THE EFFECT OF RECEIVER CAPTURE AND COCHANNEL INTERFERENCE ON PRMA

M. D. Orange, K. W. Sowerby
E-mail: m.orange@auckland.ac.nz & kw.sowerby@auckland.ac.nz

Radio Systems Group, Department of Electrical & Electronic Engineering, The University of Auckland, Private Bag 92019, Auckland, New Zealand

Abstract: The performance of a speech-only cellular PRMA system is analysed in an interference limited Rayleigh faded environment. The inter-cell interference present in such a system can significantly increase the probability of packet loss as well as reduce the overall system utilisation. In this paper an ideal 2-branch SIR selection diversity scheme has been shown to be effective in combating the level of inter-cell interference. A cellular PRMA system with a cluster size of 4 was found to be the most efficient, in terms of system utilisation and number of users per cell able to be supported.

I. Introduction

Packet Reservation Multiple Access (PRMA) [1,2] is a random-access technique which has been proposed as a possible terminal-to-base-station access technology for future wireless networks. In this paper, the performance of PRMA is examined in an interference-limited cellular architecture in which packets experience Rayleigh fading. With a Rayleigh fading channel, packets arrive at the base station receiver with different power levels so that in the event of a collision between multiple packets, the packet with the strongest power may still be capable of capturing the base station receiver due to the capture effect. In addition to the desired packet, interfering packets may arrive from other users in the same cell (intra-cell interference) and from users in cochannel cells (inter-cell interference). In the context of cellular PRMA, intra-cell interference only occurs during available timeslots, whereas inter-cell interference may occur during both available and reserved slots.

In the simulation study of [3] it was suggested that methods should be investigated to reduce the impact of inter-cell interference. In this paper an ideal SIR (signal to interference ratio) selection diversity scheme is investigated for its ability to combat the effect of inter-cell interference during reserved timeslots. In order to facilitate the analysis of a speech-only cellular PRMA system, a Markov method has been adopted as the primary analysis technique. From this analysis the packet dropping probability, interference probability, throughput per cell and system utilisation have been studied for various system configurations. The interference probability is the probability of a reserved packet being corrupted by inter-cell interference. The overall packet loss probability is therefore the total number of dropped packets plus interfered packets.

II. Cellular PRMA

An overview of PRMA's operation will not be provided here, however a complete explanation is provided in [2]. An idealised hexagonal cell structure is assumed, where the central cell has six equi-distant cochannel interferer cells. Inter-cell interference from cells beyond this “inner ring” is assumed to be negligible. The number of users per cell, M, with active connections to their base station is considered constant for all cells. It is assumed that the central base station only accepts packets from users inside its cell: packets from cochannel cells are considered to be interference. The base stations and users share global frame and slot synchronisation. Propagation delays are neglected as the propagation distances are assumed relatively short. The area mean power of a received packet is considered to be related to its transmission distance (r) according to an $r^\beta$ dependency where $\beta=4$ is the propagation exponent assumed in this study.

At any moment in time there will be $C (0 \leq C \leq M)$ central cell users actively attempting to obtain a slot reservation (ie. in the contention state). Given that a particular slot is available, it is possible that $n (0 \leq n \leq C)$ of these contending users may transmit their packets to the central base station. Of the $n$ transmitting users, one is denoted as the desired user while the other $n-1$ users constitute intra-cell interferers. With reference to Fig.1, the desired user is considered to be at a radius $r_0$ from the central base station while the $x$th ($0 \leq x \leq n-1$) intra-cell interferer is at a radius $r_x$. For a normalised cell radius and uniform spatial user density, the probability density function of a central cell user's radius ($r$) with respect to its base station is given by $f_r(r) = 2r$ ($0 \leq r \leq 1$).

Fig.1: System Model
In addition to the desired user and \( n-1 \) intra-cell interferers, \( m \) (where \( 0 \leq m \leq M \)) inter-cell interferers may also transmit in the same slot. The \( y \)th \((0 \leq y \leq m)\) inter-cell interferer is assumed, for simplicity, to be located at the centre of its respective cochannel cell so that the distance between it and the central base station is the frequency reuse distance, \( D \). For hexagonal cells \( D = \sqrt{3N_c} \), where \( N_c \) is the cluster size.

### III. Effect of Inter-Cell Interference

Inter-cell interference affects the performance of cellular PRMA during both available and reserved timeslots. During available slots, the central base station's ability to decipher the desired packet is reduced, due to the presence of not only intra-cell but also inter-cell interference. This leads to greater delay in the desired user obtaining a reservation, which in turn increases the probability of packet dropping. During reserved slots, inter-cell interference may cause the desired packet to be corrupted. If, during a reserved slot, the central base station is unable to decipher the desired packet, one of two possible events may have occurred, namely:

1. no desired packet was transmitted (i.e. the talkspurt had finished) and only inter-cell interference was received or;
2. inter-cell interference corrupted the desired packet to the extent that it could not be received correctly.

The central base station must be capable of discriminating between these two events so that it can then take appropriate action. In the first instance, the base station should relinquish the reservation. However, if the desired user’s packet has been corrupted by inter-cell interference the base station should continue to assign the reservation to the user involved. This would save the user having to attempt re-transmission of the corrupted packet which in turn would reduce further interference to other users in the system.

In order to discriminate between the two events, a comparison between the total power received in the current slot with the total power received in the same slot in previous frames could be used. If the desired packet had been transmitted, the total power received would be significantly greater in most cases than if it had not. In this study perfect discrimination between the two events is assumed.

In order to reduce the effect of inter-cell interference during reserved slots, selection diversity may be incorporated into the system design. In a Rayleigh fading environment, selection diversity exploits the fact that if two or more independent inputs (branches) to a radio receiver are available it is possible, by switching between inputs, to achieve better reception quality than is possible with just a single input. The probabilities of the desired packet capturing the central base station for both single branch and dual branch base station receivers are calculated in the following section.

### IV. Capture Model

The probability of the desired packet capturing the central base station during both available and reserved timeslots is presented in the following sub-sections. In both cases the momentary signal to interference ratio is assumed to be constant over the duration of a single timeslot.

#### A. Available Timeslots

In addition to the desired packet being transmitted in an available timeslot, it is possible that \( n-1 \) intra-cell and \( m \) inter-cell interfering packets may also be transmitted. The desired packet is considered to capture the central base station if its momentary power exceeds the momentary composite interference power by a specified threshold (or capture ratio) \( z \). The probability of the desired packet capturing the central base station from a distance \( r_0 \) is given by \[ q_{n-1,m}(r_0) = 1 - r_0^2 \sqrt{2} \arctan\left(\frac{1}{r_0^2} \right)^{n-1+1} \left(\frac{D^2}{D^2 + zr_0^2}\right)^m, \] where all packets are assumed to experience independent Rayleigh fading. The number of inter-cell interfering packets, \( m \), received at the central base station in a timeslot is approximated by the Poisson distribution given in [4], namely \[ f_m(m) = \frac{\left[\frac{6Mt_1}{N(t_1+t_2)}\right]^m}{m!} \exp\left[-\frac{6Mt_1}{N(t_1+t_2)}\right], \] where \( t_1 \) and \( t_2 \) are the mean durations of talkspurts and silent gaps respectively and \( N \) is the number of timeslots per frame (i.e. the number of logical uplink channels per cell). By averaging (1) over the probability of \( m \) inter-cell interferers and integrating over the distance, \( r_0 \), between the desired user and the central base station, the probability, \( q_{n-1} \), of the desired user capturing the base station in the presence of \( n-1 \) intra-cell interferers is \[ q_{n-1} = \int_0^\infty q_{n-1,m}(r_0)f_m(m)f_c(r_0)dr_0. \] The probability of any one of the \( n \) independent central cell transmitted packets capturing the central BS is given by \[ U_n = nq_{n-1}. \]

#### B. Reserved Timeslot

In addition to the desired packet being transmitted in a reserved timeslot, it is possible that \( m \) inter-cell interfering packets may also be transmitted.
For reserved slots, the capture probability reduces from (1) to

\[ q_{0,m}(r_0) = \left( \frac{D^4}{D^4 + z r_0^4} \right)^m . \]  

(5)

By averaging (5) over the probability of \( m \) inter-cell interferers and integrating over the distance, \( r_0 \), between the desired user and the central base station, the probability, \( q_{0,\text{div}}^m \), of the desired user capturing the central base station in the absence of intra-cell interferers is

\[ q_{0,\text{div}}^m = \int_0^1 \left[ \frac{6 M t_1}{N(t_1 + t_2)} \left( \frac{D^4}{D^4 + z r_0^4} \right)^2 - 1 \right] f_r(r_0) dr_0 , \]  

(6)

where a single branch receiver is considered at the central base station. With ideal SIR selection diversity [5], the capture probability, \( q_{0,\text{div}}^m \), for a dual branch system is

\[ q_0^{\text{div}} = 2 q_{0,\text{div}}^m \]  

(7)

\[ - \int_0^1 \left[ \frac{6 M t_1}{N(t_1 + t_2)} \left( \frac{D^4}{D^4 + z r_0^4} \right)^2 - 1 \right] f_r(r_0) dr_0 . \]

V. Markov Analysis

This section presents a brief overview of the Markov analysis used to analyse the performance of cellular PRMA. Further mathematical detail is provided in [4]. The Markov analysis technique employed is similar to that of [6] such that an individual speech user in the central cell is modelled by the \( N+2 \) state Markov chain of Fig.2, with states silent (SIL), contention (CON) and reservation (RES), \( 1 \leq i \leq N \). The probability of a silent-to-contention and contention-to-silent transition is given by \( \gamma \) and \( \sigma \) respectively. The probability of a contention-to-reserved transition is \( \phi \) while a reserved-to-silent transition is given by the probability \( \eta \).

The steps taken in the Markov analysis are as follows. From the model of Fig.2, the one-step transition probability, \( \Pr(i,j|C,R) \), is determined. The one-step transition probability is the probability that in going from the current timeslot to the next timeslot, the number of users in the CON state will go from \( C \) to \( i \) and the number of users in the RES states will go from \( R \) to \( j \). Having determined the one-step transition probability, the stationary probability distribution \( \Pr(C,R) \) is calculated. \( \Pr(C,R) \) is the probability of having \( C \) users in the CON state and \( R \) users in the RES states. With the aid of the stationary probability distribution, the packet dropping probability, interference probability, total packet loss, throughput per cell and system utilisation can be determined.

A. Packet Dropping Probability

The packet dropping probability, \( P_{\text{drop}} \), is the probability of a packet being dropped from a speech user's buffer due to excessive delay in obtaining a reservation. An expression for \( P_{\text{drop}} \) for a speech-only system is given in [2].

B. Interference Probability

The interference probability is the probability of a reserved packet being subjected to excessive inter-cell interference. The interference probability, \( P_{\text{int}}(C,R) \), in a particular state \( (C,R) \), for a single branch central base station receiver is

\[ P_{\text{int}}(C,R) = \frac{R}{N} (1 - \gamma_j) (1 - q_{0,\text{div}}^m) , \]  

(8)

where \( q_{0,\text{div}}^m \) is given in (6). If the ideal SIR selection 2-branch diversity scheme is incorporated, the term \( q_{0,\text{div}}^m \) should be replaced with \( q_0^{\text{div}} \) of (7). The average interference probability, \( P_{\text{int}} \), for \( M \) active central cell users is

\[ P_{\text{int}} = \sum_{R=0}^{N} \sum_{C=0}^{M-R} P_{\text{int}}(C,R) \Pr(C,R) . \]  

(9)

C. Packet Loss Probability

The total packet loss is the proportion of packets generated by users which are either dropped or interfered. The packet loss probability, \( P_{\text{loss}} \), is given by

\[ P_{\text{loss}} = P_{\text{drop}} + (1 - P_{\text{drop}}) P_{\text{int}} . \]  

(10)

For short packets (for eg. 16 ms of speech information) packet loss probabilities up to 0.01 are acceptable [2].

D. Throughput per Cell

The throughput per cell relates to the proportion of slots in a cell which are utilised successfully (ie. those which result in a packet being successfully received by the central base station). For a given state \( (C,R) \) the throughput, \( \eta(C,R) \), is given by
for a single branch central base station receiver. If an ideal SIR selection 2-branch diversity scheme is employed, the term \(q_{0.5}^{\text{div}}\) should be replaced by \(q_{0.5}^{\text{div}}\). The average throughput per cell, \(\eta\), for \(M\) active central cell users is

\[
\eta = \frac{R}{N} \left(1 - \gamma \right) q_{0.5}^{\text{div}} \sum_{C=0}^{\infty} \left( C \right) p^C (1 - p)^{C - \infty} U_n.
\]

for a single branch central base station receiver. If an ideal SIR selection 2-branch diversity scheme is employed, the term \(q_{0.5}^{\text{div}}\) should be replaced by \(q_{0.5}^{\text{div}}\). The average throughput per cell, \(\eta\), for \(M\) active central cell users is

\[
\eta = \frac{R}{N} \left(1 - \gamma \right) q_{0.5}^{\text{div}} \sum_{C=0}^{\infty} \left( C \right) p^C (1 - p)^{C - \infty} U_n.
\]

for a single branch central base station receiver. If an ideal SIR selection 2-branch diversity scheme is employed, the term \(q_{0.5}^{\text{div}}\) should be replaced by \(q_{0.5}^{\text{div}}\). The average throughput per cell, \(\eta\), for \(M\) active central cell users is

\[
\eta = \frac{R}{N} \left(1 - \gamma \right) q_{0.5}^{\text{div}} \sum_{C=0}^{\infty} \left( C \right) p^C (1 - p)^{C - \infty} U_n.
\]

The overall system utilisation, \(\psi\), of a cellular PRMA system is given by

\[
\psi = \frac{\eta}{N_c}.
\]

The use of smaller cluster sizes allows the number of channels (slots) per cell to be increased, hence the number of supported users per cell may also be increased. In order to facilitate comparison between systems the results are plotted as a function of \(C\), the number of active users per cell divided by the number of slots per cell, ie. \(C=M/N\). For a given system, the number of slots per cell is constant. The results obtained from the Markov analysis have been verified by computer simulation results.

VI. Results & Discussion

In this study, three cellular PRMA systems have been considered, each with a different cluster size. The total bandwidth requirement for each configuration is approximately constant to enable fair comparison. The parameter values for each system are given in Table 1.

Table 1: Cellular PRMA system parameters

<table>
<thead>
<tr>
<th>System</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster size, (N_c)</td>
<td>3</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Total channel rate, (R_f) (Mb/s)</td>
<td>2.916</td>
<td>2.88</td>
<td>2.772</td>
</tr>
<tr>
<td>Channel rate/cell, (R_c) (kb/s)</td>
<td>972</td>
<td>720</td>
<td>396</td>
</tr>
<tr>
<td>Frame duration, (T) (ms)</td>
<td>16</td>
<td>16</td>
<td>16</td>
</tr>
<tr>
<td>Slots/frame, (N) (each cell)</td>
<td>27</td>
<td>20</td>
<td>11</td>
</tr>
<tr>
<td>Slot duration, (\tau) (ms)</td>
<td>0.5926</td>
<td>0.8</td>
<td>1.4545</td>
</tr>
<tr>
<td>Speech users/cell, (M)</td>
<td>27 - 68</td>
<td>20 - 50</td>
<td>11 - 28</td>
</tr>
<tr>
<td>Speech rate, (R_s) (kb/s)</td>
<td>32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Packet size (bits)</td>
<td>576 (header: 64, info:512)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Delay limit, (D_{\text{max}}) (ms)</td>
<td>32</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Permission probability, (p)</td>
<td>0.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean talkspurt duration, (t_1) (s)</td>
<td>1.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean silence duration, (t_2) (s)</td>
<td>1.35</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Capture ratio, (z) (dB)</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The use of smaller cluster sizes allows the number of channels (slots) per cell to be increased, hence the number of supported users per cell may also be increased. In order to facilitate comparison between systems the results are plotted as a function of \(C\), the number of active users per cell divided by the number of slots per cell, ie. \(C=M/N\). For a given system, the number of slots per cell is constant. The results obtained from the Markov analysis have been verified by computer simulation results.

Fig.3 plots the packet dropping probability versus \(\zeta\) for various cluster sizes. When \(\zeta\) is low, the packet dropping probability is lower for small cluster sizes. This is because small cell clusters allow for more channels per cell, hence there are more chances for a user to obtain a reservation.

![Fig.3: Packet Dropping Probability](image)

Fig.4 plots the interference probability versus \(\zeta\) for various cluster sizes. The probability of interference is greatest for small cell cluster sizes where the frequency reuse distances are relatively short. The improvement in interference probability due to an ideal 2-branch SIR selection diversity scheme is also evident. The interference probability can be reduced significantly with a 2-branch system. For example, for \(N_c=4\) and \(\zeta=1.8\), the interference probability can be reduced from 6.9% to 1.3% with selection diversity.

![Fig.4: Interference Probability](image)

Fig.5 plots the packet loss probability versus \(\zeta\) for various cluster sizes. The improvement in the packet loss probability due to an ideal 2-branch SIR selection diversity scheme is also presented. The capacity of cellular PRMA can be estimated as the number of users per cell which can be supported at a 1% packet loss probability. Table 2 presents the system capacities of the three systems based on the results of Fig.5, where selection diversity is assumed. Without selection diversity these system capacities are significantly lower. The gain in capacity obtained with small clusters is
limited by the increase in the level of inter-cell interference which occurs. For example a cluster size of 4 has 20 channels per cell and can support 30.2 active users per cell, whereas a cluster size of 3 has 27 channels per cell but can only support 29.7 users per cell.

<table>
<thead>
<tr>
<th>System</th>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cluster size, $N_c$</td>
<td>3</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>Active users/cell, $M$, supported at the 1% packet loss level</td>
<td>29.7</td>
<td>30.2</td>
<td>16.3</td>
</tr>
<tr>
<td>Throughput per cell, $\eta$, at the 1% packet loss level</td>
<td>46%</td>
<td>63%</td>
<td>62%</td>
</tr>
<tr>
<td>System utilisation, $\psi$, at the 1% packet loss level</td>
<td>15.3%</td>
<td>15.8%</td>
<td>8.9%</td>
</tr>
</tbody>
</table>

Table 2 presents the throughputs per cell which can be supported by the various systems at a packet loss probability of 1%. In addition, the system utilisations, $\psi$, are presented.

Based on the results presented in Table 2 and for the particular parameters and assumptions considered in this paper, the most appropriate cluster size for cellular PRMA appears to be 4.

VII. Conclusion

PRMA is a flexible terminal-to-base-station access protocol. Its analysis in a Rayleigh faded cellular environment has revealed that its performance is limited by the level of inter-cell interference. Excessive inter-cell interference present in systems with small cluster sizes can significantly increase the packet loss probability and reduce the overall system utilisation. An ideal 2-branch SIR selection diversity scheme would be effective in combating the level of inter-cell interference under such circumstances. From the analysis, a system with a cluster size of 4 was found to be the most efficient: able to support 1.5 active users per cell for every timeslot available while providing an overall system utilisation of 15.8%.

Ongoing investigations are aimed at analysing the performance of cellular PRMA over combined lognormal and Rayleigh faded channels as well as the role of power control to further mitigate inter-cell interference in this type of random access packet network.

References