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Adverse Selection and Insurance Contracting: A Rank-Dependent Utility Analysis

Matthew Joseph Ryan*

Rhema Vaithianathan†

*University of Auckland, m.ryan@auckland.ac.nz

†University of Auckland, r.vaithianathan@auckland.ac.nz

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Adverse Selection and Insurance Contracting: A Rank-Dependent Utility Analysis

Matthew Joseph Ryan and Rhema Vaithianathan

Abstract

Stiglitz (1977) established three well-known features of monopoly insurance markets subject to adverse selection: (i) at least one market segment is served, despite the informational asymmetry; (ii) there is always some screening of risk classes; and (iii) efficiency is sacrificed to achieve screening. We modify Stiglitz's model, replacing his expected utility assumption on consumer behavior with a version of Quiggin's (1982) rank-dependent utility model that has received strong experimental support. We show that none of the conclusions (i)—(iii) is robust to this revision. In particular, asymmetric information need not lead to any loss in efficiency.

KEYWORDS: Insurance, Rank-Dependent Utility, Adverse Selection

1 Introduction

A growing body of experimental work has placed increasing strain on the axioms of classical expected utility (EU) theory. In an attempt to represent the sort of behavior observed by these experimenters, various generalizations of EU have been developed. One such model is *rank-dependent utility (RDU)*, originally formulated by John Quiggin (1982).¹ In this paper, we explore the implications of RDU behaviour for insurance contracting under asymmetric information.

Since RDU *generalizes* the EU model, it is possible to experimentally test the hypothesis of EU under a maintained assumption of RDU preferences. Such tests regularly reject EU – see section 2.2. More surprisingly, observed deviations from EU show remarkable consistency across individual subjects and experiments. There is a systematic tendency for extreme (best or worst) outcomes having low probability to exert an influence on choice that far exceeds their relative likelihoods. Subjects are therefore “overly pessimistic” about the possibility of suffering rare disasters; but “overly optimistic” about the prospect of unlikely windfalls.

RDU is able to capture this behavioral propensity through an inverse-S transformation of the (decumulative) probabilities attached to outcomes. Such a transformation accentuates the weights on extreme outcomes, and de-emphasizes those on intermediate outcomes. This is precisely the version of RDU favored by Quiggin in his original proposal (Quiggin, 1982).

RDU behavior of this type has significant implications for insurance contracting. Pessimism with respect to adverse events increases the gains from trade with risk-neutral insurance companies. Pessimistic consumers may even purchase full insurance on actuarially unfair terms. By contrast, EU predicts that full insurance is never demanded at an actuarially unfair premium. This implication of EU has long been regarded as empirically and intuitively implausible. Kunreuther and Hogarth (1989) further observe that the propensity to pay above the fair premium for full insurance is especially marked for consumers exposed to low levels of risk. This, too, is consistent with RDU and an inverse-S probability transformation. As the likelihood of the adverse event increases, pessimism diminishes. Wakker, Thaler and Tversky (1997) show that the commonly observed aversion to “probabilistic insurance” – insurance for which there is a small probability of default – is incompatible with EU, but explicable if consumers conform to RDU with an inverse-S transformation.

This probability-dependent propensity toward pessimism or optimism also underpins our results on adverse selection. Low risk consumers are more pessimistic than high risks, and the latter may even exhibit optimism. Since screening requires that high risks be offered greater coverage than low risks, this relative pessimism effect can work against screening in two ways. First, if low-risk sales are particularly lucrative and low risks are

¹Quiggin called his theory “anticipated utility”.

a large fraction of the market, then it may be optimal to fully insure both types. In this case, all agents receive Pareto optimal contracts.² Second, if high risks are so optimistic that they need a subsidy in order to buy any insurance, and since it is impossible to serve only low risks under asymmetric information, pooling may again be optimal. In this case, the pooled contract will offer partial cover. It is even possible that neither type is served and the entire market breaks down.

In summary, for the class of RDU preferences that we study, one may observe pooling in a monopolized insurance market subject to adverse selection, including the extreme cases of complete market failure (neither type insured) and Pareto optimal contracting (both types fully insured). Each of these possibilities is absent from the EU version of the model (Stiglitz, 1977). A companion paper – Ryan and Vaithianathan (2003) – demonstrates that Pareto efficient contracting is also possible under moral hazard. Taken together, these two papers convey the message that asymmetric information need not undermine the efficient allocation of risk through insurance markets, contrary to received wisdom based on EU analysis.

Several authors – especially Machina (1995) – have attested to the robustness of classical insurance theory to deviations from EU under *symmetric* information. Machina notes (*ibid.*, p.36) that the area of insurance under asymmetric information had not been explored from a non-EU perspective to that time.³ In addition, Machina (1995) restricts attention to “smooth” (Fréchet differentiable) preferences, which RDU preferences, in general, are not.⁴ Smoothness excludes the phenomenon of *first-order risk-aversion*,⁵ which plays an important role here.⁶

The outline of the rest of the paper is as follows. In the next section we review the theory of RDU maximization, and the experimental evidence in favor of the inverse-S transformation. In section 3 we examine its implications for Pareto optimal contracting under symmetric information. Section 4 contains the main results on insurance under adverse selection. Section 5 concludes. Proofs are contained in the Appendix.

²Shi (1988) and Young and Browne (2000) demonstrate the possibility of Pareto optimal pooling for the case of competitive insurance markets. Shi (1988) assumes quasi-linear preferences (Chew, 1983), while Young and Browne adopt Yaari’s (1987) “dual” model – a special case of RDU in which utility is linear in wealth – and further assume that the probability transformation is convex, rather than inverse-S.

³Apparently Machina had overlooked the contribution of Shi (1988).

⁴See Karni’s (1995) discussion of Machina’s paper.

⁵Segal and Spivak (1990).

⁶See also Dupuis and Langlais (1997).

2 RDU preferences

2.1 Theory

Consider a lottery

$$L = (x_1, p_1; x_2, p_2; \dots; x_n, p_n).$$

Each $x_i \in \mathbb{R}_+$ is a monetary outcome, and $p_i \in (0, 1]$ is the probability of this outcome. As usual, we assume that $\sum_i p_i = 1$. For the RDU model, an outcome's *rank* in L is also important, so we adopt the convention that lottery prizes are listed in descending order. That is:

$$x_1 \geq x_2 \geq \dots \geq x_n.$$

The RDU preference representation involves a utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$, and a strictly increasing function $w : [0, 1] \rightarrow [0, 1]$, satisfying the normalization conditions $w(0) = 0$ and $w(1) = 1$. Adopting the convention that $\sum_{i=1}^0 p_i = 0$, the lottery L is evaluated as follows:

$$\sum_{i=1}^n u(x_i) \left[w \left(\sum_{j=1}^i p_j \right) - w \left(\sum_{j=1}^{i-1} p_j \right) \right] \quad (1)$$

Observe that

$$\sum_{i=1}^n \left[w \left(\sum_{j=1}^i p_j \right) - w \left(\sum_{j=1}^{i-1} p_j \right) \right] = 1,$$

so RDU maximizers do perform expected utility calculations, but using a (rank-dependent) transformation of the true probabilities. Moreover, if w is linear (i.e. the identity function), then (1) is a standard expected utility calculation. However, experiments consistently reject the linearity of w .

As a matter of interpretation, it is important to note that non-linear probability transformations need not imply misperception of the true probabilities, any more than non-linearity in the utility function indicates a failure to appreciate the value of a dollar. For a better understanding of the foundations for the RDU model, see the simple axiomatization of Chateauneuf and Wakker (1999), or the intuitive motivation provided by Diecidue and Wakker (2001).

2.2 Evidence on the shape of w

Wakker (2001, Appendix A) reviews the experimental literature on the shape of w , and concludes that there is strong evidence for w having the inverted-S shape of Figure 2.1. Such decision-makers tend to overweight (small) probabilities attached to very highly or very lowly ranked outcomes, and hence to underweight the probabilities of non-extreme outcomes. In particular, the transformation function depicted in Figure 2.1 implies the

violation of global risk aversion. For example, the over-weighting of small probabilities attached to very high-ranking outcomes is compatible with gambling behavior.

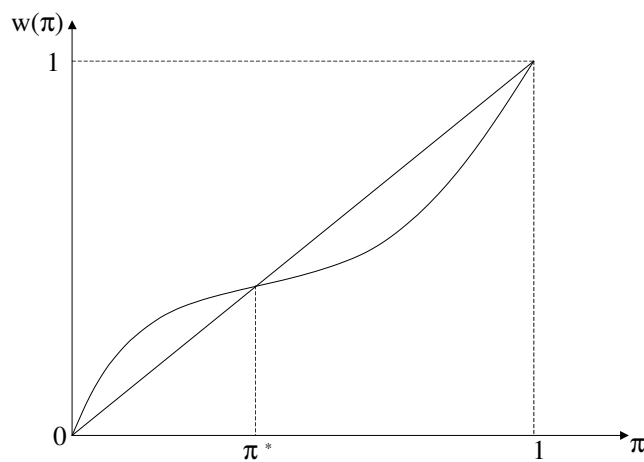


Figure 2.1: Typical probability transformation function

Wu and Gonzalez (1996) develop direct preference-based tests for the concavity and convexity of w . These tests assume RDU, but are independent of the form of the utility function. Wu and Gonzalez find significant evidence of concavity of w up to a critical probability level of between 0.3 and 0.4, with convexity thereafter. Camerer and Ho (1994), Gonzalez and Wu (1999) and Abdellaoui (2000) obtain similar results.

It is also worthy of note that Quiggin (1991) has shown that the inverse-S weighting function implies an optimal lottery design that conforms well with evidence from actual lotteries: an increasing sequence of prizes, rather than one large prize. Karni and Safra (1990) demonstrate that an inverse-S model has the potential to explain “preference reversal” phenomena in laboratory experiments.

3 Pareto optimal contracts (symmetric information)

Consider a population of RDU maximizing agents with the following common features: wealth level y , utility function u , and transformation function w . We distinguish two types of agent: a “high risk” type, with probability $p^H \in (0, 1)$ of suffering a financial loss of $c < y$; and a “low risk” type, with probability $p^L \in (0, p^H)$ of suffering the same loss. Let $\theta \in (0, 1)$ be the proportion of high risks in the population.

An uninsured high risk agent therefore faces the lottery

$$(y, 1 - p^H; y - c, p^H)$$

while low risks face the lottery

$$(y, 1 - p^L; y - c, p^L)$$

Using (1), a lottery $(y, 1 - p; y - c, p)$ is evaluated as

$$u(y - c) + w(1 - p)[u(y) - u(y - c)]$$

Let w be as in Figure 2.1, and suppose that $1 - p^H < \pi^* < 1 - p^L$. Then, relative to EU maximizers, *high risk* individuals act as if they are *over-optimistic* about their chances of avoiding the loss, while *low risk* individuals act in an *overly pessimistic* fashion. This distortion of probability information will augment the potential gains from trade (relative to the EU case) between an expected profit maximizing insurance company and a low risk client; but will squeeze potential gains in the case of high risk clients. Indeed, it is possible that gains from trade may vanish altogether in the latter case.⁷

For two-state problems such as these, it is convenient to visualize matters using a Hirshleifer-Yaari diagram. Fix two states, having probabilities p and $1 - p$ respectively. The vertical axis in the Hirshleifer-Yaari diagram will measure wealth (z_2) in the state that occurs with probability p ; while the horizontal axis measures wealth (z_1) in the other state. If u is differentiable, the slope of an RDU indifference curve at the point (z_1, z_2) is

$$\frac{-(1 - w(p))}{w(p)} \frac{u'(z_1)}{u'(z_2)} \tag{2}$$

when $z_2 > z_1$ and

$$\frac{-w(1 - p)}{1 - w(1 - p)} \frac{u'(z_1)}{u'(z_2)} \tag{3}$$

when $z_2 < z_1$. Notice that:

- (i) There is a non-differentiability (“kink”) along the 45 degree (or *certainty*) line unless $w(1 - p) + w(p) = 1$.⁸
- (ii) Upper contour sets (points on or above a given indifference curve) are convex if u is concave and $w(1 - p) + w(p) \leq 1$.

Figure 3.1 illustrates these two properties.

⁷Quiggin (1991, p.6) makes a similar observation: “the effects of probability weighting discourage insurance against high-probability events.”

⁸Non-linearity of the weighting function is not sufficient for a kink. For example, the function

$$w(p) = \frac{1}{2}(2p - 1)^3 + \frac{1}{2},$$

has the inverse-S shape, but $w(p) = 1 - w(1 - p)$ for all p , so the associated RDU indifference curves are smooth in any Hirschleifer-Yaari diagram.

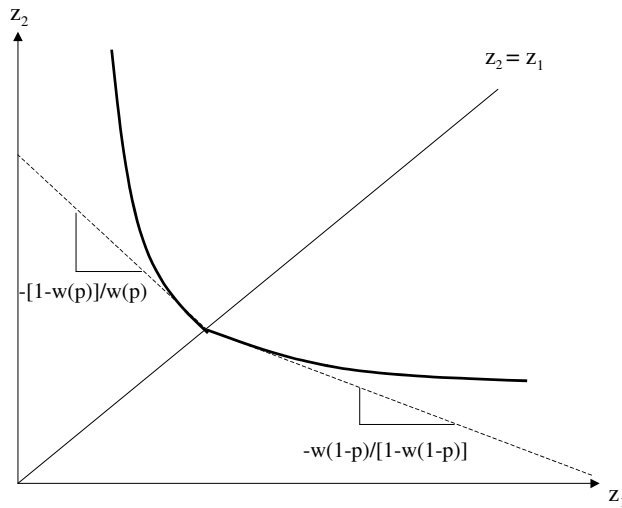


Figure 3.1: RDU indifference curve with convex upper contour set

Since we confine attention in what follows to two-outcome lotteries, the analysis generalizes immediately to a suitable sub-class of *biseparable* preferences (Ghirardato and Marinacci, 2001).⁹ These are preferences that admit a non-trivial, monotonic representation, which is of the form (1) when $n = 2$ (*ibid.*, Definition 1). Ghirardato and Marinacci provide an axiomatization of such preferences. The analysis in the present paper is compatible with any biseparable preference ordering that satisfies:

Assumption 1 *The utility function u in (1) is strictly increasing, concave and continuously differentiable, and the transformation function w has an inverse-S form with $w(1-p) + w(p) \leq 1$, for p the consumer's individual probability of loss.*

We maintain Assumption 1 throughout. It guarantees convex upper contour sets, though it does not imply risk aversion.¹⁰ Indeed, from Figure 3.1 it is clear that a Pareto optimal contract (with an expected profit maximizing insurance company) will imply:

1. over-insurance if $p < w(p)$;
2. partial (and possibly no) insurance if $1 - p < w(1 - p)$; and
3. full insurance otherwise.

⁹We thank Simon Grant for this observation.

¹⁰See Chew, Karni and Safra (1987) for sufficient conditions for (strong) risk aversion.

Figure 3.2 summarizes these three possibilities. Observe that Case 3 exists as a possibility only if $\pi^* \leq \frac{1}{2}$. However, the latter is quite consistent with the experimental evidence – see Section 2.2.

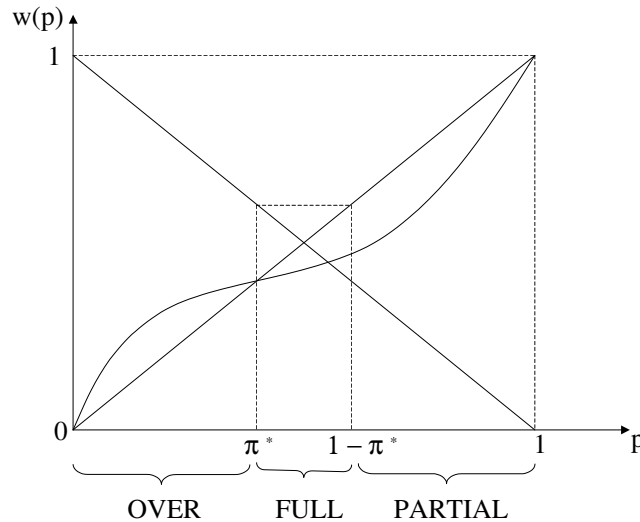


Figure 3.2: Pareto optimal coverage vs loss probability, p

Under Case 3, the decision-maker will exhibit *first-order risk-aversion* (Segal and Spivak, 1990) unless $1 - p = w(1 - p)$: full insurance is strictly preferred to any partial level of cover for a range of premia *above* the actuarially fair rate. This property is important for the discussion in Section 4.1.

4 Adverse selection

What if the consumer's risk type is private information? Models of adverse selection almost invariably assume risk-averse EU consumers. Under this assumption, Stiglitz (1977) considered the case of a risk-neutral, monopoly insurance company. He showed that the market will either (a) offer full insurance and serve high risks only; or (b) serve both types with a screening menu of contracts in which high risks receive full insurance, but low risks receive only partial cover. In particular, pooling contracts are never observed, and risks are misallocated – low risks receive Pareto sub-optimal contracts (partial or no cover).¹¹

¹¹These facts generalize to more than two types as follows: the highest risk type always receives a full-insurance contract; if more than one type is served, the highest risk type is always screened, so all other types receive an inefficient contract (partial or no cover). See Laffont and Martimort (2002, Chapter 3).

If consumers exhibit RDU (or biseparable) preferences satisfying Assumption 1, then other possibilities arise. First, one may observe pooling in which both types receive a Pareto optimal full insurance contract. Second, a pooled partial insurance contract may be offered, with low risks cross-subsidizing high risks. Because of the need for cross-subsidization, such contracts are viable only if θ (the proportion of high risks in the population) is sufficiently low. If not, then neither type is served. Therefore, the range of possible market configurations expands to include both Pareto efficiency and complete market failure (neither type served).

We now illustrate each of these possibilities. Throughout, we assume a risk-neutral, monopoly insurer.

4.1 Pareto optimal pooling

In this section we consider a scenario in which all consumers must be fully insured for risk to be allocated in a Pareto efficient manner. Crucially, we consider a case in which low risks exhibit *first-order risk-aversion*. In this case, low risks are willing to pay an actuarially unfair premium for full insurance, and pursuit of this profit may discourage insurers from offering a partial insurance contract to screen low risks.

In an EU (or any other “smooth”) model, marginal deviations from full insurance that maintain expected utility constant have zero first-order effects on expected profit. Since there are first-order gains from being able to reduce the information rent paid to risk-averse high risks, screening is optimal. However, when low risks exhibit first-order risk-aversion, this logic breaks down. Marginal deviations from full insurance now impose first-order reductions in expected profit earned on low-risk contracts. When θ (the proportion of high risks in the population) is low, these losses may outweigh any consequent savings on the information rent paid to high risks.

In the following Proposition, π^* refers to the cross-over point in Figure 3.2.

Proposition 1 *Under Assumption 1, if $\pi^* < p^L < p^H \leq 1 - \pi^*$, then a pooled full-insurance contract will be offered when θ is sufficiently close to zero. A Pareto optimal allocation of risk results.*

The proof of Proposition 1 goes through even if $\pi^* = 0$. This limiting case of the inverse-S gives a convex transformation function, and implies global risk aversion (Chew, Karni and Safra, 1987). Hence, Proposition 1 demonstrates the possibility of pooling even when *all consumers are globally risk-averse*. In particular, as the intuition given prior to the statement of Proposition 1 suggests, it is first-order risk-aversion that is crucial here, not the change in curvature of w at π^* . Conversely, the latter feature of the inverse-S RDU model underpins the next result, while first-order risk-aversion plays no role.

4.2 Inefficient pooling

Suppose that high risks are sufficiently optimistic that it is unprofitable to sell them any insurance. Such will be the case if

$$\frac{w(1 - p^H)}{[1 - w(1 - p^H)]} \frac{u'(y)}{u'(y - c)} \geq \frac{1 - p^H}{p^H} \quad (4)$$

so that the high-risk indifference curve is at least as steep as the “fair bet” line at the point $(y, y - c)$. Condition (4) requires that $p^H > 1 - \pi^*$ and that $u'(y)/u'(y - c)$ is not too small.

Given (4), it is intuitive that it will be optimal to pool the risk types. Low risks must be served if expected profit is to be non-negative, and the insurer will offer as little insurance to high risks as possible, subject to meeting the incentive compatibility constraints. The latter requires high risks to receive at least as much cover as low risks (by the usual single-crossing property), so a single contract will be offered. The optimal pooling contract will be “null” if high risks are a sufficiently large fraction of the population, since there will not be enough low risks on which to recover losses suffered on high-risk contracts.

Proposition 2 *Under Assumption 1, if $p^L \leq 1 - \pi^*$ and condition (4) obtains, there exists some $\bar{\theta} \in (0, 1)$ such that a non-null pooling contract is offered whenever $\theta < \bar{\theta}$, and neither type is served otherwise. In either case, risk sharing deviates from the Pareto optimum.*

Figure 4.1 illustrates the result. Locus L is a low-risk indifference curve, while H_1 and H_2 are high-risk indifference curves.¹² Point E represents the endowment. Low-risk types are strictly profitable to serve (recall that $p^L \leq 1 - \pi^*$). Suppose that A in Figure 4.1 is the low-risk component of a screening menu of contracts. Incentive compatibility requires that the high risk contract must lie to the northwest of A , between the indifference curves labelled L and H_2 . The most profitable of these is A itself.

Therefore, pooling is more profitable than screening. The optimal pooling contract approaches E as $\theta \rightarrow 1$, and it is always Pareto inefficient: low risks receive strictly less than their optimal level of insurance since $\theta > 0$.

¹²For simplicity, we have drawn the indifference curves without kinks at certainty, since first-order risk-aversion plays no role in the analysis.

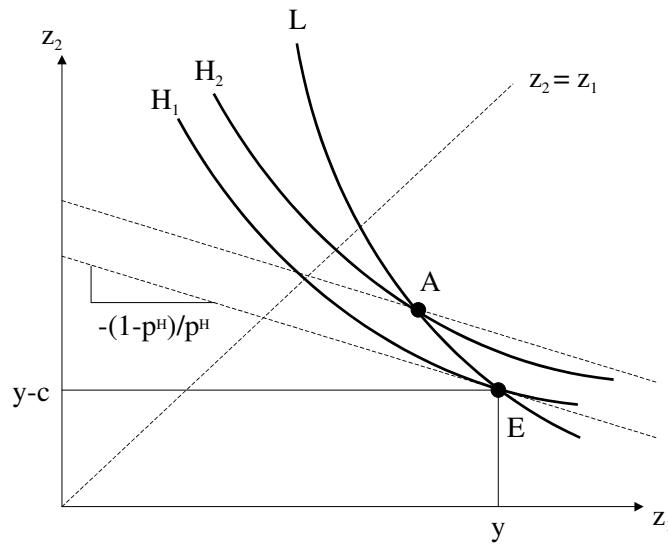


Figure 4.1

The inverse-S transformation is at work here. The implied *probability-dependent risk attitude* is what makes it possible that only low risks are profitable to serve. When this fact is combined with asymmetric information – which necessitates high risks receiving at least as much insurance cover as low risks – we immediately obtain the optimality of pooling.

5 Conclusion

In this paper we adopt the view that consumer behavior conforms to the RDU model (or biseparability) with an inverse-S transformation function. This assumption has greater experimental support than EU.

The main message of the paper is that such preferences are compatible with pooling in monopoly insurance markets subject to adverse selection. It is even possible that fully Pareto optimal risk sharing is achieved, despite the asymmetric information. Ryan and Vaithianathan (2003) establish a similar result for the case of insurance subject to moral hazard.

In summary, plausible deviations from the EU assumption fundamentally alter insurance market predictions when asymmetric information is present. This is so if consumers are assumed to be globally risk averse, or if the more experimentally supported inverse-S transformation is imposed.

6 Colophon

6.1 Acknowledgements

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6.2 Addresses

Matthew J. Ryan:

Department of Economics
Commerce A Building
University of Auckland
PB 92019
Auckland
New Zealand
Email: m.ryan@auckland.ac.nz

Rhema Vaithianathan:

Department of Economics
Commerce A Building
University of Auckland
PB 92019
Auckland
New Zealand
Email: r.vaithianathan@auckland.ac.nz

Appendix

Proof of Proposition 1: The insurance company’s options may be usefully divided into the following three categories: (i) offer a single contract attractive only to high risks; (ii) offer a single contract attractive to both types (pooling); or (iii) offer a screening

menu of contracts. Option (ii) clearly dominates option (i) for θ sufficiently close to zero. Moreover, since $p^L \in (\pi^*, 1 - \pi^*)$, P is the most profitable pooling contract for θ close to zero – see Figure A.1. It therefore suffices to show that, for θ near zero, pooling at P is strictly more profitable than any screening menu.

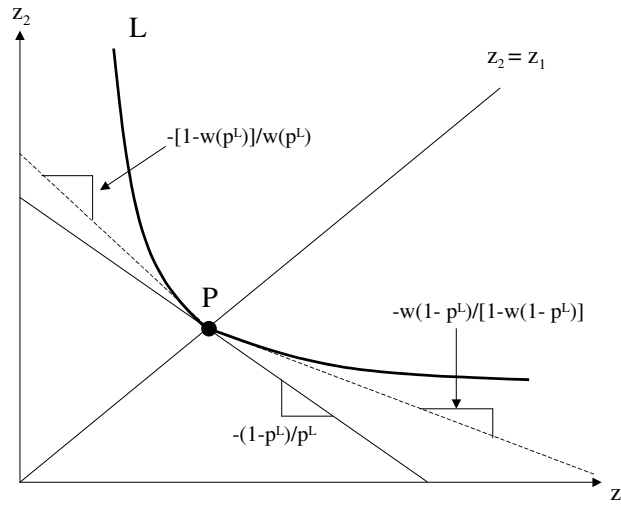


Figure A.1

Consider Figure A.2. Standard arguments – as in Stiglitz (1977) or Kreps (1990, Chapter 18) – give that an optimal menu will offer low risks a contract on L between P and E . Define the function $f^L : [a, y] \rightarrow \mathbb{R}$ by the condition

$$u(f^L(z_1)) + w(1 - p^L) [u(z_1) - u(f^L(z_1))] = u(y - c) + w(1 - p^L) [u(y) - u(y - c)].$$

That is, for each $z_1 \in [a, y]$, $(z_1, f^L(z_1))$ lies on the indifference curve L . Since u is strictly increasing and continuously differentiable, f^L is continuously differentiable. Next, define the function $f^H : [a, y] \rightarrow \mathbb{R}$ as follows:

$$u(f^H(z_1)) = u(f^L(z_1)) + (1 - p^H) [u(z_1) - u(f^L(z_1))].$$

Since, $p^H \in (p^L, 1 - \pi^*]$, standard arguments imply that $(f^H(z_1), f^H(z_1))$ is the most profitable incentive compatible contract for high risks, when $(z_1, f^L(z_1))$ is available to low risks. By the continuous differentiability of f^L and the aforementioned properties of u , f^H is also continuously differentiable.

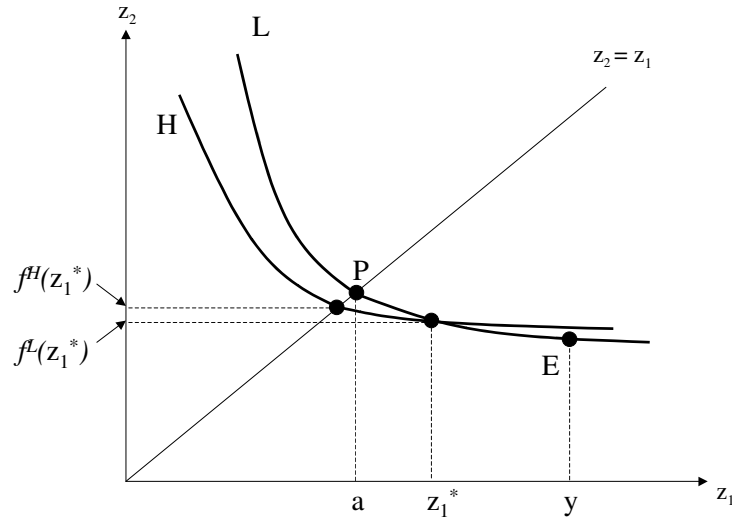


Figure A.2

Finally, let us define the following expected profit function:

$$\Pi(z_1, \theta) = \theta [y - f^H(z_1) - p^H c] + (1 - \theta) [(y - z_1) + p^L (z_1 - c - f^L(z_1))] \quad (\text{A.1})$$

The RHS of (A.1) is the expected profit (per customer) from offering the contract menu $((z_1, f^L(z_1)), (f^H(z_1), f^H(z_1)))$.

Consider the difference

$$\begin{aligned} \Pi(z_1, \theta) - \Pi(a, \theta) = \\ \theta [a - f^H(z_1)] + (1 - \theta) [(1 - p^L) (a - z_1) + p^L (a - f^L(z_1))] \end{aligned} \quad (\text{A.2})$$

This is the net gain from using the menu $((z_1, f^L(z_1)), (f^H(z_1), f^H(z_1)))$ instead of the pooling contract P . The first term on the RHS represents the average gain on the sales to high risks, since $f^H(z_1) \leq a$ (with equality if and only if $z_1 = a$). The second term represents the average loss on sales to low risks, since $f^L(z_1) \leq a$ (with equality if and only if $z_1 = a$). We wish to show that, for θ sufficiently small, $\Pi(z, \theta) - \Pi(a, \theta) < 0$, for all $z_1 \in (a, y]$. Let

$$\delta = \max_{z_1 \in [a, y]} \left| \frac{df^H(z_1)}{dz_1} \right| > 0$$

(taking one-sided derivatives when $z_1 = a$ or $z_1 = y$).¹³ Then the first term in (A.2) is bounded above by $\theta\delta(z_1 - a)$. That is:

$$\theta [a - f^H(z_1)] \leq \theta\delta(z_1 - a) \tag{A.3}$$

Similarly, we may bound the second term by noting that

$$\max_{z_1 \in [a, y]} \left| \frac{df^L(z_1)}{dz_1} \right| = (f^L)'_+(a) = \frac{w(1 - p^L)}{1 - w(1 - p^L)}.$$

Hence:

$$\frac{(1 - \theta) [(1 - p^L)(a - z_1) + p^L(a - f^L(z_1))]}{(z_1 - a)} \leq (1 - \theta) \left[\frac{p^L w(1 - p^L)}{1 - w(1 - p^L)} - (1 - p^L) \right] \tag{A.4}$$

Let

$$\beta = \left[(1 - p^L) - \frac{p^L w(1 - p^L)}{1 - w(1 - p^L)} \right] > 0.$$

Combining (A.3) and (A.4), a sufficient condition for $\Pi(z_1, \theta) - \Pi(a, \theta) < 0$ for all $z_1 \in (a, y]$ is that

$$\theta < \frac{\beta}{\delta + \beta}.$$

Hence, for such θ , $\Pi(z_1, \theta) < \Pi(a, \theta)$ for all $z_1 \in (a, y]$, so P is strictly more profitable than any screening menu of contracts. \square

Proof of Proposition 4.2: The insurance company must either serve both types, with low-types cross-subsidizing high, or neither, since it is not possible to serve low types only, and high types are unprofitable.

¹³The maximum is well defined. For example, it is equal to

$$\max \left\{ \max_{z_1 \in [a, a + \frac{1}{2}(y-a)]} |(f^H)'_+(z_1)|, \max_{z_1 \in [a + \frac{1}{2}(y-a), y]} |(f^H)'_-(z_1)| \right\},$$

which exists because each one-sided derivative is continuous over the specified compact domain.

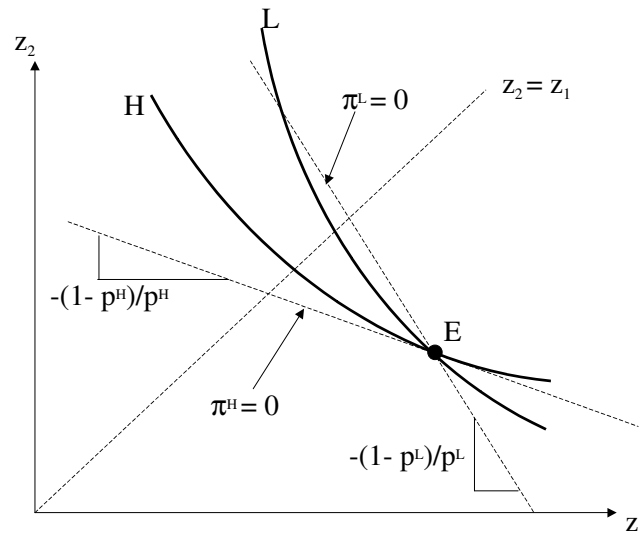


Figure A.3

Suppose, then, that the insurance company serves both types. We claim that a pooling contract will be offered. To see why, consult Figure A.3.¹⁴ Point E is the endowment: that is, the point $(y, y - c)$. Since the low-type contract must be weakly profitable and meet the low-risk's participation constraint, it must lie between the line $\pi^L = 0$, which is the set of contracts yielding zero expected profit when sold to low risks, and the indifference curve L . This region is non-empty by virtue of the fact that $p^L \leq 1 - \pi^*$.

Incentive compatibility requires that the high-risk contract lie to northwest of the low-risk contract, between the high- and low-risk indifference curves through the low-risk contract. All such points lie above the curve H and to the west of point E in Figure A.3. Since u is concave, indifference curves get steeper as we move directly northward. Therefore, condition (4) implies that, for any low-risk contract between L and the line $\pi^L = 0$, the high-risk indifference curve through that point is *steeper* than the $\pi^H = 0$ line. Therefore, given any such low-risk contract, it is optimal to offer high risks the same contract.

Thus, if both types are served, a pooling contract on L will be offered. Because $\theta > 0$, $\theta p^H + (1 - \theta) p^L > p^L$, so there will be Pareto inefficient risk sharing with low risks. Any (non-null) contract is Pareto inefficient for high risks. As θ increases, the optimal pooling contract moves down the indifference curve L toward E . If θ becomes high enough, E will be optimal – neither type will be served – since the losses on high risks cannot be recovered on the small population of low risks.

¹⁴As with Figure 4.1, we omit kinks in the indifference curves for simplicity.

The critical level $\bar{\theta}$ is the solution to the equation

$$\frac{1 - \theta p^H - (1 - \theta) p^L}{\theta p^H + (1 - \theta) p^L} = \frac{w(1 - p^L)}{1 - w(1 - p^L)} \frac{u'(y)}{u'(y - c)} \quad (\text{A.5})$$

This says that the iso-expected profit locus for pooling contracts is tangent to the low risk indifference curve at E . The left-hand side of (A.5) is strictly decreasing in θ . A unique solution to (A.5) therefore exists in $(0, 1)$ since

$$\frac{1 - p^L}{p^L} > \frac{w(1 - p^L)}{1 - w(1 - p^L)} \frac{u'(y)}{u'(y - c)}$$

by virtue of $p^L \leq 1 - \pi^*$, while condition (4) and $p^L < p^H$ imply

$$\frac{1 - p^H}{p^H} \leq \frac{w(1 - p^H)}{1 - w(1 - p^H)} \frac{u'(y)}{u'(y - c)} < \frac{w(1 - p^L)}{1 - w(1 - p^L)} \frac{u'(y)}{u'(y - c)}$$

This completes the proof. \square

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