Performance evaluation of a multiple-cell CDMA radio system

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ABSTRACT

A statistical method for evaluating the performance measures of the base to mobile link of a multiple-cell CDMA system is presented. Specifically, techniques and expressions for estimating the short term average bit-error-rate (BER), service reliability and link availability are developed. The link performance is estimated for a mobile at the vertex of multiple adjacent cells. In the analysis, the system is assumed to employ coherent BPSK modulation and direct sequence spreading and the received signals are assumed to undergo Rayleigh fading, log-normal shadowing and frequency selective fading. The effects of power control and error correction are also investigated.

I. INTRODUCTION

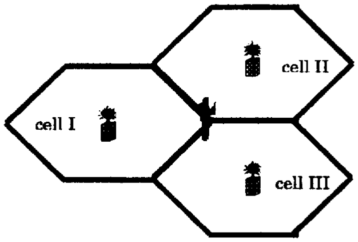
In this era of communications revolution, the demand for mobile radio services is growing rapidly. However, there is only a limited range of spectrum allocated for mobile radio system use. This leaves system planners with the problem of accommodating the rapidly growing number of new users in the limited spectrum. Code division multiple access (CDMA) is a radio communications technique well known for its ability to allow multiple users to simultaneously use the same band. CDMA offers the promise of higher system capacities which may be required to support the number of users that personal communications networks (PCNs) may attract [1,2].

In the past many papers have analysed the performance of single cell CDMA systems. This is a useful and necessary step in the analysis of a complete CDMA system, as these analyses give expressions for capacity and system performance measures under a variety of conditions. However, in order to evaluate the performance of a complete cellular system, the analyses performed for single cells have to be extended to multiple cells. This paper analyses the performance of the base-to-mobile link of a multiple-cell CDMA system.

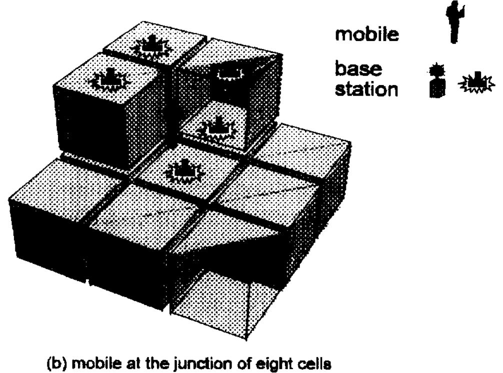
Section II describes the system analysed in this paper and outlines the derivation of the probability density functions (PDFs) of the signal variables. Appropriate system performance measures are defined in Section III. The role of error correction and power control are outlined in Section IV and the results from the analyses are presented in Section V. Finally, the conclusions drawn from this study are summarised in section VI.

II. SYSTEM MODEL

The quality of radio reception by a mobile at the junction of adjacent cells in a CDMA cellular system is estimated in this analysis, which is applicable to both two and three dimensional cellular layouts and is independent of the cell shape. Figures 1(a) and 1(b) show a mobile at the junction of two-dimensional and three-dimensional cell layouts, respectively.



(a) mobile at the junction of three cells in a two dimensional cell layout



(b) mobile at the junction of eight cells in a three dimensional cell layout

Figure 1. Mobile at the junction of multiple cells

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The signal from the base station received at the mobile is assumed to suffer Rayleigh fading, log-normal shadowing and frequency selective fading [3,4]. In some cases the wide band nature of CDMA channels may result in fading that is not Rayleigh distributed but is actually less severe. However in [5] it is reported that signals occupying a bandwidth of 1MHz can suffer Rayleigh like fades exceeding 10-30dB, therefore for the purpose of this study it is reasonable to model the signal variability by a Rayleigh distribution and consider this as a "worst" case. The signals from the different base stations are assumed to be asynchronous and to fade independently of each other. All the signals (including those intended for other users in the same cell) transmitted to by a particular base station occupy the same frequency spectrum and propagate through the same multipath channel to arrive at a mobile. Therefore the CDMA signals from a particular base station, arriving at a given mobile, fade in unison. The frequency selectivity of the channel is modelled by the correlation bandwidth of the channel. The time delay corresponding to the correlation bandwidth is measured as a multiple of the chip period T_c [4].

All the users in the system under analysis are assumed to employ coherent BPSK modulation and direct-sequence (DS) spreading [6]. All the base stations and mobile antennas are assumed to be omnidirectional. The receiver employed in this CDMA system is assumed to reject all but one of the multipath components of the desired signal [4]. Referring to Fig 1(a) it is assumed that the desired user is connected to the base station in cell I and all the other base stations act as interferers. For simplicity it is assumed that there are *K+1* users in cell I and *K* users in the other interfering cells (i.e. there are *K* interferers in each cell). The mobile at the junction of adjacent cells will experience interference from a number of cells, but the effect of the interference from the **adjacent** cells will dominate. Therefore in this analysis only interference from the adjacent cells is considered.

For a large number of users the combined interference can be approximated by a Gaussian random variable [4]. When the user is at the junction of adjacent cells the number of interferers is generally large enough to make this approximation valid. Having approximated the total interference, the momentary bit-error-rate (BER) can be approximated by the complementary error function of the signal-to-interference ratio (SIR) [6]. The SIR at the junction of *L* adjacent cells is given by [4,6]

$$SIR(\alpha_1, \mathrm{X}) = \frac{\alpha_1}{\sqrt{\mathrm{X}}}, \qquad \text{...(1)}$$

where

$$\mathrm{X} = a(K)\beta\delta\alpha_1{}^2 + b(K)\beta\delta\left(\alpha_2{}^2 \ldots + \alpha_L^2\right) + \frac{\mathrm{N}_0}{2E}. \qquad \text{...(2)}$$

In eqn. (2) *a(K)* represents the self interference from the desired base station. This comprises of unwanted multipath components of the desired user's signal and the interference from signals intended for other users in the same cell. *b(K)* represents the interference from users in the other cells. $\mathrm{N}_0/2$ is the spectral density of the double sided additive white Gaussian noise (AWGN). E is the energy per bit. α_1 is the signal strength received from base station I, α_2 is the signal strength received from base station II, etc. It is assumed that $\alpha_1, \alpha_2, \ldots \alpha_L$ are each Rayleigh distributed, and the means of the Rayleigh distributions are assumed to be log-normally distributed. β is the voice activity factor, and δ represents the interference reduction factor if power control is used. The momentary (BER), $P(e|\alpha_1, \mathrm{X})$, is given by [6]

$$P(e|\alpha_1, \mathrm{X}) \approx \mathbf{erfc}\left[\frac{\alpha_1}{\sqrt{\mathrm{X}}}\right]. \qquad \text{...(3)}$$

If $\gamma = \left(\alpha_2{}^2 + \alpha_3{}^2 + \ldots \alpha_L^2\right)\alpha_1{}^{-2}$ is substituted into eqn. (3) then the expression for the momentary BER reduces to

$$P(e|\gamma) \approx \mathbf{erfc}\left[\frac{1}{\sqrt{a(K)\beta\delta + b(K)\beta\delta\gamma}}\right]. \qquad \text{...(4)}$$

It is assumed that $\mathrm{N}_0/2E$ is small and is therefore ignored in eqn. (4).

In order to determine the actual probability of error, the PDF of γ must first be determined. This can be obtained by evaluating the PDF of $\alpha_2{}^2 + \alpha_3{}^2 + \ldots \alpha_L^2$ (using Laplace transforms) and then evaluating the PDF of $\left(\alpha_2{}^2 + \alpha_3{}^2 + \ldots \alpha_L^2\right)\alpha_1{}^{-2}$ (using Mellin convolution) [7]. The PDF of α_i is given by [3]

$$\mathrm{PDF}(\alpha_i) = \frac{2\alpha_i}{\Gamma_i}\mathbf{exp}\left[-\frac{\alpha_i{}^2}{\Gamma_i}\right], \qquad \text{...(5)}$$

where Γ_i is the mean-square value of α_i and is assumed to be log-normally distributed, that is [3]

$$\mathrm{PDF}(\Gamma_i) = \frac{\Omega}{\sqrt{2\pi}\sigma\Gamma_i}\mathbf{exp}\left[-\frac{\left(\Omega\,\mathbf{ln}\left(\frac{\pi}{4}\Gamma_i\right) - m\right)^2}{2\sigma^2}\right], \qquad \text{...(6)}$$

where m is the area mean (in dBm), σ is the standard deviation of the signal variability (in dB) and $\Omega = \frac{10}{\ln(10)}$. The PDF of γ has thus been determined to be

$$PDF_{\gamma}(\gamma) = \sum_{i=1}^{L-1} \frac{\gamma^{L-2} \Lambda_i}{(\gamma + \Lambda_i)^2} \prod_{\substack{j=1 \\ j \neq i}}^{L-1} \frac{1}{(\gamma + \Lambda_j)}, \qquad \text{...(7)}$$

where $\Lambda_1, \Lambda_2, \ldots$and Λ_{L-1} are the ratios of the mean square values of the Rayleigh variables, namely $\Lambda_1 = \Gamma_2 / \Gamma_1$, $\Lambda_2 = \Gamma_3 / \Gamma_1$, ... and $\Lambda_{L-1} = \Gamma_L / \Gamma_1$. The PDFs of Λ_i can be derived using Mellin convolution [7] and are given by

$$PDF_{\Lambda_i}(\Lambda_i) = \frac{\Omega}{2\sqrt{\pi}\sigma\Lambda_i} \exp\left[-\left[\frac{\Omega \ln(\Lambda_i)}{2\sigma} \right]^2 \right]. \qquad \text{...(8)}$$

III. ESTIMATION OF SYSTEM PERFORMANCE MEASURES

SHORT-TERM AVERAGE BIT-ERROR-RATE

The short-term average BER as a function of the number of users, $P_{av}(K)$, is defined as the momentary BER averaged over the Rayleigh fading and is given by

$$P_{av}(K) = \int_0^{\infty} \operatorname{erfc}\left[\frac{1}{\sqrt{a(K)\beta\delta + b(K)\beta\delta\gamma}} \right] \times PDF_{\gamma}(\gamma) d\gamma. \qquad \text{...(9)}$$

It is also useful to plot short-term average BER as a function of short-term average SIR. The short-term average SIR, $SIR_{av}(K)$, is defined as the momentary SIR averaged over the Rayleigh fading and can be evaluated from

$$SIR_{av}(K) = \int_0^{\infty} \frac{1}{\sqrt{a(K)\beta\delta + b(K)\beta\delta\gamma}} \times PDF_{\gamma}(\gamma) d\gamma. \qquad \text{...(10)}$$

SERVICE RELIABILITY

The service reliability $P_{ser}(K)$, is defined as the percentage of time the momentary BER is below a maximum desired level, $\mathrm{BER}_{\max}$, i.e. $P_{ser}(K) = \mathrm{Prob}\,(P(e|\gamma) < \mathrm{BER}_{\max})$. For K users per cell the service reliability can also be expressed as

$$P_{ser}(K) = \mathrm{Prob}\,(SIR_K > SIR_0), \qquad \text{...(11)}$$

where SIR_0 is given by $\operatorname{erfc}(\mathrm{SIR}_0) = \mathrm{BER}_{\max}$.

Given that

$$\mathrm{SIR} = \frac{1}{\sqrt{a(K)\beta\delta + b(K)\beta\delta\gamma}}, \qquad \text{...(12)}$$

for a given value of SIR_0 a corresponding value of γ_0 can be found. Hence the service reliability can be estimated by evaluating the probability that $\gamma < \gamma_0$ averaged over the log-normal shadowing, i.e.

$$P_{ser}(K) = \int_0^{\infty} \cdots \int_0^{\infty} \int_0^{\gamma_0} PDF_{\gamma}(\gamma) \cdot PDF_{\Lambda_1}(\Lambda_1) \cdot \ldots \cdot PDF_{\Lambda_{L-1}}(\Lambda_{L-1}) d\gamma d\Lambda_1 \ldots d\Lambda_{L-1}. \qquad \text{...(13)}$$

LINK AVAILABILITY

The link availability, $P_{lnk\_av}(K)$, is defined as the percentage of locations that the short-term average BER is below a maximum desired level $\mathrm{BER}_{\text{short}}$. The link availability can be calculated by integrating the PDFs of Λ_1, Λ_2, ...and Λ_{L-1} over all possible values for which the short-term average BER (defined in eqn. (9)) is below the threshold $\mathrm{BER}_{\text{short}}$ i.e.

$$P_{lnk\_av}(K) = \int_0^{\Lambda'_1} \int_0^{\Lambda'_2(\Lambda_1)} \cdots \int_0^{\Lambda'_{L-1}(\Lambda_1 \ldots \Lambda_{L-2})} PDF_{\Lambda_{L-1}}(\Lambda_{L-1}) \cdot \ldots \cdot PDF_{\Lambda_2}(\Lambda_2) \cdot PDF_{\Lambda_1}(\Lambda_1) d\Lambda_{L-1} \ldots d\Lambda_1, \qquad \text{...(14)}$$

where $\Lambda'_1, \Lambda'_2(\Lambda_1), \ldots, \Lambda'_{L-1}(\Lambda_1 \ldots \Lambda_{L-2})$, are values of $\Lambda_1, \Lambda_2, \ldots\ldots$and Λ_{L-1} respectively for which the short-term BER is below the threshold $\mathrm{BER}_{\text{short}}$. It should be noted that the values of $\Lambda'_2(\Lambda_1), \ldots, \Lambda'_{L-1}(\Lambda_1 \ldots \Lambda_{L-2})$, are functions of all the variables of the outer integrals in eqn. (14).

IV. THE ROLE OF ERROR CORRECTION AND POWER CONTROL

If we assume an *(l,k)* block code (i.e. for every k data bits l-k error correcting bits are added) capable of correcting all combinations of c and fewer errors, then the average BER, P_b, can be approximated by [8]

$$P_b \approx \frac{1}{l}\sum_{i=c+1}^{l} i\binom{l}{i}P_e^i(1-P_e)^{l-i}, \qquad ...(15)$$

where P_e is the corresponding BER in the absence of error correction. If the data is to be transmitted at a particular rate then as error correcting codes are added to the data the band-width occupied by the composite (base-band signal) code increases. If only a limited bandwidth is available for transmission, then the processing gain has to be reduced to compensate for the greater bandwidth of the base-band signal.

Power control is an essential requirement of CDMA systems [9]. A simple power control algorithm that could be implemented requires that the power transmitted from the base station be proportional to the distance between the base station and the user, raised to the power of the path loss exponent [9]. In general the users can be assumed to be uniformly distributed within a cell. The PDF of the distribution of the mobile at a radius, r, from the base station is given by

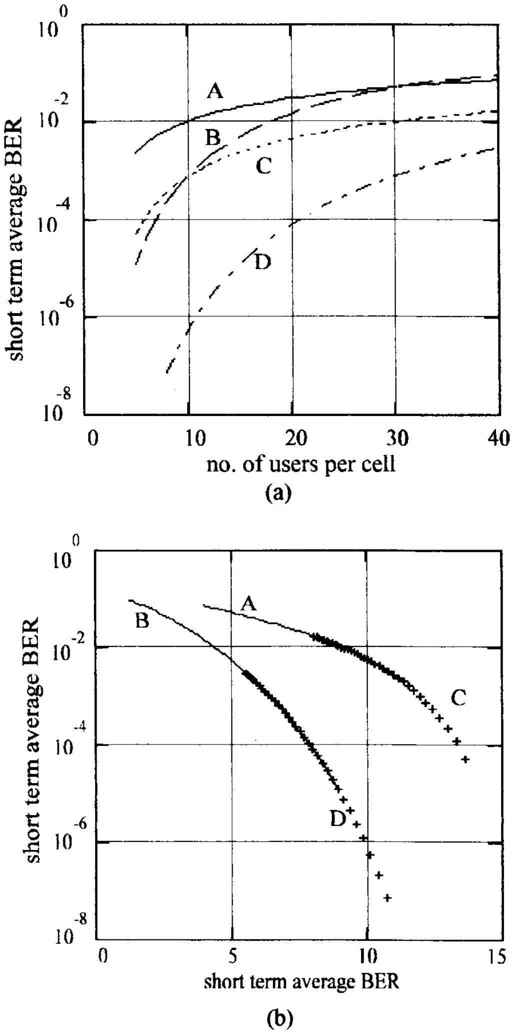
$$p_r(r) = \frac{2r}{R^2}, \qquad ...(16)$$

where R is the radius of the cell. Thus the power control factor δ is given by

$$\delta = \int_0^R \left[\frac{r}{R}\right]^n \frac{2r}{R^2}dr = \frac{2}{n+2}. \qquad ...(17)$$

V. RESULTS

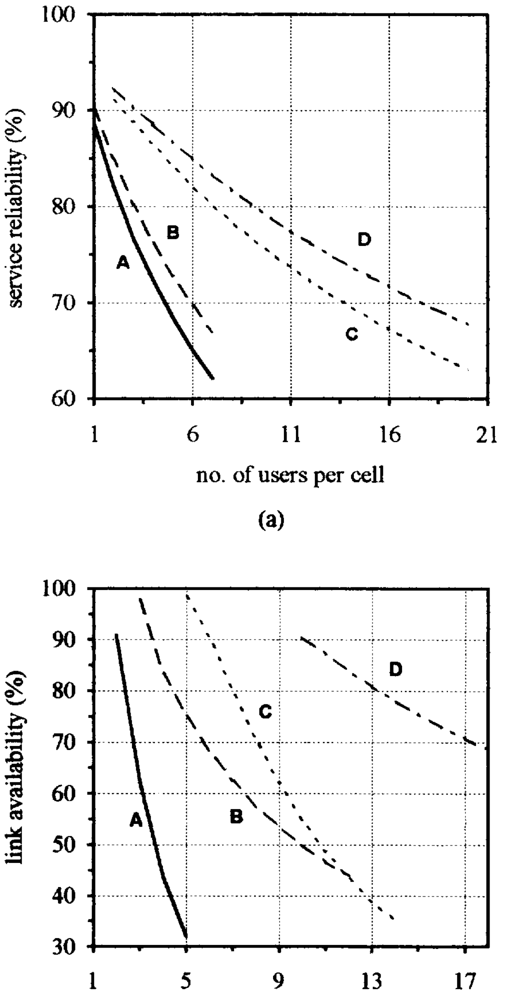
A simple three cell system has been considered (as shown in Fig. 1(a)) for this paper and the results presented in this section investigate the impact of error correction and power control on system performance. A (23,12) Golay code for error correction is assumed, for which $c = 3$. In order to maintain the same transmission bandwidth, the error corrected data is assumed to have a processing gain of 66 and the uncorrected data a processing gain of 128. A voice activity factor of 0.5 and a path loss exponent of 4 have been used in this analysis. Figure 2 presents the short term average BER as a function of the number of users per cell and as a function of the short-term average SIR.



A - with no error correction and no power control
B - with error correction and no power control
C - with power control and no error correction
D - with error correction and power control

Figure 2. Short term average BER as
(a) a function of no. of users per cell
(b) and short term average SIR

A BER threshold of 10^{-3} and σ values of 6dB were used in the estimation of service reliability and link availability. Figure 3 presents the service reliability and link availability as functions of the number of users of users per cell.



(a)

(b)

A, B, C, and D as in Figure 2.

Figure 3. (a) Service reliability
(b) link availability as a function of no. of users per cell

VI. CONCLUSIONS

From the results presented in this paper the following observations are made :

1. From Fig. 2(a) it can be seen that for a small number of users the error-corrected system performs better than the system with no error correction, but as the number of users increases, the situation reverses. (Note that both the systems occupy the same spread bandwidth).
2. For a path-loss exponent of 4, power control increases the capacity by approximately three fold.
3. The combined effect of power control and error correction increase the capacity by 200-400% depending on the performance measure used in determining the maximum allowable number of users.

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REFERENCES

[1] R. L. Pickholtz, L. B. Milstein and D. L. Schilling, "Spread spectrum for mobile communication," IEEE Trans. Vehic. Technol., pp. 313-322, May 1991.

[2] D G Smith, "Spread spectrum for wireless phone systems: the subtle interplay between technology and regulation," *IEEE Commun. Mag.*, pp. 44-46, February 1991.

[3] K W Sowerby and A G Williamson "Outage probability calculations for a mobile radio systems with multiple interferers," *Electron. Lett.*, vol. 24, No. 24, Nov. 1988, pp. 1511-1513.

[4] D E Borth and M B Pursley, "Analysis of direct-sequence spread-spectrum multiple-access communication over Rician fading channels," *IEEE Trans. Commun.*, vol. COM-27, pp. 1566-1577, Oct. 1979.

[5] D L Schilling et. al, "Broadband CDMA for personal communications system," *IEEE Commun. Mag.*, pp. 86-93, Nov. 1991.

[6] L B Milstein, T S Rappaport and R Barghouti, "Performance evaluation for cellular CDMA," *IEEE J Select. Areas Commun.*, vol 10, pp. 680-689, May 1992.

[7] M D Springer, "The Algebra of Random Variables," Wiley, New York, 1979.

[8] D J Torrieri, "The information -bit error rate for block codes," *IEEE Trans. Commun.*, vol. COM-32, pp. 474-476, April. 1984.

[9] R R Gejji, "Forward-link-power control in CDMA cellular systems," *IEEE Trans. Veh. Tech.*, vol. 41, pp. 532-536, Nov. 1992.