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ASSESSMENT OF SEISMIC DAMAGE TO CIVIL STRUCTURES USING STATISTICAL PATTERN RECOGNITION TECHNIQUES AND TIME SERIES ANALYSIS

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Abstract

The ability to estimate seismic induced damage to civil infrastructure is undoubtedly one of the most important challenges faced by structural engineers. In this research two complementary methods of damage estimation using either knowledge of the structure and earthquake or recorded structural responses were investigated. These methods gave different natured estimates, either prediction or detection, which are suitable for different applications. Firstly, damage to a structure was predicted based on analysis of structural and ground motion properties. Secondly, damage to a structure was detected and assessed by analysing the structural response under dynamic excitation.

In the first approach, basic structural and ground motion properties were used to characterise a broad group of structures and earthquakes. These properties were used as inputs into a Back-Propagation (BP) Artificial Neural Network (ANN) and related to a damage index that quantified the extent of damage to the structure. A set of prior structural analyses was required to train the ANN before useful predictions could be made. Applied to 2D Reinforced Concrete (RC) frames, the method was capable of predicting with good accuracy damage to frames of varying stiffness, strength and topology whilst subjected to a range of ground motion severities.

In the second approach, Autoregressive (AR) models were used to fit the acceleration time histories obtained when the structure was in both undamaged and damaged states. The AR coefficients were selected as damage sensitive features and statistical pattern recognition techniques were investigated for interpreting changes in the values of these features caused by damage. Initially, an offline damage detection method was developed in which BP ANNs were used for both classification and quantification tasks where the percentage remaining stiffness at a specific location was estimated. The method was applied to three experimental structures; a 3-storey bookshelf structure, the ASCE Phase II Experimental SHM Benchmark Structure and a RC column. In addition, for damage classification tasks only, the supervised classification techniques of Nearest Neighbour and Learning Vector Quantisation were found to be effective while Self-Organising Maps, an unsupervised classification method, showed promising results. Finally, an online damage detection method was developed based on recursive identification of the AR models using the forgetting factor and Kalman filter approaches. A linear 3-DOF model with time varying stiffness was investigated and the results showed that damage could be detected and quantified as it occurred. Nonlinear damage detection was addressed with the investigation of a 1-DOF bilinear oscillator and a 3-DOF Bouc-Wen hysteretic system. In both cases the on-set of nonlinearity was detected using Outlier analysis.
Acknowledgements

I would to acknowledge the support and contribution of my supervisors, Dr. Piotr Omenzetter and Dr. John Butterworth in overseeing this research. The assistance of Mr. Bastian Vaurigaud in conducting experimental work is much appreciated.

I would also like to acknowledge the efforts of the technical and laboratory staff involved in this research; Mark Byrami, Tony Daligan, Hank Moody, Noel Perinpanayagam and Mark Twiname.

Finally, I would like to express gratitude to The University of Auckland and the Earthquake Commission Research Foundation of New Zealand for the financial support of this research.
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<tbody>
<tr>
<td>ACF</td>
<td>Autocorrelation Function</td>
</tr>
<tr>
<td>ANN</td>
<td>Artificial Neural Network</td>
</tr>
<tr>
<td>AR</td>
<td>Autoregressive</td>
</tr>
<tr>
<td>ARMAX</td>
<td>Autoregressive-Moving Average with eXogenous input</td>
</tr>
<tr>
<td>ARX</td>
<td>Autoregressive with eXogenous input</td>
</tr>
<tr>
<td>ASCE</td>
<td>American Society of Civil Engineers</td>
</tr>
<tr>
<td>BP</td>
<td>Back-Propagation</td>
</tr>
<tr>
<td>CSD</td>
<td>Cross Spectral Density</td>
</tr>
<tr>
<td>DFT</td>
<td>Discrete Fourier Transform</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree Of Freedom</td>
</tr>
<tr>
<td>ERA</td>
<td>Eigenvalue Realisation Algorithm</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>FRF</td>
<td>Frequency Response Function</td>
</tr>
<tr>
<td>IFR</td>
<td>Impulse Response Function</td>
</tr>
<tr>
<td>LVQ</td>
<td>Learning Vector Quantisation</td>
</tr>
<tr>
<td>MA</td>
<td>Moving Average</td>
</tr>
<tr>
<td>MAC</td>
<td>Modal Assurance Criterion</td>
</tr>
<tr>
<td>NN</td>
<td>Nearest Neighbour</td>
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<tr>
<td>PCA</td>
<td>Principal Component Analysis</td>
</tr>
<tr>
<td>PCF</td>
<td>Partial Autocorrelation Function</td>
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<tr>
<td>PEM</td>
<td>Prediction Error Method</td>
</tr>
<tr>
<td>PGA</td>
<td>Peak Ground Acceleration</td>
</tr>
<tr>
<td>PGD</td>
<td>Peak Ground Displacement</td>
</tr>
<tr>
<td>PGV</td>
<td>Peak Ground Velocity</td>
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<tr>
<td>PSD</td>
<td>Power Spectral Density</td>
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<tr>
<td>RC</td>
<td>Reinforced Concrete</td>
</tr>
<tr>
<td>SHM</td>
<td>Structural Health Monitoring</td>
</tr>
<tr>
<td>SI</td>
<td>Spectrum Intensity</td>
</tr>
<tr>
<td>SOM</td>
<td>Self-Organising Maps</td>
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Notation

The following notation is used throughout this thesis:

0 null matrix
A state matrix
A cross-sectional area
a AR coefficient
B input matrix
b exogenous coefficient
C damping matrix, output matrix
Cov covariance operator
c MA coefficient, viscous damping coefficient
D feedthrough matrix
D Park and Ang damage index, distance between vectors
d vector of desired ANN outputs
E selection matrix
E error, expectation operator
e error vector
e error
F discrete Fourier transform of force, frequency, force
f frequency
H Hankel matrix, covariance matrix of noise
H frequency response function
I identity matrix
I second moment of area
Im imaginary part
J Jacobian matrix
K stiffness matrix
k stiffness
L gain matrix
M mass matrix
m codebook vector
m mass
na AR order
nb exogenous input order
nc MA order
o vector of ANN outputs
P estimated covariance matrix
P Partial Autocorrelation Function
Q covariance matrix of noise
q backshift operator
R matrix of singular vectors
R autocorrelation function, cross-correlation function
Re real part
r bilinear factor
S matrix of singular vectors, sensitivity matrix
S power spectral density, cross-spectral density, spectrum response
T transition matrix
T natural period
t time
u state-space input vector
u weighted sum of inputs, displacement, input
V matrix of singular vectors
Var variance operator
w ANN weights vector
X discrete Fourier transform of response
x state vector, feature vector
x input, excitation
Y matrix of previous time series output
y output vector, vector of current time series outputs
y output, time series value
Z measurement matrix
z principal component

α plastic angle, model parameter for Takeda hysteresis, Rayleigh damping, LVQ, SOM, Bouc-Wen hysteresis
β model parameter for Takeda hysteresis, Rayleigh damping, Bouc-Wen hysteresis
Φ matrix of mode shapes
ϕ mode shape
φ vector of previous time series values
γ covariance
Λ matrix of eigenvalues, matrix of singular values
λ iteration parameter, eigenvalue, forgetting factor
θ parameter vector of time series model coefficients, vector of updating parameters
ρ Autocorrelation Function
Σ covariance matrix, matrix of singular values for ERA
σ standard deviation
τ time
ω natural radial frequency
ξ damping ratio

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|| absolute value

Subscripts:
a analytical
accel accelerometers
c continuous time system, closest codebook vector, complex number
comp complex number
crit crucial
E Euclidean metric
e experimental
f input
g gross area, gross second moment of area
i index, iteration step
j index
\( k \) time step, index, iteration step
\( M \) Mahalanobis metric
\( m \) measurement
\( r \) input
\( \text{real} \) real number
\( s \) sampling frequency
\( V \) spectral velocity
\( x \) output

Superscripts:
\( T \) matrix transpose
\( ^\sim \) estimated value
\( - \) mean value
\( + \) pseudoinverse