The Combined Effects of Heat Transfer Coefficient and Thermal Diffusivity on Heating Rates of Solid Materials - A Simple Analysis

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Abstract

It is often perceived that an object with a higher thermal diffusivity will heat faster in comparison to that with a lower thermal diffusivity. This is however not always the case, as previously numerically proved by Palazoglu (2006). In this short communication, an analytical explanation of this is provided. Three generic cases are discussed which reinforce the observations made by Palazoglu (2006).

KEYWORDS: thermal diffusivity, heat transfer coefficient, heating rate, analytical solutions

Introduction

It is well known that many thermal processing operations in food industry involve heat transfer between a solid food and a fluid medium. Heat is transferred to the surface of the food product by convection and within the solid food is by conduction. As summarised by Palazoglu (2006), the rate of heat penetrating within the solid food is a function of several factors including thermal conductivity (k), density (ρ), and specific heat capacity (c) of the food. It is often perceived that an object with a higher thermal diffusivity ($\alpha = \dfrac{k}{\rho c}$) will always heat faster in comparison to that with a lower thermal diffusivity. He commented that Özişik (1993) and Singh (1982) described that the higher the thermal diffusivity, the shorter the time required for heat propagating within the solid. Özisik (1993) demonstrated this by comparing the times for several materials with different thermal diffusivity values to cool from an initial temperature to half of this value. From this analysis, he concluded that heat penetrates faster within a solid that has a higher thermal diffusivity. Incropera and De Witt (2002) also indicated that materials of higher thermal diffusivity respond more quickly to temperature changes in their environment (Palazoglu, 2006). Palazoglu (2006) has shown that this is not always true. He made the remarks that whether an object with a higher thermal diffusivity will heat faster than that with a lower thermal diffusivity is actually a function of the surface convective heat transfer coefficient. He then concluded that it is not really possible to determine which object will heat faster without knowing the heat transfer coefficient.

In the current paper, a generic and simple analysis is provided, which is analytical by nature, to capture the effect(s) suggested by Palazoglu (2006). It can be seen from the more analytical approach taken in this study, the 'not so obvious' can become much more obvious.

Cases considered

To unveil the kind of 'mystery' described above, two extreme conditions are considered in this study.

(i) Very small *Biot* number (*Bi* $\rightarrow$ *0*)

Conventionally, the *Biot* number is introduced through a scenario of steady state heat conduction in a slab with one side cooled by convection (Cengel, 1998; Incropera and DeWit, 2002; Chen, 2007). In this case, the conductive heat flux through the wall must be equal to the convection heat flux to the surrounding fluid (see Figure 1):

$$q''_x = k \cdot \frac{T_{s,1} - T_{s,2}}{L} = h \cdot \left(T_{s,2} - T_\infty \right) \tag{1}$$

The ratio of the temperature differences can then be expressed as

$$\frac{T_{s,1} - T_{s,2}}{T_{s,2} - T_\infty} = \frac{h \cdot L}{k} \tag{2}$$

The *Biot* numb*e*r is defined as

$$Bi = \frac{h \cdot L}{k} \tag{3}$$

For a spherical object, the characteristic dimension L may be set as the radius of the sphere.

On the other hand, equation (2) may be re-written as:

$$\frac{T_{s,1} - T_{s,2}}{T_{s,2} - T_\infty} = Bi = \frac{L/k}{1/h} = \frac{R_{cond}}{R_{conv}} \tag{4}$$

Equation (4) reflects the temperature ratio as in equation (2) and more importantly, the *Biot* number indicates the ratio of the internal thermal resistance (R_{cond}) to the external thermal resistance (R_{conv}).

When Bi is very small, i.e. $Bi \to 0$ (very small heat transfer coefficient for instance), say $Bi < 0.1$, the material being heated can be considered to have approximately uniform temperature throughout ($T_{s,1} = T_{s,2} = T$). This rule can be made 'tighter', as required (for instance $Bi < 0.01$). The simple heating law for this is well known, called the 'lumped capacitance' model (Incropera and DeWit, 2002), as follows:

$$mc_p \frac{dT}{dt} = -hA \left(T - T_\infty \right) \tag{5}$$

where m is mass (kg), c_p is the specific heat capacity (J.kg^{-1}.K^{-1}), T is the temperature of the object (K) and T_∞ is the bulk fluid temperature (K), h is the heat transfer coefficient (W.m^{-2}.K^{-1}) and A is the heat transfer surface area (m^2). This has a simple solution:

$$\frac{T_\infty - T}{T_\infty - T_i} = \exp\left(-\frac{hA}{mc_p} t \right) \tag{6}$$

Where T_i is the initial temperature (K) of the object.

or

$$\frac{T_\infty - T}{T_\infty - T_i} = \exp\left(-\frac{h \cdot A}{\rho c_p \cdot V} \cdot t\right) \qquad (7)$$

In the other words, under this extreme condition, and for the given geometry, the property that dominates the heating rate is in fact:

$$\beta = \frac{h}{\rho c_p} \qquad \text{(m.s}^{-1}) \qquad (8)$$

and the characteristic dimension $L_c = V/A$ would be of interest. V is the volume (m^3) and A is the surface area (m^2).
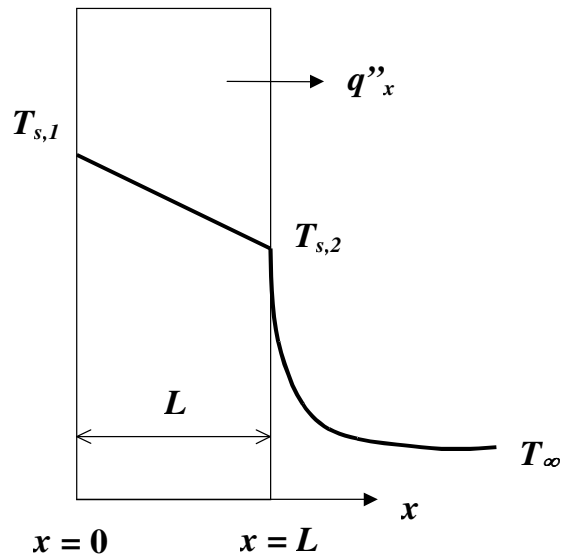

Figure 1 The definition of *Biot* number.

(ii) Very large *Biot* number ($Bi \rightarrow \infty$)
When the internal thermal resistance can not be ignored, equation (7) is no longer valid. Usually, the *Heisler* charts of standard objects may be used. When $Bi \rightarrow \infty$ (for example, very large heat transfer coefficient), the surface temperature of the object, as soon as it is subjected to heating, moves up from initial temperature T_i to T_∞, instantaneously. The surface temperature is then held at constant T_∞ for the rest of the heating period.

In this case, the heat penetration of the same object as mentioned in case (i) but for $Bi \rightarrow \infty$, is determined by the thermal diffusivity:

$$\alpha = \frac{k}{\rho c_p} \qquad \text{(m}^2.\text{s}^{-1}) \qquad (9)$$

For the very initial penetration time (or very shallow depth into the object being heated), i.e. when the sample may be viewed as a semi-infinite domain, as an illustration, it is known that the penetration depth is approximately (Welty, Wicks and Wilson, 1984):

$$\delta \approx \sqrt{12 \cdot \alpha \cdot t} \tag{10}$$

One can see that for the fixed location inside the object in this case, the rate of heat penetration is proportional to $(1/\alpha)^{0.5}$ and therefore it is not difficult to see that process is dominated by the value of thermal diffusivity.

(iii) Intermediate values of *Biot* number

The roles played by finite α and β are of interest and to this end of the analysis the approximate approach following van der Sman (2003) becomes very useful. In his approach, a critical characteristic dimension δ_c was introduced ($\delta_c = L_c/4$). It is then possible that the heating of a particle can be simulated using the overall heat transfer concept (or the overall thermal resistance concept):

$$mc_p \frac{d\overline{T}}{dt} = -UA\left(\overline{T} - T_\infty\right) \tag{11}$$

This is in contrast to equation (1) and for simplicity, the average temperature is considered here. The overall heat transfer coefficient is calculated as follows:

$$\frac{1}{U} \approx \frac{1}{h} + \frac{\delta_c}{k} \tag{12}$$

The solution of this is simple again:

$$\frac{T_\infty - \overline{T}}{T_\infty - T_i} = \exp\left(-\frac{U}{\rho c_p L_c} t\right) \tag{13}$$

This solution has been found to be suitable for accurately predicting the average temperature-time profiles at different *Biot* numbers with only small errors at the start of the heating processes (van der Sman, 2003).

$$\theta = \frac{T_\infty - \overline{T}}{T_\infty - T_i} = \exp\left(-\frac{1}{\left(\frac{1}{\beta} + \frac{\delta_c}{\alpha}\right) \cdot L_c} t\right) \tag{14}$$

Before the heat penetration reaches δ_c, the following may be used to approximate the initial warming up:

$$\delta \approx \sqrt{12 \cdot \alpha \cdot t} \tag{10}$$

therefore,

$$\theta = \frac{T_\infty - \bar{T}}{T_\infty - T_i} = \exp\left(-\frac{1}{\left(\frac{1}{\beta} + \frac{\sqrt{12t}}{\alpha^{1/2}}\right) \cdot L_c} t\right) \qquad (14a)$$

As soon as this thickness δ_c is exceeded, the fixed δ_c is used. This 'warming up' period may become more important for thicker or larger samples.

Equation (14) shows a clear understanding that when β is very large, the controlling parameter is α. Taking the equivalent sphere of the cube sample (assuming equal volume) used by Palazoglu (2006), the equivalent radius is 0.00744m. Thus the critical dimension proposed by van der Sman (2003) is δ_c =0.00744m /4≈ 0.00186m. The materials used by Palazoglu (2006) were potato tissue and polymethyl-pentene polymer (TPX) respectively. The physical properties are respectively:

Potato: ρ = 1090 kg.m^{-3}, k = 0.554 W.m^{-1}.K^{-1}, c_p = 3517 J.kg^{-1}.K^{-1}, α = 1.45 x 10^{-7} m^2.s^{-1};

TPX: ρ = 833 kg.m^{-3}, k = 0.17 W.m^{-1}.K^{-1}, c_p = 1968 J.kg^{-1}.K^{-1}, α = 1.04 x 10^{-7} m^2.s^{-1};

Two heat transfer coefficients were used to investigate the behaviour: 50 and 1000 W.m^{-2}.K^{-1} respectively. Equation (14a) was employed to predict the average temperature versus time with the two different h values. Figure 2 shows that the potato heats up more slowly than TPX when h = 50 W.m^{-2}.K^{-1}. This makes the point that the higher thermal diffusivity of potato (1.45 x 10^{-7} m^2.s^{-1}) did not make potato heats up faster. Figure 3 shows that the development of the average temperature profiles are similar when h = 1000 W.m^{-2}.K^{-1}.

One needs to note that the results shown so far and that by Palazoglu (2006) do not suggest that the conventional *Heisler* charts are incorrect, which are based on the exact solutions (Incropera and De Witt, 2002). In the *Heisler* charts, the dimensionless time is the Fourier number $F_o = \frac{\alpha t}{L_c^2}$. Furthermore, the value of the temperature history curve is determined by the value of Bi^{-1} (i.e. k/hL_c). One can see that both dimensionless parameters contain k thus the effect of k is encapsulated in both expressions. Therefore, it is not straightforward to identify the effect of h from the *Heisler* chart. A different time scale may be useful,

$$t^* = \frac{1}{\left(\frac{1}{\beta} + \frac{\delta_c}{\alpha}\right) \cdot L_c} \cdot t \qquad (15)$$

Note here δ_c may be made time-dependent for the 'warming up' period as mentioned earlier. However, the perfection of this requires more accurate approach, which is beyond the current scope of analysis.
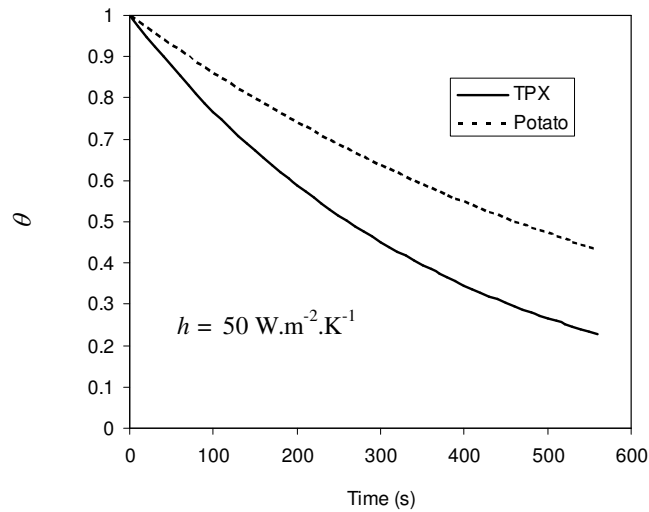

Figure 2 Average temperature versus time for $h = 50$ W.m^{-2}.K^{-1}

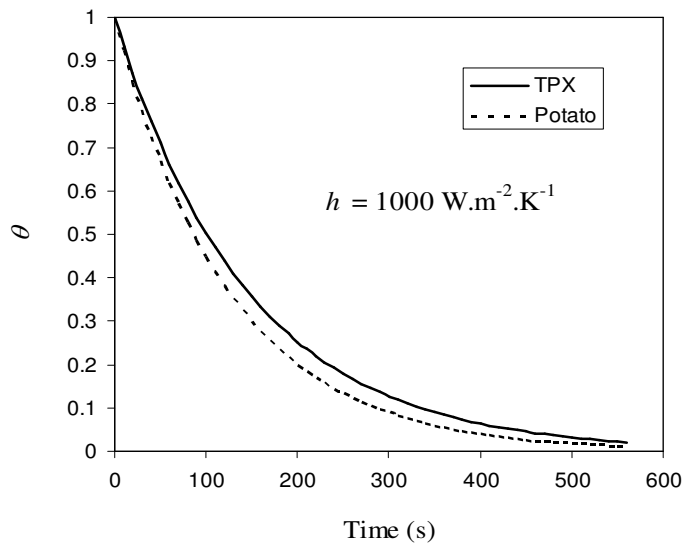Figure 3 Average temperature versus time for $h = 1000$ W.m^{-2}.K^{-1}

In any case, a plot of the temperature ratio such as θ as a function t^* would be very useful already. Actually as $\theta = \exp(t^*)$, the plot of t^* vs t (Figure 4) shows the clear trends that with higher heat transfer coefficient, dimensionless time is higher with higher thermal diffusivity. However when the heat transfer coefficient is relative lower, the opposite trend of dimensionless time vs thermal diffusivity is found.
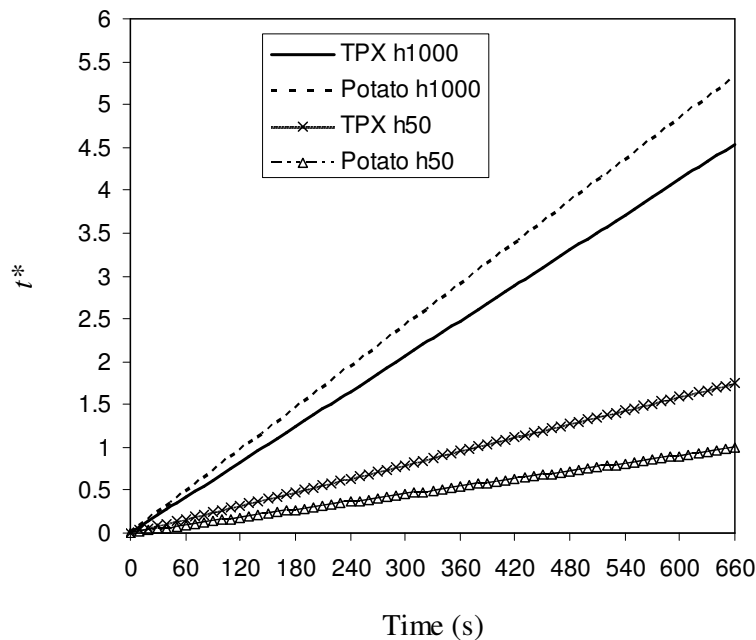

Figure 4 Dimensionless time vs time

Conclusion

In this paper, the analytical explanation of the observations made by Palazoglu (2005) is provided, which reinforces his observations in a generic way. Three cases have been considered which have adequately addressed the perception that thermal diffusivity alone can not determine the rate of heating. Two parameters α and β have been singled out which are useful in the context of the problem addressed in this study. A new t* vs t diagram has been drawn, which seems to be very instructive to show the matter of interest.

Notation

A	surface area or heat transfer area (m^2)
Bi	*Biot* number
c_p	specific heat capacity ($J.kg^{-1}.K^{-1}$)
h	heat transfer coefficient ($W.m^{-1}.K^{-1}$)
k	thermal conductivity ($W.m^{-2}.K^{-1}$)
L_c	length or dimension (m)
R	particle (sphere) radius (m)
t	time (s)
t^*	dimensionless time (s)
T	temperature (K)
T_∞	drying air temperature (K)
V	volume (m^3)
X	water content on a dry basis ($kg.kg^{-1}$)
δ_c	critical characteristic dimension (m)
ρ	density ($kg.m^{-3}$)

Subscripts

∞	bulk fluid or surrounding or infinitive value
1,2	surface 1 and surface 2
i	initial temperature
s	surface
c	charactreistic dimension

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