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Eclipsing Binaries in the MOA-II Database

Man Cheung Alex Li

Supervisor: Dr. N. J. Rattenbury

Department of Physics
University of Auckland

This dissertation is submitted for the degree of
Doctor of Philosophy

University of Auckland             February 2019
I dedicate this thesis to my loving parents.
I imagine this midnight moment’s forest:
Something else is alive
Beside the clock’s loneliness
And this blank page where my fingers move.

Through the window I see no star:
Something more near
Though deeper within darkness
Is entering the loneliness.

– Ted Hughes, *The Thought Fox*
Declaration

I hereby declare that except where specific reference is made to the work of others, the contents of this dissertation are original and have not been submitted in whole or in part for consideration for any other degree or qualification in this, or any other university. This dissertation is my own work and contains nothing which is the outcome of work done in collaboration with others, except as specified in the text and Acknowledgements. This dissertation contains fewer than 65,000 words including appendices, bibliography, footnotes, tables and equations and has fewer than 150 figures.

Man Cheung Alex Li
February 2019
I am immensely grateful to my supervisor, Nicholas Ratterbury. I thank him for all his advice, support, encouragement and the freedom he gave me over the years. I thank him for letting me participate in MOA observations. I will always remember those lonely but joyful nights on Mt. John.

I thank all the other astronomers at the University of Auckland, including JJ Eldridge, Richard Easther, Grant Christie, Martin Donachie, John Bray and Lin Xiao, who showed me how wonderful it is to study astronomy and to love the diversity of our world. I thank Ian Bond for his data preparation for my research. I also thank Gary Ferguson for proofreading this thesis.

I particularly acknowledge the contribution of the NZ eScience Infrastructure high-performance computing facilities to the results of this project. NZ’s national facilities are provided by the NeSI and funded jointly by NeSI’s collaborator institutions and through the Ministry of Business, Innovation & Employment’s Research Infrastructure Programme.

Finally, I mostly thank my parents even though they would not understand anything, even a word, in this thesis. Even though I am such a foolish son who always makes them worry, they always give me immense support and encouragement. I am eternally grateful for their sacrifices to help me achieve my selfish dream.
The present evolution of the Microlensing Observations in Astrophysics (MOA-II) project began in 2006. MOA-II monitors millions of stars simultaneously towards the densely populated regions of the Galactic bulge (GB) using a 1.8m telescope with a wide-field CCD camera and has resulted in about 100TBs of image data spanning over nine years to date. Such a large amount of data is a valuable resource for variable star research. This thesis is focused on the aspects of eclipsing binary (EB) study using the MOA-II data including identifying EBs in the MOA-II database and searching for tertiary companions in EBs via eclipse time variation (ETV) analysis. We first present the first catalogue of EBs in two MOA fields towards the GB, i.e., GB9 and GB10, in which 8733 EB candidates, mostly contact and semi-detached binaries of periods $< 1$ day, were identified using two MOA observational seasons’ worth of data. At this stage, we also identified three triple candidates among these 8733 EB candidates by detecting the light travel time effect (LTTE) signals in their ETV curves. A sample of 542 EBs with periods less than two days in two subfields in GB9 and GB10 - GB9-9 and G10-1 - were selected for further ETV analysis. For this sample we were able to obtain the full time series from MOA-II that spans 9.5 years. We discovered 91 EBs among these 542 EBs with detected LTTE signals suggesting the presence of tertiary companions of orbiting periods from 250 days to 28 years. The frequency of EBs with tertiary companions in our sample increases as the period decreases and reaches the value of 0.65 for contact binaries of periods $< 0.3$ days. If only the contact binaries of periods $< 0.26$ days are considered, the frequency even goes to the unit. Our results suggest that contact binaries with periods close to the 0.22-day contact binary limit are commonly accompanied by relatively close tertiary companions.
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**Glossary**

**AME** apsidal motion effect. 68

**AU** astronomical unit. 77, 81, 99, 120–124

**BER** beaming, ellipsoidal and reflection. xiii, 19

**BIC** Bayesian information criterion. 88, 99, 100, 103

**BJD** Barycentric Julian Date. 49, 73, 74

**CCD** charge-coupled device. 8, 15, 25, 57, 68

**CE** conditional entropy. 47, 48, 60, 63, 74

**CM** centre of mass. 11, 13, 20, 23–25

**DIA** difference imaging analysis. 5, 9


**ELVs** ellipsoidal variables. 63, 118


**GB** Galactic bulge. 4, 9, 10, 46, 51, 57, 58, 61, 81, 82, 85, 88, 89

**JD** Julian Date. 49, 73, 74

**LMC** Large Magellanic Cloud. 3, 4

**MACHO** massive compact halo object. 5, 132–134

**MBJD** Modified Barycentric Julian Date. 81, 120–124

**MCMC** Markov chain Monte Carlo. 78, 128


**OGLE** Optical Gravitational Lensing Experiment. xiii, 3, 5, 9, 28, 30, 31, 36, 39, 41–43, 46, 51, 52, 57, 58, 61–63, 127–129, 134, 135

**PCEB** post-common-envelope binary. 60

**PDM** phase dispersion minimization. 48, 51, 57

**PFA** period finding algorithm. 47

**PSF** point spread function. 9

**QTV** quadrature time variation. 118

**SMC** Small Magellanic Cloud. 3, 4
Chapter 1

Introduction

If people sat outside and looked at the stars each night, I’ll bet they’d live a lot differently.

Bill Watterson, Calvin and Hobbes

1.1 The Glowing Band in the Night Sky

The Universe is visible to our human eyes because stars burn their bodies to give light. Stars are not distributed randomly over the Universe. Rather, the vast majority of them lives in galaxies. The Sun, as the central star of our Solar System, like other visible stars in the night sky, resides in the Milky Way Galaxy, a spiral galaxy which consists of a thick disk with spiral arms and a bulge at the Galaxy’s central portion [56]. The spiral arms are associated with active star formation regions and the locations of young stellar populations, while the Galactic bulge is mainly populated by old stars \( \sim 10 \text{Gyr} \) in age [44, 148]. The Milky Way Galaxy contains hundred billion stars. The Sun lies in the inner part of the Orion Arm, located at a distance of 8.32kpc from the Galactic centre [73]. As viewed from the Earth, the Galactic disk appears as a glowing band arching across the night sky, which the Romans named Via Lactia. When we take a photograph of the Milky Way, the brightest regions should be around the edge of the Galactic bulge, while the central part of the Milky Way appears as a bulk of darkness because of a large amount of dust in the Galactic plane.

In spite of the Galactic bulge fields being the brightest regions, indicating the highest number of stars, they are also infamous for high interstellar extinction [146] and severe crowdedness for optical surveys [169]. The latter suggests that stars in the
Galactic bulge fields have high probability of suffering from blending by nearby stars. In the case of a variable star survey, for example, a single star blended by a background variable object can lead to a false positive detection. Also, the high fraction of stars’ spectra blended by unwanted stars might be a fatal problem for transit surveys such as the *Kepler* space mission which aims to detect planetary transits of solar-like stars [32]. A transit occurs when there is an unseen object passing in front of its host star, resulting in periodical dimming of the star’s brightness. By this technique known as the transit method, we can determine the radius ratio between the unseen object and its host star as well as the orbital period and inclination. Yet the transit method alone cannot give us information about the mass of the unseen object. It generally requires a combination of radial velocity measurements of the star via spectroscopy and the photometric transit light curve for the mass of the unseen object to be measured unambiguously and, therefore, its planetary identity conclusively verified (see Section 1.4.3). The contamination in the target stars’ spectra due to blending by unwanted stars might ruin the possibility of extracting spectroscopic information and, thus, undermine the rate of confirmed planet detection in a transit survey [74]. Concerning this issue, the Galactic bulge fields were disfavoured for transit surveys.

### 1.2 Background & Motivation

Interestingly, the Galactic bulge fields which are not ideal for planetary transit surveys are nevertheless ideal places for finding planets via microlensing. Indeed, all active ground-based microlensing surveys today such as the Optical Gravitational Lensing Experiment (OGLE), Microlensing Observations in Astrophysics (MOA) and Korea Microlensing Telescope Network (KMTNet) are dedicating most of their telescope time to the detection of planetary signals in microlensing events towards the Galactic bulge. The usefulness of microlensing, however, became appreciated only in recent decades. Microlensing is a transient phenomenon which occurs when two point sources such as two stars align along the line of sight and results in a variation in the background star’s flux. The probability of observing a microlensing event depends on the density of a field being monitored. For fields towards the Magellanic Clouds, the probability is roughly $10^{-6}$ at any time [150]. Just like many other astrophysical phenomena, when and where a microlensing event will emerge is practically unpredictable. Because of their rarity and randomness, the detection of microlensing events requires constantly monitoring a densely populated region which contains millions of stars. A microlensing survey,
therefore, would also be intrinsically a massive photometric survey for the variability in the sky. In fact, Bohdan Paczynski, the pioneer of microlensing surveys as well as the founder of the OGLE collaboration, ended his classic paper on the detection of massive compact halo objects (MACHOs) [150] with a paragraph stating that one of the attractive by-products of a microlensing observational project would be a systematic discovery of a large number of variable stars in a nearby galaxy even though no lensing events might be discovered. He also provided a review on the prospects of stellar research that microlensing surveys could enable [151]. The aspect that interested me, amongst all his suggestions, is the possibility of obtaining a catalogue of complete, homogeneous samples of various types of variable stars.

Over the last two decades, the OGLE group has discovered a vast amount of variable objects including different types of Cepheids (i.e., classical, type II and anomalous Cepheids), RR Lyrae stars and eclipsing binaries in the fields towards the Magellanic Clouds (LMC and SMC) and Galactic bulge and disk. In particular, the number of eclipsing binaries identified in the Galaxy has exploded unprecedentedly. The OGLE project alone has discovered over 450,000 eclipsing binaries [190], which is 156 times more than the total eclipsing binaries identified from the entire Kepler data set [105]. Meanwhile, the OGLE database also contains thousands of Cepheids and over 45,000 RR Lyrae stars [196], known to be useful as distance indicators, towards the Magellanic Clouds. Among them, unusual RR Lyrae and Cepheids of double or triple mode were found (e.g. [192, 187, 191]). The OGLE group also discovered six heartbeat stars in the LMC [145], a subclass of eccentric binaries, with ellipsoidal variations that mimic electrocardiograms. The OGLE public data can be downloaded via its server ftp://ftp.astrouw.edu.pl/ogle and have been used to develop machine learning techniques (e.g. [153]), to derive period-luminosity relations of various types of variable stars (e.g. [82, 155]) and to serve as benchmarks for testing the Gaia space satellite performance [215]. Incorporated into spectroscopic observations, the OGLE data in the LMC fields led to the interesting discovery of a class of eclipsing binaries consisting of luminous B-type main sequence stars with closely orbiting low-mass pre-main-sequence companions [138]. Subsequently, it was proved that the OGLE sample in the LMC can serve as a homogeneous sample in terms of detection method for studying the multiplicity of main sequence stars [139].

New Zealand has been renowned for its microlensing research as the MOA collaboration, one of the microlensing research groups, has its own telescope operating at the University of Canterbury Mount John Observatory for microlensing survey. In the year
Table 1.1 Variable stars including RR Lyrae (RR Lyr), classical Cepheids (δ Cep), type II Cepheids (type II Cep), anomalous Cepheids (ACs) and eclipsing binaries (EBs) in the OGLE-III fields towards the LMC, SMC, Galactic bulge and Galactic disk.

<table>
<thead>
<tr>
<th>Field</th>
<th>RR Lyr</th>
<th>δ Cep</th>
<th>Type II Cep</th>
<th>ACs</th>
<th>EBs</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMC</td>
<td>39082</td>
<td>4620</td>
<td>197</td>
<td>141</td>
<td>40204</td>
</tr>
<tr>
<td>SMC</td>
<td>6369</td>
<td>4915</td>
<td>43</td>
<td>109</td>
<td>8401</td>
</tr>
<tr>
<td>GB</td>
<td>38257</td>
<td>87</td>
<td>924</td>
<td>20</td>
<td>450598</td>
</tr>
<tr>
<td>GD</td>
<td>45</td>
<td>20</td>
<td>–</td>
<td>–</td>
<td>11589</td>
</tr>
</tbody>
</table>

I joined the MOA group as a PhD student, the MOA-II observation has been carried out for eight years and the MOA-II database already contained about 100TBs of image data. Yet the MOA-II database has never been used for study other than microlensing research. For the benefit of the MOA project, it was reasonable to investigate other potential research directions that could benefit from a large amount of the MOA data. For me, it would be interesting to study a topic that was related to a variety of subjects. As suggested by my supervisor in the provisional year, I decided to work on search for planetary signals in eclipsing binaries in the MOA-II data. Nonetheless, I could not start any serious work until the middle of my second year because of the unforeseen troubles in generating the full light curves from the raw images as well as storage issues. In the end, there were only two observational seasons’ worth of data for each MOA field and the full nine year’s data for two subfields (i.e, GB9-9 and GB10-1) available for my purpose. Although I could not exploit the full MOA database as I had originally planned, the amount of data I was able to access was already substantial, and, fortunately, I found that the quality of the MOA data was good enough for eclipse time variation (ETV) analysis for short period eclipsing binaries.

This thesis contains six chapters. In the rest of Chapter 1, I will give an overview of the MOA project and review the methods that can be used to identify binary systems. In Chapter 2, I will review the types of variable stars that are concerned and present the physics of eclipsing binaries that is relevant to understanding the work in this thesis. In Chapter 3, the first eclipsing binary catalogue from the MOA-II database is presented. The preliminary study of ETVs using two observational seasons’ worth of data is presented in Chapter 4. The results of the search for light travel time effect in short period MOA eclipsing binaries are presented in Chapter 5. Finally, I will give the conclusions in Chapter 6.
1.3 Microlensing Observations in Astrophysics

The MOA collaboration [203] is a Japan and New Zealand research group formed in 1995. In the early time of the project, the MOA collaboration used the facilities at the Mt. John University Observatory in New Zealand including the 61 cm Boller and Chivens telescope with a wide-field camera, MOA-cam1, installed for the sake of microlensing survey. As the interest in microlensing observation was shifted from search for MACHOs to planet detection, the MOA project was evolved to its second stage. In order to enhance performance, the 1.8 m MOA-II telescope was built in Japan and installed at the Mt. John University Observatory in 2005 (see Figure 1.2). The upgraded wide field camera, MOA-cam3, consisting of ten E2V CCD4482 chips, each having $2k \times 4k$ pixels, was also attached to the telescope, giving a field of view as large as 2.2 deg$^2$ with a single exposure (Figure 1.3) [176]. The second state of the MOA project (MOA-II) was dedicated entirely to planet hunting, different from OGLE-IV [216], the current phase of the OGLE project, which is also interested in doing sky variability surveys. Regular MOA-II observations began in 2006. Instead of surveying over a large portion of the Milky Way like the OGLE project, the MOA group decided to concentrate the telescope time on monitoring 22 Galactic bulge fields (see Figure 1.1). During the observational season from February to November in which the Galactic bulge was at altitude with proper seeing, the telescope time was totally spent on taking images of these 22 Galactic bulge fields through the custom MOA-Red wide-band filter, which is roughly equivalent to the combination of the standard Kron/Cousins $R$ and $I$ bands, from 600 nm to 900 nm (see Figure 1.4), automatically following the preprogrammed procedures. Images of the most crowded fields were taken every 10 to 15 minutes, while for other fields the cadence was from 30 minutes to an hour [203]. The exposure time of a MOA image of a Galactic bulge field was 60 s. In the off-season period, the telescope time was spent for observations towards the Magellanic Clouds as well as a few transit targets.

The difference imaging analysis (DIA) method was used to reduce the MOA images. The DIA method extracts variable objects in an observed image by subtracting the observed image from a high quality, good seeing, reference image that is transformed to have the same seeing and photometric scaling as the observed image beforehand. As a result, the DIA method gives the measurement of the relative fluxes instead of the absolute fluxes. It makes the DIA method intrinsically good at detecting transient events and faint variable objects. The DIA method achieves photometry with better
Fig. 1.1 MOA fields towards the Galactic bulge. There are 22 Galactic bulge fields that the MOA telescope monitors regularly in cadences between 10 minutes to an hour.
Fig. 1.2 1.8m MOA-II telescope, which was constructed by the Nishimura Company in Kyoto, Japan in 2003-2004 and installed at Mt. John University Observatory of the University of Canterbury in 2005. The 9m diameter dome was also built by Nishimura to cover and protect the telescope.
Fig. 1.3 MOA-cam3, which consists of ten E2V CCD4482 chips. Each CCD chip has $2k \times 4k$ pixels and they are placed to form a $2 \times 5$ array.

Fig. 1.4 MOA passbands. The MOA-$R$ band spanning from 600nm to 900nm is routinely used for the MOA-II observations. As the microlensing effect is independent of wavelength, the $V$-band observations are only carried out occasionally in order to obtain the colour difference information of microlensing source stars. In addition, as the MOA-$R$ band already covers the $I$ band, the $I$ band is no longer used.
### 1.3 Microlensing Observations in Astrophysics

Table 1.2 Coordinates of the MOA Galactic bulge (GB) fields’ centres. The first column shows the MOA GB field numbers, the fourth column gives the number of images, \( N_{\text{image}} \), for a GB field taken from February 2013 to August 2014, and \( t_{\text{exp}} \) is the exposure time.

<table>
<thead>
<tr>
<th>GB</th>
<th>R.A. (J2000) (h:m:s)</th>
<th>Dec (J2000) (d:m:s)</th>
<th>( N_{\text{image}} )</th>
<th>( t_{\text{exp}} ) (sec)</th>
</tr>
</thead>
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<td>-23:53:31.1</td>
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Precision than other methods towards very crowded fields, for example, the Galactic bulge fields, because almost all stars in such fields cannot be resolved individually in practice, by straight PSF-fitting routines such as DOPHOT36. It is verified by re-analyzing the MACHO and OGLE databases that the detection rates of microlensing events are significantly improved by the DIA method which overcomes the problems of undetectability due to the blending and faintness of the microlensing source stars in the crowded fields [4, 3].

MOA-II has collected \(~100\ TBs of image data of the GB fields since 2006. In order to reduce the pressure on data storage, the light curves, i.e. the time series of photometric measurements, of all resolved stars are not maintained in the database, but their positions in the subtracted images are. The light curves of variable objects
Table 1.3 Orbital parameters.

<table>
<thead>
<tr>
<th>Name (Symbol)</th>
<th>Explanation</th>
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<tr>
<td>true anomaly ($\nu$)</td>
<td>an angle from the periastron to a position on the ellipse</td>
</tr>
<tr>
<td>period ($P$)</td>
<td>the time to complete a full orbit</td>
</tr>
<tr>
<td>semi-major axis ($a$)</td>
<td>half the distance between the periastron and apastron</td>
</tr>
<tr>
<td>eccentricity ($e$)</td>
<td>the measure of deviation from circularity</td>
</tr>
<tr>
<td>argument of periastron ($\omega$)</td>
<td>the angle from the ascending node to the periastron</td>
</tr>
<tr>
<td>inclination ($i$)</td>
<td>the ellipse’s inclination in respect of the reference plane</td>
</tr>
<tr>
<td>time of periastron ($\tau$)</td>
<td>the reference time of the periastron passage</td>
</tr>
<tr>
<td>longitude of ascending node ($\Omega$)</td>
<td>the angle of the ascending node on the reference plane</td>
</tr>
</tbody>
</table>

thus will only be produced if requested. And, when requested, their photometric measurements, i.e. the relative fluxes, are extracted from the subtracted images using aperture photometry with an aperture radius of 6 pixels. The coordinates of the centres of all GB fields are listed in Table 1.2.

1.4 Stars with Companions

Stars in the Galaxy do not usually exist in isolation without any gravitationally interacting partner. Most of them form binary systems, even triple or higher hierarchical systems. If planets, brown dwarfs and compact stars (i.e. white dwarfs, neutron stars and black holes) are included, we can safely assume that all stars in the Galaxy host at least a companion (e.g. [139, 35]).

The existence of binary and multiple stellar systems was first verified by William Herschel who performed an all-sky survey using the largest telescope he owned, and measured the parallax and proper motions of every visible star. After observation over nearly three decades, he demonstrated that many double stars (a double star is a pair of stars which appears as a single star to the naked eyes) had common proper motions that must be explained by the two stars being gravitationally bound [9]. Not only did his work prove that stellar binaries exist, but it also verified the universality of Newton’s law of gravitation throughout the Galaxy.

1.4.1 Two-body orbital motion

Newton’s law of gravitation states that two bodies of masses, $m_1$ and $m_2$, will experience attractive forces, from each other, of magnitude inversely proportional to square of the separation ($r$) and the product of the masses, i.e, $F = -\frac{Gm_1m_2}{r^2}$, where $G$ is the
1.4 Stars with Companions

Fig. 1.5 Orbital plane of a body in a binary system in the centre of mass frame. The centre of mass of the binary system is located at the focus of the elliptical orbit.

The gravitational constant. For a stellar binary, the gravitational forces make the two stars travelling in Keplerian orbits, either elliptical or circular, about the centre of mass (CM) of the system, following Kepler’s third law, i.e.,

$$\frac{P^2}{a^3} = \frac{4\pi^2}{G(m_1 + m_2)},$$

(1.1)

where $a = a_1 + a_2$ is the relative semi-major axis. Note that $a_1$ and $a_2$ are the semi-major axes of each body’s orbit around the CM, and they are related to each other via

$$a_1m_1 = a_2m_2.$$  \hspace{1cm} (1.2)

The general equation for an ellipse is

$$r(\nu) = \frac{a(1 - e^2)}{1 + e\cos \nu}$$

(1.3)
in polar coordinates, where $r$ is the distance from the focus located at the CM of the system and $\nu$ is the true anomaly. The Keplerian orbit in the sky is determined by six orbital parameters: semi-major axis ($a$), eccentricity ($e$), period ($P$), argument of periastron ($\omega$), inclination ($i$), and true anomaly ($\nu$). The argument of periastron is the
Fig. 1.6 Eccentric anomaly of an elliptic orbit. The eccentric anomaly $E$ of a point P of an elliptic orbit (brown) is defined as the angle $FCP'$, where F and C are the focus and centre of the elliptic orbit, respectively. $P'$ is the point P projected onto the outer concentric circle (blue) with radius same as the elliptic orbit’s semi-major axis $a$. The inner concentric circle (green) with radius of $b$ equal to the semi-minor axis of the elliptic orbit is also shown for reference. Note that $e$ is the eccentricity of the elliptic orbit and $\nu$ is the true anomaly of the point P.
angle from the ascending node to the periastron in the orbital direction. The ascending node is the intersection point of the orbit with the reference plane, an imaginary plane in the sky perpendicular to the line of sight, as shown in Figure 1.5. The true anomaly can be further expressed as a function of time if we introduce mean anomaly \((M)\), i.e, \(M = (2\pi/P)(t - \tau)\), where \(t\) is time and \(\tau\) is the time of periastron, and eccentric anomaly \((E)\) (see Figure 1.6) such that

\[
\nu = \sqrt{\frac{1+e}{1-e}} \tan \left( \frac{E}{2} \right).
\]

Solving Kepler’s equation,

\[
M = E - e \sin E,
\]

and using eq.(1.4), the true anomaly of the body can be calculated, and thus we can work out the orbital position at any time.

1.4.2 Triple and multiple systems

When the system consists of more than two objects, its orbital behaviour will no longer be as simple and predictable as the two-body situation. The long-term behaviour of triple systems, in particular, has been intriguing many scientists. It has been known that the three-body equations of motion do not have closed-form solutions. They have to be solved numerically. In general a multiple system would be unstable. Simulations of multiple stellar systems in star clusters have shown that unstable systems will disintegrate into lower order systems in the time scale of Myr [218]. Almost all multiple stellar systems that can survive in the long-term after the formation have hierarchical structures such that the stars in the hierarchical multiple system can be divided into subgroups which can be considered as self-governing in the sense that the components in a subgroup are orbiting around their own CM, while each subgroup orbits the CM of the whole system. In a triple system, there is only one hierarchical configuration; i.e. a tertiary star orbits around the two inner stars, which form a stellar binary. For higher multiple stellar systems, there are more possible and complicated arrangements, which can be illustrated by Evans’s mobile diagram [62].

As systems of multiplicity higher than three are not what we are concerned about in this thesis, we shall restrict ourselves to binary and triple systems. For hierarchical stellar triples to be stable, the distance between the two inner stars generally has to be substantially smaller than the distances of the outermost star from them by at least
Fig. 1.7 Mobile diagram of hierarchical configurations of multiple stellar or planetary systems proposed by Evans (1968). (a) A solar system with three planets. (b) A binary system. (c) A triple system of hierarchy 2+1; e.g., a planet-moon system which orbits around its host star. (d) A quadruple system of hierarchy 2+2; i.e. two binary systems orbit each other. (e) A quadruple system of hierarchy 3+1; i.e. a 2+1 triple system with an orbiting companion. (f) A quintuple system of hierarchy 4+1; i.e. a quadruple system of hierarchy 3+1 with an orbiting companion.

an order of magnitude during the system’s lifetime [31]. This in turn means that the triple system can be treated as a system having two 2-body components - the inner and outer binary components, each of which follows Keplerian orbits. The inner binary component consists of the two inner stars in the triple system, while the outermost star and the inner binary’s barycentre constitute the outer binary component. The orbital dynamics of the triple system can then be investigated in the framework of the perturbed motion of these two binary components [31, 25].

1.4.3 Detection methods

There are various methods to detect an orbiting companion in a star or identify a binary system depending on what quantities of the target stars are supposed to be measured. The oldest method is the proper motion measurement (that is what William Herschel applied for the determination of any physical association between two visually close stars) to measure the changes in the apparent positions of stars in the sky. If two stars are bound, they will travel through space in similar directions, exhibiting a common proper motion. Despite being straightforward, proper motion measurements can be carried out only for resolved stars. For close binaries, say, of angular separations
less than 0.1 arcsec, they cannot be resolved in general. Therefore, detection of a close companion relies on indirect methods which always requires observing the influence of the companion on the primary star’s light, photometrically or spectroscopically.

**Photometry**

Photometry is the science of measuring the flux of electromagnetic radiation we receive from celestial objects \[171\]. In photometry, the presence of a companion in a star can be deduced by observing the change in the observed star’s flux. The well-known related phenomenon is transits, which occur when the companion passes in front of the star from an observer’s point of view, blocking the star and reducing the flux as a consequence. In fact, the “Demon Star”, Algol, which was thought to be associated with a monster Medusa in Greek mythology, might arguably be the first stellar binary that humans have ever identified photometrically, using our natural photometric detector - human eyes - since its periodical brightness variation was already documented by the ancient Egyptians \[99\]. The variability of Algol was rediscovered by Italian astronomer Geminiano Montanari in 1667. Although the identity of Algol had been speculated to be a stellar binary, there was no indisputable evidence showing that Algol consists of two stars. It was not until the advent of spectroscopy that Algol was finally confirmed to be a binary system by Potsdam astronomer Hermann Carl Vogel who observed periodic Doppler shifts in Algol’s spectra \[17\]. Algol is the first stellar binary identified photometrically as well as spectroscopically. Technically, photometric observations are easier to be carried out than spectroscopic observations since, in principle, the photometric measurement is simply to count the number of photons received by a CCD camera in each exposure (although data reduction must be carried out to extract useful values in reality). The precision in photometry with a CCD camera has reached the level that allows the detection of tiny change in stellar flux. For example, the *Kepler* space telescope has a photometric precision of the order of 10 ppm \[32\]. The realization of such high photometric precision is one of the reasons that the transit survey for exoplanets by the *Kepler* space telescope was exceptionally successful.

Transits are useful to determine orbital period. If successive transits are observed, the orbital period can be determined by measuring the time difference between the middles of transits. The complete coverage of a transit, which might last a few hours to days, however, would be usually unachievable by ground-based observations, owing to the day-night cycles and, particularly, bad weather conditions that prevent observing a transit constantly. The orbital period nonetheless can be derived by folding the
light curve at a range of trial periods. The orbital period will then correspond to the trial period at which the good shape of transit signal emerges. Several statistical measures have been proposed and used to evaluate the goodness of the shape of the folded light curve such as phase dispersion minimization [200] and conditional entropy [81]. The trial period that minimizes the values of these measures would be expected to correspond to the orbital period. However, transits are uncommon as they will arise only if the normal of the orbital plane is close to perpendicular to the line of sight, meaning that, assuming random orientation of the orbits, the transit probability, \( P_t \), will be equal to

\[
P_t(a) = \frac{R_1 + R_2}{a}
\]

for the star and the companion of radii, \( R_1 \) and \( R_2 \), respectively, orbiting each other in circular orbit at the orbital distance, \( a \) [152]. Using eq.(1.1), i.e. Kepler’s third law, eq.(1.6) can be expressed in terms of the orbital period, \( P \), such that

\[
P_t(P) = (R_1 + R_2) \left( \frac{2\pi}{P} \right)^{2/3} \frac{G^{-1/3}(m_1 + m_2)^{-1/3}}{103}.
\]

The probability of a binary system of a given total mass exhibiting transits can be derived by integrating eq.(1.7) over \( P \) given that a probability distribution function of orbital period, PDF(\( P \)), is known; that is, \( P_t = \int P_t(P) \text{PDF}(P) dP \) [103]. According to eq.(1.6) and eq.(1.7), the probability will be higher if the companion is bigger and closer to the star.

In general, transits due to secondary stars are called eclipses and the associated binary systems are categorized as eclipsing binaries, while the term “transit” is specifically used for the transiting objects considerably smaller and fainter than their host stars, for example, planets or brown dwarfs, which do not emit light by nuclear fusion and usually would exhibit only shallow secondary transits, often undetectable, when they are behind the host stars. A typical type of transits has flat bottoms, apart from the relatively small effects of stellar limb darkening [140], and has four contact points in the light curves, indicating that the companions’ discs had been totally inside their host stars’ discs (see Figure 1.8). For eclipsing binaries, this corresponds to total eclipses. It is, however, possible that the star’s disk would never completely embrace the companion’s disk, resulting in a grazing transit, in which the second and third contact points are missing from the light curve.
Fig. 1.8 Schematic diagram of a transit event. An ingress is the region between contact points 1 and 2, while an egress is the region between contact points 3 and 4. For a grazing transit, there are no second and third contact points but the point of minimum instead. $R_*$ and $R_p$ are the radii of the star and planet, respectively. $b$ is the impact parameter and $\delta$ is the transit depth. $D$ is the time interval between the outer contact points, while $d$ is the time interval between the inner contact points.
For a planetary transit, the transit depth ($\delta$), i.e., the difference between the obscured to unobscured flux, is related to the radius ratio such that

$$\delta = \left(\frac{R_p}{R_*}\right)^2,$$

(1.8)

where $R_*$ and $R_p$ are the radii of the star and planet, respectively. Although no information about the planet’s mass can be derived from the transit light curve, given the measurements of transit depth as well as the time interval between the first and the fourth contact point ($D$) and that between the second and third contact points ($d$), Seager and Mallén-Ornelas (2003) showed that the unique solutions for the impact parameter ($b$) and the orbital distance ($a$) in units of $R_*$ can be determined:

$$b = \frac{a}{R_*} \cos i = \left[\frac{\sin^2 \Omega_D (1 - \sqrt{\delta})^2 - \sin^2 \Omega_d (1 + \sqrt{\delta})^2}{\sin^2 \Omega_D - \sin^2 \Omega_d}\right]^{1/2},$$

(1.9)

$$a = \frac{R_*}{\Omega_D} = \left[\frac{(1 + \sqrt{\delta})^2 - b^2 (1 - \sin^2 \Omega_D)}{\sin^2 \Omega_D}\right]^{1/2},$$

(1.10)

where $\Omega_D = \pi D/P$ and $\Omega_d = \pi d/P$. Combining eq.(1.9) and eq.(1.10), we can calculate the orbital inclination ($i$), which is the essential parameter in order to determine the true mass of the orbiting companion from the minimum mass obtained from the radial velocity curve.

Multiple transits in a star will definitely manifest the existence of an orbiting companion. Yet not every star with companions will exhibit transits. For a star with a close orbiting companion, ellipsoidal variation in the flux might be observed because of the tidal distortion of the star due to the companion. Such distortion will produce non-uniform surface brightness. As a consequence, the observed flux will be varying as the star is orbiting and showing different cross sections with respect to the observer. Ellipsoidal variation [224] is often present in the light curves of stellar binaries of periods less than two days. Additional sinusoidal flux variations arising from reflection [223] and beaming effects [233] might be present and detectable in the light curves of close binaries as well. The former is induced by the side of the companion which is heated by the star’s radiation and becomes luminous. The latter, on the other hand, is the result of Doppler boosting, which increases (decreases) the brightness of the companion when it is approaching (receding from) the observer. In photometry, the presence of close orbiting companions might still be verifiable by the detection of
1.4 Stars with Companions

beaming, ellipsoidal and reflection (BER) modulations in the light curves when no transit is observed [63]. Figure 1.9 shows the BER modulations in terms of orbital phase. As they are not in phase, each component would be distinguishable from the other two. Given the high precision in the photometry, BER modulations have been observed in the Kepler light curves of very short period white dwarf binaries, and even in the short period transiting Kepler light curves associated with companions of mass in the brown dwarf domain [118].

**Spectroscopy**

Spectroscopy is the study of stellar spectra and a stellar spectrum contains spectral lines corresponding to elements that are present in a star’s atmosphere. As the star is apparently approaching (or receding from) an observer owing to its orbital motion, its spectral lines would shift towards the blue (or red) end of the spectrum accordingly. This phenomenon to the spectral lines of a moving star is known as the Doppler shift [19]. In the idealized situation where the two stars in a binary system have a single spectral line at the same wavelength in the rest frame, their orbital motions would
split that spectral line into two lines, of which each will shift up and down periodically, and out of phase. Such splitting and shifting of a spectral line would be a clear sign of the presence of two stellar components in the system. When the spectral lines from only one component is observed, it is called a single-lined spectroscopic binary. If the spectral lines from both components are present, it is called a double-lined spectroscopic binary.

By measuring the shift of the spectral lines with respect to the rest reference, the radial velocity \(v_r\) of the star can be calculated via the Doppler equation,

\[
\frac{v_r}{c} = \frac{\Delta \lambda}{\lambda_i},
\]

where \(\Delta \lambda = \lambda_o - \lambda_i\), \(\lambda_o\) is the observed wavelength and \(\lambda_i\) is the wavelength in the rest reference. Considering a binary system of which the two components are orbiting in elliptical orbits around the CM, then the radial velocity of each component in terms of the orbital parameters is given by

\[
v_{r,1} = \frac{2\pi a_1 \sin i}{P\sqrt{1-e^2}} \left[\cos(\nu + \omega) + e \cos \omega\right],
\]

and

\[
v_{r,2} = \frac{2\pi a_2 \sin i}{P\sqrt{1-e^2}} \left[\cos(\nu + \omega) + e \cos \omega\right],
\]

where

\[
K_j = \frac{2\pi a_j \sin i}{P\sqrt{1-e^2}}, \quad j = 1, 2
\]

is the semi-amplitude of the radial velocity of the \(j^{th}\) component. The orbital parameters, i.e., \(a_j \sin i\), \(P\) and \(e\) of the component can be derived by fitting its radial velocity curve, i.e., the time-series of the radial velocity measurement, by eq.(1.12) or eq.(1.13), accordingly. As a result, we can obtain the mass ratio \((q)\) from the amplitude ratio such that

\[
q = m_2/m_1 = a_1/a_2 = K_1/K_2.
\]

Further, using eqs.(1.1) and (1.2) to eliminate \(m_2\), an upper limit for \(m_1\) can be obtained in terms of \(K_1\) and \(K_2\) as

\[
m_1 \sin^3 i = \frac{4\pi^2 K_2}{P^2 G(K_1+K_2)} \left(\frac{\sqrt{1-e^2}}{2\pi} P\right)^3 (K_1 + K_2)^3
= \frac{P}{2\pi G} (1-e^2)^{3/2}(K_1 + K_2)^2 K_2.
\]
Meanwhile, an upper limit for \( m_2 \), i.e. \( m_2 \sin^3 i \), can also be found by simply interchanging 1 and 2 in eq.(1.16). Hence, as long as the inclination is known, the true mass of each component can be determined. On the other hand, the stellar temperature and luminosity (as well as the mass) can be estimated by fitting the spectrum, i.e., determining the depth of different spectral lines. Therefore, it is hard not to appreciate the importance of double-lined spectroscopic binaries that are also eclipsing binaries for which the stellar and orbital parameters can be fully determined.

If only \( K_1 \) is available because of missing the radial velocity measurements of the other component, for example, due to the faintness of its spectral lines in the spectrum, we can still calculate the mass function \( f(m) = m_2^3 \sin^3 i / (m_1 + m_2)^2 \). From eq.(1.15), we can determine \( K_2 \) in terms of \( m_1, m_2 \) and \( K_1 \); i.e. \( K_2 = (m_1/m_2)K_1 \). Substituting this expression of \( K_2 \) into eq.(1.16), we obtain

\[
m_2 \sin^3 i = \frac{PK_1^3}{2\pi G} (1 - e^2)^{3/2} \left( \frac{m_1 + m_2}{m_2} \right)^2.
\]

Hence, the mass function \( f(m) \) can be expressed in terms of the parameters \( P, K_1 \) and \( e \) which can be determined from the radial velocity curve; i.e.,

\[
f(m) = \frac{m_2^3 \sin^3 i}{(m_1 + m_2)^2} = \frac{PK_1^3}{2\pi G} (1 - e^2)^{3/2}.
\]

**Imaging**

All the methods mentioned so far are indirect methods. A direct method would be to see whether a system contains a companion by resolving the light of the companion from that of the host star. Yet not every binary system is resolvable because of the limit in telescope resolution power. The main difficulty of having high resolution images of stars from ground-based telescopes always comes from the Earth’s turbulent atmosphere that induces a blurring of the images. The adoption of adaptive optics on the telescope, which corrects atmospheric distortion by measuring the wave fronts of a natural or a guide star’s light, has allowed the achievement of the spatial resolution of the order of 0.01 arcs [49]. For example, Figure 1.10 shows the images of binary star HIC 59206 taken by the 8.2-m VLT KUEYEN telescope in the infrared \( K \)-band (wavelength 2.2µm) when the adaptive optics system, i.e., the MACAO-VLTI system, was switched off and on, respectively. With the correction by the MACAO-VLTI system, the system was able to be resolved into two stars with separation of 0.120 arcs. Other methods that have been applied to reduce the atmospheric effect include speckle imaging [95],
Introduction

Fig. 1.10 Infrared images of binary star HIC 59206 with (right) and without (left) correction by adaptive optics system, taken by MACAO-VLTI system at the 8.2-m VLT KUEYEN telescope in the infrared $K$-band (wavelength 2.2 $\mu$m). Using the adaptive optics system, the system was clearly resolved into two stars separated by 0.120 arcs. These images are credited to ESO.

aperture synthesis [167] and lucky imaging [77]. For non-stellar companions, the challenge to detect via imaging comes from their faintness. The possible imaging relies on receiving their reflection light or irradiation which would often be overwhelmed by the light of their host stars. Their visibility might be enhanced by blocking the stars’ light by placing physical masks (i.e. cornographs) on the stars (e.g. [125]), or using the nulling interferometry technique [7] which introduces destructive interference to darken the stars. Another technique that can produce a high contrast image of a companion is angular differential imaging [124] by which quasi-static speckle noise is reduced, improving the detectability of any nearby companion.

Timing

The presence of an unrecognized companion in a system may be deduced by detecting abnormality of strictly periodical phenomena from the system, for example, radio radiation from pulsars, pulsations of variable stars, transits and eclipses in binary systems, etc. As the intrinsic sources of the periodicity of these phenomena are usually well-known, the occurrence times of successive events of these phenomena would be predictable. Deviations from the predicted occurrence times, therefore, will indicate the presence of additional sources that also govern the periodicity of the system. For
example, detecting anomalies in pulsation periods by timing pulsation times has been used to discover planets in pulsars (e.g. [68], [225]) and secondary objects in variable stars (e.g. [15], [184]).

For binary systems showing eclipses (i.e., eclipsing binary systems), timing eclipses is a method which can be used to detect variations of their orbital periods. Supposing such a binary system is a two-body system purely governed by Newton’s gravitational law, eclipsing events towards it should occur repeatedly and constantly after the same orbital period. Perturbations in the orbital motion due to additional companions would, therefore, alter the periodicity of eclipsing events. Search for perturbations in the orbital motion by analysing variations in eclipse times can provide the possibility of detecting additional planetary or stellar components. These perturbations manifest themselves in non-linear variations of the difference between the observed and the predicted time of mid-eclipses, and can be unveiled using the observed-minus-calculated (O–C) diagram, which is simply the plot of the observed minus the calculated mid-eclipse times with respect to the cycle numbers. The O–C diagram has been a main tool to detect variations in eclipsing period, or eclipse time variations (ETVs), and study effects that cause the period to vary over time. Some of these effects cause secular or cyclic ETVs which are related to angular momentum loss due to magnetic baking [141], non-conservative mass transfer between components [142], magnetic cycles [111, 113, 8], and apsidal motion [31, 76]. The ETVs which are relevant to the presence of orbiting companions are usually considered to consist in two parts from the light travel time effect and dynamical effect, respectively [28].

The light travel time effect (LTTE) arises from the reflex motion of the CM of a binary system [130], which is present so long as the binary system is accompanied by a tertiary companion which causes the CM to orbit around the barycentre of the whole three-body system. In this case, the distance of the binary from the observer would vary in response to the orbital motion of its CM (see Figure 1.11), and, owing to the finite speed of light, the orbit motion would lead to the early arrival or delay of the light from the binary. If the binary we observe is an eclipsing binary, we would observe the occurrence times of eclipsing events to be different from the expected times, showing periodic behaviour as illustrated by Figure 1.12. Since the inner binary’s orbit remains Keplerian (i.e., the LTTE does not alter the eccentricity and semi-major axis), the LTTE is simply considered as a geometrical effect. The application of the LTTE became a serious subject after pioneering papers of Chandler (1892), Hertzsprung (1922), Woltjer (1922) and Irwin (1952) were published.
Fig. 1.11 Schematic diagram of light travel time effect in a distant eclipsing binary due to the presence of a tertiary companion. As the CM of the eclipsing binary orbits around the barycentre of the triple system, the distances between the observer and the eclipsing binary will also differ. Therefore, the light from eclipsing events occurring at different orbital positions will take different times to reach the observer. \( E_1, E_2, E_2 \) and \( E_4 \) represent the eclipsing events at four different orbital positions.

Fig. 1.12 Illustrative diagram of eclipse time variation due to light travel time effect corresponding to Figure 1.11. The occurrence times of repeating eclipsing events, labeled as \( E_1, E_2, E_2 \) and \( E_4 \), are shifted periodically with respect to the orbital motion of the CM of the eclipsing binary around the barycentre, being earlier or later than the expected times.
Fourier series might be used to approximate a LTTE signal [26, 30]. Yet there is an analytical equation of ETV due to the LTTE. To derive the LTTE equation, we shall adopt the reference plane as a plane which is perpendicular to the line of sight of an observer and passes through the triple system’s barycentre. In this configuration, the CM of the eclipsing binary and the tertiary companion both orbit around the barycentre, and the perpendicular distance, $z$, of the CM of the eclipsing binary from the barycentre will be given by $z = r \sin i_2 \sin (\nu_2 + \omega_2)$, where $r$ is the orbital distance of the CM of the eclipsing binary from the barycentre, and $i_2, \nu_2$ and $\omega_2$ are the inclination, true anomaly and argument of periastron of the tertiary companion’s orbit, respectively. Therefore, the time delay of the light from the eclipsing binary due to its orbital motion, i.e., the ETV due to the LTTE, $\Delta_{\text{LTTE}}$, will be

$$\Delta_{\text{LTTE}} = \frac{z}{c} = \frac{r \sin i_2 \sin (\nu_2 + \omega_2)}{c} = \frac{a_{\text{AB}} \sin i_2 \sin (\nu_2 + \omega_2)}{c} \frac{1 - e_2^2 \cos \nu_2}{1 + e_2 \cos \nu_2},$$

(1.19)

with the amplitude defined by $A_{\text{LTTE}} = (a_{\text{AB}} \sin i_2 / c) \sqrt{1 - e_2^2 \cos^2 \omega_2}$. Note that $r = a_{\text{AB}} (1 - e_2^2) / (1 + e_2 \cos \nu_2)$, where $a_{\text{AB}} = (m_C / m_{\text{ABC}}) a_2$; $m_C$ is the mass of the tertiary companion, $m_{\text{ABC}}$ is the total mass of the triple system, and $a_2$ is the semi-major axis of the tertiary companion’s orbit around the barycentre. Times of eclipse minima of eclipsing binaries have long been collected by ground-based surveys in the use of photography, photoelectric and CCD photometry [104]. So far over hundred eclipsing binaries of different types were observed to exhibit LTTE signals in their O–C diagrams (e.g. [197, 92, 30, 130, 1, 70]). In a certain number of cases, the existence of tertiary companions was further confirmed by imaging or spectroscopy (e.g. [231, 131]). In recent decades, timing accuracy has been substantially improved thanks to the advent of high-speed photometry with CCD camera, and the method of eclipse timing has been successfully applied to search for circumbinary planets in post-common-envelope binary (PCEB) systems by ground-based telescopes (e.g. [13] and [114]). The launch of space telescopes has also made the detection of triple system with short period tertiary components become achievable. For example, using photometric data from the *Kepler* space telescope, Borkovits et al. (2016) have identified 222 eclipsing binaries as triple systems via eclipse timing, and no less than 14 triple systems in their discovery have outer period shorter than a year.
The dynamical effect from tertiary companions may become important, and detectable, if the tertiary companions closely orbit around their accompanying binaries forming so-called compact hierarchical triples [31]. The gravitational influence of the tertiary companions in this kind of triple systems is so significant that the inner binaries are perturbed to follow non-Keplerian orbits for which the orbital elements are no longer fixed but vary over time. Eclipsing binaries in compact hierarchical triples would exhibit complex ETVs which are attributable to the dynamical effect on top of the LTTE. The amplitude of ETV due to the dynamical effect is given approximately by
\[ A_{\text{dyn}} = \frac{1}{2\pi} \frac{m_C}{m_{ABC}} \frac{P_1^2}{P_2} (1 - e_2^2)^{-3/2}, \]
where the periods (i.e., \( P_1 \) and \( P_2 \)) and the outer eccentricity are known from the LTTE solution [29]. The contribution of the dynamical effect to the ETV would be significant only if \( A_{\text{dyn}}/A_{\text{LTTE}} \) is close to or greater than unity. This condition can be satisfied if the triple system has the outer period (\( P_2 \)) comparable to the inner period (\( P_1 \)), or the tertiary mass (\( m_C \)) comparable to the total mass (\( m_{ABC} \)). The dynamical effect has never been observed by ground-based telescopes. It was not until the launch of the *Kepler* space telescope and CoRoT spacecraft that the dynamical effect was detected towards 67 eclipsing systems [83, 29]. In studies of planetary systems, the dynamical effect is considered as perturbations on transiting planets due to the closest neighboring objects and induces variations in the transit times [25, 91, 2, 94]. Although the LTTE should also be present in multi-planetary systems, perturbations due to the LTTE are usually negligible because of small reflex motions of planet host stars [2]. Analysis of transit time variations has helped to discover multi-planetary systems and estimate the masses of their planets (e.g. [122, 75, 93]).

Finally, it is worth noting that eclipse timing, as pointed out in a few papers on circumbinary planets (e.g. [126] and [116]), is a plausible method to unveil any circumbinary planet with a highly inclined orbit with respect to the orbital plane of its host binary, which would be hidden from the transit and radial velocity methods.
Chapter 2

Variable Stars

It seems to me the natural world is the greatest source of excitement; the greatest source of visual beauty, the greatest source of intellectual interest. It is the greatest source of so much in life that makes life worth living.

David Attenborough

2.1 Introduction

Stars are natural light generators, yet they are never the objects with constant luminosity like the light bulbs we use to light up the darkness at night. Every star shows variation in brightness in different evolutionary stages of their lifetimes. Even our Sun, as a main-sequence star in hydrostatic equilibrium state, shows cyclic variations in its surface brightness with a period of about 11 years owing to the appearance and disappearance of dark spots associated with the Sun’s magnetic activities [86]. Although all stars can be considered to be variable in brightness, only stars with variable brightness with timescales ranging from seconds to years that humans can observe and measure would be classified as variable stars. According to the General Catalogue of Variable Stars (GCVS), variable stars are classified into six main types including pulsating, eruptive, cataclysmic, rotating variables, eclipsing binary systems and intense variable X-ray sources depending on the physical processes that produce the variability [177]. Variable stars are also traditionally grouped into two families called intrinsic and extrinsic variables [36]. The former includes pulsating, eruptive, cataclysmic and
rotational variables involving the internal mechanisms in the stars that generate the variability, while the variability of the latter is due to external processes associated with the presence of multiple objects in the systems. Basically, the entire family of extrinsic variables is represented by eclipsing binary variables. Among various types of variable stars, pulsating variables (including RR Lyrae stars and Cepheids) and eclipsing binaries usually have characteristic, regular light curve shapes, allowing them to be identified easily in photometry. The periods of their variability can also be determined by calculating time differences between successive repeating maxima or minima of the light curves. Therefore, eclipsing binaries and pulsating variables are abundant in photometric surveys for variability in the sky such as OGLE.

2.2 Pulsating Variables

Classical pulsating variables including RR Lyrae stars and Cepheids are the variable stars located in the instability strip in the Hertzsprung-Russell diagram (see Figure 2.1). Their brightness is observed to increase and decrease continuously and periodically, having the light curve shapes which can be reconstructed using Fourier decomposition [185]. The variability in their brightness results from radial pulsations corresponding to cyclic expansion and compression of their surface layers, which are driven mainly by the $\kappa$ and $\gamma$ mechanisms [46].

Through the $\kappa$ mechanism, stellar layers in the zones of partially ionized hydrogen and helium can gain heat during compression as the opacity increases, while the temperature change keeps relatively small due to the thermostatic effect because of the demand for ionizing hydrogen and helium. Subsequently, through the $\gamma$ mechanism, additional heat from the hotter surroundings will be also added to the compressed layers via conduction or by radiation. The heat gained in this way will then release when the layers expand, exciting pulsations with large radial amplitudes. The depth of the partial ionization zone is sensitive to the surface temperature of the star, and the effectiveness of the $\kappa$ and $\gamma$ mechanisms would be weakened by convection. Stars later than F type would have partial ionization zones located in the convective regions, while stars earlier than G type would have the ionization zones so deep that the excited pulsations would barely penetrate to the surface, failing to produce surface oscillation large enough to induce significant brightness variations. Indeed, the instability strip is narrow in spectral types, only spanning from F to G type. These problems of driving
2.2 Pulsating Variables

Fig. 2.1 Instability strip in the Hertzsprung–Russell diagram. The region of instability strip is the region covered by the dashed line. RR Lyrae stars are located at the lower end of the instability region, while Cepheids are located from the middle to the upper regions of instability strip. This figure is taken from Wikimedia Commons and credited to Rursus.
pulsations by the $\kappa$ and $\gamma$ mechanisms have been considered as the explanation for the narrowness of the instability strip.

### 2.2.1 RR Lyrae

RR Lyrae stars are the most abundant types of pulsating variable stars in the OGLE catalogue (see Table 1.1 and [196]). They are located at the lower end of the instability strip, corresponding to low mass stars of spectral types of A2 to F6 in the red giant branch (see Figure 2.1). RR Lyrae stars are evolved from solar-like main-sequence stars of masses around $0.8M_\odot$ [186]. Since the lower mass main-sequence stars would have longer lifetimes in the main-sequence state, RR Lyrae stars represent the old stellar population, typically older than 10 Gyr, and they can be used as standard candles for distance estimation of old stellar clusters. The pulsation periods of RR Lyrae stars are generally between 0.2 and 1.0 days, having amplitudes of 0.3 up to 2 magnitudes in optical wavebands and mean effective temperatures from $\geq 6000$K to $\geq 7250$K. According to the light curve shape, pulsation amplitude and luminosity, RR Lyrae stars are usually classified into three subtypes called RRa, RRb and RRc. The first two types (RRa and RRb) pulsate in fundamental modes, whereas the third type (RRc) pulsates in the first overtone. The typical light curve shapes of RRa, RRb and RRc are illustrated in Figure 2.2, in which RRa and RRb types have similar light curve shapes, whereas RRc type appears flatter. The fact that there are two distinct types of RR Lyrae stars can be easily demonstrated by a plot of amplitude against period, in which RRa and RRb types form a single cluster, while other types, including RRc, RRd and RRe, are clustered in a distinct region (see Figure 2.3). RRd and RRe are rare types of RR Lyrae stars in which the former exhibits double mode pulsation and the latter pulsates in the second overtone.

### 2.2.2 Cepheids

Moving upwards along the instability strip, we come across Cepheid variable stars. The most abundant type of Cepheids is classical Cepheids, sometimes also termed $\delta$ Cepheids or type I Cepheids. The spectral types of classical Cepheids range from F6 for the faintest to K2 for the brightest Cepheids [213]. Yet their effective temperatures and spectral types vary as they pulsate. Pulsation periods for classical Cepheids often lie between 1 and 100 days, but exceptions do exist. Ulaczyk et al. (2013) found classical Cepheids with periods as long as 135 days in the LMC. They also discovered a number
2.2 Pulsating Variables

Fig. 2.2 Representative light curves of different types of RR Lyrae stars according to the Bailey classification. (Top) RRa, (middle) RRb and (bottom) RRc. RRa and RRb types both pulsate in fundamental modes and are usually classified into a single group called RRab. On the other hand, RRc type pulsates in the first harmonic and has a flatter light curve shape. The examples were selected from the OGLE RR Lyrae catalogue. The plots were generated using the OGLE-III data [189, 193, 194], and are credited to Kozłowski (2007).
Variable Stars

Fig. 2.3 Period-amplitude diagram of RR Lyrae stars in the $I$ band. It can be easily seen there are two distinct clusters. Type a and b RR Lyrae (RRab) stars form a single cluster on the right side, while the one on the left side is constituted by all other types of RR Lyrae stars. This figure is credited to Soszyński et al. (2009).

of classical Cepheids in the LMC which pulsate in first overtone modes with periods shorter than 0.5 days. The pulsation of classical Cepheids is excited by the $\kappa$ and $\gamma$ mechanisms like RR Lyrae stars, but the $\kappa$ and $\gamma$ mechanisms in classical Cepheids are triggered in the zone where helium atoms are doubly ionized rather than singly ionized [36].

Classical Cepheids are more massive than the Sun, having evolved from main sequence stars of 2 to 20 $M_\odot$ [213]. However, many classical Cepheids in the Milky Way have masses in the more restricted range of 4 to 9 $M_\odot$. The theoretical evolutionary path of a 5 $M_\odot$ star that becomes a Cepheid is depicted in Figure 2.4. Such a star pulsates as a Cepheid first when it crosses the instability strip on its way to the red giant branch and, later, during a blue loop as it fuses helium in its core. Since the progenitors of Cepheids are massive stars which have short lifetimes in their main sequence stage, so that although classical Cepheids are evolving stars away from the main sequence stage, they are still relatively young stars, with ages ranging from about $10^7$ years for the brightest and most massive ones to a few times $10^8$ years for the faintest [36]. Because of this, classical Cepheids are only found in systems that have experienced recent star formation. In the Milky Way, they belong to the young disk population. Classical Cepheids have been identified in other nearby galaxies that contain young
Fig. 2.4 Evolutionary track of $5\,M_\odot$ main sequence star. The star will pulsate as a Cepheid when it crosses the instability strip during the stage in the subgiant branch as well as during the time of core-helium burning loops (blue loops) which occur after leaving the red giant branch and before reaching the asymptotic giant branch (AGB). This figure is taken from Wikimedia Commons and credited to Lithopsian.
stars, including the Magellanic Clouds (e.g. [195]). In the period-luminosity (P-L) diagram, there are two groups of classical Cepheids, exhibiting two different P-L relationships (see Figure 2.5). The two relations are indeed associated with classical Cepheids pulsating in first overtone and fundamental modes, respectively. Because of the strictness of the P-L relations, classical Cepheids are viable standard candles for measuring the distances from the Galactic centre (e.g. [22]) and neighbouring galaxies in the Local group (e.g. [64], [65]).

In addition to classical Cepheids, there is another type of Cepheids, called type II Cepheids which follow a striking different P-L relationship from those for classical Cepheids (see Figure 2.5). As the name suggests, type II Cepheids are population II stars with relatively little metal, and are old evolved stars of low mass (about 0.5-0.6\,M\odot). The typical pulsation periods of type II Cepheids range from 1 to 50 days. Historically, type II Cepheids are divided into three subclasses in terms of period (\(P\)) including BL Herculis (BL Her), W Virginis (W Vir) and RV Tauri (RV Tau). Between the clusters of classical and type II Cepheids in the P-L diagram, i.e., Figure 2.5, there are clusters of Cepheid-like pulsating stars that have periods less than two days and also follow distinct P-L relationships. These Cepheid-like stars are known as anomalous Cepheids, which was first termed by Zinn and Searle (1976). Anomalous Cepheids have light curves which resemble RR Lyrae stars, but anomalous Cepheids are more luminous than RR Lyrae stars.

### 2.3 Eclipsing Binary Systems

In Chapter 1, we mentioned that the presence of eclipses in the light curve of a stellar system is the feature showing the existence of an orbiting companion. Such stellar systems are called eclipsing binaries, and are cataloged into the group of extrinsic variables that have variability owing to the amount of light from the system being reduced by an orbiting companion.

Eclipsing binaries are particularly interesting and useful variable objects. Stellar binaries are common in our Galaxy and they might be the origins of many astrophysical phenomena such as supernova explosions, gamma-ray bursts and accretion disks, etc. [103, 115, 40, 50]. However, determining the properties of a stellar binary, or even identifying a binary system, is usually unachievable without an observed periodicity either in photometry or spectroscopy. In this sense, eclipsing binaries are crucial as we can determine their orbital periods by observing the eclipses repeatedly. Combining
Fig. 2.5 Period-luminosity relationship for Cepheids in the $I$ band. Excluding anomalous Cepheids, there are three distinctive groups associated with three different linear relationships between $\log P$ and magnitude. The upper two groups are associated with classical Cepheids, while the type II Cepheids including BL Her, W Vir and RV Tau form a single group with a different linear relationship from those of classical Cepheids. This figure is credited to Bernard et al. (2008).

this with high-quality radial velocity measurements using spectroscopy can further allow a complete modelling of the light curve of an eclipsing binary, and a determination of fundamental parameters, such as temperature, mass, radius and luminosity of each stellar component with high accuracy, which can be used to test current stellar models [128].

In addition, long-term observations of eclipsing binaries can provide an opportunity to detect the change in the orbital state and probe the interior of an eclipsing binary; for example, the detection of apsidal motion in an eccentric binary by accurate eclipse timings can act as an indirect way to determine the internal structure of the star, allowing stellar and evolution model testing (see, e.g., [43] and references therein). Since every essential orbital and stellar parameter of eclipsing binaries can be potentially determined from photometry and spectroscopy, their distances can be potentially derived as well. Eclipsing binaries can, therefore, serve as distance estimators of local galaxies (e.g. [79], [158] and [219]) and stellar clusters (e.g. [134]), and be used to study the spatial distribution of stars in the Galaxy [87].

Given the interesting and special role of eclipsing binaries amongst different types of variable stars, most wide-field survey projects have made efforts to identify and catalogue eclipsing binaries in their databases. Over the last decade, the number
of eclipsing binaries identified in the Galaxy has dramatically increased thanks to ground-based and space wide-field surveys, including the OGLE collaboration and NASA *Kepler* mission. The latter published the *Kepler* Eclipsing Binary Catalog that contains over 2,800 eclipsing binaries in the *Kepler* fields \([105]\), while the OGLE collection of variable stars contains over 450,000 and 11,589 eclipsing binaries towards the Galactic bulge \([190]\) and Galactic disk fields \([156]\), respectively. The triumph of such works, in addition to the sheer number of various types of new eclipsing binaries, is the discovery of a new class of eccentric binaries called heartbeat stars (see Section 2.5), which show periodic pulsations arising from the tidal interaction between two stellar components \([207]\).

**Roche lobes**

Depending on how the components are filling their Roche lobes, eclipsing binaries are generally classified into three physical groups: detached, semi-detached and contact binaries. The Roche lobe is the equipotential surface that crosses the Lagrangian point located between the two components.

The equipotential surface of a binary system can be derived if we deal with it as a restricted three-body system in which the gas consisting in the stellar envelopes is treated as a third body of infinitesimally small mass. We assume the primary and
secondary stars have masses equal to \( m_1 \) and \( m_2 \), respectively, in the co-rotating frame wherein all the bodies are rotating together in angular velocity, \( \omega \), about the origin located at the centre of mass of the binary system. The rotation axis is in the \( z \)-direction. The rotation plane of the two stars lies on the \( x-y \) plane and the two stars are set to be on the \( x \)-axis. The angular velocity, by definition, is

\[
\omega = \frac{2\pi}{P} = \sqrt{\frac{G(m_1 + m_2)}{a^3}},
\]

where \( a = a_1 + a_2 \) is the relative semi-major axis. Then, in this non-inertial reference, the potential at the location of the test particle, \( \mathbf{r} \), from the origin is given by

\[
\Phi(\mathbf{r}) = -\frac{Gm_1}{|\mathbf{r} - \mathbf{a}_1|} - \left( -\frac{Gm_2}{|\mathbf{r} - \mathbf{a}_2|} \right) - \frac{1}{2}(\omega \times \mathbf{r})^2
\]

Note that \( \mathbf{a}_1 \) and \( \mathbf{a}_2 \) are the distances of the primary and secondary stars from the origin, respectively (see Figure 2.6). Incorporating Kepler’s law and defining the mass ratio \( q \) such that \( 0 < q \leq 1 \) (i.e. \( q = m_2/m_1 \) if \( m_2 < m_1 \)), eq.(2.2) can be rewritten as
the normalized potential, called the Roche potential,

\[ \Phi_n = \frac{2}{r_1(1+q)} + \frac{2q}{r_2(1+q)} + \left( x - \frac{q}{1+q} \right)^2 + y \]  

(2.3)

where \( r_1 = |r_1| = \sqrt{x^2 + y^2} \) and \( r_2 = |r_2| = \sqrt{x^2 + (y+1)^2} \). Here we adapted the length scale so that \( a = 1 \). The equipotential surfaces can be determined by setting \( \Phi_n = C \), where \( C \) is a constant value. Five stationary points, called Lagrangian points, exist around the stars, satisfying the condition that \( \nabla \Phi_n(r) = 0 \). Three of them, labeled by \( L_1, L_2 \) and \( L_3 \), lie along the line on the x-y plane connecting the two stars. \( L_2 \) and \( L_3 \) are the outer Lagrange points in which \( L_3 \) is assigned to the one with higher potential. \( L_1 \) is located between the two components and is the Lagrangian point that represents the intersection point on the Roche lobe. The Roche lobe represents the boundary surface within which the test particle is bound to the closest component of the binary system. In other words, the Roche lobe is the maximum volume within which the component is self-bound that no material should be attracted out of it by the other component’s gravitational force.

**Physical classification**

When the components are separate from each other at a large distance so that each of them fills the volume well inside the Roche lobe, then these two components can be treated as isolated systems evolved independently. Such binary systems are classified as detached binaries. Since the components in detached binaries lack interactions that could alter their stellar properties, the eclipsing binaries that are detached binaries are valuable for testing stellar theories as the stellar parameters such as sizes, masses, temperatures and luminosity, etc., can be accurately derived by modelling their photometric light curve, spectra and radial velocity curves. Information about stellar mass and luminosity would allow us to calculate the distances of the binaries. Therefore, detached binaries can be used as distance indicators like Cepheids and RR Lyrae stars.

The equipotential surfaces according to the Roche potential, i.e., eq.(2.2), are usually used to define the stellar surfaces of the components in a binary system. When the separation is so large that the surface potential of each component is far below the Roche lobe, the stellar shape would be close to a sphere as equipotential surfaces over there are nearly spherical. However, as the separation decreases, and the surface
When one of the components fills its Roche lobe, its material above the Roche lobe can now be transferred to the other component across the Lagrangian point $L_1$. This process of mass transfer is called the Roche-lobe overflow. In this situation, the binary system is classified as a semi-detached binary. If the Roche lobe is overfilled by both components, then their envelopes will become overlapping with each other, forming a contact binary with the shared envelopes. Figure 2.8 illustrates the physical appearance of these three types of binary systems, and the plot of the Roche potential along the x-axis, demonstrating the existence of asymmetric potential wells with the boundaries at $L_1$, $L_2$ and $L_3$ that hold the stellar masses. The Roche-lobe overflow occurs inevitably once the surface potential of either component is above the potential at $L_1$ as would be expected to happen in semi-detached and contact binaries.

Contact and semi-detached binaries are usually close binary systems with periods less than three days. They are particularly abundant in the catalogues of eclipsing binaries from photometric surveys (e.g. the OGLE catalogue.) Since mass transfer happens in contact and semi-detached binaries, these binary systems can undergo complicated or unusual evolutionary processes. Hence, they are believed to be the origins of many astrophysical phenomena and the progenitors of many unusual stellar objects.

**Observational classification**

In photometry, according to the light curve shapes, eclipsing binaries may be classified into three observational types: EA (or Algol), EB (or $\beta$ Lyrae) and EW (or W Ursae Majoris) (see Figure 2.9).

The EA or Algol type is the type of eclipsing binaries named for the prototype star Algol ($\beta$ Persei), the first eclipsing binary ever discovered by humans. This type of eclipsing binaries has well-defined eclipses in which the ingress and egress points can be clearly identified. The difference between the depths of primary and secondary eclipses is usually very large. Even the secondary eclipses may be absent because they are too shallow to be observed. The overall brightness outside the eclipse phases is relatively constant, although smooth variation may be noticeable.

The EB or $\beta$ Lyrae-type eclipsing binaries exhibit continuous variation in the light curve’s brightness outside the eclipse phases. In general, the primary and secondary eclipses have different depths clearly.
Fig. 2.8 Profile of Roche potential along the x-axis. When the surface potential of a component is below the Roche lobe (orange), the material of that component is self-gravitationally bound. In the case that the material of both components is self-gravitationally bound, the system is a detached binary in which each component can be approximately recognized as an isolated system. When one of the components overfills the Roche lobe (red), then its material can transfer into the other component and the binary system becomes semi-detached. When both components overfill the Roche lobe (brown), they share the envelope and the binary system is a contact binary.
Fig. 2.9 Representative light curves of three different observational types of eclipsing binaries (EA, EB and EW). The examples were selected from the OGLE catalogue. The plots were generated using the OGLE-III data in the I band [156, 154, 80], and are credited to Kozłowski (2007).
The EW or W Ursae Majoris (W UMa) has continuous variation in brightness, preventing identification of ingress and egress points that define the eclipse regions. Generally, primary and secondary eclipses have little difference in depth. Most of them have periods less than a day.

### 2.4 RS Canum Venaticorum

The non-uniform surface temperature of a rotating system can produce observable periodic brightness variation in the light curve as long as the rotational axis does not coincide with the line of sight. Such non-uniformity of surface temperature may be due to the presence of cool spots on the star associated with strong magnetic activity. As the star is rotating, the spots will be in and out of the observer’s view cyclically. Therefore, the stellar brightness varies cyclically as well. Rotational variability is found in RS Canum Venaticorum (RS CVn) stars, a type of binary systems, having active chromospheres with large stellar spots. They are easily identified if they are also eclipsing binaries because the light curves would be branched if they were folded at the eclipsing periods as a result of deviations between stellar rotational and orbital periods. Figure 2.10 shows the folded light curves of RS CVn eclipsing binaries in the OGLE catalogue. As can be seen, they are branched as if there were two overlapping light curves in each case.

### 2.5 Heartbeat Stars

One of the new discoveries by massive photometric surveys in the last decade is a subclass of eccentric ellipsoidal variables (ELVs) called heartbeat stars. The prominent feature of the heartbeat star’s light curve is the increase in brightness at the phase of periastron as a result of stellar surface distortion by tidal interaction \(^{109}\). The shape of the resultant brightness variation (i.e., eccentric ellipsoidal variation) strongly depends on the inclination, eccentricity and argument of periastron of the orbit as shown in Figure 2.11. Over 170 heartbeat stars in the Galaxy were discovered by the *Kepler* space telescope \(^{105}\) and a few in the LMC were discovered by the OGLE collaboration \(^{145}\). Many of them contain the components of A-F type stars. In addition to the eccentric ellipsoidal variations, many heartbeat stars also exhibit observable sinusoidal pulsations which have frequencies that are the multiples of their orbital frequencies in their light curves \(^{207}\). Such pulsations are induced when the orbital frequency is so close to an
Fig. 2.10 Phased light curves of three RS CVn eclipsing binaries in the OGLE catalogue using the data collected from 2002 to 2009. Branching can be seen when the light curves are phased at the eclipsing periods. The red curves represent the variation components induced by star spots. Note that $P_{\text{orb}}$ is orbital period. This figure is credited to Pietrukowicz et al. (2013).

Fig. 2.11 Synthetic light curves of heartbeat stars generated from the analytical model purposed by Kumar et al. (1995) with different values of eccentricity ($e$), inclination ($i$) and argument of periastron ($\omega$). The light curves are normalized such that the amplitudes equal one.
eigenmode of the stellar component that the star behaves as a forced oscillator with the amplitude of the mode being amplified. Because of these pulsation phenomena, the heartbeat stars are interesting objects for testing stellar pulsation theories. On the other hand, approximately half of the heartbeat stars in the *Kepler* catalogue have periods between 3 and 15 days and high eccentricities ($e > 0.3$). The tidal dissipation in close eccentric binaries is so effective that the circularization timescales are much smaller than the lifetime of hot main-sequence stars that many heartbeat stars contain. Hence, the highly eccentric orbits of many heartbeat stars should not be maintained. The presence of tertiary companions which excite the orbits of these heartbeat stars by, e.g., the Kozai effect, and thus counteract the tidal dissipation, has been suggested to explain the problem [207].
Chapter 3

The First Eclipsing Binary Catalogue from the MOA-II Database

There is no such thing on earth as an uninteresting subject; the only thing that can exist is an uninterested person.

G.K. Chesterton, Heretics

Search for companions in eclipsing binaries (EBs) is the ultimate interest in this thesis. Unfortunately, although MOA-II has collected \( \sim 100 \, \text{TBs} \) of image data toward the Galactic bulge (GB) since 2006, there was no existing EB catalogue from the MOA database that we could work on. Because of this, the first half of the research period for this project was almost entirely dedicated to work on the identification of EBs in the MOA-II database. In this chapter, we present the results of this aspect that was published in the paper of Li et al. (2017).

3.1 Identification of Eclipsing Binaries

Since there are only raw image data available in the MOA database, for our research purpose, we had to first request Ian Bond at Massey University, who is in charge of maintaining the MOA database, to generate photometric data from the MOA raw images using the algorithm he developed. In return, we obtained the two observational
seasons’ worth of data of each field, collected from February 2013 to August 2014, from Ian Bond. Nonetheless, generating the light curves of every star in a GB field even from the data provided by Ian Bond is time-consuming. For this reason, and for the purposes of demonstrating our method, we decided to inspect only the GB9 and GB10 fields in order to search for EB candidates. The GB9 and GB10 fields were selected for inspection simply because these fields were observed most frequently. They also overlap with the OGLE fields [206] which allows us to cross reference the two databases.

The GB9 and GB10 fields alone already contained \( \sim 8,000,000 \) variable objects. Machine learning or semi-automatic algorithms are desirable to search for EB candidates in such a great deal of data. Unfortunately, the data sets also included spurious variable objects which result from contamination by nearby variable stars, or imperfect image subtraction at the stars’ positions due to the effects of differential refraction [24, 4]. This made the existing automatic classification algorithms used by other research groups such as the OGLE (e.g. [133]) and *Kepler* (e.g. [10]) teams less applicable to our data sets. We did not attempt therefore to develop our own automatic or semi-automatic algorithm to do the task, given that no EB catalogue of MOA-II existed before for testing. Instead, we adopted a straightforward and tedious strategy; that is, we performed a period analysis of the data sets and folded the light curves at the calculated periods, and we then identified EB candidates by inspecting their shapes by eye.

### 3.1.1 Cleaning light curves

Before finding periodic signals, we first cleaned every light curve by iterating twice an outlier rejection algorithm that filtered out the data points with values 4.0σ above or 9.0σ below the relative flux mean, and detrended the light curves by linear regression. The asymmetric criterion in relative flux was taken to avoid rejecting data points corresponding to an eclipse. As we expected, the error bar of a measurement point should not be much larger than the overall relative flux standard deviation, data points with errors > \( \sqrt{3} \sigma \) were discarded as unreliable measurements. Note that \( \sigma \) is the standard deviation of all relative flux values in the light curve. After cleaning and checking a certain number of the resultant light curves, we ignored the light curves with \(< 1000 \) good data points as being poor objects.
3.1 Identification of Eclipsing Binaries

3.1.2 Period analysis

Conditional entropy method

As we wanted to take all good data points of a light curve into account to search for periodic signals, we needed a period finding algorithm (PFA) that was still relatively fast, and of course robust, even when handling large time series data. The MOA observation adopted high cadences towards the densely-populated fields towards the GB, e.g. GB9 and GB10, and resulted time-series of variable objects in those fields of about 3000 flux measurement points in a MOA observational season. The Lomb-Scargle periodogram \cite{120, 179}, Fourier transform \cite{66} and the phase dispersion minimization (PDM) method \cite{200} are PFAs which have been regularly used to identify periodic signals of a time series which contains a few thousands of data points. However, in the case of MOA GB9 and GB10 fields, the number of measurement points doubled as the time span doubled because the fields were monitored regularly by the MOA telescope. Typically, there were about 6000 measurement points per time series of our data sets over two MOA observational seasons. The number of measurement points increased to about 30000 if the data sets of the full observational time span, i.e., 9.5 years, were used. Those conventional PFAs mentioned before were not satisfactory in terms of their computational speeds if we considered to take the full time series into account to determine the period of a variable object. We thus adopted the conditional entropy (CE) method proposed by Graham et al. (2013) to determine the light curve periods.

The CE method is a modified version of the Shannon entropy method that was first introduced by Cincotta et al. (1995). It determines the period of a light curve by minimizing the conditional entropy over a range of trial periods at which the light curve is folded, based on the idea that the most ordered arrangement of a light curve, corresponding to the signal with the highest recognizable shape in the phase-flux plane, should be established when it is folded at the correct period. The conditional entropy, $H_c = H(m|\phi)$, which should be minimized is defined by:

$$H_c = \sum_{i,j} p(m_i, \phi_j) \ln \left( \frac{p(\phi_j)}{p(m_i, \phi_j)} \right),$$

where $p(m_i, \phi_j)$ is the occupation probability for the $i^{th}$ partition in flux and the $j^{th}$ partition in phase and $p(\phi_j)$ is the occupation probability of the $j^{th}$ phase partition,
which for rectangular partitioning is just

\[ p(\phi_j) = \sum_i p(m_i, \phi_j). \quad (3.2) \]

We divided the phase-flux plane into 20 × 20 grids of equal size. The boundaries of the plane were taken as the maximum and minimum fluxes of the light curve. The trial periods ranged from 0.05 to 600 days. The occupation probability was simply calculated by counting the number of points in the grids involved over the total number of points. The CE method gave faster performance than the PDM method with a large number of data points, while they both produced similar results for almost every light curve in the unpublished catalogue of unclassified MOA variable objects.

**Phase dispersion minimization method**

However, conditional entropy is not a useful measure to rank the periodic signals of different light curves in order to easily pick out obvious EB candidates, because the minimized conditional entropy values just tell us how compact the folded light curves are but do not give an absolute sense regarding the sharpness of their shapes. As a consequence, the PDM statistic, \( \Theta \), defined by:

\[ \Theta = s^2/\sigma^2, \quad (3.3) \]

was calculated for the light curve folded at the period determined by the CE method as the ranking measure, where \( s^2 \) is the overall variance of the partitions of the folded light curve in phase, given by

\[ s^2 = \frac{\sum_{j=1}^{M}(n_j - 1)s_j^2}{\sum_{j=1}^{M} n_j - M}, \quad (3.4) \]

and \( \sigma^2 = \sum_{i=1}^{N}(x_i - \bar{x})^2/(N - 1) \) is the variance of the folded light curve of \( N \) data points. Note that, in eq.(3.4), \( s_j^2 \) and \( n_j \) are the variance and number of points in the \( j^{th} \) partition in phase, respectively, and \( M \) is the total number of partitions, i.e. the number of bins in phase. We used \( M = 20 \) bins in our calculations. The PDM statistic, \( \Theta \), is appropriate for our purpose since it intrinsically measures how much the shape of a folded light curve is different from a horizontal straight line. The larger the value of \( \Theta \), the more identical the folded light curve is to a horizontal straight line. Therefore,
those folded light curves with the lowest values of $\Theta$ would go to the top in our list of candidates.

One thing that should be mentioned is that the observation time of a MOA image was taken as the midpoint between the start and end times of exposure, which were recorded up to six decimal places in Julian Date. The difference between Julian Date (JD) and Barycentric Julian Date (BJD), i.e., $\text{BJD} - \text{JD} = \Delta_{R\odot} + \Delta_C$, where $\Delta_{R\odot}$ is the Rømer delay (i.e., the time correction when the barycentre of our Solar system is taken as the reference point) and $\Delta_C$ is the clock correction from Coordinated Universal Time (UTC) to Barycentric Dynamical Time (TDB), varies over time and can be as much as about nine minutes to date [57]. Nevertheless, the systematic time errors that are induced when using JD instead of BJD are negligible compared to a typical period of contact binaries, let alone detached binaries. We, therefore, did not convert JD to BJD in the period analysis at the stage of searching for EB candidates. Nonetheless, we did apply BJD corrections and recalculated the periods of all EB candidates afterward, and the eclipsing periods provided throughout this chapter are the recalculated periods after the BJD corrections. Since BJD correction is essential for eclipse time variation analysis, we will go into detail on this aspect in Section 4.3.1 instead.

### 3.1.3 Light curve inspection

Since the MOA observations, as every ground-based optical observation, always suffer from discontinuity due to day-night cycles which induce 0.5d aliases as the strongest periodic signals in a large majority of our data sets, a way to eliminate the folded light curves of 0.5d aliases was needed. To do this the light curves were folded at double their calculated periods for inspection. Those with phase gaps larger than 0.03, after folding, were then ignored. This strategy is workable and helpful first because there were usually sufficient data points covering the phases of our light curves well, even when folded at double the calculated periods, and second because the true periods were often double the calculated periods for EB candidates.

All EB-like folded light curves were listed in the first round of inspection. We then rejected those false positives, which are in fact Cepheids or artifacts, by examining the shapes of the listed light curves more carefully and determined the multiples of the calculated periods that corresponded to the true periods of EB candidates. The common artifacts we recognized in the MOA data are those of periods between 0.99 and 1 days. Almost every periodic signal that fell into this range turned out to be artifacts. In order to reduce the workload of inspection, we thus decided to ignore
Fig. 3.1 Examples of periodic signals at 0.332d that we considered as aliases.

Fig. 3.2 Examples of repeated EB candidates with the same period but different coordinates in the MOA database. $P$ is the period in unit of day. RA and Dec coordinates are shown in the top left-hand corner in each graph.

every signal we found that fell into the range between 0.99 and 1 days. In addition, suspicious periodic signals that looked like the ones shown in Figure 3.1 were also frequently discovered at period $\sim 0.332$d. We do not know the source of this type of periodic signals; however, given their similarity in both shape and period, we strongly suspect they are simply another type of artifacts, and thus decided to ignore any signal found at 0.332d as well.

On top of the problem of substantial numbers of artifacts, we also recognized that many neighbouring EB candidates in our list were exactly or nearly identical in terms of their periods and folded light curve shapes. This indicated that most of the EB candidates we included in the list were spurious. For example, Figure 3.2 shows the three light curves of the EB candidates identified in the first round of inspection. They all have exactly the same period and light curve shape, except their coordinates are different but close. This suggests that they are the same object that was erroneously repeatedly extracted from the images at different image positions. Without reanalyzing the photometry of the light curves, whenever suspicious identical neighbouring EB
candidates were found within six arcsec from each other, we simply chose the one among them with the lowest value of PDM statistic, Θ, as the genuine one.

### 3.2 Eclipsing Binary Catalogue

Using the method described in Section 3.1, we identified 8733 EB candidates with periods ranging from 0.09 to 66 days in the GB9 and GB10 fields. Figure 3.3 shows the histogram of the EB candidates’ periods with logarithmic bins, in which a cut-off in period exists at \( \sim 30 \) days (excluding the four EB candidates of periods > 40 days), indicating that our search was biased towards EBs with periods < 30 days. On the other hand, the majority of EB candidates discovered are of periods < 1 day and, unsurprisingly, belong to contact or semi-detached binaries. It is manifest that detecting EBs, particularly detached binaries, of periods > 30 days is difficult using only two observational seasons’ worth of data. The identification rate of such long period binaries should be improved using the data with a longer time base. Since the GB9 and GB10 fields both overlap with the OGLE fields, we thus cross-checked our candidates with the OGLE collections of EBs towards the GB from the OGLE-II, OGLE-III and OGLE-IV surveys [190]. There are 486 EB candidates in our catalogue which have no OGLE EB counterpart within four arcsec.
Fig. 3.3 Period histogram of EB candidates in the GB9 and GB10 fields with logarithmic bins. The number of counts, $N_{\text{count}}$, is presented in the linear (upper panel) and logarithmic scale (lower panel), respectively. A total of 8733 EB candidates were identified in these two fields, using the data from two MOA observational seasons. Among them, 486 EB candidates have no OGLE EB counterpart within 4 arcsec.
Fig. 3.4 Examples of the folded light curves of EBs in the GB9 and GB10 fields, including those with dominant ellipsoidal modulations (first row), or reflection effects (second row), those showing strong O’Connell effects\(^1\) (third row), and the three EBs with unusual phase modulations potentially involving multiple effects in addition to reflection effect (fourth row). The bottom row shows three examples of binaries with giant or sub-giant star components. The eclipsing periods are in days. The minimum of the primary eclipse of each folded light curve was adjusted to be located at the zero phase.
Fig. 3.5 Folded light curves of EBs, in the GB9 and GB10 fields, with phase modulations potentially associated with ellipsoidal variations modified by strong reflection effects. The eclipsing periods are in days. The minimum of the primary eclipse of each folded light curve was adjusted to be located at the zero phase.

Fig. 3.6 Folded light curves of six eccentric binaries in the GB9 and GB10 fields. The eclipsing periods are in days. The minimum of the primary eclipse of each folded light curve was adjusted to be located at the zero phase.
Fig. 3.7 Folded light curves of six eclipsing RS Canum Venaticorum (RS CVn) type star candidates in the GB9 and GB10 fields. The eclipsing periods are in days. The minimum of the primary eclipse of each folded light curve was adjusted to be located at the zero phase.
Fig. 3.8 Folded light curves of four EBs in the GB9 and GB10 fields with periods below the 0.22-day contact binary minimum. MOA-314103-GB10-5 is clearly a contact binary and MOA-50256-GB9-4 is a post-common-envelope binary candidate consisting of a main-sequence star and a subdwarf, judging from their light curve morphology, while the identities of MOA-234255-GB9-6 and MOA-340135-GB9-10 are unknown. Note that the eclipsing periods are in days. The minimum of the primary eclipse of each folded light curve was adjusted to be located at the zero phase.
Table 3.1 Interesting EBs, including eccentric binaries, eclipsing RS Canum Venaticorum (RS CVn) type stars, binaries with noteworthy phase modulations as well as binaries with post main-sequence stellar components, in the GB9 and GB10 fields. The first column shows the MOA number of each EB candidate, the second column shows their corresponding GB fields and the third column gives the CCD chip numbers at which the EB candidates are located. Note that $H_c$ is the conditional entropy in eq.(3.1) and $\Theta$ is the PDM statistic in eq.(3.3); $T_p$ and $T_s$ are the reference epochs of primary and secondary eclipse minima, respectively, measured by the template method in Section 4.4. The last column shows the OGLE EB counterparts of the MOA EB candidates within 4 arcsec; note that * is within 1 arcsec and ** is within 2 arcsec.

| No.   | GB chip | Period (d) | $H_c$  | $\Theta$  | RA (h:m:s) | Dec (d:m:s) | $|T_p - T_s|$ (BJD) | OGLE Object       |
|-------|---------|------------|--------|-----------|------------|-------------|-----------------|-------------------|
| 260591| 9       | 2          | 1.2581 | 1.3925    | 0.1270     | +17:58:56.82| -28:55:19.50    | --                |
| 263861| 10      | 4          | 1.7900 | 1.1014    | 0.0586     | +17:58:52.60| -27:57:16.63    | --                |
| 255102| 9       | 1          | 2.9331 | 1.2296    | 0.1573     | +17:59:11.29| -28:32:49.91    | --                |
| 100092| 10      | 1          | 0.8526 | 1.3759    | 0.5736     | +17:58:45.89| -26:53:12.64    | --                |
| 315721| 9       | 2          | 3.2436 | 1.2507    | 0.2843     | +17:59:19.52| -29:01:13.96    | --                |
| 393777| 9       | 3          | 4.8285 | 1.1393    | 0.5220     | +17:59:37.52| -29:13:20.04    | --                |
| 23676 | 9       | 7          | 2.1793 | 1.4319    | 0.8533     | +18:02:57.08| -29:35:22.17    | --                |
| 42742 | 9       | 8          | 2.2359 | 1.3244    | 0.6708     | +18:02:47.89| -29:19:45.30    | --                |
| 350526| 9       | 8          | 5.2286 | 1.3003    | 0.0765     | +18:00:39.56| -29:14:39.34    | --                |
| 349294| 9       | 7          | 2.3968 | 1.8205    | 0.3710     | +18:00:43.97| -27:47:42.92    | --                |
| 146114| 9       | 7          | 3.6452 | 1.1635    | 0.3001     | +18:01:59.68| -29:26:49.62    | --                |
| 170603| 10      | 8          | 4.5938 | 1.5166    | 0.3023     | +18:01:49.94| -27:38:01.43    | --                |
| 57615 | 10      | 1          | 18.2139| 1.4984    | 0.0822     | +17:58:13.26| -26:49:45.86    | --                |
| 68403 | 9       | 3          | 19.5865| 1.0055    | 0.2275     | +17:57:27.67| -29:22:26.36    | --                |
| 170700| 9       | 2          | 34.1678| 1.2526    | 0.0746     | +17:58:18.76| -28:49:18.71    | --                |
| 312054| 9       | 2          | 5.5753 | 0.9452    | 0.9098     | +17:59:18.56| -28:44:26.82    | --                |
| 157137| 9       | 10         | 6.6259 | 1.3546    | 0.7577     | +18:01:50.26| -28:25:51.79    | --                |
| 238852| 9       | 5          | 20.4211| 1.3200    | 0.5753     | +17:58:57.69| -29:59:39.40    | --                |
| 164457| 9       | 4          | 3.8996 | 1.1218    | 0.1631     | +17:58:10.09| -29:31:09.27    | --                |
| 189457| 9       | 3          | 7.6329 | 1.2755    | 0.2396     | +17:58:17.67| -29:19:37.28    | --                |

*Continue on the next page*
Table 3.1 Interesting EBs in the GB9 and GB10 fields (cont.)

| No.  | GB chip | Period (d) | $H_e$ | $\Theta$ | RA (h:m:s) | Dec (d:m:s) | $|T_p - T_s|$ (BJD) | OGLE Object                  |
|------|---------|------------|------|---------|-----------|------------|-----------------|-----------------------------|
| 282091 | 9       | 17.6026    | 1.5346 | 0.2778  | +18:00:56.10 | -28:52:33.01 | --              | OGLE-BLG-ECL-248557*       |
| 204923 | 9       | 5.4672     | 1.9721 | 0.6582  | +17:58:23.83 | -29:23:14.60 | --              | OGLE-BLG-ECL-219864*       |
| 129368 | 10      | 4.5422     | 1.5755 | 0.6452  | +17:59:05.39 | -26:52:13.39 | --              |                             |
| 45015  | 10      | 6.7270     | 1.3525 | 0.7198  | +18:02:39.23 | -26:51:56.80 | --              | OGLE-BLG-ECL-267353*       |
| 111131 | 9       | 8.1225     | 1.7073 | 0.8977  | +18:02:17.51 | -29:05:18.55 | --              | OGLE-BLG-ECL-263293*       |
| 147629 | 10      | 9.0413     | 1.1516 | 0.6541  | +17:59:17.86 | -26:51:13.97 | --              | OGLE-BLG-ECL-230190*       |
| 138632 | 9       | 19.1310    | 1.3043 | 0.9214  | +18:02:03.19 | -29:40:28.47 | --              | OGLE-BLG-ECL-260748*       |
| 244397 | 9       | 0.9612     | 1.5803 | 0.4159  | +18:01:22.59 | -29:21:20.06 | --              |                             |
| 175628 | 9       | 4.6249     | 1.6673 | 0.2966  | +17:58:27.24 | -29:52:02.39 | --              | OGLE-BLG-ECL-220539*       |
| 305199 | 9       | 4.1276     | 0.9349 | 0.6976  | +17:59:36.48 | -28:40:50.44 | --              | OGLE-BLG-ECL-233520*       |
| 356601 | 9       | 4.1813     | 1.0998 | 0.6422  | +18:00:23.29 | -29:56:56.28 | --              | OGLE-BLG-ECL-242433*       |
| 268839 | 9       | 4.1528     | 1.6834 | 0.3196  | +18:01:08.13 | -29:38:53.95 | --              | OGLE-BLG-ECL-250942*       |
| 207541 | 9       | 7.8195     | 1.3760 | 0.1397  | +18:01:30.01 | -28:57:00.47 | --              | OGLE-BLG-ECL-254823*       |
| 314103 | 10      | 0.2182     | 1.0595 | 0.4664  | +17:59:51.53 | -28:05:07.24 | --              | OGLE-BLG-ECL-000106*       |
| 50256  | 9       | 0.1849     | 1.1874 | 0.8021  | +17:57:20.60 | -29:33:50.31 | --              | OGLE-BLG-ECL-207454*       |
| 234255 | 9       | 0.1569     | 1.3045 | 0.9545  | +18:01:17.51 | -29:57:21.00 | --              |                             |
| 340135 | 9       | 0.0950     | 1.4823 | 0.8985  | +18:00:23.24 | -28:23:41.14 | --              | OGLE-BLG-ECL-000110**      |
| 129173 | 10      | 0.5603     | 1.2395 | 0.0567  | +17:59:05.20 | -26:53:45.98 | 0.279105        | OGLE-BLG-ECL-227801*       |
| 360325 | 10      | 0.2995     | 1.6164 | 0.1658  | +18:00:40.33 | -27:44:28.88 | 0.150731        | OGLE-BLG-ECL-245557*       |
| 115233 | 10      | 0.3315     | 1.5493 | 0.0776  | +18:02:00.84 | -27:06:42.65 | 0.163787        | OGLE-BLG-ECL-260381*       |
3.2 Eclipsing Binary Catalogue

Fig. 3.9 Folded light curves of doubly EB candidates in the GB10 field. (a) The main eclipsing signals. (b) The additional eclipsing signals. Note that the eclipsing periods are in days. The minimum of the primary eclipse of each folded light curve was adjusted to be located at the zero phase.

Eclipsing Binaries with Various Phase Modulations

In spite of lacking long period binaries, our catalogue already contains a large variety of EB candidates in terms of the light curve morphology. Many of them have phase modulations obviously dominated by ellipsoidal variations due to stellar surface distortion by tidal interaction [224] (e.g. MOA-263861-GB10-4 and MOA-255102-GB9-1 in Figure 3.4) or dominated by reflection effects [223] (e.g. MOA-100092-GB10-1 and MOA-315721-GB9-2 in Figure 3.4). The O’Connell effect [48, 147]¹ was observed in a large number of ellipsoidal EB candidates, amongst which a few extreme cases were discovered (e.g. MOA-23676-GB9-7, MOA-42742-GB9-8, and MOA-350526-GB9-8 in Figure 3.4). Also, we noticed certain numbers of EB candidates having unusual phase modulations to which multiple effects might take significant contributions in addition to the reflection effect (see Figure 3.4). Meanwhile, our discovery includes a group of EB candidates showing similar sinusoidal phase modulations, where the conjunction phases, in which eclipses would occur, happen to be midway between the maxima and minima of the sinusoidal curves, potentially resulting from ellipsoidal modulations modified by strong reflection effects (e.g. [118]) (see Figure 3.5).

¹The O’Connell effect is asymmetries of out-of-eclipse brightness maxima of close EBs, first systematically studied by the astronomer D. J. K. O’Connell [147]. Brightness maxima of an EB should be the same when the components switch positions every half a period. However, unequal maxima have been observed in many close EBs’ photometric light curves. In the case that the primary maximum after the primary eclipse is brighter than the secondary maximum after the secondary eclipse, the O’Connell effect is said to be positive, whereas the opposite case is referred to as negative. Causes of the O’Connell effect are still mysteries. Models which have been suggested to explain the effect include the hot spot model [182] and the model involving interaction of EBs’ components with their circumstellar materials [119].
Eccentric and Other Interesting Eclipsing Binaries

We also discovered hundreds of detached binaries. Among them, at least six eccentric binaries were identified by inspecting their folded light curves (see Figure 3.6), in which the phase differences between their own primary and secondary eclipses were noticeably different from 0.5 phase, indicating non-zero eccentricity of their orbits.

Other interesting EB candidates include those with giant or sub-giant star components (e.g. MOA-170700-GB9-2 in Figure 3.4), PCEBs (e.g. MOA-50256-GB9-4 in Figure 3.8), and eclipsing RS Canum Venaticorum (RS CVn) type star candidates, where their identities were deduced by their periods and the characteristics of their folded light curve shapes recognized by eye. For example, the ultra-short period plus the strong reflection modulation of MOA-50256-GB9-4, which is clearly a detached binary from the manifest ingress and egress, and the very short phase duration of its primary eclipse, satisfy the characteristics of the typical PCEB light curve shape (e.g. [52]); and a group of EB candidates shown in Figure 3.7 are considered as eclipsing RS CVn type stars because of the quasi-periodic brightness variations in the out-of-eclipse phase regions of their folded light curves, which are the characteristic features of RS CVn type star light curves owing to their active star spots (e.g. [170]).

Doubly Eclipsing Binaries

We further searched for any hidden eclipsing signals on top of the strongest eclipsing signals as follows: we calculated the mean flux values of a folded light curve with 200 bins; we then subtracted the mean flux values from the folded light curve and unfolded it afterwards; we then determined any additional eclipsing period for the subtracted light curve using the CE method, and inspected the light curves folded at the new periods. As a result, three doubly EB candidates, listed in Table 3.2, were discovered in the data sets. The folded light curves of their two eclipsing signals are shown in Figure 3.9. Given the fact that they lie in the most crowded fields, they might not be genuine but result from the blending of nearby EBs.
Table 3.2 Doubly EB candidates in the GB10 field. Note that * is within 1 arcsec and ** is within 2 arcsec

<table>
<thead>
<tr>
<th>No.</th>
<th>GB chip</th>
<th>$P_a$ (d)</th>
<th>$(H_e)_a$</th>
<th>$\theta_a$</th>
<th>$P_b$ (d)</th>
<th>$(H_e)_b$</th>
<th>$\theta_b$</th>
<th>RA (h:m:s)</th>
<th>Dec (d:m:s)</th>
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3.3 Discussion and Conclusions

The MOA project was evolved to its second stage in 2006. We presented the first MOA-II catalogue of EB candidates identified in the two Galactic bulge fields, GB9 and GB10, using the data collected from February 2013 to August 2014. The MOA-II EB catalogue contains over 8000 EB candidates with periods ranging from 0.09 to 66 days. Most of them are contact or semi-detached binaries of periods less than a day. Detached binaries of periods $> 30$ days were difficult to identify using just two MOA observational seasons’ worth of data. The identification rate of long period detached binaries can be increased if the complete data sets are used. However, it will cause substantial computational pressure on the light curve generation and data analysis, especially for fields with high cadences such as GB9, GB10, and GB5, in which there are no less than 30000 images in total to date. Another challenge to the EB identification from the 100 TB MOA database is to develop a robust automated or machine learning algorithm that can effectively filter out the artifacts and 0.5d aliases which came up substantially in the manner described in Section 3.1, and replace the tedious and time-consuming light curve inspection by eye that has limited our study to only two MOA fields from a total of 22 fields. The possible solution to this problem is to develop our own machine learning algorithm, for example, using random decision forests [53] for the MOA database. Other than contact and semi-detached binaries, there are certain numbers of eccentric binaries, binaries with complicated phase modulations and eclipsing RS CVn type stars as well as a few ultra-short period binaries discovered in the GB9 and GB10 fields. Additionally, three EB light curves were discovered to have extra eclipsing signals under their main eclipsing signals indicating their doubly EB identity.

The paper on the study in this chapter was published in Monthly Notices of the Royal Astronomical Society (MNRAS) [117], and it interested Radek Poleski, a postdoctoral researcher at the Ohio State University, soon after the publication. Radek Poleski further cross-matched our MOA EB candidates with the OGLE catalogue of EBs in terms of period and coordinates. He obtained 288 matches between our MOA EB candidates and the OGLE EBs which have similar periods. Among them, 2166 of the MOA EB candidates have similar periods to their OGLE counterparts. For the rest, the periods of the MOA EB candidates are twice as long as those of the OGLE counterparts in most of the cases. The mismatch between the MOA and OGLE periods in those cases might be due to their secondary eclipses being so shallow that
we failed to recognize them and, therefore, overestimated their periods. On the other hand, Radek Poleski also realized that the first false match occurred at 0.52 arcsec when matching radii (i.e. the angular separation between a MOA EB candidate and an OGLE EB). In our private conversation, he suggested that the optimal matching radius is around 2.5 arcsec, and the highest separation match he found was 3.6 arcsec. He eventually identified 112 new objects in our MOA EB catalogue and added them to the catalogue of variable stars in the Kepler 2 Campaign 9 (K2 C9) field.

Our search for EBs was carried out using datasets of the GB9 and GB10 fields only, but we have already identified a total of 8733 EB candidates. Despite this, there is a possibility that we failed to discover a certain number of EB candidates, in particular, with period close to 0.5 days or 1 day as we intentionally ignored every light curve which was found to have the main periodic signal with period close to 0.5 days or within the range from 0.99 to 1 days. Our catalogue might also contain pulsating single stars which have light curve morphology similar to EW-type EBs or ELVs and were misidentified as EB candidates. Usually duration of individual eclipses for a binary is a few hours unless the binary has a component being a giant star. In this sense, eclipses would appear narrower in phase as eclipsing period increases. In our period analysis in which the CE method was applied to search for eclipsing signals, we adopted 20 bins in phase to bin the folded light curves. Given this number of bins, the CE method might only be able to resolve an eclipse of duration in phase of the order of 0.05. For a long period EB, say, of period 10 days, if the eclipse duration is about 6 hours, the width of its eclipse in phase will be only 0.025, which is under the phase resolution (of 0.05). Hence, the CE method might fail to pick up eclipsing signals associated with long period, detached main sequence star binaries because of the narrowness of their eclipses in phase. It means that not only did our method to search for EBs tend to discover short period EBs, but it was also biased towards EBs with giant star components when searching for long period EBs.

We have published the first EB catalogue from the MOA-II database which contains 8733 EB candidates in two MOA fields, GB9 and GB10. The potential number of EBs in the full MOA database is expected to be over 80000 given the number of EBs in the GB9 and GB10 fields. Such a large number of samples should be beneficial to statistical study of stellar population and multiplicity [139, 188], which is important to understand the formation of binary and multiple stellar systems. We also believe that our catalogue contains a diverse population of EBs, including many interesting EBs such as highly eccentric binaries and RS CVn stars, which should be beneficial
to studies of stellar physics. Another potential research using our EB catalogue is to study the multiplicity of contact binaries, which is also another aspect of this thesis we investigated.
Chapter 4

Preliminary Search for Light Travel Time Effect in MOA Eclipsing Binaries

Look at the stars
Look how they shine for you
And everything you do
Yeah, they were all yellow.

Coldplay, Yellow

This thesis chapter originally appeared in the literature as Li et al., Monthly Notices of the Royal Astronomical Society, 470, 539-550, 2017

4.1 Introduction

In Chapter 3, we presented the work on the search for eclipsing binaries (EBs) in the MOA-II database. Concerned with limited computational resources, our search was restricted to the two particularly densely populated fields, GB9 and GB10, in which we discovered 8733 EBs of periods ranging from 0.09 to 66 days. Given such a large number of EBs, it would also be interesting to identify triple systems among them subsequently.

Identifying triple systems from stellar binary population is important as to correctly estimate stellar multiplicity which is a ubiquitous outcome of the star formation
process [54]. Statistical study (e.g., [210, 209]) of stellar multiplicity using data from the Hipparcos catalog [61] has shown that triple systems are relatively common among stellar systems with solar-type main-sequence stars, constituting about 10% of the stellar population in the solar neighbourhood. In the survey towards a sample of 165 known solar-type spectroscopic binaries, Tokovinin et al. (2006) found that 96% of spectroscopic binaries with period < 3d in the sample were accompanied by tertiary companions, indicating that close binaries often belong in triple systems. On the other hand, the formation process of close binaries has not been understood. A possible scenario to form a close binary is via the interaction between a wide binary and its tertiary companions which causes the wide binary’s orbit to shrink to its current orbital separation through Lidov-Kozai oscillations with tidal friction [59, 58, 107]. Therefore, understanding population of close binaries with tertiary companions is a crucial step to understand the formation process of close binaries.

Nonetheless, detecting tertiary companions around binary systems is challenging. The radial velocity measurement would be the way that could provide unambiguous evidence for the presence of any tertiary object orbiting around a binary system (e.g. [129]). However, it would require applying for observational times of other telescopes with a spectroscopic instrument and this approach was certainly impractical given the time available in this project for any follow-up research. Fortunately, EBs, as a special type of binary systems showing recurring eclipsing events, allow the investigation of eclipse time variation (ETV) as a unique channel to detect tertiary companions.

The analysis of ETV has been traditionally performed using the observed-minus-calculated (O−C) diagram [201]. The term “observed” means the observed occurrence times of eclipses, while the term “calculated” means the predicted occurrence times assuming the eclipsing period to be constant. As eclipses are the phenomena uniquely observed in photometry, the construction of O−C diagrams does not demand spectroscopic measurement and can be done using photometric data alone. Such advantage implies that we can in principle carry out the search for companions in every MOA EB we discovered so far just using the MOA observational data without requiring extra data, either photometric or spectroscopic, from other telescopes.

ETVs can arise from physical processes that would redistribute the angular momenta of EBs’ components or cause the angular momentum loss (AML) of the systems, resulting in the actual changes of their orbital periods. For a semi-detached or contact binary, the angular momenta between the components can be exchanged by the mass
transfer via Roche overflow [142]. This would be expected to produce a monotonic increase in the orbital period, while variable mass transfer rates may induce erratic ETVs in addition [29].

On the other hand, mass loss may exist due to stellar winds. When the EB’s components are magnetically active (e.g. late-type stars with convective envelopes), the interaction between the outflowing magnetized stellar wind from one component and the magnetic field of the other component can cause the system to lose its orbital angular momentum via magnetic braking [141]. Magnetic braking has been considered as the major mechanism to reduce the orbital periods of moderately close binaries to form very close binary systems such as EW-type binaries after the tidal locking had been established and when the nuclear evolution of the stars was not important [164]. Mass loss would have the effect of increasing the orbital period, while magnetic braking would result in the opposite. These effects as well as the mass transfer would produce secular ETVs, and their resultant O–C diagrams can be represented as quadratic functions of time (or cycle).

Close binaries may also exhibit long-term recurring ETVs when their orbits are coupled to their shapes via the gravitational quadrupole moments so that the orbital motions vary in response to changes in the gravitational quadrupole moments due to the magnetic cycles of the stars. This effect, known as the Applegate effect\(^1\) [8], has been applied to explain the long-term orbital period variations observed in Algol type binaries [33, 221]. As orbital variations due to the Applegate effect\(^1\) are correlated with stellar magnetic cycles, a recurring period of this type of ETVs is typically over decades. Given the time span of the full MOA data which is less than 10 years, ETVs arising from the Applegate effect\(^1\) should behave approximately as quadratic variations in O–C diagrams.

Instead of long-term secular ETV, mono-periodic ETV may appear in the O–C diagram as the result of the apparent change in the orbital period induced by the dynamical motions of the EB’s orbit. The effects that can give rise to such ETV include the light travel time effect (LTTE) and the apsidal motion effect (AME) [29, 31]. The

\(^1\)The Applegate effect (or the Applegate mechanism) was proposed by Applegate (1992) to explain long-term orbital period variations observed in certain EBs containing main sequence star components. He suggested that the outer layers of main sequence stars undergoing magnetic cycles would be subject to magnetic torques which cause the angular momenta to redistribute. Redistribution of angular momentum should also result in change in stellar oblateness or shape. Since stellar binaries can be gravitationally coupled to the shapes via the components’ quadrupole moments, which would vary according to the magnetic cycles, they should experience orbital period modulations (typically on the order of \(\Delta P/P \sim 10^5\)) on the same time scale as the magnetic cycles (typically on the order of decades).
LTTE is associated with the presence of a third body orbiting around the EB, while the AME is a relativistic phenomenon that might be observed in eccentric binaries. In the case of the AME, the ETV curve for the primary eclipse will be anti-correlated with that for the secondary eclipse. In contrary, the LTTE will result in consistent ETV curves for primary and secondary eclipses. The intrinsic variability of the EB’s components, for example, from star spots and pulsations, can distort the light curve. Such distortion may influence the accuracy of eclipse timing, leading to spurious ETV signals that show periodic or quasi-periodic patterns [212].

For spectroscopic binaries with confirmed tertiary companions, it might be possible to derive orbital parameters of the tertiary companions such as outer period and eccentricity and constrain their tertiary masses by analyzing the ETVs (e.g., [88, 129]). Although eclipse timing has been applied to search for tertiary companions around EBs before the era of space telescope surveys, it was usually only capable of detecting LTTE signals of period longer than several years or decades because of poor precision in ground-based photometry and lack of regular eclipse timings in those days. Since the advent of CCD camera, photometric precision has been improved dramatically. The development of automatic wide-field observation techniques has also allowed telescopes to regularly monitor the night sky and take images of variable objects in higher cadences. These technological advances have made accurate and efficient ground-based eclipse timings possible.

The MOA data spanning two observational seasons are available for all the EB candidates identified in Chapter 3, and the nine observational seasons’ worth of data are available only for the samples from the subfields GB9-9 and GB10-1. Our purpose is to identify triple systems in the MOA data using the ETV method. The evidence that an EB belongs to a triple system would come from the LTTE signal detected in the associated O–C diagram. Before looking for LTTE signals using the full MOA data which were not available for every EB candidate, we decided to use the same data sets to carry out the investigation of eclipse timing for all EB candidates in our catalogue. The data sets covered only two MOA observational seasons and the ETV study using these data sets was considered to be the preliminary work for the ETV study presented in Chapter 5. Here we present the results of this preliminary work which originally appeared in Li et al. (2017) in this chapter.
4.2 Light Travel Time Effect

The LTTE is an effect associated with the change in orbital motion which appears in an EB with a tertiary companion wherein the centre of mass of the EB is no longer stationary but moving around the barycentre of the whole three body system. From an observer’s point of view, the movement of the EB’s centre of mass might be reflected by the measurement of the times of eclipse minima which occur later or earlier cyclically than expected due to the finite speed of light and varying distances between the conjunction and the observer. Analyzing ETVs via O–C diagrams, otherwise known as the ETV method, has been a traditional technique to detect LTTE in EBs, with or without spectroscopic information. Changes in orbital periods were already observed in many EBs a century ago. Chandler [37], suggested that the observed period change in Algol resulted from the LTTE due to the presence of a tertiary object. But this was after Woltjer [226] who was first able to perform the LTTE calculation that the LTTE was seriously considered as the plausible explanation. Later, Irwin [97] proposed the analytical model of the LTTE to the O–C diagram in terms of stellar masses and orbital parameters.

The O–C diagram represents the variations of the times of eclipse minima, which are calculated by the following equation,

\[ \Delta = T_o(E) - T_c(E) = T_o(E) - T_0 - P_sE \]  

(4.1)

where \( T_o(E) \) and \( T_c(E) \) denote the observed and calculated times of the \( E \)-th eclipse minimum, \( T_0 \) represents the reference epoch and \( P_s \) denotes the average eclipsing period. The general ETV model involving the LTTE is defined by:

\[ \Delta = c_0 + c_1E + c_2E^2 - \frac{a_{AB}\sin i_2}{c} \left( \frac{1 - e_2^2}{1 + e_2 \cos \nu_2} \right) \left( \sin(\nu_2 + \omega_2) \right) \]  

(4.2)

where the zeroth and first order coefficients, \( c_0 \) and \( c_1 \), in the polynomial of \( E \) provide the corrections in \( T_0 \) and \( P_s \), respectively, while the second order coefficient, \( c_2 \), is equal to half the rate of change in period, regardless of its origin. The parameters in the LTTE term, i.e. the last term in eq.(4.2), include eccentricity (\( e_2 \)), true anomaly (\( \nu_2 \)), argument of periastron (\( \omega_2 \)), inclination (\( i_2 \)) and the semi-major axis of its absolute orbit, \( a_{AB} \), equal to \( (m_C/m_{ABC})a_2 \). The period (\( P_2 \)) and the time of periastron (\( \tau_2 \)), of the tertiary object are also needed implicitly when calculating \( \nu_2 \). The LTTE term, therefore, depends on six parameters. Note that \( m_C \) is the mass of the tertiary object,
**m_{ABC}** is the total mass of the triple system, and **a_2** is the semi-major axis of the tertiary object’s orbit around the EB and **c** is the speed of light. The amplitude of the LTTE is defined by

$$A_{LTTE} = \frac{a_{AB} \sin i_2}{c} \sqrt{1 - e_2^2 \cos^2 \omega_2}. \quad (4.3)$$

Unfortunately, the semi-major axis of the absolute orbit (**a_{AB}**) and the inclination (**i_2**) are degenerate in the LTTE model. Yet the mass function, **f(m_C)**, defined as

$$f(m_C) = \frac{m_C^3 \sin^3 i_2}{m_{ABC}^2} = \frac{4\pi^2 a_{AB}^3 \sin^3 i_2}{GP^2_2}, \quad (4.4)$$

can be calculated when the LTTE solution is known. Then we can calculate the amplitude of the LTTE via the approximation equation given by

$$A_{LTTE} \approx 1.1 \times 10^{-4} f(m_C)^{1/3} P_2^{2/3} \sqrt{1 - e_2^2 \cos^2 \omega_2}. \quad (4.5)$$

Note that the period and amplitude are in days and the masses are in units of **M_⊙**. From eq.(4.3) we know that the LTTE amplitude decreases as the outer period (i.e., **P_2**) decreases. Because of this, and owing to insufficient precision in ground-based photometry and difficulty in observing eclipsing events continuously as well as having eclipse time measurements frequently enough on the Earth to satisfactorily cover a short period LTTE cycle, LTTEs associated with triple systems with outer periods less than two years were rarely detected by ground-based telescopes, and stellar triples identified on the Earth by the ETV method tend to be those with periods longer than several years or even decades.

Additional dynamical perturbations may become significant and observable in the O–C diagram on top of the perturbation due to the LTTE, if a tertiary companion tightly interacts with the inner binary [29], causing the inner binary motion to deviate from purely Keplerian and affecting the occurrence times of the eclipses. When the inner binary is circular, the ETVs due to this kind of dynamical perturbations will have the amplitude approximately given by:

$$A_{dyn} = \frac{1}{2\pi} \frac{m_C}{m_{ABC}} \frac{P_C^2}{P_2^2} (1 - e_2^2)^{-3/2}, \quad (4.6)$$

where the periods (i.e., **P_1** and **P_2**) and the outer eccentricity (**e_2**) are known from the LTTE solution [29]. The ratio of **A_{dyn}** to **A_{LTTE}**, i.e., **A_{dyn}/A_{LTTE}**, as can be seen from eq.(4.6) and (4.5), is proportional to **P_2^{-5/2}**. In this sense, ETV contributions
from the dynamical perturbations would not be a concern unless $P_2$ is small and, from eq.(4.6), comparable to $P_1$.

In the case of the inner binary being eccentric, the ETV term corresponding to apsidal motion may have to be included as well. The apsidal motion may be simply regarded as linear variation in $\omega_1$ as a result of the apsidal line of the inner binary’s orbit rotating with a constant angular velocity in the direction of the orbital motion arising from the tidal deformation of the shapes of the binary components or relativistic effects [31, 25, 202, 45]. Nonetheless, the presence of the tertiary companion may induce the apsidal motion of the inner binary to behave in a complicated manner in which no orbital parameters, except the semimajor axes, would remain constant (see [31, 143] and further references therein). Thus, the eccentric EBs in our MOA EB catalogue were ignored for ETV analysis in this thesis. Also, as long as we restricted our study to short period binaries for which circular orbits should be established, apsidal motions should be negligible.

Although the detection of a LTTE signal with multiple cycles is strong evidence for the existence of a tertiary companion in an EB, several mechanisms such as the mass transfer between the EB’s components, magnetic braking and the Applegate effect\(^1\) that we mentioned earlier can produce quadratic variation in the orbital period that may be confused with the LTTE cycle of period longer than the data time span. Star spots can produce spurious ETVs that mimic LTTE behaviour as well. In order to justify the plausibility of a LTTE solution to the ETV curve, Frieboes-Conde and Herczeg [70] suggested four general criteria: (1) the shape of the ETV curve must follow the analytical form of a LTTE solution; (2) the ETVs of the secondary and primary minima must be consistent with each other in both phase and amplitude; (3) The estimated mass or lower limit to the mass of the tertiary companion derived from the mass function must be in accord with photometric measurements or limits on the third light in the system; (4) the variation of the radial velocity of the system must be in accord with the LTTE solution. Obviously, without radial velocity data, criterion (4) could not be satisfied. Also, without the spectra, we did not have information about the temperatures of the MOA EBs and could not estimate the third light from light curve modeling, so criterion (3) could not be satisfied as well. Yet it is worth identifying EBs which have periodic ETV signals that satisfy the criteria (1) and (2) as the candidates of triple systems for future follow-up observation.
Fig. 4.1 Römer delay. The Römer delay for an observer on a planet with respect to a new reference, i.e., the central star in this diagram is $\Delta R = \vec{r} \cdot \hat{n} / c$, where $c$ is the speed of light, $\hat{n}$ is the unit vector from the observer to the source and $\vec{r}$ is the position vector of the observer from the central star.

### 4.3 Light Curve Preparation for Eclipse Timing

#### 4.3.1 Time corrections

For precise eclipse timing, careful treatment for time has to be taken. Indeed, timing astrophysical events is not simply meant to record the observation times, which would usually be done automatically by a computer nowadays, because the Earth on which the observations are taken is not an inertial frame. The Earth is indeed orbiting around the Sun, while the Sun is also orbiting around the barycentre of the Solar system. Therefore, the distance between an observer on the Earth and a distant target would vary due to the Earth’s orbital motion, causing the light from the target to arrive at the observer earlier or later by as much as eight minutes. This effect known as the Römer delay will always induce periodic spurious ETV of amplitude equal to eight minutes. Considering a triple system of solar mass components, the tertiary component of outer period of a year will induce the LTTE of amplitude about $0.0027 \text{d} (\sim 3.8 \text{ min})$ which will be overwhelmed by the spurious ETV arising from the Römer delay. To unveil any LTTE signal, time correction for the Römer delay $\Delta R$ is essential. The
Rømer delay is given by

\[ \Delta_R = \vec{r} \cdot \hat{n} / c, \]  

where \( c \) is the speed of light, \( \hat{n} \) is the unit vector from the observer to the source and \( \vec{r} \) is the position vector of the observer from the origin of the new reference frame that is concerned. Additional uncertainty would also come from the improper time basis adopted by the clock, i.e., the clock correction \( \Delta C \). In the case of MOA observation, all times were recorded in JD in the UTC time basis\(^2\) (i.e., \( JD_{UTC} \)). UTC changed irregularly because of the addition or subtraction of leap seconds to it in accord with the slowing or speeding of the Earth’s rotation. For microlensing analysis, the required time precision would be of the order of 10s which can be achieved by converting the time in JD to Heliocentric Julian Date (HJD); i.e., \( HJD = JD + \Delta R_H + \Delta C \), where \( \Delta R_H \) is the Rømer delay with respect to the Sun as the new reference frame. However, the time precision down to an order of 0.1s can be further achieved if Barycentric Julian Date (BJD) in Barycentric Dynamical Time (TDB) (i.e., \( BJD_{TDB} \)) is adopted instead. In the other words, the Rømer delay was calculated with respect to the barycentre of the Solar system as the new reference frame. Not taking the Rømer delay and the clock correction into account, the systematic error in timing can be as large as nine minutes to date \cite{57}.

As mentioned above all MOA light curves provided were recorded in JD. We must convert the times in JD to BJD in order to eliminate spurious ETVs from the heliocentric motion of the Earth as well as the Sun’s motion around the barycentre of the Solar system and remove the systematic time errors arising from the time differences between UTC and TDB. The time correction was done as follows. We first used astropy \cite{11}, a Python module for astronomy, to perform the time conversion from UTC to TDB. Then we calculated the Rømer delay with respect to the barycenter of the Solar system, for which the position of the Earth in the Barycentric reference frame was calculated using the Python package jplephem with the ephemerides of the Moon and the barycentre of the Earth-Moon system given by the Jet Propulsion Laboratory (JPL) Planetary and Lunar Ephemeris DE423 \cite{67}. We also cross-checked

\(^2\text{UTC is currently used as the international time standard for regulating clocks and times. Every modern, network-connected computer that is set to be synchronized to a Network Time Protocol (NTP) server would automatically display and record time in UTC. A leap second may be added or subtracted to UTC every six months at the end of 31 December and 30 June by the International Earth Rotation and Reference Systems Service (IERS) in order to keep the difference between UTC and UT1 less than 0.9s. Note that UT1 is a time standard to specify the length of a day (or the mean solar day) based on the Earth’s rotation.}
the BJD times calculated following the algorithm above with the times calculated via the online applet\(^3\) created by Eastman et al. (2010). The differences were always less 20 ms, so we were convinced that the algorithm we adopted produced the correct time conversion from \(J_{\text{UTC}}\) to \(B_{\text{TDB}}\). From now on, we implicitly mean BJD as \(B_{\text{TDB}}\) and JD as \(J_{\text{UTC}}\).

### 4.3.2 Cleaning light curves and period reanalysis

Since the true eclipsing periods after the time correction could be significantly different from the values calculated before, it was necessary to recalculate the eclipsing periods. Meanwhile, we decided to further clean the EB light curves by the following procedure:

1. Fold the light curve at the eclipsing period, \(P_s'\), obtained in Section 3.1.2
2. Bin the folded light curve into 200 bins in phase
3. Calculate the weighted mean and standard deviation of flux in each bin
4. Remove the data points three weighted standard deviations above or below the weighted flux mean

After this additional cleaning process, the eclipsing periods were recalculated using the CE method by minimizing the conditional entropy over trial periods within \(P_s' \pm 0.05\), where \(P_s'\) is the original eclipsing period. The recalculated eclipsing periods were used as the input periods into the ETV analysis described below. Data points with errors > 2000 flux units were rejected as being poor measurement points, as suggested by Ian Bond in a private conversation. We thought his advice was appropriate since most MOA data points have errors < 1000 flux units, indicating that the data points with errors > 2000 flux units are comparatively less reliable.

### 4.4 Eclipse Timing Methods

The orbital motion of an EB, if it is purely a two-body system, should be exactly described and predicted by Kepler’s equation as long as the apsidal motion is not concerned. Thus, EBs can be used as precise clocks in astronomy. Yet accurate eclipse timing is challenging on the Earth. Individual eclipses last usually a few hours. Ground-based observation often fails to obtain the complete coverage of an eclipse because of the

\(^3\)http://astroutils.astronomy.ohio-state.edu/time/utc2bjd.html
poor condition of the night sky. Traditionally, the time of an eclipse minimum would be derived using the Kwee-van Woerden method [110]. Several recent studies to look for circumbinary planets in post-common-envelope binaries applied this method to derive the times of the eclipse minima (e.g., [13] and [114]). The Kwee-van Woerden method, however, cannot work properly if an eclipse is not symmetric about its minimum, or the distribution of the data points over the eclipse is not even (i.e., the observations over the eclipse were not taken in regular cadences), or the number of data points covering the eclipse is too low. The Kwee-van Woerden method also, as mentioned in Pribulla et al. (2012), usually underestimates the uncertainties in the derived times. The eclipse template method, as far as we know, turned out to be an alternative method commonly used nowadays. Various ways to create an eclipse template were proposed and used by different research groups. The high-order polynomial fit (polyfit) was used by the Kepler group in order to create an approximate eclipse template. If the radial velocity curve of a target is available, deriving a realistic eclipse template by fitting the photometric light curve and the radial velocity curve simultaneously is always preferred using PHOEBE, an EB modeling package [161].
The template generation methods mentioned so far are, however, either impractical or unsatisfactory for our study. Although the template of a grazing eclipse can be appropriately generated by a quartic polynomial using the `polyfit` code, we found that it has trouble producing an appropriate template for a total eclipse. A higher-order polynomial might be adopted to generate templates for total eclipses, but then it often yielded the templates with rippling bottoms and the minima did not appropriately correspond to the eclipse minima. We desired a template generation method that was workable for grazing and total eclipses without spectroscopic information. For these reasons, we finally decided to adopt the phenomenological light curve model of EBs proposed by Mikulášek (2015). Considering only the portion of the light curve belonging to either the primary or secondary eclipse, the model is reduced to the function of five parameters defined by

\[
f(t_i, \theta) = \alpha_0 + \alpha_1 \psi(t_i, t_0, d, \Gamma)
\]

(4.8)

where \(\alpha_0\) is the magnitude zero-point shift (i.e., the relative flux baseline level in our study) and \(\alpha_1 < 0\) is a negative multiplication constant of the eclipse profile function, i.e.,

\[
\psi(t_i, t_0, d, \Gamma) = 1 - \left\{ 1 - \exp \left[ 1 - \cosh \left( \frac{t_i - t_0}{d} \right) \right] \right\}^\Gamma.
\]

(4.9)

Note that \(t_0\) is the time of the minimum of an eclipse, \(d > 0\) is the minimum width and \(\Gamma > 0\) is the parameter specifying the pointedness of the minimum such that \(\Gamma > 1\) corresponds to the flat minimum associated with a total eclipse. The portions corresponding to primary and secondary eclipses were determined by estimating the minima of the second derivative of the folded light curve, which would correspond to the ingress and egress phases of the eclipses (detailed descriptions on the procedures to determine eclipse portions in a light curve are presented in Section 5.4.) We then fitted each eclipse having at least four data points that were distributed across the eclipse minimum with the template, using `emcee`, an extensible, pure-Python implementation of Goodman & Weare’s Affine Invariant Markov chain Monte Carlo (MCMC) Ensemble sampler [69]. In this fitting we allowed only \(t_0\), \(\alpha_0\) and \(\alpha_1\) to vary. In terms of the template method, the reference epoch, \(T_0\), in eq.(4.1), was defined by \(T_0 = \phi_0 P + \tau_0\), where \(\phi_0\) is the phase of the minimum of the eclipse template with respect to the time zero, \(\tau_0\). The time zero was taken such that \(\tau_0 < t_{\text{obs}}\), where \(t_{\text{obs}}\) is an observation time, and both primary and secondary eclipses are not broken in phase when the light
Table 4.1 Boundaries of the parameters of the ETV model eq.(4.2). Note that $d$ is day, $d/c$ is day/cycle, AU is astronomical unit, and $P_1$ is eclipsing period.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_0 \ (d)$</td>
<td>-0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$c_1 \ (d)$</td>
<td>-0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$c_2 \ (d/c)$</td>
<td>-0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>$P_2 \ (d)$</td>
<td>$10P_1$</td>
<td>1000</td>
</tr>
<tr>
<td>$e_2$</td>
<td>0</td>
<td>0.999</td>
</tr>
<tr>
<td>$\omega_2$</td>
<td>0</td>
<td>$2\pi$</td>
</tr>
<tr>
<td>$\tau_2$</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$a_{\text{AB}} \ (\text{AU})$</td>
<td>0</td>
<td>100</td>
</tr>
</tbody>
</table>

curve is folded with respect to $\tau_0$. The time of an eclipse minimum is the median of the projected posterior on $t_0$. The uncertainty in $t_0$ was taken as the 1-$\sigma$ confidence interval from the median.

4.5 LTTE Model Fitting

We attempted to construct O–C diagrams for every EB candidate we identified and searched for the candidates which showed promising periodic signals in their O–C diagrams by eye. We did not inspect the O–C diagrams of every candidate as it was a time-consuming job. We decided to focus on the EB candidates having sharp eclipsing signals as indicated by their low values of the PDM statistic ($\theta < 0.7$). As a result, we discovered three EB candidates, MOA-115233-GB10-9, MOA-129173-GB10-1 and MOA-360325-GB10-7, which show obvious periodic patterns in their O–C diagrams.

The selected EB candidates all turned out to be contact binaries after inspecting their folded light curves. As pointed out by Tran et al. (2013), star spots, which are common on contact binaries, on an EB can produce spurious ETVs on primary and secondary eclipses that appear to be anti-correlated, or out of phase from each other. In general, averaging the ETVs of primary and secondary eclipses is required to eliminate such spurious ETVs. Unfortunately, in our EB candidates’ light curves, usually only one of the primary or secondary eclipses was present in a cycle. This means that averaging was not possible for most of the cases. Nonetheless, our three selected EB candidates did not establish obvious anti-correlated behaviours between the ETV curves of their primary and secondary eclipses based on our visual inspection. Therefore, explaining the observed ETVs as being due to star spots is not favoured and no averaging was carried out. We then fitted their O–C diagrams by the ETV
model (i.e., eq.(4.1)) and searched for the best-fitting solutions using \texttt{pymc} [14], another Python module of MCMC fitting algorithms, taking the ETV measurements of both primary and secondary eclipse times into the parameter search at the same time. Here we adopted \texttt{pymc} for LTTE model fitting instead of \texttt{emcee} simply because we recognized the performance of \texttt{pymc} was faster than \texttt{emcee} when many free parameters were involved. We adopted the built-in normal distribution function in \texttt{pymc} as the likelihood function and assumed a uniform prior for each parameter over the boundaries (see Table 4.1) that we assumed to be appropriate. The initial guess values of $e_2$, $\omega_2$ and $\tau_2$ were taken to be the middles of their boundaries, while $a_{AB}\sin i_2$ (in AU) was set to be 0.3. The default sampling algorithm in \texttt{pymc}, i.e., Metropolis-Hastings algorithm [85, 135], was used for sampling the posterior distribution. Yet we found that the posterior distributions only started to converge when the number of iterations was set to be greater than 3,000,000. In theory, as the number of iterations approaches infinity, convergence will be guaranteed. We, therefore, ran the Metropolis-Hastings algorithm with 4,500,000 iterations and discarded the first 3,500,000 samples. We then thinned the Markov chain by 20; that is, after discarding the first 3,500,000 samples, we discarded all but every 20th sample in order to reduce autocorrelation in the samples. The resultant simple size after this procedure was 1,75,000, and we were able to obtain low autocorrelations for most of the parameters except $P_2$ and $\tau_2$. The autocorrelations for $P_2$ and $\tau_2$ were barely below 1.0 even at lag 100. Nonetheless, given the sample size, their effective sample sizes (EES), calculated by the equation

$$\text{EES} = \frac{n}{1 + 2\sum_{k=1}^{\infty}{\rho(k)^2}}, \quad (4.10)$$

where $n$ is the number of samples and $\rho(k)$ is the correlation at lag $k$, were larger than 870, indicating the quality of the posterior estimate was acceptable [34, 168]. The LTTE solutions for these three selected EB candidates are shown in Table 4.2 and their best-fitted plots are presented in Figure 4.3. The uncertainty of each parameter was taken as 1-\sigma confidence interval from the median.

Period changes (of the order of $10^{-10}$ to $10^{-9}$ days per cycle) are detected in these three EB candidates according to their best-fitting ETV solutions, indicating mass transfer might be occurring in them. On the other hand, the time differences between their own primary and secondary eclipse minima (see Table 3.1) are all close to half of their orbital periods. Their eccentricities are thus safely assumed to be zero, excluding apsidal motion as a main source for the observed ETVs. In their residual curves, no
Fig. 4.3 (Left) The folded light curves of three EB candidates in our catalogue with detected light travel time effect (LTTE) signals; the minimum of the primary eclipse of each folded light curve was adjusted to be located at the zero phase. (Right) The ETV curves of these three EB candidates; $P_1$ is the period of the inner binary determined by the conditional entropy method, while $P_2$ is the period of the tertiary companion given by the LTTE solution; the blue points are the ETV measurements of the primary eclipses and the red points are those of the secondary eclipses; the green lines represent the best fits of the ETV model defined by eq. (4.2); the bottom panels show the residuals of the fits. Note that the periods are in days.
periodic signals were noticeable, so the dynamical effect should be insignificant. We, therefore, accepted the LTTE as the main source for their observed ETVs.
Table 4.2 The orbital elements from the light travel time effect solutions for three selected EBs in our catalogue with the uncertainties given by 1-σ confidence intervals from the medians. Note that $P_1$ is the period of the inner binary determined by the conditional entropy method plus the correction, $c_1$, given by the best fit of eq.(4.2) to the ETV curve, and $\Delta P_1 = 2c_2$, where $c_2$ is the second order coefficient in eq.(4.2), is the change in inner binary orbital period per orbital cycle in units of [day/cycle], and $m_{AB}$ was taken as $2M_\odot$ when calculating $(mc)_{\min}$.

<table>
<thead>
<tr>
<th>No.</th>
<th>GB</th>
<th>chip</th>
<th>$P_1$</th>
<th>$\Delta P_1$</th>
<th>$P_2$</th>
<th>$e_2$</th>
<th>$\omega_2$</th>
<th>$\tau_2$</th>
<th>$a_{AB}\sin i_2$</th>
<th>$(mc)_{\min}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\times 10^{-6}$ (d)</td>
<td>$\times 10^{-10}$ (d/c)</td>
<td>$\times 10^{-2}$ (d)</td>
<td></td>
<td>(deg)</td>
<td>(MBJD)</td>
<td>(AU)</td>
<td>($M_\odot$)</td>
</tr>
<tr>
<td>129173</td>
<td>10</td>
<td>1</td>
<td>560311.71$^{+0.86}_{-0.93}$</td>
<td>58.3$^{+16.6}_{-15.4}$</td>
<td>24651.1$^{+0.7}_{-0.6}$</td>
<td>20.6$^{+3.5}_{-3.7}$</td>
<td>261.6$^{+9.5}_{-10.6}$</td>
<td>56322.80$^{+0.08}_{-0.08}$</td>
<td>0.302$^{+0.001}_{-0.007}$</td>
<td>0.76</td>
</tr>
<tr>
<td>360325</td>
<td>10</td>
<td>7</td>
<td>299511.32$^{+0.67}_{-0.62}$</td>
<td>9.2$^{+5.9}_{-6.4}$</td>
<td>48180.7$^{+11.1}_{-12.6}$</td>
<td>25.3$^{+4.4}_{-4.9}$</td>
<td>191.9$^{+8.4}_{-11.5}$</td>
<td>56323.61$^{+0.15}_{-0.19}$</td>
<td>0.941$^{+0.032}_{-0.038}$</td>
<td>1.97</td>
</tr>
<tr>
<td>115233</td>
<td>10</td>
<td>9</td>
<td>331521.24$^{+0.66}_{-0.63}$</td>
<td>5.6$^{+6.3}_{-6.6}$</td>
<td>42678.8$^{+1.9}_{-1.7}$</td>
<td>30.3$^{+5.4}_{-5.2}$</td>
<td>306.1$^{+13.2}_{-12.3}$</td>
<td>56323.57$^{+0.11}_{-0.10}$</td>
<td>0.391$^{+0.014}_{-0.015}$</td>
<td>0.65</td>
</tr>
</tbody>
</table>
Because of the big gaps in the MOA data sets due to the off-season periods in which the altitude of the GB was too low to allow good seeing observations, our selection of O–C diagrams for ETV analysis was obviously biased towards the systems of outer periods ~ 1.5 years, for which a ETV cycle can be recognized by eye without difficulty. In fact, a large number of good eclipse time measurement points would be achieved only for very short period MOA EBs because day-night cycles and weather conditions often prevent from good coverage for eclipses lasting longer than half a day. On the other hand, detached binaries are usually of periods longer than a day and the number of cycles decreases as the period increases. All these factors make it difficult to obtain the ETV curve for a detached binary dense enough to show any promising feature that would hint at the presence of the LTTE within the time span of just two MOA observational seasons. Our selection was thus naturally biased towards the contact binaries of periods < 0.6d. Although the tertiary mass cannot be determined from the LTTE solution, the LTTE amplitude, i.e., eq.(4.3), allows estimation of the mass function, \( f(m_C) \), via the approximation equation, i.e., eq.(4.5). Assuming the components of the three selected EB candidates are all solar, and thus taking \( m_{AB} \), the total mass of each selected EB candidate, equal to \( 2M_\odot \), then the minimum masses of their tertiary companions can be calculated. The estimated minimum masses of their tertiary companions are shown in Table 4.2.

4.6 Summary and Discussion

We presented the ETV method that we adopted to search for triple systems in the MOA EB catalogue using two observational seasons’ worth of data. Among 8733 MOA EBs, LTTE signals were detected in the O–C diagrams of three short period EBs (\( P < 1 \) day) suggesting the presence of tertiary companions with outer periods between 250 and 480 days. The amplitudes of their LTTE solutions indicate that the tertiary companions are stellar assuming the masses of the inner binaries are \( 2M_\odot \). In our preliminary investigation, we noticed there were two intrinsic problems which would limit the usefulness of the MOA data for LTTE detection. First, there are always big gaps of width of about 100 days present in the O–C diagram. Such gaps exist because of the off-season periods during which the telescope was not used to observe the GB fields. As a consequence, over-fitting happened often if the lower bound of the period was set too small (say, 100 days). Second, the number of measurement points of eclipse timing decreases approximately linearly as the period increases if both are
4.6 Summary and Discussion

Fig. 4.4 Relation between the period and the number of eclipses that were accepted for eclipse timing according to the criterion that there are at least four data points across an eclipse minimum for the MOA EBs candidates in the GB9 and GB10 fields. Plotted in log scale, as shown in Figure 4.4. This tendency can be partly explained by the fact that eclipsing events are repeated less frequently for longer period EBs. But the non-linear relationship also reflected that the rate of cycles being accepted for eclipse timing drops significantly for periods longer than a day owing to the inevitable poor coverage of eclipses with long durations by the MOA observations. Regarding these issues, the detection of tertiary companions via eclipse timing would be heavily biased towards those orbiting about EBs of periods less a day. This implied that the MOA data is not particularly useful for detecting LTTE towards long period EBs, but it should be promising towards EBs of periods less a day. As we already succeeded in detecting LTTEs in short period EBs using the MOA data sets spanning just two observational seasons, it is evident that further ETV study using the full MOA time series is worth executing. Indeed we attempted to search for LTTE in contact binaries in the MOA catalogue using the data with longer time spans and the results are presented in Chapter 5.
Chapter 5

Tertiary Companions in MOA
Short Period Eclipsing Binaries

Change will not come if we wait for some other person or some other time. We are the ones we’ve been waiting for. We are the change that we seek.

Barack Obama

5.1 Introduction

The MOA-II project has reached its 12th observational season since establishment in 2006. The MOA database is unique in the sense that it has maintained over 100 TB photometric data towards 22 Galactic bulge (GB) fields which span over nine years to date. However, the MOA database has never been exploited for scientific research other than microlensing. This means 99.9999% stars (given that the occurrence probability of microlensing events is 1 in a million) in the MOA database have been ignored for a decade. The MOA eclipsing binary (EB) catalogue presented in Chapter 3 is the first substantial achievement from the MOA database for a non-microlensing study. Although only two GB fields were exploited, we have already identified over 8000 EBs. Among them, we further discovered three short period EBs with tertiary companions via the eclipse time variation (ETV) method using two observational seasons’ worth of data, suggesting that more triple candidates could be discovered if the full MOA data were used.
As discussed in Chapter 4, given the quality of the MOA data, the detection of tertiary companions via the ETV method would be heavily biased towards the MOA EBs of periods less than a day. In fact, EBs in this period range are the most numerous in the database of a photometric survey. It is the same in the case of MOA as can be seen in the period histogram of the MOA EBs (i.e. Figure 3.3) in Chapter 3. Meanwhile, the majority of such EBs are contact binaries. As described in Chapter 2, contact binaries are stellar binaries having two components with a shared envelope. They are thought to merge into single stars eventually. The merging of the contact binaries is also considered as a possible evolutionary channel to form blue stragglers in a stellar cluster, which seem to be main sequence stars given by their color-luminosity relationships but peculiarly appear above the turn-off point of the main sequence in the Hertzsprung-Russell diagram [199, 178]. Contact binaries with components of massive stars might be the origin of many types of transient phenomena such as nova [214], supernova [102] and gamma-ray bursts [6]. The components of contact binaries are so close that they cannot be born at their current positions from the core collapse of molecular clouds. The formation of contact binaries is not fully understood. It had been thought that close and contact binaries could be formed by the fission of rapidly rotating protostars [165, 172]. Yet numerical simulations have shown that such fission would not occur but the protostar should be deformed to have an elongated shape like a bar. Then, the gravitational torques from the bar instability led to the transfer of excess angular momentum outward, ending up as a single object surrounded by a disc [55]. Although the fission mechanism cannot produce close binaries with detached components, it has not been ruled out as a possible mechanism for the contact binary formation. But there is only one observational study of contact binaries by Bilir et al. (2005) which suggested that a group of contact binaries with young kinematic ages might be formed by a fission process. Nonetheless, they also pointed out that those young age contact binaries could be also formed by rapid inward spiraling of two detached components.

As the fission mechanism was disfavoured by numerical simulations and lacks support from observations, it has been believed that close binaries were originated from wide binaries, which were formed first by either the fragmentation of collapsing clouds or discs, and then undergone orbital shrinkage to establish the current separations [100, 220]. There are several mechanisms that can cause the binaries to lose their orbital angular momenta. In the case of the disk fragmentation, the secondary star would be born inside the gas disc, and the interaction between the disk and the secondary star
such as disk accretion and fiction during the protostellar state could make the star drift inward [16]. On the other hand, if a binary system had non-zero eccentricity, it would suffer tidal dissipation causing the orbital period to decrease until the circular orbit was established [230]. Yet the circular orbit should be established via the tidal dissipation before the orbital period down to 5.7 days for a solar mass binary [127, 132]. Thus, binaries with shorter orbital periods could not be established generally by the tidal dissipation alone. The Kozai cycles induced by tertiary companions in addition to tidal friction has been considered as a plausible orbital shrinkage mechanism to establish short period binary systems [106]. Once the periods are short enough, further orbital shrinkage can be driven by the loss of angular momentum via magnetic braking down to the separation at which the components overfill the Roche lobes to become semi-detached or contact binaries [100, 198, 137]. In this scenario, the presence of tertiary companions is a key to the formation of close and contact binaries. To verify this idea, a search for tertiary companions in well-known contact binaries via spectroscopic analysis, imaging with adaptive optics, radial velocity method and detection of light travel time effect (LTTE) had been carried out (e.g. [174], [47], [160]). Their results all indicated tertiary companions are common in contact binaries.

In the preliminary study of ETVs in the MOA EBs using two observational seasons’ worth of data, we were able to detect LTTE signals in the O-C diagrams of three MOA EBs which would be induced by tertiary companions of periods between 250 and 480 days according to the model fitting by the ETV model using MCMC. The preliminary results are encouraging and show that the MOA data have the outstanding quality in terms of both photometric precision and the density of measurement points that a certain number of tertiary stellar companions of periods > 400 days should become detectable by eclipse timing if the full MOA time series are used. Before the era of space telescope surveys, the number of EBs with detected LTTE signals was limited and the triple candidates identified by the ETV method tended to be of outer periods longer than several years or decades because of poor precision in ground-based photometry and insufficient frequency of eclipse timings on the Earth. The majority of the triple candidates identified via the ETV method, unsurprisingly, comes from the Kepler [29]. However, stellar triples with outer periods longer than four years are obviously deficient in the Kepler triple candidates due to the limited duration (i.e. 1470 days) of the mission. Such bias in the population of stellar triples identified by the ETV method may be reduced using the photometric data of long-term ground-based surveys such as MOA.
The obvious problem that can be foreseen in our study is that the third and fourth criteria for the acceptance of the LTTE detection mentioned in Chapter 3 cannot be satisfied because of lacking spectroscopic information. In order to avoid false positive detection, we shall employ the Bayesian information criterion (BIC) [180] as a model selection criterion between two models,

$$\text{BIC} = n \ln \left( \frac{1}{n} \sum_{i=1}^{n} (x_i - \hat{x}_i)^2 \right) + k \ln n,$$

where \(x_i\) are the measurement values, \(\hat{x}_i\) are the calculated values from the model fit, \(n\) is the number of measurement points, and \(k\) is the number of variable parameters in the model fit. Eq.(5.1) is BIC in the Gaussian special case in which each model error is assumed to be independent and identically distributed according to a normal distribution. The goodness of BIC as a model selection criterion is that it includes the penalty term, i.e., \(k \ln n\), to disfavour the case of over-fitting by adding parameters. We shall accept the detected LTTE signals as genuine if they satisfy the first and second criteria as well as have lower values of BIC compared to the quadratic fits associated with ETVs produced by other mechanisms.

MOA-II has adopted an observational strategy in which most of its time in the sky is dedicated to routinely monitoring the same fields towards the GB every clear night with high cadences. This gives MOA-II an advantage over other microlensing surveys to obtain eclipse time measurements from short period EBs that would be frequent enough to reveal any short period ETV. From the preliminary investigation using the MOA data spanning two observational seasons, we know that there are typically \(\sim 100\) eclipse time measurement points for MOA EBs of periods less than 1 day using the method described in Section 5.4, though this number declines to about \(\sim 19\) for the MOA EBs of periods between 2 and 10 days. This implies that the MOA data should be useful for stellar companion detection and the study of the frequency of tertiary companions in contact or semi-contact binaries.

Nonetheless, we also realized that the number of good eclipse time measurement points drops substantially when the binary period is larger than a day. This means there would be a significant shortage of eclipse time measurement points for long period EBs to cover any LTTE signal, and the search for tertiary companions is strongly biased toward EBs of periods less than a day. Fortunately, the discovery of EBs is also strongly biased towards EBs of periods less a day owing to day-night cycles on the Earth. About 50\% of the MOA EBs are of periods < 1 day (see Figure 5.1).
5.2 Observation and Data Reduction

Interestingly, almost all EBs of periods < 1 day are contact binaries. This implies that the search for tertiary companions in the MOA EBs of periods < 1 day is equivalent to studying the frequency of contact binaries with tertiary companions. This chapter is dedicated to present the work on the ETV analysis of the MOA EB sample of periods less than two days using the full MOA light curves. In Section 5.2, we present details about the MOA observations. We then present the methods and procedures of period analysis, eclipse time measurement and LTTE analysis in Section 5.3, 5.4 and 5.5, respectively, in detail. The results will be presented in Section 5.6 and we discuss and conclude our results in Section 5.7.

5.2 Observation and Data Reduction

In Chapter 3 and 4, we used the MOA light curves generated from the image data collected during the observational period from February 2013 to August 2014 for the study. In fact, the MOA team has carried routine observations towards the GB fields every year since 2006. Image data spanning nine years have been maintained in the MOA database. Since the main purpose of the MOA project is to search for planetary signals in microlensing events, the MOA collaboration has adopted to keep taking
images with high cadences towards the densely populated fields such as GB9 and GB10. Because of this observational strategy, the density of the MOA data from these fields is also high enough that a large number of eclipse time measurement points and, thus, densely sampled ETV curves are obtainable. Concerned with the homogeneity of the sample, we focused on studying the MOA EBs of periods < 2 days and attempted to obtain the full light curves of the MOA EBs within this period range. Unfortunately, generating the full light curves from the raw image data was computationally expensive and time-consuming. There are over 4000 EBs in the MOA EB catalogue which are of periods < 2 days. Generating the full light curves for all MOA EBs that we initially requested was impractical. Therefore, we further restricted our study to the two subfields, GB9-9 and GB10-1, from which the full light curves could be generated easily. There were 542 EBs of periods less than two days in the GB9-9 and GB10-1 fields. The data sets we obtained in the end span 9.5 years and were collected from February to November every year since 2006. The image reduction was done following the same procedures described in Section 1.3 using the difference imaging analysis (DIA) method. The density of the light curves from the GB9-9 field is approximately uniform, while the density of the light curves from the GB10-1 field is low during the period of the first three observational seasons (from 2006 to 2008), although big gaps exist as expected due to the off-season periods. The exposure time of 60s was taken for both fields over the entire observational time span. An observation time were recorded in JD and calculated to be the middle between the start and end times of an exposure.

5.3 Period Analysis

In the beginning, all the light curves of our EB samples were cleaned following the cleaning procedure same as in Li et al. (2017). In a nutshell, we discarded outliers that are 4.0σ above or 9.0σ below the relative flux mean as well as detrended the light curves via linear regression. This cleaning procedure was iterated twice before going into the light curve analysis. Meanwhile, we corrected the times from Julian Day (JD) to Barycentric Julian Day (BJD). Despite the time span of over 9 years, we did not divide the light curves into segments with shorter time spans in general, except several cases for which careful treatments in eclipse time measurement were needed (see Section 5.4).

Since the MOA fields towards which our EBs are located are densely populated, blending with nearby stars might be present. On the other hand, stellar pulsations
might be present in our EBs; particularly, a component being Cepheid or RR Lyrae, which would pulsate regularly with a period comparative to the eclipse duration, will distort an eclipse shape, causing the measurement of the time of eclipse minimum to be inaccurate. Because of these problems, we attempted first to search for an additional eclipsing or hidden pulsation signal under the main eclipsing signal. To do so, we first determined the average eclipsing period with which the 9.5 year light curve could be properly folded using conditional entropy with trial periods $P_s' \pm 0.01$, where $P_s'$ is the average eclipsing period over two MOA observational seasons provided in the MOA EB catalogue. Once the new average eclipsing period was determined, and after checking the resultant folded light curve by eye to see if it was folded properly, we binned the folded light curve in 200 bins, and created an approximate curve by calculating the mean flux value in each bin. We then produced the residual curve by subtracting the approximate curve from the folded light curve and unfolded it afterwards. The residual curve was then gone through the period analysis by the condition entropy algorithm with trial periods ranging from 0.05 to 600 days. The residual curve folded with the output period was inspected by eye afterwards.

In this manner, we discovered three residual light curves that exhibited periodic signals. Figure 5.2 shows the main eclipsing and additional periodic signals of these three EBs. The additional periodic signals in MOA-330424-GB9-9 and MOA-89673-GB9-9 are obviously associated with an EB with period of 0.421 days and an ellipsoidal binary with period of 1.316 days, respectively, while we suspect the additional periodic signal in MOA-249394-GB9-9 was an artifact due to contamination by a nearby pulsating bright star or a bright EB that caused imperfect image subtraction in the DIA. The detected additional signals were subtracted from the original light curves, and the average eclipsing periods were recalculated after subtraction.

5.4 Eclipse Time Measurement

To measure the times of eclipse minima, the template method using Mikulášek’s model to generate the eclipse templates was applied. The corresponding eclipse regions were determined by calculating the pairs of minima of the second derivative of the folded light curve which corresponds to the ingress and egress phases of the eclipses. If no valid minima were obtained from the second derivative curve, we took a pair of minima of the first derivative curve between which the eclipse minimum was located as the
Fig. 5.2 Folded light curves of the three EBs in the MOA samples from the GB9-9 and GB10-1 fields which were discovered to have additional regular periodic signals under their main eclipsing signals. (a) The main eclipsing signals. (b) The additional periodic signals. The secondary periodic signals in MOA-330424-GB9-9 and MOA-89673-GB9-9 are the eclipsing and ellipsoidal variation curves, respectively, likely associated with EBs near them. However, the source of the secondary periodic signal in MOA-249394-GB9-9 is uncertain. We suspect that it might be an artifact from imperfect image subtraction in difference image analysis owing to a bright variable star close to the EB in the images.
5.4 Eclipse Time Measurement

boundaries of the eclipse region. The procedures for identifying the ingress and egress of an eclipse are as follows:

1. Derive the mean light curve by binning the folded light curve into 20 bins and calculating the mean flux in each bin

2. Derive the first derivative curve by calculating the gradient of the mean light curve using the function gradient() in numpy

3. Derive the second derivative curve by calculating the gradient of the first derivative curve using the function gradient() in numpy

4. smooth the curves in each step above using the method of Locally Weighted Scatterplot Smoothing (LOSS) provided in the Python module statsmodels, and calculate the phases of maximum and minimum points using the function argrelextrem() in scipy

5. Take the pair of minimum points in the second derivative curve that contain the eclipse minimum as the points corresponding to ingress and egress of the eclipse

To illustrate the situations for different types of EB light curves, we take MOA-108463-GB9-9, MOA-124700-GB10-1 and MOA-238532-GB9-9 as examples. Their mean light curves and first and second derivative curves are shown in Figure 5.3. MOA-108463-GB9-9 is a Lyrae-type EB as the turning points in its mean light curve corresponding to the ingress and egress of both eclipses can be easily recognized by eye. Its first derivative curve resembles an electrocardiogram, while its second derivative curve contains two Mexican hat features associated with the primary and secondary eclipses. Unsurprisingly, the points of ingress and egress of the eclipse can be easily determined by calculating the maximum and minimum in its first derivative curve as well as the pair of minima in its second derivative curve that the eclipse minimum lies in between, respectively. In the case of MOA-124700-GB10-1, we can see that the second derivative curve fails to yield a proper Mexican hat feature for the secondary eclipse that allows us to determine the ingress and egress of the eclipse. Therefore, we took the phases of the maximum and minimum in the first derivative curve as the ingress and egress that define the boundaries of the secondary eclipse\(^1\). In the case of

\(^1\)In our sample of 542 MOA EBs, there were only a few cases in which we failed to find the ingress and egress of eclipses from the second derivative curves, and they all seemed to be either W UMa EBs or ellipsoidal binaries after inspecting their folded light curves by eye. For them, it might be more appropriate to use the light curves’ maxima to define the boundaries of their eclipse regions. Nevertheless, we did not find it would significantly impact the accuracy of our eclipse timing.
Fig. 5.3 Mean light curves (left), first derivative curves (middle) and second derivative curves (right) of three representative MOA EBs. The points of ingress and egress of an eclipse are determined by calculating a pair of minima in the second derivative curve that can well define the boundaries of the eclipse; i.e. the pair of minima of the Mexican hat feature in the second derivative curve. If there are no proper minima in the second derivative curve that correspond to the ingress and egress of the eclipse, then we use the maximum and minimum in the first derivative curve to define the eclipse region, e.g., the case of MOA-124700-GB10-1.
MOA-238532-GB9-9, instead of yielding typical Mexican hat features which have single peaks at the middles for both eclipses in the second derivative curve, double peaks were produced, indicating the presence of four contact points, which would be present in the case of total eclipsing, in both eclipses. As usual, the phases of the minima at the bottom of the Mexican hat were used to define the boundaries of the eclipse.

Once the boundaries of the eclipse region were determined, the best-fit template was derived by fitting Mikulášek’s model, i.e., eq.(4.8) to the portion of the folded light between the boundaries using emcee, a Python implementation of the affine-invariant ensemble sampler for Markov chain Monte Carlo (MCMC) [69]. The reference epoch, $T_0$ in eq.(4.1), in regard to the derived template, was thus defined accordingly as $T_0 = \phi_0 P_s + \tau_0$, where $\phi_0$ is the phase of the template minimum with respect to the time zero, $\tau_0$, which we set to be $\tau_0 < t_{\text{obs}}$, where $t_{\text{obs}}$ is an observation time, and both primary and secondary eclipses were not broken in phase when the light curve was folded with respect to $\tau_0$. The best-fit templates of six EBs in our MOA samples are shown in Figure 5.4 as examples to demonstrate the usefulness of our template generating method for different shapes of eclipses. In the case of contact binaries, e.g. MOA-124700-GB10-1 in Figure 5.4, even though we failed to derive the ingress and egress phases of the eclipses and had to take the maxima of the light curves as the eclipse boundaries, Mikulášek’s model, i.e. eq.(4.8), still generated templates which
represent the eclipses practically well enough. Once the template was generated, we unfolded the light curve, and then fitted each eclipse which had at least four data points across the eclipse minimum with the template. The general idea of the template method is to obtain the time of the eclipse minimum by shifting the template horizontally until the template best fits the eclipse. In reality, however, the brightness of a star may vary over time, and hence the template parameters $t_0$ as well as $\alpha_0$ and $\alpha_1$ were required to vary as well to search for the best fit. Again, the best-fit parameter search was executed using emcee. As a result, the time of the eclipse minimum was determined by the median of the projected posterior on $t_0$. The uncertainty in $t_0$ was taken as the 1-$\sigma$ confidence interval from the median.

The eclipse timing process described in the previous paragraph worked properly for most of the MOA EBs we studied. There are, however, six MOA EBs we could never derive periods with which they could be folded satisfactorily. Such a problem indicated that these EBs suffered significant ETVs. As improper folding could induce significant errors in the derived eclipse templates, we thus divided each of these EBs’ light curves into three segments in which the first two segments evenly span the first seven years (i.e., from April 2006 to March 2013) and the third segment spans the last 2.5 years (i.e., from April 2013 to July 2015). We then calculated the average eclipsing period for each segment and generated the corresponding templates and performed the eclipse time measurement following the same process as mentioned in the previous paragraph. We ignored the measurement points with very large uncertainties and inspected the resultant ETV curves afterwards. As expected, the ETV curves of shorter period EBs were generally denser than those of longer period EBs given that there were more eclipsing cycles for the shorter period EBs.

Star spots might be present. And they would distort eclipse shapes and induce spurious ETVs. Whether the eclipse shapes changed over time or not can be investigated by comparing the light curves of EBs from season to season. Yet Tran et al. (2013) showed that anti-correlated, random walk-like behaviours of primary and secondary ETVs would manifest the presence of active star spots. The quasai-periods of these anti-correlated ETVs are about 200 days or less according to the study of Tran et al. (2013) using the Kepler data. Quasi-cyclic variations on the time scale shorter than two MOA observation seasons ($\approx 2yr$) were noticeable in the O–C diagrams of several MOA EBs. A few of them exhibit quasi-periodic ETVs for their primary and secondary eclipses which were highly anti-correlated, for example, MOA-36543-GB9-9 (see Figure 5.5). It was obvious by comparing the O–C diagram of MOA-36543-GB9-9 with the
5.4 Eclipse Time Measurement

Fig. 5.5 Examples of the O–C diagrams of the MOA EBs with spurious ETVs. (left) The O–C diagram of MOA-36543-GB9-9, in which the ETV curves for the primary and secondary eclipses vary cyclically and behave anti-correlated to each other. (right) The O–C diagram of MOA-47495-GB10-1, which appears to have two separate curves or rapidly oscillating ETVs. The blue (diamond) points are the ETV measurements of the primary eclipses and the red (circle) points are those of the secondary eclipses. The average uncertainties for the primary and secondary eclipses are represented by the red and blue error bars, respectively, on the top-left corner of each figure. Note that the periods are in days.

O–C diagrams of the Kepler sample in Tran et al. (2013) that the anti-correlated ETVs in MOA-36543-GB9-9 were due to the presence of active star spots. Since these highly anti-correlated ETVs would cause serious trouble for ETV analysis, we rejected any of them for our search for LTTE signals unless they showed evident long-term variations.

Scattering of the ETV points comparable to the average error bars on the time scale of one MOA observational season was also common in the O–C diagrams of our sample. In some cases, the ETV points dispersed such that the O–C diagrams seemed to have two separate curves, or exhibit very rapid oscillations on very short time scales (< 100 days), e.g., MOA-47495-GB10-1 (see Figure 5.5). Orbital perturbations due to very short period tertiary companions could produce oscillating ETVs, which have been observed in the Kepler triples [29]. However, we also suspect that this kind of ETVs might be spurious arising from stellar oscillations or pulsations [27]. Given the frequency of and accuracy in eclipse timing from the MOA data, the proper coverage of a ETV cycle shorter than 200 days was expected to be unachievable. Considering the problem of over-fitting scattering of ETV points, as well as possible spurious ETVs discussed in the previous paragraph, we decided to further restrict the search for the
LTTE cycles of periods longer than 200 days in order to avoid false detection. Further discussion about this issue is presented in Section 5.6.

5.5 Light Travel Time Effect Analysis

The O–C diagram of each EB was constructed according to eq.(4.1) with $P_s$ being the average eclipsing periods over the full data time span and $T_0$ being the time of the eclipse template minima, except the six special EBs mentioned in Section 5.4 that the average eclipsing periods and the times of the eclipse template minima associated with the segments of the first 3.5 years (i.e., from April 2006 to September 2009) were used instead. We discarded the ETV measurement points with uncertainties $> 0.01$ days (except MOA-146280-GB9-9 and MOA-43392-GB9-9 for which we accepted the ETV measurement points with uncertainties up to 0.02 days instead in order to include more measurement points for model fitting). We then fitted the LTTE model, including or excluding the quadratic term of $E$, to the primary and secondary ETV curves simultaneously, using pymc [14], another Python module of MCMC fitting algorithms. For consistency between the calculations of the polynomial terms of $E$ in eq.(4.2) for primary and secondary eclipses, we added the phase difference ($\delta$) between the minima of the eclipse templates to the cycles ($E$) whose corresponding eclipse is in the secondary half of the folded light curve; in other words, if the secondary (or primary) eclipse in the folded light curve is located between 0.5 and 1 in phase, we shall add $\delta$ to $E$ for each cycle of the secondary (or primary) eclipse. The LTTE term in eq.(4.2) depends implicitly on $P_2$ and $\tau_2$ via the true anomaly, $\nu_2$, which must be calculated by solving Kepler’s equation iteratively using a numerical method. We used Halley’s method in this study (see Appendix B). The calculation of $\nu_2$ in fact caused serious speed issues in the parameter search using pymc. To improve the computational speed, the calculation of $\nu_2$ was done using the code written in Cython instead of Python/numpy.

The Metropolis-Hastings algorithm is used in pymc for distribution sampling. We adopted the built-in normal distribution function in pymc as the likelihood function and assumed a uniform prior for each parameter over the boundaries that we assumed to be appropriate (see Table 5.1). After testing the model fitting algorithm, we realized that the likelihood function might not be able to converge, or it might converge incorrectly to a local minimum, if the initial guess of the outer period value was not close to the true value. The difficulty in having a good guess of $P_2$ happened particularly when only the partial LTTE cycle was observed. Concerning these problems, and with the usage of
Table 5.1 Boundaries of the parameters of the ETV model eq.(4.2). Note that \( d \) is day, \( d/c \) is day/cycle and AU is astronomical unit.

<table>
<thead>
<tr>
<th>Parameter (unit)</th>
<th>Lower</th>
<th>Upper</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_0 ) (d)</td>
<td>-0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( c_1 ) (d)</td>
<td>-0.1</td>
<td>0.1</td>
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<tr>
<td>( c_2 ) (d/c)</td>
<td>-0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>( \log(P_2) ) (d)</td>
<td>( \log(P_2/2) )</td>
<td>( \log(2P_2) )</td>
</tr>
<tr>
<td>( e_2 )</td>
<td>0</td>
<td>0.999</td>
</tr>
<tr>
<td>( \omega_2 )</td>
<td>0</td>
<td>2\pi</td>
</tr>
<tr>
<td>( \tau_2 )</td>
<td>0</td>
<td>1</td>
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<td>( a_{AB} ) (AU)</td>
<td>0</td>
<td>100</td>
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</tbody>
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the New Zealand eScience Infrastructure (NeSI) high performance computing facilities, the parameter search was carried out over a set of initial values of \( P_2 \) as long as we had no confident estimation of the value of \( P_2 \) by eye. For a ETV curve with a potential LTTE signal of periods longer than 3000 days, for example, we ran the model fitting with initial values of \( P_2 \) from 2000, 3000, 4000, 5000, 6000, 7000, 8000 and 10000 days, respectively. For convenience, the search was over the \( \log(P_2) \) space instead, bounded between \( \log(P_2/2) \) and \( \log(2P_2) \). The initial guess values of other orbital parameters including \( e_2 \), \( \omega_2 \) and \( \tau_2 \) were taken to be the middles of their boundaries in principle, while the projected semi-major axis of the absolute orbit of a tertiary companion, i.e., \( a_{AB}\sin i_2 \), in AU is set to be 0.5 as the initial guess value based on the properties of Kepler triple candidates discovered by Borkovits et al. (2016) which are typically of \( P_2 < 4 \) years and \( a_{AB}\sin i_2 < 1 \)AU. Based on the model fitting experience in Section 4.5, we decided to run the Metropolis-Hastings algorithm with 6,500,000 iterations (because a larger \( P_2 \) space was searched through), discarded the first 5,500,000 samples and then thinned the Markov chain by 20 in order to ensure the posterior distribution would converge and reduce autocorrelation among the parameters.

Although the mass transfer would happen in contact and semi-contact binaries, the reliability of the best-fit solution from the LTTE model incorporating the quadratic term of \( E \) in eq.(4.1) might be questionable because such a combination can easily produce a satisfactory fit to a long-term ETV curve that led to false positive LTTE detection. Therefore, we always preferred the best-fit solution of the LTTE model without the quadratic term unless the BIC value of the best fit with the quadratic term was lower than that without the quadratic term by at least 10, indicating the former is highly favourable. In addition, the detection of the LTTE was accepted to be
genuine only if the BIC value of the best-fit LTTE solution was lower than that of the best-fit solution of the quadratic equation of $E$, which was also derived using pymc.

5.6 Results

5.6.1 The reliability of the results

We attempted to search for LTTE in all MOA EBs of periods less than two days in the GB9-9 and GB10-1 fields. In these two fields, there are 575 EBs within the period range we were interested in. Among them, there were 542 EBs we were able to derive the eclipse templates for both primary and secondary eclipses. We inspected the cases for which the template generation failed and found that they failed either due to the light curves omitted because they had too few (<1000) good photometric measurement points (i.e., with uncertainties <2000) or our algorithm failed to derive the ingress and egress points from the stationary points of both first and secondary derivatives. The latter problems occurred for total eclipses, for which our code rejected as exceptions because their second derivative curves had the central minima of the Mexican hat features below the zero level. Our code was unable to distinguish such central minima from other local minima outside the Mexican hats and, thus, failed to sort out and extract the minima corresponding to the eclipse’s ingress and egress points. On the other hand, there were a few cases in which the failure of the template generation was due to the recalculated periods falling into the 0.5d aliases because of the true periods being close to 0.5 or 1 day. We did not attempt to fix these problems or determine the ingress and egress points manually for those failed cases as it would have required a lot of additional time and work. We decided to simply exclude them from the final EB sample and perform the ETV analysis only for the rest of 542 EBs, among which we have 436 and 106 samples from the GB9-9 and GB10-1 fields, respectively.

Following the procedures of the ETV analysis in the precious section, we discovered 91 out of these 542 EBs with the LTTE solutions preferable to their ETV curves. 65 triple candidates were identified in the GB9-9 field, while 26 were in the GB10-1 field. The derived orbital parameters of the triple candidates are shown in Table 5.2 and Table 5.3, respectively. Whether the LTTE solutions are reliable is always questionable as several mechanisms can produce ETV curves that mimic the LTTE. In particular, ETVs for primary and secondary eclipses which seemed be anti-correlated with each other were observed in many of our EB samples. In the O-C diagrams
5.6 Results

Fig. 5.6 ETV curves of MOA-289148-GB9-9 and MOA-351777-GB9-9. $P_1$ is the period of the inner binary determined by the conditional entropy method, while $P_2$ is the period of the tertiary companion given by the LTTE solution. The blue points are the ETV measurements of the primary eclipses and the red points are those of the secondary eclipses, while the green lines represent the best fits of the ETV model defined by eq. (4.2). The ETVs for the primary and secondary eclipses were observed to vary cyclically and behave anti-correlated to each other in the time scale of a year, while the long-term trends were consistent. The bottom panels show the unfolded light curves. Note that the periods are in days.

of MOA-289148-GB9-9 and MOA-351777-GB9-9 (see Figure 5.6), for example, the ETVs for primary and secondary eclipses were observed to vary cyclically and behave anti-correlated to each other in a time scale of a year, while the long-term trends were consistent. We suspect the anti-correlated behaviours in the ETVs were due to the presence of active star spots [212]. Generally, averaging the ETVs of the primary and secondary eclipses might reduce the contribution of such spurious ETV. But either the primary or secondary eclipse would usually be missing in a cycle, thus averaging was not applicable for the majority of the MOA samples. Nonetheless, we recognized that the best fit obtained by pymc would roughly represent the solution to the mean ETV curve if we fit the ETV curves of primary and secondary eclipses simultaneously, provided that the uncertainties in ETVs for primary and secondary eclipses are comparable.

In addition, the LTTE solution might represent the over-fitting to the ETV curve when the uncertainties in the times of eclipse minima were overall larger than the LTTE amplitude. Particularly, the model of quadratic ETV plus LTTE could easily provide a good fit to a ETV curve, leading to false positive detection of LTTE. To avoid over-fitting, we used the Bayesian information criterion (BIC) to decide whether to accept or reject the solution from the model with more free parameters. In our ETV analysis, we accepted the solution of the LTTE model plus the quadratic term of $E$
Fig. 5.7 Amplitude of light travel time effect (LTTE), $A_{\text{LTTE}}$, vs root mean square of uncertainty in eclipse timing for primary eclipses, $\text{rms} (\sigma_p)$, for 91 triple candidates identified in the MOA EB sample of periods $< 2$ days from the GB9-9 and GB10-1 fields. The red line represents $A_{\text{LTTE}}$ equal to $\text{rms} (\sigma_p)$. About half of these 91 triple candidates have $A_{\text{LTTE}}$ larger than $\text{rms} (\sigma_p)$. Bayesian information criteria (BIC) was used to decide whether the ETV model with the LTTE was accepted or not. Note that $\Delta \text{BIC} = \text{BIC}(\text{P}(E)) - \text{BIC}(\text{P}(E) + \text{LTTE})$, where $\text{P}(E)$ represents the polynomial of $E$ in eq.(4.2) and $E$ is cycle. 81 of these triple candidates have $\Delta \text{BIC} > 10$, indicating the best fits of the ETV model with the LTTE were strongly preferable. These 81 triple candidates include all those with inner periods $P_1 < 0.26$ days and all those with outer eccentricities $e_2 > 0.9$. 
as the best fit only if its BIC value was lower than that excluding the quadratic term of $E$ by 10. In addition, the detection of LTTE was accepted finally only if the BIC value of the LTTE solution is lower than that of the parabolic solution. In this way, we accepted the ETV curves of 22 samples to be best fitted by the LTTE model plus the quadratic terms, while the fits by the LTTE model without the quadratic terms were preferred for 69 samples. Figure 5.7 shows the plot of LTTE amplitudes, $A_{LTTE}$, against root-mean-square errors in eclipse timing for primary eclipses, $\text{rms} (\sigma_p)$. There were only about half of the detected LTTE signals with amplitude greater than $\text{rms} (\sigma_p)$. Nonetheless, among these 91 triple candidates, 88 of them have differences between the BIC values of the LTTE and parabolic solutions higher than 10, indicating the LTTE solutions are very strongly preferable. On the other hand, there are two of them, i.e. MOA-40690-GB9-9 and MOA-7772-GB9-9, which have the BIC differences barely above 0, indicating the statistical evidence is weak, although they are still included in the list of EBs with detected LTTE signals.

5.6.2 Statistics and distribution

Since we selected the EBs from the subfields GB9-9 and GB10-1 only in terms of period alone, it represents a homogeneous sample of EBs of periods less than two days. Therefore, it is worth examining distributions and statistics of several interesting orbital parameters.

Tertiary period

The advantage of the ETV method is that we can derive the orbital periods and eccentricities of tertiary companions from the LTTE solutions. The distribution of the tertiary period of all triple candidates in our sample is shown in Figure 5.8, while the distribution of the triple candidates in the GB9-9 and GB10-1 fields are shown in Figure 5.9 and Figure 5.10, respectively, for comparison. The logarithmic bin of 20 was used to bin the tertiary period from $\log(P_2) = 0$ to $\log(P_2) = 5$. The tertiary period distribution peaked at $\log(P_2 \approx 3.4)$, which is close to the time span of the MOA data, i.e., 3420 days. Since the LTTE signal of period longer than 3420 days would only have a portion of its cycle seen in the O-C diagram, it would usually be indistinguishable from the parabolic ETV unless the portion of the LTTE curve seen in the O-C diagram has a curvature significantly different from that of a parabolic curve. Therefore, we suspected the lack of triple candidates of longer outer periods is due to
the limited time span of the data. On the other side, there is almost no detection of tertiary companions of periods less than 600 days. MOA-129173-GB10-1 is the only one having a tertiary companion of period less than 600 days\(^2\).

The lack of tertiary companions of the period less than 600 days might be related to the general formation process of contact binaries. However, we have to also point out that the LTTE amplitude increases as the outer period increases or the mass of the tertiary companion increases, so short period and low mass tertiary companions might be simply undetectable given the uncertainties in ETV measurements from the MOA data. Also, the existence of regular gaps between two MOA observational seasons in the data always results in regular gaps in the ETV curves which in turn

\(^2\)In fact, MOA-129173-GB10-1 is one of the three triple candidates discovered in our preliminary ETV investigation. The other two are MOA-115233-GB10-9 and MOA-360325-GB10-7 which have tertiary companions of periods 427 and 482 days, respectively. However, concerning the homogeneity, we did not include these two in our sample of MOA triple candidates for statistical analysis.
5.6 Results

Fig. 5.9 Distribution of tertiary periods of triple candidates identified in the GB9-9 field. The tertiary periods are binned in logarithmic bins of 20 from $10^2$ to $10^5$ days. The red line represents the time span of the MOA data which is about 3420 days. The distribution peaks at 2660 days.

Fig. 5.10 Distribution of tertiary periods of 26 triple candidates identified in the GB10-1 field. The tertiary periods are binned in logarithmic bins of 20 from $10^2$ to $10^5$ days. The red line represents the time span of the MOA data which is about 3420 days. The distribution appears to be bimodal with one peak at $P_2 \approx 3700$ and the other at $P_2 \approx 1300$. The presence of a double mode is likely due to the non-uniformity of the density of the data sets from the GB10-1 field.
Fig. 5.11 Binary period $P_1$ vs tertiary period $P_2$ for 91 triple candidates identified in the GB9-9 and GB10-1 fields. All the triple candidates have $\log(P_2/P_1)$ between 3 and 5, except MOA-129173-GB10-1 which has a close tertiary companion of $P_2$ about 247 days according to its best-fit light travel time effect (LTTE) solution.

prevents the proper coverage of short period LTTE signals and might make the short period LTTE signals difficult to be detected. In addition, the triple candidates in the GB9-9 and GB10-1 fields follow distinctive outer period distributions. Particularly, the tertiary period distribution of the GB10-1 sample seemed to be bimodal with a peak at $P_2 \approx 3700$ days and the other at $P_2 \approx 1300$ days. It could be just the effect due to the small sample size, but we suspected the peak at $P_2 \approx 1300$ days resulted from the non-uniform density of the light curves of the GB10-1 sample, in which there are fewer data points in the period of the first three observational seasons (from 2006 to 2008) because lower cadences for imaging were taken towards GB10 during that period.

We also plotted the tertiary period ($P_2$) against the inner binary period ($P_1$) for the 91 triple candidates as shown in Figure 5.11. All of the triple candidates have the period ratios $P_2/P_1$ between $10^3$ and $10^5$, except MOA-129173-GB10-1 of which the period ratio is below $10^3$. We mentioned in Section 4.2, the dynamical effect would be important and have significant contributions to the ETV variations if $A_{\text{dyn}}$
is comparable to $A_{\text{LTTE}}$. That is, for moderate values of $P_2/P_1$ ratio the dynamical contributions should be of great concern. As the $P_2/P_1$ ratios are greater than $> 10^2$ for all our triple candidates, we expected the contributions due to the dynamical effect were very small. This is further demonstrated by the fact that no periodic feature are seen in the residual curves after subtracting the LTTE contributions from the ETV curves (see Figure 5.6, 5.16, 5.17 and Appendix C). We, therefore, concluded that the dynamical effect is negligible in our triple candidates.

**Frequency of eclipsing binaries with tertiary companions**

Figure 5.12 shows the distribution of the period of our EB sample. The peak occurs at around 0.5 day and the number of EBs declines rapidly when period is longer than 0.5 day. On the other side, there is a cut-off at $\sim 0.2$ days. The lack of contact binaries below 0.2 days in the MOA EBs is consistent with the idea of the existence of a physical lower limit of the period of contact binaries [173]. Looking at the period distribution of the 91 EBs with detected LTTE signals, 69 of them (i.e. 75%) are of period less 0.5 days, while none of them are of period longer than 1.5 days. The overall frequency of EBs with detected LTTE signals is $91/542 = 0.168$. If we look at the distribution of the frequency of EBs with detected LTTE signals as shown in Figure 5.13, it is interesting to note that the frequency basically increases as the period decreases, and the frequency reaches $13/20 = 0.65$ when the period is less than 0.3 days. When we further zoomed into the period range between 0.2 and 0.4 days (see Figure 5.14 and 5.15), there are six EBs in our sample of periods less than 0.26 days and they all have the LTTE signals detected in their O-C diagrams, giving the frequency of having tertiary companions equal to 1. Figure 5.16 shows the ETV curves of these six EBs. The periods of their tertiary companions range from $\sim 1500$ days (or 4 years) to $\sim 8000$ days (or 22 years). We have to emphasize that our estimation of tertiary companion frequency is very preliminary. To obtain robust estimation of the frequency of contact binaries with tertiary companions, the corrections which take all the selection effects and detection limitations into account have to be estimated through the population synthesis. Unfortunately, it would require substantial follow-up work which could not be done properly by the end of my PhD and, therefore, is out of the scope of this thesis.
Fig. 5.12 Period histogram of the MOA EB sample which contains 542 EBs of periods < 2 days from the GB9-9 and GB10-1 fields. The portion of the whole EB sample with detected LTTE signals was filled with yellow, while the rest was hatched with diagonal lines.

Fig. 5.13 Frequency of the MOA EBs in the GB9-9 and GB10-1 fields which are of periods < 2 days with detected LTTE signals. We binned the periods from 0 to 2 days into 20 bins. Frequency is defined as the number of EBs with detected LTTE signals over the total number of EBs in each bin.
5.6 Results

Fig. 5.14 Period histogram of the MOA EB sample of periods ranging from 0.2 to 0.4 days in the GB9-9 and GB10-1 fields. There are six EBs in the sample which are of periods $< 0.26$ days. Signals of light travel time effect (LTTE) were detected in all their ETV curves, indicating the presence of tertiary companions orbiting around them.

Fig. 5.15 Frequency of the MOA EBs in the GB9-9 and GB10-1 fields which are of periods ranging from 0.2 to 0.4 days with detected light travel time effect (LTTE) signals. We binned the periods from 0.2 to 0.4 days into 20 bins. Frequency is defined as the number of EBs with detected LTTE signals over the total number of EBs in each bin.
Fig. 5.16 ETV curves of six EBs of periods less 0.26 days. $P_1$ is the period of the inner binary determined by the conditional entropy method, while $P_2$ is the period of the tertiary companion given by the LTTE solution. The blue points are the ETV measurements of the primary eclipses and the red points are those of the secondary eclipses, while the green lines represent the best fits of the ETV model defined by eq. (4.2); the bottom panels show the unfolded light curves. Note that the periods are in days.
5.6 Results

**Outer eccentricity**

Another interesting property to look at is the distribution of the outer eccentricities. We plotted the outer eccentricity distributions with the number of bins of 10 and 20. In the case of outer periods binned into 10 bins, the distribution is characterized by a peak at $e_2 = 0.7$, while, interestingly, the second peak which contains 10 triple candidates is seen at $e_2 = 0.9$. When we binned the outer eccentricity into 20 bins instead, an excess was even clearly noticed at $e_2 > 0.95$. Taking uncertainties in the eccentricity into account, the outer eccentricities of these 10 triple candidates all still fall into the range of $e_2 > 0.9$ except one which just fell into the range of $e_2$ from 0.8 to 0.9. Since the excess at $e_2 > 0.9$ is still preserved for our triple candidates when the uncertainties are concerned, such an excess is not an artifact resulting from binning.

Nonetheless, such high eccentric companions are expected to be so unstable that they would not survive owing to long-term instability or their eccentricities would not be still maintained to be so large if they formed with the inner binary systems roughly at the same time, given that contact binaries such as W UMa variables belong to old populations of ages about 4.4-4.6 Gyr [229]. Thus, whether the derived LTTE solutions were physical has to be examined carefully. We inspected the O-C diagrams of every EB with detected LTTE signals by eye. The LTTE solutions associated with $e_2 > 0.9$ turned out to have unique shapes with sharp turning points (see Figure 5.17).

In particular, such sudden period changes are already noticeable in the O-C diagrams of MOA-284305-GB9-9, MOA-108463-GB9-9 and MOA-249394-GB9-9. Although the values of $\Delta \text{BIC}^3$ of their LTTE fits are much larger than 10, it should be emphasized that the high value of $\Delta \text{BIC}$ simply means that the LTTE model, eq.(4.1), which can be recognized as a mathematical model containing combination of sinusoidal terms, gives a better description than the pure parabolic model and does not guarantee that the LTTE fit is physically reliable. Since the LTTE solutions with extremely high outer eccentricities are probably unphysical, other reasons might be more appropriate to explain the observed ETVs of these ten MOA EBs. Abrupt changes in or sudden jumps of orbit periods are, in fact, not a rare phenomenon in close binaries. Dozens of close binaries, which belong to Algol- or W UMa-type, were reported to exhibit sudden jumps in their O-C diagrams (e.g., [162, 163, 144]). Mechanisms which might induce such sudden period jumps include sudden mass exchange [89] or mass loss [228] via stellar flares, variations in the internal structures (i.e., convective envelopes) of binaries’

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3See Figure 5.7.
active components [162], and the rapid accretion of binaries from the circumstellar matter [228]. Also, the periodicity of the O–C diagrams might come from magnetic cycles arising from, e.g., the Applegate effect\(^1\), which can produce quasi-cyclic ETVs, instead of LTTE from unseen tertiary companions. Despite the questionable reliability of the LTTE solutions, these ten MOA EBs show very interesting ETVs, which are worth taking notice of.

The cumulative distribution of the outer eccentricity of the MOA triple candidates was calculated (see Figure 5.18). If all 91 triple candidates are taken into account, the calculated distribution lies between the uniform distribution and the thermal distribution\(^4\). However, as the reliability of the LTTE solutions of extremely high outer eccentricities are quite questionable, inclusion of the triple candidates with \(e_2 > 0.9\) might lead to an incorrect conclusion. We, therefore, excluded the triple candidates with \(e_2 > 0.9\) and recalculated cumulative distribution. The recalculated cumulative distribution, in contrast to the case when all the triple candidates were included, resembles neither a linear nor a flat distribution, indicating that the issue of whether the detection of the triple candidates with very high outer eccentricities was real or not would lead to very different conclusions.

The plot of outer eccentricity against tertiary period is shown in Figure 5.20. The correlation coefficient was calculated to be 0.042, indicating no correlation between the outer eccentricity and tertiary period for our MOA sample.

### 5.7 Discussion and Conclusions

We carried out ETV analysis for the sample of MOA EBs of periods < 2 days in two MOA subfields, GB9-9 and GB10-1, using the MOA-II data spanning 9.5 years. The sample contains 524 EBs, 436 and 106 in the GB9-9 and GB10-1 fields, respectively. The Bayesian information criterion was used as a measure for the model selection between ETV models with and without the LTTE term. In this way, we discovered 91 MOA EBs with detected LTTE signals, indicating the presence of tertiary orbiting companions. The distribution of tertiary period for our 91 triple candidates peaked sharply at 2660 days (or 7.2 years), while there were no EB in the sample with any tertiary companion of orbiting period \(P_2 > 30\) years. Given the fact that the data

\(^4\)Thermal eccentricity distribution refers to the distribution of eccentricities of a population of binary stars, where every member has interacted with each other and reached statistical equilibrium. The normalized distribution of such a population as a function of eccentricity is \(f(e) = 2e de\), where \(e\) is eccentricity, derived by Jeans (1919).
Fig. 5.17 ETV curves of ten triple candidates with outer eccentricities $e_2 > 0.9$. $P_1$ is the period of the inner binary determined by the conditional entropy method, while $P_2$ is the period of the tertiary companion given by the LTTE solution. The blue points are the ETV measurements of the primary eclipses and the red points are those of the secondary eclipses, while the green lines represent the best fits of the ETV model defined by eq. (4.2). The bottom panels show the unfolded light curves. Note that the periods are in days.
Fig. 5.18 Cumulative distribution of outer eccentricity for the triple candidates identified in the GB9-9 and GB10-1 fields. The green curve represents the cumulative distribution for uniform distribution of eccentricity from 0 to 1. The blue curve represents the cumulative distribution for thermal (or linear) eccentricity distribution derived by Jeans (1919). The cumulative distributions of the outer eccentricity of the triple candidates, excluding and including the triple candidates with $e_2 > 0.9$, in our sample are represented by the red and black curves, respectively, and their underlaying distributions are distinct from each other.
Fig. 5.19 Distribution of outer eccentricity for 91 triple candidates in the GB9-9 and G10-1 fields. The distribution is binned into 10 (white) and 20 (yellow) bins, respectively, and they are plotted on top of each other in the same graph. The distribution increases as eccentricity increases and peaks at about $e_2 = 0.7 - 0.8$. The excess of eccentricity is observed at $e > 0.9$ in the triple candidates we identified.
Fig. 5.20 Tertiary period $P_2$ vs Outer eccentricity $e_2$ for 91 triple candidates in the GB9-9 and GB10-1 fields. The red curve is the best linear fit which has a correlation coefficient of 0.042, indicating there is no significant correlation between $P_2$ and $e_2$ for the group of these 91 triple candidates.
5.7 Discussion and Conclusions

Fig. 5.21 Light curve (above) and ETV curve (below) of MOA-182318-GB9-9, whose ETVs seemed to vary in accordance with the long-term flux variation over the data time span (i.e., 9.5 years). The blue points are the ETV measurements of the primary eclipses and the red points are those of the secondary eclipses. Note that the periods are in days.

spanned only 9.5 years, it is obvious that the lack of detection of tertiary companions of $P_2 > 30$ years is a consequence of the data time span being not long enough. In addition, we suspect that the peak being at 2660 days also resulted from a selection effect due to the data time span. Nonetheless, the significant decline in the distribution for $P_2 < 10^3$ days might be related to the formation of close and contact binaries although it might be also due to the presence of regular gaps in the ETV curves associated with the off-season periods.

As our sample was homogeneous in terms of period, it would be interesting to see how the frequency of EBs with tertiary companions varies as a function of the inner binary period $P_1$. Particularly, the group of EBs of periods $< 0.5$ days represented a homogeneous sample of contact binaries and the detection of LTTEs in the contact binaries in this period range should suffer from the least selection effect due to day-night cycles as indicated by the number of eclipse time measurement points we obtained.
For our sample, there is an obvious tendency for short period contact binaries to be likely accompanied by tertiary companions. The frequency of our EBs with tertiary companions increases as $P_1$ decreases. For our 13 contact binaries of $P_1 < 0.3$ days, the frequency reaches a value of 0.65. Looking into these 13 contact binaries, we further found that all six contact binaries of $P_1 < 0.26$ days are with tertiary binaries. Since all our detected tertiary companions are of orbiting periods $<10^4$, our results suggest that contact binaries of periods close to the 0.22-day contact binary limit are commonly accompanied by relatively close tertiary companions. Meanwhile, the outer eccentricity distribution for our 91 triple candidates behaved approximately as a linear function, but an excess at $e_2 > 0.9$ was observed.

It should be emphasised that false positive detection of LTTE signals is possible since we relied on the ETV analysis only. Follow-up observations in spectroscopy and third-light analysis via light curve modeling are needed to falsify our discoveries. However, it should also be pointed out that analysis of variations in times of ellipsoidal maxima, known as quadrature time variations (QTVs) [29], could be conducted to check the validity of the LTTE signals in our EB sample. The QTV curves for LTTE have the same formula as that of ETVs given by eq. (4.2) and their behaviours should be consistent with each other because the QTVs and ETVs are produced by the same manners. As demonstrated in the study of Borkovits et al. (2016) to identify triple systems amongst the Kepler EBs, the incorporation of QTVs into the O–C diagram examination is helpful as to identify false positives of triple candidates which are in fact wide binary systems with pulsating components. That is, pulsating stars having the sinusoidal light curves similar to ELVs were misclassified as EBs in the first place. The light curves of such pulsating stars would have alternating maxima and minima with the same amplitude. Fortunately, none of EBs with detected LTTEs in our sample have the same amplitudes for primary and secondary maxima and minima in their folded light curves, indicating the unlikelihood of being false positive EBs. Indeed, being able to derive QTV curves which are consistent with the ETV curves would manifest the genuineness of the observed ETVs, but it does not provide any new information about the sources of the ETVs. Because of these reasons, as well as considering the time limit and computational resources for this project, we did not carry out the QTV analysis.

In addition, long-term flux variations were seen in the light curves of most of our triple candidates. In a few cases, the flux variations were seemingly correlated with the ETVs as inspected by eye (e.g., MOA-182318-GB9-9; see Figure 5.21). This kind of flux variation was also observed in OGLE EBs with cyclic ETVs [157]. The long-term
flux variations might come from the third light from bright stars which orbit around the EBs (e.g. [51]). Nonetheless, such variations might otherwise originate from changing luminosity of EBs’ components associated with stellar magnetic activities or pulsations. In particular, the Applegate mechanism\(^1\) predicts cyclic variations in the luminosity and colours which are correlated with the orbital period variations (i.e., the \(O-C\) cycles) \([112, 113, 8]\). Since there is a possibility that the detected \(O-C\) cycles for those MOA samples were driven by the Applegate effect\(^1\), it will be necessary to examine the correlation between the long-term flux variations and ETVs in order to have better judgment on the origins of their ETVs.
Table 5.2 Orbital elements from the light travel time effect (LTTE) solutions for 65 EBs in the GB9-9 field. Note that $P_1$ is the period of the inner binary determined by the conditional entropy method plus the correction, $c_1$, given by the best fit of eq.(4.2) to the ETV curve, and $\Delta P_1 = 2c_2$, where $c_2$ is the second order coefficient in eq.(4.2), is the change in inner binary orbital period per orbital cycle in units of $[\text{day/cycle}]$, and $m_{AB}$ was taken as $2M_\odot$ when calculating $(m_C)_{\text{min}}$.

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<th>$\Delta P_1$ $10^{-10}$ (d/c)</th>
<th>$P_2$ (d)</th>
<th>$e_2$</th>
<th>$\omega_2$ (deg)</th>
<th>$\tau_2$ (MBJD)</th>
<th>$a_{AB}\sin i_2$ AU</th>
<th>$f(m_C)$</th>
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Table 5.2 Orbital elements from the LTTE solutions for 65 EBs in GB9-9 (cont.)

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* - with additional periodic signal
Table 5.3 Orbital elements from the light travel time effect (LTTE) solutions for 26 EBs in the GB10-1 field. Note that $P_1$ is the period of the inner binary determined by the conditional entropy method plus the correction, $c_1$, given by the best fit of eq.(4.2) to the ETV curve, and $\Delta P_1 = 2c_2$, where $c_2$ is the second order coefficient in eq.(4.2), is the change in inner binary orbital period per orbital cycle in units of [day/cycle], and $m_{AB}$ was taken as $2M_\odot$ when calculating $(m_C)_{min}$.

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<td>–</td>
<td>224(220)</td>
<td>0.2(2)</td>
<td>276(91)</td>
<td>54879.1(6)</td>
<td>0.29(4)</td>
<td>0.00006(3)</td>
<td>0.14</td>
<td>27.47</td>
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<td>0.73(5)</td>
<td>170(2)</td>
<td>55179.94(1)</td>
<td>0.48(4)</td>
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<tr>
<td>174776</td>
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<td>–</td>
<td>480(489)</td>
<td>0.22(5)</td>
<td>151(11)</td>
<td>58491.4(1)</td>
<td>3.4(5)</td>
<td>0.2(1)</td>
<td>1.36</td>
<td>261.24</td>
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Table 5.3 Orbital elements from the LTTE solutions for 26 EBs in GB10-1 (cont.)

<table>
<thead>
<tr>
<th>No.</th>
<th>$P_1$ (d)</th>
<th>$\Delta P_1 \times 10^{-10}$ (d/c)</th>
<th>$P_2$ (d)</th>
<th>$e_2$</th>
<th>$\omega_2$ (deg)</th>
<th>$\tau_2$ (MBJD)</th>
<th>$a_{AB} \sin i_2$ (AU)</th>
<th>$f (m_C)$</th>
<th>$(m_C)<em>{\text{min}}$ (M$</em>\odot$)</th>
<th>ΔBIC</th>
</tr>
</thead>
<tbody>
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<td>102925</td>
<td>0.3926504(1)</td>
<td>–</td>
<td>3480(32)</td>
<td>0.383(2)</td>
<td>242.1(7)</td>
<td>56860.06(1)</td>
<td>3.19(4)</td>
<td>0.36(1)</td>
<td>1.7</td>
<td>3119.59</td>
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<td>63946*</td>
<td>0.41931334(2)</td>
<td>–</td>
<td>1827(51)</td>
<td>0.9(1)</td>
<td>1(2)</td>
<td>54515.31(10)</td>
<td>0.23(10)</td>
<td>0.0005(6)</td>
<td>0.13</td>
<td>5.76</td>
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<tr>
<td>69632*</td>
<td>0.4569038(2)</td>
<td>–</td>
<td>3971(241)</td>
<td>0.78(2)</td>
<td>65(4)</td>
<td>57200.09(8)</td>
<td>0.63(6)</td>
<td>0.0021(6)</td>
<td>0.22</td>
<td>4.19</td>
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</tbody>
</table>

* - with additional periodic signal
Chapter 6

Conclusions

We are going to die, and that makes us the lucky ones. Most people are never going to die because they are never going to be born. The potential people who could have been here in my place but who will in fact never see the light of day outnumber the sand grains of Arabia.

Richard Dawkins, *Unweaving the Rainbow: Science, Delusion and the Appetite for Wonder*

The achievement of sufficient detection rate of microlensing events relies on the capability of regularly monitoring millions of stars. To further detect planetary microlensing events, imaging in high cadences has to be adopted as well. Because of these requirements, it should be expected that a microlensing survey would intrinsically result in a large amount of photometric data of a million objects in the sky that have high quality in terms of precision and data density. Considering the fact that most of the ground-based microlensing surveys such as OGLE and MOA have monitored the sky over a decade, not only do their data contain a huge number of objects but also cover time spans longer than any other sky survey. The databases of microlensing surveys, therefore, are also valuable resources for other scientific research. This thesis verified this idea.

An intrinsic by-product of a microlensing survey is the massive photometric data of variable stars in the sky. For example, OGLE, one of active ground-based microlensing surveys, is also renowned as a world-leading sky survey for variability providing variable star catalogues which contain already millions of objects. Since 2006 MOA has routinely monitored towards the Galactic bulge (GB) and collected $\sim 100$ TBs of image data in
its database. Inspired by OGLE, the MOA database should also be useful for variable star research. The type of variable stars that interested us was eclipsing binaries (EBs). The attempt to establish an EB catalogue from the MOA database was carried out. We restricted ourselves to investigating only two MOA fields, GB9 and GB10, and used the MOA data spanning two observational seasons only, concerning the problems of limited time for the project and other technical issues. To identity EB candidates in the data, we folded the light curves at the periods calculated by the conditional entropy method and then inspected their shapes by eye. We recognized that the majority of variable objects with regular periodicity found in the MOA data were artifacts, which likely resulted from erroneous image subtraction which could occur when there were bright stars near the image positions. After substantial and tedious work, we identified 8733 EB candidates with periods ranging from 0.09 to 66 days.

The most interesting thing about EBs in our perspective is that there is a unique way to detect a tertiary companion orbiting an EB via eclipse time variation (ETV) analysis. The idea is to detect signals of light travel time effect (LTTE) in the EBs’ ETV curves. The ETV curves were constructed by measuring the differences between the observed and predicted occurrence times of eclipses. Eclipse timing was carried out using a template method, in which we generated the template of an eclipse by fitting Mikulášek’s phenomenological model to the folded light curve and then measured the occurrence times of the eclipses by fitting the template to the eclipses in the unfolded light curve. Once the ETV curve was obtained, we fitted the ETV models with and without the LTTE term to the ETV curve. The search for best-fit solutions was done using the MCMC algorithm. In the preliminary study in which we used only two observational seasons’ worth of data, we discovered three contact binaries with detected LTTE signals, associated with tertiary companions of orbiting periods between 250 and 480 days, among the 8733 EBs found in the GB9 and GB10 fields. These results suggested that the MOA data would be useful for a search for tertiary companions in very short period EBs and it would be likely to be able to find more triple candidates if the MOA data covering a longer time span were used.

It turned out that the main achievement in this project was the discovery of 91 triple candidates in the sample of 542 EBs with periods less than 2 days selected from the two subfields, GB9-9 and GB10-1, using the MOA data spanning 9.5 years. The tertiary period ($P_2$) for our 91 triple candidates had the distribution with a sharp peak at 2660 days (or 7.2 years). No LTTE signal corresponding to tertiary companions of $P_2 > 30$ years were detected. The lack of detection of tertiary companions of $P_2 > 30$...
years and the peak at 2660 days in the $P_2$ distribution might be the consequences of the limited data time span (i.e., 9.5 years). Also, the $P_2$ distribution was observed to decline significantly for $P_2 < 10^3$ days. Whether such a decline is related to the formation of close and contact binaries is an interesting problem to investigate in the future. We also examined the frequency of EBs with tertiary companions in our sample in terms of the inner binary period ($P_1$). We found that the frequency increases as $P_1$ decreases in our sample showing that short period EBs are more likely accompanied by tertiary companions. In particular, the frequency reaches a value of 0.65 for EBs of $P_1 < 0.3$ days. As we looked into the EBs in this period range, we further found that LTTE signals were detected in the ETV curves of all the EBs of $P_1 < 0.26$ days showing the possibility of accompanying tertiary companions. Because all the EBs of $P_1 < 0.26$ in our sample are contact binaries and the outer periods $P_2$ given by their LTTE solutions are shorter than $10^4$ days, our results suggest that contact binaries of periods close to the 0.22-day contact binary limit are commonly accompanied by relatively close tertiary companions. Meanwhile, an excess in the outer eccentricity distribution at $e_2 > 0.9$ was observed. Such highly eccentric outer orbits should be unstable in the long term, so their LTTE solutions might be unphysical and unreliable. Fortunately, none of these questionable triple candidates were of $P_1 < 0.26$ days. Therefore, either including or excluding these triple candidates with $e_2 > 0.9$ would not alter our conclusion about the tertiary companion frequency of contact binaries close to the 0.22-day limit. In addition, most of our triple candidates showed long-term flux variations in their light curves. And a few of them appeared to be correlated with their ETVs. The sources of these variations might be the third light from bright stars which orbit around the EBs. Nonetheless, without spectroscopic data, we could not perform thorough light curve modelling and test our hypothesis.

There is much follow-up work that can be carried out in the future. First of all, given the fact that the MOA fields we investigated overlap the OGLE fields and the OGLE observations began earlier than MOA, it is worth investigating the possibility of including the OGLE data to extend the time span for ETV analysis. On the other hand, since several mechanisms such as mass transfer and the Applegate mechanism which would be often present in contact binaries could induce long-term ETVs, the possibility of false positive detection of LTTE in our sample has to be a concern. In this sense, radial velocity measurements or direct imaging would be desirable to confirm our discoveries. Also, we investigated short period binaries only in the two MOA subfields, GB9-9 and GB-10-1. We did not exploit the entire MOA EB catalogue
that was established in this project. Therefore, the study of the multiplicity of contact binaries using a larger sample from the current MOA EB catalogue should be a task that can be carried out in the near future.
Appendix A

Overview of Gravitational Micro-lensing

Gravitational microlensing, usually just referred to as microlensing, is a type of gravitational lensing effect which occurs when two point sources such as two stars align along the line of sight. This effect exists because light can be bent by a massive object according to Einstein’s Theory of General Relativity. The first study of gravitational lensing dates back to 1924 when Russian physicist Orest Chwolson published a paper in which he demonstrated how this effect could produce multiple images of a star [41]. In 1936, Einstein himself, at the request of R. W. Mandl, also performed a simple calculation for the case of two distant stars being perfectly aligned along the line of sight. His calculation predicted the presence of a luminous ring as seen by an observer. However, such a ring (now known as the Einstein ring) would have an angular radius much less than an arcsecond, beyond the capability of the telescopes at that time to resolve. Einstein, therefore, described the gravitational lensing as an uninteresting subject with no hope of direct observation [60].

Lacking interest in the scientific community, the detailed mathematical description on the gravitational lensing by a star, though in hindsight mathematically relatively simple compared to other problems in General Relativity, was missing until 1964 when Sjur Refsdal published a paper where the analytical model of the effect was provided [166]. On the other hand, it wasn’t until 1979 when the gravitational lensing event was observed for the first time; however, that event was not caused by a star lensing a background object as Einstein originally considered, but by a galaxy lensing a distant quasar to produce twin images [222]. Later that year, Kyongae Chang and Sjur Refsdal proposed a type of gravitational lensing effect associated with stars in a galaxy acting as
tiny lenses on a distant quasar [39]. This gravitational lensing would produce Einstein rings of radii on order of $10^{-5}$ arcsecond and, instead of having resolvable multiple images of the quasar, would end up producing fluctuation in the quasar’s flux in the order of months. Given the resultant Einstein ring’s radius, this type of gravitational lensing effect due to a tiny lens (that is, the lens can be treated as a point source) was first termed “microlensing” by Polish astronomer Bohdan Paczynski in his paper on the numerical study of the lensing effect at large optical depth in 1986 [149].

In the same year, Paczynski also published his influential paper on microlensing [150] in which he suggested that microlensing could be used to search for massive compact objects that could be a candidate of dark matter in the Galactic halo if a large number of stars in the Magellanic Clouds were observed. Such massive compact objects in the Galactic halo, also called massive compact halo objects (MACHOs), if they were existent, might pass in front of the distant background stars and act as gravitational lenses to amplify the observed stellar flux. Since microlensing produces unresolvable images of the background star, the changing flux of the star during the passage of the MACHOs in front of it would be the only observable effect. The microlensing light curve of a single lens event, i.e, the plot of flux measurements of the lensed star over time, behaves as a smoothly varying, symmetric, single-peaked curve with the maximum at the time of the closest approach of the background star to the lens as seen from the observer. Figure A.1 shows the set of theoretical light curves of microlensing events corresponding to different values of impact parameter, $u_0$, which is the shortest projected separation between the lens and the background star in units of Einstein radius $R_E$ occurring at time $t_0$. The observable flux variation is characterized by the total magnification, $A$, of the background star’s images, i.e.,

$$A = A_+ + A_- = \frac{u_0^2 + 2}{u \sqrt{u^2 + 4}}, \quad (A.1)$$

where

$$u = u(t) = \sqrt{u_0^2 + \left(\frac{t - t_0}{t_E}\right)^2} \quad (A.2)$$

and $t_E = R_E/v_\perp$ is the duration of the background star crossing the Einstein ring radius. From eq.(A.1) and eq.(A.2), the magnification effect would be stronger when the impact parameter is smaller. In the case that $u_0$ is close to zero corresponding to perfect alignment between the lens and the background star, $A$ varies approximately as $1/u$, resulting in very high magnification at $t_0$. The information of the lens mass is
Fig. A.1 Configurations of the source star and lens in microlensing events associated with different values of impact parameter $u_0$ (left) and their magnification light curves (right). The gray shaded arcs, $I_+$ and $I_-$, in the diagram on the left are the two images of the source star (S) created by the lens (L) in the case of $u_0$ equal to 0.2. The image $I_+$ is outside the Einstein ring (the dashed circle) and on the same side of the lens, while $I_-$ is inside the Einstein ring and on the opposite side of the lens. The induced images are not resolved and only the total magnification is measurable generally. The total magnification is a function of $u$ and varies as the source star crosses the Einstein ring as shown in the plot on the right. This figure is taken from Gaudi (2012).

embedded in $R_E$, which in turn can be determined from the measurement of $t_E$ by fitting the observational light curve with the analytical model, i.e., eq. (A.1). However, the lens mass cannot be determined unambiguously unless their transverse velocities are measured as well. Nonetheless, assuming every observed event towards the Galactic halo was produced by the MACHOs, the density of the MACHOs would be proportional to the frequency at which the microlensing events were detected. Therefore, a microlensing survey could at least provide constraint on the mass range of the MACHOs.

To promote the survey of MACHOs by microlensing, Paczynski calculated the optical depths towards the galaxies in the Local Group. Optical depth is related to the density of matter toward a field. It could, therefore, represent the probability of the occurrence of a microlensing event in that field. According to Paczynski’s calculation, the optical depths were about $10^{-6}$ towards the Magellanic Clouds [150]. This in turn implied the probability of a microlensing event occurring towards the Magellanic Clouds at any time was roughly $10^{-6}$, provided the Galactic halo was made of MACHOs larger than $10^{-8} M_\odot$. Such a low probability suggested that the capability of monitoring millions of stars at the same time should be the essential criterion to ensure the practicability of the microlensing survey. Fortunately, the wide-field
observation technology was available soon after the publication of Paczynski’s paper. In 1992, the OGLE collaboration, as the first research group for microlensing survey, was founded in Warsaw and used a telescope at the Las Campanas Observatory in Chile to search for MACHOs [205]. Other collaborations including EROS, MACHO, MOA and SuperMACHO also joined the campaign to search for MACHOs in early 90s. All these microlensing surveys, however, had no detection of MACHOs, and the null results from them have suggested that the MACHOs cannot be the significant contributors to the dark matter in our Galaxy [227, 208, 5].

Since the early 2000s, attention in microlensing research has been focused on the application for exoplanet detection. The idea that gravitational microlensing could be used as a planet hunting method was first proposed by Mao and Paczynski in 1991 [123]. They showed that the alignment of two stars in the Galaxy could generate a microlensing light curve similar to that of a single lens event due to a MACHO. If the lens star was accompanied by an orbiting planet, such a planet could cause a dramatic deviation from the single lens light curve, provided the planet’s position is close to the Einstein ring. The perturbation by a planet in a mass range from 1 to 10 $M_{\odot}$ would typically last for a few hours. The magnitude and duration of the perturbation and, thus, the overall shape of the light curve depend on the mass and position of the planet as well as other geometrical parameters such as the lens distance and the separation between the lens and source. Hence, the properties of the lens such as the mass ratio could then be derived by modelling the light curve, although additional effects such as the finite source effect and microlensing parallax would have to be detected as well to resolve the degeneracy in the parameters. Figure A.2 shows the observational light curve of the microlensing event OGLE-2005-BLG-390 in which the planetary perturbation, associated with a super Earth of 5.5 $M_{\oplus}$, was observed 10 days after the peak [18].

However, different to single lens events for which the analytical model exists, the light curves of binary or multiple lens events cannot be studied analytically. The major challenge of modelling a planetary microlensing light curve comes from expensive treatment in computation and the problem of parameter degeneracy, especially in the cases of caustic-crossing microlensing events. A caustic represents the set of positions of a background star (treated as a point source) at which the magnification of its flux becomes infinite [84]. For a star-planet lens system, the planetary perturbation arises from the background star traveling across the caustic induced by the planet (i.e. the planetary caustic), while the central peak is associated with the crossing of the
Fig. A.2 Light curve of the microlensing event OGLE-2005-BLG-390. There was a second peak observed 10 days after the central peak. The overall light curve was best fitted by the model of the lens star with a planet of $5.5M_\odot$. This figure is taken from Beaulieu et al. (2006).

background star through the central caustic induced by the star. Indeed the planetary caustic might overlap the central caustic. Depending on the crossing trajectory, a complex magnification light curve could be established for a caustic-crossing event. Analysis of caustic-crossing events has always been a major challenge and active research in theoretical study of microlensing (e.g. [101, 175, 121]). On the other hand, since microlensing events are unique and do not repeat themselves, the capability of real time observations are also of a great concern. Given the brevity of planetary deviation that may emerge at any time during a microlensing event, it is essential that the microlensing event has to be monitored continuously and the images have to be taken in cadences less than an hour. The satisfactory coverage of a light curve in real time cannot be achieved by telescopes at the same location because of the typical time span of a microlensing event longer than a day. Telescopes over the globe, therefore, have to be involved to obtain the time series of the flux measurements well covering the event for light curve modelling. For this purpose, the success in microlensing observations has always been the result of collective efforts, involving telescopes of professional microlensing survey groups such as OGLE and MOA, and the world-wide network of follow-up telescopes, often operated by amateur astronomer groups, for example, the Microlensing Follow-Up Network (MicroFUN) [78] in New Zealand.
The microlensing method, among all available planet detection methods, is bizarre and unique since it does not rely on the detection of light from the planets or their host stars as transit, radial velocity and imaging methods. It can, therefore, be applied to detect planets around every type of stars in principle and is the only method which is capable of finding free-floating planets which are not orbiting around any star. In addition, the microlensing method is particularly sensitive to planets located beyond the snow line, where massive planets are expected to form initially, which are usually out of reach by other methods [183]. For these reasons, the microlensing survey for planets is of great value in order to provide a complete picture of planet populations, test the theories of and understand planetary formation. So far there are over 50 confirmed exoplanets [204], including the circumbinary planet in OGLE-2007-BLG-349 [20], as well as a lot of free-floating planets [12] discovered via microlensing. In the future, the space microlensing survey by the Wide Field InfraRed Survey Telescope (WFIRST), which is planned to be launched in 2020, will likely be able to detect thousands of planets beyond the habitable zone, completing the statistical census of exoplanets with the achievement of the transit surveys by the Kepler space telescope and its future successors [72].
Appendix B

Solving Kepler’s Equation

To determine the true anomaly of a body in an eccentric orbit, given that the mean anomaly $M$ is known, we have to first calculate the eccentric anomaly $E$ by solving Kepler’s equation

$$M = E - e \sin E,$$  \hspace{1cm}  (B.1)

which is a transcendental equation so that it cannot be solved for $E$ algebraically. We, therefore, applied Halley’s method, an iterative method, to calculate $E$ from eq. (B.1). We first rewrite eq. (B.1) as

$$0 = E - e \sin E - M,$$  \hspace{1cm}  (B.2)

and then define a function $f(x)$ as

$$f(x) := x - e \sin x - M.$$  \hspace{1cm}  (B.3)

Comparing eq. (B.2) with eq. (B.3), it can be seen that the root of eq. (B.3) represents the solution of Kepler’s equation for a given value of $M$, which can be computed using the iteration equation

$$x_{i+1} = x_i + \Delta x_i,$$  \hspace{1cm}  (B.4)

with the correction term, $\Delta x_i$, equal to

$$\Delta x_i = -\frac{ff'}{(f')^2 - \frac{1}{2}ff''}.$$  \hspace{1cm}  (B.5)

The advantage of Halley’s method is that its convergence region is large with regard to eccentricity $e$. Also, it does not depend too much on the initial value $x_0$. In our
calculations, therefore, the upper boundary of $e$ could be set to be as large as 0.999 and we always set $x_0 = M - e \sin M$. 
Appendix C

Eclipse Time Variation Curves of MOA Triples
Fig. C.1 ETV curves of all the other 73 MOA triple candidates.
Fig. C.1 (continued)
Fig. C.1 (continued)
Fig. C.1 (continued)
Eclipse Time Variation Curves of MOA Triples

Fig. C.1 (continued)
References


lations on the time scale of the orbital period of the perturber. Astronomy and Astrophysics, 528:A53.


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