

Generalised outage probability and BER estimation using dual protection margins

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Abstract: The design of tomorrow's mobile communications systems will require accurate mathematical tools for estimating the performance of potential system configurations. Ideally these tools will be easy to apply and be based upon parameters that can be estimated by a simple appraisal of the propagation environment. In this paper a general outage probability technique for considering the effect of noise and cochannel interference on reception reliability is presented. The technique is used to assess the compatibility and applicability of previously published techniques that either treat noise as cochannel interference or consider a minimum detectable received signal threshold. A simple yet accurate threshold-based technique is described and used to predict bit error rates (BER) for a variety of conditions. Specifically, outage probability is used to accurately predict average BER.

I. INTRODUCTION

System planners require techniques for estimating the reception reliability of high capacity digital mobile radio systems. These techniques must take into account the many factors that constrain reception reliability, primarily including the effects of background noise, cochannel interfering signals, modulation schemes and power delay spread. Ideally, the variables incorporated in these techniques should be easily predicted from the environment. The great variability in the received signal strength, caused by the constantly changing nature of the mobile radio channel, makes reception reliability estimation a complicated issue. Generally, the more complex the reception reliability model, the more expensive is its implementation in terms of time and computational resources.

Due to the large variability in the radio channel, system designers require a measure of reception reliability that is an indication of the percentage of time that the communications quality is acceptable. *Outage probability*, the probability of failing to achieve adequate signal reception, is such a measure. In general terms, "adequate" reception is achieved when the short-term desired signal strength is

sufficiently greater than both the background noise and the sum of cochannel interfering signal strengths.

Previously published outage probability techniques have treated "noise" in one of two ways. In early outage probability work (e.g. [1]) the effect of noise was included implicitly by setting a minimum reception threshold for the desired signal. This approach was also used in later analyses of various multiple interferer situations [2-4]. Adequate reception is achieved when the desired signal power is stronger than the minimum required signal level of the receiver and, simultaneously, stronger than the sum of the interfering signal powers by a margin known as the *cochannel interference protection ratio*. The general form of these outage probability expressions is

$$P_{out} = 1 - \int_{w_{min}}^{\infty} p_d(w_d) \int_0^{w_d/r_p} p_I(w_{inf}) dw_{inf} dw_d, \quad \dots(1)$$

where w_d is the (momentary) desired signal power, $p_d(w_d)$ is its probability density function and w_{inf} is the momentary power of the sum of all the interfering signals with pdf $p_I(w_{inf})$. The integral limits reflect the requirement that the desired signal power be simultaneously greater than the minimum required signal power, w_{min} , and the sum of the interfering signals by the interference protection ratio, r_p .

The second way to include the effect of noise is to treat the noise as interference [5]. In this paper, this second method is generalised by including *dual interference protection margins*. In this technique the noise is added to the interference in the calculation of outage probability. Separate protection margins (ratios) are used for the "noise" and the cochannel interference, making the expressions more general than previous techniques. A dual margin technique may be useful in situations where the characteristics of the noise are sufficiently different from those of the cochannel interference and may have a different effect on reception. In this case, the receiver may require a different degree of protection from the noise than is required from the cochannel interference, thus warranting the use of different protection margins.

II. DUAL PROTECTION MARGIN OUTAGE PROBABILITY

In *dual protection margin outage probability*, the momentary desired signal power is required to be larger than both the noise power by the *noise power protection margin*, r_n , and the total interference power by the *cochannel interference power margin*, r_p , expressed mathematically as:

$$w_d > r_n N + r_p w_{inf}, \quad \dots(2)$$

where N is the total noise power, and w_{inf} the total cochannel interference power. Now there is a combined requirement on the signal strength that depends on both the noise and the cochannel interference. In other words, the noise is included as interference in the calculation of outage. The general outage probability expression for the dual margin case can be written as:

$$P_{out} = 1 - \int_{r_n N}^{\infty} p_{w_d}(w_d) \int_0^{r_p} p_I(w_{inf}) dw_{inf} dw_d \dots(3)$$

Using a substitution of variables, ($x = w_d - r_n N$), the dual margin outage probability expression can be written in a similar form to the general expression for outage probability given in equation (1), that is

$$P_{out} = 1 - \int_0^{\infty} p_{w_d}(x + r_n N) \int_0^{x/r_p} p_I(w_{inf}) dw_{inf} dx \dots(4)$$

By considering the individual pdf of each cochannel interfering signal, the outage probability expression of equation (4) can be expanded to

$$P_{out}^n = 1 - \int_0^{\infty} p_{w_d}(x + r_n N) \int_0^{x/r_p} p_1(y_1) \int_0^{x/r_p - y_1} p_2(y_2) \dots \int_0^{x/r_p - y_1 - y_2} p_n(y_n) dy_n \dots dy_1 dx \dots(5)$$

where the superscript n in P_{out}^n refers to the number of cochannel interferers and $p_i(y_i)$ is the pdf for the i th interferer. Note that equation (5) is entirely general in that the actual signal strength pdfs have not been specified.

EXAMPLE

To illustrate the differences and similarities between the dual protection margin outage probability expressions and the previous single margin cases, consider the general Rayleigh fading case where the desired signal power is exponentially distributed with a pdf given by

$$p_{w_d}(w_d) = \frac{1}{\Gamma} \exp\left[-\frac{w_d}{\Gamma}\right], \quad \dots(6)$$

where Γ is the mean desired signal power. In this case the outage probability expression can be written as

$$P_{out}^n = 1 - \exp\left[-\frac{r_n N}{\Gamma}\right] \int_0^{\infty} p_{w_d}(x) \int_0^{x/r_p} p_I(w_{inf}) dw_{inf} dx \dots(7)$$

It is worth noting that the double integral in equation (7) is the "cochannel interference only" service reliability expression [6]. For the special case where $r_n = r_p$ and $p_I(w_{inf})$ is the pdf of the sum of a number of exponential distributions, equation (7) reduces to the expression presented by Linnartz [5].

The outage probability calculated for any given scenario is dependent on whether noise is included as interference or a minimum desired signal threshold is used. Essentially the difference in the two cases relates to whether the term $r_n N$ is included in the upper limit of the inner integral in equation (3). When equation (3) is evaluated for the case where there are n exponentially distributed cochannel interfering signals represented by

$$p_i(y_i) = \frac{1}{\Gamma_i} \exp\left[-\frac{y_i}{\Gamma_i}\right] = \frac{\Lambda_i}{\Gamma} \exp\left[-\frac{y_i \Lambda_i}{\Gamma}\right], \quad \dots(8)$$

where $\Lambda_i = \Gamma / \Gamma_i$ is the mean CIR of the desired signal with respect to the i th interferer, the final form of the dual protection margin outage probability expression is

$$P_{out}^n = 1 - \exp\left[-\frac{r_n N}{\Gamma}\right] \prod_{i=1}^n \frac{\Lambda_i}{r_p + \Lambda_i} \dots(9)$$

If the term $r_n N$ in the upper limit of the inner integral is absent from equation (3), the outage probability expression can be shown to reduce to

$$P_{out}^n = \exp\left[-\frac{r_n N}{\Gamma}\right] + \sum_{j=1}^n \frac{a_j}{1 + \frac{\Lambda_j}{r_p}} \exp\left[-\frac{r_n N}{\Gamma} \left[1 + \frac{\Lambda_j}{r_p}\right]\right], \quad \dots(10)$$

$$\text{where } a_j = \prod_{\substack{k=1 \\ k \neq j}}^n \frac{1}{1 - \frac{\Lambda_k}{\Lambda_j}}.$$

The resultant difference in outage probabilities is illustrated in Fig. 1, for which an exponentially distributed desired signal and a single exponentially distributed interfering signal are considered.

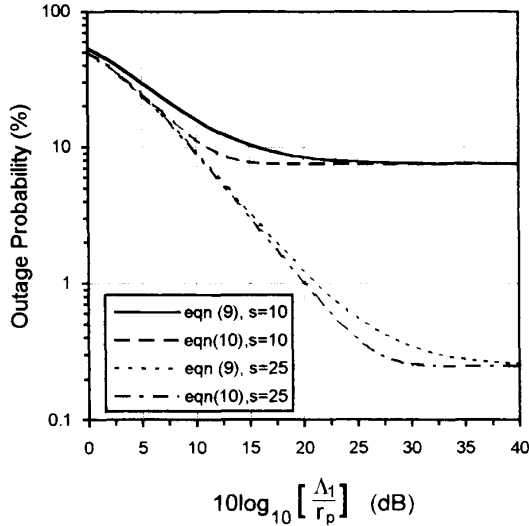


Fig. 1 Plot of outage probability equations (9) and (10) for the case of an exponentially distributed desired signal and one exponentially distributed cochannel interfering signal as a function of CIR for $s = 10$ dB and 25 dB. where $s = -10 \log_{10} [4r_n N \cdot (\Gamma \pi)]$

The accuracy of outage probability expressions can be assessed by comparing expected outage rates to those that occur in the field. However, this paper does not aim to determine the accuracy of such techniques explicitly, but rather, offers an improved, more flexible model for outage probability estimation. The *dual margin outage probability* technique has separate protection ratios for the noise and cochannel interference and therefore has the flexibility to model real situations more precisely than outage probability techniques with a single protection ratio. The generalised outage probability expressions can be applied to a large range of situations by substituting in the most appropriate pdfs for the signal variability.

III AVERAGE BER PREDICTION

In a digital mobile radio system, the average bit error rate (average BER) may be a more appropriate measure of reception reliability than outage probability. However, many average BER expressions are mathematically complex and involve significant computational resources,

thus making them somewhat impractical to use in large scale system planning. There is a need for faster, more efficient techniques for estimating average BER in digital mobile radio systems, especially in situations where cochannel interference is present.

Outage probability expressions may be ideal for estimating average BER since they are mathematically less complex than their counterpart average BER expressions. Also, in general terms, outage probability and average BER are highly correlated. A high average BER corresponds to a high probability of outage, while a small average BER relates to a low outage probability. This section investigates this relationship between outage probability and average BER - or more precisely, using the former to approximate the later.

When no cochannel interference is present, only the characteristics of the desired signal need to be considered in determining the average BER. At a particular value of signal to noise ratio (SNR), γ , the BER is given by $P_e(\gamma)$. The variability of the desired signal, $p(\gamma)$, might be modelled using, say, an exponential, Rician or gamma distribution. If the SNR is the only factor influencing the BER, then the average BER, BER_{av} , is given by

$$BER_{av} = \int_0^{\infty} P_e(\gamma) p(\gamma) d\gamma. \quad \dots(11)$$

The relationship between the momentary BER and the momentary SNR is dependent on the type of digital modulation scheme used, and therefore $P_e(\gamma)$ is different for each modulation scheme [7].

The calculation of the average BER where cochannel interference is present is more complicated than in the interference-free case because the distributions of the interfering signals must also be considered. There are three basic steps involved in calculating the average BER when multiple interfering signals are included.

Firstly, a pdf for the total interference power from n interferers is determined. An expression for the signal to interference ratio (SIR) is then developed using this pdf. Finally, the average BER expression is determined using equation (11), but where $p(\gamma)$ is now the distribution of the SIR rather than the SNR.

For example, using this methodology it can be shown that the average BER expression for the case of DPSK modulation assuming an exponential desired signal and I independent but otherwise identically distributed

exponential interfering signals (with the same mean power) is [7]

$$BER_{av} = \frac{1}{2(I-1)!} \sum_{k=1}^I (k-1)! (-\Lambda)^{I-k} \dots - \frac{(-\Lambda)^I}{2(I-1)!} \exp(\Lambda) Ei(-\Lambda), \quad \dots(12)$$

where Λ is the signal to interference ratio and $Ei(z)$ is the Exponential integral function defined in [8].

IV. AVERAGE BER APPROXIMATIONS

By approximating the average BER expressions, the computer processing time required to make a BER estimate can be drastically reduced, enabling many system scenarios to be considered over a smaller period of time with minimal loss in accuracy. In [5] it was suggested that it may be possible to use outage probability expressions to approximate the average BER. Research presented in this section has proven that accurate estimates of BER can be achieved by this method.

For a particular modulation scheme operating under non-fading (static) conditions, the transition between very high BER (≈ 0.3 , defined as α) and very low BER (≈ 0), as a function of signal to noise ratio, may be very rapid. Therefore the BER tends to be close to either α or zero depending on the value of the momentary SNR. Because the momentary SNR typically varies rapidly over a range of about 30 dB due to multipath fading, a simple average BER approximation can be made, as shown in Fig. 2.

The approximation simply assumes that if the momentary SNR is below a threshold S_o then the BER is approximately α . If the momentary SNR is above S_o then the BER is approximately 0. Hence the average BER is closely related to the fraction of time that the momentary SNR is below the threshold, S_o , namely the outage probability.

The general expression for the approximate average BER, BER_{approx} , is

$$BER_{approx} = \alpha \int_0^{S_o} p(\gamma) d\gamma, \quad \dots(13)$$

where $p(\gamma)$ is the pdf of the momentary SNR. When cochannel interference is considered, the signal to interference ratio (SIR) needs to be determined rather than the SNR. The expression for the approximate average BER

for an exponentially distributed desired signal and I cochannel interfering signals (with identical mean powers) is found by substituting the exponential distribution into the BER approximation of equation (13) to yield [7]

$$BER_{approx} = \alpha \left[1 - \exp\left(\frac{-S_o I}{\Lambda}\right) \right], \quad \dots(14)$$

where Λ is the (local-mean) SIR with respect to each interferer. Approximate values for S_o and α need to be determined for each modulation scheme. The approximate average BER is significantly easier to calculate than evaluating the "exact" average BER expression in equation (12) at only a slight cost in accuracy. An added advantage is that the complexity of the approximation is independent of the number of identical interferers.

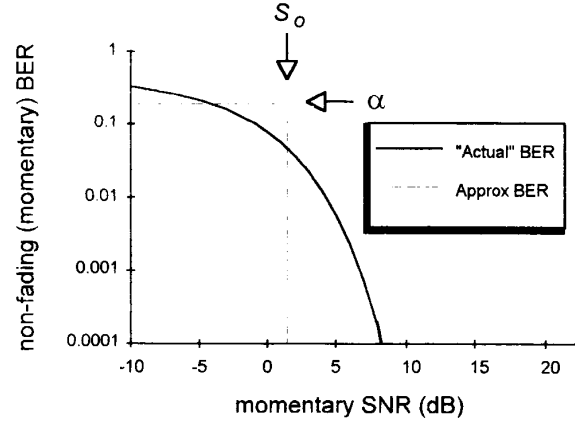


Fig. 2 Momentary and approximate BER versus SNR. The graph shows a typical relationship denoted "actual" and a rectangular approximation.

The optimum values for the outage threshold (S_o) and the BER constant (α) are dependent on the number of interferers, the modulation scheme and the range of SIR (or SNR) over which these two variables have been optimised. In this paper, the approximation variables (α and S_o) have been optimised using a least mean squares error approach over the SIR range of 0 to 30 dB [7].

In order to demonstrate the accuracy of the approximation, the "exact" and approximate average BER for six interferers have been plotted in Fig. 3. The approximation has been plotted using the optimum values for the variables α , and S_o for six interferers over the range 0 to 30 dB. It can be seen from Fig. 3 that the approximation is accurate across the entire range of SIR and particularly at average BER values typically of most interest i.e. $10^{-2} - 10^{-3}$.

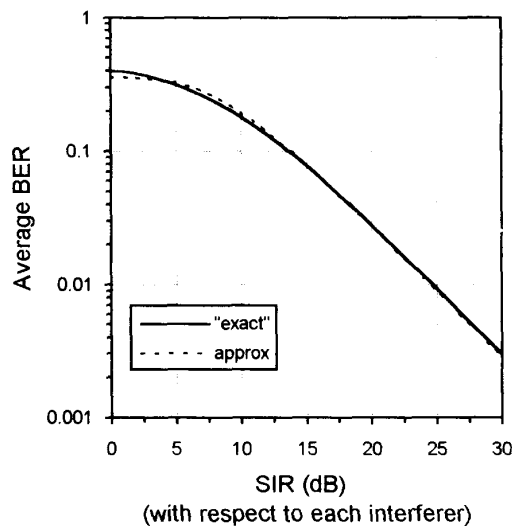


Fig. 3. "Exact" and approximate average BER versus SIR for 6 interfering signals for DPSK modulation.

The significance of the results presented in Fig. 3 is that a single accurate average BER expression can be given for DPSK with an exponentially distributed desired signal and three or more exponential interferers (with the same mean power). Near optimum values for the variables in the approximation can be quoted for each modulation scheme. These values are approximately the same for any number of interferers. Similar results can be obtained for other modulation schemes, different statistical models for desired and interfering signals and different ranges of SIR [7].

V. CONCLUSIONS

Dual protection margin outage probability expressions that take into consideration both the effect of "noise" and cochannel interference have been presented in this paper. The dual margins make the outage probability expressions more flexible than single margin expressions in modelling real situations.

For digital mobile radio systems, a simple approximation for the average BER based on outage probability has been presented and its accuracy found to be high. In the approximation, the calculation requirements were independent of the number of interferers and the approximation parameters were nearly constant for three or more interferers. Other ranges of SIR, different modulation schemes and different received signal power distributions can also be treated using this approximation.

VI. ACKNOWLEDGMENT

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