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Auctions and Posted Prices in Directed Search Equilibrium

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Benoit Julien, John Kennes, and Ian Paul King

Abstract

We compare equilibrium allocations in directed search models where prices are determined alternatively by posting and by competing auctions, with the following results. With finite numbers of players, sellers' expected payoffs are higher when all sellers auction than when all sellers post. This difference is largest in the 2-by-2 case, where payoffs to sellers are 1/3 higher if they auction. The difference in the payoffs decreases rapidly with market size and vanishes in the limit "large" economy. When sellers can choose whether to post prices or auction in the 2-by-2- case, all combinations (auction-auction, post-post, and auction-post) can occur in equilibrium if sellers choose mechanism and price simultaneously. However, if sellers choose mechanism before price then the dominant strategy equilibrium has both sellers auctioning.

KEYWORDS: Matching, Directed Search, Coordination, Competing Auctions

INTRODUCTION

Individuals are constrained by the number of hours in a day that they have to work with. Also, employers are often constrained by the number of jobs that they have to fill. These facts make the analysis of equilibria with capacity constraints, developed by Peters (1984), particularly relevant to the labor market. Recent work by Montgomery (1991), Julien, Kennes, and King (2000), and Burdett, Shi, and Wright (2001), and has explored the implications of this insight.[1] One major implication is that, when buyers randomize in their decision about which seller to approach, the equilibrium matching function generated by this process in large economies shares many properties with the matching function commonly used in macroeconomics (for example, Pissarides (2000)). Thus, these "directed search" models can be used to analyse factors that influence this matching process.

Although the models presented in these papers all imply the same matching function, there are some significant differences in the models. In particular, in Montgomery (1991) and Burdett, Shi, and Wright (2001), firms play the role of sellers -- selling jobs to workers. In Julien, Kennes, and King (2000), workers sell labor to firms. Also, in the first two papers, sellers post prices (wages) that they commit to, regardless of the number of buyers that approach them. In the third paper, instead, sellers commit to a bidding game where the good (labor) is sold to the highest bidder.[2] In general, the expected payoffs to buyers and sellers are not invariant to these distinctions. However, in large economies of this type (with infinite numbers of buyers and sellers), Kultti (1999) has shown that the expected payoff to sellers is the same if they post prices or auction.

In this paper, we examine the choice of sales mechanism in this type of environment where the numbers of buyers and sellers is finite. We first consider a general model, with arbitrary numbers of agents on each side of the market, and compare the expected payoffs for buyers and sellers under both price-posting and competing-auction assumptions. We find that, for any finite number of agents, the expected payoffs for sellers are always higher in the competing-auction game than in the price-posting game. This difference in payoffs is most significant in the 2-by-2 case, where sellers receive 1/3 more when they auction. However, the difference gets small quite quickly as either the scale of the economy increases or as the buyer-seller ratio increases, and vanishes in both limits.

We then consider the problem of sellers choosing which sales mechanism to use (price-posting or competing-auction) in a strategic setting with other sellers. Here, we restrict attention to the 2-by-2 case, where the difference in the payoffs is most significant. We consider two settings. In the first, sellers choose their mechanism and price simultaneously. In the second, sellers choose their mechanisms first, then (once the mechanism choice is revealed to all players) choose their prices. We find that, in the first setting, all possible combinations of mechanisms (both sellers posting, both sellers

[1] See, also, Shimer (1999) and Shi (2001a,b).
[2] Following Julien (1997) and Burguet and Sakovics (1999) this is referred to as the "competing auction" framework.

auctioning, and one seller auctioning while the other posts) can exist in equilibrium. Thus, even though both sellers are significantly better off if both auction in small markets, we can observe both auctioning and posting in equilibrium. However, in the second setting, the unique subgame perfect equilibrium has both sellers choosing to auction.

The remainder of the paper is organized as follows. The general model, with arbitrary numbers of buyers and sellers is laid out in Section 1. Section 2 presents the 2-by-2 game where sellers can choose sales mechanisms. Finally, Section 3 presents the conclusion and some discussion.

1. THE GENERAL MODEL

The market consists of $N \geq 1$ identical risk neutral sellers and $M = bN$ identical risk neutral buyers, where $b > 0$ is a parameter. Each seller has one unit of the good to sell. If she sells at price p then she obtains payoff p; if she does not sell, she receives a payoff of zero. Each buyer wants to buy one unit of the good and is willing to pay up to his reservation price, which is normalized to 1. If a buyer purchases at price p, then his payoff is $1 - p$. If he is unable to buy, his payoff is zero. Each buyer has only one opportunity to buy, and must choose only one seller to visit.[3]

1.1 *The Price-Posting Game*

The price-posting game has the following structure. First, each seller announces her price and rationing rule. The seller is committed to selling at the announced price if at least one buyer makes an offer at that price. If $m \geq 1$ buyers offer to buy from her at that price, each buyer will be able to purchase from her with probability $1/m$. Second, after observing all of the announced prices, each buyer chooses which seller to visit (that is, which seller to make an offer of purchase to). Finally, once buyers have been allocated across sellers, the good is allocated to buyers according to the rationing rule. Equilibria are computed by backward induction.

Multiple asymmetric pure strategy equilibria exist in this model but, as in Montgomery (1991), Burdett, Shi and Wright (2001) and Shi (2001a,b), attention is focussed here on the unique symmetric mixed strategy equilibrium in which each seller charges the same price and buyers randomize over sellers -- visiting each seller with the same probability $\pi \in (0,1)$.[4] In this section, we report the equations derived in Burdett, Shi and Wright (2001).

[3] For dynamic versions of the price-posting and competing-auction games, where buyers can visit one seller in each period, see Cao and Shi (2000) and Julien, Kennes and King (2000) respectively.
[4] This is usually justified by pointing out the high degree of coordination among buyers required to implement the pure strategy equilibria. Experimental evidence also suggests that buyers have trouble coordinating in this way, even in very small markets (Ochs, 1990). See Cao and Shi (2000) for further discussion on this point.

The probability that each buyer assigns to visiting each seller is:

$$\pi = 1/N \tag{1.1}$$

The expected number of matches (or the "matching function") is given by:

$$x(N,b) = N(1-(1-1/N)^{bN}) \tag{1.2}$$

The equilbrium price of the good is:

$$p_P(N,b) = \frac{N - N\left(1+\dfrac{bN}{N-1}\right)(1-1/N)^{bN}}{N-\left(N+\dfrac{bN}{N-1}\right)(1-1/N)^{bN}} \tag{1.3}$$

The expected payoff to each seller is:

$$S_P(N,b) = (1-(1-1/N)^{bN})p_P(N,b) \tag{1.4}$$

A key feature to note in this equilibrium is that the sellers receive the price $p_P(N,b)$ when *at least one* buyer approaches them. This occurs with probability $(1-(1-1/N)^{bN})$, (where $(1-1/N)^{bN}$ is the probability of no buyers approaching). The expected payoff to the seller is simply the product of these two expressions.

1.2 *The Competing-Auction Game*

This game has the following structure. First, each seller announces a reserve price and a rationing rule. The seller is committed to selling the good in a bidding game with the following rules. If only one buyer approaches the seller, then she will sell the good to that buyer at the reserve price. If more than one buyer approaches, then the good will be sold to the highest bidder. If this price is offered by $m \geq 2$ buyers then each of these buyers will receive the good with probability $1/m$. Second, once buyers have observed all announced reserve prices, each buyer chooses which seller to visit. Finally, once buyers have been allocated across sellers, the good is allocated according to the bidding game. Again, equilibria are computed by backwards induction, and attention is restricted to the unique symmetric mixed strategy equilibrium. In this equilibrium each seller announces the same reserve price *r* and buyers randomize over sellers, visiting each seller with the same probability $\pi \in (0,1)$.

From Julien, Kennes and King (2000), this equilibrium has the following properties. Exactly as in the price-posting game, the probability that each buyer assigns to each seller is given in equation (1.1). Also, the equilibrium matching function is given in equation (1.2). However, in the competing-auction game, there is price dispersion in equilibrium: sellers that have only one buyer visit them receive only the reserve price, while sellers that have more than one buyer visit are able to receive the entire value of the surplus (unity). The equilibrium reserve price of the good is:

$$r(N,b)=\frac{bN-1}{bN+N(N-2)} \tag{1.5}$$

The equilibrium price of the good is:

$$p_A(N,b)=\begin{cases} r(N,b) & if \quad m=1 \\ 1 & if \quad m\geq 2 \end{cases} \tag{1.6}$$

The expected payoff to each seller is simply the weighted sum of the prices the seller receives, where the weights are the probabilities of each event. If no buyers arrive, the seller receives nothing. This occurs with probability $(1-1/N)^{bN}$.[5] If one buyer arrives, the price is $r(N,b)$. The probability of this happening is $b(1-1/N)^{bN-1}$. If at least two arrive then the price is unity. The probability associated with this event is $1-(1-1/N)^{bN}-b(1-1/N)^{bN-1}$. Collecting terms, one obtains:

$$S_A(N,b)=1-(1-1/N)^{bN}-b(1-1/N)^{bN-1}\left(1-r(N,b)\right) \tag{1.7}$$

1.3 *Comparing Equilibrium Payoffs in the Two Games*

The difference between the expected payoffs that sellers receive, in the competing-auction and the price-posting games, can now be found from equations (1.4) and (1.7):

$$\begin{aligned} S_A(N,b)-S_P(N,b)=1-(1-1/N)^{bN}-b(1-1/N)^{bN-1}\left(1-r(N,b)\right) \\ -\left(1-(1-1/N)^{bN}\right)p_P(N,b) \end{aligned} \tag{1.8}$$

where $r(N,b)$ and $p_P(N,b)$ are given in (1.5) and (1.3) respectively. Figure 1 illustrates this difference, for different values of *N* and *b*.

[5] When workers are sellers, this probability is the expected unemployment rate.

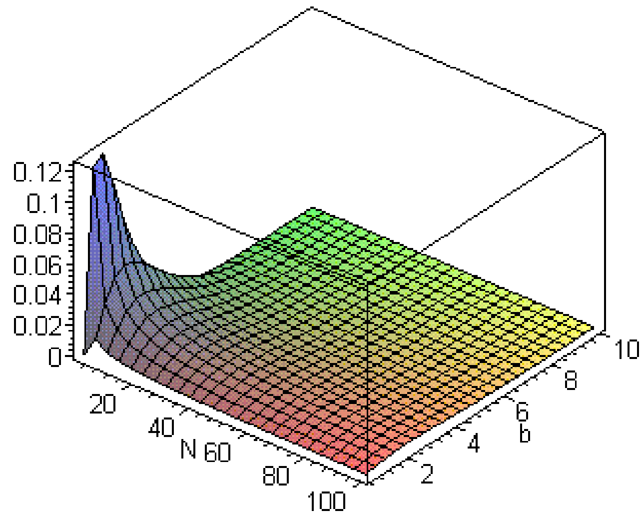

Figure 1: The Difference of the Expected Payoffs

This figure shows that, for all finite N and b, where $N \geq 2$ and $M \geq 2$, sellers' expected payoffs in the competing-auction game are strictly larger than in the price-posting game. This difference is maximized in the 2-by-2 case (where N = 2, b = 1). In this case, the expected payoffs under auctioning and price posting are 0.5 and 0.375 respectively. That is, sellers earn 1/3 more if they auction. This premium drops off precipitously as either N or b increase. The difference is less than 1% of the value of the surplus for all N > 30 and all b > 5 and converges uniformly to zero as $N \to \infty$ and as $b \to \infty$.

The reasoning behind the result that the payoffs converge as $b \to \infty$ is very straightforward: in either game, as the number of buyers per seller gets arbitrarily large, each seller is able to extract the entire surplus. It is less obvious why the payoffs converge as $N \to \infty$. As N increases, this erodes the advantage that sellers have when announcing their reserve prices. In finite-sized markets, where the reserve price is positive, the auction gives sellers the advantage of being able to exploit the *ex post* allocation of buyers, when more than one buyer approaches the seller. (For example, in the 2-by-2 case, both r and p_P equal 1/2 but sellers in the competing-auction receive a premium if more than one buyer approaches the seller -- the price equals one.) As the market increases, however, the reserve price is driven down to zero (while the posted price converges to a positive number: $(1-e^{-b}-be^{-b})/(1-e^{-b})$). Posted price sellers care only about generating at least one offer while competing-auction sellers care about generating at least two offers in order to extract the surplus. For this reason, sellers using the auction are more aggressive at reducing equilibrium reserve prices, and the advantage of being able to exploit *ex post* opportunities disappears.

2. CHOOSING A SALES MECHANISM IN THE 2-BY-2 ECONOMY

In this section, we allow sellers in a market to choose whether to post a price or to auction their good. We confine our attention to the 2-by-2 case, where the differences in the payoffs that agents receive are the most dramatic. We analyze the problem by the sequential development of three key games in which (i) both sellers post prices, (ii) both sellers auction, and (iii) one seller posts a price and the other auctions. The games (i) and (ii) are identical to the games considered in sections 1.1 and 1.2 respectively above, with M and N set equal to 2. As will become clear below, it is useful to work through the 2-by-2 cases because the reaction functions will be used when considering the equilibrium choices of sales mechanism.[6]

2.1 *Both Sellers Posting Prices*

Here, each seller chooses a posted price p_j where $j \in \{1,2\}$ is used to index sellers. Each buyer then chooses to visit either seller based on the posted price of each seller and the expected behavior of the other buyer. The probability that a buyer visits seller j is given by π_i^j where i is used to index buyers. Buyer i's expected payoff from visiting seller j is given by:

$$B_P(i, j) = (1-\pi_{-i}^j)(1-p_j) + \pi_{-i}^j(1-p_j)/2 \tag{2.1}$$

where π_{-i}^j is the probability that seller i is not the only potential buyer visiting seller j, in which case the good is rationed according to the symmetric rationing rule. In the mixed strategy equilibrium each buyer is indifferent about which seller he visits. Moreover, since each buyer faces the same vector of posted prices, the probability that each buyer visits seller j is the same. Thus, $B_P(i,1) = B_P(i,2)$ and $\pi_i^j = \pi^j$ for all i. Using these conditions, together with (2.1), the probability π^j that any particular buyer visits seller j becomes:

$$\pi^j = \frac{1-2p_j + p_{-j}}{2-p_j - p_{-j}} \tag{2.2}$$

Expression (2.2) is the reaction function of buyers to the posted prices of sellers. The probability that seller j is able to sell her good is:

$$q(p_j, p_{-j}) = 2\pi^j(1-\pi^j) + (\pi^j)^2$$

[6] At this point, it is useful to make a slight change in notation. In the previous sections of this paper, the arguments in the functions emphasized the numbers of players on each side of the market. (Attention was restricted entirely to symmetric equilibria, it was unnecessary to index particular agents.) In this section, the numbers of players on each side of the market are set at two, but we allow for asymmetric equilibria. For this reason, the arguments in the functions are used to index particular agents.

where $2\pi^j(1-\pi^j)$ is the probability that the seller j faces one potential buyer and $(\pi^j)^2$ is the probability that she faces two (in which case, she can sell only to one). We can think of $q(p_j, p_{-j})$ as the demand function for good j. A key feature of this demand function is that it is continuous and differentiable. Moreover, under posted prices, the seller has the same payoff if one or two potential buyers come to visit. Therefore, the expected payoff to seller j from offering posted price p_j is given by:

$$S_P(p_j, p_{-j}) = q(p_j, p_{-j})p_j \tag{2.3}$$

The solution to each seller's problem is found simply by differentiating her payoff function with respect to her posted price, and setting this derivative equal to zero. In this way we obtain the following reaction function for seller j:

$$p_j(p_{-j}) = \frac{(p_{-j}+1)(p_{-j}-2)}{5p_{-j}-7} \tag{2.4}$$

The intersection of the sellers' reaction functions gives the equilibrium posted price of $p_j^* = 1/2$. The equilibrium prices of this subgame can then be substituted into the payoff functions of each seller to give the equilibrium payoff of:[7]

$$S_P(p_j^*, p_{-j}^*) = 0.375. \tag{2.5}$$

2.2 *Both Sellers Auctioning*

As in section 3 above, each seller chooses a *reserve* price r_j. Each buyer then chooses to visit either seller based on the reserve price of each seller and the expected behavior of the other buyer. Buyer i's expected utility from visiting seller j is given by:

$$B_A(i, j) = (1-\pi_{-i}^j)(1-r_j) \tag{2.6}$$

where $(1-\pi_{-i}^j)$ is the probability that the buyer is alone at this auction. If the buyer is not alone, then competitive bidding ensures that the sale price of the good is unity, in which case the buyer does not obtain any utility from purchasing the good. In the mixed strategy equilibrium, each buyer is indifferent about which seller he visits. Moreover, since each buyer faces the same vector of reserve prices, the probability that any particular buyer visits seller j is the same. Therefore, $B_A(i,1) = B_A(i,2)$ and $\pi_i^j = \pi^j$ for all i.

[7] Notice that, using the notation from Section 1.1, from (1.4) we get: $S_P(2,1) = .375$.

Consequently, using (2.6), the probability π^j that any particular buyer visits seller j is given by:

$$\pi^j = \frac{1-r_j}{2-r_j-r_{-j}} \tag{2.7}$$

This expression is the reaction function of the buyers to the reserve prices of the sellers. Sellers exploit this function in their reserve price decision. The probability that seller j has exactly one potential buyer visit is given by $q^1(r_j,r_{-j})=2\pi^j(1-\pi^j)$, while the probability that she has two potential buyers visit is $q^2(r_j,r_{-j})=(\pi^j)^2$. Whereas in the posted price case, the number of potential buyers who visit has no bearing on the price at which the good is sold (as long as this number is not zero), in the auction case, this is not true. In particular, the good is sold at the reserve price r_j if only one potential buyer visits seller j, but sold at the price of unity (the buyer's valuation) if more than one potential buyer visits. Thus, the payoff $S_A(r_j,r_{-j})$ to seller j offering reserve price r_j is given by:

$$S_A(r_j,r_{-j}) = q^1(r_j,r_{-j})r_j + q^2(r_j,r_{-j}) \tag{2.8}$$

The solution to the seller's problem is found by differentiating the seller's payoff function (2.8) with respect to r_j and setting the result equal to zero. Doing so, we find the following reaction function for seller j:[8]

$$r_j(r_{-j}) = 1/2 \tag{2.9}$$

The expected payoff function of the seller (2.8) is continuous and concave, given the other seller is choosing the equilibrium reserve price. Therefore, the vector of reserve prices satisfying the reaction functions (2.9) is an equilibrium of this subgame. The reaction functions determine the equilibrium reserve price for each seller at $r_j^* = 1/2$. These choices can now be substituted back into the payoff functions to obtain the equilibrium expected payoff to each seller:[9]

$$S_A(r_j^*,r_{-j}^*) = 1/2. \tag{2.10}$$

[8] The non-responsiveness of this reaction function to the reserve price of the other seller deserves some comment. A higher reserve price by the other seller clearly raises the probability that a buyer will visit seller j. However, seller j chooses not to raise her reserve price in response because she would then discourage potential buyers from visiting her own auction. When the number of buyers equals the number of sellers (as in this 2 by 2 case) these effects exactly cancel out. More generally, when the number of buyers and sellers differ, these reaction functions have a positive slope.

[9] Using the notation of Section 1.2, from (1.7) this payoff is: $S_A(2,1) = 1/2$.

2.3 *One Seller Posting a Price, the Other Auctioning*

The auction and posted-price subgame, developed here, is similar to the previous two subgames, except that one seller posts a price while the other conducts an auction. We denote the posted price seller by her posted price *p*, and the auction seller by her reserve price r. The probability that buyer i visits the *posted price* seller is denoted by π_i. The expected utility that buyer i receives if he visits the posted price seller is given by:

$$B_{PA}(i,P) = (1-\pi_{-i})(1-p) + \pi_{-i}(1-p)/2 \tag{2.11}$$

while the expected utility from visiting the seller who auctions is given by:

$$B_{PA}(i,A) = \pi_{-i}(1-r) \tag{2.12}$$

As in the previous two subgames, an equilibrium in mixed strategies exists for this subgame such that each buyer is indifferent about which seller to visit. Also, since each buyer faces the same vector of prices, the probability that any particular buyer will visit a particular seller is the same. Therefore, $B_{PA}(i,P) = B_{PA}(i,A)$ and $\pi_i = \pi$ for all i. These two conditions allow us to solve for the probability π that any particular buyer will visit the posted price seller:

$$\pi = \frac{1-p}{3/2-r-p/2} \tag{2.13}$$

Equation (2.13) expresses the reaction function of the buyers to the reserve and posted prices of the two sellers. Both sellers exploit this function when making their posted/reserve price decisions. The expected payoff to a seller offering posted price *p* is given by:

$$S_{PA}(p,r) = q(p,r)p \tag{2.14}$$

where $q(p,r) = 2\pi(1-\pi) + (\pi)^2$ is the probability that the posted price seller receives a visit from at least one buyer. The expected payoff of the seller offering the reserve price r is given by:

$$S_{PA}(r,p) = q^1(r,p)r + q^2(r,p) \tag{2.15}$$

where $q^1(r,p) = 2\pi(1-\pi)$ is the probability that one buyer will visit the auctioning seller, and $q^2(r,p) = (1-\pi)^2$ is the probability that two will.

Differentiating the payoff functions of each seller with respect to the relevant decision variable and setting these derivatives equal to zero gives two reaction functions. First, for the price-posting seller, we have:

$$p(r) = \frac{2r-3}{4r-5} \tag{2.16}$$

and second, for the auctioning seller, we have:

$$r(p) = \frac{1+p}{6} \tag{2.17}$$

The intersection of these two reaction functions yields the equilibrium reserve price $r^* = 0.2713$ and posted price $p^* = 0.6277$. Substituting these choices into the sellers' expected payoff functions yields the expected payoff for the price-posting seller as:

$$S_{PA}(p^*, r^*) = 0.4069 \tag{2.18}$$

and the expected payoff for the auctioning seller as:

$$S_{PA}(r^*, p^*) = 0.4827 \tag{2.19}$$

2.4 *Equilibrium*

The three games presented in sections 2.1 - 2.3 determine the reaction functions of each seller, and the equilibrium prices and payoffs, given the selling mechanism of both sellers. In this section, we determine the choice of the selling mechanism in two different settings. First, we consider equilibria the when sellers choose both mechanism and price simultaneously. We then consider subgame perfect equilibria in a sequential game where sellers first choose mechanisms, then choose prices. The first proposition summarizes the result in the simultaneous choice game.

***Proposition* 1:**

When sellers choose both mechanism and price simultaneously, all possible combinations (post-post, auction-auction, post-auction, and auction-post) can exist in equilibrium.

***Proof*:**

We consider deviations from each candidate equilibrium in turn. If both sellers post and choose the equilibrium price in that case, then the equilibrium prices and payoffs for both sellers are as given in Section 2.1: $p_j^* = 1/2$, $S_P(p_j^*, p_{-j}^*) = 0.375$. Consider now a seller who considers deviating by choosing to auction and picking the reserve price that is the optimal response to the other seller posting $p = 1/2$. From the reaction function (2.17) we have: $r(1/2) = 1/4$. Now substituting p = 1/2 and r = 1/4 into equation (2.13) one obtains: $\pi = 1/2$. Using this in equation (2.15) yields: $S_{PA}(r = 1/4, p = 1/2) = .375$. Hence, deviation by either player from this candidate equilibrium will not increase the payoff for either player, so this is an equilibrium.

If both sellers auction and choose the equilibrium reserve price in that case, then the equilibrium reserve prices and payoffs for both sellers are as given in Section 2.2: $r_j^* = 1/2$, $S_A(r_j^*, r_{-j}^*) = 1/2$. Consider now a seller who considers deviating by choosing to post and picking the posted price that is the optimal response to the other seller using $r = 1/2$. From the reaction function (2.16) we have: $p(1/2) = 2/3$. Now substituting $p = 2/3$ and $r = 1/2$ into equation (2.13) one obtains: $\pi = 1/2$. Using this in equation (2.14) yields: $S_{PA}(p = 2/3, r = 1/2) = 1/2$. Hence, deviation by either player from this candidate equilibrium will not increase the payoff for either player, so this is an equilibrium.

If one seller auctions and the other posts a price, using the equilibrium prices, then the equilibrium reserve prices, posted prices, and payoffs are as given in Section 2.3: $r^* = .2713$, $p^* = .6277$, $S_{PA}(r^*, p^*) = .4827$, $S_{PA}(p^*, r^*) = .4069$. Suppose that the auctioning seller deviates by choosing to post, assuming that the other seller continues to post, but at the posted price in the candidate equilibrium. From the reaction function (2.4) we have: $p(.6277) = .5785$. Using this in (2.2) we obtain: $\pi = .5931$. Using this in (2.3), yields: $S_P(.5785, .6277) = .4827$. Thus, the auctioning seller is made no better off by deviating. Similarly, suppose the posting seller chooses to deviate by auctioning, assuming that the other seller continues to auction, but at the reserve price in the candidate equilibrium. From the reaction function (2.9) we have: $r(.2713) = 1/2$. Using these reserve prices in (2.7), we obtain: $\pi = .4069$. Using this in (2.8) yields: $S_A(.5, .2713) = .4069$. Thus, the posting seller is made no better off by deviating. ■

***Proposition* 2:**

In the sequential game, where sellers choose mechanisms before prices, there is a unique subgame perfect equilibrium. In this equilibrium, both sellers auction.

***Proof*:**

Equations (2.5) and (2.10), respectively, give the payoffs to sellers in the *subgames* where both sellers post and auction, choosing their prices (reserve prices) simultaneously. Equation (2.18) gives the payoff to a seller who posts in the subgame where one seller posts, while the other auctions, when both sellers choose their price (reserve price) simultaneously. Similarly, equation (2.19) gives the payoff to a seller who auctions in the subgame where one seller posts, while the other auctions, when both sellers choose their price (reserve price) simultaneously. Using the payoffs from the subgames to construct a normal form for the first stage of the game, we have:

	Auction	Price Post
Auction	.5, .5	.4827, .4069
Price Post	.4069, .4827	.375, .375

The result follows from this matrix, where (top, left) is the unique equilibrium. ■

2.5 Discussion

Clearly, both sellers are better of if they both auction. However, if they choose both mechanism and price simultaneously, multiple equilibria exist[10] and sellers face a coordination problem. In fact, as shown in the proof of Proposition 1, when considering deviations from any candidate equilibrium, sellers are exactly indifferent between equilibrium and deviant actions. This arises from the fact that any deviation induces a different participation strategy from buyers – buyers understand the tradeoff between price and probability of trade and adjust their visit probabilities accordingly.

Arguably, though, the auction-auction equilibrium represents a natural focal point. This intuition is reinforced by the fact that, if sellers *sequence* their decisions (mechanism first, price second) the coordination problem can be eliminated in the sense that the unique subgame perfect equilibrium has all sellers auctioning.

3 CONCLUSIONS AND DISCUSSION

Given the coordination problem that is assumed among buyers here, sellers are better off if they all auction their goods, rather than post prices. This provides a justification for using auctions even in the absence of informational asymmetries. The premium that sellers get from auctioning is quite significant in the smallest, 2-by-2 case, but declines smoothly and rapidly with market size. In the limit, as the market size increases, the expected payoffs to sellers under the two different mechanisms converge. In the smallest market that we consider here, (the 2-by-2 market), a coordination problem also exists for sellers if they choose mechanism and price simultaneously. However, if the mechanism is chosen before the price, the unique subgame perfect equilibrium has both sellers choosing to auction.[11]

In the context of the labor market, we should expect to see workers in smaller, more specialized, markets auctioning their services. In this analysis, we abstracted from the costs of conducting auctions and posting prices. Given that auctions are more costly, it makes sense for sellers in larger markets to forgo the premium from the auction, to avoid the extra expense.

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[10] Coles and Eeckhout (2001) find similar results, but where the focus is on the continuum of mechanisms that lie between price-posting and auctions.

[11] An astute referee pointed out that these results could change if agents are risk averse – because auctions are riskier than posted prices. However, the degree of risk aversion required to change these results would have to be quite severe. The results are preserved, for example, if utility takes the square root function.

The literature on directed search has grown rapidly. For example, Coles and Eeckhout (1999), Acemoglu and Shimer (2000), Cao and Shi (2000) and Shi (2001a,b), among others, have explored the influence of heterogeneity in this type of model. Both the simplicity of the framework and the constrained efficiency of the equilibrium in large markets make this a promising direction for future research.

Colophon

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