Robust Stabilization of Time Delay Nonlinear Systems With a Triangular Structure

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ABSTRACT

In this paper, we examine the problem of robust stabilization of time-delay nonlinear systems which are in parametric strict feedback (PSF) form. We propose an iterative procedure of stabilizing controller construction similar to backstepping procedure for this class of time-delay nonlinear systems.

1. INTRODUCTION

Robust stabilization for nonlinear structural systems has long been an interesting and challenging problem. Since the inception of the matching conditions in the control literature by Leitmann's paper [7], a number of systematic design procedures has been developed to stabilize nonlinear systems satisfying these conditions; see [3, 2, 1, 4, 11, 5, 8] for example. With the recent development of geometric theory for nonlinear feedback systems, there emerges a number of robust controller design techniques for linearizable nonlinear systems with mismatched nonlinearities/uncertainties. In [6], an iterative procedure known as backstepping has been developed to design adaptive controllers which globally (locally) stabilize systems which are in so-called parametric-strict-feedback (parametric-pure-feedback) form. In [9], the authors considered systems which are of a much more general form than the PSF. An iterative procedure, similar to backstepping procedure has been used to construct a globally stabilizing state feedback controllers for those systems. Nonlinear systems with block-triangular structure have been further studied in [10] where the backstepping procedure is also used to provide global stabilization.

Recently, the problem of stabilization of uncertain time-delay systems has been of great interest to many researchers. The motivation of this paper stems from the fact that all the aforementioned results assumed that the nonlinear systems under investigation are free of time-delay. As we know that, in general, the existence of time delay degrades the control performance and sometimes makes the closed-loop stabilization difficult, especially when the systems are nonlinear. What we intend to do in this paper is to propose an iterative procedure for designing a stabilizing controller for uncertain time-delay nonlinear systems which are in PSF form.

The rest of the paper is organised as follows: In Section 2, we give a brief description of nonlinear systems which will be discussed throughout the paper. In Section 3 we propose an iterative robust controller design procedure for the class of nonlinear systems outlined in Section 2. Some conclusions are drawn in Section 4.

2. SYSTEM AND PRELIMINARIES

The class of single input time delay nonlinear systems to be considered in this paper is given by

\[
\begin{align*}
    \dot{x}_i(t) &= F_i(w_i(t)) + H_i(w_i(t - \tau)) + G_i(w_i(t))x_{i+1} + \sum_{j=1}^{i-1} x_{j}K_{F_{ij}}(w_j(t)) \\
    \dot{x}_n(t) &= F_n(w_n(t)) + H_n(w_n(t - \tau)) + G_n(w_n(t))u
\end{align*}
\]

where \(x_i\) are the state variables, \(w_i(t) = [x_1(t), \ldots, x_i(t)]\) are the state vectors, \(w_i(t - \tau) = [x_1(t - \tau), \ldots, x_i(t - \tau)]\) are the delay state vector and \(u \in \mathbb{R}\) is the control input of the system. The nonlinear functions \(F_i(\cdot), H_i(\cdot), G_i(\cdot)\) and \(H_i(\cdot)\) with \(F_i(0) = 0\) and \(H_i(0) = 0\) are assumed to satisfy the following conditions:

C1: The nonlinear function \(F_i(w_i(t))\) is assumed to be a Carathéodory function and to satisfy

\[
|F_i(w_i(t))| \leq \sum_{j=1}^{i} |x_j|K_{F_{ij}}(w_j(t))
\]

where \(K_{F_{ij}}(w_j(t))\) are known smooth nonlinear functions.
C2: The nonlinear function $H_i(w_j(t - \tau))$ is assumed to be a Carathéodory function and to satisfy

$$|H_i(w_j(t - \tau))| \leq \sum_{j=1}^{i} |x_j(t - \tau)|K_{H_i}(w_j(t - \tau))$$

where $K_{H_i}(w_j(t - \tau))$ are known smooth nonlinear functions.

C3: $0 < G_i(w_i) \leq \gamma_i$ where $\gamma_i$ are known positive constants.

To simplify our controller design, we introduce the following nonlinear coordinates transformation:

$$T \begin{cases} x_1 = z_1 \\ x_i = z_i - \phi_{i-1}(r_{i-1}); \quad i = 2, \ldots, n. \end{cases}$$

where $r_i(t) = [z_1(t), \ldots, z_i(t)]$ and $\phi_{i-1}(r_{i-1})$ with $\phi_{i-1}(0) = 0$ are some smooth functions. The system $\Sigma$ under transformation $T$ becomes

$$\dot{z}_i = f_i(r_i(t)) + h_i(r_i(t - \tau)) + g_i(r_i(t)) [z_{i+1} + \phi_i(r_i(t))] + \psi_i(r_i(t)) + \sum_{p=1}^{i-1} \frac{\partial \phi_{i-1}(r_{i-1})}{\partial z_p} h_p(r_p(t - \tau))$$

$$\dot{z}_n = f_n(r_n(t)) + h_n(r_n(t - \tau)) + g_n(r_n(t)) u + \psi_{n-1}(r_{n-1}(t)) + \sum_{p=1}^{n-1} \frac{\partial \phi_{n-1}(r_{n-1})}{\partial z_p} h_p(r_p(t - \tau))$$

where $f_i(r_i(t)), h_i(r_i(t - \tau))$ and $g_i(r_i(t))$ are functions $F_i(w_i(t)), H_i(w_i(t - \tau))$ and $G_i(w_i(t))$ in new coordinates, respectively. Also

$$\psi_{i-1}(r_{i-1}(t)) = \sum_{p=1}^{i-1} \frac{\partial \phi_{i-1}(r_{i-1})}{\partial z_p} \{f_p(r_p) + g_p(r_p)[z_{p+1} + \phi_p(r_p)]\}$$

It is readily seen that Conditions C1 - C3 under transformation $T$ are given as:

- TC1: $|f_i(r_i(t))| \leq \sum_{j=1}^{i} |z_j| \rho_{ij}(r_j(t))$ where $\rho_{ij}(w_j(t))$ are known smooth nonlinear functions.

- TC2: $|h_i(r_i(t - \tau))| \leq \sum_{j=1}^{i} |z_j(t - \tau)| \omega_{ij}(r_j(t - \tau))$ where $\omega_{ij}(z_j(t - \tau))$ are known smooth nonlinear functions.

- TC3: $0 < \gamma_i$ where $\gamma_i$ are known positive constants.

Remark 1. We stress that conditions given in TC1 and TC2 are very weak. As the matter of fact, for any given smooth function, say $M(x_1, x_2, \ldots, x_i)$ with $M(0) = 0$, there always exist smooth functions $m_i(x_1, \ldots, x_i)$ such that

$$M(x_1, x_2, \ldots, x_i) = x_1 m_1(x_1) + x_2 m_2(x_1, x_2) + \cdots + x_i m_i(x_1, \ldots, x_i).$$

3. MAIN RESULT

In this section, we present a design procedure, similar to the backstepping procedure for constructing a nonlinear asymptotically stabilizing controller for the system $\Sigma$ satisfying Conditions C1-C3. The proof is given by explicitly show how to construct a nonlinear asymptotically stabilizing controller.

Theorem 3.1. If the system $(\Sigma)$ satisfies Conditions C1-C3, then there exists a state feedback controller such that the closed-loop system is asymptotically stable.

Proof. Construction of a $\Sigma$'s controller: The design procedure adopted here is very similar to backstepping procedure [10, 6, 9].

Step 1. Under transformation $T$, the system $\Sigma$'s first equation becomes

$$\dot{z}_1 = f_1(z_1(t)) + h_1(z_1(t - \tau)) + g_1(z_1(t)) [z_2 + \phi_1(z_1(t))] + S(z_1(t)) - S(z_1(t - \tau))$$

First let ignore the term $g_1(z_1(t))z_2$ and choose the Lyapunov function $V_1(z_1) = \frac{1}{2} z_1^2(t) + \int_{t_0}^{t} S(z_1(\sigma)) d\sigma$ where $S(z_1(\sigma))$ is a positive function yet to be determined. The time-derivative of $V_1(z_1)$ along (3.1) reads

$$\dot{V}_1(z_1) \leq z_1(t)[f_1(z_1(t)) + h_1(z_1(t - \tau)) + g_1(z_1(t)) \phi_1(z_1(t))] + S(z_1(t)) - S(z_1(t - \tau))$$

By triangular inequality and Conditions TC1 and TC2, (3.2) becomes

$$\dot{V}_1(z_1) \leq \frac{1}{2} z_1^2(t) \rho_1(z_1(t)) + \frac{1}{2} z_1^2(t - \tau) \omega_1^2(z_1(t - \tau)) + S(z_1(t)) - S(z_1(t - \tau))$$

Choose

$$S(z_1(\sigma)) = \frac{1}{2} z_1^2(\sigma) \left[ n g_{11}^2(z_1(\sigma)) + \sum_{i=2}^{n} g_i^2(z_1(\sigma)) \right]$$

we have

$$\dot{V}_1(z_1) \leq \frac{1}{2} z_1^2(t) \rho_1(z_1(t)) + \frac{1}{2} \left[ n g_{11}^2(z_1(t)) + \sum_{i=2}^{n} g_i^2(z_1(t)) \right]$$

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In order to have $V_1(z_1) \leq 0$, we may select $\phi_1(z_1)$ as

$$\phi_1(z_1) = \frac{1}{n_1} z_1(t) \left[ \frac{n}{2} c_1 + \varphi(z_1(t)) \right]$$  \hspace{1cm} (3.5)$$

where

$$\varphi(z_1(t)) = \frac{1}{2} \rho_1(z_1(t)) + \frac{1}{2} \left[ n \eta_{z_1}^2(z_1(t)) + \sum_{i=2}^{n} \eta_{z_i}^2(z_1(t)) \right]$$  \hspace{1cm} (3.6)$$

and $c_1$ is a positive constant. Substitute (3.5) into (3.4) yields

$$V_1(z_1) \leq \frac{n}{2} c_1 z_1^2(t) - \frac{1}{2} z_1^2(t - \tau) \times$$

$$\left[ (n - 1) \eta_{z_1}^2(z_1(t)) + \sum_{i=2}^{n} \eta_{z_i}^2(z_1(t)) \right]$$  \hspace{1cm} (3.7)$$

$$\leq 0.$$  

Step 2. The second equation of system $\Sigma$ under transformation $T$ becomes

$$\dot{z}_2 = f_2(r_2(t)) + h_2(r_2(t - \tau))$$

$$+ g_2(r_2(t)) [z_2 + \phi_2(r_2(t))]$$

$$+ \psi_1(r_2) + \frac{\partial \phi_1(z_1)}{\partial z_1} h_1(z_1(t - \tau))$$  \hspace{1cm} (3.8)$$

where

$$\psi_1(r_2) = \frac{\partial \phi_1(z_1)}{\partial z_1} \{ f_1(z_1(t))$$

$$+ g_1(z_1(t)) [z_2 + \phi_1(z_1(t))].$$  \hspace{1cm} (3.9)$$

Choose a new Lyapunov function as

$$V_2(r_2) = V_1(z_1) + \frac{1}{2} z_2^2 + \int_{t-\tau}^{t} S_2(r_2(\sigma)) \ d\sigma$$  \hspace{1cm} (3.10)$$

where $S_2(r_2(\sigma))$ is a positive function yet to be computed. Using (3.7), it is easy to show that the time derivative of (3.10) along (3.1) and (3.8) is given by

$$\dot{V}_2(r_2) \leq -\frac{n}{2} c_1 z_1^2 - \frac{1}{2} z_1^2(t - \tau) \times$$

$$\left[ \sum_{i=1}^{n} \eta_{z_i}^2(z_1(t)) + (n - 1) \eta_{z_1}^2(z_1(t - \tau)) \right]$$

$$+ z_1 g_1(z_1(t)) z_2 + z_2 \{ f_2(r_2(t)) + h_2(z_1(t - \tau))$$

$$+ g_2(r_2(t)) [z_2 + \phi_2(r_2(t))] + \psi_1(r_2(t))$$

$$+ \frac{\partial \phi_1(z_1)}{\partial z_1} h_1(z_1(t - \tau)) \}$$

$$+ S_2(r_2(t)) + S_2(r_2(t - \tau))$$  \hspace{1cm} (3.11)$$

Again using triangular inequality and Conditions TC1 and TC2, (3.11) becomes

$$\dot{V}_2(r_2) \leq -\frac{n}{2} c_1 z_1^2 - \frac{1}{2} z_1^2(t - \tau) \times$$

$$\left[ \sum_{i=1}^{n} \eta_{z_i}^2(z_1(t)) + (n - 2) \eta_{z_1}^2(z_1(t - \tau)) \right]$$

$$+ z_1^2 \{ g_2(r_2(t)) + 1 + \frac{1}{2} z_2^2(t - \tau) \} \eta_{z_2}^2(r_2(t - \tau)) +$$

$$+ z_2 \psi_1(r_2) + z_2 g_1(z_1) z_2 + \frac{1}{2} z_2^2 \left( \frac{\partial \phi_1(z_1)}{\partial z_1} \right)^2$$

$$+ z_2(t) g_2(r_2(t)) [z_2 + \phi_2(r_2(t))]$$

$$+ S_2(r_2(t)) - S_2(r_2(t - \tau))$$  \hspace{1cm} (3.12)$$

Let us derive an upper bound for the term $z_2 \psi_1(z_1) + z_2 g_1(z_1) z_2$. Using triangular inequality, we have

$$|z_2 \psi_1(z_1) + z_2 g_1(z_1)| \leq$$

$$\frac{1}{2} z_2^2 \left( \frac{\partial \phi_1(z_1)}{\partial z_1} \right)^2$$

$$+ \left( \frac{n}{2} c_1 + \varphi(z_1) \right) \{ z_2 + \phi_2(r_2(t)) \}$$

$$+ 2 \left( \frac{\partial \phi_1(z_1)}{\partial z_1} \right) \left[ \frac{\partial \phi_1(z_1)}{\partial z_1} \right]$$

$$+ \frac{1}{2} z_1^2 \Phi_1(z_1) + \frac{1}{2} z_1^2$$  \hspace{1cm} (3.13)$$

where $\Phi_1(z_1)$ is a known smooth function (smoothness of $\Phi_1(z_1)$ is assured by Conditions TC1 and TC2). Choose

$$S(r_2(\sigma)) = \frac{1}{2} z_2^2(\sigma) [(n - 1) \eta_{z_2}^2(r_2(\sigma))$$

$$+ \sum_{i=3}^{n} \eta_{z_i}^2(r_2(\sigma))]$$  \hspace{1cm} (3.14)$$

and with the upper bound derived in (3.13) and the choice of $S(r_2(\sigma))$ given in (3.14), (3.12) becomes

$$\dot{V}_2(r_2) \leq -\frac{n}{2} c_1 z_1^2 - \frac{1}{2} z_1^2(t - \tau) \times$$

$$\left[ \sum_{i=1}^{n} \eta_{z_i}^2(z_1(t)) + (n - 2) \eta_{z_1}^2(z_1(t - \tau)) \right]$$

$$+ z_1^2 \eta_{z_2}^2(r_2(t)) + 1 + \frac{1}{2} z_2^2(t - \tau) \eta_{z_2}^2(r_2(t)) \times$$

$$+ \frac{1}{2} z_2^2 \left( \frac{\partial \phi_1(z_1)}{\partial z_1} \right)^2$$

$$+ z_2(t) g_2(r_2(t)) [z_2 + \phi_2(r_2(t))]$$

$$+ S_2(r_2(t)) - S_2(r_2(t - \tau))$$  \hspace{1cm} (3.12)$$
\[
\left[ (n - 2)q_{22}^2(r_2(t - \tau)) + \sum_{i=3}^{n} q_{i2}^2(r_2(t - \tau)) \right] 
\]

(3.15)

Similiar to Step 1, we first forget about the term containing \( z_3 \). Choose

\[
\phi_2(r_2) = -\frac{1}{2} \zeta_2 \left[ \frac{n - 1}{2} c_1 z_2 + \varphi_2(r_2) \right]
\]

(3.16)

where

\[
\varphi_2(r_2) = \rho_2(r_2) + 1 + \left( \frac{\partial \phi_1(z_1)}{\partial z_1} \right)^2 + \frac{1}{2} \Phi_1(z_1)
\]

and with the choice of \( \varphi_2(r_2) \) given in (3.16), (3.15) becomes

\[
\dot{V}_2(r_2) \leq -\frac{(n - 1)c_1}{2} \| z_2 \|^2 - \frac{1}{2} z_2^2(t - \tau) \times \left[ \left( n - 2 \right) q_{11}^2(z_1(t - \tau)) \sum_{i=3}^{n} q_{i2}^2(z_1(t - \tau)) \right] \]

\[
- \frac{1}{2} z_2^2(t - \tau) [\sum_{i=3}^{n} q_{i2}^2(r_2(t - \tau)) ] + (n - 2) q_{22}^2(r_2(t - \tau)).
\]

(3.17)

Step \( k \) (3 \( \leq k \leq n - 1 \)). Under transformation \( T \), the system \( \Sigma \)'s \( k \)th equation becomes

\[
\dot{z}_k = f_k(r_k) + h_k(r_k(t - \tau)) + \psi_{k-1}(r_k(t)) + g_k(r_k(t))[z_{k+1} + \phi_k(r_k(t))] + \sum_{p=1}^{k-1} \frac{\partial \psi_{k-1}(r_k-1)}{\partial z_p} h_p(r_r(t - \tau))
\]

(3.19)

where

\[
\psi_{k-1}(r_k) = \sum_{p=1}^{k-1} \frac{\partial \psi_{k-1}(r_k-1)}{\partial z_p} \left\{ f_p(r_p) + g_p(r_p)[z_{p+1} + \phi_p(r_p)] \right\}
\]

(3.20)

Choose a new Lyapunov function as

\[
V_k(r_k) = V_{k-1}(r_k-1) + \frac{1}{2} z_k^2 + \int_{t-\tau}^{t} S_k(r_k(\sigma)) \, d\sigma
\]

(3.21)

where \( S_k(r_k(\sigma)) \) is a function yet to be found. Follow from Steps 1 and 2, it can be shown that the time-derivative of \( V_k(r_k) \) reads

\[
\dot{V}_k(r_k) \leq -\frac{(n + 2 - k)}{2} c_1 \sum_{p=1}^{k-1} z_p^2 + z_k^2 g_k(r_k)[z_{k+1} + \phi_k(r_k)]
\]

(3.22)

Again by triangular inequality and Conditions TC1 and TC2, Eq. (3.22) becomes

\[
\dot{V}_k(r_k) \leq -\frac{(n + 2 - k)}{2} c_1 \sum_{p=1}^{k-1} z_p^2 - \frac{1}{2} \sum_{j=1}^{k-1} z_j^2(t - \tau) + z_k g_k(r_k)[z_{k+1} + \phi_k(r_k)]
\]

(3.23)

Let us derive an upper bound for the term \( z_k \psi_{k-1}(r_k-1) + z_k g_k(r_k-1)z_k \). By using triangular inequality, we obtain

\[
|z_k \psi_{k-1}(r_k-1) + z_k g_k(r_k-1)z_k| \leq \frac{1}{2} \sum_{p=1}^{k-1} z_p^2 + \frac{1}{2} \sum_{p=1}^{k-1} \left( \frac{\partial \phi_{k-1}(r_{k-1})}{\partial z_p} \right)^2
\]

(3.24)

where \( \Phi_{k-1}(r_{k-1}) \) is some smooth nonlinear function. Once again, forget about the term containing \( z_{k+1} \). Choose

\[
S_k(r_k(\sigma)) = \frac{1}{2} z_k^2(\sigma)[(n + k - 1) \partial \phi_k(r_k(\sigma))]
\]

(3.25)
and with the choice of $S_k(r_k(\sigma))$ given in (3.25) and the upper obtained in (3.24), we have

$$
\dot{V}_k(r_k) \leq -\frac{(n-k)}{2} c_1 \sum_{j=1}^{k} z^2_{k-1} + \frac{1}{2} \sum_{j=1}^{k} z^2_j (t - \tau) \left\{ (n-k) \frac{\partial s_k}{\partial r_k}(r_k(t - \tau)) + \frac{1}{2} \sum_{j=k+1}^{n} \frac{\partial s_j}{\partial r_k}(r_k(t - \tau)) \right\} + x_k g_k(r_k(\sigma))
$$

(3.26)

Choose $\phi_k(r_k)$ as

$$
\phi_k(r_k) = -\frac{1}{\gamma_k} \frac{1}{2} c_1 + \frac{n+k}{2} \phi_k(r_k)
$$

(3.27)

where

$$
\phi_k(r_k) = \rho_k(r_k) + \frac{k}{2} + \frac{1}{2} \Phi_k(r_k) - \frac{1}{2} \sum_{j=1}^{k} z^2_j (t - \tau)
$$

(3.28)

Follow the same analysis as in $k = 2$ case, we have

$$
\dot{V}_k(r_k) \leq -\frac{(n-k)}{2} c_1 \sum_{j=1}^{k} z^2_{k-1} + \frac{1}{2} \sum_{j=1}^{k} z^2_j (t - \tau) \left\{ (n-k) \frac{\partial s_k}{\partial r_k}(r_k(t - \tau)) + \frac{1}{2} \sum_{j=k+1}^{n} \frac{\partial s_j}{\partial r_k}(r_k(t - \tau)) \right\} + x_k g_k(r_k(\sigma))
$$

(3.29)

Step n. Under transformation $\mathcal{T}$, the last equation becomes

$$
\dot{z}_n = f_n(r_n) + h_n(r_n(t - \tau)) + g_n(r_n) u + \psi_n-1(r_n) + \sum_{p=1}^{n-1} \frac{\partial \Phi_n-1(r_n)}{\partial z_p} h_p(r_n(t - \tau)).
$$

(3.30)

where

$$
\psi_n-1(r_n) = \sum_{p=1}^{n-1} \frac{\partial \Phi_n-1(r_n)}{\partial z_p} [f_p(r_p) + g_p(r_p) + \phi_p(r_p)]
$$

(3.31)

Choose a new Lyapunov function as

$$
V_n(r_n) = V_{n-1}(r_n) + \frac{1}{2} z_n^2 + \int_{t-\tau}^{t} S_n(r_n(\sigma)) d\sigma(3.32)
$$

where $S_n(r_n(\sigma))$ is a function yet to be determined. It can be shown that the time derivative of (3.32) reads

$$
\dot{V}_n(r_n) \leq -c_1 \sum_{p=1}^{n-1} z^2_p + \frac{1}{2} \sum_{j=1}^{n-1} z^2_j (t - \tau) \times \left\{ \frac{\partial s_j}{\partial r_k}(r_k(t - \tau)) + \frac{1}{2} \sum_{j=k+1}^{n} \frac{\partial s_j}{\partial r_k}(r_k(t - \tau)) \right\} + x_n g_n(r_n) z_n + \sum_{p=1}^{n-1} \frac{\partial \Phi_n-1(r_n)}{\partial z_p} h_p(z_p(t - \tau)) + \psi_n-1(r_n) + S_n(r_n(t)) - S_n(r_n(t - \tau)).
$$

(3.33)

With triangular inequality and Conditions TC1 and TC2, (3.22) becomes

$$
\dot{V}_n(r_n) \leq -c_1 \sum_{p=1}^{n-1} z^2_p + x_n g_n(r_n) z_n + \frac{1}{2} \sum_{p=1}^{n-1} \frac{\partial \Phi_n-1(r_n)}{\partial z_p} h_p(z_p(t - \tau)) + \frac{1}{2} \sum_{j=1}^{n-1} z^2_j (t - \tau) \times \left\{ \frac{\partial s_j}{\partial r_k}(r_k(t - \tau)) + \frac{1}{2} \sum_{j=k+1}^{n} \frac{\partial s_j}{\partial r_k}(r_k(t - \tau)) \right\} + x_n g_n(r_n) u + \psi_n-1(r_n) + S_n(r_n(t)) - S_n(r_n(t - \tau)).
$$

(3.34)

Again, let us derive an upper bound for the term $z_n \psi_n-1(r_n) + x_n g_n(r_n) z_n$. By using triangular inequality, we have

$$
|z_n \psi_n-1(r_n) + x_n g_n(r_n) z_n| \leq \frac{1}{2} \sum_{p=1}^{n-1} z^2_p + \frac{1}{2} \sum_{p=1}^{n-1} \left\{ \frac{\partial \Phi_n-1(r_n)}{\partial z_p} h_p(z_p(t - \tau)) + x_n g_n(r_n) u + \psi_n-1(r_n) + S_n(r_n(t)) - S_n(r_n(t - \tau)).
$$

(3.35)

and with the choice of $S_n(r_n(\sigma))$ given in (3.36) and the upper bound derived in (3.35), we have

$$
\dot{V}_n(r_n) \leq -c_1 \sum_{p=1}^{n-1} z^2_p + \frac{1}{2} \sum_{p=1}^{n-1} \frac{\partial \Phi_n-1(r_n)}{\partial z_p} h_p(z_p(t - \tau)) + x_n g_n(r_n) u + \frac{1}{2} \sum_{p=1}^{n-1} \left\{ \frac{\partial \Phi_n-1(r_n)}{\partial z_p} h_p(z_p(t - \tau)) + x_n g_n(r_n) u + \psi_n-1(r_n) + S_n(r_n(t)) - S_n(r_n(t - \tau)).
$$

(3.37)

Now choose $u(t)$ as

$$
\dot{u}(t) = -\frac{1}{\gamma_n} \frac{1}{2} c_1 + \frac{n+k}{2} \phi_n(r_n)
$$

(3.38)

where

$$
\phi_n(r_n) = \rho_n(r_n) + \frac{n}{2} + \frac{1}{2} \Phi_n-1(r_n-1)
$$

(3.39)

Follow the same analysis as in $k = 2$ case, we obtain

$$
\dot{V}_n(r_n) \leq -c_1 \sum_{p=1}^{n-1} z^2_p.
$$

(3.40)

Clearly, (3.40) implies that the variables $z_t$ are exponentially stable. Now all we need to do is to show that
system $\Sigma$ is asymptotically stable or $x_i$ are asymptotically stable. From the transformation defined in (2.1), we have the following relationship:

\[
\begin{align*}
|x_1| &= |z_1| \\
|x_i| &= |z_i + \varphi_i(r_i)| \\
&\leq |z_i| + |\varphi_i(r_i)|, \quad \text{for } i = 2, 3, \ldots, n
\end{align*}
\]

(3.41)

(3.42)

Note that $\varphi_i(0)$ are zero because of $\varphi_i(0) = 0$ and $\varphi_i(0) = 0$. Clearly from (3.42), we deduce that $x_i$ are asymptotically stable.

4. CONCLUSIONS

In this paper, the problem of robust stabilization of time delay nonlinear system with a triangular structure has been addressed. An iterative procedure with similar to backstepping procedure has been proposed to construct a robust control which stabilizes this class of time-delay nonlinear systems.

REFERENCES


