Adaptive Fuzzy Control

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Abstract

In this paper, adaptive fuzzy logic control (FLC) will be designed by using a simple reference model. This design approach is based on our new methodology "design rule base qualitatively and data base quantitatively". If the linear rule base is used, the model of FLC can be obtained. It is actually a nonlinear function with only three scaling gains need to be designed and tuned. The conventional control theory can thus be used. This model reference adaptive fuzzy control (MRAFC) requires less restriction on the reference model, but often achieves a more robust performance than its classical counterpart.

1. Introduction

Different types of fuzzy logic control (FLC) should be used for different applications [8]. The most popular type of fuzzy control is the feedback error type. In this paper, we only discuss the adaptive approach for this type of FLC.

In industry, FLC is designed based on experience. Sound knowledge of the process is often needed. The tuning for matching linguistic rules and numerical input/output is normally done by rule adjustment. This qualitative design is entirely heuristic, and thus difficult to obtain the systematic design. In academic area, there is research in artificial neural network (NNW) to possess self-learning capability. No or limited initial knowledge is needed. The quantitative training is carried out to generate the rule base. Except for the time-consuming, however, this pure quantitative approach may lose the original linguistic interpretation. There is another research in developing adaptive fuzzy control by combining NNW and fuzzy logic [2,9]. Fuzzy logic is used to build rule base and NNW to tune the data base. This approach is still complex and time consuming.

There is another simple methodology that attempts to bridge qualitative and quantitative design by using conventional control theory [5]. The basic concept is to *design rule base qualitatively and tune the data base quantitatively*. The rule base is designed based on the general dynamics of the process [3], and preserved during the operation. By using this general rule base, the tuning should be left to the data base, mainly, the scaling gains [7]. The mathematical model of FLC obtained under the special conditions shows that FLC is actually a nonlinear variable structure control (VSC) [4]. The rest of FLC design is actually to design the scaling gains. Then conventional control theory could be used for this quantitative tuning.

The classical model reference adaptive control (MRAC) can be derived from the gradient method [1]. Though FLC is a nonlinear control, it can be approximated as a linear one around the equilibrium state. Then the idea for the classical control can be copied to FLC.

The model of FLC is presented first. Then, model reference adaptive fuzzy control (MRAFC) is constructed based on the classical gradient method. In theory, these adaptive schemes are only valid around the equilibrium state and for linear plant, however, they can be extended to a more general situation in the reality. As it requires less restriction on the reference model, a simple first-order model can be used for a wide range of processes. Finally, performance simulation demonstrates the viability of this approach.

2. Mathematical model of FLC

A basic structure of fuzzy two-term control is shown below. The model of FLC can be derived based on following assumptions.

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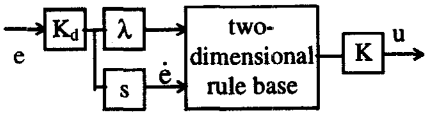


Figure 1: Basic structure of a fuzzy two-term control

<u>Assumptions</u>

1) A linear rule base is used as shown in Figure 2, which has seven labels for each input and output.
2) The triangular membership functions (MFs) are used with equal spread 2A for input and 2B for output.
3) Mamdani's max-min inference method is used.

$\dot{E} = K_d \dot{e}$

$\dot{E}$ \ E	NL	NM	NS	ZR	PS	PM	PL
PL	zr	ps	pm	pl	pl	pl	pl
PM	ns	zr	ps	pm	pl	pl	pl
PS	nm	ns	zr	ps	pm	pl	pl
ZR	nl	nm	ns	zr	ps	pm	pl
NS	nl	nl	nm	ns	zr	ps	pm
NM	nl	nl	nl	nm	ns	zr	ps
NL	nl	nl	nl	nl	nm	ns	zr

$E = K_e e$

Figure 2: A 2-dimensional linear rule base

The linear rule base can be divided into many inference cells (ICs) as shown in Figure 3. The mathematical model of a two-dimensional rule base can be derived from ICs as shown in (1) [4,10].

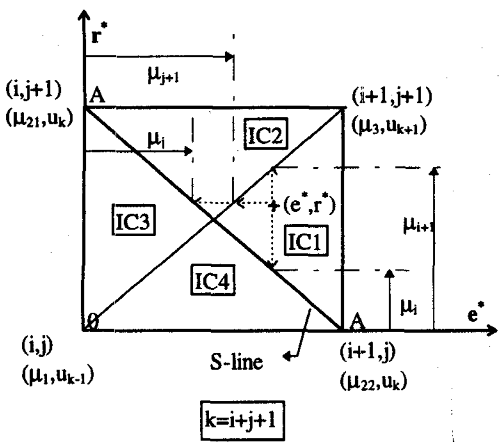


Figure 3: Functional composition of the inference cell

$$u_l = \frac{B}{A}\gamma_l S + kB(1-\gamma_l) \quad (l=1,2,3,4) \tag{1}$$

with $S = E + \dot{E} = K_d(\lambda e + \dot{e})$
$k = i+j+1$
γ_l is the nonlinear parameter at sub-region IC_l.

For the simplicity, the index l is omitted in the rest of paper.

3. Adaptive gain design based on the approximate model

Gradient method

Consider an unknown plant described by

$$y = Pu$$

Assume that its reference model is described by

$$y_m = G_m u_c$$

The model-plant mismatch is chosen as:

$$\varepsilon = y - y_m$$

and the criterion is chosen as:

$$J(\theta) = \frac{1}{2}\varepsilon^2 \tag{2}$$

The parameter θ should be adjusted in the direction of the negative gradient of J [1].

$$\frac{d\theta}{dt} = -\eta\frac{\partial J}{\partial\theta} = -\eta\varepsilon\frac{\partial\varepsilon}{\partial\theta} = -\eta\varepsilon\frac{\partial y}{\partial\theta} \tag{3}$$

Standard MRAFC

<u>*Adaptive scheme for output gain K*</u>

FZ-PD can be simplified as a PD control F*e plus a relay term R as shown in (4). The relay term R goes to zero when the system approaches the equilibrium point. Thus, FZ-PD can be approximated as linear control around the equilibrium state.

$$u = K(R + F^*e) \tag{4}$$

with $F = \frac{B}{A}\gamma K_d(\lambda + p)$, $p = \frac{d}{dt}$

The gradient method can be used to construct the model reference adaptive fuzzy control (MRAFC) for the linear plant around the equilibrium state. The results can be later extended to the nonlinear plant globally.

By approximating γ as constant, the closed-loop system around the equilibrium state can be approximated as:

$$y = \frac{KFP}{1+KFP} u_c$$

and
$$e = \frac{1}{1+KFP} u_c$$

From (3),

$$\frac{\partial \varepsilon}{\partial K} = \frac{\partial y}{\partial K} = \frac{FP}{(1+KFP)^2} u_c = \frac{KFP}{1+KFP} \frac{e}{K} \approx G_m \frac{e}{K} \qquad (5)$$

The approximation $\frac{KFP}{1+KFP} \approx G_m$ is made by assuming y approaches y_m. From (3), the adaptation law for the output gain of FLC is approximated as below:

$$\frac{dK}{dt} = -\eta G_m \frac{\varepsilon e}{K}$$

If the reference model is chosen as a first-order system as:

$$G_m = \frac{b_m}{a_m + p} \qquad (6)$$

then, the adaptation law is approximated as:

$$\ddot{K} + a_m \dot{K} = -\eta \frac{\varepsilon e}{K} \qquad (7)$$

where b_m is absorbed in η. The MRAFC system can be constructed as shown in Figure 4.

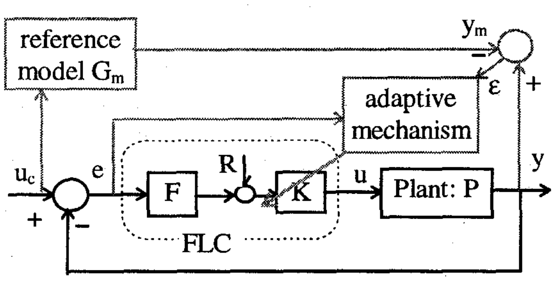


Figure 4: MRAFC for output gain

Feedforward type MRAFC

The equilibrium state of the above MRAFC is not the reference model, which may require smaller gain for the system to avoid too fast adaptation around the reference. A slight modification can be made to improve the adaptation by the feedforward MRAFC shown in Figure 5. The error e between the input and the output is replaced by the model-plant mismatch ε. Then the equilibrium state becomes the reference model. The controlled output should converge to the reference model. The closed-loop output y is:

$$y = \frac{KFP}{1+KFP} y_m$$

and the model-plant mismatch is: $\varepsilon = \frac{1}{1+KFP} y_m$

Adaptive scheme for the output gain K

From (5), we have

$$\frac{\partial \varepsilon}{\partial K} = -\frac{\partial y}{\partial K} = -\frac{FP}{(1+KFP)^2} y_m = -\frac{KFP}{1+KFP} \frac{\varepsilon}{K} \approx -\frac{\varepsilon}{K} \qquad (8)$$

after the approximation $\frac{KFP}{1+KFP} \approx 1$ by assuming y approaches y_m. The new adaptation law for the output gain is obtained from (3).

$$\frac{dK}{dt} = \eta \frac{\varepsilon^2}{K} \qquad (9)$$

As ε replaces e, the new adaptation (9) can keep the fast convergence in the beginning and slow down when approaching the reference model. Practically, it can keep the better model-following performance than the adaptation (7). Similarly, the new adaptation does not require a perfect model match and also tolerates more uncertainties than MRAC. Thus, a first-order model can be used for a wide range of plant.

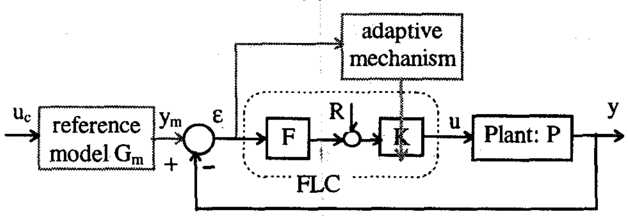


Figure 5: Feedforward MRAFC with output gain only

Adaptive scheme for the input gain

Among input gains, λ is more important. FLC becomes linear in the equilibrium state; then from (4), we have

$$\frac{\partial F}{\partial \lambda} = B\gamma = \frac{F}{\lambda + p}$$

Similarly from (8), we have:

$$\frac{\partial \varepsilon}{\partial \lambda} = -\frac{\partial y}{\partial \lambda} = -\frac{KP}{(1+KFP)^2}\frac{\partial F}{\partial \lambda} y_m = -\frac{KFP}{1+KFP}\frac{\varepsilon}{\lambda + p} \approx -\frac{\varepsilon}{\lambda + p}$$

then, the adaptation for the input gain ratio λ can be approximated from (3) as:

$$\ddot{\lambda} + \lambda\dot{\lambda} = \eta\varepsilon^2 \qquad (10)$$

4. Simulation

In this section, some simulations will be carried out to compare MRAC and MRAFC for plants with unmodelled dynamics. The simulation is completed by using OMRON fuzzy inference board FB-30at and the simulation software packet (DCS) developed in our control laboratory [6].

About the plant and the reference model

The reference model is chosen as the first-order linear model in (6) with $a_m = b_m = 1$.

The plant includes:

1) a first-order linear plant with unknown parameters which provides only parameter mismatch.

$$P(s) = \frac{2}{1+0.4s}$$

2) a third-order linear plant with unknown orders and parameters, which provides structure mismatch.

$$P(s) = \frac{1}{1+s}\left(\frac{458}{s^2 + 30s + 229}\right)$$

3) a nonlinear plant with unknown time-varying parameters, which provides dynamic mismatch.

$$0.4\dot{x} + (1 + 0.2\sin(0.1t))x = 2u - \mathrm{sgn}(\dot{x})$$

The command signal u_c is the square wave and the sampling step is 0.1 second.

Performance comparison

MRAC

For the first-order linear plant, the performance of MRAC with η=0.1 is shown in Figure 6. It has a perfect model following after 500 seconds.

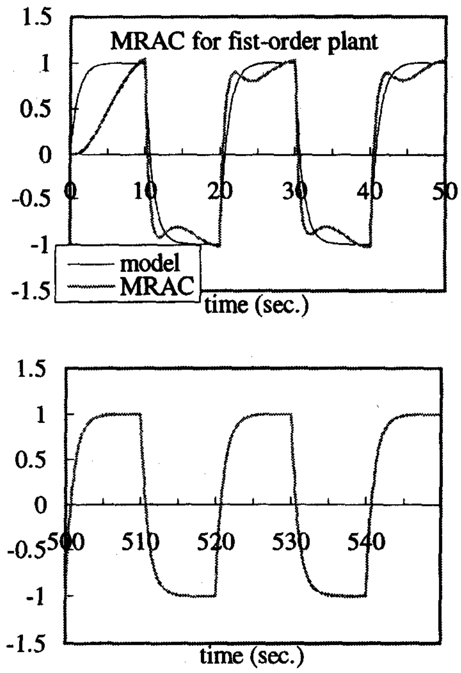
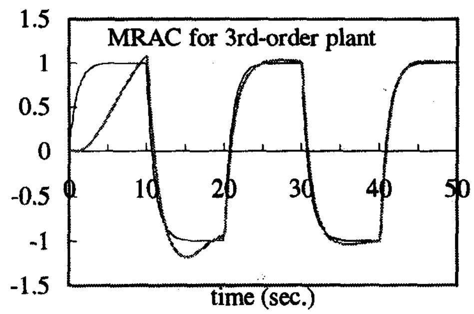


Figure 6: Performance of MRAC for 1st-order plant

For the third-order linear plant, MRAC can still handle this partially known system as shown in Figure 7. However, the model following performance is deteriorated much from the above.

For the nonlinear time-varying plant, MRAC can not achieve satisfactory performance.



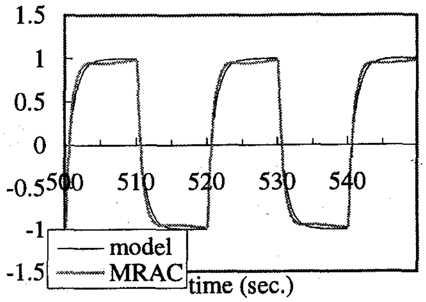


Figure 7: Performance of MRAC for 3rd-order plant

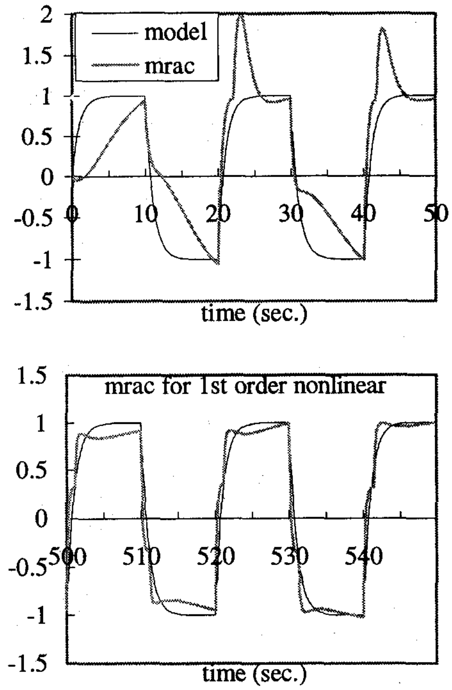


Figure 8: Performance of MRAC for 1st-order nonlinear time-varying plant

MRAFC

MRAFC is chosen as FZ-PI with the feedforward type adaptation. The adaptation gains for K and λ are chosen as η_K=0.5 and η_λ=0.1.

For the first-order linear plant, the performance of MRAFC is satisfactory. The process output will follow the reference model quite well, though not as perfect as MRAC.

For the third-order linear plant, the performance of MRAFC is not much different from the first-order plant. MRAFC achieves better performance than MRAC.

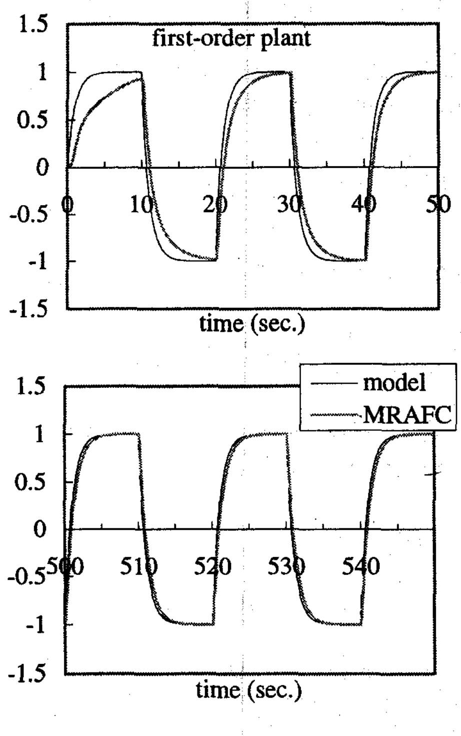


Figure 9: Performance of MRAFC for 1st-order plant

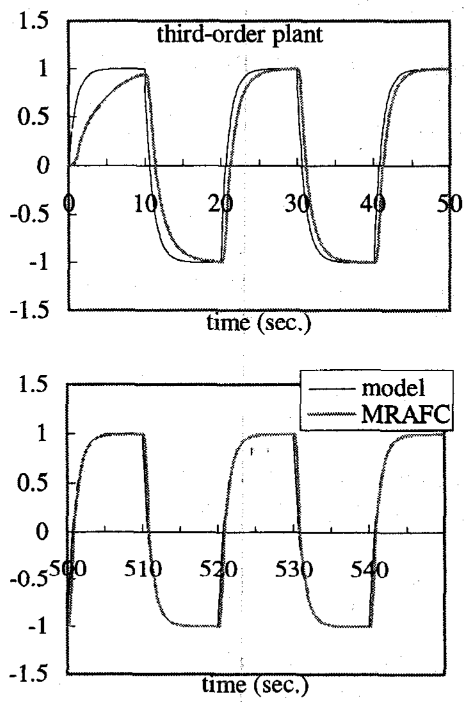


Figure 10: Performance of MRAFC for 3rd-order plant

For the nonlinear time-varying plant, the performance of MRAFC is still not much different from the first-order plant. MRAFC achieves much better performance than MRAC.

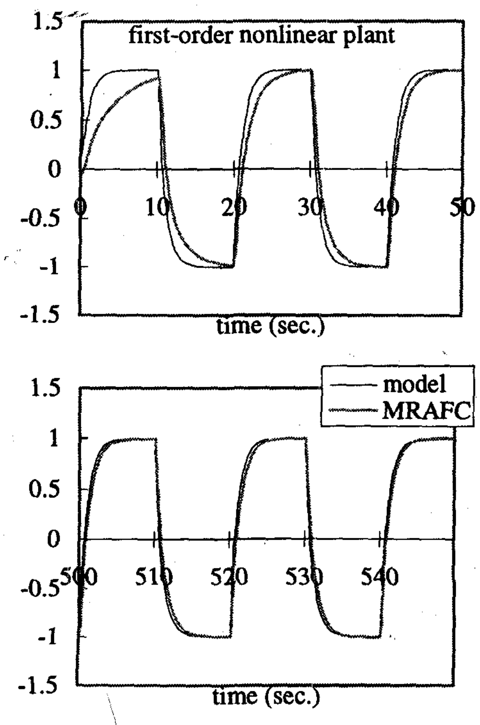


Figure 11: Performance of MRAFC for 1st-order nonlinear time-varying plant

5. Conclusions

The conventional FLC can be considered as a nonlinear function with three scaling gains need to be designed and tuned. Of these three gains, only the input gain ratio λ and the output gain K are critical based on the VSC theory [4]. By using the gradient method from the classical MRAC, the adaptive mechanism for these gains can be derived for the linear plant on the equilibrium state. The structure of this MRAFC is similar to MRAC. To improve the model-following performance, the feedforward type MRAFC is designed to keep the equilibrium state on the reference model. This feedforward type MRAFC is simple in structure, fast in convergence and better in performance.

Though MRAFC is derived for the linear plant on the equilibrium state, however, it does not require the perfect model matching. Thus, it can be applied to systems globally with large model-plant mismatch, including plants with some nonlinear or time-varying features. Successful simulations demonstrate that MRAFC can achieve more robust performance than MRAC, especially for plant with the unmodelled dynamics. MRAC can only work well with plants having mainly parameter mismatch because of its time-invariant features. MRAFC shows the superior performance with the nonlinear and time-varying plant because of its nonlinear time-varying features.

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