Integrated Technology
In The Undergraduate
Mathematics Curriculum:
A Case Study of Computer Algebra Systems

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degree of Doctor of Philosophy in Mathematics Education,
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The effective integration of technology into the teaching and learning of mathematics remains one of the critical challenges facing tertiary mathematics, which has traditionally been slow to respond to technological innovation. This thesis reveals that the term integration is widely used in the literature with respect to technology and the curriculum, although its meaning can vary substantially, and furthermore, the term is seldom well defined. A review of the literature provides the basis for a survey of undergraduate mathematics educators, to determine their use of technology, their views of what an Integrated Technology Mathematics Curriculum (ITMC) may resemble, and how it may be achieved. Responses to this survey, and factors identified in the literature, are used to construct a taxonomy of integrated technology. The taxonomy identifies six defining characteristics of an ITMC, each with a number of associated elements. A visual model using radar diagrams is developed to compare courses against the taxonomy, and to identify aspects needing attention in individual courses.

Evidence from an observational study of initiatives to introduce Computer Algebra Systems into undergraduate mathematics courses at The University of Auckland, firstly using CAS-calculators and latterly computer software, is examined against the taxonomy. A number of critical issues influencing the integration of these technologies are identified. These include mandating technology use in official departmental policy, attention to congruency and fairness in assessment, re-evaluating the value of topics in the curriculum, re-establishing the goals of undergraduate courses, and developing the pedagogical technical knowledge of teaching staff.

The thesis concludes that effective integration of technology in undergraduate mathematics requires a recognition of, and comprehensive attention to, the interdependence of the taxonomy components. An integrated, holistic approach, which aims for curricular congruency across all elements of the taxonomy, provides the basis for a more consistent, effective and sustainable ITMC.
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