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# Non-Congruent Views about Signal Precision in Collective Decisions

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#### Abstract:

We relax the standard assumptions in collective decision-making models that voters can not only derive a perfect view about the accuracy of the information at their disposal before casting their votes, but can, in addition, also correctly assess other voters' views about it. We assume that decision-makers hold potentially differing views, while remaining ignorant about such differences, if any. In this setting, we find that information aggregation works well with voting rules other than simple majority: as voters vote less often against their information than in conventional models, they can deliver higher-quality decisions, including in the canonical 12 jurors case. We obtain voting equilibria with many instances, in which other voting rules, including unanimity, clearly outperform simple majority.

**Keywords:** group decision-making, voting rules, non-congruent views, jury trial, referenda, simulations, humansubjects laboratory experiment

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## 1 Introduction

Much real world negotiation and decision-making takes place in groups voting over binary choices. These collective decisions determine outcomes that matter to single individuals, households, businesses and organizations, as well as communities and wider society. They impact, for instance, the lives of organ recipients, the fates of defendants in jury trials, the allocation of research funding, as well as the outcomes of various referenda. Consequently, much research in the fields of social choice, political economy, and political science is devoted to identifying the processes and rules that achieve the best collective decisions. Existing studies concur that majority voting should be preferred in many collective decision settings.

This consensus is rooted in two major findings, the Condorcet Jury Theorem (de Condorcet 1785) and what we refer to as the Jury Paradox. The Condorcet Jury Theorem establishes that collective decisions generated by majority voting have a higher probability of selecting the correct alternative than the decision made by a single expert, especially as the size of the group grows. The Jury Paradox refers to the superiority of the majority rule over the unanimity rule because even well-intentioned voters can be strategic and rationally disregard their information. See for example, Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998). This incentive is the strongest for the unanimity rule, reinforcing the case for using simple majority voting.

In this paper, we posit that the superiority of the majority rule hinges upon the unrealistic assumption that individuals cast their votes based on information whose reliability is commonly known. In contrast, we consider a modification of the jury model of Feddersen and Pesendorfer (1998), in which we allow voters to hold potentially non-congruent views about the accuracy of the information they receive before casting their votes. More precisely, we assume that voters (i) adhere to subjective views about the accuracy of the information at their disposal before casting their votes and (ii) assign to others the same views as theirs.

The assumption that voters form subjective views on the accuracy of their information should not be controversial. Binmore (2016) postulates that decision-makers often need to formulate subjective probabilities for events they are not well informed about. Sometimes, they may not even have sufficiently many empirical frequencies of the occurrence of these events to be able to form measurable probabilities for them. See also Binmore

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(2009, p. 140). The assumption that voters have potentially differing views, yet are unable to recognize them, relates to models of situations in which individuals lack *transpersonal understanding*. Their different experiences not only lead individuals to adopting personal views but also preclude them from transposing someone else's experiences onto themselves. Kaneko and Matsui (1999) uses this assumption to model persistent prejudice and discrimination and, building on this earlier work, Kaneko and Kline (2009, 2013) study games with partial views and derived memories.<sup>1</sup> To require of voters to fully recognize their limitations in assigning probabilities to relevant events, to admit their potential biases, as well as to acknowledge that such limitations also apply to other voters is very demanding. However, these restrictive assumptions are typically used in studies of collective decision-making.

We show how relaxing these assumptions affects the formation of the voters' posterior beliefs and their voting behaviour. We show how it impacts on the quality of the collective decisions as a function of the voting rule. We demonstrate that, within our more realistic environment, information aggregation works well with voting rules other than simple majority, including unanimity. This is in contrast to the received literature. We derive conditions under which voters vote more often in accordance with their information than in conventional models, thereby delivering higher-quality decisions, including in the canonical case of 12 jurors. We obtain a richer set of voting equilibria than in received jury models, including equilibria in which some voters vote informatively while others randomize their votes to convict or acquit. The instances in which non-unanimous voting rules and even unanimity clearly outperform simple majority include the classical jury voting examples provided in Feddersen and Pesendorfer (1998), that is, when the jury size equals twelve and the level of the reasonable doubt is relatively high. Specifically, we highlight the effect derived in our model in terms of the induced probabilities of convicting the innocent (type I error) and acquitting the guilty (type II error) when compared to those obtained in the canonical jury trial model. This leads to the result that, for large juries, type I errors are minimized if jurors are 'trusting' of the information they receive; whereas for smaller juries, they are minimized if jurors are 'distrusting' of that information.

Our results are obtained in a model with strategic voting in juries as Austen-Smith and Banks (1996), Feddersen and Pesendorfer (1998), and Berend and Paroush (1998) or Laslier and Weibull (2013). We retain the assumption that voters only vote according to their information if that maximizes their expected utility. Further, jurors vote, thinking they share a common assessment of the precision of the signal they receive before casting their votes contributing to the group's decision when, in fact, they may not. In this environment, voting strategies are still defined by maximizing subjective expected utility. Our results, derived relaxing the congruent views assumption, have a similar flavour to those obtained in Fabrizi and Pan (2016). Despite adhering to subjective expected utility maximization and not incorporating ambiguity and ambiguity averse voters, unlike in Fabrizi and Pan (2016), our study also shows that it is possible to restore some efficiency in the information aggregation even if adhering to voting rules other than the simple majority one.

To complement our theoretical analysis, we report results from a laboratory experiment aimed at testing our theoretical predictions in terms of the effects of different voting rules on the outcome of the collective decisions when voters have (potentially) non-congruent views about the quality of their private information, and they disregard those differences across them. To that end, we employed a  $2 \times 2 \times 2$  design. We varied (i) the information structure for the accuracy of private information received (common or non-congruent views), (ii) two different institutional designs for determining the group decision (simple majority or unanimity), and (iii) the availability of free-form communication prior to subjects making their choices (no communication or free-form communication).

The Jury Paradox should not manifest itself in a small group committee, and, as expected also in the experiment, with small decision-making groups of size of n = 5, we found that the realized type I errors under unanimity voting are smaller than those under majority voting. This result persists across differing information structures. When multiple views are possible and as long as not everyone is fully trusting the quality of the information to be the highest among two possible values provided, both type I and type II errors are reduced. This is a novel result, which suggests the importance of personal views on the precision of the information provided in the collective decision-making. Our experimental results also have normative implications, since the choice of the voting rule matters for the quality of the collective decision outcome. It matters in the sense that the optimal choice of a voting rule for a voting situation might depend, beyond the 'true' precision level of that information (if known), and the jury size, as usual, also on the additional factor of the likely composition of voters' differing views about the precision of their information, at least as long as voters are ignorant of such differences.

Section 2 presents our theoretical results. Sections 3 and 4 report on our simulations and our experiment. All proofs and simulation tables are relegated to the Appendices **A** and **B**.

## 2 Theory

#### Setup

Feddersen and Pesendorfer (1998), FP from here on, assume there are *n* jurors, j = 1, ..., n, who decide the fate of a defendant. The jurors' common prior assigned to the defendant's guilt is  $Pr(G) = Pr(I) = \frac{1}{2}$ . Before casting their votes, each juror receives a private, informative signal about the defendant's guilt, denoted  $s_j = \{i, g\}$ , with i = innocent or g = guilty. Their signals are independently drawn from a common distribution with precision Pr(g|G) = Pr(i|I) = p. All jurors share the same *p* and this is common knowledge.

Based on the jurors' votes, the trial results in a verdict to convict, *C*, or acquit, *A*, the defendant. Voting rules are denoted by the number of jurors needed to convict,  $\hat{k} \le n$ . Simple majority, hence, is  $\hat{k} = (n + 1)/2$  and unanimity is  $\hat{k} = n$ . Jurors have common preferences over the outcome of the verdict,

$$u(A, I) = u(C, G) = 0,$$
  
 $u(C, I) = -q,$   
 $u(A, G) = -(1 - q),$ 

where the parameter *q* is interpreted as a threshold of reasonable doubt.

Before describing FP's main results, let us recall the classes of equilibria considered and some related, as well as useful, definitions.

#### **Definition 1.**

Informative voting occurs when all voters cast their votes in accordance to their signals: each voter casts a guilty vote when receiving a guilty signal and an innocent vote when receiving an innocent signal.

#### **Definition 2.**

Strategic voting occurs when voters cast the vote that maximizes their subjective expected utility.

Therefore, strategic voting might lead to a voter casting a vote against their signal.

#### **Definition 3.**

*Symmetric* Nash Equilibria are those in which all voters who receive the same signal follow the same (possibly mixed) strategy.

#### **Definition 4.**

Responsive Nash Equilibria are those in which voters choose different strategies for different signals.

In this setting, FP derive the following main results. First, with *informative* voting, the unanimity rule minimizes the type I error of convicting innocent defendants; but it maximizes the type II error of acquitting guilty defendants. Second, informative voting is not always a Nash equilibrium. When it is not, there exists a unique symmetric responsive Nash equilibrium in which jurors with a guilty signal vote to convict and voters with an innocent signal randomize between voting to acquit and voting to convict. In this equilibrium, the probability of committing both errors under unanimous voting is bounded away from zero, even for large jury sizes, while there exist non-unanimous rules for which both approach zero as the jury size grows. Hence, FP predicts that the unanimity rule, which was meant to protect innocent defendants, leads to more miscarriages of justice than non-unanimous rules as voting informatively is not an equilibrium in many voting situations. In what follows, we relax FP's assumption that jurors share the same signal precision *p* and that this is

In what follows, we relax FP's assumption that jurors share the same signal precision p and that this is common knowledge. In our modification, jurors disagree in the assessment of the precision, p, of their signal. Further, they do not have common knowledge of the signal precisions used to update the prior. This modification to FP's setup accounts for situations, in which individuals do not have sufficiently many occurrences of the possible events such that objective probabilities for them cannot be obtained.<sup>2</sup> In these situations, Knight (1921) suggests that individuals eventually assign a subjective probability estimate towards the events and feel toward this estimate as toward any other probabilities. Given the nature of the information going into jury decisions, it is often reasonable to assume that this estimate diverges among jurors and that jurors are uninformed about this divergence.

It follows that, even though different voters may have different beliefs about the precision, p, every voter believes that it is commonly believed that everybody believes in the same precision as herself. In this sense, from each player's point of view it is as if the game that is played were consistent with sharing the same, commonly known, parameter p, that is as if the game were the one described in FP, even when it is not. More formally, a player believing in p has a belief hierarchy that puts positive probability (at all orders of beliefs) to the event

that everybody in the game also has parameter p. Therefore, she is not responding in equilibrium to the actual strategies chosen by the other voters, but rather to the strategies she would believe others would play in case they were of the same type as herself.<sup>3</sup>

We formalize our relaxation of FP in Assumptions 1 and 2.

#### Assumption 1.

*Jurors are either skeptical or trusting. For skeptical jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = p*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = P*, and for trusting jurors,* Pr(g|G) = Pr(i|I) = P*, and for trusting jurors,* Pr(g|G) = P*, and for trusting jurory,* Pr(g|G) = P*, and for trusting jurory,* Pr(g|G) = $Pr(i|I) = \bar{p}; with 1/2$ 

Given Assumption 1, we will refer to juror j's type as follows

$$t_j = \begin{cases} (s_j, \overline{p}) & \text{if skeptical,} \\ (s_j, p) & \text{if trusting,} \end{cases}$$

where  $s_i \in \{i, g\}$ .

#### Assumption 2.

Each juror believes that the signal is of a common precision and that this precision is common knowledge among them.

Given Assumption 2, a trusting juror believes that all jurors are of type  $t_i \in \{(i, \overline{p}), (g, \overline{p})\}$  and a skeptical juror believes that all jurors are of type  $t_i \in \{(i, p), (g, p)\}$ . Both dimensions of a juror's type determine whether, in light of all the evidence, it is optimal for that juror to vote to convict or acquit.

We denote the fraction of skeptical and trusting jurors that make up a given jury by m and 1 - m, with  $0 \le m \le 1$ . If m = 0, all jurors are skeptical and believe *p*; if m = 1, all jurors are trusting and believe  $\bar{p}$ .

Other than that, we maintain all of FP's assumptions. In particular, as in FP, the jurors' signals are still drawn from one and the same distribution, that is, there is one objective probability, denoted  $\hat{p}$ , that the signal jurors receive is correct. We will use this objective probability when we compute type I and II errors.

Similarly to FP, we write the condition for a vote to convict to be optimal for a juror of type  $t_i$  as:

$$E[u_j(C,\cdot)|piv_j,t_j] \ge E[u_j(A,\cdot)|piv_j,t_j]$$

$$\Rightarrow Pr(G|piv_j, t_j)u(C, G) + (1 - Pr(G|piv_j, t_j)u(C, I)$$
  
$$\geq Pr(G|piv_j, t_j)u(A, G) + (1 - Pr(G|piv_j, t_j)u(A, I)$$

$$\Leftrightarrow Pr(G|piv_j, t_j) \ge q.$$

Denote the posterior probability  $Pr(G|piv_i, t_i)$  that, conditional on being pivotal, a juror of type  $t_i$  holds about a defendant being guilty as  $\beta^{t_j}$ . Depending on the voting rule adopted, there will be exactly  $\hat{k}$  votes needed for a conviction to be reached. Hence, for any juror to be pivotal,  $\hat{k} - 1$  among the n - 1 voters would need to cast guilty votes. However, whether a juror correctly anticipates to be in a position of being pivotal, depends on combination of their belief about the 'true' precision of the signal and that of other jurors. Using Bayes' rule, we can easily derive an expression for this 'perceived' posterior probability that a juror of type  $t_{ii}$  holds about a defendant being guilty, conditional on believing that they are pivotal:

$$\beta^{t_j} = \frac{Pr(piv_j, t_j|G)}{Pr(piv_j, t_j|G) + Pr(piv_j, t_j|I)} = \frac{Pr(t_j|G)Pr(piv_j|G)}{Pr(t_j|G)Pr(piv_j|G) + Pr(t_j|I)Pr(piv_j|I)}$$

For simplicity, we will avoid to distinguish whether a juror is actually pivotal – that is whether a juror correctly believes to be pivotal – and instead refer to a juror being pivotal as a situation in which the juror (perhaps wrongly) believes to be pivotal.

Define the strategy of a juror *j* as a rule that prescribes with which probabilities the juror should vote to convict as a function of each of the possible signals to be received and given their idiosyncratic belief about the precision of those signals. Denote this strategy for a skeptical and a trusting juror *j*, respectively, as functions  $\sigma_i^{(s,p)}: \{i,g\} \rightarrow [0,1] \text{ and } \sigma_i^{(s,\overline{p})}: \{i,g\} \rightarrow [0,1].$ 

Equilibrium Concept We restrict attention to the class of equilibria, which are symmetric and responsive, although, unlike in FP, these restrictions need to hold type by type, that is in the following sense.

# Assumption 3. *Symmetric*

- For any two jurors j and 
$$-j$$
 of type  $(s, \underline{p})$ ,  $\sigma_j^{(i,\underline{p})} = \sigma_{-j}^{(i,\underline{p})} = \sigma_{-j}^{(i,\underline{p})}$  and  $\sigma_j^{(g,\underline{p})} = \sigma_{-j}^{(g,\underline{p})} = \sigma_{-j}^{(g,\underline{p})} = \sigma_{-j}^{(g,\underline{p})}$ ;

- For any two jurors j and -j of type  $(s,\overline{p})$ ,  $\sigma_j^{(i,\overline{p})} = \sigma_{-j}^{(i,\overline{p})} = \sigma_j^{(i,\overline{p})}$  and  $\sigma_j^{(g,\overline{p})} = \sigma_{-j}^{(g,\overline{p})} = \sigma_j^{(g,\overline{p})}$ .

## Assumption 4.

Responsive

- For any juror of type  $t^{(s,\underline{p})}$ ,  $\sigma^{(i,\underline{p})} \neq \sigma^{(g,\underline{p})}$
- For any juror of type  $t^{(s,\bar{p})}$ ,  $\sigma^{(i,\bar{p})} \neq \sigma^{(g,\bar{p})}$ .

Assumption 3 and Assumption 4 state that, given the same signal, any two skeptical (trusting) jurors vote to convict the defendant with the same probability; and that any skeptical (trusting) juror votes to convict with a different probability when confronted with different signals.

We derive the probability for a skeptical juror to vote to convict when the defendant is innocent as

$$\gamma_{I}^{(s,\underline{p})} = (1-\underline{p})\sigma^{(g,\underline{p})} + \underline{p}\sigma^{(i,\underline{p})},$$

and the probability that a skeptical juror votes to convict when the defendant is guilty as:

$$\gamma_G^{(s,\underline{p})} = \underline{p}\sigma^{(g,\underline{p})} + (1-\underline{p})\sigma^{(i,\underline{p})}.$$

Since a skeptical (trusting) juror assumes everybody else to be skeptical (trusting), the posterior probability a juror assigns to the event that a defendant is guilty, conditional on being pivotal given any voting rule  $0 < \hat{k} \le n$ , equals

$$\beta^{(s,p)}(\hat{k}) = \frac{Pr((s,p)|G)(\gamma_G^{(s,p)})^{k-1}(1-\gamma_G^{(s,p)})^{n-k}}{Pr((s,p)|G)(\gamma_G^{(s,p)})^{\hat{k}-1}(1-\gamma_G^{(s,p)})^{n-\hat{k}} + Pr((s,p)|I)(\gamma_I^{(s,p)})^{\hat{k}-1}(1-\gamma_I^{(s,p)})^{n-\hat{k}}}, \\ \forall p \in \{p, \bar{p}\}.$$

#### **Unanimous Voting Rule**

We now analyze the equilibria of our modified collective decision-making game for the *unanimous* voting rule. We examine the jurors' incentives to vote *informatively*, in accordance to their signal, or *strategically*, that is, randomizing their vote, and possibly casting it against their signal.

#### Informative Voting Equilibrium

We start by investigating the conditions for an informative voting equilibrium to exist when jury decisions are made with the unanimity rule. In an informative equilibrium, skeptical and trusting jurors vote to convict if their signal is guilty and to acquit if their signal is innocent.

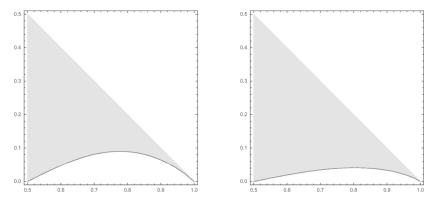
A juror subjectively believes all jurors to have received a correct signal with probability p and a wrong signal with probability 1 - p, where p = p for a skeptical and  $p = \overline{p}$  for a trusting juror. In any informative equilibrium it must be the optimal strategy for any juror to vote as follows:  $\sigma^{(i,p)} = 0$  and  $\sigma^{(g,p)} = 1$ , where, again, p = p for a skeptical and  $p = \overline{p}$  for a trusting juror. Therefore, the probability that a juror votes to convict when the defendant is innocent equals his subjective probability of receiving the wrong signal g, that is  $\gamma_1^{(s,p)} = 1 - p$ , and the probability that a juror votes to convict when the defendant is guilty equals his subjective probability of having received the right signal g, that is,  $\gamma_G^{(s,p)} = p$ , where, once more, p = p for a skeptical and  $p = \overline{p}$  for a trusting juror.

#### **Proposition 1.**

For the unanimity rule,  $\hat{k} = n$ , informative voting is an equilibrium if and only if  $q \in [\beta^{(i,p)}(n), \beta^{(g,p)}(n)]$  for each type  $p \in \{p, \bar{p}\}$ . Such an interval exists if and only if

$$\left(\frac{1-\bar{p}}{\bar{p}}\right)^{\frac{n-2}{n}} > \frac{1-\underline{p}}{\underline{p}}.$$

Note that compared to FP, there is an additional reason for the informative equilibrium not to exist, namely that for finite *n*, the posterior of a trusting voter with an innocent signal may be larger than that of a skeptical voter with a guilty signal. Figure 1 illustrates the existence of a *q* for which there is an informative equilibrium with the unanimity rule as a function of *p* and  $\Delta p \equiv \bar{p} - p$  for n = 6 and n = 12. For finite *n*, such a *q* exists if, for a given *p*, the  $\Delta p$  is not too large. As *n* increases, the  $\Delta p$  for which such a *q* exists becomes smaller. As  $n \to \infty$ , there is no *q* for which informative voting is an equilibrium irrespective of  $\Delta p$ .



**Figure 1:** Existence of informative equilibrium for n = 6 (left) and n = 12 (right) as a function of  $\underline{p}$  (horizontal axes) and  $\Delta p$  (vertical axes). For any pair  $(\underline{p}, \Delta p)$  in the shaded area, there is no q for which an informative equilibrium exists. For any pair  $(\underline{p}, \Delta p)$  in the white area below the black line, there is a q for which an informative equilibrium exists. Note that  $\overline{p} < 1$  implies  $\Delta p < 1 - p$ , so any pair  $(p, \Delta p)$  above the shaded area is not relevant.

#### Corollary 1.

For the unanimity rule, if the jury size tends to infinity,  $n \rightarrow \infty$ , informative voting is not an equilibrium for any q.

#### Voting Equilibria with Randomization

We now consider situations in which informative voting is not an equilibrium. Now, jurors may randomize and, possibly, vote against their signals.

We have two classes of symmetric, responsive equilibria in which at least one type mixes. In one class, both types mix when they receive an innocent signal and vote to convict when they receive a guilty signal. Whereas in the other class, which we call semi-mixed only the trusting voters mix when they receive an innocent signal and vote to convict if they receive a guilty signal and the skeptical voters acquit after an innocent and convict after a guilty signal. We classify those equilibrium classes more formally, using the following definitions.

#### **Definition 5.**

Define as E1 the class of equilibria for which  $0 < \sigma^{(i,\underline{p})} < 1$ ,  $\sigma^{(g,\underline{p})} = 1$ ,  $0 < \sigma^{(i,\overline{p})} < 1$ ,  $\sigma^{(g,\overline{p})} = 1$ .

#### **Definition 6.**

Define as E2 the class of equilibria for which  $\sigma^{(i,p)} = 0$ ,  $\sigma^{(g,p)} = 1$ ,  $0 < \sigma^{(i,\bar{p})} < 1$ ,  $\sigma^{(g,\bar{p})} = 1$ .

#### **Proposition 2.**

For the unanimity rule,  $\hat{k} = n$ , the symmetric responsive Nash equilibrium is of class

- 1. *E*1 if  $q < \beta^{(i,p)}(n)$  for each type  $p \in \{p, \bar{p}\}$  and q > 1 p; and
- 2. *E2* if  $\beta^{(i,\underline{p})}(n) < q < \min\{\beta^{(i,\overline{p})}(n), \beta^{(g,\underline{p})}(n)\}$  and  $q > 1 \overline{p}$ ,

where  $\beta^{(s,p)}(n)$  is evaluated at  $\sigma^{(i,p)} = 0$  and  $\sigma^{(g,p)} = 1$ , for every relevant  $p \in \{p, \bar{p}\}$ .

To consider mixing strategies equilibria, informative voting cannot be an equilibrium. Therefore, the conditions specified in Proposition 2 say, if a juror that could observe all the signals, of which n - 1 are g, they would prefer to convict the defendant (hence, we evaluate the threshold posterior beliefs in Proposition 2 assuming jurors cast their vote according to their signals).

For  $n \to \infty$ , Corollary 1 implied informative voting is never an equilibrium, irrespective of the level of *q*. Further to that, the semi-mixed equilibria also do not exist because the conditions for either type of juror to vote informatively are violated. The only surviving class of symmetric responsive equilibria in large juries is summarized in Proposition 3.

For the unanimity rule,  $\hat{k} = n$ , with  $n \to \infty$ , there only symmetric responsive equilibrium with potentially both skeptical and trusting voters is a mixed strategy equilibrium of class E1.

#### Induced Type I and Type II Errors

We now derive analytical expressions for the induced type I and type II errors for collective decisions with the unanimity rule. They are computed using the equilibrium voting strategies obtained and expressed as functions of the jurors' personal views,  $\overline{p}$  and  $\underline{p}$ , the underlying 'true' precision of the signals,  $\hat{p}$ , the composition, m, as well as the size, n of the jury.

Even though the jurors' personal views determine their optimal voting strategies, it is the 'effective' probabilities of voting to convict innocent and guilty defendants,  $\hat{\gamma}_{I}^{(\cdot,\cdot)}$  and  $\hat{\gamma}_{G}^{(\cdot,\cdot)}$ , that dictate the induced type I and type II errors. These effective probabilities are derived from the jurors' optimal voting strategies and the underlying 'true' precision,  $\hat{p}$ , with which signals are generated. We add the 'hat' to these probabilities to remind ourselves that the underlying 'true' precision determines them, and not the one(s) jurors believe to be true.

Then the type I and type II errors can be expressed as weighted functions of the probabilities of skeptical and trusting jurors to vote to convict if the defendant is innocent or guilty, where the weights correspond to the proportion of skeptical and trusting jurors, m and 1 - m, for a given jury size, n, as follows

$$Pr(C|I) = ((\hat{\gamma}_{I}^{(s,\bar{p})})^{m} (\hat{\gamma}_{I}^{(s,\bar{p})})^{1-m})^{n}, \text{ and}$$
$$Pr(A|G) = ((\hat{\gamma}_{G}^{(s,p)})^{m} (\hat{\gamma}_{G}^{(s,\bar{p})})^{1-m})^{n},$$

Remember that, in finite jury sizes, there were two classes of equilibria, one class in which  $0 < \sigma^{(i,\underline{p})} < 1$ ,  $\sigma^{(g,\underline{p})} = 1$ ,  $0 < \sigma^{(i,\underline{p})} < 1$ ,  $\sigma^{(g,\underline{p})} = 1$ , and one in which  $\sigma^{(i,\underline{p})} = 0$ ,  $\sigma^{(g,\underline{p})} = 1$ ,  $0 < \sigma^{(i,\underline{p})} < 1$ ,  $\sigma^{(g,\underline{p})} = 1$ . We denoted the first by *E*1 and the second by *E*2. Remember also that, for  $n \to \infty$ , *E*1 is the only class of symmetric, responsive equilibria.

In *E*1, the probabilities of any (skeptical or trusting) juror to vote to convict respectively if the defendant is innocent or guilty, can be computed as

$$\begin{split} \hat{\gamma}_{I}^{(s,p)} &= \hat{p}\sigma^{(i,p)} + (1-\hat{p}) = \frac{\left[\frac{(1-q)(1-p)}{qp}\right]^{\frac{1}{(n-1)}}(p+\hat{p}-1)+p-\hat{p}}{p-(1-p)\left[\frac{(1-q)(1-p)}{qp}\right]^{\frac{1}{(n-1)}}},\\ \hat{\gamma}_{G}^{(s,p)} &= \hat{p} + (1-\hat{p})\sigma^{(i,p)} = \frac{\left[\frac{(1-q)(1-p)}{qp}\right]^{\frac{1}{(n-1)}}(p-\hat{p})+(p+\hat{p}-1)}{p-(1-p)\left[\frac{(1-q)(1-p)}{qp}\right]^{\frac{1}{(n-1)}}}, \end{split}$$

where p = p for a skeptical and  $p = \overline{p}$  for a trusting juror.

In *E*2, the probabilities of skeptical jurors to convict respectively if the defendant is innocent or guilty, is computed as

$$\hat{\gamma}_{I}^{(s,p)} = 1 - \hat{p}$$

$$\hat{\gamma}_{G}^{(s,p)} = \hat{p}.$$

whereas for trusting jurors they are as in *E*1.

For  $n \to \infty$ , only *E*1 exists and the type I and type II errors become

$$\lim_{n \to \infty} Pr(C|I) = \left(\frac{(1-q)(1-\underline{p})}{q\underline{p}}\right)^{\frac{\underline{p}m}{2\underline{p}-1}} \left(\frac{(1-q)(1-\underline{p})}{q\overline{p}}\right)^{\frac{\underline{p}(1-m)}{2\underline{p}-1}}, \text{ and}$$
$$\lim_{n \to \infty} Pr(A|G) = 1 - \left(\frac{(1-q)(1-\underline{p})}{qp}\right)^{\frac{(1-\underline{p})m}{2\underline{p}-1}} \left(\frac{(1-q)(1-\overline{p})}{q\overline{p}}\right)^{\frac{(1-\underline{p})(1-m)}{2\overline{p}-1}},$$

and, when we compute these limit type I and II errors for the extreme cases of m = 0 and m = 1, we get

$$\lim_{n \to \infty} \Pr(C|I)|_{m=0} = \left(\frac{(1-q)(1-\bar{p})}{q\bar{p}}\right)^{\frac{\bar{p}}{2\bar{p}-1}},$$
$$\lim_{n \to \infty} \Pr(C|I)|_{m=1} = \left(\frac{(1-q)(1-\bar{p})}{q\underline{p}}\right)^{\frac{\bar{p}}{2\underline{p}-1}},$$

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for the type I errors and

$$\begin{split} \lim_{n \to \infty} \Pr(A|G)|_{m=0} &= 1 - \left(\frac{(1-q)(1-\bar{p})}{q\bar{p}}\right)^{\frac{(1-\bar{p})}{2\bar{p}-1}},\\ \lim_{n \to \infty} \Pr(A|G)|_{m=1} &= 1 - \left(\frac{(1-q)(1-\underline{p})}{qp}\right)^{\frac{(1-\bar{p})}{2\bar{p}-1}}, \end{split}$$

for the type II errors. A quick inspection reveals that, as  $n \to \infty$ , type I errors are lower for m = 0 than for m = 1; whereas type II errors are lower for m = 1 than for m = 0. In a general voting population (e.g., for referenda) if all voters were trusting of the information they receive, they would commit fewer type I and more type II errors than if all voters were skeptical of their information. These results are summarized in the proposition below.

#### **Proposition 4.**

For voting under unanimity,  $\hat{k} = n$ , with a proportion of skeptical and trusting voters of m and 1 - m, if  $n \to \infty$ , the equilibrium voting strategy is of class E1, and

- 1.  $Pr(C|I)|_{m=1} > Pr(C|I)|_{m=0}$ ; and
- 2.  $Pr(A|G)|_{m=1} < Pr(A|G)|_{m=0}$ .

Finally, for finite *n*, it is difficult to compare the analytical expressions for the induced type I and type II errors. However, we can derive exact expressions for specific parameter values of the jury size, *n*, the skeptical and trusting jurors' beliefs, *p* and  $\bar{p}$ , the threshold of reasonable doubt, *q*, the 'true' precision of the signals,  $\hat{p}$ , and the jury composition, *m*. In Section 3 and Appendix B, we provide such a numerical analysis of the equilibrium strategy profiles and the magnitude of the type I and II errors induced across various combinations of the underlying parameters of interest of our model.

#### **Non-unanimous Voting Rules**

#### Informative Voting Equilibrium and Voting Equilibria with Randomization

We now turn to non-unanimous voting rules. We denote them by  $\hat{k} = \alpha n$ , with  $1/2 \le \alpha < 1$ . As before, we first characterize the conditions for an informative voting equilibrium to exist. We then characterize symmetric and responsive strategic voting equilibria for the case in which informative voting is not an equilibrium action. In this case, we find the same two classes of symmetric responsive voting equilibria, *E*1 and *E*2, as well as a third one, in which the skeptical voters randomize their vote, whereas the trusting ones do not.

#### **Definition 7.**

Define as E3 the class of equilibria for which  $0 < \sigma^{(i,p)} < 1$ ,  $\sigma^{(g,p)} = 1$ ,  $\sigma^{(i,\bar{p})} = 0$ ,  $\sigma^{(g,\bar{p})} = 1$ .

Depending on whether the size of a jury is an odd or an even number, the conditions for those equilibria to exist are slightly different. To simplify the exposition of our results, in the remainder of this subsection we present analytical results only for the odd numbered jury size case. In Section 3 and Appendix B, when we provide simulations for the induced type I and type II errors for the canonical jury size of n = 12, we resort to the relevant conditions for even jury sizes.

Proposition 5 gives conditions for the informative equilibrium to exist and Proposition 6 provides conditions for the existence of voting equilibria with randomization.

#### **Proposition 5.**

For non-unanimous voting rules  $\hat{k} = \alpha n$  and n odd, informative voting is an equilibrium if and only if  $q \in [\beta^{(i,p)}(\alpha n), \beta^{(g,p)}(\alpha n)]$  for each type  $p \in \{p, \bar{p}\}$ . Such an interval exists for

1. minority rules,  $\hat{k} < \frac{n}{2}$ , if and only if  $\left(\frac{1-p}{\underline{p}}\right)^{2\hat{k}-n-2} > \left(\frac{1-\bar{p}}{\bar{p}}\right)^{2\hat{k}-n}$ ;

2. the simple majority rule, 
$$\frac{n}{2} < \hat{k} < \frac{n}{2} + 1$$
, if and only if  $\left(\frac{1-p}{\underline{p}}\right)^{2\hat{k}-n-2} > \left(\frac{1-p}{\underline{p}}\right)^{2\hat{k}-n}$ ;

3. super-majority rules, 
$$\frac{n}{2} + 1 < \hat{k}$$
, if and only if  $\left(\frac{1-\bar{p}}{\bar{p}}\right)^{2\hat{k}-n-2} > \left(\frac{1-p}{\underline{p}}\right)^{2\hat{k}-n}$ .

Note that the condition for  $q \in [\beta^{(i,p)}(\alpha n), \beta^{(g,p)}(\alpha n)]$  to be non-empty holds for any *finite n* for the simple majority rule; whereas the corresponding conditions do not necessarily hold for finite *n* for super-majority and minority rules.

#### Corollary 2.

For non-unanimous voting rules, if the jury size tends to infinity,  $n \rightarrow \infty$ , informative voting is not an equilibrium for any *q*.

Proposition 6 follows directly from Proposition 5.

#### **Proposition 6.**

For non-unanimous voting rules  $\hat{k} = \alpha n$  and n odd, the symmetric responsive Nash equilibrium is of class

1. E1 if  $q < \beta^{(i,p)}$  for each  $p \in \{\underline{p}, \overline{p}\}$  and  $q < \frac{1}{1 + \left(\frac{p}{1-p}\right)^{n-\overline{k}+1}}$  for super-majority rules; and  $q < \frac{1}{1 + \left(\frac{\overline{p}}{1-p}\right)^{n-\overline{k}+1}}$  for minority

and simple majority rules;

- 2. E2 if  $\beta^{(i,\underline{p})} < q < \min\{\beta^{(i,\overline{p})}, \beta^{(\underline{g},\underline{p})}\}$  and  $q < \frac{1}{1 + \left(\frac{\overline{p}}{1-\overline{p}}\right)^{n-\hat{k}+1}}$  for super-majority rules;
- 3. E3 if  $\beta^{(i,\bar{p})} < q < \min\{\beta^{(i,\underline{p})}, \beta^{(g,\bar{p})}\}$  and  $q < \frac{1}{1 + \left(\frac{p}{1-p}\right)^{n-\hat{k}+1}}$  for minority and simple majority rules;

where  $\beta^{(s,p)}(n)$  is evaluated at  $\sigma^{(i,p)} = 0$  and  $\sigma^{(g,p)} = 1$ , for every relevant  $p \in \{\underline{p}, \overline{p}\}$ .

#### Induced Type I and Type II Errors

As in FP, also in our model, it is always possible to find a specific non-unanimous voting rule outperforming any other voting rule. To the extent that it is feasible to pick such an 'ideal' voting rule, among any non-unanimous voting rules, that is, it is possible to select what the minimum number of votes for a conviction verdict to be reached should be, type I and type II errors can be driven to zero as the size of the jury grows sufficiently large.

#### **Proposition 7.**

A sufficient condition for both types of errors to approach zero as  $n \to \infty$  for  $\hat{k} = \alpha n$ , with  $1/2 \le \alpha < 1$  and a proportion of skeptical and trusting voters of m and 1 - m, respectively, is that  $\hat{\gamma}_G^{(s,p)} > \hat{\gamma}_G^{(s,\bar{p})} > \alpha > \hat{\gamma}_I^{(s,\bar{p})} > \hat{\gamma}_I^{(s,\bar{p})}$ .

## 3 Simulations

In this section, we resort to numerical simulations to compare the errors generated in a finitely sized jury. In particular, we illustrate these errors for n = 12, the standard jury size as analyzed and discussed in the canonical jury voting model in FP.

Let  $n_1 = mn$  and  $n_2 = (1 - m)n$ , where *m* stands for the fraction of skeptical voters in the jury, and denote the number of votes to convict by skeptical and trusting jurors as  $m_1$  and  $m_2$ . Then, a voting rule  $\hat{k}$  implies that a conviction verdict is reached if an only if  $m_1 + m_2 \ge \hat{k}$ . Using this notation, we compute the type I and type II errors for each voting rule,  $\hat{k} \in [\frac{n}{2} + 1, n]$ , for given numbers  $n_1$  and  $n_2$  of skeptical and trusting jurors of any given, finite, jury size *n* as:

$$\begin{aligned} Pr(C|I) &= \sum_{m_1+m_2=k}^n \binom{n_1}{m_1} \binom{n_2}{m_2} (\hat{\gamma}_I^{(s,p)})^{m_1} (1-\hat{\gamma}_I^{(s,p)})^{n_1-m_1} (\hat{\gamma}_I^{(s,\bar{p})})^{m_2} (1-\hat{\gamma}_I^{(s,\bar{p})})^{n_2-m_2} \\ Pr(A|G) &= 1-\sum_{m_1+m_2=k}^n \binom{n_1}{m_1} \binom{n_2}{m_2} (\hat{\gamma}_G^{(s,p)})^{m_1} (1-\hat{\gamma}_G^{(s,p)})^{n_1-m_1} (\hat{\gamma}_G^{(s,\bar{p})})^{m_2} (1-\hat{\gamma}_G^{(s,\bar{p})})^{n_2-m_2} \end{aligned}$$

In **Appendix** B, we provide a series of tables for the induced type I and type II errors in our model for a jury of size n = 12, q = 0.90 and  $\hat{p} = 0.8$ , which is the unique and commonly known precision of the signals for the canonical voting model studied in FP. We do so, to contrast our results against those obtained in FP.

Table 5, Table 6, Table 7, Table 8 and Table 9 illustrate in bold all instances in which our model induces lower type I errors with unanimity than with alternative voting rules. For example, Table 5 shows that, with a jury

size of n = 12, unanimity generates lower type I errors than any other voting rule, including simple majority, as long as there is at least one skeptical juror who sufficiently discounts the precision of the signals. Table 6, Table 7, Table 8 and Table 9 show that this is not the only case in which this happens. This result is in stark contrast to the prediction of FP that, in a jury of size 12, the unanimity rule generates the highest type I errors.

Similarly, Table 10, Table 11, Table 12, Table 13 and Table 14 highlight that the unanimity rule can also induce lower type II errors than other voting rules if there is at least one skeptical juror as long as this juror does not discount the signal's precision too severely. The most striking improvement in terms of type II errors under our model is when adhering to unanimity as opposed to majority voting. Unlike in the standard jury voting model, where type II errors under unanimity in a jury size of n = 12 were the highest, once there exist some skeptical voters, relying on unanimity as opposed to simple majority can considerably help reduce type II errors. For instance, if we have a 50/50 split in a jury between skeptical and trusting jurors, unanimity always does at least as well as majority, and can decrease errors up to 33% in some cases, which is not negligible: from everyone being acquitted even if guilty, only 67% of defendants who are guilty are acquitted in that case.

Lastly, our simulations show that for a finite jury size the level of the induced type I errors is at its lowest under unanimity for  $n_1 = 12$ , that is, when all voters are skeptical. Vice versa, for a finite jury size the level of the induced type II errors is at its lowest under unanimity for  $n_1 = 0$ , that is, when all voters are trusting.

## 4 **Experimentation**

We next report results from a laboratory experiment, which we conducted to test the predictions from our model and to contrast them against the predictions obtained in a canonical model.

## **Experimental Design and Execution**

In the experiment, subjects were confronted with a series of collective decisions between one of two alternatives. The alternatives represented neutral metaphors for an acquittal or a conviction of a defendant. Before casting their votes, subjects received private signals regarding the true state of the world, analogous to evidence produced in a trial. Individual votes translated into the group decision as a function of the voting rule applied. Subjects received rewards reflecting the quality of the group decisions they were part of, that is, whether their group decisions were correct, or wrong.

Specifically, our experiment employed a  $2 \times 2 \times 2$  design to emulate such a jury decision-making process. Our treatments involved (i) two informational structures for the accuracy of signals (congruent and possibly non-congruent views), (ii) two voting rules (simple majority and unanimity), and (iii) two communication protocols (no communication or free-form communication). The latter treatments were included in our experiments, to create a parallel with existing experimental evidence regarding voting with communication, and to highlight any difference in the predictions of standard voting models with communication, such as Gerardi and Yariv (2007) and Goeree and Yariv (2011), as opposed to the actual behavior of voters in our environment if also allowed to communicate before casting their votes.<sup>4</sup> However, since our model does not incorporate explicitly the assumption of deliberation (communication across voters before the casting of their votes) for the purpose of this study we concentrate on reporting experimental results from the other treatments, those without communication and leave the communication treatments for further research.<sup>5</sup>

Based on our experimental design,<sup>6</sup> we conducted our experiments on 2–9 October 2015 in the *DECIDE Laboratory for Business Decision Making* at the University of Auckland. We recruited overall 165 subjects to participate in the experiments through ORSEE (Greiner 2004). The ORSEE subject pool consists of students at the University of Auckland, spanning across all areas. In the course of the experimental sessions, we collected 3300 individual decision-making results and 660 group decision-making outcomes.

Each experimental session involved 20 rounds of group decisions. At the start of each round, subjects were randomly and anonymously matched into groups of five, and each group was assigned a jar. A jar contained 10 balls with either a majority of red balls (a *red jar*) or with a majority of blue balls (a *blue jar*). In the beginning of each round, subjects were informed that either the red jar or the blue jar was selected at random with equal probability. In each round, each subject was shown a ball that was drawn independently with replacement from the jar selected for their group for that round. The probability of that randomly drawn ball to match the color of the jar represents the accuracy of the subjects' information. The color of the ball drawn for a particular subject was privately revealed to that subject only. Subjects then cast their individual votes, contributing to the group-decision on the color of the chosen jar. The default group decision with unanimity was blue. That is, if with the unanimity rule less than 5 out of 5 group members chose Red, then the decision was Blue. Before entering the next round, subjects neither received feedback regarding the other group members' individual decisions, nor

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did they receive feedback regarding the group decision and its quality. Only after the 20th round did subjects receive feedback regarding their individual and group decision and its quality for one randomly selected round for the experiment. Their payments were computed using that randomly selected round. In particular, subjects received a NZ\$10 show up fee and a bonus of NZ\$15 if, in a randomly selected round for the experiment, their group decision was correct. Otherwise, their bonus was NZ\$5.<sup>7</sup>

Our treatments applied two alternative *information structures*, one in which all subjects had congruent views and one in which they had possibly non-congruent views of the informativeness of the evidence provided to them. As mentioned, in the beginning of each round, subjects were informed that either the red jar or the blue jar was selected at random with equal probability.

At that point, in the treatment with *congruent views*, subjects were shown one red and one blue jar. The red jar contained 6 red balls and 4 blue balls, and the blue jar contained 6 blue balls and 4 red ones. Thus, subjects knew that the color of any randomly drawn ball matched the jar's color with probability p = 0.6 (or 60%).

In contrast, in the treatment with possibly *non-congruent views* we showed the subjects two different red and two different blue jars, with different compositions of red and blue balls. The two alternative red jars contained either 6 red and 4 blue balls or 9 red balls and 1 blue ball and the two different blue jars contained either 6 blue and 4 red balls or 9 blue balls and 1 red ball. Following Stecher, Shields, and Dickhaut (2011), when we displayed the four jars to the subjects, we provided them with a graph that included 50 histograms for the occurrence of either of the compositions for the red or the blue jars, each compiled from 1000 draws. These histograms showed no patterns or regularities of the objective probability distribution of the possible compositions, so that in this setting subjects had to form their own view on the composition of the jar that applied to that particular round.<sup>8</sup> In each round, subjects were then asked to decide which of the compositions, p = 0.6 or p = 0.9, they thought their group's jar had. After this decision, each subject was displayed one red and one blue jar with the composition s/he had just expressed views about for that round, either two jars with p = 0.6, or two jars with p = 0.9. While these two jars were displayed, each subject was shown a ball drawn randomly from the jar that had been selected for their group in that round; and they were asked to vote whether that ball was drawn from the red or the blue jar. By showing each subject only the two jars that were of the composition that subject had expressed views about, we essentially framed subjects to believe that the relevant composition of the jar they were voting on corresponded to one with the composition they expressed views about.<sup>5</sup>

All experimental sessions were computerized using z-Tree (Fischbacher 2007). For each of the treatments, after all subjects arrived, instructions were provided by the experimenters and time was given to subjects to read those instructions and to answer open questions, before the experiment started. Preceding each such session, subjects participated in a trial experiment to familiarize themselves with the process.<sup>10</sup>

#### **Analysis and Results**

Table 1 and Table 2 summarize the subjects' strategies and the realized errors for all treatments. The theoretical predictions of our model for our experimental setting (i.e., for a jury of size n = 5 and threshold of reasonable doubt q = 1/2 and a true precision of  $\hat{p} \in \{0.6, 0.9\}$ , for which  $q > 1 - \hat{p}$ ) are presented in round parentheses. A visual inspection of these tables shows that the evidence is in line with what the theory predicts for both the congruent and non-congruent views treatments. Reaching a unanimous decision is not easy. Therefore, and as expected with the unanimity rule, given the small number of jurors and the default set to blue (our metaphor for acquittal), we observe low type I errors. We find that, in both treatments with congruent and non-congruent views, subjects vote more often for red when they have a red signal than when they have a blue signal. Further, in both treatments, and as the theory predicts, type I errors are lower with the unanimity rule. Table 3 reports the so predicted and realized type I and II errors depending on the realized precision of the jurors' signals. In addition, in the lower four rows, it distinguishes juries with a majority of trusting jurors (m < 0.5) and a majority of skeptical jurors (m > 0.5).

Table 1: Experimental realizations and theory predictions in the congruent views treatment.

n = 5	$\hat{k} = 3$	$\hat{k} = 5$
Number of individual decisions	400	500
Number of group decisions	80	100
Red votes with red signals	82% (100%)	76% (100%)
Red votes with blue signals	17% (0%)	33% (24%)
Wrong jury outcomes	36% (32%)	35% (43%)
True jar blue (Type I error)	37% (32%)	2% (8%)
True jar red (Type II error)	36% (32%)	81% (79%)

Numbers in the round parentheses correspond to the theory predictions.

Table 2: Experimental realizations and theory predictions in the non-congruent views treatment.

n = 5	$\hat{k} = 3$	$\hat{k} = 5$
Number of individual decisions	300	500
Number of group decisions	60	100
Skeptical jurors		
Red votes with red signals	63% (100%)	74% (100%)
Red votes with blue signals	26% (0%)	49% (60%)
Trusting jurors		
Red votes with red signals	93% (100%)	89% (100%)
Red votes with blue signals	3% (0%)	21% (50%)
Wrong jury outcomes	28% (16%)	38% (31%)
True jar blue (Type I error)	20% (15%)	5% (13%)
True jar red (Type II error)	37% (16%)	80% (50%)

Numbers in the round parentheses correspond to theory predictions.

n = 5		$\hat{k} = 3$		$\hat{k} = 5$
	<i>p</i> = 0.6	<i>p</i> = 0.9	<i>p</i> = 0.6	<i>p</i> = 0.9
Type I error	36% (32%) 67% (32%)	6% (1%) 7% (1%)	8% (20%) 90% (65%)	3% (7%) 60% (22%)
Type II error	p = 0.6	p = 0.9	p = 0.6	p = 0.9
Type I error				
m < 0.5	42% (32%)	7% (1%)	11% (19%)	0% (6%)
m > 0.5	0% (32%)	0% (1%)	0% (22%)	14% (8%)
Type II error		. ,	. ,	
m < 0.5	64% (32%)	8% (1%)	88% (65%)	62% (22%)
m > 0.5	75% (32%)	0% (1%)	100% (62%)	50% (19%)

**Table 3:** Experimental realizations and theory predictions in the non-congruent views treatment by realized composition of Jars, *p*.

Numbers in the round parentheses correspond to the theory predictions.

Given the relatively low number of observations at the group level, especially when controlling for their actual compositions of skeptical and trusting subjects, we decided not to conduct a statistical analysis that compares type I and type II errors across treatments and group compositions. Instead, we perform an econometric analysis to estimate what explains individual votes for red. The first two columns of Table 4 reports the results of Probit estimations. As expected, with and without congruent views, receiving a red signal significantly increases the probability of a vote for red. With non-congruent views, also the voting rule matters. Under the simple majority rule, the marginal effect of a red signal is larger than it is under unanimity. This means, with non-congruent views, jurors with a guilty signal would be less likely to convict with the unanimity rule than with the majority rule. The marginal effect of a red signal furthermore increases slightly in the last five rounds of the experimental sessions. However, this increase is only marginally significant. This is so, in spite of the fact that, contrary to Goeree and Yariv (2011), our subjects did not receive feedback after each round on what the

correct decision would have been. The fact that, a decision that was late in the game at most marginally impacts the decision to vote for red, supports that individual subjects had no opportunity to influence each others' later choices, reinforcing the treatment of individual and group choices as independent observations in our analysis. We report the results of a linear probability model in the last two columns as a robustness check. The results are in line with those of the Probit model.

	Prob	oit (marginal effects)	Linear Probability Mode			
Views	Congruent	Non-congruent	Congruent	Non-congruent		
Red signal	0.654***	0.724***	0.656***	0.707***		
C	(0.0870)	(0.0743)	(0.0874)	(0.0718)		
Unanimity	0.165	0.316***	0.116	0.220***		
2	(0.109)	(0.111)	(0.0819)	(0.0809)		
Red signal ×	-0.0483	-0.306**	-0.0431	-0.202*		
Unanimity						
-	(0.179)	(0.151)	(0.121)	(0.105)		
Late	0.141*	-0.226	0.0998	-0.107		
	(0.0835)	(0.152)	(0.0672)	(0.0671)		
Late × Red signal	-0.0194	0.184**	-0.0207	0.113*		
0	(0.0801)	(0.0929)	(0.0562)	(0.0620)		
Late × Unanimity	-0.116	0.0788	-0.0777	-0.00292		
-	(0.0926)	(0.155)	(0.0655)	(0.0783)		
Constant	Included	Included	0.145***	0.119**		
			(0.0410)	(0.0484)		
Pseudo- $R^2 / R^2$	0.3183	0.3193	0.4022	0.3963		
Observations	900	800	900	800		

Table 4: Estimations that explain individual votes for red.

Robust standard errors are given in parentheses. Standard errors are clustered at the individual level. \*\*\*Significant at 1% level, \*\*significant at 5% level, \*significant at 10% level.

In summary, our experimental results show that (1) the voters tend to vote strategically, consistent with the existing literature; (2) the Jury Paradox was not manifesting itself, given the relatively small size of the group voting collectively and consistent with the existing literature; (3) when group members have non-congruent views, there are instances in which both type I and II errors are reduced compared to the treatments with congruent views.<sup>11</sup> This is a novel result, which suggests the importance of personal views on the precision of the information provided in the collective decision-making. These first results call for further investigation.<sup>12</sup>

# 5 Conclusion

We relaxed the assumptions in collective decision-making models that voters can not only derive a perfect view about the accuracy of the information at their disposal, but can, in addition, also correctly assess other voters' views about it. We thereby demonstrated both theoretically and experimentally, that, contrary to what is normally accepted, information aggregation works well with voting rules other than simple majority, including unanimity. Within our model, we derived conditions for voters to vote less often against their information than in conventional models, delivering higher-quality decisions, including, among others, for the canonical 12 jurors case. Our analysis provided us with richer sets of voting equilibria, than for standard collective decision-making models, namely those in which some voters vote informatively, whereas others randomize their votes.

These novel results shed light on cases in which voters hold non-congruent views and are potentially ignorant of such differences. Hence, our study addresses scenarios in which voters are called upon making decisions over issues for which they may be divided upon not because they do not share the same common goal (all voters may seek to reach the same commonly valued outcomes), but rather because of their reading of how much the information provided to them is to be trusted to begin with. Those are scenarios in which voters may not even be aware of their differing positions, while they cast their votes. Our results also have implications for general election cases, such as referenda, in which the level of trust or distrust in the information available to voters may have serious consequences, which depend on the adopted voting rule.

For instance, after the Brexit referendum, it became apparent that different voters had very different experiences of Europe due to their different age groups and locations. Their partial views based on different experiences might have translated into varying degrees of trust in the information provided by the two campaigns and, hence, into very different regional outcomes. Our theory posits that, in this situation, democracy might have not been served best by relying on simple majority voting. Institutions may need to be put to a tougher test. They need to be robust to voters that are oblivious to other voter's positions and subjective views of the world.

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# A Proofs

#### **Proof of Proposition 1.**

If the voting equilibrium is informative, each vote will have to reveal exactly what signal each juror receives. Assume there to be a skeptical juror who receives an innocent signal *i*. The posterior probability that this juror holds about a defendant being guilty, conditional on being pivotal, and given unanimity voting, evaluated at  $\sigma^{(s,\underline{p})} = \{\sigma^{(i,\underline{p})} = 0, \sigma^{(g,\underline{p})} = 1\}$  would reduce to

$$\beta^{(i,\underline{p})}(n) = \frac{1}{1 + \frac{p(1-\underline{p})^{n-1}}{(1-\underline{p})p^{n-1}}}.$$
(A)

Hence, this juror will vote for acquittal if and only if  $\beta^{(i,p)}(n) \leq q$ .

Vice versa, if this juror received a guilty signal *g*, the posterior probability that this juror would hold about a defendant being guilty, conditional on being pivotal, and given unanimity voting evaluated at  $\sigma^{(s,\underline{p})} = \{\sigma^{(i,\underline{p})} = 0, \sigma^{(g,\underline{p})} = 1\}$  would be

$$\beta^{(g,\underline{p})}(n) = \frac{1}{1 + \frac{(1-\underline{p})^n}{p^n}}.$$
(B)

This demonstrates that this same juror would have voted for conviction if and only if  $\beta^{(g,\underline{p})}(n) > q$ . Therefore, for a skeptical juror to always vote in accordance with the signal received, both conditions need to be satisfied at the same time, which happens if and only if  $\beta^{(i,\underline{p})}(n) \le q < \beta^{(g,\underline{p})}(n)$ .

Consider equilibrium candidate  $\sigma^{(i,p)} = 0$  and  $0 \le \sigma^{(g,p)} < 1$ . If this were an equilibrium, we would have

$$\beta^{(g,\underline{p})}(n) = \frac{1}{1 + \frac{(1-\underline{p})(\gamma_I^{(g,\underline{p})})^{n-1}}{\underline{p}(\gamma_G^{(g,\underline{p})})^{n-1}}} > \frac{1}{1 + \frac{(1-\bar{p})^n}{\bar{p}^n}} \ge q,$$

which implies that a voter with a guilty signal would have a strict incentive to vote to convict, contradicting  $0 \le \sigma^{(g,\underline{p})} < 1$ . So  $\sigma^{(i,\underline{p})} = 0$  and  $0 \le \sigma^{(g,\underline{p})} < 1$  is not an equilibrium.

Consider equilibrium candidate  $0 < \sigma^{(i,\underline{p})} \leq 1$  and  $\sigma^{(g,\underline{p})} = 1$ . If this were an equilibrium, we would have

$$\beta^{(i,\underline{p})}(n) = \frac{1}{1 + \frac{\underline{p}(\gamma_I^{(s,\underline{p})})^{n-1}}{(1-p)(\gamma_G^{(s,\underline{p})})^{n-1}}} < \frac{1}{1 + \frac{(1-\overline{p})^n}{\overline{p}^n}} \le q,$$

which implies that a voter with an innocent signal would have a strict incentive to acquit, contradicting  $0 < \sigma^{(i,\underline{p})} \leq 1$ . So  $0 < \sigma^{(i,\underline{p})} \leq 1$  and  $\sigma^{(g,\underline{p})} = 1$  is not an equilibrium.

Similarly, we can derive conditions for the trusting juror type to vote informatively. These will dictate that  $\beta^{(i,\tilde{p})}(n) \leq q$  and  $\beta^{(g,\tilde{p})}(n) > q$ , where

$$\beta^{(i,\bar{p})}(n) = \frac{1}{1 + \frac{\bar{p}(1-\bar{p})^{n-1}}{(1-\bar{p})\bar{p}^{n-1}}} \le q;$$
(C)

and

$$\beta^{(g,\bar{p})}(n) = \frac{1}{1 + \frac{(1-\bar{p})^n}{\bar{p}^n}} > q,$$
(D)

which is equivalent to  $\beta^{(i,\bar{p})}(n) \le q < \beta^{(g,\bar{p})}(n)$ .

Such interval for the posterior beliefs exists if and only if

$$\max\{\beta^{(i,p)}(n),\beta^{(i,\bar{p})}(n)\} < \min\{\beta^{(g,p)}(n),\beta^{(g,\bar{p})}(n)\}.$$

We know that (1)  $\min\{\beta^{(g,\underline{p})}(n), \beta^{(g,\underline{p})}(n)\} = \beta^{(g,\underline{p})}(n)$  and (2)  $\beta^{(g,\underline{p})}(n) > \beta^{(i,\underline{p})}(n)$ . Hence, there exists such a q for which informative voting is an equilibrium if

$$\beta^{(i,\bar{p})}(n) < \beta^{(g,\underline{p})}(n) \Leftrightarrow \frac{1}{1 + \left(\frac{1-\bar{p}}{\bar{p}}\right)^{n-2}} < \frac{1}{1 + \left(\frac{1-\bar{p}}{\underline{p}}\right)^n} \Leftrightarrow \left(\frac{1-\bar{p}}{\bar{p}}\right)^{\frac{n-2}{n}} > \frac{1-\underline{p}}{\underline{p}}.$$

For *n* finite there are pairs  $(p, \bar{p})$  for which this inequality holds.

#### **Proof of Corollary 1**

By Proposition 1, informative voting is an equilibrium if and only if  $q \in [\beta^{(i,p)}(n), \beta^{(g,p)}(n)]$  for each  $p \in \{p, \bar{p}\}$ . There is no such q if

$$\max\{\beta^{(i,p)}(n),\beta^{(i,\bar{p})}(n)\} > \min\{\beta^{(g,p)}(n),\beta^{(g,\bar{p})}(n)\}.$$

We know that (1)  $\min\{\beta^{(g,\underline{p})}(n), \beta^{(g,\overline{p})}(n)\} = \beta^{(g,\underline{p})}(n)$  and (2)  $\beta^{(g,\underline{p})}(n) > \beta^{(i,\underline{p})}(n)$ . Hence, there exists no q for which informative voting is an equilibrium if

$$\beta^{(i,\bar{p})}(n) > \beta^{(g,\underline{p})}(n) \Leftrightarrow \frac{1}{1 + \left(\frac{1-\bar{p}}{\bar{p}}\right)^{n-2}} > \frac{1}{1 + \left(\frac{1-\underline{p}}{\underline{p}}\right)^n} \Leftrightarrow \left(\frac{1-\bar{p}}{\bar{p}}\right)^{\frac{n-2}{n}} < \frac{1-\underline{p}}{\underline{p}}.$$

Because  $\lim_{n\to\infty} \left(\frac{1-\bar{p}}{\bar{p}}\right)^{\frac{n-2}{n}} = \frac{1-\bar{p}}{\bar{p}}$ , this inequality holds for any  $\underline{p} < \bar{p}$ .

#### **Proof of Proposition 2.**

If  $\beta^{(i,p)}(n) = q < \beta^{(g,p)}(n)$ , we must have  $0 < \sigma^{(i,p)} < 1$ , that is

$$\frac{1}{1 + \frac{\underline{p}(\gamma_I^{(s,\underline{p})})^{n-1}}{(1-\underline{p})(\gamma_G^{(s,\underline{p})})^{n-1}}} = q$$

which implies

$$\left(\frac{\gamma_{I}^{(s,\underline{p})}}{\gamma_{C}^{(s,\underline{p})}}\right)^{n-1} = \frac{1-q}{q} \frac{1-\underline{p}}{\underline{p}}.$$

Thus, in the case when the pivotal voter receives g, it must be true that

$$\frac{1}{1 + \frac{(1-\underline{p})(\gamma_{I}^{(g,\underline{p})})^{n-1}}{p(\gamma_{G}^{(g,\underline{p})})^{n-1}}} = \frac{1}{1 + \frac{1-q}{q}\frac{1-\underline{p}}{\underline{p}}\frac{1-\underline{p}}{\underline{p}}} > q,$$

that is  $\beta^{(g,\underline{p})}(n) > q$ , which implies  $\sigma^{(g,\underline{p})} = 1$ . And when  $0 < \sigma^{(i,\underline{p})} < 1$  and  $\sigma^{(g,\underline{p})} = 1$ ,

$$\beta^{(i,\underline{p})}(n) = \frac{1}{1 + (\frac{p}{1-\underline{p}})(\frac{\underline{p}\sigma^{(i,\underline{p})} + (1-\underline{p})}{\underline{p} + (1-\underline{p})\sigma^{(i,\underline{p})}})^{n-1}} = q,$$

which implies that

$$\sigma^{(i,\underline{p})} = \frac{\left[\frac{(1-q)(1-\underline{p})}{q\underline{p}}\right]^{\frac{1}{(n-1)}}\underline{p} - (1-\underline{p})}{\underline{p} - \left[\frac{(1-q)(1-\underline{p})}{q\underline{p}}\right]^{\frac{1}{(n-1)}}(1-\underline{p})}.$$

Because  $\sigma^{(i,p)}$  has to be strictly between 0 and 1, we require q > 1 - p.

If  $\beta^{(i,p)}(n) < q < \beta^{(g,p)}(n)$ , we must have  $\sigma^{(i,p)} = 0$  and  $\sigma^{(g,p)} = 1$  following the equilibrium condition. If  $\beta^{(g,p)}(n) = q$ , we must have  $0 < \sigma^{(g,p)} < 1$ , that is

$$\frac{1}{1 + \frac{(1-\underline{p})(\gamma_I^{(s,\underline{p})})^{n-1}}{\underline{p}(\gamma_G^{(s,\underline{p})})^{n-1}}} = q_I$$

which implies

$$\left(\frac{\gamma_I^{(s,\underline{p})}}{\gamma_G^{(s,\underline{p})}}\right)^{n-1} = \frac{1-q}{q}\frac{\underline{p}}{1-\underline{p}}.$$

Then, in the case when the pivotal voter's private signal is *i*, it must be true that

$$\frac{1}{1 + \frac{\underline{p}(\gamma_I^{(i,\underline{p})})^{n-1}}{(1-p)(\gamma_G^{(i,\underline{p})})^{n-1}}} = \frac{1}{1 + \frac{1-q}{q}\frac{\underline{p}}{1-\underline{p}}\frac{\underline{p}}{1-\underline{p}}} < q,$$

that is  $\beta^{(i,p)}(n) < q$ , which implies  $\sigma^{(i,p)} = 0$ .

However, this is not an equilibrium as we have shown in the proof of Proposition 1, i.e., given  $\sigma^{(i,p)} = 0$ , we must have  $\sigma^{(g,p)} = 1$ , which contradicts the assumption  $0 < \sigma^{(g,p)} < 1$ . Therefore,  $\sigma^{(i,p)} = 0$  and  $0 < \sigma^{(g,p)} < 1$  is not a responsive equilibrium.

If  $q < \beta^{(i,p)}(n)$ , according to the equilibrium condition, we must have  $\sigma^{(i,p)} = 1$  and  $\sigma^{(g,p)} = 1$ . However, this is not a responsive voting strategy.

If  $\beta^{(g,\underline{p})}(n) < q$ , we must have  $\sigma^{(i,\underline{p})} = 0$  and  $\sigma^{(g,\underline{p})} = 0$ , which is not a responsive voting strategy either. The same logic applies to type  $(i,\overline{p})$  voters and type  $(g,\overline{p})$  voters. Therefore, any responsive voting equilibrium rium for those who believe in  $\bar{p}$  must be of the form  $0 \le \sigma^{(i,\bar{p})} < 1$  and  $\sigma^{(g,\bar{p})} = 1$ . And  $\sigma^{(i,\bar{p})}$  is between 0 and 1 as long as  $q > 1 - \bar{p}$ , where

$$\sigma^{(i,\bar{p})} = \frac{\left[\frac{(1-q)(1-\bar{p})}{q\bar{p}}\right]^{\frac{1}{(n-1)}}\bar{p} - (1-\bar{p})}{\bar{p} - \left[\frac{(1-q)(1-\bar{p})}{q\bar{p}}\right]^{\frac{1}{(n-1)}}(1-\bar{p})}.$$

#### **Proof of Proposition 5.**

Assume a supermajority rule,  $\frac{n}{2} + 1 < \hat{k}$ . Then  $\beta^{(i,\underline{p})}(\alpha n) < \beta^{(i,\overline{p})}(\alpha n)$  and  $\beta^{(g,\underline{p})}(\alpha n) < \beta^{(g,\overline{p})}(\alpha n)$ . We have  $\beta^{(i,\bar{p})}(\alpha n) < \beta^{(g,\underline{p})}(\alpha n)$  if and only if

$$\begin{split} \mathcal{B}^{(i,\bar{p})}(\alpha n) &= \frac{1}{1 + \frac{\bar{p}}{1 - \bar{p}} \left(\frac{1 - \bar{p}}{\bar{p}}\right)^{\hat{k} - 1} \left(\frac{\bar{p}}{1 - \bar{p}}\right)^{n - \hat{k}}} \quad < \frac{1}{1 + \frac{1 - p}{\underline{p}} \left(\frac{1 - p}{\underline{p}}\right)^{\hat{k} - 1} \left(\frac{p}{1 - \underline{p}}\right)^{n - \hat{k}}} = \beta^{(g, \underline{p})}(\alpha n) \\ &\Leftrightarrow \left(\frac{1 - \bar{p}}{\bar{p}}\right)^{2\hat{k} - n - 2} \qquad > \left(\frac{1 - p}{\underline{p}}\right)^{2\hat{k} - n}. \end{split}$$

Suppose  $\beta^{(i,\bar{p})}(\alpha n) \leq q < \beta^{(g,\underline{p})}(\alpha n)$  and  $\left(\frac{1-\bar{p}}{\bar{p}}\right)^{2\hat{k}-n-2} > \left(\frac{1-p}{\underline{p}}\right)^{2\hat{k}-n}$  are violated. In this case, we can demonstrate that voters who believe in  $\bar{p}$  prefer to randomize their vote to convict.

First assume that, if the juror knew all *n* signals,  $\hat{k} - 1$  of which were *g*, he would strictly prefer to vote to convict, that is,

$$q < \frac{(1-\bar{p})(\bar{p})^{\hat{k}-1}(1-\bar{p})^{n-\hat{k}}}{(1-\bar{p})(\bar{p})^{\hat{k}-1}(1-\bar{p})^{n-\hat{k}} + \bar{p}(1-\bar{p})^{\hat{k}-1}\bar{p}^{n-\hat{k}}}.$$

Then, if  $\beta^{(i,\bar{p})}(\alpha n) = q < \beta^{(g,\bar{p})}(\alpha n)$ , we must have  $0 < \sigma^{(i,\bar{p})} < 1$ , that is

$$\frac{1}{1+\frac{\bar{p}}{1-\bar{p}}(\frac{\gamma_{I}^{(s,\bar{p})}}{\gamma_{G}^{(s,\bar{p})}})^{\hat{k}-1}(\frac{1-\gamma_{I}^{(s,\bar{p})}}{1-\gamma_{G}^{(s,\bar{p})}})^{n-\hat{k}}}=q,$$

and we can derive

$$(\frac{\gamma_{I}^{(s,\bar{p})}}{\gamma_{C}^{(s,\bar{p})}})^{\hat{k}-1}(\frac{1-\gamma_{I}^{(s,\bar{p})}}{1-\gamma_{C}^{(s,\bar{p})}})^{n-\hat{k}} = \frac{(1-q)(1-\bar{p})}{q\bar{p}}$$

Thus, in the case when the pivotal voter receives signal *g*, it must be true that

$$\frac{1}{1+\frac{1-\bar{p}}{\bar{p}}(\frac{\gamma_{I}^{(s,\bar{p})}}{\gamma_{G}^{(s,\bar{p})}})^{\hat{k}-1}(\frac{1-\gamma_{I}^{(s,\bar{p})}}{1-\gamma_{G}^{(s,\bar{p})}})^{n-\hat{k}}} = \frac{1}{1+\frac{1-q}{q}\frac{1-\bar{p}}{\bar{p}}\frac{1-\bar{p}}{\bar{p}}} > q,$$

that is  $\beta^{(g,\bar{p})}(\alpha n) > q$ , which implies  $\sigma^{(g,\bar{p})} = 1$ .

Therefore, since for  $0 < \sigma^{(i,\bar{p})} < 1$  and  $\sigma^{(g,\bar{p})} = 1$  we have  $\gamma_I^{(s,\bar{p})} = 1 - \bar{p} + \bar{p}\sigma^{(i,\bar{p})}$  and  $\gamma_G^{(s,\bar{p})} = \bar{p} + (1 - \bar{p})\sigma^{(i,\bar{p})}$ , we obtain

$$\frac{1}{1+\frac{\bar{p}}{1-\bar{p}}(\frac{1-\bar{p}+\bar{p}\sigma^{(i,\bar{p})}}{\bar{p}+(1-\bar{p})\sigma^{(i,\bar{p})}})^{\hat{k}-1}(\frac{\bar{p}-\bar{p}\sigma^{(i,\bar{p})}}{1-\bar{p}-(1-\bar{p})\sigma^{(i,\bar{p})}})^{n-\hat{k}}}=q,$$

which implies that

$$\sigma^{(i,\bar{p})} = \frac{1 - \bar{p}(1 + ((\frac{1-q}{q})(\frac{\bar{p}}{1-\bar{p}})^{\hat{k}-n-1})^{\frac{1}{\bar{k}-1}})}{(1 - \bar{p})((\frac{1-q}{q})(\frac{\bar{p}}{1-\bar{p}})^{\hat{k}-n-1})^{\frac{1}{\bar{k}-1}} - \bar{p}}$$

Thus,

$$\sigma^{(i,\bar{p})} = \frac{\bar{p}(1+f(\bar{p},k)) - 1}{\bar{p} - f(\bar{p},\hat{k})(1-\bar{p})},$$

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where

$$f(\bar{p},\hat{k}) = (\frac{(1-q)}{q}(\frac{(1-\bar{p})}{\bar{p}})^{n-\hat{k}+1})^{1/(\hat{k}-1)}.$$

Because  $\sigma^{(i,\bar{p})}$  has to be strictly between 0 and 1, we require  $q < \frac{1}{1+(\frac{\bar{p}}{1-\bar{p}})^{n-\bar{k}+1}}$ .

The proofs for the minority and simple majority rules are analogous.

#### Proof of Corollary 2.

Informative voting is an equilibrium if and only if  $q \in [\beta^{(i,p)}(\alpha n), \beta^{(g,p)}(\alpha n)]$  for each type  $p \in \{p, \bar{p}\}$ . (1) Assume a minority rule,  $\hat{k} < \frac{n}{2}$ . Then, keeping in mind  $2\hat{k} - n - 2 < 2\hat{k} - n < 0$ , for  $n \to \infty$ , such an interval does not exist, because

$$\lim_{n \to \infty} \left(\frac{1-\underline{p}}{\underline{p}}\right)^{2k-n-2} \leq \lim_{n \to \infty} \left(\frac{1-\overline{p}}{\overline{p}}\right)^{2\hat{k}-n} \Leftrightarrow \underline{p} \leq \overline{p},$$

holds. (2) Assume the simple majority rule,  $\frac{n}{2} < \hat{k} < \frac{n}{2} + 1$ . Then, keeping in mind  $2\hat{k} - n - 2 < 0 < 2\hat{k} - n$ , for  $n \to \infty$ , such an interval does not exist, because

$$\lim_{n \to \infty} \left(\frac{1-\underline{p}}{\underline{p}}\right)^{2\hat{k}-n-2} \le \lim_{n \to \infty} \left(\frac{1-\underline{p}}{\underline{p}}\right)^{2\hat{k}-n} \Leftrightarrow \underline{p} \le \underline{p},$$

holds. (3) Assume a supermajority rule,  $\frac{n}{2} + 1 < \hat{k}$ . Then, keeping in mind  $0 < 2\hat{k} - n - 2 < 2\hat{k} - n$ , for  $n \to \infty$ , such an interval does not exist, because

$$\lim_{n \to \infty} \left( \frac{1 - \bar{p}}{\bar{p}} \right)^{2\hat{k} - n - 2} \le \lim_{n \to \infty} \left( \frac{1 - \underline{p}}{\underline{p}} \right)^{2k - n} \Leftrightarrow \underline{p} \le \bar{p},$$

holds.

#### **Proof of Proposition 7.**

With a proportion *m* of skeptical voters and a proportion 1 - m of trusting voters, the probability that a defendant is indeed guilty, given a conviction verdict is pronounced is

$$Pr(G|C) = \frac{1}{1 + (\frac{(\hat{\gamma}_{I}^{(s,\bar{p})})^{m\alpha}(\hat{\gamma}_{I}^{(s,\bar{p})})^{(1-m)\alpha}(1-\hat{\gamma}_{I}^{(s,\bar{p})})^{m(1-\alpha)}(1-\hat{\gamma}_{I}^{(s,\bar{p})})^{(1-m)(1-\alpha)}}{(\hat{\gamma}_{G}^{(s,\bar{p})})^{m\alpha}(\hat{\gamma}_{G}^{(s,\bar{p})})^{(1-m)\alpha}(1-\hat{\gamma}_{G}^{(s,\bar{p})})^{m(1-\alpha)}(1-\hat{\gamma}_{G}^{(s,\bar{p})})^{(1-m)(1-\alpha)}})^{n}}$$

For  $n \rightarrow \infty$  this probability that a defendant is guilty if convicted approaches one; and the probability that a defendant is innocent if convicted approaches zero.

This is so, since when  $n \to \infty$ , we have

$$\lim_{n \to \infty} f(\underline{p}, \hat{k}) = \lim_{n \to \infty} \left( \frac{(1-q)}{q} \left( \frac{(1-\underline{p})}{\underline{p}} \right)^{n-\hat{k}+1} \right)^{1/(\hat{k}-1)} = \left( \frac{1-\underline{p}}{\underline{p}} \right)^{\frac{1-\alpha}{\alpha}},$$

and

$$\lim_{n \to \infty} f(\bar{p}, \hat{k}) = \lim_{n \to \infty} (\frac{(1-q)}{q} (\frac{(1-\bar{p})}{\bar{p}})^{n-\hat{k}+1})^{1/(\hat{k}-1)} = (\frac{1-\bar{p}}{\bar{p}})^{\frac{1-\alpha}{\alpha}}.$$

and, therefore, we have

$$\sigma^{(i,\underline{p})} = \frac{\underline{p}(1+f(\underline{p},\hat{k})) - 1}{\underline{p} - f(\underline{p},\hat{k})(1-\underline{p})} = \frac{\underline{p}(1+(\frac{1-\underline{p}}{\underline{p}})^{(1-\alpha)/\alpha}) - 1}{\underline{p} - (\frac{1-\underline{p}}{\underline{p}})^{(1-\alpha)/\alpha})(1-\underline{p})}$$

and

$$\sigma^{(i,\bar{p})} = \frac{\bar{p}(1+f(\bar{p},\hat{k}))-1}{\bar{p}-f(\bar{p},\hat{k})(1-\bar{p})} = \frac{\bar{p}(1+(\frac{1-\bar{p}}{\bar{p}})^{(1-\alpha)/\alpha})-1}{\bar{p}-(\frac{1-\bar{p}}{\bar{p}})^{(1-\alpha)/\alpha}(1-\bar{p})}$$

Therefore, in the limit case of  $n \to \infty$ , we must have

$$\begin{split} \hat{\gamma}_{G}^{(s,p)} &= \hat{p} + (1-\hat{p}) \frac{\underline{p}(1+(\frac{1-p}{\underline{p}})^{(1-\alpha)/\alpha})-1}{\underline{p}-(\frac{1-p}{\underline{p}})^{(1-\alpha)/\alpha})(1-\underline{p})}, \\ \hat{\gamma}_{I}^{(s,p)} &= \hat{p} \frac{\underline{p}(1+(\frac{1-p}{\underline{p}})^{(1-\alpha)/\alpha})-1}{\underline{p}-(\frac{1-p}{\underline{p}})^{(1-\alpha)/\alpha})(1-\underline{p})} + (1-\hat{p}), \\ \hat{\gamma}_{G}^{(s,\bar{p})} &= \hat{p} + (1-\hat{p}) \frac{\bar{p}(1+(\frac{1-\bar{p}}{\underline{p}})^{(1-\alpha)/\alpha})-1}{\bar{p}-(\frac{1-\bar{p}}{\underline{p}})^{(1-\alpha)/\alpha})(1-\bar{p})}, \\ \hat{\gamma}_{I}^{(s,\bar{p})} &= \hat{p} \frac{\bar{p}(1+(\frac{1-\bar{p}}{\underline{p}})^{(1-\alpha)/\alpha})-1}{\bar{p}-(\frac{1-\bar{p}}{\underline{p}})^{(1-\alpha)/\alpha})(1-\bar{p})} + (1-\hat{p}). \end{split}$$

Hence, it is easy to verify that  $\hat{\gamma}_{G}^{(s,p)} > \hat{\gamma}_{G}^{(s,\bar{p})} > \hat{\gamma}_{I}^{(s,\bar{p})} > \hat{\gamma}_{I}^{(s,\bar{p})}$ . Therefore, a sufficient condition for

$$\lim_{n \to \infty} \Pr(G|C) = \lim_{n \to \infty} \frac{1}{1 + (\frac{\hat{\gamma}_{I}^{(s,\bar{p})}}{\hat{\gamma}_{G}^{(s,\bar{p})}})^{(1-m)\alpha n} (\frac{\hat{\gamma}_{I}^{(s,\underline{p})}}{\hat{\gamma}_{G}^{(s,\underline{p})}})^{m\alpha n} (\frac{1-\hat{\gamma}_{I}^{(s,\bar{p})}}{1-\hat{\gamma}_{G}^{(s,\bar{p})}})^{(1-m)(1-\alpha)n} (\frac{1-\hat{\gamma}_{I}^{(s,\underline{p})}}{1-\hat{\gamma}_{G}^{(s,\underline{p})}})^{m(1-\alpha)n}} = 1$$

is  $(1-m)\gamma_G^{(s,\bar{p})} + m\gamma_G^{(s,p)} > \alpha$  and  $(1-m)\gamma_I^{(s,\bar{p})} + m\gamma_I^{(s,p)} < \alpha$ . If  $\hat{\gamma}_G^{(s,p)} > \hat{\gamma}_G^{(s,\bar{p})} > \alpha > \hat{\gamma}_I^{(s,p)} > \hat{\gamma}_I^{(s,\bar{p})}$ , we have:

$$\lim_{n \to \infty} \left(\frac{\hat{\gamma}_{I}^{(s,\bar{p})}}{\hat{\gamma}_{G}^{(s,\bar{p})}}\right)^{(1-m)\alpha n} \left(\frac{\hat{\gamma}_{I}^{(s,\bar{p})}}{\hat{\gamma}_{G}^{(s,p)}}\right)^{m\alpha n} \left(\frac{1-\hat{\gamma}_{I}^{(s,\bar{p})}}{1-\hat{\gamma}_{G}^{(s,\bar{p})}}\right)^{(1-m)(1-\alpha)n} \left(\frac{1-\hat{\gamma}_{I}^{(s,\bar{p})}}{1-\hat{\gamma}_{G}^{(s,p)}}\right)^{m(1-\alpha)n} = 0$$

implying  $\lim_{n\to\infty} Pr(G|C) = 1$ .

# **B** Simulations

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**Table 5:** Type I errors of the 12-Person Jury case, given  $\hat{p} = \bar{p} = 0.8$ , p = 0.52, q = 0.9 and different voting rules.

12	11	10	9	8	7	ĥ
C	0	0	0	0	0	$\sigma^{(i,\underline{p})}$
C	0	0	0	0	0	$\sigma^{(g,\underline{p})}$
0.5759	0.422645	0.277769	0.143552	0.022976	0	$\sigma^{(i, \bar{p})}$
1	1	1	1	1	1	$\sigma^{(g, \tilde{p})}$
		ror	Type I er			<i>n</i> <sub>1</sub>
C	0	0	0	0	0	12
0	0	0	0	0	0	11
0	0	0	0	0	0	10
C	0	0	0	0	0	9
0	0	0	0	0	0	8
(	0	0	0	0	0	7
0	0	0	0	0	0	6
(	0	0	0	0	1.28E-05	5
C	0	0	0	5.17E-06	8.45E-05	4
0	0	0	3.04E-05	3.75E-05	0.000314	3
0	0	0.00018	0.000218	0.000151	0.000864	2
0	0.0011	0.0012	0.00086	0.000448	0.002	1
0.0069	0.0067	0.0045	0.0025	0.0011	0.0039	0

† The corresponding voting equilibrium for the case of n = 12 and  $\hat{k} = 7$  is not responsive, that is  $\sigma^{(i,\underline{p})} = \sigma^{(g,\underline{p})} = \sigma^{(i,\overline{p})} = 0$  and  $\sigma^{(g,\overline{p})} = 1$ , since  $2\hat{k} - n - 2 = 0$  and  $\beta^{(i,\underline{p})} = \beta^{(i,\overline{p})} < \beta^{(g,\underline{p})} \le q < \beta^{(g,\overline{p})}$ . †The corresponding voting equilibrium for the case of n = 12 and  $\hat{k}$  is either equal to 8, 9, 10, 11, or 12 is not responsive, that is  $\sigma^{(i,\underline{p})} = \sigma^{(g,\underline{p})} = 0$ ,  $0 < \sigma^{(i,\overline{p})} < 1$ , and  $\sigma^{(g,\overline{p})} = 1$ , since  $2\hat{k} - n - 2 > 0$ ,  $q < 1/(1 + (p/(1-p))^{n-\hat{k}-1})$  and  $\beta^{(i,\underline{p})} < \beta^{(g,\underline{p})} \le q < \beta^{(g,\overline{p})}$ , with  $p < 1/(1 + ((1-\overline{p})/\overline{p})^{(2\hat{k}-n-2)/(2\hat{k}-n)})$ .

71		, ,	, , <u>,</u>		0	
ĥ	7	8	9	10	11	12
$\sigma^{(i,p)}$	0	0	0	0	0	0
$\sigma^{(g,p)}$	0	0	0	0	0	1
$\sigma^{(i,\bar{p})}$	0	0.022976	0.143552	0.277769	0.422645	0.5759
$\sigma^{(g,\bar{p})}$	1	1	1	1	1	1
$\overline{n_1}$			Type I e	rror		
12	0	0	0	0	0	4.1E-09
11	0	0	0	0	0	1.35E-08
10	0	0	0	0	0	4.47E-08
9	0	0	0	0	0	1.48E-07
8	0	0	0	0	0	4.88E-07
7	0	0	0	0	0	1.61E-06
6	0	0	0	0	0	5.33E-06
5	1.28E-05	0	0	0	0	1.76E-05
4	8.45E-05	5.17E-06	0	0	0	5.81E-05
3	0.000314	3.75E-05	3.04E-05	0	0	0.000192
2	0.000864	0.000151	0.000218	0.00018	0	0.000634
1	0.002	0.000448	0.00086	0.0012	0.0011	0.0021
0	0.0039	0.0011	0.0025	0.0045	0.0067	0.0069

**Table 6:** Type I errors of the 12-Person Jury case, given  $\hat{p} = \bar{p} = 0.8$ , p = 0.55, q = 0.9 and different voting rules.

+ The corresponding voting equilibrium for the case of n = 12 and  $\hat{k} = 12$  is semi-mixed, that is  $\sigma^{(i,\underline{p})} = 0$ ,  $\sigma^{(g,\underline{p})} = 1$ ,  $0 < \sigma^{(i,\overline{p})} < 1$ , and  $\sigma^{(g,\overline{p})} = 1$ , since  $2\hat{k} - n - 2 > 0$ ,  $q < 1/(1 + (\underline{p}/(1 - \underline{p}))^{n-\hat{k}-1})$  and  $\beta^{(i,\underline{p})} \le q < \beta^{(g,\underline{p})} \le \beta^{(i,\overline{p})} < \beta^{(g,\overline{p})}$ , with  $p \le 1/(1 + ((1 - \overline{p})/\overline{p})^{(2\hat{k} - n - 2)/(2\hat{k} - n)})$ .

Table 7: Type I errors of the 12	Person Jury case, giv	ren $\hat{p} = \bar{p} = 0.8, p =$	0.6, $q = 0.9$ and di	fferent voting rules.
	, , , , , , , , , , , , , , , , , , , ,			0

	0		1 · · ·			51
12	11	10	9	8	7	ĥ
0.258831	0.145201	0.032513	0	0	0	$\sigma^{(i,\underline{p})}$
1	1	1	1	0	0	$\sigma^{(g,\underline{p})}$
0.575916	0.422645	0.277769	0.143552	0.022976	0	$\sigma^{(i,\bar{p})}$
1	1	1	1	1	1	$\sigma^{(g, \bar{p})}$
		ror	Type I er			$\overline{n_1}$
2.07E-05	2.69E-05	1.45E-05	6.22E-05	0	0	12
3.36E-05	4.35E-05	2.44E-05	8.7E-05	0	0	11
5.45E-05	7.04E-05	0.000041	0.000121	0	0	10
8.85E-05	0.000113	6.84E-05	0.000168	0	0	9
0.000144	0.000182	0.000113	0.000232	0	0	8
0.000233	0.000291	0.000186	0.000319	0	0	7
0.000379	0.000464	0.000304	0.000436	0	0	6
0.000614	0.000736	0.000491	0.000591	0	0.000028	5
0.000997	0.0012	0.000786	0.000798	5.17E-06	8.45E-05	4
0.0016	0.0018	0.0012	0.0011	3.75E-05	0.000314	3
0.0026	0.0028	0.0019	0.0014	0.000151	0.000864	2
0.0043	0.0044	0.003	0.0019	0.000448	0.002	1
0.0069	0.0067	0.0045	0.0025	0.0011	0.0039	0

+ The corresponding voting equilibrium for the case of n = 12 and  $\hat{k}$  equal to either 10, 11, or 12 is a randomizing one, that is  $0 < \sigma^{(i,\underline{p})} < 1$ ,  $\sigma^{(\underline{g},\underline{p})} = 1$ ,  $0 < \sigma^{(i,\bar{p})} < 1$ , and  $\sigma^{(\underline{g},\bar{p})} = 1$ , since  $2\hat{k} - n - 2 > 0$ ,  $q < 1/(1 + (\bar{p}/(1-\bar{p}))^{n-\hat{k}-1})$  and  $q < \beta^{(i,\underline{p})} < \beta^{(\underline{g},\underline{p})} < \beta^{(\underline{g},\underline{p})}$ , with  $\underline{p} \le 1/(1 + ((1-\bar{p})/\bar{p})^{(2\hat{k}-n-2)/(2\hat{k}-n)})$ .

				-		
ĥ	7	8	9	10	11	12
$\sigma^{(i,\underline{p})}$	0	0	0.087515	0.21698	0.35096	0.488343
$\sigma^{(g,\underline{p})}$	0	1	1	1	1	1
$\sigma^{(i,ar{p})}$	0	0.041675	0.146284	0.277094	0.430939	0.575916
$\sigma^{(g, \bar{p})}$	1	1	1	1	1	1
$\overline{n_1}$			Type I er	rror		
12	0	0.000581	0.00073	0.0015	0.0021	0.0018
11	0	0.000639	0.000816	0.0017	0.0024	0.002
10	0	0.000702	0.00091	0.0018	0.0026	0.0023
9	0	0.00077	0.001	0.002	0.0029	0.0025
8	0	0.000845	0.0011	0.0022	0.0033	0.0028
7	0	0.000926	0.0013	0.0024	0.0036	0.0032
6	0	0.001	0.0014	0.0026	0.004	0.0035
5	1.28E-05	0.0011	0.0016	0.0029	0.0045	0.004
4	8.45E-05	0.0012	0.0017	0.0032	0.005	0.0044
3	0.000314	0.0013	0.0019	0.0034	0.0055	0.0049
2	0.000864	0.0015	0.0021	0.0038	0.0061	0.0055
1	0.002	0.0016	0.0024	0.0041	0.0068	0.0062
0	0.0039	0.0017	0.0026	0.0045	0.0075	0.0069

**Table 8:** Type I errors of the 12-Person Jury case, given  $\hat{p} = 0.8$ ,  $\bar{p} = 0.9$ , p = 0.7, q = 0.9 and different voting rules.

**Table 9:** Type I errors of the 12-Person Jury case, given  $\hat{p} = \bar{p} = 0.8$ , p = 0.75, q = 0.9 and different voting rule.

51		, , , ,		· 1	0	
ĥ	7	8	9	10	11	12
$\sigma^{(i,\underline{p})}$	0	0	0.123403	0.256223	0.396154	0.541545
$\sigma^{(g,\underline{p})}$	0	1	1	1	1	1
$\sigma^{(i,\bar{p})}$	0	0.996109	0.143552	0.277769	0.422645	0.575916
$\sigma^{(g, \bar{p})}$	1	1	1	1	1	1
$\overline{n_1}$			Type I er	rror		
12	0	0.000581	0.0016	0.0031	0.0044	0.0042
11	0	0.000613	0.0017	0.0032	0.0046	0.0043
10	0	0.000646	0.0018	0.0033	0.0048	0.0045
9	0	0.000682	0.0018	0.0034	0.0049	0.0042
8	0	0.000718	0.0019	0.0035	0.0051	0.0049
7	0	0.000757	0.0019	0.0037	0.0053	0.005
6	0	0.000797	0.002	0.0038	0.0054	0.0054
5	1.28E-05	0.00084	0.0021	0.0039	0.0056	0.005
4	8.45E-05	0.000884	0.0022	0.004	0.0058	0.0058
3	0.000314	0.000931	0.0022	0.0041	0.006	0.006
2	0.000864	0.00098	0.0023	0.0043	0.0062	0.0064
1	0.002	0.001	0.0024	0.0044	0.0064	0.0066
0	0.0039	0.0011	0.0025	0.0045	0.0067	0.0069

<sup>†</sup>The corresponding voting equilibrium for the case of n = 12 and  $\hat{k} = 8$  is semi-mixed, that is  $\sigma^{(i,\underline{p})} = 0$ ,  $\sigma^{(g,\underline{p})} = 1$ ,  $0 < \sigma^{(i,\overline{p})} < 1$ , and  $\sigma^{(g,\overline{p})} = 1$ , since  $2\hat{k} - n - 2 > 0$ ,  $q < 1/(1 + (\underline{p}/(1 - \underline{p}))^{n - \hat{k} - 1})$  and  $\beta^{(i,\underline{p})} \le q < \beta^{(i,\overline{p})} < \beta^{(g,\underline{p})} < \beta^{(g,\overline{p})}$ , with

 $\underline{p} > 1/(1 + ((1 - \bar{p})/\bar{p})^{(2\hat{k} - n - 2)/(2\hat{k} - n)}).$  The corresponding voting equilibrium for the case of n = 12 and  $\hat{k}$  equal to either 9 or 10 is a randomizing one, that is  $0 < \sigma^{(i,\underline{p})} < 1$ ,  $\sigma^{(g,\underline{p})} = 1$ ,  $0 < \sigma^{(i,\bar{p})} < 1$ , and  $\sigma^{(g,\bar{p})} = 1$ , since  $2\hat{k} - n - 2 > 0$ ,  $q < 1/(1 + (\bar{p}/(1 - \bar{p}))^{n - \hat{k} - 1})$  and  $q < \beta^{(i,\underline{p})} < \beta^{(i,\underline{p})} < \beta^{(g,\underline{p})} < \beta^{(g,\underline{p})}$ , with  $p > 1/(1 + ((1 - \bar{p})/\bar{p})^{(2\hat{k} - n - 2)/(2\hat{k} - n)}).$ 

ĥ	7	8	9	10	11	12
$\overline{n_1}$			Type II er	ror		
12	1	1	1	1	1	1
11	1	1	1	1	1	1
10	1	1	1	1	1	1
9	1	1	1	1	1	1
8	1	1	1	1	1	1
7	1	1	1	1	1	1
6	1	1	1	1	1	1
5	0.7903	1	1	1	1	1
4	0.4967	0.8244	1	1	1	1
3	0.2618	0.5498	0.8157	1	1	1
2	0.1209	0.3084	0.5315	0.7899	1	1
1	0.0504	0.1511	0.2881	0.4864	0.7407	1
0	0.0194	0.0666	0.1352	0.2452	0.4113	0.6548

**Table 10:** Type II errors of the 12-Person Jury case, given  $\hat{p} = \bar{p} = 0.8$ , p = 0.52, q = 0.9, and different voting rules.

**Table 11:** Type II errors of the 12-Person Jury case, given  $\hat{p} = \bar{p} = 0.8$ , p = 0.55, q = 0.9 and different voting rules.

ĥ	7	8	9	10	11	12
<i>n</i> <sub>1</sub>			Type II er	ror		
12	1	1	1	1	1	0.9313
11	1	1	1	1	1	0.9214
10	1	1	1	1	1	0.9101
9	1	1	1	1	1	0.8971
8	1	1	1	1	1	0.8823
7	1	1	1	1	1	0.8654
6	1	1	1	1	1	0.846
5	0.7903	1	1	1	1	0.8238
4	0.4967	0.8244	1	1	1	0.7984
3	0.2618	0.5498	0.8157	1	1	0.7694
2	0.1209	0.3084	0.5315	0.7899	1	0.7362
1	0.0504	0.1511	0.2881	0.4864	0.7407	0.6982
0	0.0194	0.0666	0.1352	0.2453	0.4113	0.6548

ĥ	7	8	9	10	11	12
$\overline{n_1}$			Type II er	ror		
12	1	1	0.2054	0.4185	0.6337	0.8542
11	1	1	0.1991	0.404	0.6177	0.8433
10	1	1	0.1928	0.3893	0.6012	0.8316
9	1	1	0.1866	0.3747	0.5842	0.8191
8	1	1	0.1805	0.36	0.5667	0.8056
7	1	1	0.1775	0.3453	0.5487	0.7912
6	1	1	0.1686	0.3307	0.5303	0.7756
5	0.7903	1	0.1628	0.3161	0.5114	0.7589
4	0.4967	0.8244	0.1571	0.3017	0.4921	0.741
3	0.2618	0.5498	0.1514	0.2873	0.4724	0.7217
2	0.1209	0.3084	0.1459	0.2731	0.4524	0.701
1	0.0504	0.1511	0.1405	0.2591	0.432	0.6787
0	0.0194	0.0666	0.1352	0.2453	0.4113	0.6548

**Table 12:** Type II errors of the 12-Person Jury case, given  $\hat{p} = \bar{p} = 0.8$ , p = 0.6, q = 0.9 and different voting rules.

**Table 13:** Type II errors of the 12-Person Jury case, given  $\hat{p} = 0.8$ ,  $\bar{p} = 0.9$ , p = 0.7, q = 0.9 and different voting rules.

ĥ	7	8	9	10	11	12
$\overline{n_1}$			Type II er	ror		
12	1	0.0726	0.1611	0.2871	0.474	0.7262
11	1	0.0716	0.1587	0.2836	0.4683	0.7209
10	1	0.0707	0.1564	0.2801	0.4626	0.7154
9	1	0.0698	0.1541	0.2766	0.4568	0.7099
8	1	0.0689	0.1518	0.2731	0.4511	0.7042
7	1	0.068	0.1495	0.2697	0.4453	0.6985
6	1	0.0671	0.1472	0.2662	0.4394	0.6926
5	0.7903	0.0663	0.145	0.2628	0.4336	0.6866
4	0.4967	0.0654	0.1428	0.2593	0.4277	0.6805
3	0.2618	0.0645	0.1405	0.2559	0.4218	0.6742
2	0.1209	0.0637	0.1384	0.2525	0.4158	0.6679
1	0.0504	0.0628	0.1362	0.2491	0.4099	0.6614
0	0.0194	0.062	0.134	0.2457	0.4039	0.6548

**Table 14:** Type II errors of the 12-Person Jury case, given  $\hat{p} = \bar{p} = 0.8$ , p = 0.75, q = 0.9 and different voting rules.

12	11	10	9	8	7	ĥ
		or	Type II err			$\overline{n_1}$
0.6846	0.4348	0.2599	0.1443	0.0726	1	12
0.6823	0.4329	0.2587	0.1435	0.072	1	11
0.6799	0.4309	0.2574	0.1428	0.0715	1	10
0.6774	0.429	0.2562	0.142	0.071	1	9
0.675	0.427	0.255	0.1412	0.0705	1	8
0.6725	0.4251	0.2538	0.1405	0.071	1	7
0.67	0.4231	0.2525	0.1397	0.07	1	6
0.6676	0.4211	0.2513	0.139	0.0695	0.7903	5
0.665	0.4192	0.2501	0.1382	0.0686	0.4967	4
0.6625	0.4172	0.2489	0.1375	0.0681	0.2618	3
0.6599	0.4152	0.2477	0.1367	0.0676	0.1209	2
0.6574	0.4133	0.2464	0.136	0.0671	0.0504	1
0.6548	0.4113	0.2452	0.1352	0.0666	0.0194	0

## Notes

1 Experiential knowledge has also been analyzed in other decision-making contexts and traits related to *familiarity biases* have been discussed and documented also in experimental settings (see, for instance, Chew and Sagi (2008), Chew, Ebstein, and Zhong (2012), and Chew, Miao, and Zhong (2014). This is also compatible with the notion of ignorance or prejudice. A prejudice can be thought of as an unfounded belief, not rooted in exact fact-checks, but instead grounded on some reinforced beliefs based on a partial view of the world, or reality, as one experiences it. Elbittar et al. (2014a, 2014b) studies voters with biased views. In these studies, the prejudice/bias that voters hold is with respect to the prior probability of a defendant being guilty or innocent. Biased voters have the option to better inform themselves by performing some costly information acquisition. Voters privately know their idiosyncratic cost of acquiring information as well as the ex-ante distribution of those costs across voters. In this environment, voting equilibria with and without costly information acquisition, depending on the degree of the biases some voters possess, as well as abstention from voting can be rationalized.

2 Our consequence of this assumption is weaker than that in Binmore (2016) where, as a consequence of insufficient occurrences of possible events, decision-makers might not even know the whole state space.

3 We are grateful to an anonymous referee for stressing this point, thereby helping us clarify further how our model differs from FP and, hence, putting our study in the appropriate context within the existing literature.

4 In an environment with mistrials and communication, Coughlan (2000) finds that unanimity outperforms majority because jurors vote informatively.

5 The results from these treatments are available upon request from the authors.

6 Ethical approvals to conduct this research with human subjects were obtained from both the Massey University Human Ethics Committee and the University of Auckland Human Participants Ethics Committee on the 5th of March 2015 and on the 18th of June 2015 (with reference number 014565), respectively. Both ethical clearances are valid for a duration of three years.

7 Paying the same bonus for group decisions leading to a type I or a type II error corresponds in our model to a threshold of reasonable doubt of q = 1/2.

8 To understand this further, think of the parallel example of two possibly unfair coins, which could be selected: one of them is tilted to show one side 60% of the time, when flipping it many times; whereas the other unfair coin shows that same side 90% of the time, when flipped many times. And, one does not know which of the two coins is selected, prior to flipping it many times, making it impossible to predict the exact frequency of the possible realizations of heads and tails, a priori.

<sup>9</sup> Without deceiving subjects, this is the closest we can bring subjects towards possibly holding non-congruent views regarding the signal's precision and towards applying this subjective measure to all other group members.

10 The detailed instructions for each of the treatments analyzed in this study are available as supplementary material.

11 Furthermore, in this environment free-form communication helps to reduce both type I errors and type II errors. Additionally, freeform communication helps to eliminate the differences across different voting rules. These last results need further theoretical investigation, since, as discussed, our model does not provide testable predictions for voting behavior with communication.

12 Fabrizi and Pan (2016) provides a theoretical study of voters who, confronted with information the precision of which varies within an interval, form subjective beliefs according to the Maxmin approach. Similar to the present paper, Fabrizi and Pan (2016) finds that unanimity may outperform majority in an ambiguous world. This study provides theoretical support to a conjecture that our experimental results might hold in more general ambiguous settings.

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