Diagram](image)

**Figure 4.14** – Experimental setup of the laser with a subcavity for supermode noise reduction. HNLF: highly nonlinear fibre, WDM: wavelength division multiplexer, FBG: fibre Bragg grating.

The main point of criticism about the composite cavity laser is, that the composite cavity acts...
like a Mach-Zehnder filter. However the response function of a Mach-Zehnder filter is sinusoidal, thus only a relatively small number of cavity modes are filtered. In this setup we integrate the subcavity in a slightly different configuration. The experimental setup is depicted in figure 4.14. The subcavity consists of a fibre ring with two identical fibre couplers. The light is coupled into the ring with one coupler and is coupled out of the ring in the same direction using the second coupler. Instead of a Mach-Zehnder transfer function, this subcavity exhibits a transfer function equivalent to the transmission of a Fabry-Perot filter. The finesse and FSR of the filter are determined by the coupler ratio and the length of the subcavity respectively. However increasing the coupling ratio increases the impact of the cavity losses as the light propagates more times around the cavity, the maximum transmission therefore reduces with higher coupling ratio. Selecting a coupling ratio has to therefore balance the desire to achieve a high finesse and the acceptable losses of the filter (see reference [199] for a detailed discussion of fibre based filters). The all-fibre nature of the filter has the advantage that it can safely operate at very high power levels, while ensuring low insertion losses compared to a fibre-pigtailed bulk Fabry-Perot etalon.

**Power spectrum**

The RF-spectrum of the laser incorporating a 440 cm long subcavity based on two 10/90 fibre couplers is depicted in Figure 4.15. The 440 cm correspond to a FSR of the filter of approximately 45 MHz. To ease the comparison with the spectral density of the laser without a filter, the pump power is adjusted so that the average optical output power is always 10 mW.

We observe that the broad continuous 10 GHz peak observed in the RF spectrum of the original laser [figure 4.13] is replaced by a discrete number of peaks separated by approximately 45 MHz corresponding to the FSR of the sub-cavity. Each individual peak has a full width of around 5 MHz. Although the signal-to-noise ratio is improved by 10 dB, the number of supermodes is still significant. The still large number of peaks which contains around 20 supermodes each lead to more than 1000 supermodes overall.

We have also built two additional filters with different free spectral ranges; one filter with
Figure 4.15 – RF-Spectral density of the 10 GHz repetition rate laser with a 440 cm long subcavity. The main window shows the spectrum centred at 10 GHz while the inset shows a small section of the spectrum demonstrating the 45 MHz separation between the modes. The resolution bandwidth (RBW) and video bandwidth (VBW) is 1 MHz in both cases.

Figure 4.16 – Power spectrum of the laser with (a) L = 60 cm long subcavity and (b) L = 424 cm long subcavity. The RBW and VBW were 1 MHz in both cases and the pump power was adjusted to yield 10 mW output power.
an approximate length of 60 cm, corresponding to about 290 MHz FSR, and a second filter with a length of around 424 cm slightly shorter than the 440 cm filter and a FSR of about 47 MHz. The RF-spectra of the laser with each of these filters are shown in figure 4.16(a) and (b) respectively. As expected the laser exhibits a power spectrum with peaks separated by the FSR of the respective filters. Because the finesse of the two filters is around 30, we see that there is a significant amount of modes within each of the peaks. This is particularly observable in the power spectrum of the 290 MHz subcavity. Similar to the 45 MHz filter, both filters yield an increase in the signal-to-noise ratio of about 5 dB for the 290 MHz subcavity and 10 dB for the 47 MHz subcavity.

To achieve a higher supermode noise suppression the three filters can be combined to form a filter with larger FSR and higher finesse. The combination of either the 47 or 45 MHz subcavity with the 290 MHz subcavity is straightforward. The large FSR filter creates modes spaced at 290 MHz, however due to the small finesse of the filter the width of the individual peaks is still large. When this filter is combined with one of the low FSR subcavities, the width of the individual peaks is reduced to the width of the peaks of the low FSR filter. The resulting RF-spectrum of this combination is depicted in figure 4.17. The number of individual peaks is significantly smaller compared to the RF-spectra of the single subcavity setups and the 2 GHz envelope is also not clearly distinguishable. Instead there is a low number of peaks with varying power. Using the main peak, we see that the signal-to-noise ratio is increased by almost 30 dB compared to the setup without subcavities.

The combination of the 45 MHz and 47 MHz filters is even more appealing. The two filters superimpose according to the Vernier principle. The length of the 47 MHz filter is chosen such that its FSR is 21/20th of the 45 MHz filter (Note that the ratio of 47/45 is not exactly 21/20 however the 47 MHz filter is in reality closer to 47.25 MHz which does match the 21/20th of 45 MHz). The principle behind the Vernier superposition is shown in figure 4.18(a) and (b). Figure (a) depicts the transmission of two Fabry-Perot filters with an FSR ratio of 11/10. Thus every 10 or 11 peaks respectively the peaks of both filters fully overlap, while in between the filters are detuned from one another. Multiplying both filters therefore leads to a transmission function as shown in (b) for an FSR ratio of 21/20. The
Figure 4.17 – RF-spectrum of the laser with two subcavities with length $L = 60$ cm and $L = 424$ cm.

Figure 4.18 – (a) Transmission of two Fabry-Perot filters with a FSR-ratio of $11/10$ demonstrating Vernier superposition. (b) Transmission of the superposition of two Fabry-Perot filters with a FSR-ratio of $21/20$ and an individual finesse of 50.
Chapter 4 Supermode noise

filter exhibits peaks at a multiple of the FSR of the individual filters, while the width of the peaks is identical to that of the individual filters. The finesse of the resulting filter is thus significantly larger than that of the individual filters. However the filter does exhibit smaller peaks where the filter peaks partially overlap, but these are significantly smaller than the main transmission peaks.

Here the superposition of the 45 MHz and 47 MHz filter yields a filter with a FSR of 945 MHz and a finesse of over 600. The RF-spectrum of the laser with these two subcavities is shown in figure 4.19. Examining the spectrum we see one central peak at around 10.4 GHz and two smaller sidebands separated by approximately 80 MHz to either side of the main peak. The signal-to-noise ratio is around 50 dB using the main peak. The reduction in modes compared to the individual filters is significant. Considering the theoretical width of the peaks calculated from the transmission function of the filters, each of the peaks might contain around 20 cavity modes. Thus with the two subcavities the laser possibly contains a maximum of 60 supermodes. However compared to the number of supermodes in the laser without any additional filter this is a reduction by a factor of over 100. The position of the two sidebands does not correspond to the position where the two sidebands in the theoretical superposition of the filters appear. This can be attributed to the fact, that the FSR-ratio is not exactly 21/20 but an irrational number instead.

Although the Vernier superposition of the filters does yield a strong reduction in supermode noise, it does exhibit problems as well. The fact that small changes in the length of the two filters can have a strong effect on the length ratio of the filters and thus on the FSR-ratio causes the two subcavity configuration to be susceptible to fluctuations due to environmental influences. The result is that the laser becomes temporally unstable. The average output power fluctuates and when monitoring the RF-spectrum, we observe the modes jumping, i.e. the position and power of the peaks changes abruptly.

It should also be noted that inclusion of the sub-cavity filters comes at a cost of a reduction in output power. This is not entirely surprising, as the subcavities introduce losses and the transmission of the filters cannot be 100%. The output power as a function of pump power for different subcavity combinations is depicted in figure 4.20. It is evident that the slope
efficiency does not significantly change. However the lasing threshold increases when subcavities are used. The increased lasing threshold does result in a lower maximum output power. Nevertheless, although the changes are significant they are not prohibitively large. It is also possible to reduce the impact of the subcavities by more carefully optimizing their loss properties. We should note that the setup with the Vernier superposition of subcavities is not included in the graph due to the temporal instabilities and the resulting power fluctuations which prevent a consistent power measurement.

**Autocorrelations**

Finally we want to examine the influence of the subcavities on the pulse train in the temporal domain by performing autocorrelation measurements. For a better comparison against previous autocorrelation results we used the 100 GHz laser instead of the 10 GHz laser. No autocorrelation measurements could be taken of the laser with the two Vernier superim-
Figure 4.20 – Measured (symbols) average optical output power of the 10 GHz laser with linear fit (lines) with no subcavities (circles, solid line), the 60 cm subcavity (squares, dashed line) and combination of the 60 cm and 440 cm subcavities (diamonds, dotted line).

posed subcavities, because the temporal instability of the Vernier superposition prevented any autocorrelation measurements. The output power fluctuated significantly during an autocorrelation measurement, causing the autocorrelation to fluctuate as well. This happens on a timescale of seconds, and therefore significantly alters the appearance of the autocorrelation trace, which takes about 30 s to record with the FROG.

When comparing the different autocorrelation traces in figure 4.21, we see some differences between the curves. Both the trace taken from the laser without a subcavity and with the 60 cm subcavity exhibit a Gaussian-shaped envelope over the autocorrelation, with the higher-order crosscorrelation peaks having lower power than the central autocorrelation peak. The width of the autocorrelation envelope is slightly smaller for the unfiltered laser. The difference is however only subtle, but can be distinguished when comparing the peak power of the peaks at $-50$ ps: the peak in figure (b) is slightly more powerful. The autocorrelation of the laser with the combination of the 440 cm and 60 cm subcavities at the same
Figure 4.21 – Autocorrelation of the 100 GHz laser with (a) no subcavity, (b) the 60 cm subcavity, (c) combination of the 60 cm and 440 cm subcavities, at 40 mW output power.

Power does not exhibit a clear envelope. There is some peak power fluctuations, however there is no Gaussian-type envelope and the fluctuations appear to be less regular. In addition to the difference between the envelopes, the three traces vary in width of the central autocorrelation peak and background level. Although not large, the differences are significant as can be seen in Table 4.1. The increase in background and pulse width with filtering is counter-intuitive, for bright pulses we expect a decrease in background when supermode noise is reduced. However it is important to remember that by adding the subcavities we also alter the cavity dispersion and loss. As the pulse shape of the laser is dependent on the


Table 4.1 – Table of the difference in pulse width and background level of the autocorrelation of the 100 GHz laser.

<table>
<thead>
<tr>
<th>Filter</th>
<th>FWHM</th>
<th>Background</th>
</tr>
</thead>
<tbody>
<tr>
<td>No filter</td>
<td>1.14 ps</td>
<td>0.3</td>
</tr>
<tr>
<td>60 cm filter</td>
<td>1.52 ps</td>
<td>0.36</td>
</tr>
<tr>
<td>60 and 440 cm filter</td>
<td>1.9 ps</td>
<td>0.41</td>
</tr>
</tbody>
</table>

cavity dispersion, we effectively change the laser characteristics, and pulse shapes from the different setups are not entirely comparable.

4.6 Summary

Supermode noise presents a problem for all lasers mode-locked at high harmonics of the cavity resonance frequency. Although the principles of supermode noise formation in the DFWM passively mode-locked laser differ from the more common case of supermode noise in actively, harmonically mode-locked lasers, it is desirable to understand supermode noise effects and eliminate supermode noise in both cases. As the many passively mode-locked lasers operate at repetition rates beyond the abilities of traditional measurement techniques which are limited by electronic bandwidths, we need to find new methods for the study of noise effects in these lasers. We have presented a numerical method which allows the simulation of the influence of supermode noise on the laser, providing valuable information about the dynamics and limitations of the laser. The numerical results show, that if supermode noise is included in the treatment, the laser light consists of bunches of pulses instead of a constant pulse train. The bunches not only differ in amplitude and width of the bunch envelope, but also in the phase and more importantly in the frequency of the underlying pulse-train. When the bunches overlap we observe beating of the pulse-trains. These numerical results agree with the phenomenal description of the effects of supermode noise; The cavity modes separated by slightly different frequencies create new modes through four-wave mixing. Thus sets of modes at different frequencies are created causing mode-locking at the different frequencies. The numerically attained autocorrelation traces qualitatively resemble the autocorrelations obtained from the experimental setup, accu-
rately reproducing the autocorrelation envelope. The numerical simulations demonstrate that the pulse fluctuations which are measured by the width of the autocorrelation envelope do not show a dependence on the sampling of the filter. Increasing the filter sampling beyond a threshold value where pulse fluctuations due to supermode noise appear does not cause stronger pulse fluctuations. However the pulse fluctuations depend strongly on the gain parameter. When the gain is increased, fluctuations due to supermode noise grow simultaneously. A pulse energy fluctuation measurement calculated from the simulations agrees well with the observations from the autocorrelation traces for the variation of the filter sampling and the variation of the gain parameter.

In addition to accurately reproducing the envelope of the experimental autocorrelation, the numerical simulations with supermode noise also reflect the experimentally observed behaviour of the pulse width when the gain is increased. In section 3.4.1 we noted that the width of the autocorrelation pulse decreases exponentially with pump power. In contrast to the simulations without supermode noise the simulations presented in this chapter, which take supermode noise into account, exhibit the same decrease. With respect to the pulse shape we have found the supermode noise is also responsible for the discrepancy between the experiment and simulations in chapter 3. The autocorrelations from simulations with supermode noise exhibit a Lorentzian shaped pulse similar to the experimental one shown in figure 3.22. We can conclude, that the numerical model with a provision for supermode noise accurately describes the experimental laser. It does account for all essential features observed in the experiment and gives valuable insight into the dynamics of the system.

To be able to measure the effects of supermode noise more quantitatively we constructed a laser with 10 GHz repetition rate using a different FBG filter. The lower repetition rate of this laser permits the use of traditional electronic equipment for more direct noise measurements. The results reveal a significant influence of the noise on the power spectrum of the laser, which exhibits a ∼ 2 GHz broad peak centred at the repetition rate instead of a narrow mode. The width of the peak is determined and closely matches the FWHM of the individual reflection bands of the FBG filter. Because the width of the reflection bands does not change for filters with a larger FSR, the noise peak also does not change significantly.
Therefore, although the relative width of the peak is significant at around 20% of the repetition rate for the 10 GHz laser, it is an order of magnitude smaller in the case of the 100 GHz laser at 2%.

Finally in order to suppress the impact of supermode noise on the laser dynamics, we presented a possible reduction technique by integrating subcavities into the setup. These subcavities act as additional Fabry-Perot filters reducing the number of cavity modes. The RF-spectra of the 10 GHz laser incorporating this filtering technique show that the amount of supermodes is reduced when a subcavity is integrated into the laser. However including only one subcavity still leaves a significant amount of supermode noise inside the cavity and we therefore combined two subcavities to form a one filter to further suppress the noise. Combining a small and large FSR filter already showed a great improvement over the single subcavity setups by improving the signal to noise ratio by 30 dB over the unfiltered case, while one subcavity only lead to 10 dB improvement. The number of supermodes was also significantly reduced, however the greatest reduction in supermodes could be attained with a Vernier superposition of two subcavities. The number of supermodes was reduced from over 10000 to under 100. However the reduction comes at the drawback of the introduction of temporal instabilities to the laser output, because the two subcavities fluctuate in their relative lengths. As the Vernier superposition depends on the length ratio of the subcavities, these fluctuations lead to large changes to the spectral response of the combined filters, causing the laser output power and RF-spectrum to fluctuate on a timescale of a few seconds. The lasing threshold is also influenced by the additional loss due to the subcavities. Nevertheless the losses are not prohibitive and can be improved with a more careful loss optimization of the setup. The comparison of the autocorrelation of the filtered to the unfiltered laser reveals that once a significant amount of supermode noise is filtered from the laser, the envelope disappears. However there is still some supermode beating present, which causes slow fluctuations which can be identified by some residual changes in the crosscorrelation peak power.

The presented work is the first study of the noise influence on a passively mode-locked laser at such a high repetition rate. Although the results are mostly qualitative in nature
due to either being performed with numerical simulations or based on a interference from low repetition rates to higher repetition rates, they nevertheless provide valuable information and give a reasonably accurate picture of the nature of the laser dynamics under the influence of noise.

In the future further investigation of the supermode noise of the laser should concentrate on different suppression techniques, rendering the laser more viable for possible applications. This work has already laid the foundation for such research. The stabilization of the Vernier superposition of the subcavities provides a good starting point and should be possible using a feedback mechanism altering the subcavity ratio with a piezo crystal for example. An initial experiment had already been setup, however due to equipment failure rendering the pump laser unusable it had to be abandoned. With a functioning stabilization the Vernier superposition of the subcavities provides an effective filtering technique. Additional suppression methods could try to improve the filtering either with supplemental filters, possibly FBG filters instead of the subcavity based filters, or increasing the main filter finesse, which might become feasible with improved FBG manufacturing techniques.

Another approach is to increase the cavity resonance frequency. This involves reducing the cavity length, requiring propagation media providing the necessary cavity gain and nonlinearity. Again with the advances in the design of speciality fibres, this might soon become viable. In particular chalcogenide glass based fibres or photonic crystal fibres might offer the desired characteristics, however they are currently limited by their prohibitively high losses which prevent their application in the current setup. A medium which already offers the desired characteristics are semiconductor optical amplifiers (SOA) which provide very large gains and nonlinearity over very short lengths. However the gain and nonlinear processes in SOAs are significantly more complicated than in fibres. They therefore warrant an entire investigation of their own and are beyond the scope of this thesis.
In the discussion about the noise properties of the passively mode-locked Raman fibre laser in chapter 4 we have excluded the possible influence of pump laser noise. This omission was deliberate, as its inclusion would have significantly complicated the discussion of the noise of the mode-locked laser. Furthermore the passively mode-locked laser is currently dominated by supermode noise thus fluctuations transferred to the laser from the pump are negligible, especially because the laser is pumped in a counter-propagating configuration. In this chapter we investigate the noise properties of continuous wave Raman fibre lasers such as the one acting as the pump laser in the experiments of the previous chapters. We will introduce a novel method of measuring fast intensity fluctuations and show that the Raman fibre laser exhibits fast, high-contrast intensity fluctuations. Finally we compare the experimental results with numerical simulations which show a good agreement.
5.1 Background and motivation

Continuous-wave (cw) cascaded Raman fibre lasers (RFLs) are efficient high power light sources and fixed wavelength, tunable RFLs are commercially available with average output powers above 10 W. They often act as pump sources for fibre amplifiers and, like Erbium-doped fibre amplifiers, are of critical importance in modern long-haul telecommunications systems [200]. More recently cw RFLs have become popular as pump sources for supercontinuum (SC) generation using long dispersion-shifted or highly nonlinear fibres [48, 201–203]. In contrast to the common SC generation schemes using femtosecond laser pump sources, cw RFLs offer the possibility to obtain ultra-broadband spectra in the cw regime providing advantages for applications such as optical coherence tomography [204].

The work presented in this chapter was inspired by a publication from Vanholsbeeck and co-workers [48]. In their article the authors developed a numerical model of cw SC generation taking into account the role of the partial coherent nature of the cw cascaded RFL pump. It was found that good agreement between the numerical model and the experimentally measured cw SC spectra was only obtained by assuming the existence of fast high-contrast intensity fluctuations of the pump, similar to the hypothetical fluctuating intensity shown in figure 5.1(a). The timescale of the fluctuations was assumed to be determined by the spectral width of the pump laser to be around 10 ps. Due to the limitations imposed by the electronic bandwidths, it is therefore not possible to measure the noise using traditional methods. The authors however confirmed the strong fluctuations indirectly by performing an intensity autocorrelation measurement of the RFL output which is shown in figure 5.1(b). The autocorrelation displays a peak with a width of several tens of picoseconds and a peak power of about two times the background level. Because the autocorrelation of a true continuous wave laser is flat and an intensity autocorrelation does not react to phase noise, the results provide convincing evidence that the laser used by the authors indeed exhibits intensity fluctuations on a timescale of tens of picoseconds.

This result contradicts the traditional phase-diffusion model [205, 206]. In the phase-diffu-
sion model cw lasers are generally assumed to exhibit mainly phase noise and only a limited amount of amplitude or intensity noise. Furthermore, modern cw RFLs are usually known for the low value of the relative intensity noise (RIN), which is defined as the ratio of the variance of the power fluctuations over the mean power squared ($\frac{\langle \delta P^2 \rangle}{P_0^2}$). In the case of the experiment in reference [48] the specified RIN of the pump laser was as low as $-110$ dBc/Hz at 1 GHz, which translates to about 2% fluctuations at a maximum frequency of 1 GHz. Very little is known about fluctuations significantly faster than 1 GHz and generally no data is provided by the manufacturers detailing such noise.

![Figure 5.1](image)

**Figure 5.1** – (a) Temporal distribution of the cw pump beam used in the simulations in reference [48] (b) autocorrelation of the experimentally used cw RFL. (From [48])

To understand the origin of the noise, let us first describe a cw laser in the spectral and time domain. In general, a cw laser exhibits a finite width in the spectral domain, measured either in units of wavelengths ($\Delta \lambda$) or frequency ($\Delta \nu$). Via the Fourier theorem the spectral width is associated with fluctuations in the time domain; If the modes of the laser are incoherent, the spectral width of the laser is directly related to its temporal coherence which is associated with random phase and/or amplitude fluctuations. The timescale of the fluctuations is on the order of the inverse of the spectral bandwidth, usually denoted the coherence time of the light source ($\tau_c \sim 1/\Delta \nu$). In the work of the authors above the spectral bandwidth of the pump source was approximately $\Delta \lambda = 1.1$ nm or $\Delta \nu = 150$ GHz, corresponding to a coherence time in the order of ten picoseconds.
Since the publication by Vanholsbeeck et al. several authors have investigated the influence of the pump coherence on cw SC generation. They come to significantly different conclusions regarding the structure of the pump laser. In contrast to reference [48] Frosz et al. propagate a model similar to the phase-diffusion model, featuring a Lorentzian shaped spectrum and predominantly phase fluctuations [46]. More recently some authors have proposed to use the output of a numerically modelled laser as the pump for the SC generation [47]. They reach the conclusion that the input pump does exhibit intensity fluctuations, albeit on a longer timescale than proposed by [48]. Simultaneous to the investigations of pump laser coherence for the purpose of SC generation, Babin et al. have developed a model to account for the spectral bandwidth of RFLs [207]. The same authors have also experimentally observed beating modes in the RF spectrum of a RFL [43].

The numerical investigations of the impact of pump coherence on SC generation give only a qualitative indication of the nature of the pump coherence. The work by Babin et al. on the other hand demonstrates fluctuations of cascaded RFLs in the megahertz region. However, both reference [48] and [47] found much faster high-contrast fluctuations to be necessary to accurately model the SC generation. Unfortunately, it is difficult to measure fluctuations in the range of a few picoseconds. Traditional noise measuring techniques use RF-equipment, yet this limits the measurable timescale of the fluctuations to less than 100 GHz due to the limitations of the electronic bandwidth. However a measurement to determine if these high-contrast fluctuations exist is highly desirable, not only in the context of SC generation. The fluctuations will also have an increased impact on amplification schemes in lightwave telecommunications systems as bit-rates increase.

### 5.2 Principle and experimental setup

In this section we introduce a novel method for measuring fast intensity fluctuations of a laser. Although we utilize the technique to measure the noise of an RFL it can be easily applied to pump lasers based on other gain mechanisms such as Ytterbium-doped lasers for example. Traditional measurement techniques of the RIN of a laser based on a fast pho-
Principle and experimental setup

todiode and an RF spectrum analyser are limited to timescales compatible with electronic bandwidths and thus fail to detect the fast fluctuations propagated in reference [48]. To circumvent this problem it is therefore necessary to transfer the noise from the laser to a signal at a low repetition rate, which can then be measured using the traditional techniques. The method we propose is based on the transfer of the noise from a strong pump laser to a weak pulse train supplied by a mode-locked laser through amplification via stimulated Raman scattering. Because Raman amplification is dependent on the pump power, the fluctuations of the pump translate to gain fluctuations and the pump noise is thus transferred to the signal. A number of requirements have to be met for this scheme to work:

- The duration of the signal pulses has to be of a similar timescale as the pump fluctuations.
- The group velocity of the signal has to be the same as the group velocity of the pump wave ($v_g^{(\text{pump})} = v_g^{(\text{signal})}$), i.e. the amplifier operates at zero walk-off.
- The repetition rate of the signal pulses has to be low enough so that traditional RF-based methods can be used to measure its noise characteristics.
- Ideally the amplification should operate in the undepleted pump regime.
- The signal power has to be small so that the signal propagates in the linear regime, i.e. we can ignore nonlinear effects such as self-phase modulation (SPM).

A schema of the principle of the noise transfer is depicted in figure 5.2 with the left-hand side representing the input of the fibre and the right-hand side representing the output. The noisy pump source and a train of pulses co-propagate through a piece of fibre (upper row of figure 5.2). If both signal and pump propagate at the same group velocity, some signal pulses overlap with high instantaneous pump intensities and experience a large gain, while other signal pulses overlap with low intensities and experience negligible gain (middle row of figure 5.2). From the figure we easily see why it is necessary for the width of the pulses to be of the same order as the the pump fluctuations timescale. If the pulses
are significantly wider, one signal pulse experiences a gain derived from the averaging over multiple pump fluctuations. Finally, the lower row of figure 5.2 depicts the case when the group velocities of signal and pump differ. A large walk-off between pump and signal causes the pulses to “walk through” the fluctuations and they experience a gain based on the average of many pump fluctuations. The same is also true if the amplifier is operated such that signal and pump propagate in different directions, which is equivalent to a very large walk-off between pump and signal. This is the reason why most Raman amplifiers operate in a counter-propagating configuration to minimize noise transfer. Because the walk-off determines how much fluctuations the signal experiences, we can use it to measure the timescale of the pump noise. If the signal “walks through” a large amount of fluctuations it experiences a gain based on the average of the fluctuations, thus less pump noise is transferred to the signal. If in contrast the walk-off is zero, all fluctuations of the pump directly transfer to the signal as the signal will experience a gain based on the instantaneous pump intensity. Experimentally a change in walk-off can be accomplished by either a change in the dispersion or length of the fibre, or by tuning the pump to signal wavelength separation. As the former is difficult to achieve in practice we have opted to use a tunable pump laser to change the separation between the pump and signal wavelength. We should note that the effect of walk-off on the RIN transfer in Raman amplifiers has been studied before [171, 172]. However the intent of those studies was markedly different. The authors measured the influence of a known pump modulation on a signal wave amplified by that pump. The aim of the present work is to measure the unknown intensity fluctuations of a pump using a similar scheme. Moreover the work in references [171, 172] is restricted to weak modulations at relatively low repetition rates (< 20 GHz).

5.2.1 Setup

The full experimental setup is depicted in figure 5.3. The pump laser is a cw cascaded RFL generating up to 4 W of pump power at a wavelength tunable between 1410 and 1490 nm. The pulsed signal is obtained from a figure-of-eight passively mode-locked Erbium doped fibre laser (ML-EDFL) running at 1534 nm. The ML-EDFL produces pulses with a pulse
Figure 5.2 – Schema of the principle behind the RIN transfer in a Raman amplifier. Top row: A low power signal from a mode-locked laser (red) and a noisy cw pump laser propagate through a fibre simultaneously. Middle row: If the group velocities of signal and pump are the same, the signal pulses that overlap with different instantaneous pump intensities at the input (left) experience a different gain and thus have different intensities at the output (right). Bottom row: If the group velocities are different the signal pulses move through different instantaneous pump intensities and thus experience an average gain. All pulses are amplified by the average pump power.
duration of around 4 ps, which is slightly shorter than the estimated timescale of the pump intensity fluctuations. The repetition rate of the ML-EDFL is about 3.82 MHz and the output can therefore easily be analysed using traditional RF-equipment. The average power of the signal was adjusted using either an erbium-doped amplifier (EDFA) to increase the power, or a variable attenuator to decrease it (both not shown in the setup). These two light sources are combined through a wavelength-division multiplexer (WDM) before being launched into a length of large effective area fibre (LEAF). The fibre was chosen for its particular dispersion profile which has been measured by white light interferometry (the parameters of the fibre are given in the appendix A). The zero-dispersion wavelength (ZDW) of the fibre is at around 1490 nm in between the wavelengths of the RFL and the ML-EDFL. It is therefore possible to obtain group velocity matching between the signal and pump by tuning the pump to about 1452 nm, which is well within the tunability of the RFL. The wavelength
Preliminary results

separation is also well within the Raman gain bandwidth, allowing for an efficient amplification of the signal. The output is measured using a 10 MHz photodiode after the signal has been separated from the pump by using a prism in free-space. The results are then analysed using either a RF-spectrum analyser or a digital oscilloscope. In some of the experiments a second WDM was placed before the prism coupling stage to be able to reconfigure the setup into a backward pumping configuration.

5.3 Preliminary results

The RIN transfer from the pump wave to the weak signal differs significantly if the signal is amplified in a co- or counter-propagating configuration. Counter-propagation of pump and signal averages the pump fluctuations and the signal experiences a gain based on the average pump power. The counter-propagation thus creates a low-pass filter for the pump fluctuations. It can easily be seen that the width of the low-pass filter depends on the length of the fibre. For a given fluctuation frequency a signal propagating through a longer fibre will average over more fluctuations, thus less noise is transferred from the pump to the signal. The −3dB frequency of the low-pass filter is given by [171]:

\[ f_c = \frac{v_g^{(signal)}}{4\pi L_{eff}} \]  

(5.1)

Where \( L_{eff} \) is the effective length of the fibre and \( v_g^{(signal)} \) is the group velocity of the signal. Equation 5.1 basically says, that if the fluctuations are significantly faster than the propagation time through the fibre, the signal only experiences an average of the fluctuations. Choosing a relatively short fibre length therefore allows us to find a lower boundary of the frequency of the pump noise.

We use a LEAF of approximately 150 m in this experiment. The average power of the ML-EDFL is about 1 mW. With the Raman gain of the LEAF being approximately \( g_R \approx 0.7 / \text{W/km} \) and an average RFL power of about 4 W this corresponds to a small signal gain of around 1.8 dB. This gain is easily detectable using the traditional RF-equipment, while due to the
short fibre length we can still assume the signal and pump to be in the linear regime when propagating individually through the fibre. Figure 5.4 shows (a) the power spectrum and (b)

![Figure 5.4](image)

Figure 5.4 – (a) Power spectrum and (b) oscilloscope traces of the signal in counter-propagating configuration. The power spectrum shows the non-amplified (dotted) spectra for comparison. The oscilloscope figure shows 30 overlaid single-shot traces plus the non-amplified signal in grey. The RFL power is 4 W for the power spectrum and 3 W for the oscilloscope traces. The RFL wavelength is 1444 nm.

the oscilloscope traces of the signal when the RFL pump is turned off, and when the signal is pumped in a counter-propagating configuration with a pump wavelength of 1444 nm. Note that wavelength differs slightly from the calculated zero walk-off wavelength, however for a fibre length of 150 m the difference in pump-to-signal walk-off is relatively small between 1444 nm and 1452 nm pump wavelength. Furthermore the 1444 nm pump wavelength resulted in the maximum noise in the forward pumping direction. We can see that the RF-spectra of the amplified and non-amplified pulse train are almost indistinguishable. The spectrum displays a peak at ~ 3.8 MHz, the repetition rate of the ML-EDFL. The peak of the signal counter-propagating to the pump exhibits a higher peak power due to the fact the signal is amplified, however the noise level corresponding to the noise floor of the detection system is identical in both cases. Figure 5.4(b) shows an overlay of 30 single-shot oscilloscope traces for both the amplified and non-amplified signal. It displays the same situation as the RF-spectra; the pulses do not exhibit significant fluctuations and are similar except for the higher peak power of the amplified pulses. The signal does therefore not
acquire any additional noise from the RFL laser. In comparison the signal pumped in a co-propagating configuration acquires a significant amount of noise, as can be seen in the power spectrum in figure 5.5(a). A strong > 10 dB amplification of the noise is observed in the co-propagating pump configuration. The noise amplification factor is much larger than the average gain of the amplifier, which can be measured by comparing the level of the main peak at 3.8 MHz with the level of the same peak in the non-amplified case. The average gain is approximately 1.7 dB the same as the amplification in the counter-propagating configuration and is within the theoretically expected range. The observed amplified noise is a broadband white noise, which gives an indication of the fast random nature of the noise. Note that two observed sidelobes at approximately 0.8 MHz around the main peak can be attributed to slow transient relaxation oscillations of the RFL and they appear and disappear randomly. The magnitude of the amplitude jitter caused by the noise transfer can be appreciated by observing the temporal output of the pulse train on a sampling oscilloscope. As in for the counter-propagating configuration we show an overlay of 30 single-shot traces in figure 5.5(b). Comparing the amplified traces to the non-amplified ones, we see that the pulses vary greatly in peak power. Some pulses show almost no amplification while other

**Figure 5.5** – (a) Power spectrum and (b) oscilloscope traces of the amplified signal. The power spectrum also shows the counter-pumped (dotted) and non-amplified (dashed) spectra for comparison. The oscilloscope figure shows 30 overlaid traces plus the non-amplified signal in grey. The RFL power is 4 W and the RFL wavelength 1444 nm.
pulses have been amplified to almost twice the peak power, illustrating the high-contrast nature of the pump noise. A particular observation to note, is, that the amplification between subsequent pulses differs significantly, varying from about 3 dB amplification for one pulse to almost zero amplification for the next pulse, as can be seen in the inset of figure 5.5(b) which shows a single-shot trace of the amplified signal.

The presence of pulses which experience no amplification can only be explained by the fact that the instantaneous pump power co-propagating with these pulses must be close to zero. Similarly the pulses with an amplification which is much larger than average must be propagating with a pump power which is much higher than the average pump power.

It is therefore evident that the pump exhibits high-contrast fluctuations. Because the large differences between the amplification factors occur between subsequent pulses we can further conclude that the fluctuations must be larger than the signal repetition rate of 3.8 MHz. Additionally, as we do not see any evidence of noise transfer in the counter-propagating configuration, we can conclude that the contribution of pump noise with a timescale lower than $f_c$ in equation 5.1 is negligible. As $f_c$ calculates to 1 MHz this does not however give us additional information over the specifications of the laser and fast photodiode measurements of the pump noise which do not show significant amount of noise at timescales of 20 GHz. In the next section we will discuss a method to achieve a better indication of the timescale of the fluctuations.

### 5.4 RIN measurements for varying walk-off

The results in the previous section provide strong evidence for high-contrast intensity fluctuations of the RFL. However the laser specifications and fast-photodiode measurements do not show significant evidence of noise of timescales up to 1 GHz or 20 GHz respectively. The frequency of the fluctuations must therefore be larger than the maximum frequency of the traditional measurement methods. In this section we therefore present a method to measure the frequency of the RFL noise by more rigorous measurements of the RIN of the signal for varying pump wavelengths, i.e. different walk-off between pump and signal.
From the dependence of the RIN on the walk-off we can then deduce the timescale of the fluctuations. For the fibre length in the previous section we had to vary the RFL wavelength significantly to achieve a walk-off of several pulse widths which is necessary to observe a significant change in RIN transfer. However by changing the wavelength of the RFL we are also changing the Raman gain of the signal because the pump-to-signal separation varies. Additionally the WDM in the setup is wavelength dependent and changing the RFL wavelength also changes the pump power inside the fibre. The range of the RFL wavelengths is thus limited by the Raman gain shape and the transfer-function the WDM. In order to achieve significant walk-off for the available RFL wavelength range we increased the length of the LEAF to 1078 m in this experiment. Due to the longer length of fibre both signal and pump power have to be reduced compared to the previous section to avoid nonlinear effects. The exact values for the powers are given with the corresponding experimental result. The output was measured with a fast photodiode and analysed using a Tektronix DPO 4104 fast digital sampling oscilloscope. The oscilloscope has a bandwidth of 1 GHz and a sampling rate of 5 GS/s. To measure the noise on the pulse train we take a long oscilloscope trace with a width of 200 µs containing about 700 pulses. We use an algorithm to detect the pulse maxima and calculate the mean and variance of the peak power of the pulses. The RIN of the signal can be calculated as the ratio of the variance over the mean squared [171]:

\[ r_s = \frac{\langle \delta P^2_s \rangle}{P^2_{s0}} \]  

(5.2)

Because the coupling efficiency of the WDM is wavelength dependent and the Raman gain of the signal also depends on the wavelength of the pump laser it is important to compensate for these effects in order to achieve an similar average gain over all the pump wavelengths. Figure 5.6 depicts the RIN as a function of pump wavelength where two different techniques were used to compensate the changing average gain when the RFL wavelength is varied. In figure 5.6(a) the power of the RFL was adjusted so that the average signal power at the position of the photodiode was constant. The power of the signal was measured using a slow power meter in this case. However further investigation revealed that there was
Figure 5.6 – RIN versus pump wavelength with two different techniques for compensating the changing gain. (a) Keep the average output power of the signal constant, the signal was measured with a power meter, (b) measure the average output power of the signal with an OSA and adjust pump accordingly. In (b) an additional FBG and a 1430/1530 and 1460/1530 WDM are used to achieve better separation of the signal from the pump. The input power of the signal into the fibre was (a) 2 µW and (b) 6 µW.

still a non-negligible amount of stray pump light present after the free-space propagation. The remaining pump light did therefore distort the average power measurements as it accounted for a significant proportion of the overall average power. In order to separate the remaining pump from the signal we added two additional WDMs and a broadband fibre Bragg grating (FBG) in front of the photodiode. The FBG consisted of a broadband reflection spectrum around the signal wavelength and was coupled into the setup with a polarization insensitive circulator. The output was monitored with an optical spectrum analyser (OSA) and only the average power in the signal band was measured using the same OSA. The pump power was then adjusted to yield a constant signal power. Note that this method required a higher average signal power due to the increased loss stemming from the additional components. Figure 5.6(b) depicts the results from this measurement. We see that the two measurements display similar characteristics. The RIN is largest at a pump wavelength of around 1449 nm and the −3 dB wavelength is at about 1445 nm on the low wavelength side. The high wavelength side of the maximum the RIN seems to be exhibit slightly
larger RIN and the $-3$ dB value is around 1455 nm. We should note that the absolute values of the RIN are not directly comparable between the graphs, due to the different pump and signal powers that were required by the different techniques. The characteristics of the two graphs are comparable however. Although the power measurements for figure 5.6(a) were distorted by the presence of residual pump light, the graph is qualitatively similar to figure 5.6(b) we can therefore conclude that the change in RIN is dominated by the walk-off and not by small differences in pump power and gain.

The timescale of the pump noise can be calculated when we convert the values for the pump wavelength into the walk-off parameter $d_{sp} = \beta_1(\text{signal}) - \beta_1(\text{pump})$. In figure 5.7(a) and 5.7(b) we show the RIN from figure 5.6(b) as a function of walk-off parameter which was calculated using measured dispersion parameters and the dispersion parameters specified by the fibre manufacturer respectively. We can see that although the maximum in both fig-

![Graph](image)

**Figure 5.7** – RIN as a function of walk-off parameter calculated using (a) the measured dispersion and (b) the dispersion according to manufacturers specifications. It is evident that the two set of dispersion parameters result in a shift of the zero walk-off length.

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bly explains the difference between theoretical position of the maximum and experimental observations. Considering the differences between measured and specified dispersion the position of the RIN maximum can be assumed to be at zero walk-off within the measurement error of the actual dispersion. Calculating the 3 – dB-bandwidth from the two graphs results in $d_{\text{FWHM}} = 16.9\ \text{ps}$ and $d_{\text{FWHM}} = 18.1\ \text{ps}$ for figure 5.7(a) and 5.7(b) respectively. Using similar considerations as presented in section 5.3 we calculate a corner-frequency for the low-pass filter induced by the walk-off [171] as,

$$f_c = \frac{\alpha_p}{2\pi D \Delta \lambda}.$$  \hspace{1cm} (5.3)

Here $\alpha_p = 1/L_{\text{eff}}$ is the fibre attenuation as seen by the pump, $D$ is the dispersion and $\Delta \lambda$ is the wavelength separation between signal and pump. $\alpha_p/(D \Delta \lambda)$ corresponds to the walk-off between the signal and pump. We can use this formula to calculate the timescale of the pump noise. Using $d_{\text{FWHM}}/2$ we calculate the corner frequency to be approximately $f_c \approx 20\ \text{GHz}$. The above formula is however only valid for long fibre lengths in the range of several kilometres. For shorter fibre lengths Fludger et al. predict that we should observe periodic dips superimposed on the RIN transfer-function. Examining figure 5.6(a) and (b) again, we indeed observe dips in the RIN of the signal. To explain the dips lets assume a periodically modulated pump. A signal propagating with a pump will experience a transfer of the pump modulations depending on the walk-off. However if the signal walks through an exact integer number of pump modulations it experiences no net transfer of the modulations, thus no noise is transferred to the signal. Note that although the pump modulation is random in the experiment these considerations are still valid if the pump fluctuations have a dominant frequency, resulting in slightly reduced RIN transfer if the pulses walk through an integer multiple of the dominant frequency of the fluctuations. Using the formula for the position of the dips

$$f_n = \frac{N}{D L \Delta \lambda},$$ \hspace{1cm} (5.4)

where $N$ is an integer, we can calculate a second estimate of the frequency of the pulse fluctuations. The position of the first dip appearing at a walk-off between 12 ps and 15 ps,
results in an approximate frequency of $\sim 70$ GHz.

The results show that the RFL laser does indeed exhibit very fast intensity noise, however the timescale of $20 - 80$ GHz is not quite as fast as indicated by the width of the spectrum (150 GHz). The theoretical considerations used for the calculations strictly only apply to small fluctuations of the pump [171], nevertheless they provide strong evidence about the nature of the fluctuations. In the next section we will perform numerical simulations to investigate the validity of the above considerations for high-contrast fluctuations. In particular we will examine the behaviour with respect to pump depletion.

5.5 Numerical analysis

For a better understanding of the experimental results we perform numerical simulations using a simple model of the evolution of the signal and pump power. One aspect of particular interest is the effect of depletion of the instantaneous pump power. Consider a train of pulses that is amplified by a noisy pump and both pump and pulse train propagate at the same group velocity. If the pump noise is of high-contrast, the temporal distribution is composed of parts with very high and very low instantaneous power. Signal pulses coinciding with low instantaneous pump power quickly deplete the instantaneous pump power and the RIN transfer-function changes. Depending on their power, the pulses can deplete parts of low instantaneous pump power even if the walk-off between pump and signal is not zero. The curve of the RIN transfer as a function of walk-off could thus be significantly altered. The key parameter to quantify the influence of instantaneous pump depletion is the signal power. A low signal power only leads to the depletion of instantaneous pump powers which are very small, and for a larger walk-off pump depletion is negligible due to the relatively large duty cycle of the signal pulse train. However high power signal pulses can significantly deplete the instantaneous pump even if its power is relatively large. In particular high power pulses can lead to pump depletion even for significant walk-off. The effect of pump depletion on the RIN transfer in fibre amplifiers has been studied previously, however only in the context of small pump modulations [173].
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For the model we consider a signal with power $P_s$ and a pump with power $P_p$ propagating along a fibre in a coordinate system travelling with the signal. If both waves propagate in the linear regime individually and we ignore all higher-order dispersion effects we can write the equations for the evolution of pump and signal power as [173]:

$$\frac{\partial P_s}{\partial z} = -\alpha P_s + C_R P_s P_p$$  \hspace{1cm} (5.5)

$$\left(\frac{\partial}{\partial z} - d_{sp} \frac{\partial}{\partial t}\right) P_p = -\alpha P_p - C_R P_s P_p$$  \hspace{1cm} (5.6)

Here we assumed the loss parameter $\alpha$ to be the same at the signal and pump wavelength. $C_R$ is the Raman gain coefficient and $d_{sp} = \beta_1(\lambda_{\text{signal}}) - \beta_1(\lambda_{\text{pump}})$ is the walk-off coefficient.

Note that $P_s$ and $P_p$ are functions of time, i.e. they are modelled with a temporal distribution. The signal field is modelled by a sech$^2$-shaped pulse with a pulse width similar to the width of the pulses from the EDFL in the experiment. The simulation time window was chosen to match the inverse of the repetition rate of the signal, we thus simulate the propagation of a single signal pulse only. The pump laser was modelled by a sech$^2$-shaped spectrum with a FWHM of 150 GHz multiplied by a random phase similar to the method used by Vanholsbeeck et al. to model the laser for supercontinuum generation [48]. The temporal distribution of the pump field is created through a Fourier transformation of the spectrum and a typical distribution looks very similar to figure 5.1(a). All other parameters were chosen similar to the experiment. The simulations are carried out using the well known split-step propagation algorithm [34] and the numerical step-size was chosen in such a way, that the pump field does not move more than half a pulse width from one propagation step to the next. To simulate the noise transfer to a train of pulses we performed 50 simulations with a different initial random phase of the pump spectrum for each value of the dispersion or walk-off parameter and calculated the noise statistics from the 50 resulting pulses. Figure 5.8 depicts the RIN as a function of pump wavelength and walk-off, using the specified dispersion parameters. Although the qualitative behaviour of the numerical results is similar to the experimental observations, there are some quantitative differences. The discrepancy of the zero walk-off wavelength can easily be attributed to the uncertainty
Figure 5.8 – (a) RIN as a function of wavelength (b) RIN as a function walk-off for the specified dispersion parameters. Other simulation parameters are: average pump power $P_{\text{pump}} = 0.8$ W, average signal power $P_{\text{pump}} = 6$ $\mu$W all other parameters as the experimental ones.

in the experimental dispersion parameters as has already been pointed out in the previous section. However the maximum RIN is about 5 dB larger in the numerical simulations than in the experimental traces and the numerical results also display a stronger decrease of the RIN for increasing walk-off and the minimum RIN is significantly lower than in the experiment. The $-3$ dB walk-off width of the RIN is also about 12 – 14 ps compared to the experimentally observed 17 – 18 ps. However, there are significant uncertainties with respect to the experimental parameters as well as the assumptions of the model. The numerical model does not include any initial RIN of the signal and ignores any nonlinear effects which might influence the RIN, this could possibly explain the significant difference in the observed RIN for large walk-offs between numerical simulations and experiments. Furthermore the exact power of the signal pulses at the input of the fibre is only known approximately in particular the peak power of the pulses. As we have pointed out earlier, a change of the signal power can significantly alter the shape of the measured RIN curve, due to the effect of instantaneous pump depletion.

To observe the effect of a change in instantaneous pump depletion, we conducted additional simulations with a slightly higher average signal power (9 $\mu$W). Comparing the new high signal power simulations with the previous simulations, should give us insight into the
impact of varying instantaneous pump depletion. The RIN as a function of wavelength and walk-off is shown in figures 5.9(a) and figure 5.9(b) respectively. The results nicely demon-

![Graphs showing RIN versus pump wavelength and walk-off](image)

Figure 5.9 – (a) RIN as a function of wavelength (b) RIN as a function walk-off for the specified dispersion parameters and a higher signal power. The simulation parameters are pump power $P_{pump} = 0.8$ W, signal power $P_{pump} = 9 \mu W$ all other parameters as the experimental ones.

strate the effect of increased instantaneous pump depletion on the RIN transfer. If there is no walk-off the pulses propagating with a low instantaneous pump power deplete the pump upon propagation. If the signal power is larger, parts of the pump with a higher instantaneous power are depleted as well. Additionally instantaneous pump depletion can occur even for small walk-off values. The pump depletion causes an equalisation of the transferred noise, thus the calculated RIN value is smaller. We therefore expect the RIN for zero and small walk-off values to reduce. However, the effect of pump depletion reduces with walk-off as the signal “walks through” the low instantaneous pump power parts before it can deplete them. We should therefore observe a flattening of the peak of the graph in figure 5.8. The expected behaviour is nicely reflected in the graphs in figure 5.9. The peak RIN value reduced from about $-5$ dB for the low signal power simulations, to about $-10$ dB in the high signal power ones and the value of $-10$ dB agrees much closer with the experimental measurements. Additionally we observe the expected flattening of the curve. The graph appears broader because the RIN does not decrease significantly for larger walk-off
values, while the peak RIN reduces. The 3 dB walk-off width of 20 ps is also much closer to the experimentally measured results.

The results incorporating a slightly higher signal power, show significantly improved quantitative agreement with the experimental observations. The discrepancies are within the uncertainties of the experimental parameters, which are quite high, in particular because the effects of less pump power and a slightly higher signal power are cumulative and parameters such as the Raman gain coefficient also affect pump depletion. Furthermore the numerical simulations demonstrate that pump depletion significantly alters the RIN transfer thus complicating the deduction of quantitative RIN values for the pump from the signal measurements. Finally it highlights the limitations of the theoretical considerations in the previous section. The pump fluctuations are calculated from a spectrum with a width of 150 GHz which determines the mean timescale of the fluctuations. However, the theoretical considerations point to a timescale between 20 and 70 GHz, about half of what is expected. This clearly demonstrates that the theory developed for small pump modulations is only limitedly applicable to high-contrast fluctuations and to gain a more accurate estimate a new theory that takes pump depletion into account is needed.

Finally, we would like to stress again the comparison of our results to the investigation of RIN transfer under the effect of pump power reported by Mermelstein et al. [173]. Although the authors also observe a decrease in the maximum RIN transfer, their results are only applicable to small modulations of the pump compared to the high-contrast fluctuations present here.

5.6 Summary and discussion

In this chapter we presented a novel method for measuring ultra-fast (> 50 GHz), high-contrast intensity noise of a cw Raman pump laser. The technique involves transferring the fast noise from the pump to a low repetition rate mode-locked signal laser by Raman amplification in a zero walk-off configuration. The fast pump noise cannot be measured using traditional RF-pulse measurement techniques, due to the limitations imposed by electronic
Chapter 5  Fast intensity fluctuations of a Raman fibre laser

bandwidths. However the transfer of the noise to the low repetition rate signal re-enables us to utilize common RF RIN measurement techniques. We have shown that our cascaded cw RFL does indeed exhibit high-contrast fluctuations by examining the transferred noise on the signal in oscilloscope traces. The noise measurement technique can further be enhanced by varying the pump wavelength thus enabling us to measure the timescale of the pump noise. A theoretical estimate of the timescale based on a small pump modulation model, places the frequency of the pump noise in the range of 20 – 70 GHz. Numerical simulations from a more accurate model which includes pump depletion, confirm that once pump depletion is taken into account, the observed measurements are consistent with random intensity noise from a 150 GHz bandwidth laser with a random phase. Ultra-fast, high-contrast fluctuations similar to the ones observed in our laser are very likely to be present in a broad range of cascaded cw RFLs. The measurements also affirm the observation of reference [48] that high-contrast pump noise plays a crucial role in the process of cw SC generation and choosing a pump source can have significant implications on the width and the quality of the generated SC. The numerical simulations also uncovered the limitations of the theoretical model and the experiment in general. We see a significant discrepancy between the timescale calculated using the theory and the numerical timescale. The theoretical model fails to give accurate estimates of the frequency of high-contrast fluctuations, as it fails to account for the depletion of the instantaneous pump power. The numerical simulations demonstrate, that due to the high-contrast nature of the pump fluctuations, the depletion of the instantaneous pump plays a significant role in the RIN measurements. A small change in the average signal power, the average pump power or the gain can decisively alter the RIN measurements and thus the estimate of the frequency of the pump noise. This also imposes further limitations on the accuracy of the experimental measurements, because the uncertainties of the experimental parameters introduces significant difficulties for estimating the exact timescale of the fluctuations.

Let us also point out the delicate balance of the experimental parameters for designing the experiment. On one hand it is desired to achieve a large walk-off for small changes of the RFL wavelength to minimize the impact of gain changes due to the transfer-function of the
WDM and the changes in Raman gain for the signal. Thus a long fibre length is advantageous. However, a longer fibre length results in higher fibre loss, i.e. a smaller signal power at the output, thus increasing the difficulties of detecting the signal. Additionally a longer fibre also exacerbates the effect of nonlinearities. For the noise transfer technique to provide reliable results, the pump and signal must not experience significant nonlinear transformations when propagating individually. The ability to increase the signal strength or the pump power to compensate for the higher fibre loss is therefore considerably limited and a decrease in signal and pump power is often required, further enhancing the difficulties of detecting the signal.

Nevertheless, despite the difficulties and uncertainties in the measurements our experimental measurements proof that cascaded, cw RFLs exhibit ultra-fast, high-contrast intensity fluctuations. Although we are not able measure the dominant frequency of the RFL intensity noise with high accuracy, we obtain a reasonable estimate for the timescale to be around 100 GHz. To further increase the accuracy of this technique requires an improved theoretical model which takes into account pump depletion and is not limited to small modulation depth fluctuations only. The emerging use of ultra-fast repetition rate lasers should provide further incentive for further research into pump to signal noise transfer in the case of gigahertz, high-contrast pump fluctuations and the possible development of such a model.

Finally, we want to point out some recent results published by Hammani et al. [208]. The authors use a technique similar to ours to demonstrate the occurrence of so-called “rogue-waves”, i.e. very large intensity spikes of the signal, in pump-to-signal noise transfer with Raman amplification. Although the authors point out significant differences between the statistics for a large and small walk-off they do not utilize a zero walk-off configuration, as both signal and pump propagate in the normal dispersion regime. It would be interesting to investigate the dependence of the amplified signal statistics on the walk-off between pump and signal using our data, this is however beyond the scope of this thesis.
Concluding remarks

The aim of this thesis was to investigate nonlinear effects in optical fibres at the boundary between parametric and non-parametric processes. We have concentrated on three aspects of the interaction between two of the key phenomena in nonlinear fibre optics, i.e. Raman scattering and four-wave mixing. The investigations ranged from the fundamental, an examination of simultaneous four-wave mixing processes in the presence of Raman scattering, to more applied topics, i.e. the combined use of four-wave mixing and Raman scattering in a passively mode-locked Raman fibre laser and the measurement of ultra-fast, high-contrast intensity noise of a continuous wave Raman fibre laser.

In the following we will give a brief overview of the key results for each of the three topics treated in this thesis. Furthermore we will identify directions of interests as well as open questions for future research efforts.

- In chapter 2 on four-wave mixing processes we conducted experiments which demonstrated that multiple, independent four-wave mixing processes can exist simultaneously despite the stringent requirements of phase matching. We could identify a number of created waves resulting from a mixture of phasematched and non-phasematched four-wave mixing processes. New frequency components were created from spontaneous modulation instability both in the normal and anomalous dispersion region, from a phasematched four-wave mixing process between a strong signal a pump within an amplified spontaneous emission noise band and a newly created idler wave and from non-phasematched, degenerate four-wave mixing between the same signal wave, a narrow pump and a second created idler wave. The presence of
the second idler wave in particular can be attributed to the presence of Raman scattering assisting the phase-matching to create gain at the idler frequency. The experimental results are the first observation of this process competing with a simultaneous phasematched process.

- The presence of the idler wave created by four-wave mixing with the incoherent pump wave from the amplified spontaneous emission band prompted a further investigation of four-wave mixing with an incoherent pump. The experimental results reveal that when a signal wave is launched in combination with a broadband incoherent pump with a centre wavelength close to the zero-dispersion wavelength of the fibre, an idler wave is created through four-wave mixing. The result in itself is not obvious, because most theoretical descriptions of four-wave mixing presume idealized, perfectly coherent, continuous wave components. Furthermore we could show that the process is effective even for relatively low pump or signal powers, over a broad range of signal wavelengths, as is highly desirable for wavelength conversion applications. Additionally the four-wave mixing process can yield an idler wave with a smaller bandwidth than the signal wave, attributed to the narrow gain bandwidth for four-wave mixing due to fourth-order dispersion when the pump is in the normal dispersion regime. The results point to an interpretation that the choice of the signal wavelength determines a pump frequency inside the band of the incoherent amplified spontaneous emission spectrum, which fulfils the phase matching requirements for a degenerate four-wave mixing process which creates the idler wave. The process is very effective for a broad range of signal wavelengths, because the pump frequency inside the incoherent amplified spontaneous emission band needs to change only slightly for large changes of the signal wavelength. If the zero-dispersion wavelength of the fibre is known, this method can also in principle be used for measurements of the ratio of $\beta_3$ to $\beta_4$ by relating the pump detuning to the position of the created idler.

- The main research effort during this thesis has gone into the design and implementation and characterisation of a passively mode-locked Raman fibre laser. We demon-
strated that by combining the advantages of Raman scattering as a gain process with the passive mode-locking technique of dissipative four-wave mixing, we are able to create a laser with repetition rates of up to 500 GHz and average output powers of almost one Watt. The experimental results show the average output power to depend linearly on the pump power and the slope efficiency of the laser can be increased up to 30% by the use of different output couplers. Furthermore we showed that pumping in a backward configuration is highly advantageous compared to forward pump, yielding higher output powers and a much better pulse quality. The theoretical and numerical description of the laser reproduces many of the observed phenomena and offers improved insights into the laser dynamics compared to the original model by Quiroga-Teixeiro [39]. In particular the numerical model explains that the experimentally observed asymmetry of the spectrum can be attributed to a combination of intrapulse, stimulated Raman scattering and third-order dispersion. Both the numerical and experimental model also identify several stability regimes of the laser: A stable regime where a continuous pulse train is created, two semi-stable regimes with periodic and quasi-periodic pulse fluctuations and finally an unstable regime where the laser output becomes seemingly chaotic. Although these regimes could not be conclusively observed in experiments, their presence is indicated by the appearance of subpulses in the autocorrelation traces.

- After demonstrating the successful mode-locking of the laser, we identified the presence of supermode noise as one of the key hindrances for adoption in applications. Because laser operates at repetition rates beyond the capabilities of traditional noise measurement techniques, we had to resort to numerical modelling to gain indirect information about the laser dynamics with respect to supermode noise. The novel numerical model taking into account the presence of supermodes reproduced a number of key characteristics of the experimental results which could not be observed in the noise-less model. In particular we did reproduce the envelope over the autocorrelation traces and the exponential decrease of the pulse width as a function of pump power. In addition we performed measurements with a Raman laser mode-locked at
Concluding remarks

a lower repetition rate, to evaluate the effectiveness of a supermode noise suppression technique based on adding subcavities functioning like additional Fabry-Perot filters. We could show that the amount of supermodes can be reduced by more than a factor of a hundred through the Vernier-superposition of two subcavities with a ratio of 20/21, eliminating the majority of supermode noise.

- The last results of this thesis we demonstrated that contrary to common conception continuous wave Raman fibre lasers exhibit strong, high-contrast, ultra-fast intensity fluctuations. Because the fluctuations cannot be measured using traditional noise measurement techniques as they exceed electronic bandwidths, we used a noise transfer technique. The noise from the Raman laser is transferred to a low repetition rate mode-locked laser by Raman amplification in a zero walk-off configuration. A measurement of the timescale of the fluctuations is carried out by varying the walk-off between the pump and the signal. The results show that fluctuations are on a timescale about 20 – 80 GHz, in the same region as the timescale of 150 GHz deduced from the spectral width of the Raman laser. However numerical simulations demonstrate the detrimental effect of depletion of the instantaneous pump power, which is difficult to avoid.

Although we have gained valuable insights into the research areas presented in this thesis, there are a number of open problems and questions which could not be addressed. Let us now mention some of these questions explicitly, so that they can be used as links for research in the future.

Although four-wave mixing has been the subject of extensive research, we could show that there still are a number unanswered problems. For instance, regarding the question of four-wave mixing with an incoherent pump wave, the interpretation of the underlying process by Chavez Boggio et al. [96] differs significantly from our own interpretation. There clearly is a need for further investigations into the exact nature of the process. Furthermore the work of Sauter et al. [94], predicts a dependency of the shape of the modulation instability gain on the coherence of the pump which has not been experimentally investigated. The
influence of pump coherence on four-wave mixing still also is still not well understood and more research is needed to clarify the underlying mechanisms. The process becomes even more complicated if we include the influence of Raman scattering and when signal and pump wave are of similar powers.

The main problem of the passive mode-locked Raman fibre laser remains the presence of supermode noise. Although the amount of supermode noise can be significantly reduced by including subcavities to the setup, their inclusion introduces a susceptibility to environmental fluctuations, rendering the laser temporally unstable. Nevertheless we are confident that this problem can be overcome or reduced with a feedback mechanism which adjusts the length ratio of the subcavities and this could be one of the first steps to tackle the problem. With improvements of the fibre Bragg grating fabrication techniques the bandwidth of the reflection bands may be reduced, making subcavities possibly obsolete. Additionally the advancements on the subject of tunable gratings [165] could make a tunable laser feasible. A different approach to supermode noise reduction is to shorten the cavity length. This could be achieved by using stronger pump sources and fibres with a larger nonlinearity, or by using other gain materials such as Ytterbium which has produced lasers with cavity length of several centimetres [134] (however the question of sufficient nonlinearity would still remain). In particular implementing the laser using semiconductor optical amplifiers is very intriguing, as these provide a very large gain and nonlinearity in a length of millimetres [209]. Finally we should say that at the current level the amount of supermode noise makes this laser unsuitable for most applications. In particular applications in the telecommunications field rely on low intensity noise and low timing jitter. Currently the passively mode-locked Raman laser can not compete with the current techniques used in this area, such as active modulation of the output from distributed feedback lasers. Nevertheless the current drawbacks can be overcome and the laser will become more suitable for applications especially in areas where laser noise is less crucial.

Measurements of ultra-fast fluctuations at timescales exceeding the electronic bandwidths remain a problem. Although the noise transfer method we implemented was successful in moving the noise to the slow signal, the interpretation in particular with respect to the
timescale of the fluctuations remains difficult, due to the influence of the depletion of the instantaneous pump. In order to better interpret the results a theoretical description of noise transfer for the case of high-contrast fluctuations is necessary.

Let us conclude by saying that this work on the boundary of fundamental research and application has improved our understanding of four-wave mixing, Raman scattering and their interplay. At the same time it has provided us with some key pointers for the design and development of a laser which is able to fulfil the requirements of future telecommunication networks. In this light we should view the unanswered questions which arose as opportunities to pursue, to find new answers. And these answers in turn will uncover more questions. I think science is as much about finding new questions as it is about finding the answers and fortunately we do not seem to be running out of questions any time soon.
In the following we provide the fibres parameters and parameters for the fibre Bragg gratings which were used in the experiments in this thesis.

### A.1 Fibre parameters

<table>
<thead>
<tr>
<th>Fibre</th>
<th>Manufacturer</th>
<th>Length [m]</th>
<th>$\lambda_0$ [nm]</th>
<th>$D \text{ @ } 1550 \text{ nm [pskm}^{-1}\text{nm}^{-1}]$</th>
<th>$D_s \text{ @ } 1550 \text{ nm [pskm}^{-1}\text{nm}^{-2}]$</th>
<th>$\gamma \text{ [Wkm}^{-1}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>HNLF 1</td>
<td>Sumitomo</td>
<td>1000</td>
<td>1555</td>
<td>$-0.16$</td>
<td>0.031</td>
<td>14 – 20</td>
</tr>
<tr>
<td>HNLF 2</td>
<td>Sumitomo</td>
<td>500</td>
<td>1551.4</td>
<td>$-0.046$</td>
<td>0.033</td>
<td>11.8</td>
</tr>
<tr>
<td>DSF</td>
<td>Alcatel</td>
<td>3100</td>
<td>1549.2</td>
<td>0.057</td>
<td>0.071</td>
<td>10</td>
</tr>
<tr>
<td>LEAF (specified)</td>
<td>Corning</td>
<td>1000</td>
<td>1495.3</td>
<td>4.44</td>
<td>0.08</td>
<td>1.3</td>
</tr>
<tr>
<td>LEAF (measured)</td>
<td>Corning</td>
<td>1000</td>
<td>1486.3</td>
<td>4.81</td>
<td>0.075</td>
<td>1.3</td>
</tr>
</tbody>
</table>

**Table A.1** – Parameters for the fibres used in the experiments in this thesis.

### A.2 Fibre-Bragg grating parameters

<table>
<thead>
<tr>
<th>Name</th>
<th>FSR [GHz]</th>
<th>$\nu_{\text{FWHM}}$ [GHz]</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>FBG 1</td>
<td>100</td>
<td>250</td>
<td>A.1</td>
</tr>
<tr>
<td>FBG 2</td>
<td>160</td>
<td>400</td>
<td>N/A</td>
</tr>
<tr>
<td>FBG 3</td>
<td>500</td>
<td>500</td>
<td>A.2</td>
</tr>
<tr>
<td>FBG 4</td>
<td>10</td>
<td>25</td>
<td>A.3</td>
</tr>
</tbody>
</table>

**Table A.2** – Table of the fibre Bragg gratings used in the DFWM experiments
Chapter A Specifications

Figure A.1 – Reflection spectrum of FBG 1.

Figure A.2 – Reflection spectrum of FBG 3.
Figure A.3 – Reflection spectrum of FBG 4.
In this appendix we discuss some of the numerical methods used throughout this thesis. In particular we present the technique for solving the nonlinear Schrödinger type equations which describe the light propagation inside the laser cavity.

Although there exist many different approaches for solving the nonlinear Schrödinger equation (NLSE) the most commonly used method is the so-called split-step Fourier method [210], which was also adopted for the numerical simulations in this work. The split-step Fourier method is a so-called pseudo-spectral method which relies heavily on the fast-Fourier-transform (FFT) algorithm. Because FFTs are extensively used in signal processing they have been strongly optimised and the split-step Fourier method can be up to two orders of magnitude faster than other methods for solving the NLSE. We will not discuss the details of the implementation of the algorithm, but instead the interested reader is pointed to Agrawal’s Nonlinear Fiber Optics [34], which contains an introduction to the split-step Fourier method and serves as an excellent starting point. Instead we discuss some of the subtleties of the implementation with respect to the simulations in this thesis, in particular how to simulate the laser especially the cavity configuration and the contribution of stimulated Raman scattering. Finally we will discuss how convergence of the algorithm has been judged and how appropriate step sizes can be chosen.
B.1 Simulating the laser cavity

The Ginzburg-Landau type equation [equation 3.33] used for modelling the passively modelocked Raman laser differs from a basic NLSE equation by the additional saturated gain and the loss terms. However, because these terms have the form of a simple multiplication of the field by a constant factor (although this factor might vary for each step), they can be easily incorporated into the simulations by adding them to the dispersive step of the split-step algorithm and the strategy for solving the NLSE remains the same. A second difference between the simulation of laser cavity and numerically solving basic NLSE propagation along a fibre is that the light recirculates inside a laser cavity in contrast to propagation along a fibre where it does not. It is therefore necessary to use two numerical loops in the laser simulation, an inner loop for the propagation along the cavity length, and an outer loop to simulate a number of cavity roundtrips. Furthermore once every cavity roundtrip we need to apply the spectral filter which corresponds to the fibre Bragg grating in the experiment. Listing B.1 displays the implementation of the laser cavity simulation in pseudo code.

Listing B.1 – Pseudo code of the numerical procedure to simulate the laser cavity

```plaintext
L = ... # cavity length
roundtripmax = ... # No of cavity roundtrips
h = ... # stepsize
z = 0 # propagation distance
rountrip = 0 # number of roundtrips
field = ... # initialize the optical field
field_ft = FFT(field) # initialize the fourier transform of the optical field
filter = ... # initialize the spectral filter

# further initialization

while roundtrip <= roundtripmax:
    while z <= L:
        field_ft = DispersionStep(field_ft)
        field = IFFT(field_ft) # perform inverse Fourier transform
        field = NonlinearStep(field_ft)
```

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Simulating the laser cavity

```plaintext
field_ft = FFT(field)  # perform Fourier transform
z = z + h
field_ft = field_ft * filter_ft
roundtrip = roundtrip + 1
z = 0
```

Note that inside the DispersionStep and NonlinearStep functions there are possibly a number of operations being performed, such as the multiplication with the dispersion, the calculation of the average power of the field for the gain saturation etc..

### B.1.1 Numerical discretization

The size of the simulation window and the numerical discretization of the field has to be chosen in such a way as to avoid aliasing effects in the frequency domain and simulating a temporal field with a number of pulses of at least about ten. This implies choosing a width of the frequency field several times the width of the spectral filter so that no modes are generated very close to the edge of the simulation window. Furthermore the discretization has to be adjusted such that the maxima of the Fabry-Perot filter coincide with the numerical discretization points to avoid inducing an additional filtering effect due to undersampling. The number of pulses inside the simulated time window is then determined by the number discretization points between two Fabry-Perot filter maxima. Finally let us add that it is advantageous to choose the number of discretization points as a power of two, as this will greatly improve the efficiency of the FFT algorithms, thus significantly reducing simulation time. Typical values chosen in the simulations in chapter 3.6 were $2^{12}$ simulation points and at least 20 points between two Fabry-Perot maxima.

### B.1.2 Calculating the contribution of Raman scattering

After discussing the basic strategy for simulating the laser cavity let us now discuss the specific problem of how to integrate stimulated Raman scattering into the simulations. If we reexamine the equation 3.34, we see that the nonlinear step contains the instantaneous Kerr effect and the contribution of Raman scattering given by the integral. Generally nu-
merically calculating an integral requires specific numerical algorithms which offer different numerical accuracy depending on the algorithm and the type of the integral. However in this case the integral is in fact a convolution between the Raman response function and the field intensity. There are different strategies for calculating the convolution, arguably the simplest method is to calculate it in Fourier space. The convolution thus becomes a multiplication of the Fourier transform of the field intensity with the Raman gain curve. In pseudo code the calculation of the nonlinear step looks like listing B.2

Listing B.2 – Pseudo code of the nonlinear step with Raman scattering contribution

```plaintext
field squared = abs(field) ** 2
Raman = IFFT(Raman_gain * FFT(field squared))  # perform the convolution
field = field * exp(i * h * ((1-fr) * field squared + fr * Raman))
```

Note that due to the normalization in equation 3.34 we do not multiply by the nonlinear parameter $\gamma$.

### B.1.3 Noise

In the experiment the laser signal is seeded by quantum fluctuations and spontaneous Raman scattering, which are then amplified by stimulated Raman scattering. In the numerical simulations it is therefore necessary to initialize the propagation field with a certain amount of noise as otherwise there is no field for the gain to act upon. The initial noise of the optical field is often simulated by injecting a noise seed with the power of one photon per mode [211]. The exact magnitude of the noise is particularly important when one wants to gather quantitative measurements from the numerical experiment. However in the case of the laser, controlling the noise power this precisely is not necessary. The output field of the laser does not depend strongly on the precise initial conditions of the simulations, as the laser solution depends strongly on the inherent saturation mechanism, due to the spectral filtering and the simulation over many roundtrips. The main consequence of increasing the noise strength in the simulations was a slightly faster convergence of the laser to a steady state. We therefore used a uniform distribution of the noise amplitude with a normalized power $|U|^2 \approx 10^{-8} – 10^{-4}$ and a random phase [see equation 3.33].
To accommodate for gain fluctuations we further added a small noise which varied randomly with frequency to the gain term. This term was either recalculated every roundtrip, or taken constant over the whole laser propagation, however we found that in both cases fluctuations of a strength of even about a tenth of the gain strength did not have a significant influence on the steady state output of the laser.

B.2 Convergence and step size

There are a number of strategies for choosing an appropriate step size in split-step simulations, ranging from simple methods using a constant step size, often given by a fraction of the nonlinear length, or the dispersion length to more elaborate schemes such as choosing a logarithmic distribution of the step size [212], or adaptive methods such as the local error method [213]. However we found that there was no significant reduction in simulation time when adopting any of the more elaborate stepsize methods. Instead we found that the best method to choose an appropriate step size was by performing a number of initial simulations at different step sizes. If there was no significant change in the laser behaviour when reducing the step size, we would adopt this step size for simulations with similar parameters. It should also be noted that due to the normalization of the modelling equations the number of steps is significantly smaller than when using a non-normalized model. This method resulted in a typical step size of 0.01 (Note that this value is normalized dimensionless distance).

Generally there is no straight forward criterion for judging the convergence of the laser simulations, as the laser behaviour can converge into a non-stationary solution, thus rendering typical methods which compare the relative error between roundtrips useless. We therefore adopted a more qualitative approach. To ensure the convergence of the laser behaviour a number of around 200 cavity roundtrips was usually simulated. If the laser output had approached a steady state, i.e. the change of the output intensity distribution from roundtrip to roundtrip was minimal, the laser behaviour was assumed to have converged. This was typically judged qualitatively by plotting either the trajectory in phase space or the three
Chapter B  Numerical solution to the nonlinear Schrödinger equation

dimensional plot of the temporal distribution of the output power [see e.g. figure 3.35]. If the laser exhibited variations [such as the ones in figure 3.36] of the output distribution, the number of roundtrips was increased to ensure the variations were not of a transient nature. In all the simulations that were carried out the laser approached a steady state significantly before 200 roundtrips and we did not observe a change of dynamics even when extending the number of roundtrips to up to 500. Although this method can not guarantee that the laser behaviour has converged, it forms a good compromise between computational cost and certainty.


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