A Consensus Value Approach for Influence Maximization in Social Networks

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***Abstract*—Information diffusion has been a crucial issue in social network research. The influence maximization problem asks for ways to identify nodes that are able to maximally diffuse information through a network. Recently, game-theoretical methods emerged that partitions a network into coalitions, and facilitate influence maximization. A notable example is a method based on Shapley value [13]. A major disadvantage with this approach is the lack of consideration of externality on coalition formation. In this paper, we propose *consensus value for influence maximization* (CVIM) to overcome this limitation. After forming coalitions using consensus value, the method selects initial coalitions based on the gain of each coalition, and finally chooses seed nodes in the selected coalitions. Empirical evidence over several real-world networks demonstrates that CVIM produces better results of influence maximization while significantly reducing the running time as compared to existing approaches.**

***Index Terms*—Consensus value; Coalitional game theory; Influence maximization; Social network**

I. Introduction

Online social networks such as Facebook, Twitter and Weibo play a vital role in people's daily lives. Rapid expansion has made these social platforms important channels of information and communication. Through "word of mouth" and "viral marketing", any individual has the potential to influence public knowledge, opinion and taste. In the case of viral marketing, a natural question arises as to how the marketer may effectively capitalize influence through social networking. One strategy involves picking a well-selected sets of individuals (i.e., the seeds) to target on, in order to initiate a cascade of information through the social network. The selection of a fixed number of seeds which maximize the effect of the information cascade is called the *influence maximization problem* [1].

Influence maximization has been extensively studied. Richardson and Domingos first introduced the problem to social network research [1]. The celebrated work of Kempe, Kleinberg and Tardos formalized influence maximization problem as a discrete optimization problem. They show that the problem is NP-hard, and proposed a greedy algorithm with an approximation guarantee arbitrarily close to $(1-1/e)$ [2]. Chen et. al. followed suit to propose the MixGreedy algorithm [3]: the algorithm has two rounds. The first round reduces the graph size by removing edges that will not contribute to propagation, the second round uses the CELF optimization algorithm introduced in [4] to select seeds. The method significantly improves the efficiency of the greedy algorithm.

Nevertheless, a greedy approach for influence maximization still proves to be too inefficient in large scale networks. One way to improve efficiency is through *community structure*: real-world networks consists of dense subgraphs which are sparsely connected externally; these subgraphs are called *communities* [5], [6]. Methods emerged that incorporate coalition detection with influence maximization [7], [8]. Based on properties of the community structures, one can effectively avoid overlapped information. Galstyan et al. proved that for models with critical behavior, the structural properties of the network, and specifically its coalition structure, have great significance for the influence maximization problem. Wang et al. proposed a community-based greedy algorithm for mining the top-k influential nodes in mobile social networks. Many researchers have proposed a variety of community detection methods [9], [10]. But these algorithms need to set in advance the size of the network and the number of coalitions, which will be difficult to apply to the real networks.

More recently, an paradigm based on *coalition formation* has been proposed to identify reasonable network partitions. Here, each node in the network is treated as a player of a *coalitional game* [11]. A fundamental assumption is that the players carry out certain joint task. By forming into coalitions, the players would complete the task more effectively. A coalition will thus receive a collective gain which will then be divided among its members. As the players are self-interested, i.e., they prefer outcomes that give themselves higher gain, the question is how the players may arrive at a reasonable coalition division. In the context of *networked coalitional games*, each coalition is a subgraph and the gain of the coalitions are affected by the graph topology [12]. A solution concept of the game corresponds to a stable division of the network into coalitions, which can in turn be used to solve influence maximization.

Shapley value is a classical normative solution concept of coalitional games. Recently, Zhou and Cheng proposed an algorithm for coalition detection that is based on the concept of Shapley value on networked coalitional games [13]. This approach has a major limitation: Suppose a player v moves in or out of a coalition C. As the decision is made thanks to Shapley value, no concern is paid to other players in the coalition C. As a result, the method does not truthfully reflect realistic coalitional structures.

Contribution: We posit that the preferences of other members of the coalition C constitute important exogenous variables and should be taken into account by the player v. Thus the players should act as 'externality-aware' agents. With this intuition in mind, we propose a new solution concept, called *consensus value*, which not only considers gains obtained by v, but gains by other players in the coalition.

We next emphasize on applying consensus value to influence maximization. We propose *consensus value for influence maximization* (CVIM). The method first selects initial coalitions based on each coalition's gain proportion; it then uses the MaxDegree algorithm and propagation mechanisms to identify seeds in the selected coalitions [2], [14]–[17]. An advantage of CVIM is that it bridge a link between graph partition and information diffusion where not only influential nodes, but also influential coalitions are calculated.

Finally, we test our algorithm over several real world datasets. Through the experimental and theoretical analysis, we show that (i) CVIM can achieve comparable performance as the greedy algorithm for influence maximization with linear threshold model; (ii) CVIM significantly improves running time comparing with the greedy algorithm.

Paper organization: Sec. II presents basic notions of influence maximization and coalitional games. Sec. III introduces consensus value as a novel solution concept for coalitional games. Sec. IV presents the CVIM algorithm. Sec. V discusses our experimental results. Sec. VI concludes the work with future works.

II. Preliminaries

A. Influence Maximization on LTM

A *social network* is a directed graph $G = (V, E)$ where V is a set of nodes and $E \subseteq V^2 \setminus \{(u, u) \mid u \in V\}$ is a set of directed edges on V. If $(i, j) \in E$ then i and j are *adjacent*. We define $a_{i,j} = 1$ if $\{i, j\} \notin E$ and $a_{i,j} = 0$ otherwise. The *in-neighborhood* of i is $N(i) = \{j \mid a_{j,i} = 1\}$. The *in-degree* of a node $i \in V$ is $d(i) = |N(i)|$.

An *information diffusion* model describes the way in which a piece of information spreads in the network. In this paper we focus on the *linear threshold model* (LTM) which is one of the most well-studied information diffusion model [2]. This model assigns a *threshold* $\theta_i \in [0, 1]$ for each node $i \in V$ which is normally uniformly randomly chosen. For any set $S \subseteq V$ and any node $i \in V$, set $\alpha_S(i) = |N(i) \cap S|/|N(i)|$.

During information diffusion, a node can be either *active* or *inactive*. The process starts with a set $S_0 \subseteq V$ of active nodes, called *seeds*, while other nodes are initially inactive. The process follows a sequence of discrete time steps. At any step t, say S_{t-1} is the set of active nodes after step $t-1$, an inactive node $i \notin S_{t-1}$ may turn active if $\alpha_{S_{t-1}}(i) \geq \theta_i$. Thus the process generates a sequence $S_0 \subseteq S_1 \subseteq \cdots$ such that each $S_t \subseteq V$ and for any $t > 0$, we have

$$S_{t+1} = S_t \cup \{i \in V \mid \alpha_{S_{t-1}}(i) \geq \theta_i\}.$$

Note that once a node is activated, it stays active during all subsequent time steps. Since G is finite, there is a (minimum) time step $t \geq 0$ where $S_t = S_{t'}$ for all $t' > t$. We then call S_t the *activated set*. The size of the activated set captures the effectiveness of information diffusion and clearly depends on the seeds S_0 and thresholds θ_i for $i \in \mathcal{V}$.

For $A \subseteq V$, we use $\sigma(A)$ to denote the expected size of activated set if the process starts with seeds $S_0 = A$. The *influence maximization (IM)* problem is stated as:

INPUT: A network $G = (V, E)$, number $K \in \mathbb{N}$.
OUTPUT: Subset $S = \arg\max_{A \subseteq V, |A| = K} \sigma(A)$.

B. Coalitional Games

Coalition games are used to model the coalition formation process among networked agents. Let $N = \{1, 2, \ldots, n\}$ be a set of players. A *coalition* is simply a subset $S \subseteq N$. A *coalition structure* is a partition of N into coalitions. A *characteristic function* $\nu \colon 2^N \to \mathbb{R}$ assigns a *payoff* value $\nu(S)$ to every coalition S; we require $\nu(\varnothing) = 0$. A *coalitional game* is a pair (N, ν). Now take a network $G = (V, E)$, we regard V as the set N of players. The following characteristic function reflects the network topology:

$$\nu(S) = \begin{cases} 0 & \text{if } |S| \leq 1 \\ \sum_{i \in S, d(i) > 0} \sum_{j \in S, j \neq i} \frac{a_{i,j}}{d(i)} & \text{if } |S| \geq 2. \end{cases}$$

The *Shapley value* is a solution concept that provides a way to distribute collective gains among players. For this, Shapley value takes into account the relative importance (marginal contribution) of each player in the game. More formally, for any coalition $S \subseteq N$, Shapley value defines a vector $\phi(S) = (\phi_1, \ldots, \phi_{|S|}) \in \mathbb{R}^{|S|}$ where ϕ_i denotes the payoff to player i if i in the coalition S [18].

Using Shapley value, one may partition the set of players N into coalitions according to the social network's structure by following the procedure below [19]: The initial configuration arranges every player in a distinct singleton "coalition". During the coalition formation process, the players take actions in turn. When it is a player i's turn, i examines all other non-empty coalitions in the current game. If the payoff received by i by joining another coalition is higher than the payoff received in the current coalition, i will have a tendency to withdraw from the original coalition and join the other coalition. The player i eventually will join the coalition that gives her the highest increase in payoff. The process is repeated until no more changes happens in the game, at which moment the game forms a coalition structure.

III. Consensus Value

While Shapley value is widely recognized for its ability to generate fair utility for individuals within a coalition, it is not entirely reasonable to use it in a mechanism for coalition formation. This is because that using Shapley value, a player's decision for joining and leaving coalition is independent from the effect on other players in the same coalition. To make up for this limitation, we introduce the *consensus value*, a new solution concept which calculates the gain of each player in the new coalition. Consensus value is derived from 2-person games: Imagine a player i is deciding between an original coalition S and another S'; the gain formed by the two coalitions are evenly distributed, and then weighted average method is made among all possible permutations. Using a similar mechanism for coalition formation described above for Shapley values, the players reach consensus and form stable coalitions. Such a method is called the *standardized remainder rule*; see def. below [21].

More precisely, in the game (N, ν), let $\nu(S)$ denote the marginal contribution value of coalition S where $S \subseteq N$. $\Pi(N)$ is the set of all permutation $\pi\colon \{1, 2, \cdots, |N|\} \to N$. For a given permutation $\pi(N)$ and $k \in \{1, 2, \cdots |N|\}$, coalition S_k^π is the set of players $\{\pi(1), \pi(2), \cdots, \pi(|k|)\} \subset N$ and $S_k^\pi = \varnothing$ [21].

Definition 1: The value $r(S_k^\pi)$ is the *standardized remainder* for coalition S_k^π: the value left for S_k^π after allocating surpluses to earlier leavers $N\ S_k^\pi$: $r(S_k^\pi)$ for a coalition S_k^π $(1 \leq k \leq |N|)$ is defined as in [21]:

$$r(S_k^\pi) = \begin{cases} \nu(S_k^\pi) + \frac{r(S_{k+1}^\pi) - \nu(S_k^\pi) - \nu\{\pi(k+1)\}}{2} & \text{if } k < |N| \\ \nu(N) & \text{if } k = |N| \end{cases}$$

Players enter into the coalition based on the sequence $\pi(1), \pi(2) \cdots, \pi(|N|)$. Allocating gain for each player, in addition to the personal gain, should be supplemented by half of the standardized remainder $r(S_k^\pi)$ in the coalition [21]. Thus, we then define the following:

Definition 2: In the game (N, ν), the *individual standardized remainder* $S_{\pi(k)}^\pi(\nu)$ is the gain of each player generated by the coalition:

$$S_{\pi(k)}^\pi(\nu) = \begin{cases} r(S_1^\pi) & \text{if } k = 1 \\ \nu(\{\pi(k)\}) + \frac{r(S_k^\pi) - \nu(S_{k-1}^\pi) - \nu\{\pi(k)\}}{2} & \text{if } k \geq 2 \end{cases}$$

Players join the coalition in different order, and the individual standard remainder that the players eventually get is also different. The average value of the individual standard remainder defines the *gain* of each player $\pi(k)$, which is captured by the *consensus value*.

Definition 3: The *consensus value* $\varphi(\nu)$ of the game (N, ν) is the average of the individual standard remainder $S_{\pi(v)}$, which is defined as [21]:

$$\varphi(\nu) = \frac{1}{|N|!} \sum_{\pi \in \Pi(N)} S_{\pi(\nu)}$$

IV. Consensus Value for Influence Maximization

This section proposes our approach to solve IM which we call *consensus value for influence maximization* (CVIM). CVIM comprises three phases: (i) Apply consensus value to identify coalitions; (ii) Initial coalitions are selected based on the proportion of gain from each coalition; and (iii) Seed nodes are finalized in those selected coalitions.

A. Coalition Formation

Similar to the Shapley value-based approach for coalition formation described in Sec. II.B, we start with every player in its own singleton coalition. The players then iteratively decides to move from their coalition with others'. One can easily generalize definition of $\varphi(\nu)$ to any subgame of the form $(S, \nu \restriction S)$ where $S \subseteq N$; the gain of members of each member of S is given by the consensus value.

At any step, a player i moves by deciding whether to join any other coalitions. When facing a coalition T, i is essentially in a 2-player game consisting of i and T as players (here, T acts as a single player); Each player has two strategies, *not join* and *join*. The respective payoffs to each player are: the payoffs of i equal to ith gain before and after joining T; the payoffs of T equal to the original coalition gains and the gains from the coalition $\{i\} \cup T$. Player i has an intension to join T only when *join* give higher payoff to both players. Player i then chooses among the best among all coalitions whom it has an intension to join.

In the same way as the coalition formation rule for Shapley value, the players iteratively decides on optimal coalition to join (or not join any). In this way, all nodes can choose the coalition that makes these nodes get largest gain to cooperate with. The concept not only maximizes its own gain, but also will increase the entire coalition value, which can achieve a win-win result through cooperation. In this way, we can achieve a more reasonable coalition formation.

B. Select Initial coalitions

According to the consensus value, we can obtain the results of coalition structure T. Assume the characteristic function ν represents the marginal contribution value of the all nodes in the coalition. The marginal contribution value of coalition in each coalition is regarded as payoff of entire network. The greater than marginal contribution value of a coalition, the more closely players in the coalition are connected, thus resulting in a higher influence if players in the coalition are selected as seed nodes. Therefore, we can use ν as a guidance to select initial coalitions.

Definition 4: Let $T_1, T_2, \ldots, T_m$ be the coalitions where $T = \{T_1, T_2, \ldots, T_m\}$, $T_i \bigcap T_{i+1} = \emptyset$, m is the number of coalition. Say $\nu(T_i)$ is the marginal contribution value of the coalition T_i. The probability of assigning seed nodes P_{T_i} in the coalition T_i is in the coalition T_i is:

$$P_{T_i} = \nu(T_i) / \sum_{i=1}^{m} \nu(T_i).$$

We arrange $T_1, T_2, \ldots$ such that the sequence of P_{T_i} is in descending order. The K seed nodes are assigned to the K coalitions with large marginal contribution value.

C. Finding Seed Nodes

In order to locate seed nodes in the social network, the K seed nodes are assigned to the corresponding K coalitions respectively. We apply MaxDegree algorithm to find a seed node in each coalition.

Definition 5: Let $M(T_i)$ be the seed node chosen in the coalition T_i by MaxDegree. Define the *seed sets* A as:

$$A = \bigcup_{i=1}^{K} M(T_i)$$

Then we apply function $\sigma(A)$ to calculate the final influence number of seed node set A.

D. Algorithm Pseudo-code

Based on the above description for each phase, we present the CVIM algorithm below. We first describe the variables used in CVIM:

- There is a boolean variable $Flag[i]$ for each player i; the default value is false.

- $N = \{1, 2, \cdots n\}$ is the set of players.

- $M(T_i)$ represents a seed node in the coalition T_i chosen by one step of MaxDegree in coalition T_i.

- $\sigma(A)$ represents the total numbers of active nodes generated by LTM starting from A.

- $\varphi(\nu) = \varphi_x(\nu) \cup \varphi_i(\nu)$. $\varphi'_i(\nu)$ is the consensus value of the player i when i chooses to join a new coalition. $\varphi_i(\nu)$ is the consensus value of the player i in the original coalition.

- $\varphi'_x(\nu)$ is the consensus value of player x after the player i chooses to join x's coalition. $\varphi_x(\nu)$ is the consensus value of other player x before i chooses to join the coalition.

Algorithm 1 CVIM Algorithm

```
Input: social network G and the number of initial nodesK.
Output: coalition partition results T, seed nodes set A and σ(A)
Set T = {{1}, {2}, ..., {n}}
Flag[1···n] = true;
for i = 1 to N do
  while Flag[1] or Flag[2] or ··· or Flag[n] do
    Flag[1 ···n]=false;
    for i = 1 to N do
      if Max(φ'_i(ν) > φ_i(ν) and φ'_x(ν) > φ_x(ν)
       then
        The player i chooses to withdraw from the original
        alliance, and join the new coalition.
        Flag[i]=true;
      end if
    end for
  end while
end for
Let P_{T_i} array in descending order.
Select K initial coalitions based on P_{T_i} value.
Combine with all M(T_i), and find seed nodes A
Calculate σ(A)
return  T and A and σ(A).
```

V. Experiments

In order to evaluate the efficiency and feasibility of the algorithm CVIM, the random algorithm, the climbing greedy algorithm, the MaxDegree algorithm and the Shapley Value algorithm are implemented for comparison. All algorithms are implemented in the C++ language and designed based on the linear threshold model and independent cascade model. The comprehensive performance study is conducted on several classical real world datasets.

A. Experimental setup

Datasets: We perform some experiments on several real world datasets: (1) Karate Club Network [22] (2) CollegeMsg Network [23] (3) Gnutella Network [23] (4) Wikipedia Network [23] (5) Hepth Network [23]. The parameters of several networks are shown in Table 1.

TABLE I
PARAMETERS OF NETWORKS

Networks	Nodes	Edges
Karate Club	34	78
CollegeMsg	1899	20296
Gnutella	6301	20777
Wikipedia	7115	103689
Hepth	9877	25998

Algorithms and Parameters: we compare our algorithm CVIM with several algorithms that appear in the paper. The following is a list of algorithms we evaluate in our experiments.

Random algorithm: As a baseline comparison, simply select K random vertices in the graph [15].

Climbing Greedy algorithm: The original greedy algorithm [2] with the lazy-forward optimization of Leskovec et al. [4]. For each candidate seed set A, 2000 simulations is run to obtain an accurate estimate of $\sigma(A)$.

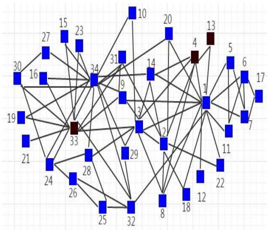
Maxdegree algorithm: The algorithm chooses nodes u in order of decreasing degrees [2].

In the linear threshold model, $\alpha_S(i)$ for an edge (i, j) is set as is the in-degree of node i. θ is random chosen and $\theta_i \in [0, 1]$.

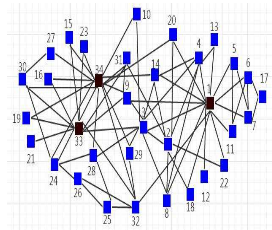
B. Experimental Results

In the experiment, we compare the seed nodes, influence spread, running time of different algorithms. The results are shown as follows:

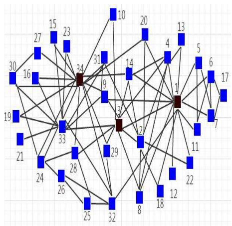
The Feasibility of the Algorithm: In order to evaluate the feasibility and reflect coalition structure, we compare seed nodes of different algorithms in Karate Club data. The black points represent the seed nodes. The results are shown in Figures 1:



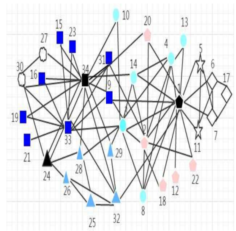
(a)Random Algorithm



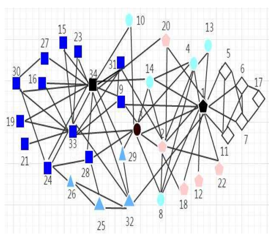
(b)MaxDegree Algorithm



(c)Climbing Greedy Algorithm



(d)Shapley Value Algorithm

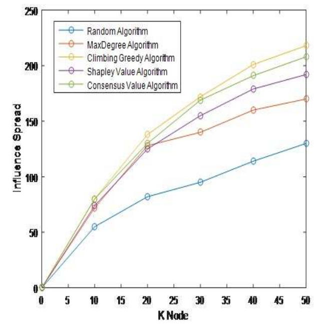


(e)Consensus Value Algorithm

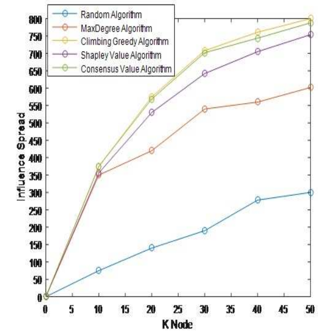
Fig. 1. Seed nodes of different algorithms in Karate Club data

Fig. 1 shows the seed nodes of different algorithms in the Karate Club data. In Fig. 1(c) the greedy algorithm has proven to be the best for influence maximization. In Fig. 1(a) the random algorithm demonstrates much variation, and thus has limited applicability. In Fig. 1(b) the MaxDegree algorithm will overlap information when we select seed nodes. Node 34 is selected as a seed first, before it activates node 33. The second largest degree node is node 33, we will select node 33 as seed node, which naturally leads to redundant seed selection. In Fig. 1(d) the Shapley value algorithm divides the Karate Club network into seven different categories of coalitions with the same color, and the seed set is $\{34, 1, 24\}$. In Fig. 1(e) the consensus value algorithm divides the Karate Club network into five different categories of coalitions, and then selects the three coalitions with the largest proportionality value from the marginal contribution ratio of each coalition. Finally, seed node is selected in the selected each coalition, and seed node set is $\{34, 1, 3\}$. The seed node set chosen by consensus value algorithm is consistent with the greedy algorithm, and we can see the obvious coalition structure.

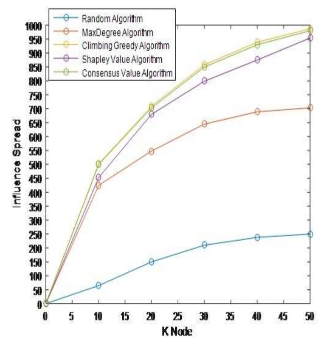
Comparison of Influence Spread: We use four real datasets to test efficiency of different algorithms. The number of seed nodes is from 0 to 50, and the total number of active nodes is found by an independent cascade model or a weight linear threshold propagation model, and some parameters of model are presented by section algorithms and parameters. The results are shown by Figures .2:



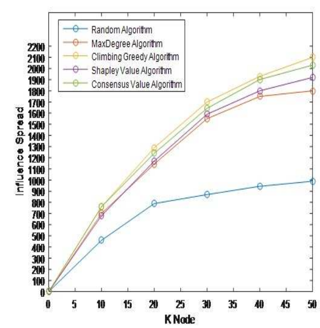
(a)CollegeMsg Network



(b)Gnutella Network



(C)Wikipedia Network



(d)Hepth Network

Fig. 2. Influence spread with different algorithms

Figures .2(a) show influence spread in the CollegeMsg Network. The x-axis represents the number of seed nodes, and the y-axis represents influencing node number. The efficiency of random algorithm is low. As the seed node increasing, the MaxDegree algorithm chooses the largest degree as the seed node, which may be mutual activation in the seed node. It will repeat the overlay nodes. Shapley value algorithm only takes into account the gain of new participants and does not take into account the gains of other players, the results of the coalition division are different from the consensus value algorithm. Therefore, the seed node set is also different . The impact of consensus value algorithm is close to the greedy algorithm. In Fig. 2(b), when the number of seed nodes grows from 0 to 10, the influence spread of Shapley value algorithm, the consensus value algorithm, and the greedy algorithm are the nearly same. The reason is that Shapley value algorithm and consensus value algorithm are almost identical when the top10 nodes are selected. Consensus value algorithm for coalition division can get a good result. In Fig. 2(c) the seed nodes are from 0 to 10, the number of activated nodes starts to grow rapidly, and the latter tends to keep a steady state in MaxDegree algorithm. The consensus value algorithm can achieve almost the efficiency of the greedy algorithm. Figures .2.(d) show influence spread in the Hepth Network. With the increasing of data, the effect of our algorithm is obvious.

The Running time of Algorithms: In the four networks, we use different algorithms to solve the influence maximization. The running time is shown in Fig. 3.

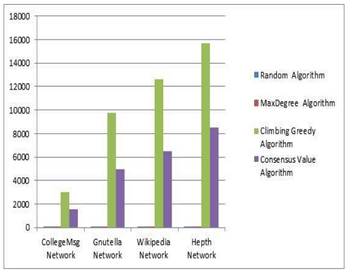


Fig. 3. Running time on different networks

Fig. 3. shows the running time of different algorithms on the Networks. The random algorithm only needs to use the random function to select K seed nodes. The algorithm can be completed in short time. We apply the adjacency matrix to calculate the degree of each node, and then use the bubble sort method to sort the degree of the node. Then we select the top K nodes as seed nodes. The MaxDegree algorithm runs for a short time. In greedy algorithm we need combine the influence diffusion mechanism to calculate the influence of each node, from which to select the most influential node as a seed node. In the selection process, if there are the same nodes of influence, we must traverse these nodes to choose one of the nodes, so greedy algorithm needs to consume a lot of time. consensus value algorithm based on coalitional game theory divides the whole network into different coalitions, we only need to select a seed node in top-K coalition with large marginal contribution. Compared with the greedy algorithm, consensus value algorithm can reduce running time.

VI. Conclusion

We present a new approach, CVIM, for coalition formation and influence maximization. Consensus value algorithm takes the externality of coalitional game into account to overcome the limitations of Shapley value. An approach based on the gain proportion of each coalition coalition is proposed to select initial coalitions. According to each coalition marginal contribution rate, the initial coalitions can be found. Compared with the greedy algorithm, our approach is to select only one node in a certain coalition, which reduces the consumption of time. In the selected coalitions, we use the MaxDegree algorithm and propagate mechanisms to find seed nodes set. The experimental results on several classical real world datasets show that our CVIM algorithm spends less running time than greedy algorithm. Much work is needed to expand understanding of consensus value as a solution concept of coalitional games. In the future, we will apply the approach to IM in dynamic social networks. This is a worthwhile study to explore the dynamic coalition structure to address influence maximization problem. Another interesting future work is to investigate the possibility of applying consensus value to community detections. This involves comparing the coalition structure generated by consensus value with established notions of communities in networks.

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