

HIGH RESOLUTION IMAGE RECONSTRUCTION USING MEAN FIELD ANNEALING

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Abstract: A high resolution image can be reconstructed from a sequence of lower resolution frames of the same scene where each frame taken by the camera is offset by a subpixel displacement. In this paper, it is shown that such a reconstruction task can be cast as an optimisation problem, and that a reconstruction can be found using the mean field annealing algorithm. The proposed technique has the added advantage over existing techniques of not requiring the registration of the displacement of each low resolution frame. In addition, the proposed technique greatly reduces the required computation as compared to a simulated annealing approach.

INTRODUCTION

Many image processing applications, such as satellite remote sensing, industrial quality control and scientific or medical imaging, require a high resolution image in which the use of commercial video camera seems rather limiting. Increasing the resolution requires an increase in the sampling rate, and thus its implementation by sensor modification is usually undesirable. Therefore, attention has turned to obtaining higher resolution images using signal processing techniques instead. One promising approach is to reconstruct a high resolution still-frame image from a sequence of lower resolution frames of the same scene where each frame taken by the camera is offset by a subpixel displacement. This reconstruction problem has been addressed by several researchers, and various reconstruction techniques have also been proposed [1-7].

In spite of their apparent variety, the existing techniques have a common structure and contemporary reconstruction procedures usually consist of two main parts; the registration phase and the reconstruction phase. In practice, the best possible reconstruction quality is, however, unlikely to be obtained due to the limitation of currently available registration and reconstruction methods. Indeed, undersampled images include aliased frequency components

which cause errors in the registration phase. On the other hand, the accuracy of this estimation influences the reconstruction quality, since most existing reconstruction methods are based on the assumption that the displacements are correctly estimated [2, 5, 6].

In an effort to resolve these difficulties, the authors demonstrated [8] that the high resolution image reconstruction task can be recast as an optimisation problem in which the registration and reconstruction phases can be performed simultaneously, and that a solution can be found using the simulated annealing algorithm. Nevertheless, this algorithm is still not suitable for real-world images due to its large computational effort.

In this paper, the authors have applied the mean field annealing algorithm [9, 10] to the high resolution image reconstruction problem. By using this new approach, the reconstruction can still be achieved without requiring a separate registration phase, but with much less computational effort compared with the simulated annealing algorithm.

AN OPTIMISATION APPROACH FOR HIGH RESOLUTION IMAGE RECONSTRUCTION

The concepts involved in reconstructing a high resolution image from multiple low resolution images may be elucidated by considering the process of obtaining a low resolution image, $g(m, n)$, from a higher resolution image, $f(k, l)$, as illustrated in Fig. 1, in which the relationship between the image pixels $g(m, n)$ and $f(k, l)$ can be expressed as [4]:

$$g(m, n) = \sum_{k=0}^{K-1} \sum_{l=0}^{L-1} f(k, l)h(m, n; k, l), \quad (1)$$

where

$$h(m, n; k, l) = \frac{\mathcal{A}\{S_h(k, l) \cap S_l(m, n)\}}{\mathcal{A}\{S_l(m, n)\}}, \quad (2)$$

denotes the point spread function (PSF), \mathcal{A} denotes the area of its argument, and $S_h(., .)$ and $S_l(., .)$ denote the support of the high resolution and low resolution sensors centred around the pixel $(., .)$, respectively.

Supposing that the PSF of the imaging system is known, one can obtain a number of low resolution images, $\hat{g}_i(m, n)$, from an estimated high resolution image, $\hat{f}(k, l)$, using such an imaging process. If $\hat{f}(k, l)$ is identical to the correct high resolution image, then the estimated image $\hat{g}(m, n)$ should be identical to the given image $g(m, n)$. This hypothesis can also be applied to the case of multiple low resolution images of the same scene where each frame taken by the camera is shifted by a subpixel displacement. In the latter case, each low resolution frame has different PSF, $h_i(m, n; k, l)$, which can be determined from its corresponding displacement. In addition, if these displacements are unknown, then the estimated image $\hat{g}(m, n)$ will be identical

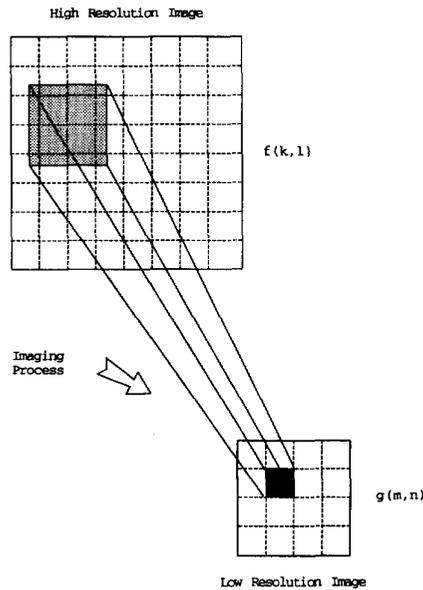


Figure 1: A schematic diagram simulating a process of obtaining a low resolution image.

to the given image $g(m, n)$ only if both the high resolution image and the displacements are correctly estimated.

Therefore, the high resolution image reconstruction task can be cast so as to find the *maximum a posteriori* (MAP) estimate of the high resolution image and the displacements when a sequence of low resolution images is given. That is

Estimate \mathbf{f} and \mathbf{d} such that
 $P(\mathbf{f}, \mathbf{d} | \mathbf{G})$
 is maximized

where \mathbf{f} denotes a high resolution image, \mathbf{G} denotes a sequence of low resolution images, and \mathbf{d} denotes a sequence of displacements corresponding to each low resolution image, that is

$$\mathbf{d} = \{\delta_{xi}, \delta_{yi}; i = 1, \dots, P\},$$

where P denotes the number of available images, and δ_{xi} and δ_{yi} are the displacements of the i^{th} low resolution image along the x and y direction respectively.

In general, an image can be modeled as a Markov random field (MRF), and the posterior probability can be described by the Gibbs distribution as follows [11]:

$$P(\mathbf{f}) = \frac{1}{Z} \exp \left[-\frac{U(\mathbf{f})}{T} \right], \quad (3)$$

where Z is the normalization constant (also called partition function), and $U(\mathbf{f})$ is the energy function of the form

$$U(\mathbf{f}) = \sum_c V_c(\mathbf{f}), \quad (4)$$

c being the set of cliques associated with the neighbourhood. Thus it can be shown that [12] the MAP can be given as:

$$P(\mathbf{f}, \mathbf{d} | \mathbf{G}) = \frac{1}{Z} \exp \left[-\frac{U(\mathbf{f}, \mathbf{d} | \mathbf{G})}{T} \right]. \quad (5)$$

Moreover, the energy function is given as:

$$U(\mathbf{f}, \mathbf{d} | \mathbf{G}) = c \cdot \left[\sum_{i=1}^P \sum_m \sum_n \left| g_i(m, n) - \sum_k \sum_l \hat{f}(k, l) \hat{h}_i(m, n; k, l) \right|^2 + \lambda \sum_k \sum_l \left\{ \sum_{u \in N_k} \sum_{v \in N_l} \left| \hat{f}(u, v) - \hat{f}(k, l) \right|^2 \right\} \right], \quad (6)$$

where $\hat{h}_i(m, n; k, l)$ is the estimated PSF of the i^{th} low resolution image, c is a constant and λ is the regularising parameter, and the partition function is given as

$$Z = \sum_{\{\mathbf{f}, \mathbf{d}\}} \exp \left[-\frac{U(\mathbf{f}, \mathbf{d} | \mathbf{G})}{T} \right], \quad (7)$$

where $\sum_{\{\mathbf{f}, \mathbf{d}\}}$ means the sum over all the possible configurations $\{\mathbf{f}, \mathbf{d}\}$.

In summary, the reconstruction of a high resolution image is the solution that maximises $P(\mathbf{f}, \mathbf{d} | \mathbf{G})$ which coincides with minimising the cost function given in (6). The authors have demonstrated that the solution of this reconstruction problem can be successfully obtained using the simulated annealing algorithm [8]. Using this reconstruction approach, it has the advantage of not requiring a separate registration phase. It is therefore possible to obtain higher resolution images even if the displacements of the low resolution images are unknown.

MEAN FIELD ANNEALING

It is well known that the major disadvantage of simulated annealing is its large computational effort. To avoid this computational burden, this section proposes the mean field annealing algorithm [13] to solve the high resolution image reconstruction problem. Mean field annealing is an optimisation technique which can be derived from two different perspectives: statistical mechanics [9] and information theory [13], and it has been shown to provide good results much faster than simulated annealing [10, 14]. In the field of

image processing, mean field annealing has been employed in several applications, notably image restoration [10, 14, 15], motion estimation [16], image segmentation [17], and etc. [18, 19].

Mean field annealing uses a deterministic approach to find the mean of the Gibbs distribution which is an approximation of the thermal equilibrium distribution of the temperature T . In terms of the high resolution image reconstruction problem, the mean of image pixel $f(k, l)$ can be given as:

$$\begin{aligned}\langle f(k, l) \rangle &= \sum_{\{f, d\}} f(k, l) P(f, d | G) \\ &= \frac{1}{Z} \sum_{\{f, d\}} f(k, l) \exp \left[-\frac{U(f, d | G)}{T} \right].\end{aligned}\quad (8)$$

It can be seen that the calculation of the above equation is not possible, or at least infeasible, since it involves interaction between all the possible configurations. The mean field theory suggests an approximation of (8) by the assumption that the mean of the field $f(k, l)$ can be updated by the mean values of its neighbours, and the mean value can also be approximated by its local energy, that is

$$\langle f(k, l) \rangle = \frac{1}{Z_{kl}} \sum_{f(k, l) \in R_D} f(k, l) \exp \left[-\frac{U(f_{kl}, \langle d \rangle | G)}{T} \right].\quad (9)$$

The terms $U(f_{kl}, \langle d \rangle | G)$ and Z_{kl} are called the mean field local energy and local partition function at pixel (k, l) , respectively. These can be written as

$$\begin{aligned}U(f_{kl}, \langle d \rangle | G) &= c \cdot \left[\sum_{i=1}^P \sum_{m \in Y_k} \sum_{n \in Y_l} \left| g_i(m, n) - \sum_u \sum_v \langle \hat{f}(u, v) \rangle \langle \hat{h}_i(m, n; u, v) \rangle \right|^2 + \right. \\ &\quad \left. \lambda \left\{ \sum_{u \in N_k} \sum_{v \in N_l} \left| \hat{f}(u, v) - \hat{f}(k, l) \right|^2 \right\} \right],\end{aligned}\quad (10)$$

and

$$Z_{kl} = \sum_{f(k, l) \in R_D} \exp \left[-\frac{U(f_{kl}, \langle d \rangle | G)}{T} \right].\quad (11)$$

Note that, $m \in Y_k$ and $n \in Y_l$ mean a low resolution pixel (m, n) which is influenced by a high resolution pixel (k, l) .

The mean of the displacement δ_{xi} can be approximated by:

$$\langle \delta_{xi} \rangle = \frac{1}{Z_d} \sum_{\delta_{xi} \in R_D} \delta_{xi} \exp \left[-\frac{U(\langle f \rangle, \delta_{xi} | G)}{T} \right],\quad (12)$$

where the mean field local energy and local partition function are defined as

$$U(\langle \mathbf{f} \rangle, \delta_{xi} | \mathbf{G}) = c \cdot \left[\sum_m \sum_n \left| g_i(m, n) - \sum_u \sum_v \langle \hat{f}(u, v) \rangle \langle \hat{h}_i(m, n; u, v) \rangle \right|^2 \right], \quad (13)$$

and

$$Z_d = \sum_{\delta_{xi} \in R_D} \delta_{xi} \exp \left[-\frac{U(\langle \mathbf{f} \rangle, \delta_{xi} | \mathbf{G})}{T} \right], \quad (14)$$

respectively. The mean of the displacement δ_{yi} can be approximated in a similar manner.

In addition, the image intensities and the displacements are continuous values which implies that the summations $\sum_{f(k,l) \in R_D}$, $\sum_{\delta_{xi} \in R_D}$ and $\sum_{\delta_{yi} \in R_D}$ may be replaced by integral equivalents.

By using the above approximations, the equilibrium state at each temperature T can be obtained through the mean field. As the temperature $T \rightarrow 0$, this approximation will coincide with the exact distribution, and the image \mathbf{f} will be equal to $\langle \mathbf{f} \rangle$. In summary, the mean field annealing algorithm for high resolution image reconstruction can be stated as:

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 $T \leftarrow$  initial temperature
while ( $T > T_{\min}$ )
  do until (a steady state is reached)
    for all image pixels
      Calculate  $U(f_{kl}, \langle \mathbf{d} \rangle | \mathbf{G})$ 
      Calculate the mean  $\langle f(k, l) \rangle$ .
    end
    for all image displacements
      Calculate  $U(\langle \mathbf{f} \rangle, \delta_{xi} | \mathbf{G})$  and  $U(\langle \mathbf{f} \rangle, \delta_{yi} | \mathbf{G})$ 
      Calculate the mean  $\langle \delta_{xi} \rangle$ , and alternately  $\langle \delta_{yi} \rangle$ .
    end
  end
  Decrease  $T$ 
end

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EXPERIMENTAL RESULTS

In order to evaluate the performance of the mean field annealing algorithm, a picture of characters with different sizes was imaged by a *charge-coupled device* (CCD) camera. The pixel size in each digital image was measured to be 3.06 and 2.19 mm along the x and y directions, respectively. The picture was placed on a mechanical device which can be shifted on both directions with an accuracy of 0.1 mm. By shifting this mechanical device, 16 low resolution

images of size 64×64 pixels were taken with different displacements, and Fig. 2a illustrates one of the low resolution images.

Mean field annealing was applied to the low resolution images in order to improve the resolution. The algorithm was implemented with $T_0 = 150$, $c = 0.001$, $\lambda = 0.0001$, and the temperature was decreased using an exponential rule $T_n = 0.95 \times T_{n-1}$. The initial estimated image was defined as the constant pattern. In addition, the algorithm was terminated when the temperature was lower than 0.01.

Two higher resolution images were reconstructed, and are shown in Fig. 2c and 2e with resolution increase of two-fold and four-fold, respectively. From the results, it can be seen that invisible detail in the low resolution images becomes clearly apparent in both reconstructions. In terms of the frequency domain, Fig. 2b illustrates the spatial frequency spectrum of the low resolution image shown in Fig. 2a. Whereas Fig. 2d and 2f illustrate the spatial spectrum of the corresponding high resolution images. From these illustrations, it is obvious that some of the distorted high frequency spectrum can be recovered when applied this reconstruction algorithm.

CONCLUSIONS

This paper has considered the problem of increasing image resolution from multiple low resolution images. To avoid the limitations of existing methods, this paper has demonstrated that the high resolution image reconstruction task can be formulated as an optimisation problem, and that a solution can be found using the mean field annealing algorithm. This new technique has the advantage over existing techniques in that it does not require a separate registration phase. It is therefore possible to obtain higher resolution images even if the displacements of the low resolution images are unknown. In addition, the experimental results have demonstrated the success of this new algorithm for real-world images.

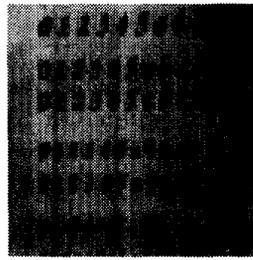
In summary, the contributions of this paper are: (i) to present an optimisation approach for reconstructing a high resolution image, and (ii) to show that the proposed technique can be successfully applied to real-world images.

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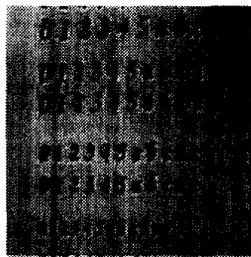
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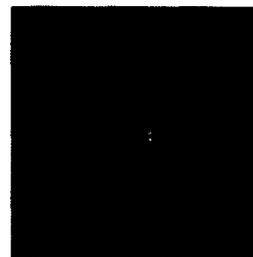
(a)



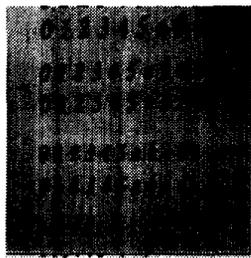
(b)



(c)



(d)



(e)



(f)

Figure 2: (a) A real-world low resolution image of size 64×64 pixels, (b) its corresponding frequency spectrum, (c) a reconstruction of size 128×128 pixels, (d) its corresponding frequency spectrum, (e) a reconstruction of size 256×256 pixels, (f) its corresponding frequency spectrum.

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