Modelling Myocardial Excitation Wavefront Propagation in Ventricles by Finite Element Solution of an Eikonal Equation

K. A. Tomlinson, A. J. Pullan, P. J. Hunter.
Department of Engineering Science, The University of Auckland, New Zealand.
email: k.tomlinson@auckland.ac.nz.

Abstract—The spreading of excitation in ventricular myocardium is modelled by treating the thin region of rapidly depolarizing tissue as a propagating wavefront. The model determines tissue excitation time using an eikonal equation that includes the effects of wavefront orientation in the myocardial structure and wavefront curvature.

Use of a Petrov–Galerkin finite element method with a no-inflow boundary condition enables the eikonal equation to be solved on reasonably coarse meshes of cubic Hermite elements. The method is applied successfully on a model of the complete canine ventricular myocardium.

I. INTRODUCTION

One application for a model of the spreading of depolarization in myocardium is in the determining of the activity and condition of the heart from electrical potentials recorded on the body surface. This inverse problem is ill-conditioned so regularization is necessary when attempting to reconstruct epicardial potentials. Unfortunately this usually involves constraining epicardial potentials to be smooth which is not physically realistic. Using instead a model of the heart's activity would allow application of physically realistic constraints and determination of more physiologically useful parameters such as tissue conduction velocities or initial excitation times.

Modelling the complex interaction of ions between connected myocardial cells and the surrounding tissue and fluid is very computationally demanding due to the small scale on which this activity takes place. Most of the electrical activity during depolarization, however, occurs in a thin region of tissue. Often it is sufficient to model the propagation of this thin wavefront of activity. Far field potentials, for example, are not so much affected by the individual currents as by the dramatic change in potential across the wavefront. If the motion of this excitation wavefront is known, it provides a very good indication of the function of the myocardium.

II. MODEL DETAILS

Solving for the position of the wavefront at each point in time is difficult due to the need for an evolving mesh, so a solution is obtained for the excitation time $u(x)$, defined as the time at which the wavefront passes the point x. The position of the wavefront at any time t is then given by the surface $u(x) = t$.

The governing equation is the eikonal equation,

$$c_0\sqrt{\nabla u \cdot D\nabla u} - \nabla \cdot (D\nabla u) = \tau_m,$$

where τ_m is the membrane time constant, D is a tensor representing the square of the space constant in each direction and c_0 is a dimensionless constant dependent on active currents. This can be derived from the reaction-diffusion system for a bidomain model [1]. The first term is an advection term based on the reciprocal of the propagation speed. The second term is a diffusion term which includes curvature effects.

A finite element method is used to solve the equation numerically. The influence of the diffusion term is small, so oscillations corrupt the solution if simple Galerkin weighting functions are used. Galerkin weights are therefore supplemented with their derivatives in the direction of propagation. The small influence of diffusion also means that the natural no-flux boundary condition is only weakly enforced, allowing propagation to initiate at arbitrary points on the boundary. An additional no-inflow term is included in the finite element equations to prevent this.

As the eikonal equation is non-linear, a continuation method introducing the non-linear term gradually is used to provide starting points for Newton iterations.

III. RESULTS

The method was used to run simulations on a 180 cubic Hermite element mesh of canine ventricular myocardium based on measured geometry and fibre and sheet structure [2]. A snapshot of the excitation wavefront calculated in one simulation is is shown in figure 1.

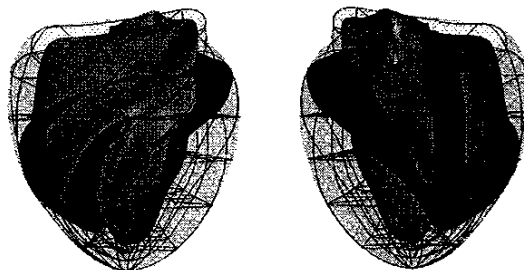Fig. 1. A wavefront 100 ms after LV free wall epicardial stimulus.

REFERENCES

[1] P Colli Franzone and L Guerri, "Models of the spreading of excitation in myocardial tissue," *Crit. Rev. Biomed. Eng.*, vol. 20, pp. 211, 1992.
[2] I J Le Grice, B H Smaill, L Z Chai, S G Edgar, J B Gavin, and P J Hunter, "Laminar structure of the heart I: Cellular organisation and connective tissue architecture in ventricular myocardium," *Am. J. Physiol.*, 1995.