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Integrated Technology
In The Undergraduate
Mathematics Curriculum:
A Case Study of Computer Algebra Systems

Greg Oates

A thesis submitted in fulfilment of the requirements for the
degree of Doctor of Philosophy in Mathematics Education,
The University of Auckland, 2009
The effective integration of technology into the teaching and learning of mathematics remains one of the critical challenges facing tertiary mathematics, which has traditionally been slow to respond to technological innovation. This thesis reveals that the term *integration* is widely used in the literature with respect to technology and the curriculum, although its meaning can vary substantially, and furthermore, the term is seldom well defined. A review of the literature provides the basis for a survey of undergraduate mathematics educators, to determine their use of technology, their views of what an *Integrated Technology Mathematics Curriculum* (ITMC) may resemble, and how it may be achieved. Responses to this survey, and factors identified in the literature, are used to construct a taxonomy of integrated technology. The taxonomy identifies six defining characteristics of an ITMC, each with a number of associated elements. A visual model using radar diagrams is developed to compare courses against the taxonomy, and to identify aspects needing attention in individual courses.

Evidence from an observational study of initiatives to introduce Computer Algebra Systems into undergraduate mathematics courses at The University of Auckland, firstly using CAS-calculators and latterly computer software, is examined against the taxonomy. A number of critical issues influencing the integration of these technologies are identified. These include mandating technology use in official departmental policy, attention to congruency and fairness in assessment, re-evaluating the value of topics in the curriculum, re-establishing the goals of undergraduate courses, and developing the pedagogical technical knowledge of teaching staff.

The thesis concludes that effective integration of technology in undergraduate mathematics requires a recognition of, and comprehensive attention to, the interdependence of the taxonomy components. An integrated, holistic approach, which aims for curricular congruency across all elements of the taxonomy, provides the basis for a more consistent, effective and sustainable ITMC.
This thesis is dedicated to the memory, love and support of my mum and dad, Jenny and Stuart, to whom I made a promise to finish. Thanks for everything.
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“Computers are useless. They only give you answers.”

(Picasso, in Eisen, 2003)

1.1 THESIS BACKGROUND

Although it is unclear in exactly what context he was referring to the use of computers, the view of computers espoused by Picasso in the above quote is highly indicative of arguments on one side of the debate over use of technology in mathematics education. There has long been a strong divide between opponents and advocates of technology in mathematics education. Indeed, technology issues were often central to what has been labelled the “maths wars” in the United States. Oates and Thomas (2001) illustrate the often highly emotive nature of this debate using comments from two opposing articles that appeared in The Mathematical Intelligencer in 1996. In the first of these articles, Koblitz (1996, p. 10) argues that “…computers should not be a major component in math education reform”, describing the movement towards technology as an example of gimmickry, labelling it as computermania. He condemns much of the effort to introduce technology in the classroom as being profit-driven, stating that “…the intrinsic value of a pedagogical idea is not considered as important as its saleability…The technology-in-education movement has some of the characteristics of a religious evangelical campaign, fuelled by corporate and foundation money” (Koblitz, 1996, p. 14). In their response, Dubinsky and Noss (1996) counter much of Koblitz’s arguments as being “simplistic”, noting that there is a growing body of literature describing evidence of (computer) effectiveness. They generally regard Koblitz’s arguments as unhelpful to the technology debate, stating that what is really needed, now that “…the initial flush of enthusiasm over ‘new technologies’ is beginning to pass,…is a dispassionate, well-informed examination of costs, benefits,
Chapter One

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and difficulties” (Dubinsky & Noss, 1996, p. 17). In support of Dubinsky and Noss (1996), Oates and Thomas (2001) observe that there is now a substantial and expanding body of research that supports the use of technology in mathematics education, and although the great majority of studies into technology are still focused at the secondary level, there is an increase in research at the tertiary level demonstrating that technology may be of substantial assistance pedagogically, with particular benefits in developing students’ understanding of concepts (e.g. see Hong & Thomas, 1997, 1998; Hollar & Norwood, 1999; Cretchley, Harman, Ellerton & Fogarty, 1999, 2000; Galbraith & Pemberton, 2002; Bradley, Kemp & Kissane, 2003; Stewart & Thomas, 2003; Fay & Joubert, 2005).

Koblitz’s arguments carry little weight with curriculum developers and educators at the primary and secondary educational level, where support for the use of graphics calculators and Computer Algebra System-calculators (CAS) in mathematics teaching has been growing since the early 1990’s in several countries. In New Zealand primary and secondary schools, the use of technology is prescribed in the National Curriculum (Ministry of Education, 1992), and graphics calculators have been permitted in Year 13 (final year of secondary schooling) national external examinations since that time (a policy for CAS usage was still being developed in 2007). The New Zealand Ministry of Education states that technology such as calculators should be an integral part of the learning process, and the following quote illustrates this philosophy on calculator use:

This curriculum statement assumes that calculators and computers will be available and used in the teaching and learning of mathematics at all levels. . . Graphic calculators, and computer software such as graphing packages and spreadsheets are tools which enable students to concentrate on mathematical ideas rather than on routine mechanical manipulation, which often intrudes on the real point of particular learning situations . . . (They) are learning tools which students can use to discover and reinforce new ideas. (Ministry of Education, 1992, p. 14)

This curriculum statement clearly puts a strong emphasis on the use of calculators in all learning situations, although it is vague about how such an integral state of technology use should be accomplished. Many countries have similar policies and some (eg Denmark; State of Victoria, Australia – See Stacey, McCrae, Chick et al., 2000) have more extensive policies where CAS calculators are actively incorporated into all aspects of the curriculum from classroom teaching to assessment.
However, at the tertiary level, it appears that the debate over the relative merits of the use of technology is by no means resolved, and it is usually a simple matter in many mathematics departments to find members of the mathematics teaching faculty who sympathise strongly with Koblitz’s (1996) views. Indeed, discussions by the researcher with tertiary mathematics teaching colleagues at such forums as MERGA conferences (Mathematics Education Group of Australasia) and DELTA symposia (for the teaching and learning of undergraduate mathematics), and comments in the mathematics education literature (e.g. *Mathematics Teacher*, Waits & Demana, 1992; Tucker & Leitzel, 1995; *Notices of The American Mathematical Society*, Ralston, 2004), all indicate that there is considerable variation between tertiary institutions nationally and internationally with respect to the acceptance and use of technology in the teaching and learning of mathematics at the tertiary level. It seems many mathematicians who regularly use technology themselves in their own research domains do not favour similar usage by their students. They appear to regard technology largely as a useful, manipulative tool, suitable for routine mechanical operations, repetitive number-crunching, and perhaps for accessing previously inaccessible content areas (such as chaos theory), but remain oblivious in their teaching to any seeming pedagogical or cognitive benefits it may provide for them and their students. Kissane (2000a) believes that one possible explanation for tertiary mathematicians’ reluctance to make use of graphics calculators in their teaching may be that:

Most college or university mathematicians have spent no time at all with a graphing calculator and are not inclined to spend the start-up time of an hour or two to learn, especially since they are unlikely to use graphing calculators on a regular basis outside the classroom. A bridge is needed for this gap between mathematics students (and secondary school teachers) on the one side and college faculty on the other. (Tucker, 1999 in Kissane, 2000a, p. 66)

Opposition to the use of technology is most evident in early undergraduate algebra and calculus courses, where some mathematicians believe, for example, that the use of calculators and/or computers may impair students’ development of basic and essential mathematical and cognitive skills. Applied courses (e.g. numerical methods, statistics) seem to be more supportive of technology, and indeed prescribe it in their courses, but even then, many share the scepticism described by Tucker above for super-calculators,
regarding them as sophisticated “toys”. Any attempt to introduce technology into tertiary courses must inevitably take place within this controversial environment, involving vigorous and often emotive debate as has been illustrated in this discussion.

At the start of this investigation in early 2000, such arguments were apparent in the various undergraduate mathematics courses at The University of Auckland, where the use of technology in first-year courses was extremely varied. First year statistics courses made extensive use of technology, in the delivery of the courses (power-point presentations, CD-ROM of coursework, computer-simulations in lectures etc), in tutorials (computer laboratories), and in the assessment, where part of the students’ grades come from computer-based projects, and the use of computer-software and graphics calculators is still actively encouraged. Perhaps the presence in the first-year teaching courses of several former secondary school teachers, as well as the widely accepted and required use of computer software in statistical research, have provided the bridge described earlier by Kissane (2000a). Students in the first-year applied mathematics course, MATHS 162 *Introduction to Applied and Computational Mathematics* also spend a significant part of their time in computer laboratories, using MATLAB with, for example, numerical methods, mathematical modelling, and differential equation solving. MATLAB is a commercially available and commonly used software package that combines CAS-capability with powerful mathematical analysis tools. However, this MATLAB usage is limited to assignments, and does not extend to lectures or formative assessment measures.

Until recently though, the use in any of the first-year *pure* mathematics courses offered at The University of Auckland was much more limited, especially in the principal linear algebra and calculus courses. Prior to 1997, the use of technology in any course was restricted to the use of simple scientific calculators, but even these were not permitted in tests and examinations in the principal courses. The process of implementation of technology started with the introduction of graphics calculators into the bridging mathematics course *Mathematics Two: Functioning in Mathematics* (see Barton & Oates, 1997). The later extension of CAS-calculators into the principal courses is elaborated on in the next section, when the background to the thesis is described, and is investigated again in more depth in later chapters (see also Oates & Thomas, 2002). However, it is worth noting here that restrictions on technology use were not limited to mathematics. With respect to calculators, the university-wide policy
Chapter One

Background and Overview

On the use of calculators in examinations prohibited the use of any calculator with alpha-numeric capability until 1999, when representatives of the mathematics and statistics departments instigated a change in the policy to permit the use of graphics calculators. The policy currently still prohibits the use of devices with “qwerty” keyboards, thus denying the use of such CAS-machines as the TI-92. There are many reports in the literature describing other institutions’ attempts to integrate technology into their courses that in many ways resemble the Auckland experiences (e.g. see Pence, 1996; Schwingendorf, McCabe & Kuhn, 2000; Balas, Goulet & Smith, 2002).

Even if arguments against the use of technology such as those described in the previous discussion are overcome, there remains many other obstacles to the successful integration of technology into mathematics teaching and learning. These issues will be examined in the review of the literature in Chapter Two and Three. Kissane (2000b, p. 3) provides a useful conclusion to this controversial debate, when he suggests that:

Rather than insist that the traditional skills are necessarily of permanent importance to all pupils for all time, we might use such observations to help us decide which things really are important, and be able to argue why they are important. We will need a better argument than the one making vague reference to needing to know how to do something at some later time…or not really understanding until you can complete the procedure the traditional way. Frankly, I find none of these arguments convincing, and suspect that many pupils find them not so persuasive either.

What should happen once a department, faculty, or tertiary institution decides to use technology in their courses? There are many decisions required with respect to the resulting curricula. For example, will technology be permitted, expected, or prohibited in assessment, and in which forms of assessment, for example examinations, assignments, tutorials? Which type of technology will be used; computer software packages, graphing calculators, CAS-calculators? What access to the technology do we require or expect of students? Will any changes be made to the content of the course, or the order in which the topics are taught? What allowances need to be made for training both teaching staff and students in the use of the technology? Are there any special attributes of the technology that need to be considered with respect to helping or hindering students’ learning? These and other factors are widely identified in the literature as having significant impact on the implementation of technology into mathematics teaching and learning. Oates (2004a) suggests that a consideration of these
factors is equivalent to an analysis of the degree to which technology is subsequently integrated into the resultant curriculum. However, most studies that examine such factors largely consider the individual factors in isolation (e.g. assessment issues in studies by Boyd & Cutler Ross, 1996; Hong, Thomas & Kiernan, 2000), and seldom attempt a holistic evaluation that sees technology use examined from a truly integrated perspective. With respect to graphics calculators in particular, Penglase and Arnold (1996, p. 79) concluded that very few studies actually investigated curriculum design and development, noting that:

…the majority of those studies which purported to investigate curriculum issues…have been seen to be instead studies of pedagogy, as the use of the tool remained intertwined with the effect of the learning context. Of more benefit, then, may be those studies, which directly attempt to address the issues of graphics calculator use within particular learning environments.

Oates (2004a) further observes that although there are many studies which claim to report on the integration of technology, the term integrated is used in a wide variety of ways with respect to technology and requires further clarification.

The need to quantify the extent of technology use is also clearly indicated in the literature, where frequent references are made using such descriptions as integrated technology (e.g. see Kissane, 2000a; Kawski, 2003), but seldom are such terms clearly defined. Indeed, most often the ensuing description is more about how technology has been simply incorporated into the existing curriculum as an additional pedagogical tool, with little subsequent change to the overall curriculum. It seems the term integrated is most often used to evoke ideas that are expected to be universally understood at some vague intuitive level, without attempting to rigorously define what is meant by the term. (Oates, 2004a, p. 282)

It is issues such as those outlined in this introduction, relating to the implementation of technology into the teaching and learning of mathematics, and the relationship to curriculum development, that this thesis aims to address. The next section, 1.2 outlines the background of the thesis more fully, in particular how the researcher became involved in the use of technology in teaching at The University of Auckland, and how this involvement led to his interest in this thesis.
1.2 A PERSONAL PERSPECTIVE

Becker (1998) notes that qualitative research has considerable potential for developing new understandings of mathematics. He cautions, however, that there are problematic ethical and personal judgment issues that might not be evident in quantitative research. Consequently, a researcher must declare their own perspective "so that readers of their research can judge their conclusions in that light" (Becker, 1998, p. 255). Given that much of this thesis is of a qualitative nature, this section will provide a brief biography of the researcher, to place the thesis in its appropriate context, in order to facilitate this judgment.

In 1993, after some ten years experience as a secondary and intermediate school mathematics teacher, I returned to university study at The University of Auckland to pursue an interest in higher education, enrolling in a Masters of Education Degree in mathematics education. This period coincided with the implementation of the new national curriculum in mathematics released by the Ministry of Education the previous year (see Ministry of Education, 1992). I therefore had only a brief experience of the new technology objectives outlined in this curriculum document, before my return to university. I was tremendously excited by the potential for new teaching and learning opportunities that the whole curriculum offered, and in particular the chance to now legitimately use the graphics calculators that we had been experimenting with for a few years in the department of mathematics where I was then teaching.

During my early Masters’ studies, I became involved in the teaching of the foundation calculus course Mathematics Two mentioned earlier in section 1.1, and on the completion in 1997 of my thesis investigation into the use of collaborative learning in tertiary mathematics, I was invited to remain in the department, sharing responsibility for coordinating and teaching the Mathematics Two course. My involvement with the introduction of graphics calculators to Mathematics Two in 1997 sparked the initial interest in the current investigation. Further impetus was provided by the Department’s decision to introduce CAS-calculators into the principal first-year pure mathematics course, MATHS 151, in semester one 2001. My personal position on technology is highly relevant to this thesis as an enthusiastic supporter of technology within the department. This enthusiasm was based largely on my experiences with graphics calculators in Mathematics Two, and the positive reinforcement for this initiative gained
from questioning students and teaching staff involved with the course (see Barton & Oates, 1997). My interactions at conferences and other education forums with colleagues involved in teaching and technology research, and my personal reading in the mathematics and technology field also contributed to my interest.

Given my experiences in Mathematics Two, and my enthusiasm for the positive potential of CAS-calculators to greatly aid student learning, I was a key motivator in the initiative to implement CAS-calculators in MATHS 151, even though I was not personally teaching the course. Additional motivation to become personally involved was provided by my departmental role as Director of The Mathematics Learning Centre, where there was clearly an intersection between the provision of student support and assistance, and the proposed changes in the course. I was keen to make sure the implementation was as supportive of students and teaching staff as possible, and to improve on our mixed experiences in introducing graphics calculators into Mathematics Two. An obvious example of this was the need to ensure tutors working in the small class tutorials were familiar with the technology, and able to assist students in its use. Despite great enthusiasm on behalf of several members of the MATHS 151 teaching team, and support from colleagues with extensive experience in research and teaching with technology, the evidence is that the initial implementation of technology was not entirely successful (Oates & Thomas, 2002). Recognition of this can be seen in the Department’s establishment in 2004 of a technology committee, charged with developing a coordinated policy for technology use. From a personal perspective, there was a sense of frustration and disappointment with the pace of progress and the equivocal results of the implementation. Identifying the factors influencing effective progress, and facilitating more successful outcomes in technology implementation can be seen as the main motivating forces behind this investigation.

This section has described my involvement in the teaching and introduction of technology to first year mathematics courses at The University of Auckland. It has provided a personal perspective on my experiences with technology, outlined my personal motivations for the thesis, and described the context within which the current investigation is conducted. The implementation of technology in Mathematics Two and MATHS 151 is elaborated on in the next section, to explain the rationale for, and significance of the thesis.
1.3 RATIONALE AND SIGNIFICANCE

Section 1.1 cited Penglase and Arnold (1996), who concluded that there was a need for more studies investigating curriculum issues and technology. Similar calls for curriculum change are widespread in the mathematics education literature, including eminent theorists (e.g. Kaput & Roschelle, 1999; Artigue, Hillel, Holton & Schoenfeld, 2002), avid proponents of technology (e.g. Arnold, 1998; Kissane, 2000a), national and international educational authorities (e.g. National Council of Teachers of Mathematics, 2000; Leigh-Lancaster, 2002), and university teachers (e.g. Pemberton, 2002; Kawski, 2003). For example, the revision of the NCTM Standards highlighted technology in a special ‘technology principle’ which stated:

Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances student learning…Students can learn more mathematics more deeply with the appropriate use of technology…In mathematics-instruction programs, technology should be used widely and responsibly, with the goal of enriching students’ learning of mathematics. The existence, versatility and power of technology make it possible and necessary to re-examine what mathematics students should learn as well as how best they can learn it.

(National Council of Teachers of Mathematics, 2000, pp. 24-25)

At secondary school level, Kissane (2000a) argues that difficult curriculum decisions now involve deciding under which circumstances the new facilities (graphics and CAS-capability) should be ignored, whether or not they should be banned, and whether (and when) we should help pupils to use them. He supports the call for general curriculum change with respect to technology when he states that:

As Kennedy (1995) has argued so eloquently, a good deal of what is in the secondary school curriculum used to be necessary for any pupils who wished to progress further in mathematics. But is less clear now that this is the case, and the time has come for a careful re-evaluation of what we have come to take for granted. … It seems inevitable that a reconsideration of what is important and central about mathematics ought to take place as technologies such as calculators become available. It is a reflection of the inertia and the natural conservatism of school curricula that they seem to be so resistant to change of this kind.

(Kissane, 2000a, pp. 64-65)
Chapter One

Background and Overview

Given the expanding number of studies into technology and mathematics education referred to by Oates and Thomas (2002), it seems reasonable to expect that the call for more investigation into technology and curriculum change by Penglase and Arnold (1996) may have been suitably or at least partially addressed by now. However, one can see from the dates of the above examples (National Council of Teachers of Mathematics Standards, 2000; Kissane, 2000a) that many educationalists still saw the same need some four years later, and indeed, Arnold (2004, p. 21) reinforces this another four years further on, when he states that:

Even today, there remain widespread reservations regarding not only the best ways to incorporate such tools into teaching and learning, but fundamental questions regarding their appropriateness…The questions which practitioners have been asking (What will be left to teach if students have access to tools which factorise, solve, and do calculus? What about their manipulative skills? What will we ask them to do in examinations?) were the same questions asked a few years ago regarding graphing calculators. In fact, they were precisely the same questions asked twenty years ago regarding student access to traditional calculators.

This view is confirmed in the introductory discussion document of the seventeenth ICMI study Digital technologies and mathematics teaching and learning: Rethinking the terrain (Hoyles & Lagrange, 2005). This is the second study to focus on the use of computers in mathematics education, following the first ever ICMI study some twenty years earlier in 1985. While some aspects discussed at the first ICMI have moved quickly (e.g. development of technological tools and diversity of use), others have remained static, including little evidence of any significant impact on the mathematics curriculum of secondary schools and universities. It is clear the co-chairs of the 2005 study still see a need for critical reflection, identifying seven different themes to be addressed by the study, including “designing learning environments and curricula” and “implementation in curricula and in classrooms” (Hoyles & Lagrange, 2005, p. 5).

While the discussion so far has suggested the need for more studies into technology and curriculum change in general, the need is even more evident at the tertiary level, where few studies exist. Anthony and Walshaw (2004), Wood (2004), and Arnold (2004), all remark on the general lack of studies at the tertiary level in several major research forums. In the 43 studies reported on in the extensive worldwide review
of handheld-graphing technology commissioned by Texas Instruments in 2002, “the focus was almost entirely focused at years 10-12” (Arnold, 2004, p. 20). Anthony and Walshaw (2004) note that tertiary studies make up only a small proportion of all Mathematics Education Research Group of Australasia (MERGA, Australia and New Zealand) studies, and that this figure is diminishing (from nearly 18% in 1994 to 5% in 2003). They cite Wood (2004) to support the claim that this trend is an enduring one (Anthony & Walshaw, 2004, p. 8). This is further compounded if studies specific to technology and curriculum are considered. In the 2003 MERGA proceedings, studies of technology and curriculum at any level were under-represented (10% and 5% of all studies respectively, Anthony & Walshaw, 2004, p. 5). A similar position exists in the 2007 Psychology of Mathematics Education Proceedings, and the situation is only marginally better in forums that focus specifically on the tertiary sector, for example the biannual Delta Southern Hemisphere symposium gathering together many international tertiary mathematicians and mathematics educators. An inspection of the Proceedings of the Delta ’03 symposium on the teaching and learning of undergraduate mathematics reveals that although technology studies are better represented than in the 2003 MERGA proceedings (11 of 57 articles are directly associated with technology), only three of these make reference to curriculum issues, and none of these has curriculum as its primary focus (see Thomas & Oates, 2003; Barton, Oates, Reilly & Thomas, 2003).

Specific technology forums obviously present a greater number of studies with a technology and curriculum focus. For mathematics education, these include the TIME 2000 conference at The University of Auckland (Technology in Mathematics Education), and ATCM (Asian Technology Conference in Mathematics). Those with a specific tertiary focus include ICTME (International Conference on the Teaching of Mathematics at the Undergraduate Level), ICTCM (Annual International Conference on Technology in Collegiate Mathematics), and the series of studies Research in Collegiate Mathematics Education published by The American Mathematical Society. Some studies from these forums do examine over-arching issues of technology and curriculum change (e.g. Kissane, 2000a; Stacey, Asp & McCrae, 2000; Bloom, Forster & Mueller, 2001; Butler, Cnop, Isoda et al., 2003). However, the majority fall into the category of reports on efforts to integrate technology into individual programmes or specific content areas (e.g. Pence, 1996; Brown, 2001; Hong & Thomas, 2001). These studies also demonstrate the large variation in the extent and ways in which technology is used in
different courses, as described in the introduction to this chapter. This variation supports
the need to establish an appropriate model to compare the use of technology between
courses. There is also considerable variation in the degree of success of the technology
implementations reported in these studies. Carlson (1996), for example, reports on a
successful transition to a calculator integrated college algebra curriculum at the
University of Kansas, where graphing technology has been fully integrated into the
curriculum for all sections of college and intermediate algebra. Other studies report
mixed experiences, where the results of implementation are sometimes less than
anticipated (e.g. see examples in Roberts, 1996; Ganter, 1999).

Even within institutions and individual mathematics departments, the extent of
technology integration and the success of the implementation may vary greatly. The
Mathematics Two course at The University of Auckland incorporated the use of
graphing calculators into the existing curriculum, with only cosmetic changes to the
content of the course, and little change to the assessment other than to remove examples
from the tests and examinations that were obviously made trivial by the technology. Use
of the graphics calculators (predominantly TI 83’s) consisted mainly in their use by
instructors in lecture demonstrations, although students were permitted and actively
courage to use them in all class activities and course assessment including final
examinations. As Barton and Oates (1997) report, the take-up rate for students
purchasing calculators was not great (10-20%), but those students with the calculators
largely reported positive experiences, and were grateful that they had access to the
technology.

By way of contrast, the MATHS 151 CAS-calculator implementation hoped to
achieve a greater level of integration, especially given the extensive overhaul of the
entire undergraduate mathematics programme in 2003. The restructuring included major
changes to the principal first-year course curricula, with the specific inclusion of CAS-
calculator technology. A survey following the first semester of use in 2001 elicited
largely favourable responses from students towards the TI-89’s chosen for the course
(see Oates & Thomas, 2002). However, several years afterwards, the level of
technology use had actually tapered off. The calculators were no longer used by most
teaching staff in both the principal first-year courses, and few students bothered to
obtain a CAS-calculator. The reasons for these observed effects are clearly complex,
and an investigation of these is one of the motivations for this current investigation.
The preceding discussions clearly indicate a pressing need for further investigation into technology and curriculum change, especially at the tertiary level. It is suggested that we should identify the factors that affect the implementation of technology, successful or not, and that we need a means of assessing the level of technology use in different courses to help us identify and quantify these factors. There is much support for such investigations in the literature, implying that a framework for describing, measuring and comparing technology use in mathematics courses would make a significant contribution to the tertiary mathematics education domain. It would allow tertiary teachers to examine their current curricula from a technology perspective, compare existing practice, and ultimately adapt their courses to better reflect the potential pedagogical advantages offered by technological advances. The next section defines the scope of this thesis, and frames specific research questions which the thesis aims to address.

1.4 THE RESEARCH FOCUS: QUESTIONS AND ASSUMPTIONS

The preceding discussions have indicated several areas of interest that this thesis seeks to investigate. There is widespread use of terminology such as integrated technology in the literature to describe the ways in which technology has been incorporated into the teaching and learning of mathematics. Many researchers and studies imply that an ideal learning environment in this technological age is one where technology is fully integrated into the curriculum. Two issues of interest stem directly from this implication, firstly the type of technology usage that this describes varies greatly, and there is no evidence in the literature of any rigorous definition of what is meant by integrated technology. It is therefore difficult to know what individual studies mean when they refer to attempts to integrate technology into courses, or make claims that such initiatives have been successful. To what extent they succeeded surely depends greatly on what degree of usage they were seeking. It is nearly impossible given this ambiguity over the definition of integrated technology to make any useful judgements or comparisons between different courses. There is a need to identify the various elements that constitute an integrated technology course, and hence the first question that this thesis will address is what might an integrated technology mathematics curriculum resemble (ITMC).
Once a framework for what is meant by an ITMC has been established, the immediate issue that follows from this is to ask on what basis the claim for integrated technology is made. While there are many studies that show positive cognitive and pedagogical benefits for the use of technology in certain contexts, it is rather preemptive to assume that an ITMC may provide an effective learning environment, especially given that it has yet to be defined. Nevertheless, there is a considerable body of supporting literature for the potential learning benefits of technology, and a survey of these studies will be conducted, to justify an integrated technology mathematics curriculum as a favourable learning environment.

The next issue of interest raised in the introductory discussions is how we might successfully implement technology into the tertiary mathematics curriculum to a satisfactory degree of integration. The technology implementations at The University of Auckland in their first-year courses have been briefly described, and it has been suggested that the difficulties they experienced are quite common. Certainly the introductory discussion has suggested there are many factors to be considered in the implementation process if the implementation of technology is to be ultimately successful. This thesis aims to identify these factors, and provide a framework for successfully integrating technology into mathematics curricula.

The key research questions this thesis aims to investigate are thus:

1. What are the characteristics of a tertiary Integrated Technology Mathematics Curriculum, and how might we measure the nature of such technology integration?

2. How can we facilitate the effective and sustainable implementation of a tertiary Integrated Technology Mathematics Curriculum?

The first question assumes that an ITMC is indeed what curriculum designers and developers should be aiming to achieve, and the literature reviews in Chapters Two and Three will look to support this assumption. Any instances where this assumption may be questioned will be clearly identified in the findings. The second key research question follows the identification of the elements of an Integrated Technology Mathematics Curriculum resulting from the first question.

A third question, “What happens to student learning as a result of an Integrated Technology Mathematics Curriculum?” is clearly of considerable interest, investigation
of this aspect depends on the first two questions framed above, and lies outside the scope of the current investigation.

The two key research questions for this thesis will be principally focused on first-year undergraduate algebra and calculus courses, using computer algebra systems (CAS), either computer software, or calculators. The investigation will undoubtedly gather information from other sources, for example applied mathematics courses (statistics, numerical analysis, engineering), higher level mathematics courses, and other forms of technology, such as the provision of courses via the internet. Information received in these additional domains will be included and used to help inform the findings where appropriate. However, the principal focus of the thesis will be as described. The next section summarises the discussions in this chapter, and provides an overall framework for the structure of the thesis.

1.5 CHAPTER SUMMARY AND STRUCTURE OF THE THESIS

This chapter has provided an introduction to this thesis, with a description of the researcher’s personal interest in the investigation, and a brief history of the use of technology in first year mathematics courses at The University of Auckland that provided the impetus for the thesis. Section 1.3 detailed the rationale for the thesis, as providing a significant contribution to the critical issue of the implementation of technology into mathematics curricula at the tertiary level. The underlying assumptions of the thesis, and the research questions the thesis aims to answer were detailed in section 1.4. Six chapters follow this introduction. The next two chapters survey the literature to provide a theoretical basis for the thesis. Studies considered in this review of the literature are largely to the end of 2006, when the principal part of this process was concluded. Some papers from significant forums in the technology and tertiary mathematics fields in 2007 have been considered at a later stage, to ensure that the thesis is grounded in the literature as recently as possible.

Chapter Two: The Curriculum and Technology reviews the literature on curriculum studies, both generally, and from a technological perspective. A definition of the way in which the term ‘curriculum’ is used in this thesis is provided, since as Begg (1998) suggests, the term is used in a widely differing number of ways by politicians, researchers and educators alike, ranging from the macro-level (national and institutional
policy statements), to the micro-level (content-based individual class lesson structures). Theoretical issues of curriculum design and development, and their relationship to technological developments, are examined, and some possible models of curriculum development for use within this thesis are considered. Teachers’ beliefs about the nature of mathematics and knowledge and technology on curriculum design and pedagogy are examined, and a possible model for the goals of mathematics education within a technological environment is proposed. Issues of implementation and the wide varieties of ways in which integration is used in the literature are identified. Finally, this chapter examines factors of curriculum change and professional development, to identify those factors that may assist or impede effective development of an ITMC.

Chapter Three: Theoretical Issues in Technology examines studies that consider the theoretical basis for the use of technology in mathematics education. The chapter is framed posing three leading questions: Why use technology? Which technology should be used? How might that technology be implemented? The first question considers the place of technology in mathematics education in the context of wider societal expectations about mathematics and technology, and examines the relationship between technology and traditional theories of learning and teaching. This section concludes by looking at studies that examine evidence for the potential pedagogical and cognitive benefits, to provide support for the underlying assumption of this thesis, that an ITMC is indeed an effective learning environment to which curriculum developers should aspire. The second question, ‘Which technology’, focuses on issues involved in choosing a particular form of technology, including the choice between graphics and/or CAS-calculators, compared to computer software packages. The final question, ‘How technology’, considers studies and reports on efforts to implement technology into mathematics teaching, principally at the tertiary level. The varying degrees of integration, and the subsequent levels of success of the implementation, are both considered in this part of the discussion.

Chapter Four: Research Design details the design of the investigation, justifies the procedures and instruments adopted in this thesis, and provides measures of trustworthiness to evaluate and justify the findings. There is a strong theoretical basis, and an empirical dimension to this thesis. The purpose of the data collection was to both inform, and provide a check on the emerging theoretical model proposed for an integrated technology mathematics curriculum. The models and implications presented
in the findings chapters were produced both from the theory as established in the review of the literature, and the collected data.

Chapters Five and Six collectively present and interpret the results of this investigation, with each chapter building on the results of previous chapters in the formulation of instruments and subsequent discussion. The structure of these two chapters thus reflects the iterative process of qualitative inquiry: a constant matching of data and literature until a satisfactory explanation of both is achieved (Keeves & Sowden, 1997). Chapter Five: A Model for Integrated Technology details the progression of surveys principally used to address the first research question, “What would a tertiary Integrated Technology Mathematics Curriculum (ITMC) look like?” It identifies the defining elements of an ITMC, and develops successive models for measuring the level of integration of technology within tertiary mathematics courses. A taxonomy of technology use is proposed, which is then used to examine technology use in selected courses.

Chapter Six: An Observational Study of Technology Implementation documents the experiences of the Department of Mathematics at The University of Auckland, in its efforts to implement new technologies into its principal undergraduate mathematics courses. The implementation process is compared against the taxonomy developed in the previous chapter. In addition, sample transcripts from interviews with representative staff from the observational study, and some survey respondents, are provided. These results, along with responses to questions in the third survey specifically examining difficulties with the use of technology, are used to identify significant features affecting the effective integration of technology. The chapter concludes with a summary of the key integration issues facing those charged with technology implementation, in order to facilitate questions about pedagogical intentions, mathematical objectives, specific technology requirements and other curricula-related phenomena. The objective is to move more easily towards a successful ITMC.

Chapter Seven: Review and Implications summarises the findings of the thesis, and connects these with the themes described in the review of the literature. The significance of the models developed in the thesis is emphasised, and the implications for future technology implementation are considered. The discussion concludes with suggestions for future areas of investigation.
“We might say that the mathematical curriculum is invariant under intellectual revolutions.”

(Steen, 1987, p. 7)

2.1 INTRODUCTION

The introductory chapter described the belief that revision of tertiary mathematics curricula is urgently required, with world-wide calls by many researchers and university educators for curriculum change, in order to reflect technological advances spanning a large number of years. Fey (1989b), for example, suggested that revision of curricula and teaching methods, to take advantage of electronic information technology, was one of the most important tasks in mathematics education today. However, as the quote from Steen (1987) that prefaces this chapter alludes, the impact of technology on mathematics curricula has historically been somewhat minimal. Early “Calculus Reform” discussions (Fey & Heid, 1984; Fey, 1989b; Steen, 1987) made it clear that the goal of calculus reform was to revise the content of calculus, to make it more relevant to applications and contemporary mathematics, and to reflect new technology. Despite such goals, Steen (1987) points out that the previous 25 years of the computer revolution have led to no change in the mathematics curriculum at the university level. Although there are more recent examples where some change has occurred (see for example Tucker & Leitzel, 1995; Roberts, 1996; Ganter, 1999; Ganter & Jiroutec, 2000), the overall impact of technology on the curriculum has continued to fall short of the expectations of early reformers, as can be seen by frequent, continual calls for change in subsequent years (e.g. see Dugdale, Thompson, Harvey et al., 1995; Waits & Demana, 1992; Noss, 1998; Hillel, 2001a; Kaput & Roschelle, 1999; Steen, 2001; Harman, 2003).
The historical extent of this debate is evidenced in the similar wording of many reports from 1982 through to 2005. In 1982, the Cockcroft Report stated that:

...there are two fundamental matters which need to be considered. The first concerns the ways in which calculators and microcomputers can be used to assist and improve the teaching of mathematics in the classroom. The second concerns the extent to which the availability of (these) should change the content of what is taught or the relative stress which is placed on different topics within the mathematics syllabus.  (Cockcroft, 1982, paragraph 374)

Mann and Tall (1992) believed the questions posed by Cockcroft were still very relevant ten years later, and added a third aspect, suggesting we should question “not only what is taught but the way in which it is presented and developed in a whole range of classroom activities and experiences” (Mann & Tall, 1992, p. 111). A further thirteen years on, Ruthven (2005) clearly believes not much has changed when he argues that:

Advocacy of the educational use of new technologies often seems to suggest that their value is evident, their adoption urgent, their implementation unproblematic, and their impact transformative. Reviewing over eighty years of such claims, Larry Cuban (1989, 2001) has demonstrated that this is far from being the case; indeed, he detects a recurring cycle governing the evolving reception of each new technology in which exhilaration then credibility give way to disappointment then blame. He reports that while new technologies have broadened teachers’ instructional repertoires to a degree, they remain relatively marginal to classroom practice, and are rarely used for more than a fraction of the school week. Cuban suggests that disciplinary tradition, school organization and external regulation all encourage teachers to behave as academic specialists whose primary concern is with curriculum coverage; with these conditions unchanged, he predicts that technology will typically precipitate only a gradual accumulation of incremental adaptations largely sustaining existing classroom practices.  (Ruthven, 2005, p. 12)

The discussion document of the second ICMI study to investigate technology in mathematics education adds further support to this stance (Hoyles & Lagrange, 2005). The discussion document raises a large number of questions that it regards as still needing attention following the first ICMI technology study some twenty years earlier. These include:
• What theoretical frameworks and methodologies are helpful in understanding how design issues impact upon the teaching and learning of mathematics?
• How can technology-integrated learning environments be designed so as to influence and change curriculum, and how can this be achieved over time?
• How have mathematics curricula changed to reflect developments in mathematics afforded by digital technologies?

(Hoyles & Lagrange, 2005, pp. 9-10)

As a specific example of how technology may affect the curriculum, Kilpatrick and Davis (1993) consider the impact of technology in its role as a reorganiser, stating that:

The computer is not merely an amplifier of general curriculum issues specific to the mathematics curriculum...It changes certain fundamental questions one needs to consider in any attempt to revise the mathematics curriculum by making the subject matter itself more problematic. What is mathematics? What knowledge of mathematics does tomorrow’s society demand? What mathematics should this pupil learn so as to be a wise and humane citizen of that society?

(Kilpatrick & Davis, 1993, p. 204)

Leigh-Lancaster (2002, p. 24) adds weight to this assertion when he claims that, central to the responsibilities and work of curriculum and assessment authorities, is “a principled and coherent response to the natural questions of what mathematics? (selection from discipline and domain knowledge, theory and application); for whom? (subsets of the student cohort); how? (curriculum and assessment requirements and possible pedagogies); and why? (rationale and purpose)”.  

The issues identified in the introductory discussion to this chapter are examined from the various perspectives of general curriculum theory and design, mathematics pedagogy, beliefs about the nature of mathematics and mathematics teaching, and curriculum change and professional development. Connections to technology issues from each of these perspectives will be established throughout the discussion, but a more thorough examination of theoretical issues from a technological perspective will be conducted in Chapter 3. One significant problem identified in the introductory chapter was that the majority of studies in the areas of technology and curriculum are at the secondary level, as revealed by an examination of several extensive analyses of
major research publications (e.g. see Anthony, 2004; Arnold, 2004; Wood, 2004). The relevance to the tertiary level of several such studies at the secondary level will be considered throughout this discussion, and connections will be drawn to the relatively few studies identified at the tertiary level.

2.2 CURRICULUM THEORY, DESIGN AND DEVELOPMENT

2.2.1 A Proposed Definition of Curriculum

Most writers and researchers in the field of curriculum design and development in mathematics acknowledge that what is meant by curriculum depends largely on one’s point of view. Wong (2003, p. 278) notes that “there are a lot of definitions of ‘curriculum’, each carrying a different perspective, or even conception, of it.” In his exploration of regional curriculum development issues in New Zealand, Begg (1998) states that:

In talking about curriculum one needs to define one’s terms. (In this study) I was thinking of a regional curriculum document together with associated teacher guide material, any compulsory assessment initiatives, and any regulations that influence what teachers do. I acknowledge that there are many other levels of curriculum such as the school curriculum, lesson plans, and the implemented curriculum that all rely on the teacher as curriculum developer. (Begg, 1998, p. 98)

Kilpatrick and Davis (1993, p. 205) similarly describe the need to “locate the curriculum”, considering the different perspectives of curriculum held by officials, teachers and students. For the Minister of Education, “the curriculum may be embodied in an official document approved by the government and published by the ministry”; for the teacher in a school, the curriculum is more likely to be defined by “the textbook chosen for use in instruction, in the examinations their pupils will face, and in the lesson plans they draw up”. However, the curriculum the child experiences may be quite different from the official documents or the teacher’s plans. Kilpatrick and Davis acknowledge the student’s perspective of the curriculum, referring to it as the lived curriculum, “the set of experiences through which mathematics is actually learned” (Kilpatrick & Davis 1993, p. 205). Frymier (1986, p. 59) encompasses these various perspectives in a broad view of curriculum that “includes what is taught, how it is
taught, and why it is taught”. While the exact terminology used may differ, similar categorisations have been used in many studies to distinguish between these differing perspectives. Goodland (1979, in Wong, 2003, p. 278) identified the aspects of ideological, formal, perceived, operational, and experiential curricula, while Burkhardt, Fraser and Ridgway (1989, p. 408) describe six categories or labels: ideal (what experts propound), available (what teaching materials, methods can be used), adopted (as directed by state or institutional authorities), implemented (what teachers actually teach), achieved (what students actually learn), and tested curriculum (e.g. public test results etc). More recent studies often refer to the similar but narrower range of categorisations used by the IEA (International Association of the Evaluation of Educational Achievement) to distinguish three levels of curriculum, intended (reflects governmental, societal and cultural imperatives, e.g. goals, philosophy, content), implemented (reflects institutional and pedagogical imperatives, e.g. text-book availability and design, teacher training) and attained or achieved (individual student perspective, nature of assessment) curriculum (see for example the SIMS study, in Bishop, 1993, p. 227; Thomas & Holton, 2003).

A consistent theme emerging from all of the above discussion is the complexity involved in any discussion of curriculum. Hillel (2001a, p. 59) proffers one potential definition which considers curriculum in the widest sense to mean “matters pertaining to the purposes, goals and content of mathematics education,...as well as the means of achieving curricula goals.” This definition suggests a distinction between the concepts of the structure of the curriculum (e.g. its components such as content, teaching methods, assessment, and the underlying theories), and the process of producing the actual curriculum itself (e.g. curriculum research, consultation, writing, resource production, implementation). In this thesis, the terms curriculum design and curriculum development are used to distinguish between the two concepts of structure and process respectively.

An added complication for this current investigation is that the majority of the studies cited in the preceding discussion (with the exception of Thomas and Holton, 2003) are focused on school curricula. As Wong (2003) notes, curricula can play very different roles in different systems, for example in the variations between countries with respect to a national school curriculum. New Zealand has had a national curriculum for many years; the United States has never had one. In considering the aspects of
curriculum previously discussed, it seems immediately apparent that there may be significant differences between curricula at the school and tertiary levels. Despite attempts at consultative processes that seek teacher’s input (consider for example the considerable consultation that preceded the release of the 1992 New Zealand curriculum, see Department of Education, 1986), school teachers in reality have little control over political or societal decisions that influence curriculum documents, or the mathematical and educational theories that underpin such documents. Even at a local level, what and how they teach is often out of their control, determined to a large extent by such factors as departmental schemes, national assessment pre-requisites and moderation, and official inspections.

At least on the face of it, teachers at the tertiary level have a lot more autonomy with respect to curriculum design and development than do teachers in schools. Certainly they are subject to political and administrative influences from both external sources, and from within their institution. For example funding issues have been the subject of much pressure in many universities in recent years, and the discourse of the calculus reform movement has clearly affected curriculum reform in many mathematics departments in relation to the broader social, cultural, educational and political aims associated with this discourse (see for example Tucker & Leitzel, 1995; Roberts, 1996; Ganter, 1999). Lecturers, like school teachers, are also subject to departmental administrative procedures, expectations, and global course structures. However, outside of these influences, many tertiary teachers enjoy the opportunity to make their own decisions about many aspects of the curriculum in courses for which they are responsible, for example the goals, choice of content, style of teaching, and assessment. These decisions may be reached in great part on the basis of their own specialist mathematics domain, and their own personal philosophy of the nature of mathematics, knowledge, and mathematics teaching. It appears that tertiary teachers may have a far more significant role in curriculum design and development than is the case in school curricula. Considerations of their influence are thus more critical (Anguelov, Engelbrecht & Harding, 2001).

Valero-Duenas (2002) proposes a curriculum model which suggests a useful overarching framework for considering the structure of the curriculum in this thesis. Her model, reproduced in Figures 2.1, 2.2 and 2.3 (Valero-Duenas, 2002, pp 29-31), is based on a view of the curriculum which defines the curriculum “as a complex set of
principles and operational decisions deriving from them, which provides the organisation of teaching and learning in different levels” (Stenhouse, 1991 and Rico, 1997, in Valero-Duenas, 2002, p. 29).

1. A macro-social level where ideas about society, culture, education and politics are formulated in relation to (an institution’s) mathematics.

![Figure 2.1 The macro-level of curriculum.](image1)

2. An intermediate or meso-sociological level, which considers the role of the students, teachers, mathematical knowledge and the educational institution.

![Figure 2.2 The intermediate level of curriculum](image2)

3. A micro-sociological or classroom level where the specific relations between the aims, content, methodology and assessment of mathematics education are formulated.

![Figure 2.3 The micro-level of curriculum.](image3)
Figures 2.1 to 2.3 demonstrate that the Valero-Duenas model incorporates many of the separate elements of curriculum as identified in the preceding discussion, while the stratified structure of the model, with its three interconnected levels, seems especially suited to this thesis. It allows us to focus on the aspects of curriculum over which we have control, or factors of interest, at any given level, isolating those aspects that are not applicable. Considered against this framework, curriculum reform for teachers in a school can be seen as extremely problematic, given that they are expected to conform to the dictates imposed upon them at both the macro- and intermediate-levels over which they have little control. Even at the micro-level, teachers’ influence in schools may likely be limited to decisions at the classroom level about content, teaching style, and in some instances assessment, but even these are subject to external influences such as departmental schemes and national assessment prescriptions.

In the tertiary sector where this thesis is focused, lecturers are still subject to influences outside their control at the macro-level (for example the effects of calculus reform, see p. 23). However, it seems highly possible that lecturers may have considerably more influence over factors at both the intermediate- and micro-levels than their secondary counterparts. Curriculum decisions may be made at departmental and individual course level with more direct, sometimes sole, input from individual teachers, and even students can have an influence through the process common in many tertiary institutions of course evaluations, and departmental reviews. The Valero-Duenas (2002) model suggests a useful framework within which to consider both the structure and process of curriculum design and development in this thesis, theoretical aspects of which will be discussed in the next section.

2.2.2 Theoretical Issues of Curriculum Design and Development

A study by Tyler in 1950 (in Klein, 1986) is commonly regarded as one of the most influential publications in the field of curriculum research. “Tyler identified three data sources which must be used in curriculum development: society, student, and subject matter…A comprehensive curriculum must include all three” (Klein, 1986, p. 31). Interestingly, while several of the elements highlighted in the Valero-Duenas model (2002) are absent from this statement, most notably the role of teacher, Klein (1986) observes that in any case, curriculum practice and research at that time focused almost
exclusively on just one of these three data sources, the subject matter. The resultant outcome is that “curricula have been developed using only what Eisner and Vallance (1974) call the technological conception, … it has emerged into dominance over all other alternative conceptions and designs” (Klein, 1986, p. 31). This curriculum design is commonly referred to as the measured curriculum (e.g. see Doll, 1986; Frymier, 1986; Klein, 1986; Short, 1986; Zuga, 1989), and to avoid confusion between the use of ‘technological’ in this curriculum design sense, and its more frequent use to describe technological tools elsewhere, this discussion will refer to measured curriculum when describing such curriculum designs in future. The key design feature of the measured curriculum is characterised by “the reduction of (its) intended outcomes to pre-specified elements, which, when taught and learned, can be measured, and on which a definitive report of results can be made public” (Short, 1986, p. 6). Many writers and modern educational theorists are critical of the measured curriculum. There is for example an implied criticism by Short (1986), when he observes that none of the authoritative models for curriculum design recommended prior to the 1940’s incorporated such reductionist assumptions. Frymier (1986) was equally critical of conventional conceptions of curriculum, describing them as typically coercive. “Most teachers, principals, supervisors, and others presume that their own knowledge, experience, and skill entitle, in fact require, them to specify objectives, select content, determine methods…without attending to particular students’ interests, abilities or needs” (Frymier, 1986, p. 60). The measured curriculum has clearly had a significant, long-term impact on curricula at school level. As Kilpatrick and Davis (1993, p. 206) observe, the “view that the intended curriculum consisting of official documents is somehow transformed into an implemented curriculum (that can be measured) was one of the key assumptions of the Second International Mathematics Study.” This view is supported by Bishop (1993, p. 227) when he notes, with respect to school mathematics curricula world-wide, that “the overwhelmingly dominant characteristics which we find are a concern for knowledge of a particular mathematical content and the performance of particular mathematical skills.” Implicit in such assumptions is that it is possible to design one canonical curriculum, or as Bishop (1993, p. 226) calls it, a “convenient lowest common multiple of mathematics curricula”. This belief has led (world-wide) to an “extraordinary uniformity of syllabuses" (Kilpatrick & Davis, 1993, p. 207). Klein (1986), however, is less critical. He believes that “the measured curriculum should
neither be condemned nor used exclusively to direct curriculum practice and research. It must be recognised for its strengths and limitations” (Klein, 1986, p. 32).

At the tertiary level, the academic rationalist perspective is regarded as the curriculum orientation with the longest history and the most influence (Erekson, 1992). Like the measured curriculum, the academic rationalist conception is also seen as compatible with technological processes, it too is knowledge-centred, it is based on subject-knowledge as the foundation of civilisation, and it sees the role of the mathematics teacher as passing on the collective wisdom of its discipline (Zuga, 1992).

Eisner and Vallance (1974) identify a further three conceptions that may be considered in curriculum design: cognitive processes (supposedly content free, emphasises the ability to think, reason, and engage in problem-solving activities), self-actualisation (student-centred, students explore personal interests), and social reconstruction (societal centred, based on problems and dilemmas of society). Subsequent studies have either built on or adapted these conceptions, so that, as in the case of the definitions of curriculum discussed earlier, there now exists quite a variety of similar based categorisations, albeit with different terminologies (Zuga, 1989). For example, in 1979, Eisner himself adapted his and Vallance’s earlier conceptions to give five new orientations towards curriculum, and in 1986, Vallance added two new conceptions, what she calls the personal success, and the personal commitment conceptions. The personal success conception is evident at the tertiary level, where curriculum reform is designed to enhance the job placement chances of graduates. It is seen as a response to technological advances which threaten to make traditional knowledge obsolete, and an increasing competition for jobs (Vallance, 1986, p. 27). Personal commitment is seen as an evolved synthesis of all five previous conceptions, it “sees the purpose of schooling as creating a personal commitment to learning” (Vallance, 1986, p. 27). Despite such variations, Herschbach (1992, p. 15) observes that generally, theorists still agree on five basic designs, which vary from Eisner and Vallance’s only in name, for example the technological or measured design is referred to by Herschbach as competencies.

Zuga (1989) suggests a framework comprising five categories, which encapsulates most of the characteristics of the curriculum designs portrayed in the previous discussion. Her categories are Academic, Technical, Intellectual Processes, Social, and Personal (Zuga, 1989, p. 3). This framework is especially useful for this thesis, in that it
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considers the position of technology in its design, with respect to the goals and purposes, content source, and structuring elements required when planning curriculum in general. A summary of Zuga’s categorisations is reproduced in Table 2.1.

Table 2.1: Framework for Curriculum Planning Models (Zuga, 1989, p. 9).

<table>
<thead>
<tr>
<th>Design</th>
<th>Goals &amp; Purposes</th>
<th>Content Source</th>
<th>Structuring Elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Academic</td>
<td>Transmit cultural heritage</td>
<td>Constructs and concepts</td>
<td>Taxonomies</td>
</tr>
<tr>
<td>Technical</td>
<td>Develop occupational proficiency</td>
<td>Observable behaviours</td>
<td>Task analyses</td>
</tr>
<tr>
<td>Intellectual Processes</td>
<td>Improve thinking and problem solving abilities</td>
<td>Cognitive processes</td>
<td>Problem solving and trouble-shooting processes</td>
</tr>
<tr>
<td>Social</td>
<td>Reconstruct or adapt to society</td>
<td>Societal needs or successful work behaviours</td>
<td>Social problems or work adjustment skills</td>
</tr>
<tr>
<td>Personal</td>
<td>Motivate personal interests in learning</td>
<td>Student interests – within subject context</td>
<td>Student research and projects</td>
</tr>
</tbody>
</table>

Like many others (for example Doll, 1986; Vallance, 1986; Johnson, 1992; Selden & Selden, 1999; Leigh-Lancaster, 2002), Zuga suggests that the various curriculum designs depicted in Table 2.1 should not be regarded as mutually exclusive, and that indeed, all aspects of the framework should at least be considered, if not necessarily included, in the final curriculum design. Despite this preference, Zuga (1992, p. 8) observes that at that time, the technical and academic designs were still by far the most dominant, often “even those who attempt to suggest alternatives ultimately employ a technical design, … therefore, though alternative designs are suggested, few are fully described.” Elements of Table 2.1 are reflected in the detailed intellectual processes framework for designing a technology education curricula proposed by Johnson (1992). While not specifically related to mathematics, he acknowledges that the development of a curricula that addresses the goals of reinforcing academic content, enhancing higher-order thinking skills, and promoting active involvement with technology is both difficult and complex (Johnson, 1992). Zuga (1989) notes few examples of such intellectual-
processes based curricula in any discipline, including mathematics, and adds that the personal design is also elusive, while the social design is equally ephemeral.

However, even though the overarching influences of the dominant ‘technical’ design are still very much evident, other influences became much more apparent in later curricula, from the 1990’s onwards. The most significant example of this is undoubtedly the effects of constructivism, the model of knowledge that attempts to answer the primary epistemological question; how do we know what we know? (Malone, Burkhardt & Keitel., 1989; Simon, 1995; Steffe & D’Ambrosio, 1995). Constructivist theory maintains that knowledge cannot be the result of passive receiving, but originates as a result of active engagement on the part of the learner (von Glasersfeld, 1989, 1991). Malone, Burkhardt & Keitel (1989) believe constructivist theory has significant implications for curriculum design and development. The traditional homes of curriculum change in schools (e.g. Ministries of Education; Curriculum panels of tertiary educators, curriculum experts and teachers) will have to change, shifting the emphasis away from the usual top-down approach, towards a continually evolving conception of curriculum involving both teachers and students, “a model wherein the teacher, along with his or her students, are the primary actors creating mathematics curricula in the contexts of ongoing teaching and learning” (Malone, Burkhardt & Keitel, 1989, p. 327). Artigue (2006) agrees, asserting that much of the failure to make significant changes to curricula with respect to technology may be attributed to top-down strategies imposing radical changes, and she suggests we need to find a new balance between top-down and bottom-up processes. The inclusion of teachers and students as key players in curriculum design can be clearly seen to reflect the ideals of the personal curriculum design in Table 2.1 (Zuga, 1989). It also reflects Steffe’s (1989) sixth principle, one of ten principles of curriculum design, that states that “(curriculum design should recognise) that teachers and students create mathematics curricula in the context of ongoing teaching and learning” (Steffe, 1989, p. 459).

There are many examples in school curricula that demonstrate the design influences from Table 2.1, and in particular, the constructivist theories that are manifested in the personal design. The overarching structure of the curriculum document Mathematics in the New Zealand curriculum (Ministry of Education, 1992), with suggested achievement objectives and learning outcomes still reflects the technical design, while the intellectual processes design can be seen in the Mathematical
Processes strand which emphasises problem-solving, developing logic and reasoning, and communicating mathematical ideas (Ministry of Education, 1992, p. 23). The personal design, with clear constructivist influences, is evident in such statements as “new experiences cause students to refine their existing knowledge and ideas, so they can construct new knowledge…it is important that students are given explicit opportunities to relate their new learning to knowledge and skills which they have developed in the past” (Ministry of Education, 1992, p. 12). Similar examples can be found internationally in such documents as Standards for curriculum and pedagogical reform in two-year college and lower division mathematics (Cohen, D., 1993), and Principles and standards for school mathematics (National Council of Teachers of Mathematics, 2000).

The influences of constructivist theory do not seem so overtly evident at the tertiary level. This is perhaps not surprising, given the earlier observation of the predominant influences of the academic conception (Erekson, 1992). However, in recent times, theories relating to instructional design have become more prominent at the tertiary level. Selden and Selden (1999) describe four examples where mathematics research has influenced curriculum design in universities. Their first example is the effect of APOS theory (Action-Process-Object-Schema) on the traditional linear nature of mathematical understanding reflected in traditional curricula (see Dubinsky & Macdonald, 2002 for a fuller description and a discussion of APOS theory from a constructivist perspective). Other examples include Didactical Engineering and the Method of Scientific Debate, which involves students becoming active participants in judicious problems, proposing conjectures and debating their relevance and truth (Artigue, 1991, and Brousseau, 1997; in Selden & Selden, 1999, p. 8), and Freudenthal’s Realistic Mathematics Education and Local Instructional Theories, the approach where students learn mathematics by mathematising the subject matter through examining realistic problems in real contexts, (Selden & Selden, 1999, p. 9).

The implication is that curriculum designers at the tertiary level should be aware of such instructional theories, and recognise them where appropriate in their curriculum design. These learning theories will be considered again with respect to theoretical issues of technology in Chapter Three.

Lastly in this discussion is a consideration of two strong theoretical influences that Valero-Duenas (2002) believes should be stressed, particularly with respect to the
intermediate level of curriculum in her model. The first of these influences is the impact of philosophies of mathematical knowledge on the way in which mathematics is conceptualised (see for example, Lakatos, 1992, in Valero-Duenas, 2002, p. 31; Ernest, 1991). There are strong parallels between the five social groups into which Ernest (1991) categorises mathematical ideologies, and Eisner and Vallance’s (1974) conceptions of curriculum design, and Valero-Duenas believes that curriculum designers should be aware of these connections. The second influence is the impact of psychology, especially the work of Piaget and Vygotsky, in the formulation of epistemological views about the learning and teaching of mathematics. These influences are central to the development of constructivist theory in particular, and will be expanded on later in Section 2.3 when considering pedagogical issues and curriculum.

2.2.3 Curriculum Development

Whilst curriculum development should ideally have some basis in curriculum design theory, it is essentially a separate process which should be considered from its own theoretical perspective, as evidenced by Rachlin (1989), who states that “the content of curriculum change and the process of curriculum change represent distinct spheres of knowledge and expertise, both of which must be present and integrated in any reform effort” (p. 259). The content and wider structure of the curriculum is clear in the model of curriculum by Valero-Duenas (2002) adopted in Section 2.2.1. The process of curriculum development may also be considered within this model, suggesting the need to pay attention to students, teachers, knowledge, the institution, aims, contents, assessment and methodology in the development process. In his examination of curriculum trends, Hillel (2001a) describes six broad factors influencing changes in tertiary mathematics curricula, all of which may be positioned within the Valero-Duenas model. Hillel’s six factors are Changes within mathematics (new theories and mathematical tools); Changes in the pre-university curriculum (changes in content, e.g. set theory removed, less emphasis on proofs); Changing clientele (more students from more diverse backgrounds); Resources (pressure on staff, costs); Technology (e.g. effects on content and pedagogy) and External influences: Governments, Research Agencies and Business (Hillel, 2001a, pp. 61-64).
There are many specific curriculum development models identified in the literature within which factors such as those identified by Hillel (2001a) may be considered. Rachlin (1989, p. 261) initially uses the 1986 Fennema-Carpenter and Peterson model, but he later dismisses this as inadequate, observing that “there is something lacking in a model that uses research to provide input on the learner and the teacher separate from the impetus for the proposed curriculum change.” As in the call to use multiple conceptions of curriculum design described earlier (e.g. Zuga, 1992), Rachlin (1989) favours a more holistic research model for the process of curriculum development, one that encompasses teachers, students and curriculum developers. Rachlin (1989) emphasises that it is essential that this model, reproduced in Figure 2.4, is regarded as a “gestalt”, with the links between each of the three areas describing a continuous, dynamic process, ignoring any one part will adversely affect development.

![Figure 2.4 A research model for curriculum development (Rachlin, 1989, p. 263).](image)

Figure 2.4 suggests that while curriculum developers need to have a firm theoretical and philosophical foundation for the new curriculum, it is equally vital that
there is early, and regular contact between the students, teachers, and the curriculum developers. The model may however be less appropriate to the tertiary level where this thesis is focused. For example, as stated earlier, at the tertiary level, often the curriculum developers and the teacher are one and the same, which alters the entire structure of the model. Also, the notion of continuous feedback from students, in a semester-based environment where student turnover can occur every 12-17 weeks, can be problematic.

Another model for curriculum development in the United States of America is proposed by the National Research Council (1999), in its guide *Designing mathematics or science curriculum programs: A guide for using mathematics and science education standards*. While also designed ostensibly with respect to school curriculum development, this model suggests a useful way of considering tertiary curriculum development, especially given that the authors describe it as “not necessarily prescriptive”, but more a series of key steps in a suggested process of curriculum design (National Research Council, 1999, p. 31). The key steps of their model are reproduced in Figure 2.5, which illustrates the importance of involving teachers and students in the process of development and change.

![Figure 2.5 Process for designing a curriculum program.](National Research Council, 1999, p. 31)

Some of the steps in the model depicted in Figure 2.5 may be sometimes less relevant at the tertiary level, for example building a common vision in a course for which one lecturer has sole responsibility. However, the overall model seems pertinent
to this thesis, where the place of technology may be considered in each of the three latter ovals which represent factors influencing the implementation process. The model also allows for evaluation of the impact of technology, with the provision of a feedback loop to influence future decisions.

Frielick (2001) proposes a model specifically designed for teaching and learning in higher education. His model, depicted in Figure 2.6, is a complex one that involves learning theories, teachers, students and the institution in an inter-connected model that he describes as an ecological model. Recognising the complexity of the model, Frielick offers the following explanation of the model:

Like an ecosystem, the model is multi-layered with complex sets of interacting processes. It is ecologically post-modern in the sense that it does not aim at deconstruction, but rather incorporates findings from student learning research blended with concepts from other disciplines to arrive at a more complex description of the teaching/learning process. It seeks to uncover the pattern which connects the multiple variables of teaching/learning process through a process of multiple descriptions. It can be used to inform practice as well as providing a deeper understanding of theory. (Frielick, 2001, p. 7)

For example, the core layer, represented by the intertwining spiral in Figure 2.6, depicts the zone of academic development, which manifests in the relations between the teacher, students, and specific content in the act of teaching and learning. This zone of academic development is a rephrasing of Vygotsky’s (1978) well-known concept, the zone of proximal development (ZPD), which adapts Vygotsky’s ideas about child development to higher education. Frielick (2001, p. 7) describes how “Substituting academic for proximal [in this model] signals that the ZPD is not only relevant to child learning but extends through all phases of lifelong learning.” It also alludes to the revision of the ZPD undertaken in recent theorising, where Vygotsky’s original concept has been extended, emphasising “the holistic nature of the learning that takes place in the ZPD by making it clear that it involves not simply speech but a wide range of mediating means, and not simply dyads in face-to-face interaction but all participants in collaborative communities of practice” (Wells, 1999, in Frielick, 2001, p. 7).
Figure 2.6 An ecological model of curriculum development (Frielick, 2001, p. 8).

While such theoretical discussions may certainly be worth considering in curriculum development, and Frielick (2001) elaborates extensively on other features of his model and their significance in tertiary education curricula, it is somewhat unreasonable to expect all tertiary mathematics teachers to either be aware of such issues, or even if they are, to accept the challenge and responsibility of recognising them in their curriculum development (Malone, Burkhardt & Keitel, 1989). The extent to which such factors are recognised may however play a significant role in the ultimate success of any curriculum implementation.

While the models of curriculum development described here may all easily located within the Valero-Duenas (2002) curriculum structure, what seems to be not as obvious is exactly where technology issues may be located. It seems that technology has effects, or raises questions, at many different levels. It may require modifications of the proposed models, as suggested in the quote by Kilpatrick and Davis (1993) in the introduction to this chapter. Many researchers signal the special issues that need to be considered with respect to technology. For example, Konig (1994), Hoyles (1998) and Heid (2003) all refer to the need to consider possible changes to the content and
sequence of topics in the curriculum, while Schwartz (1999) suggests that technological developments have implications for the educational goals of society and the structure of mathematics curricula. Dugdale, Thompson, Harvey et al. (1995) and Heid (2003) raise questions about many issues related to teachers such as their beliefs and the practice of teaching, assessment and evaluation needs, whilst Heid (2003) also describes a number of factors that concern students, such as access to hardware, time to learn to use the tools, and preferences for learning. While these and other considerations will be examined in more detail later in this chapter, the range of issues identified here does suggest that the cited models of curriculum development need to be adapted to reflect new issues raised by technological advances. How, for example, should technology be positioned in the Frielick (2001) model illustrated in Figure 2.6, given that it has a potential impact at nearly every level? Should it just be noted as implicit throughout, or should it have another entire layer of its own, perhaps positioned within the green layer depicting the institution and society, but encompassing all the other layers?

An alternative to modifying one of the existing models may be found in the concept of zero-based curriculum, a start-from-scratch approach rather than the more common incremental approach. Paulsen and Pesau (1992) note that many higher education curricula are “an accumulation of incremental and extemporaneous changes in individual courses. Collectively, such efforts can produce an accidental curriculum in place of an intentional curriculum” (p. 211). They observe that “existing curriculum can easily contaminate a curriculum review by effectively ‘pre-structuring’ the curriculum before any review process has begun. Any changes will likely to be minor variations of what is already being done” (Paulsen & Pesau, 1992, p. 212). Ralston (1996) endorses the urgency of curriculum change with respect to technology, and proffers a zero-based curriculum as a potential solution when he states that:

(Changes have) not happened, however, mainly due to the immense inertia in the educational systems of all countries. It is, therefore, worth considering what the school mathematics curriculum would look like if it were not constrained by current practice. A zero-based curriculum does this by requiring that all subject matter and pedagogical practice be justified on grounds other than that the topic is currently in the curriculum or that the methodology is that used presently.

(Ralston, 1996, p. 145)
While Ralston’s suggestion focuses on school mathematics curricula, it seems that such an approach may be somewhat easier to enact at the tertiary level. Certainly this is clearly already the case when university mathematics departments develop new courses from scratch, but perhaps the case may be more complex when faced with redesigning existing courses. A zero-based curriculum provides a model against which current practice and proposed changes in that practice maybe measured. The practical effect of a zero-based curriculum would hence be seen over a period of time, drawing curriculum change in the direction of the zero-based ideal (Ralston, 1997). In this sense, there are many similarities between Ralston’s concept of a zero-based curriculum as an ideal model for comparison and developmental direction, and the focus of the first research question of this thesis posed in Chapter One. This question aims to identify the characteristics of an Integrated Technology Mathematics Curriculum (ITMC), and formulate a possible model for such an ITMC. Although Ralston (1997) lists technology separately to the other three key components of subject matter, pedagogy, and teachers (including teacher education), he acknowledges that technology pervades all aspects of mathematics education, and therefore, its role must be considered in each aspect of a zero-based curriculum.

This section has provided a definition of curriculum as it will be used in this thesis, and suggested models for curriculum design and development. In the next section, we will examine the nature of mathematics, knowledge, and pedagogical issues that were identified as components of these models.

2.3 CURRICULUM, MATHEMATICS AND PEDAGOGY

Many of the elements of the Valero-Duenas (2002) curriculum model described in Section 2.2.1 receive frequent mention in the literature with respect to technology. In particular, the role of the teacher, and mathematical knowledge, from the intermediate level of this model, and aims, contents, and methodology, from the micro-level, all receive much attention. For example, in commenting on the slow reactions of curricula to technology, Noss (1998, p. 44) observes that “mathematical teaching is seldom in the vanguard of pedagogic innovation, … (especially) in higher education”. However, he also believes that the issue is not simply one of pedagogic innovation, it is also one of what counts as mathematics. “This, perhaps, is the main contribution of new
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technology in mathematical teaching and learning at the undergraduate level: it provides us with an opportunity to reassess not simply how we teach, or even how students learn, but what it is that we teach them and why” (Noss, 1998, p. 45). The discussion in this section is hence divided into four themes, suggested by the Valero-Duenas (2002) curriculum model and illuminated further by the following questions, drawn from the many posed by the seventeenth ICMI study discussion document:

- What new types of mathematical knowledge and practices emerge as a result of access to digital technologies, particularly computational, dynamic visualisation and communication technologies?
- What does it mean to be mathematically literate, in a world instrumented by technology?
- How are new types of technology-mediated mathematical knowledge and practices related to current classroom curricula and values, what curriculum elements exist because older-than-digital technologies were all that were available, and how should aspects of mathematics curriculum therefore be deleted or changed?
- What kinds of pedagogical approaches and classroom organisations can be employed in technology-integrated environments and how can they be evaluated?

(Hoyles & Lagrange, 2005, pp. 8-9)

2.3.1 The Nature of Mathematics and Knowledge

An exhaustive examination of the philosophy of mathematics, beliefs about mathematical knowledge, and the effect of these on curricula and attitudes to technology, lies outside the scope of this thesis. However, most writers and researchers in mathematics education agree that one’s philosophy can have substantial effects on curricula, in for example the aims, content, and teaching methods employed, and as such, attention should be paid to this when designing curricula. As Steen (2001) observes, “as mathematicians, we tend to design curricula the same way that we design proofs. We select our goal, we think carefully and systematically about all relevant factors, and then we write down all the steps that are logically necessary to reach that goal” (Steen, 2001, p. 308). Ernest (1994) offers a possible explanation of this, when he describes a bi-directional relationship between philosophies of mathematics and
mathematics education: “First, all learning and teaching practices rest upon possibly implicit epistemologies or philosophies of mathematics”, and “Second,...any philosophy of mathematics (including personal philosophies) has powerful implications for social and educational issues, and many educational and pedagogical consequences” (Ernest, 1994, p. xiii). Schoenfeld (1992) also considers the effect on the curriculum of the ways in which mathematicians think when he cites Hoffman (1989), who observes that “whether or not one is explicit about one’s epistemological stance, what one thinks mathematics is will shape the kinds of mathematical environments one creates—and thus the kinds of mathematical understandings that one’s students will develop” (Schoenfeld, 1992, p. 22). Schoenfeld notes that one of the difficulties presented by this view is that mathematicians are hardly unanimous in their conceptions of problem-solving, and by implication therefore what constitutes mathematics. Halmos (1980, in Schoenfeld, 1992) lists “Axioms, Theorems, Proofs, Definitions, Theories, Formulas” as an answer to the question “What does mathematics really consist of?” However, he does not believe that this list provides a sufficient picture of what constitutes mathematics when he states that:

Mathematics could surely not exist without these ingredients; they are all essential. It is nevertheless a tenable point of view that none of them is at the heart of the subject, that the mathematician’s main reason for existence is to solve problems, and that, therefore, what mathematics really consists of is problems and solutions.


Given such variation in mathematicians views of mathematics, it is hardly surprising that many students, and indeed most of the general public, have very little idea of what mathematics is about, which is probably a source of great annoyance to many tertiary mathematics teachers (Selden & Selden, 1999, p. 11). Schoenfeld (1992) clearly agrees with Halmos’ belief that problems are the heart of mathematics, and extends this conceptualisation of mathematics as problem solving using Polya’s conception of mathematics as an activity. Polya “eschews the notion of mathematics as a formal and formalistic deductive discipline,...arguing that mathematics is akin to the physical sciences in its dependence on guessing, insight and discovery” (In Schoenfeld, 1992, p. 17). Schoenfeld argues for a particular view of mathematical thinking that sees mathematics as an act of sense-making, socially constructed and socially transmitted.
The social constructivist perspective that he postulates extends the notion of constructivism from the purely cognitive domain (see comparative discussion of works of Piaget and Vygotsky in Devries, 2000), to the social sphere. Social constructivism as it relates to pedagogy will be discussed in more detail in Section 2.3.3. However, the notion of socialisation (or enculturation, as used by Schoenfeld, 1992; Bishop, 1993) has important considerations for curriculum design in particular, in that it highlights the importance of perspective and point of view as core aspects of knowledge (Schoenfeld, 1992, p. 19).

Schoenfeld (1992, p. 18) suggests that the social constructivist view of mathematics is theoretically well-grounded and commonly accepted among educationalists, and its influences are certainly very evident in school curricula at least. While the same may not be so true for university teaching staff in general, and mathematics lecturers especially (Erekson, 1992; Keynes & Olson, 2001), the potential impact of an individual lecturer’s perspective on all aspects of curricula should still be considered. Ernest (1991) proposes a social constructivist framework for considering such perspectives. It has three descriptive characteristics:

1. It is descriptive rather than prescriptive.
2. It is based on quasi-empiricism, conventionalism, and constructivism (as opposed to radical constructivism).
3. It consists of both subjective and objective knowledge.

(Smith, 1994, p. 80)

The first characteristic sees the basis of mathematical knowledge as linguistic knowledge, conventions and rules, as opposed to a body of known truths which must be learnt. The second characteristic describes the focus on enacting mathematics rather than solely its justification. The generation of new knowledge can be either subjective or objective, with each linked to the other in a creative cycle. Objective knowledge consists of socially accepted forms of linguistic expression, in mathematics this concerns the fairly well defined rules, theorems, formulas, and algorithms which have become accepted in a particular culture, e.g. Calculus. Subjective knowledge consists of the unique representations of mathematical knowledge constructed by an individual, mostly from reconstructed objective knowledge (Ernest, 1991, p. 43).
Within this social constructivist framework, Ernest describes five distinct social groups, each with its associated view of mathematics, educational theories, political ideologies, aims and societal mores. These are the industrial trainer, the technological pragmatist, the old humanist, the progressive educator and the public educator (Ernest, 1991, p. 138). A comprehensive description of all of the elements of these groups is not feasible here, since for each of the five groups, Ernest lists examples of a further twelve educational aspects related to an individual’s ideological perspective. These range from political ideology, through to theories of child development, ability, learning, teaching mathematics and social diversity. The depth of Ernest’s analysis is illustrated in the sample of three aspects of the five educational ideologies summarised in Table 2.2.

Table 2.2: Comparison of Selected Aspects from Ernest’s Overview of Educational Ideologies (Ernest, 1991, pp. 138, 139)

<table>
<thead>
<tr>
<th>Social Group</th>
<th>View of Mathematics</th>
<th>Mathematical Aims</th>
<th>Theory of Teaching Mathematics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Industrial Trainer</td>
<td>Set of truths, and rules</td>
<td>‘Back to Basics’: numeracy and social training in obedience</td>
<td>Authoritarian transmission, drill, no frills</td>
</tr>
<tr>
<td>Technological Pragmatist</td>
<td>Unquestioned body of useful knowledge</td>
<td>Useful maths to appropriate level (industry centred)</td>
<td>Skills instruction, motivate through work-relevance</td>
</tr>
<tr>
<td>Old humanist</td>
<td>Body of structured pure knowledge</td>
<td>Transmit body of mathematical knowledge (maths centred)</td>
<td>Explain, motivate, pass on structure</td>
</tr>
<tr>
<td>Progressive educator</td>
<td>Process view: Personalised maths</td>
<td>Creativity, self-realisation through mathematics (child-centred)</td>
<td>Facilitate personal exploration, prevent failure</td>
</tr>
<tr>
<td>Public educator</td>
<td>Social (&amp; Radical) constructivism</td>
<td>Critical awareness and democratic citizenship via mathematics</td>
<td>Discussion, conflict, questioning of content, pedagogy</td>
</tr>
</tbody>
</table>

Many of the aspects identified in Table 2.2 can be seen as closely aligned with the elements of the curriculum model of Valero-Duenas (2002), e.g. assessment, society and aims. A number of Ernest’s aspects refer to areas outside the sphere of curriculum
design influence for most school teachers (e.g. those from the macro-level of the Valero-Duenas model: political ideology, theory of society), so the effects of these teachers’ personal perspectives may be largely limited to their individual pedagogical choices, curriculum implementation and classroom interactions. However, at the tertiary level, where individual lecturers may be wholly responsible for the entire design of their course, the effects of their personal perspective may be far more pervasive. Although Ernest (1991) acknowledges that these social groups are dynamic in nature, and the aims of several groups may be furthered in a cooperative alliance (e.g. a teaching team as opposed to one lecturer), the potential impact on curricula and teaching effected by one’s position in this framework may be illustrated by comparing just a few areas of these ideologies shown in Table 2.2. The use of large lectures as the principal means of course delivery would clearly be more favoured by those from the first three social groups, while small-groups and projects may be more likely to be favoured by the latter.

Many writers comment on the general shift in the tertiary sector towards the social-constructivist epistemology of the Public Educators. There is a trend towards reduced rigour and emphasis on proof, more focus on problem-solving and process rather than content, and greater experimentation (e.g. Schoenfeld, 1992; Artigue, 2000; Hillel, 2001a), but it is still likely that most tertiary mathematics departments contain staff from a multitude of ideologies, and it is important to be aware of their perspectives when considering curriculum change.

With respect to technology, several writers emphasise the importance of placing current changes in an historical perspective. Both Kaput (2000) and Tall (1996a) consider the historical changes in representational infrastructures. Kaput emphasises the shift from static inert media to dynamic, interactive media that facilitates “the construction of new systems of knowledge employing new representational infrastructures” (Kaput, 2000, p. 8). Tall (1996a, p. 68) describes several forms of mathematics that existed before the development of the computer, including enactive, visual, symbolic, and formal. Although Tall (2004) has since revised the descriptors for these forms (e.g. enactive and visual are now identified as embodied), his belief, about the development of computers remains valid. It has set successive agendas for mathematics education (Tall, 1996a, p. 68). At the tertiary level, mathematics education is often characterised by the specialised nature of lecturers’ research and teaching domains. It is likely that an individual lecturer’s knowledge of technology may be
limited to its use in their own particular domain, and thus be unaware of its historical significance or development in other areas (Kaput, 2000).

Tall (2000) considers cognitive development in advanced mathematics, presenting empirical evidence to show how the use of visual imagery and symbols, and the later shift to formal axiomatic theories, may be enhanced, changed, or impeded by the use of technology (Tall, 2000, p. 210). Balacheff (1990, p. 145) too perceives an effect on mathematical thinking, noting that “what is interesting and important is not only that these software products (e.g. spreadsheets) enlarge or broaden already existing or utilised ways of thinking but they demand qualitatively new ways of thinking”. These issues will be expanded on in Section 2.3.4, when examining pedagogical issues and technology in greater detail. Leigh-Lancaster and Stephens (1999) refer to the way in which technology can raise questions about the legitimacy of mathematical knowledge and activity. They observe that CAS can:

…focus our attention on what we value as mathematical content and mathematical process and our beliefs as to the nature and purpose of various forms of mathematical activity. Individual preferences (often based on our own learning experiences) become evident when we consider what sort of activities we deem legitimate for “unfettered” student use of CAS, What do we accept as “allowable” activity using CAS, and subsequently what do we come to expect as CAS free and CAS required mathematical activity?...The opportunity for CAS use challenges us to articulate our values and beliefs. (Leigh-Lancaster & Stephens, 1999, p. 72)

This quote introduces the influence that beliefs about the nature of mathematics and what constitutes knowledge may have on the values that individuals attach to technology. It should be noted here that there is often a lack of consistency in definitions of the term mathematical beliefs (Torner, 2000, p. 73). For example, Furinghetti and Pehkonen (2000, p. 42) note that some researchers do not consider it important to distinguish between knowledge and beliefs, the way in which they are used is usually evident from the context. While an in-depth examination of the affective domain of mathematics education lies outside the sphere of this thesis, there is considerable evidence that beliefs and values play a significant part in both the design and implementation of curricula, from both teachers’ and students’ perspectives (e.g. see Moriera & Noss, 1995; Goldin, 2000), and therefore some consideration should to be given to these issues. From a student’s perspective, for example, Ocean (2005)
asserts that “no matter how innovative the curriculum, implementation will fail if students’ values are not taken into account in curriculum design and delivery”. She reports how some students ceased to study mathematics because it conflicted so greatly with their values, and concludes that:

...individuals have differential preferences for what they value. Some people tend to value ‘connected’ knowledge which is characterised by intuition, creativity and experience while others tend to value ‘separate’ knowledge which is characterised by logic, rigour and abstraction. (Gilligan, 1982, in Ocean, 2005, p. 1)

Parallels can be drawn between the characterisations of knowledge adopted by Gilligan (Gilligan, 1982, 1995, 2001, in Ocean, 2005), and Ernest’s (1991) categories of objective and subjective knowledge. Like Ernest, Gilligan too associates a moral perspective with each of her different categories of knowledge. In addition to the values that mathematicians and educators place on knowledge, Ocean (2005) believes we should also pay attention to such moral perspectives.

Goldin (2000) provides an extensive typology of mathematical beliefs that could be considered when examining an individual’s perspective on the nature of mathematics, for example “beliefs about mathematical validity, or how mathematical truths are established” and “beliefs about effective mathematical reasoning methods and strategies or heuristics” (Goldin, 2000, pp. 67-68). Interestingly, while none of the beliefs in Goldin’s list refers specifically to technology, affective factors are known to have a significant effect on both teachers’ and students’ attitudes towards, and use of technology. For example, beliefs are frequently shown to be very stable and resistant to change (Goldin, 2000), and this can have significant implications for a teachers response to introducing new technology. Similarly, a negative attitude to technology can have profound affect on a student’s learning in a technological environment.

Torner (2000) proposes a model (see Figure 2.7) that considers subject-specific structuring of beliefs and belief hierarchies. This model appears especially useful in this thesis, given its specific focus on technology in tertiary mathematics. Torner separates beliefs about the subject (e.g. nature of mathematics, mathematical knowledge and pedagogy) from global beliefs (e.g. societal and educational perspectives), and in between he places domain-specific beliefs (e.g. analysis, geometry). Torner stresses that given the current state of research, the model is necessarily incomplete. For example, it
is not clear whether “…global beliefs overlay both domain-specific beliefs and subject-matter beliefs…or are single-subject beliefs stronger in some situations than global beliefs?…Is it possible that domain-specific or subject-matter beliefs come before global beliefs” (Torner, 2000, p. 87)? The “top-down” and “bottom-up” structure for beliefs in this model can be seen to mirror the two perspectives of curriculum change epitomised by the traditional and constructivist approaches respectively, described earlier in this discussion (Malone, Burkhardt & Keitel, 1989).

![Figure 2.7 Different belief structures (Torner, 2000, p. 87).](image)

Despite being incomplete, Torner’s model did prove useful in his study to help distinguish the domain-specific views of some students with respect to calculus. These included “beliefs representing views on the role of logic, application, exactness, calculation and so forth” (Torner, 2000, p. 87). Tertiary mathematics frequently comprises a number of courses that deal with distinct subject material in specific domains, and as such, may be expected to exhibit domain-specific beliefs as suggested in the model. The model may well prove just as useful in this thesis with respect to beliefs about technology. It seems plausible that beliefs about technology may vary greatly between university teachers, given that many mathematics researchers use different types of technology in their particular domains, (e.g. computational software compared to dynamic geometry), and this use in mathematical research may differ again from the pedagogical uses in the mathematics education domain (Torner, 2000; Keynes & Olson, 2001).

Furinghetti and Pehkonen (2000) provide a useful summary to this discussion of beliefs. They suggest that when dealing with beliefs and related terms, it is advisable:

- to consider two types of knowledge (the objective and the subjective)
- to consider beliefs as belonging to subjective knowledge
- to include affective factors in belief systems, and distinguish affective and cognitive beliefs, if needed
- to consider degrees of stability, and to acknowledge that beliefs are open to change
- to take note of the context (e.g. population, subject) and the research goal within which beliefs are considered.

(Furinghetti & Pehkonen, 2000, pp. 54, 55)

Beliefs will be considered again when discussing the relationship between teacher beliefs, professional development and change in Section 2.4, and again when examining theoretical issues with respect to technology in Chapter Three (e.g. in studies by Galbraith & Haines, 2000; Goos, 2003).

2.3.2 Goals

Goals are seen as an important aspect in both curriculum and technology discourses (e.g. Bishop, 1993; Schwartz, 1999). For example, they are featured in the varying conceptions of curriculum design (e.g. see Table 2.1, Zuga 1992), and appear as the first step in the curriculum development model proposed by the National Research Council (1999, see Figure 2.5). Goals are intrinsically linked to one’s philosophy, beliefs and values, as is indicated in Ernest’s (1991, pp. 38, 39) description of educational ideologies, where he associates different goals to each of the five social groups. There are, however, many different levels at which goals may be considered in the curriculum. Schwartz (1999) notes that the educational goals of society traditionally include aiding the personal growth and development of citizens, preparing people for the world of work, and transmitting the culture and values of the society, while Bishop (1993) observes that the goals for mathematics education are conventionally analysed separately into those relating to the individual, and those relating to society. Goals for society, for example, include such often-listed goals as “opportunity for all, life long learning and mathematical literacy”, while individual goals include such statements as “learning to communicate mathematically” and “becoming a mathematical problem-solver” (Bishop, 1993, p. 223). Similarly worded statements are clearly visible in the goals (or aims and objectives) of many school curricula (e.g. Ministry of Education, 1992; National Council of Teachers of Mathematics, 2000).
Valero-Duenas (2002, see Figures 2.1, 2.2, 2.3) lists aims, a term often used synonymously with goals, in the third micro-level of her curriculum model. This suggests we may consider goals as a subset of all the elements at the two upper levels, for example, the goals of society, and education at the macro-level, and the goals of the institution, teachers, and students at the intermediate level, and their relationship to contents, methodology, and assessment at the micro-level. At the macro-level, Kaput and Roschelle (1999) describe the dual challenge in mathematical education at all levels, of teaching much more mathematics, to many more people. Many writers (e.g. Engelbrecht, 1998; Steen, 2003; Hockman, 2005; Holton, 2005) especially note the effects of this at the tertiary level, in the conflicting pressures of specialisation versus democratisation. On the one hand are the needs of students from a range of other disciplines which require their students to take mathematics (e.g. finance, biology, computing), whom Hillel (2001a) labels ‘client students.’ Their needs must be balanced against those of students for whom mathematics is the primary subject (maths majors), labelled ‘math program students’ by Hillel (2001a). Often the conflict between specialisation and democratisation is the result of political or financing decisions that force departments to teach large service-style courses; sometimes it is exacerbated by complaints from client disciplines, who demand the inclusion of more applications in courses (Selden & Selden, 1999). Steen (2003), however, observes that much of the pressure is a result of the growing diversity of mathematics itself, and the expansion of mathematics into an ever-growing list of occupational and research fields. Steen links this diversification of mathematics in the information age to what he describes as the digitisation of knowledge, and he provides an extensive catalogue of publications relating to mathematics, for example robotics and biomedical science, where the mathematics is “worlds apart from (the) traditional triumvirate of algebra, geometry, and analysis” (Steen, 2003, p. 199). Technology is seen as one of the driving forces behind this increasing demand for mathematics. Society is becoming more dependent, not just on technology, but mathematically driven technology, that is producing new, highly visible forms of mathematics (Bishop, 1993, Kaput & Roschelle, 1999). Keitel (1986) calls this the ‘social needs’ argument: “mathematics education should qualify students in mathematical skills and abilities, so that they can apply mathematics appropriately and correctly in the concrete problem situations that they may encounter in their lives and work” (Keitel, 1986, p. 27).
It seems clear that an important curricula goal should address the question “a mathematics curriculum for whom?” (Hillel, 2001a, p. 59). Holton (2005) observes that there is clearly no one right curriculum. We need to carefully consider and balance the needs of our students, as “not all students are a subset of…those going on to PhDs in mathematics” (Holton, 2005, p. 306). This view is supported by Hockman (2005), who suggests that even the needs of the mathematics-major students, the group that departments frequently cite in support of traditional courses, are changing. “The majority of undergraduate mathematics students who wish to major in mathematics are not career mathematicians. These students need to acquire the sophisticated tools needed in today’s workplace” (Hockman, 2005, p. 177). It is also follows that the technological demands of each group may vary just as widely. For example, ‘client students’ may want to learn how to use mathematical software packages specific to their key discipline (e.g. finance calculators, statistical packages), while maths majors may need to learn how to use such mathematical packages as Matlab or Maple. Recognising that the curricular and technological goals of students may differ widely is one thing, but finding ways of specifically expressing these may in reality be much more difficult. As a means of survival, many departments settle for offering courses for which the goals are a compromise between the needs of the different groups of students (Hillel, 2001a; Steen, 2001, 2003; Hockman, 2005).

Other goals may be considered from a curriculum designer’s societal perspective, and their beliefs about the nature of mathematics and mathematics education. Schoenfeld (1992) discusses goals from a general mathematics perspective, detailing seven broad goals for mathematics instruction being developed by the Mathematical Association of America. These goals state, for example, that mathematics instruction should “develop students understanding of important concepts, it should be aimed at conceptual understanding, rather than at mechanical skills” and it should “provide students the opportunity to explore a broad range of problems and problem situations…employing a range of approaches and techniques” (Schoenfeld, 1992, pp. 32-33).

More specifically at the tertiary level, Smith (1998) notes that while there is broad consensus among university teachers on the wider goals of higher education (he lists seven skills-based goals), he observes that historically, higher education rarely deals directly with the goals of instruction, and usually avoids stating them in measurable
terms. Smith (1998) provides a list of seven general principles of good pedagogical practice, drawn from lessons in cognitive psychology and past experience in tertiary education, which he regards as a necessary goal of undergraduate education in general. Undergraduate education should:

1) Encourage student-faculty contact.
2) Encourage cooperation among students.
3) Encourage active learning.
4) Give prompt feedback.
5) Emphasise time on task.
6) Communicate high expectations.
7) Respect diverse talents and ways of learning.

(Smith, 1998, p. 13)

From a technological perspective, Thomas and Holton (2003) relate technology to Smith’s goals, in, for example, encouraging experimentation and problem-solving to promote active learning. Technology is not specifically identified in any of Schoenfeld’s seven goals, although it may be implicit for example in some of the statements such as “range of approaches and techniques” and many other studies suggest that technology has a significant role to play in achieving the first of these goals, i.e. emphasising conceptual understanding over mechanical skills (see e.g. Fey, 1989b; Kutzler, 2003; Ruthven, 2005). Certainly there is much support for the contention that technology requires a restructuring of mathematics education goals. Schwartz (1999), for example, believes that one goal of a new curriculum designed from a technological perspective might be to make it more intellectually stimulating and socially responsible, and he echoes the earlier-cited ‘social-needs’ argument of Keitel (1986), when he states that “we need to shape our curriculum so that it clearly maximises the ability of students to function effectively in commerce, industry and the professions” (Schwartz, 1999, p. 115). Reflecting similar goals to those described earlier by Schoenfeld (1992) from a technological perspective, Schwartz lists five key aspects of mathematical activity, for which he contends that thoughtful use of thoughtfully-crafted software will greatly enhance the ability of teachers and students to achieve these mathematical objectives (Schwartz, 1999, p. 107).

Fey (1989b) states that the most prominent changes motivated by technology are in content and process goals. He suggests there should be “decreasing attention to those
aspects of mathematical work that are readily done by machines and increasing emphasis on the conceptual thinking and planning required in any tool environment” (Fey, 1989b, p. 238). In a similar vein, Bishop (1993) reflects social-reconstructivist characteristics (Ernest, 1991; Zuga, 1992), when he notes that in a technological environment: “rather than mastery of particular mathematical contents being the goal, individuals should be empowered through developing their mathematical thinking, their creativity, their imagery, their rationality, and also their critical faculties” (Bishop, 1993, p. 224). Holton (2005) agrees that it is time for a rephrasing of tertiary mathematics goals and he generally supports the approaches described above. However, he reflects some elements of Schoenfeld’s (1992) goals in his caution about redesigning curricula. Holton believes we need to be careful to ensure that our maths majors receive some idea of the main topic areas (such as algebra, analysis, applied and discrete mathematics), as well as an understanding of what mathematics is, and what mathematicians are trying to do.

Several studies do endeavour to specifically reframe the goals of a mathematics education in a technological environment, although few of these are located in undergraduate mathematics (e.g. Leigh-Lancaster, 2000; Stacey, Asp & McRae, 2000; Pierce, Turville & Giri, 2003; Cnop, 2005). Discussing the impact of CAS on the senior school mathematics curriculum, Stacey, Asp and McCrae (2000, p. 246) offer the priority list of goals for a CAS-active course shown in Table 2.3. This is not regarded as a definitive list, but rather as one for stimulating debate. For example, they question the meaning of better learners in the third goal; does this mean being able to solve a greater range of problems, or being able to solve current problems more easily and reliably? They suggest that becoming better users of mathematics “will surely require relatively more emphasis on formulating problems in mathematical terms, and relatively less emphasis on methods of solving”, the implication being that the CAS will facilitate the latter (Stacey, Asp & McCrae, 2000, p. 247). Although Stacey, Asp and McCrae note that the situation regarding CAS in tertiary mathematics is much less clear, it does not seem unreasonable to argue that these goals may readily transfer across to other forms of technology.
Table 2.3: Goals for a CAS-Active Mathematics Curriculum.  
(Stacey, Asp & McCrae, 2000)

1. To make students better users of mathematics.
2. To increase congruence between real mathematics and school mathematics.
3. To achieve deeper learning by students.
4. To promote a less procedural view of mathematics.
5. To introduce new topics into the curriculum.

One report that does focus on technology in a tertiary mathematics curriculum is that of Pierce, Turville and Giri (2003). They established a new series of goals for their course, incorporating both students’ and staff priorities. These goals, as listed below, reflect many of the arguments in the preceding discussion:

- Students who do our units should be able to reason mathematically, communicate and solve problems, as well as master algorithms and remember facts;
- Students should understand and appreciate the role of mathematics and its applications in the real world;
- Education students should form a positive view of their potential careers as mathematics teachers;
- Each unit should incorporate up-to-date technology and utilise methods that enhance student learning.

(Pierce, Turville & Giri, 2005, p. 156)

Exactly how their goals were achieved, and an examination of the results of their study, including students’ responses to the initiatives, will be considered in more detail in Chapter Three, with issues of implementation of technology. However, even though their course was redesigned ostensibly in response to the needs of what has been described earlier as ‘client students’ (education in this instance), the stated goals do provide a useful model for framing curricular goals, from a technological perspective, in other undergraduate mathematics courses, and should provide a useful basis for considering goals in this thesis. A suitably eloquent conclusion to this discussion of goals is provided by Lesh (2000), who stresses the importance of considering the effects of technology on the goals of mathematics education when he observes that:
One of the most important influences that technology should have on mathematics education is that many of the most important goals of mathematics instruction should consist of helping students develop powerful, shareable, and reusable conceptual technologies for constructing and making sense of complex systems. (Lesh, 2000, p. 72)

2.3.3 Content: What should we teach?

While this part of the discussion will focus on the technological aspects of particular content, there are other issues of curriculum content which should be examined. For example, the relative importance of any particular piece of mathematical content depends largely on who is doing the deciding, the priority they assign to the different societal aims of education, and their beliefs in the nature and role of mathematics in addressing those aims (Schwartz, 1999). The goals of the Academic and Technical curriculum designs (Table 2.1, Zuga, 1989), and the mathematical beliefs of the Technological Pragmatist and Old Humanist social ideologies (Table 2.2, Ernest, 1991), are clearly evident in Bishop’s (1993, p. 229) observation that “the justification for much of the existence of particular topics in the canonical mathematics curriculum has come from their assumed usefulness to society.” Reflecting on the impact of technology on this assertion, Bishop goes on to say that “…the validity and relevance of this ‘utility’ argument can now be seriously disputed as it becomes clear that computers and calculators can take over many of the previously needed human skills” (Bishop, 1993, p. 229).

Fey and Heid (1984) provide a typical sequence of nine topics in an algebra course (e.g. linear equations and operations, irrational numbers and radical expressions) for which they question the worth of teaching in a technological environment. They suggest that while it may well seem unnecessary for students to solve linear equations if the calculator or computer will do it, there may well be a need to learn these skills as an essential phase in building important understandings of variables, functions, and the problem-solving applications of algebra, although they claimed this was not supported by any research evidence at the time (Fey & Heid, 1984, p. 25). Artigue (2002) and Stacey (2003) extend this when they argue for a wider perspective in considering the value of specific topics in the curriculum. They both consider the pragmatic and
epistemic values of a topic, while Stacey identifies a third aspect described as the pedagogical value. The pragmatic value recognises the usefulness of a topic, while the epistemic value measures the importance of a topic’s place in the structure and development of mathematical knowledge. Pedagogical value considers whether a topic serves a purpose not related to the content itself, such as providing an opportunity to practice skills (Stacey, 2003, p. 6).

![Diagram](image)

**Figure 2.8** Topics can have epistemic, pragmatic and pedagogical value. (Stacey, 2003, p. 6).

Stacey (2003) examines several topics within this framework, some examples of which will be considered later in this discussion, when examining technology issues.

Many writers describe how consideration should be given to students’ difficulties with particular content areas, for example limits, vector spaces, the Fundamental Theorem of Calculus, and the definition of compactness (Selden & Selden, 1999; Artigue, 2000, Stewart & Thomas, 2003). Although considerable attention is given to specific content areas, most studies focus on pedagogical issues, including the use of technology, as opposed to explicitly questioning the inclusion or sequencing of particular topics. Algebra, for example, received extensive attention in the 12th ICMI Study Conference in Melbourne in 2001, which was solely dedicated towards the future of the teaching and learning of algebra (Chick, Stacey, Vincent & Vincent, 2001). The sequence of topics does receive some mention outside of technology discussions. Dubinsky, Dautermann, Leron and Zaskis (1994) strongly challenge the traditional linear sequencing that sees courses constructed on a logically coherent sequence of
knowledge, a view endorsed by Tall (1991). Tall perceives assisting students in the
difficult transition from pre-formal to formal mathematics as one of the most critical
problems facing tertiary mathematics teachers, and he suggests that a mathematician’s
logic inhibits his or her ability to design a suitable teaching schedule:

A mathematician often takes a complex mathematical idea and “simplifies” it by
breaking it into smaller components ready to teach each component in a logical
sequence. From the expert’s viewpoint the components may be seen as parts of a
whole. But the student may see the pieces as they are presented, in isolation, like
separate pieces of a jigsaw puzzle for which no total picture is available. In fact the
scenario may be worse. As the student encounters each piece of the puzzle (s)he
forms a personal concept image from the particular context which may be at
variance with the formal idea. Thus, not only is no picture available for the puzzle,
the pieces themselves may now have different shapes so that they no longer fit.

(Tall, 1991, p. 17).

Tall gives a specific example of the formal definition of a derivative, which
requires the notion of the limit of \( \frac{f(x+h) - f(x)}{h} \) as \( h \) tends to zero. The ‘simple’ steps
in which this problem is commonly broken up can easily lead to confusion for students
(Tall, 1991, p. 17). Anguelov, Engelbrecht and Harding (2001, p. 154) suggest that the
prevalence of linearly organised axiom-theorem-proof courses in South African
universities may be a reason why technology has had little impact on the curriculum.

Courses which focus on exploration within the topical content, practical
experience, self-discovery, exposure to the problems which led to the development
of the theory, and applications which are the reason for its continued existence, can
immediately make use of computer-based mathematical tools.

Dubinsky, Dautermann, Leron, and Zaskis put forward the spiral curriculum,
initially proposed by Bruner (1964, in Dubinsky, Dautermann, Leron, and Zaskis, 1994,
p. 30), as a partial solution to sequencing problems in undergraduate mathematics. The
spiral curriculum suggests that students be introduced to full-blown mathematical
concepts earlier, albeit in naïve forms. The topics are then revisited and considered
repeatedly at successively higher levels of sophistication. However, Dubinsky,
Dautermann, Leron, and Zaskis believe that the evidence of how people learn
mathematics suggests that neither a linear nor a spiral sequence is the whole answer,
describing the example of a student who appears for a short period to be at several different levels of development simultaneously (Dubinsky et al., 1994, p. 30). In a detailed examination of elementary analysis, Artigue (2000) describes a project which has undertaken an ambitious change in the curriculum. There is a suggestion of a spiral curriculum at work when she notes that the influence of analysis has started to emerge in the syllabus at Grade 10, a year before it is officially taught (Artigue, 2000, p. 9). Although Holton (2005) does not discuss content changes in great detail, he does not personally envisage great changes to the fundamental basis of what we call mathematics (albeit that what we understand to be mathematics is difficult to define). In addition to his earlier cited position that we must retain some inclusion of major topic areas, Holton suggests we should also teach some history of mathematics, especially to mathematics major students, to provide a sense of what mathematics is, and what mathematicians do. “This may require at least one course in the history of mathematics but it might be better for history to be peppered though all courses” (Holton, 2005, p. 307).

From a technological perspective, Bishop (1993, p. 237) saw little evidence that computers have affected the content of syllabuses, noting that mostly technology is “used to illuminate and enrich the teaching of traditional material”, and this is still seen as largely the case in later studies (e.g. Tucker & Leitzel, 1995; Ganter, 1999; Arnold, 2004). Most of the debate around the subject of curriculum content associated with technology is centred on the aspects of sequencing, and inclusion or exclusion of particular topics. A question almost universally asked, particularly in respect to CAS, is “What will be left to teach if students have access to tools which draw graphs, factorise and solve equations, and perform differential calculus?” (see e.g. Fey, 1989a, 1989b; Mercer, 1992; Milou, 1999, Arnold, 2004). The potential for a re-sequencing of topics is also commonly raised, with technology enabling us to introduce topics in different, and what some argue, are more natural orders (e.g. Penglase & Arnold, 1996; Smith, 1998). Harman (2003), for example, suggests teaching integral calculus before differentiation, a view shared by others (e.g. Dick, 1992; Bergsten, 1996). Instead of the traditional approach that begins with anti-differentiation, the advanced numerical methods and graphing packages that are now readily accessible to many students mean that we can address concrete *summative* problems of integral calculus first, following on with the more abstract notions of rates of change and the fundamental theorem later (Harman, 2003, p. 93). Smith (1998) describes how the traditional sequence of topics in teaching
differential equations (the *raison d’être* of calculus) can be turned on its head using technology. The power of CAS calculators to “draw direction fields and model population growth using real-world data can allow students to explore what a differential equation object is from day-one” (Smith, 1998, p. 784). The National Council of Teachers (1991) identified three areas of mathematics where it perceived technology could have a significant impact:

- Some mathematics becomes *more important* because technology *requires* it.
- Some mathematics becomes *less important* because technology *replaces* it.
- Some mathematics becomes *possible* because technology *allows* it.

Oates and Thomas (2001, p.83) develop this list further, suggesting that the following aspects should be considered with respect to content in curriculum reform:

- Content areas that may need to be or could be taught in a different order.
- Content areas that may be trivialised or made redundant by the use of the graphics calculator. This could include for example some routine algebraic skills.
- New content areas or richer conceptual understanding accessible using the graphics calculator.
- The possibility that some of the new content areas opened up by the graphics calculator may be trivial in nature and of limited educational value.

Oates and Thomas (2001) offer finding the inverse of a matrix as an example of the second point, since most CAS calculators will find the inverse of a matrix directly, leading us to question whether teaching such processes as Gaussian elimination are still useful. They note that this question is complex, and often elicits different responses from colleagues asked to comment on it. Responses seem to depend on both the respondent’s view of mathematics, and their awareness of the capabilities and processes employed by the technology (Stacey, 2003). One way of addressing this question maybe to consider it within the framework of *pragmatic, epistemic* or *pedagogical* values described earlier, by Artigue (2002) and Stacey (2003). This suggests a potentially rich area for exploration in this thesis, which will be developed in later chapters.

There are numerous examples of studies which examine technological issues with respect to the broader content areas of algebra, calculus, linear algebra, geometry, and statistics (e.g. Fey, 1989a, 1989b; Kaput, 1992; Dugdale, Thompson, Harvey et al.,
1995; Balacheff & Kaput, 1996; Tall, 1996b; Kaput & Roschelle, 1999; Thomas, 2001; Thomas & Holton, 2003), as well as specific topics within these broader areas, for example investigating the effects of technology on students’ understanding of the Riemann Integral (Hong & Thomas, 1997, 1998), or the use of CAS in differential equations (Fay & Joubert, 2005). Again, these studies largely begin from an acceptance, which is often implied, of these topics within the curriculum. They then examine how technology may be used to improve the teaching of these topics, or investigate obstacles to learning that may result from such technology use (Tall, 1989; Trouche, 2000; Doerr, 2001).

However, some studies do consider the effect of technology on the position of specific content areas in the curriculum. The place of geometry in the curriculum has progressively lessened by the move away from rigour and formalism. Dynamic geometry software allows students to develop experimental understandings of proof, and may thus have the potential to restore geometry to a significant place in the curriculum (Vincent & McCrae, 2000). Similarly for algebra, Usiskin (2004) observes that CAS can help change the public perception that algebra is not for everyone. Usiskin’s perspective is an interesting contrast to the usual arguments against CAS on the grounds that it trivialises much of the standard algebra curriculum. He believes that we must get CAS into the curriculum specifically because it can do things such as enable students to work with symbolic manipulation, which we have progressively either avoided, or pasteurised (stripped of difficult coefficients and non-integer solutions) and homogenised (problems of certain predictable forms). Arguing for a change in attitudes towards algebra, Usiskin concludes that:

The civil right is not merely the study of algebra, but the use of any available technology that may assist that study…we need to stress that CAS makes some algebra accessible to all that has only been accessible to a few, and that CAS empowers students to do important mathematics that they could not do easily without it. (Usiskin, 2004, pp. 9-10).

Oldknow (1995) provides an extensive list of topics exemplifying the facilities that were available on graphics calculators at that time. Since then, the available functions have grown at a considerable rate, to the point where it is now difficult to find content areas within the curriculum not affected by technology (Arnold, 2004). The
explosion in technological capabilities makes the task of deciding what to do more difficult, and brings with it the risk, as Kissane (2000b) cautions, that we adopt technology just because it can perform a certain process, without examining the value in doing it. Both Kissane (2000a) and Oldknow (1995) note the enthusiasm of technology companies in developing new facilities, with the rate of development exceeding the possible rate of curriculum change. Kissane warns that the research and development teams may have undue influence on curriculum development in such circumstances:

Are we getting to the point where technology companies are making de facto curriculum decisions for us? Are they paving the way, consciously or unconsciously, for their future leadership in that process…Are we doing our job as teachers or relinquishing part of it to the electronics industry?

(Kissane, 2000a, p. 67).

Kissane (2000a) asserts that the mathematics profession needs to be proactive in addressing curriculum development, and the categories of Artigue (2002) and Stacey (2003), described earlier, suggest a suitable means of assessing the relative values of a topic within a technological environment. Stacey (2003) observes that for a few topics, for example solving problems with exact values, having CAS increases the pragmatic value. More often, however, technology reduces pragmatic value, and sometimes questions the epistemic value. Stacey (2003) gives several detailed examples of how CAS may change the relative values of a topic. For example, using calculus to make a linear approximation to a function with the formula \( f(x + h) \approx f(x) + h.f'(x) \) once had a pragmatic value in permitting ready approximation to the values of complicated functions. One could, for example, readily approximate the square root of 26.1 as 5.11 (with correct value 5.1088 to four decimal places) based on \( x = 25 \), whereas a by-hand calculation is very long. Whilst even a simple scientific calculator removes the pragmatic value of this formula for the student, Stacey (2003, p. 7) suggests that it still has immense epistemic value, because it is central to the principles underlying calculus. More subtly, it also retains a pragmatic value for its use in many algorithms involved in the design of the calculator (although this may not have value for students). Another example is seen in differentiation of complex functions, where Stacey (2003) claims, with a few small provisos, that given the reliability of CAS in performing complex differentiation, the product-rule now has little pragmatic value. There is however a
strong case for inclusion on epistemic grounds, because it demonstrates the connections to other concepts. Stacey concludes that:

The curriculum value of topics is markedly changed by the introduction of CAS. Old justifications for teaching topics, especially pragmatic justifications, will not necessarily apply...The educational community now needs to build up sophisticated rationales for curriculum areas that were not debated in the past. Justifications may be on pragmatic, epistemic, or pedagogical grounds. (Stacey, 2003, p. 7).

However, while these categorisations may indeed prove very useful in assessing the relative values of individual topics, the overall impression gained from Stacey’s study is the enormous complexity of such a process, even for a researcher with considerable experience in educational theory, mathematics, mathematics teaching, and the use of CAS technology. The task of extending this across a much greater number of topics for an entire curriculum, and arriving at a consensus between the often diverse members of a mathematics department is, to say the least, ‘challenging’ (Stacey, 2003). Despite the seeming enormity of the challenge, Artigue (2006) believes we must view such complexity as an opportunity, not an obstacle.

Notwithstanding the previous discussion on the content of the curriculum, many writers believe that it is not the content itself, but the way in which it is taught that matters most (Noss, 1998; Holton, 2005). The current educational environment and the values of those responsible for curriculum development are predominantly social-constructivist. Schwartz (1999, p. 115) comments that “for those who particularly value the importance of...personal growth and development of students, it is likely that the specific mathematical content of the curriculum will be less important than the ways in which that content is engaged.” These issues are explored next.

2.3.4 Pedagogy: How should we teach?

Most discussions about pedagogy, both in schools and at tertiary level, suggest a shift away from teaching routine procedures and skills, to a greater emphasis on process, problem-solving, and development of conceptual understanding (e.g. Cockcroft, 1982; Fey, 1989b; Ministry of Education, 1992; Schoenfeld, 1992; Holton, 2005). The National Council of Teachers of Mathematics, in its publication Curriculum and Evaluation Standards (1989, p. 125) called for:
…a shift in emphasis from a curriculum dominated by memorization of isolated facts and procedures and by proficiency with paper-and-pencil skills to one that emphasizes conceptual understandings, multiple representations and connections, mathematical modelling, and mathematical problem solving.

Such a change is usually seen as equally critical in tertiary mathematics, where teaching is frequently described as being too formal or algorithmic (Smith, 1998; King et al., 2001, Holton, 2005). Dubinsky, Dautermann, Leron & Zaskis (1994), for example, observe that for many students, their early mathematical careers consist largely of learning algorithms to solve repetitive problems. They believe that the abrupt change from learning algorithms, to a requirement to understand complex concepts, may be the principal reason why students encounter so many difficulties with abstract algebra in most universities (Dubinsky, Dautermann, Leron & Zaskis, 1994, p. 27). Ball (1988) attributes much of this algorithmic style of teaching to mathematician’s own formative experiences as pupils watching other teachers; “in all too many mathematics classrooms, the teacher (or the textbook) is the authority, theorems are proved by coercion-not reason-and confusions are addressed by repeating the steps in excruciatingly fine detail” (Ball, 1988, p. 45). In support of this observation, Kline (1977), himself a mathematician, characterises mathematics teachers as pedagogically naïve stating that:

Mathematicians have a naïve idea of pedagogy. They believe that if they state a series of concepts, theorems, and proofs correctly and clearly, with plenty of symbols, they must necessarily be understood. This is like an American speaking English loudly to a Russian who does not know English. (Kline, 1977, p. 117)

In an attempt to address such naivety, and stimulate thinking about pedagogy in tertiary mathematics teachers, Lauten, Graham and Ferrini-Mundy (1999) pose the following questions, as a basis for dialogue in initiating calculus reform in tertiary mathematics departments:

- How would you characterize your department’s approach to calculus instruction? What are the more important features, the less important?
- Is using technology the basis for your efforts? Why or why not?
- Do your efforts encourage student engagement with calculus? How do you know?
• What role do the following pedagogical methods play in your approach: cooperative groups, projects, student presentations, using concrete materials to teach calculus? What solid evidence do you have that these techniques work or that the lecture method works?

• What does "rigor" mean to you, and what importance do you give "rigor" in the teaching of calculus?

(Lauten, Graham & Ferrini-Mundy, 1999, p. 238).

Schoenfeld (2001) agrees with such an examination of practice, and suggests we need more assistance from researchers on several fronts, including descriptions of both positive and negative consequences of various forms of instruction. Holton (2005, p. 307) makes a special plea for such assistance at the tertiary level, saying “as a (tertiary) practitioner, I’d like that help now. We do have a lot to learn about teaching, learning and understanding and I could use that information today.”

Technology is perceived as playing a key role in facilitating the shift in emphasis away from routine, computationally-intensive mathematics, especially in tertiary mathematics (King, Hillel & Artigue, 2001; Thomas & Holton, 2003). Pedagogically, Leigh-Lancaster and Stephens (1999) note two key elements to the impact of CAS on teaching, firstly as a supporter and enhancer of current teaching practice, and secondly, as a stimulus for developing new teaching approaches. CAS calculators and computers perform many of the previous routine calculations and algebraic manipulations required of students, thus allowing teachers to allocate more time to conceptual thinking (Dick, 1992; Kaput, 1992; Groves, 1996; Stacey, 2003). Technology also allows for greater experimentation and focus on problem solving, visualisation, opportunity for repeated exposure to multiple dynamic representations, and the ability to mediate between these linked representations (Kaput, 1992; Lee, 1993; Kissane, 2000a; Selinger, 2001; Thomas, 2001; Kutzler, 2003). Fey (1989b), for example, signals the early promise of a revival in geometry when he suggests that dynamic geometry software such as *Cabri* (see Laborde, 1990, 2004; Vincent & McCrae, 2000) will

…restore the act of discovery and conjecturing to a geometry course that has become a deadly routine of proving things that have been well known for centuries, and to facilitate inductive reasoning by making multiple tests of conjectures easy to execute. (Fey, 1989b, p. 246)
Some studies note that CAS allows students to focus on the “big picture” (Stacey, 2003), allowing students to concentrate on higher-order concepts and overarching issues, instead of being repeatedly interrupted and constrained by complicated calculations or lower-level algebra skills, which sometimes they either don’t possess, or make simple mistakes in performing (Kutzler, 2003). Ruthven (2005), for example, notes that computer graphing helps students get over the stumbling block of having to draw the graph in the first instance, allowing them to concentrate on the features and properties of the graph. For algebra, Warren and Pierce (2004) observe that CAS enriches many students’ interactions and requires a change in the focus of exercises and the style of questions asked. In particular, CAS enables the generation of many correct examples from which students can generalise, and allows them to easily swap between representations allowing them to build schema from a number of perspectives (Hong, Thomas & Kiernan, 2000). Kutzler (2003) provides an example using Gaussian elimination where technology can be used in this way, as a means of scaffolding weaker students, supporting the learning process and traditional teaching goals, while helping students acquire intellectual and mathematical skills (Kutzler, 2003, p. 69). Thomas (2001) describes the contrast between the past and present pedagogies for CAS calculators in the algebra curriculum when he states that they enable:

…investigation of mathematical concepts both within and between different representations. This opens up the possibility for students to interact with concepts in a variety of representations in close time proximity, exploring the links between them through their manifestations. In contrast, an emphasis on procedural aspects of a concept tends to anchor (a student) in one representation, for example, the symbolic one, making it more difficult to move between representations.

(Thomas, 2001, p. 584)

The advanced graphical and algebraic features of CAS are also attributed with the ability to transform the teaching of differential equations. King, Hillel and Artigue (2001) observe that technologies that graph slope fields and direction fields enable students to engage in qualitative analyses of previously inaccessible differential equations, rather than just the traditional analytic techniques. “Thus, the focus of a ‘differential equations’ course could shift from just finding the solution functions, to graphically organising the space of solution functions….and examining the nature of the solution functions” (King, Hillel & Artigue, 2001, p. 351). Pierce, Turville and Giri
(2003) use a variety of technologies (spreadsheets, CAS calculators and dynamic geometry software) in their courses to alleviate the tedium of repetitive tasks, underpin initial weakness in algebra, explore patterns in calculus, and explore variant and invariant properties of geometric shapes. In describing their experiences with this course, Pierce, Turville and Giri (2003) concur with Lesh (2000, p. 72), who states that:

…the new conceptual tools [technologies]…involve dealing with the old topics in new ways that emphasise mathematics as communication (description, explanation) more than mathematics as rules for symbol manipulation.

A useful and frequently cited summary (see e.g. King, Hillel & Artigue, 2001; Thomas & Holton, 2003) of the potential pedagogical benefits of technology is provided by Hoyles (1998), who describes considerable evidence of the computer’s ability to:

- foster more active learning through experimental approaches;
- provoke a constructivist approach to learning;
- motivate explanations in the face of unexpected feedback;
- foster co-operative work;
- open a window on student thought processes.

(Hoyles, 1998)

Schwartz (1999) relates such benefits to the attainment of society’s educational goals, when he considers how technology may be used to enhance five aspects of mathematical activity: conjecturing and exploring; acquiring, evaluating and analysing data; modelling one’s world; conceptually grounding manipulative skills; and deepening and broadening understanding. Stacey, Asp and McCrae (2000) look to synthesise both Hoyles (1998) and Schwartz’s (1999) examples, when they suggest appropriate mechanisms for CAS in achieving each of their five proposed goals for a CAS-active mathematics curriculum (see Table 2.3). For example, they suggest that CAS can help achieve the goal of deeper student learning by: promoting positive learning strategies; providing multiple, easily-linked representations of ideas; freeing up curriculum time; and providing “trainer wheels” for learners (Stacey, Asp & McCrae, 2000, p. 250). Evidence that these benefits are being found in practice is provided in a survey of secondary classroom teachers by Ruthven (2005). His summary of classroom teacher’s perceptions of the main contributions of technology to their teaching reflects most of the elements of the preceding discussion:
• Effecting working processes and improving production.
• Supporting processes of checking, trialling and refinement.
• Enhancing the variety and appeal of classroom activity.
• Overcoming pupil difficulties and building assurance.
• Focusing on overarching issues and accentuating important features.


At the tertiary level, Thomas and Holton (2003) provide a detailed examination of the role of technology in undergraduate mathematics, which extends many of the ideas previously discussed here, and includes a theoretical discussion of the role of technology in constructing mathematical concepts. They consider, for example, the work of Tall, Thomas, Davis, Gray and Simpson (2000), and Gray and Tall (1993; 2001), on the construction of mathematical objects, which they describe as a fundamental construct of advanced mathematical thinking (Thomas & Holton, 2003, p. 354). Thomas and Holton (2003) later describe the work of Dubinsky, Dautermann, Leron and Zaskis (1994), who provide a theoretical perspective (APOS theory) on the use of technology in, for example, helping students to avoid misconceptions, and the prerequisites for knowledge construction. Thomas and Holton also describe a variety of different teaching styles incorporating technology that have been tried at the tertiary level, for example multi-media presentations, and internet and web-based teaching. A more detailed discussion of these issues will be carried out in Chapter Three.

However, while the preceding discussion suggests an exciting range of possibilities for pedagogical change using technology, most researchers and practitioners also warn of problems and difficulties associated with issues such as the indiscriminate use of technology; inadequate student and teacher facility with using the technology; and the difficulty in deciding just how much mathematics we should allow the technology to perform. With respect to the latter, for example, Hillel (1993) observes that the dilemma posed by CAS for algebra is that they do well exactly those things that we expect our students to know and on which we base our evaluation of their knowledge. The real tough pedagogical question is to discern “how much of the manipulative aspect can be eliminated while still sustaining conceptual learning” (Hillel, 1993, p. 29). Flynn, Berenson and Stacey (2002) conducted an interesting survey that examined secondary and tertiary teachers’ beliefs about the relative merits of various algebraic skills in a CAS-permitted mathematics course, such as factoring,
equation solving and differentiating. Their results, which they emphasise are only preliminary, showed an increasing trend for teachers to allocate time-consuming items to being done by CAS as opposed to by hand, and also considerable favour for doing differentiation using CAS (more than 80% of participants).

In a similar vein, but with more of a focus on what should be retained, Herget, Heugl, Kutzler and Lehmann (2000) consider indispensable manual calculation skills in a CAS environment. While they acknowledge that lower-level manual skills may be in less demand, they suggest many skills are even more necessary, and they provide several examples of each using a classification system of their own design, to distinguish between indispensable manual skills and those which could be allocated to CAS. Interestingly, even within this small group, they provide nine examples on which they were unable to agree as to the correct classification within their own framework. In calculus, while technology provides an excellent mechanism for visual representations, Tall (1991) warns that visual ideas without links to the sequential processes of computation and proof are insights which lack mathematical fulfilment, and he later expresses doubts about the value of multiple representations for students, believing that students don’t necessarily think in the same flexible manner that allows mathematicians to shift between representations using the most appropriate one for the given task (Tall, 1996c, p. 5). Tall’s views are shared by Selinger (2001) and Thomas (2001). Selinger believes that technology can lead to an over-emphasis on the visual, with the risk of neglecting the more deductive, idealised and symbolic aspects of mathematics. Thomas (2001, p. 584) states that for conceptual algebra, “even having different representations of concepts and linking these to ‘real’ problems through modelling does not mean that students will necessarily build rich conceptual schemas with links between (the) representations”.

Many studies discuss the obstacles and difficulties students encounter in the use of technology (e.g. Ruthven, 1996; Guin & Trouche, 1999; Doerr & Zangor, 2000; Drijvers, 2000, 2002, Pierce, Herbert & Giri, 2004). Trouche (2000) notes student difficulties in the interpretation of graphical representations produced by the machines, and Galbraith (2005) describes the tensions students experience between the competing authorities of machine output and mathematical integrity. Galbraith cautions against inappropriate use of technology, when he quotes Zorn’s (2002) observation that “we’ve all seen students floating untethered in the symbolic ether, blithely manipulating
symbols but seldom touching any concrete mathematical ground” (in Galbraith, 2005, p. 12). Hoyles and Noss (2003, p. 341) agree with this observation, warning that “technology-driven mathematics is not the same as maths per se and neither is the knowledge that learners develop”.

Oates and Thomas (2001) use the analogy of “throwing out the baby with the bathwater” to reflect this tension between maintaining traditional curricula objectives and adopting new technological imperatives. The key question is whether strategic knowledge can develop without prior mastery of component procedures (Hillel, 1993), and most discussions in this area now focus on a re-sequencing of skills and algorithms, as opposed to abandoning them altogether, along with the need for appropriate attention to learning how to use the technology by both teachers and students (e.g. Heid, 1988; Mercer, 1992; Artigue, 2000). One solution to concerns about students losing valuable manipulative skills was provided by Heid (1988), who suggested merely changing the traditional sequence of teaching skills and concepts, as depicted in Table 2.4.

**Table 2.4**: Sequence of Skills and Concepts in Experimental and Traditional Versions of Applied Calculus (Heid, 1988, p. 7).

<table>
<thead>
<tr>
<th>Weeks</th>
<th>Experimental</th>
<th>Traditional</th>
</tr>
</thead>
<tbody>
<tr>
<td>1–12</td>
<td>Emphasis on concepts. Computer does algorithms.</td>
<td>Emphasis on skills</td>
</tr>
<tr>
<td></td>
<td>Assignments, quizzes and examinations emphasise</td>
<td>Students do algorithms</td>
</tr>
<tr>
<td></td>
<td>concepts.</td>
<td></td>
</tr>
<tr>
<td>13–15</td>
<td>Emphasis on skills</td>
<td>Assignments, quizzes and examinations emphasise</td>
</tr>
<tr>
<td></td>
<td>Students do algorithms</td>
<td>skills.</td>
</tr>
<tr>
<td></td>
<td>Assignments, quizzes and examinations emphasise</td>
<td></td>
</tr>
<tr>
<td></td>
<td>skills.</td>
<td></td>
</tr>
</tbody>
</table>

This early proposal by Heid offered to alleviate the concerns of traditionalists, that technology would erode skills, by maintaining elements of traditional teaching. Later discussions have moved towards more fundamental changes to the curriculum that see, for example, greater congruency between the use of technology and assessment (see Leigh-Lancaster, 2000, on the use of technology in examinations). Heid (2003, p. 38) herself cites many studies (e.g. Heid, 1988, 1992, Palmiter, 1991) which provide
evidence of the role of CAS as a reorganiser, to facilitate adjustment in the balance between concepts and procedures. Students in these studies spent more time interpreting symbolic results and applying concepts, than on routine manipulative procedures, and Heid concludes that while such results can not necessarily be directly attributed to CAS, “using a CAS before developing related by-hand skills seemed to help students learn concepts in greater depth than the traditional skills-before-concepts curricula” (Heid, 2003, p. 38).

Other studies suggest that students need new skills to use technology and that these skills need to be overtly developed (e.g. Dick, 1992; Artigue, 2000; Doerr, 2001; Drijvers, 2000, 2002; Trouche, 2000; Guin & Trouche, 1999, 2000). Ruthven (1996) observes, for example, that students have to use technology frequently to maintain technical proficiency, and that regular use was associated with more positive attitudes to technology. Pierce and Stacey (2004) state that in order to realise any benefits from use of CAS, students need to learn to operate the technology effectively, and that this has both cognitive and affective aspects. Affective factors may include students’ and teachers’ attitudes towards the use of technology. For example, Kendal and Stacey (2001a) found that teacher privileging influenced student learning, with students acquiring similar preferences towards CAS to those emphasised in the personal teaching styles and attitudes of their teachers. Cognitive aspects include for example the theoretical analysis of the transformation of an artefact into an instrument, or instrumentation, in such studies as those by Guin and Trouche (1999, 2000) and Artigue (2002), that build on the work of Rabardel and others (e.g. Rabardel, 1995; Vérillon & Rabardel, 1995).

Stewart, Thomas and Hannah (2005), for example, describe how students interact with CAS systems in first and second-year undergraduate mathematics courses, identifying four primary types of students with respect to their computer instrumentation. Other studies that investigate theoretical aspects of students’ use of technology build on the work of Gibson (1977) and Kennewell (2001), examining the concepts of affordances and constraints as a means of describing and developing students understandings and interactions with technology (e.g. Doerr, 2001, Brown, Stillman, & Herbert, 2004; Brown, 2006). Theoretical issues such as these and other specific aspects of student interactions with technology will be revisited in Chapter Three.
The preceding discussion clearly demonstrates that technology also has significant pedagogical implications for teachers. Kissane (1993), for example noted that perhaps as many as 90% of traditional exercises in a random sample of upper-secondary school text books could be removed if students had access to CAS, which would necessitate a major change in teacher-directed student activity, and greatly limit teacher’s appropriate resource materials. Wong (2003) believes a paradigm shift is required for both the teacher and the school system, a view which is supported by Guin and Trouche (1999, p. 199) who said that “the new role of the teacher is to organise and encourage interaction with the computer environment”. Teaching should shift from the format of “definition → theorem → proof → corollary → application” to “problem → experiment → conjecture and idea of proof” (Cnop, 2001, p. 188). Wong (2003) proposes the model by Branson (1990), reproduced in Figure 2.9, as a means to describe this paradigm shift, from the teacher as the central pedagogical figure, to one mediated by technology. This model agrees with the general social-constructivist trend described earlier in this chapter, away from a largely text-book-based teacher-centred pedagogical approach, to a more interactive and learner-centred approach.

![Figure 2.9](image_url)

**Figure 2.9** Schooling models of the past, present, and future.

Wong (2003) is careful to note that the suggested model is not without critics or problems. Koblitz (1996), for example, is critical of such a technology-centred model,
and clearly favours a role for technology as an assistant to mathematics, as opposed to being the focal point as implied in this model. The importance of the student’s ability to interact effectively with the technology, as described earlier, can also clearly be identified in the links in this model. Another critical factor can be seen from the links with the teacher in this model, and many studies identify the need for adequate training and professional development of teachers, in order to successfully adapt such a model for effective teaching (e.g. Stacey, 2003; Thomas & Holton, 2003; Wong, 2003). Hong and Thomas (2006), for example, suggest that teaching mathematics with technology requires a specific type of knowledge, which they describe as pedagogical technology knowledge (PTK), or knowing the principles and techniques to teach mathematics using the technology. PTK is an adaptation of the concept of pedagogical content knowledge (PCK) developed initially by Shulman (1986) and later extended by others (see e.g. Chinnappan & Thomas, 2003). Teacher training and professional development will be considered in more detail later, in Section 2.4.

Integration of Technology

The model depicted in Figure 2.9, with technology (expert system) as a central part of the pedagogical process, evokes the picture of technology integrated into the curriculum that is central to this thesis. As suggested in the introductory chapter, the concept of integration of technology, although promoted by a large number of studies, is used in a wide variety of ways, and is often only vaguely defined (Oates, 2004a). In general, it seems the notion of integration is used to distinguish between the two ways in which technology may be incorporated in the mathematics curriculum. The first, as suggested by the model in Figure 2.9, is as the focus or centre of attention of part or all of the curriculum, which it seems is the notion most studies are referring to when they discuss integration. The alternative is a shallower approach, which sees the technology act as a supporter or provider of additional resources to the curriculum (Stewart, 1991). Most studies support the former position as providing the most effective use of technology, especially at the tertiary level. Kemp and Kissane (1995) believe that it is inappropriate to regard technology as an add-on to a mathematics course, since this will inevitably lead to it being regarded as a frill, to be readily neglected by many. Bishop and Davies (2000) state that “it is clear that in UK higher education institutions, the integration of technology into the curriculum, rather than employing bolt-on activities
in the area, produces the most effective results” (p. 55). Kaput (1992) similarly criticises the notion of “bolt-on activities”, describing retrofitting technology into the curriculum as a largely ineffective strategy. Hong and Thomas (1997, 1998) and Harskamp, Suhre and Van Streun (2000) all argue for a more integrated approach to technology and curricula, with the latter noting that “it would be especially interesting to design a curriculum in which the role of the graphics calculator is to bridge the gap between a procedural and a structural concept of functions” (Harskamp, Suhre & Van Streun, 2000, p. 51). Hillel (2001a, p. 67) observes that while some uses of the computer in undergraduate mathematics can still be deemed to be ‘cosmetic’, many institutions have substantially redesigned their courses to fully integrate the use of various technologies. Buteau and Muller (2006) provide an example of this in the twenty-five year evolution of integrating digital technology in the teaching and learning of mathematics at Brock University in Canada.

There are also a large variety of similar terms used to evoke similar concepts to that of technology integration. Henderson (2002) and Das (2006) both describe the process of blending or incorporating technology into the curriculum, although their discussions may imply a view of technology as a supplement to traditional teaching practices, just not quite in the “bolt-on” approach described earlier. Healy (2006) describes a similar approach using the term insertion to describe the process of integration of computers into Brazilian schools, and like Henderson (2002) and Das (2006), questions the effectiveness of this approach. “Attempts to insert digital technologies...have tended to emphasise the computer as a catalyst for pedagogical change, without acknowledging the epistemological and cognitive dimensions associated with such change or the complexity associated with the appropriation of tools into mathematical and teaching practice” (Healy, 2006, p. 23). In a similar vein, Heid (1997, p. 9) refers to “technology-intensive instruction” which assumes constant student access to technology tools, and she not only agrees with the view that this requires a redefinition of epistemological authority, but believes this realignment is desirable. Stacey, Asp and McCrae (2000) and others infer the concept of integration in their description of a CAS-active curriculum, Dana-Picard and Kidron (2006) refer to “pedagogy-embedded CAS”, and Heid (2003) makes similar inferences when she refers to CAS-intensive curricula. Arnold (1998) and Brown (2006) both describe technology-rich teaching and learning environments (TRTLE’s), which clearly invoke the concept...
of integrated technology; Noss and Hoyles (1996) present similar concepts in their descriptions of *microworlds*. However, regardless of the similarity in terminology used, few of these studies attempt to fully define what they mean by their conception of integration. Stacey, Asp and McCrae (2000) cover a large number of aspects of the curriculum (e.g. goals, assessment) in discussing their CAS-active curriculum, but do not overtly identify which of these they might regard as the necessary or defining elements of integration. Dana-Picard and Kidron (2006) focus largely on the constraints of the technology that affect student’s integration of CAS, while Heid (2003) and Brown (2006) largely consider theoretical perspectives of their proposals. Brown (2006), for example, gives some suggestions as to what a technology-rich teaching and learning environment should resemble, when she discusses the *affordances* of technology in elucidating the role of the teacher in such an environment (see Gibson, 1977; Brown, Stillman & Herbert, 2004), but there are a number of other aspects (e.g. goals, assessment) that she does not consider.

The study by Dana-Picard and Kidron (2006) also introduces another way in which the term *integration* is sometimes used with respect to technology. While most often it is used, as it is in this thesis, to describe the way in which technology is incorporated into the curriculum, it is also used in other ways, and we need to take care to distinguish such usage. Dana-Picard and Kidron (2006) use it in their study to refer to the degree to which students have *integrated* technology into their learning, or have attained *instrumental genesis* (Guin & Trouche, 1999; Artigue, 2002). Thomas and Hong (2004) use the term integration in a similar fashion, describing how students develop a partnership between themselves and the CAS technology, resulting in the integration of CAS-calculators into their mathematics learning. Similarly, Goos (2006) uses the term integration with respect to the degree in which teachers have integrated technology into their practice, noting that improved access to technology alone does not result to increased use or changes in practice. While Hoyles, Noss and Kent (2004) agree that developing ways to understand and foster the process of instrumental genesis for students and teachers is a key challenge in the overall integration of technology into the classroom and the curriculum, clearly there is a distinction between the global *integration* of technology into the curriculum, and the students’ and teachers’ individual integration, which is really a subset of the former.
Usually, what is meant by integration of technology into the curriculum is left for the reader to infer. A good example of this can be seen in two discussions of integration of technology in Singapore by Koh, Koh and Wu (2004) and Kho (2006). Both of these describe an intensive process of research and discussions in a master plan aimed at achieving curriculum change with integrated technology. While the discussions identify many areas where the place of technology in the curriculum is considered (e.g. in supporting the development of “reasoning, communications and connections” Kho, 2006), and stress such factors as professional development of teachers in achieving effective integration, there is no clear statement of what the final integrated product should resemble. Indeed, the Singapore school mathematics framework is presented in a detailed diagrammatic model that has five inter-related components, Concepts, Skills, Processes, Attitudes and Metacognition, but it does not feature technology specifically anywhere in the framework (Koh, Koh & Wu, 2004, p. 19; Kho, 2006, p. 268).

![Diagram](image)

*Figure 2.10* Creating a technology rich learning environment.


The technology rich learning environment depicted in Figure 2.10 implies a similar notion to integration as described earlier by Brown (2006). Like the Branson
model shown in Figure 2.9, Arnold (1998) places technology at the centre, but he provides more links between the technology and teachers and students, distinguishing between different ways of interacting with the technology, from pedagogical and mathematical perspectives. While Arnold’s model is based on his observations in the secondary school algebra domain, he believes that his findings offer useful information regarding the ways in which software tools may be best used for teaching and learning of mathematics at all levels (Arnold, 1998, p. 31).

There are some studies which provide suggestions for integrated technology at the tertiary level specifically. Lavicza (2006) describes an ambitious project to assess the extent of CAS use and the factors influencing its integration into university curricula, similar to one of the aims of this thesis. However, while Lavicza poses several interesting questions about the use of technology in undergraduate mathematics, again exactly what is meant by integration is not identified. Kemp and Kissane (1995) suggest that adequate integration of technology into undergraduate mathematics should consider all aspects of the course; the mathematical content, lectures, tutorials, workshops, assignments, tests and examinations. They don’t however indicate a consideration of other components of the curriculum identified in the Valero-Duenas (2002) curriculum model, for example goals and aims. Stewart, Thomas and Hannah (2005) support Leigh-Lancaster’s (2000) belief that true integration of technology necessitates a congruency between curriculum, pedagogy and assessment. While this comes closer to a definition, their evidence suggests that in addition, students require time to make progress with instrumentation (Stewart, Thomas & Hannah, 2005, p. 748). Thomas and Holton (2003) give an extensive review of the use of technology as a teaching tool at university level, which covers most aspects of curriculum considered within the Valero-Duenas (2002) model. However, while Thomas and Holton’s description is quite exhaustive, and certainly a very useful model may be gained from synthesising the various elements discussed in their review, they too do not provide a clear description of what they mean when they make reference to integrated technology. Like Thomas and Holton (2003), another meta-study, by Lagrange, Artigue, Laborde and Trouche (2003) provides an excellent coverage of the issues underlying the concept of integration. This is an extensive synthesis of the literature about research and innovation in the world-wide field of integration of technology from 1994 to 1998. They emphasise the importance of considering a plurality of perspectives when addressing the issue of integration, and
they construct a multidimensional framework and a data-analysis procedure as a basis for considering the complexity of technology integration (Lagrange, Artigue, Laborde & Trouche, 2003). Their framework identifies seven dimensions for the analysis of integration of technology, and for each of these, they designed a set of indicators as a means of statistically analysing (using cluster analysis) the dimensions, looking for trends in the findings of the studies they reviewed. Their dimensions and indicators, like the range of factors identified by Thomas and Holton (2003) provide an excellent basis for considering what might constitute integrated technology, but again the concept itself is not explicitly explained.

The preceding discussion confirms the elusiveness of integrated technology as a definitive concept, and supports the objective of this thesis to investigate this. The next section will consider the relationship of teacher professional development to implementing technology into the curriculum.

2.4 CURRICULUM CHANGE AND PROFESSIONAL DEVELOPMENT

2.4.1 Effecting Change in the Curriculum and Teacher Practice

As noted frequently within the preceding discussions, curriculum change in general, and with respect to technology in particular, is just as difficult to achieve as it is to sustain. Most research concludes that teachers are a critical element in affecting any useful change, with some reports emphasising that they are the most important ingredient (Ball, 1988; Kilpatrick & Davis, 1993; Begg, 1994; Knight & Meyer, 1996; Healy, 2006). In a report commissioned by the British Department of Education and Science into curriculum development, Ahmed (1987) states that “no effective far reaching curriculum development can take place without teacher development” (Statement 12, p. 34), and “no imported curriculum development exercise can be effective without working commitment and teacher involvement” (Statement 18, p. 43). With respect to technology in schools, Kilpatrick and Davis (1993) note that most attempts to enforce radical changes in practice have been subject to trouble and distortion and only rarely have original intentions been realised. “If innovation is to proceed more satisfactorily in future, then it is essential that we ensure better
understanding and acceptance by teachers” (Howson et al., 1981, p. 265, in Kilpatrick & Davis, 1993, p. 216). Understanding and acceptance are more easily achieved when teachers feel that they and their colleagues are part of the curriculum process, and such involvement suggests that curriculum and teacher development should be integrated (sic, yet another use of the term integrated) and mutually dependent (Begg, 1998). Begg observes however that such practice is seldom the case. The traditional linear model, of dissemination involving teacher development, following curriculum development, was the prevalent view in all of the nine international regions he surveyed. Support for a less linear approach is also evidenced by the fact that the most successful efforts in curriculum change seem to occur when the new developments are decentralised, as opposed to taking place in isolation from teachers and schools (Ahmed, 1987; Kilpatrick & Davis, 1993, Begg, 1994).

Many reports further observe that effective curriculum change requires some time, and that if change is to be sustained, then provision must be made for continued and regular professional development of teachers (Ahmed, 1987; Begg, 1994; Knight & Meyer, 1996). Although the area of professional development is a highly complex field, Mousley and Rice (1993, p. xi) conclude that it is widely accepted that professional development is more likely to be effective when:

- It is conducted in school settings over an extended period of time;
- Teachers are involved in planning activities for each other;
- Teachers collaborate with and help each other;
- Teachers are involved in independent, self-directed learning;
- Opportunities exist for trialling new ideas and receiving feedback on the process;
- There is on-going support and encouragement provided for the teachers’ work.

The preceding list illustrates factors frequently identified as critical, such as the need for adequate funding and resources, and time to attend appropriate professional development activities (e.g. Ahmed, 1987, Statement 20, p. 45; Begg, 1994, pp. 204-206; Knight & Meyer, 1996, p. 36). However, some factors identified as important in persuading teachers to change their practice are not overtly evident in the list. These include attention to affective factors, such as; teacher’s feelings about the proposed changes, their beliefs and values, confidence in enacting the changes, and their predisposition and commitment to change (Mousley & Rice, 1993; Begg; 1994,
Paterson, 2007). For example, Borko and her co-researchers (Borko, Mayfield, Marion et al., 1997) found that teachers’ knowledge and beliefs about teaching, learning and subject matter were critical determinants of whether and how they implemented new educational ideas.

For teachers to change their pedagogical practices they must have the knowledge necessary to implement the changes and the beliefs to support them. Thus efforts to help teachers make significant changes must also help them to acquire new knowledge and beliefs. At the same time, teachers come to understand new practices through their existing knowledge and beliefs. This dual role of knowledge and beliefs – as both targets of change and filters – can make fundamental changes in teaching practice difficult to achieve.

(Borko, Mayfield, Marion et al., 1997, pp. 271-272)

If teachers’ beliefs are incompatible with the intention of the development team, and they are not given an opportunity to challenge them, then teachers will either ignore them, or inappropriately assimilate them into their existing practice (Paterson, 2007). While there is much evidence that teachers can be resistant to change, this is not always the case, and even some strongly held convictions may be readily shed (Ball, 1988). Certainly just promoting change through research literature, or mandating it in official documents such as changes to syllabi, is ineffective (Ahmed, 1987; Paterson, 2007).

We should not take lightly the task of helping teachers change their practices and conceptions. Attempts to increase teachers’ knowledge by demonstrating and presenting information about pedagogical techniques have not produced the desired results. Indeed, the research…suggests that teachers’ conceptions of mathematics, of how it should be taught, and of how (students) learn it are deeply rooted…We should regard change as a long-term process resulting from the teacher testing alternatives in the classroom, (and) reflecting on their relative merits.

(Thompson, 1992, p. 143)

Pritchard and McDiarmid (2006) argue that teachers need adequate support to actively reflect on and discuss their teaching practice, in order to become autonomous learners. “Reflective practitioners do not continually look to outside agencies for professional development, rather they are instrumental in developing themselves as professionals” (Serafini, in Pritchard & McDiarmid, 2006, p. 438). This role of the teacher as “learner” is identified as a significant factor in teacher education and
professional development” (e.g. Ahmed, 1987; Ball, 1988). Paterson (2007) suggests that the change of roles from teacher to learner can be a crucial point for some teachers. She describes how perturbations and dissonance, or disturbances in a teacher’s existing views of teaching and learning that seem to occur when teachers are forced to identify themselves with a pupil, can lead to teachers re-examining their classroom practice. Thus, although teachers’ responses to being perturbed vary and the outcome clearly depends at least in part on the teacher’s beliefs and their existing mathematical knowledge, programmes that engender healthy disequilibrium are likely to achieve more effective professional development (Paterson, 2007). Goldsmith and Schifter (1997, p. 46) conclude that providing the impetus for teachers to re-examine their practice is imperative, stating that “this initial motivation is extremely important, and those who seek to provide professional development need to know how to encourage teachers to find a compelling reason to undertake the task of transforming their practice”.

Paterson (2007) further identifies teacher’s work situations, and their freedom to enact change, as an important factor in implementing that change. Teachers often know more pedagogic strategies than they use, but opt to use those that best fit their current circumstances, or those that have proved successful in the past. “Practices that are found to work – that is those that teachers find useful in helping students attain desired learning outcomes – are retained and repeated” (Guskey, 1986, p. 7). This provides teachers with a rationale for not changing, since change may lead to a less successful outcome (Paterson, 2007). Another consideration of factors effecting change in teachers’ practice is provided by Clarke and Clarke (1993), who observe that improvement in student learning is one of the most powerful factors in persuading teachers to change their practice.

2.4.2 Change and Professional Development in Tertiary Mathematics

Earlier discussions suggested that a zero-based curriculum (Ralston, 1997) may be a reasonable proposition when making technological changes to university courses, based on the premise that on the face of it at least, it should be somewhat simpler to make changes to curricula at the tertiary level, than at secondary or primary level. Certainly it is more likely that curriculum development is more localised within a
tertiary teacher’s own department, and individuals are frequently given greater control over what topics they teach and how they choose to teach their courses. There are of course restrictions placed on those responsible for courses. For example, they may have to include essential topics that are pre-requisites for other courses within the department, or be required to teach in large classes. Nevertheless, lecturers often still have significant autonomy over content choice, assessment, and delivery methods (Anguelov, Engelbrecht & Harding, 2001). However, tertiary curriculum change has its own inherent problems, some of which it seems are associated with the exact autonomy of lecturers that enables them to enact change. Changes are often not sustained from one year or semester to the next, and are confined to individual courses, instructors, or institutions. For example, despite significant changes to traditional algebra and calculus courses associated with technological advances, Balacheff and Kaput (1996) note that:

…the curricular impact of these innovations has been local rather than global in the sense of changing what takes place in courses at the various levels, but not changing the larger relations among courses or the ways in which the big ideas of these courses appear in the mathematical experience of most students, particularly at the lower levels (Kaput & Nemirovsky, 1995, in Balacheff & Kaput, 1996, p. 475).

Referring to changes in content initiated under the calculus reforms, Dubinsky (1994) believes that such a minimal impact from reform initiatives is inevitable, and that “after a fairly short period of relatively superficial manipulations, the set of topics in the calculus will settle down to pretty much what it was…what needs to be changed is not the content, but the pedagogy – not what we teach, but how we teach it” (Dubinsky, 1994, p. 1). Several studies investigate change in tertiary mathematics, especially with respect to calculus reform (e.g. Flashman, 1996; Brown, 1996; Silverberg, 1999; Ganter, 1999; Keynes & Olson, 2001). There are many similarities in the findings of these studies, and a consistency with the issues described in Section 2.4.1. For example, all the cited studies identify sufficient time to enact changes as a significant factor. “In general, regardless of the reform method used, the attitudes of students and faculty (teaching staff) seem to be negative in the first year of implementation, with steady improvement in subsequent years while making continuous revisions based on feedback” (Ganter, 1999, p. 235). Similarly, Silverberg (1999) advises those seeking
change to be patient and persevere, “be patient, changes take time, changes in long established habits take a long time” (p. 247). Other common themes are provision of training for staff (both initially, and as ongoing professional development); regular opportunities for reflection, discussion and evaluation; and a need to recognise the different teaching styles and attitudes valued by teaching staff and students. Keynes and Olson (2001, p. 124) suggest that professional development should adopt an approach to using modern pedagogy “which allows senior faculty to adapt different teaching approaches to more effectively teach the content that they as individuals value.” There are also several references to affective factors, such as attending to staff morale (Brown, 1996), and recognising that some staff may feel threatened by the changes (Flashman, 1996). Keynes and Olson (2001), for example, suggest that senior teaching staff maybe more likely to be more deeply rooted in their beliefs and values about what constitutes important mathematical content and essential computational techniques, and subsequently more resistant to change if this is at variance with their beliefs. They also believe that many tertiary mathematics teachers lack an awareness of the influence of their teaching and mathematical approaches on undergraduate students’ learning and views about mathematics, and further, also lack an understanding of a variety of different pedagogical strategies such as cooperative learning, team approaches to instruction, and mentoring (Keynes & Olson, 2001, pp 124-125). Flashman (1996, p. 51) believes that an additional factor is the presence of a “champion for reform” within the department, along with at least a small group of sympathetic faculty to assist.

Questions posed by Lauten, Graham and Ferrini-Mundy (1999) were described in Section 2.3.4, as a basis for initiating change. These questions were then used to inform the construction of a questionnaire that addresses most of the issues identified in the reports just described (See Appendix C). They suggest departments should use such a questionnaire to refine the issues of importance to them in their courses and their teaching. Their questionnaire asks questions ranging from a list of topics from which to assign priorities, through to possible classroom teaching approaches (e.g. lectures, group work), and questions about the use of technology. They suggest that in their experience, such a critical analysis of practice is essential if any useful change is to eventuate (Lauten, Graham & Ferrini-Mundy, 1999, p, 237). This questionnaire certainly provides useful pointers for ways to address the research questions of this thesis.
2.4.3 Change, Professional Development and Technology

Many of the issues described in the preceding discussions of general curriculum change apply equally to change as it relates specifically to technology. For example, a number of studies recognise that it is critical to understand the role of the teacher when attempting to integrate technology into the curriculum (e.g. Kissane, 2000a; Wong, 2003; Healy, 2006). Brown (1996) stresses the need for specialised training of staff in the use of technology, and Cheung (2000) identifies teacher readiness as one of three key factors in the potential success of IT programmes (the other two factors are the presence of adequate technical support, and the school’s approach to integration). Kissane (2000a, p. 69) agrees, saying that it is simply unreasonable to expect teachers to shoulder the responsibility of adjusting the curriculum to the influence of technology without a great deal of help. Kaput (1992) supports the view that technology itself does not promote change. He states that “the difficulty in integrating the computer work with the on-going flow of curriculum and instructional activity is closely related to the level of effort and expertise” (p. 548).

However, even adequate teacher preparation and high levels of motivation are by themselves not sufficient to ensure successful integration. Wong (2003) describes a recent survey of Hong Kong mathematics teachers that found a low use of technology, even though IT is highly regarded by the teachers (p. 311), and Guin and Trouche (1999) found a similarly low use of graphics calculators in teaching in their study (at most 15% of teachers), despite the fact that all students were required to have a graphics calculator (p. 195). Some of this may be attributed to Cheung’s (2000) other two factors identified previously, although Kaput (1992) suggests in addition that applicability of the technology is important. “Software that is hard to use is likely to be applied by only a small group of expert innovators and only in circumstances that specifically support that activity” (Kaput, 1992, p. 548). Lack of time to address the complex demands facing teachers implementing change (for example training, setting up equipment, preparing new resources) are also identified by many studies as a factor in limiting change (Flashman, 1996; Roberts, 1996; Wong, 2003), as are teachers’ beliefs about the role of technology (Kissane, 2000a). Valero (1997) observes that teachers encounter conflicts between their beliefs and experience in teaching, and the new role demanded
of them in a technological environment. Some basic elements of the curricular system are altered, for example calculators give students a new autonomy with respect to the teacher’s authority as the source of knowledge and truth, which causes a shift in the way students typically view the teacher. Such responses can lead to discomfort and resistance from teachers, and it is important to provide moral support for teachers to help them come to terms with these conflicts (Brown, 1996; Wong, 2003). Like change in general, the process of changing beliefs with respect to technology is a slow, continually evolving process. Teachers require time to reflect on their experiences, and continual motivation to maintain the transformation of their beliefs and practice (Valero, 1997, p. 192).

As a contrast to the many factors that may hinder effective change, computers may in fact act as a lever for curriculum change (Bowers & Doerr, 2001). They are useful mechanisms for initiating what was described earlier as perturbations or dissonance in a teacher’s practice (Paterson, 2007), thus causing them to reconsider their values and practice. Bowers and Doerr (2001) emphasise the importance of professional development programmes that support teachers’ interacting with technology in the role of a student. They investigated how teachers participating in the role of students think about the mathematics of change, using an exploratory computer microworld, and how this experience affected their role as a teacher, in planning, implementing and reflecting on lessons. The study revealed two critical perspectives, firstly, some powerful pedagogical insights emerged as teachers assumed the role of students, and secondly, they gained some powerful mathematical insights as teachers of mathematics (Bowers & Doerr, 2001, p. 133).

Effective teaching with technology demands specialist knowledge and specific learning skills that require appropriate training and professional development (Bowers & Doerr, 2001; Kendal & Stacey, 1999, 2001a; Thomas & Chinnappan, 2008). Artigue (2000) attributes some of the lack of success of technology innovations to the fact that “the education system does not easily recognise this fact and has little taste for dedicating the necessary time and energy to this learning” (p. 11). Thomas and Chinnappan (2008) suggest that teachers require time and assistance to develop PTK (pedagogical technology knowledge, as described earlier). PTK “necessitates a shift of mathematical focus to a broader perspective of the implications of technology for the learning of mathematics” (Thomas & Chinnappan, 2008, p. 169). They identify two key
aspects of PTK; firstly the need to appreciate the variety of ways in which technological tools may be used in their teaching, and the ways in which students may interact with the technology; and secondly, like students, the teachers themselves need to develop instrumental genesis (see Artigue, 2002) with the technology. The first aspect includes an awareness of how a teacher’s emphasis on different aspects and uses of the technology may shape a student's preferences, described earlier as teacher privileging (Kendal & Stacey, 1999, 2001a). The second aspect reflects how the teacher engages in actions and decisions to adapt the tool to a particular mathematical task, by considering the abilities of the machine, from both mechanical (or instrumental) and mathematical (conceptual) perspectives (Guin & Trouche, 1999). Morony and Stephens (2000, pp. 16-17) reflect this concept of instrumental genesis, when they argue for the development of a variety of strategies to support practitioners who have reached different stages of ‘comfort’ with graphics calculators, stages identified as ‘novices’, ‘practitioners’ and ‘creators’ in increasing order of sophistication.

Despite such evidence, studies that examine professional development at the university level and report on the outcomes of reform programmes reveal that instruction in the use of technology is usually minimal, and is seldom explicitly linked to the content material (e.g. Roberts, 1996; Thomas & Holton, 2003). While training and professional development in the use for technology for teachers in secondary schools is now commonplace, the situation for lecturers in undergraduate mathematics is far less positive:

While many countries (e.g., Canada – see Muller, 2001) now require their secondary school mathematics teachers to be trained to implement technology in teaching and in the pedagogical implications of doing so, there are far fewer initiatives in place for university lecturers to gain the same confidence and proficiency. (Thomas & Holton, 2003, p. 384)

Thomas and Holton (2003) believe that professional bodies (e.g. mathematical societies, or organisations such as DELTA (cf. p. 3, p. 11 this thesis), which focus specifically on university mathematics, can play a useful role in the provision of professional development, but they caution that more attention is needed to effective dissemination of research, and that the use of educational jargon in much of the research can be off-putting for university mathematicians. Keynes and Olson (2001) agree that
we need more effective professional development programmes in university mathematics. However, in order to effectively implement such programmes, they believe we need more carefully designed studies that demonstrate a clear relationship between the use of the technology, and improved student learning. There is also a need for studies that investigate how university mathematicians’ beliefs about content, pedagogy, and technology can influence and potentially hamper change. Such studies are critical if we are to persuade often sceptical staff to adopt new technologies in their teaching (Keynes & Olson, 2001). Keynes and Olson also believe there is a different perception of the role and use of technology by many research mathematicians, and they provide a suitable conclusion to this discussion, when they state that:

(There is a) general lack of knowledge and frequently deeply-held scepticism by senior faculty on the roles and value of technology to help students learn the mathematics that the faculty member values. For example, while many faculty are familiar with symbolic algebra programs and software in a research context, they are generally not aware of the issues involved in using them in a learning context. Thus, some faculty will use technology to visualise only complex or exotic mathematical graphs and structures, rather than using it to illustrate mainstream ideas and central examples. In addition, very few senior faculty are truly familiar with graphing (or CAS) calculators and their potential use in instruction.

(Keynes & Olson, 2001, p. 125)

2.5 CHAPTER TWO SUMMARY

Chapter Two has reviewed the literature on curriculum studies, both generally, and from a technological perspective. A definition was provided of the way in which the term ‘curriculum’ will be used in this thesis (Figures 2.1 to 2.3), along with suggested frameworks for considering curriculum design and development (Figures 2.4 to 2.6). Theoretical issues relating to curriculum design and technology were examined, and the position of technology within the proposed models of curriculum was considered, with the suggestion that a zero-based curriculum model may be useful when considering the integration of technology into the curriculum.

Section 2.3 discussed how views about the nature of mathematics, and mathematical knowledge, may impact on the design and development of curricula. The
educational ideologies of those responsible for curricular change, and their beliefs towards mathematics, education, and technology, were described as critical for effective change. Goals were also shown to be an important consideration in curriculum change, and technology was seen to require a restructuring of goals for mathematics education. It was noted that technology challenges the position in the curriculum of traditionally assumed content area, and that not just the place, but the sequence of particular topics may change. Next, the affect of technology on pedagogy was considered. Technology was seen as a way of shifting from an algorithmic and mechanic skills-based approach, to focus more on conceptual understanding, and was also shown to have positive effects and applications in many aspects of pedagogy. However, realising successful outcomes was seen to be somewhat problematic, given that there are complex factors involved. Finally in this section, the way in which the term integration will be used in this thesis was identified in the literature, as well as differentiating some alternative ways in which it is commonly used. It was suggested that in this thesis, the term integration will be used to describe the holistic incorporation of technology into the curriculum, and some examples that provide a useful basis for this description were provided.

Section 2.4 considered aspects of change, in particular the importance of training and professional development of teachers, and their beliefs about pedagogy, mathematics and technology, on effecting and sustaining change. It was suggested that particularly at the tertiary level, there is a need for improved professional development of teachers with respect to technology. It was also suggested that teachers’ experiences and beliefs about technology may be a significant impediment to successful integration of technology in tertiary mathematics.

Many of the pedagogical issues in particular presented in this chapter will be re-examined in the next chapter, when we consider wider theoretical aspects of technology-specific issues in mathematics education, look at some studies that examine the benefits and disadvantages of technology, and investigate factors affecting successful implementation and integration of technology.
"If one changes the tools of thinking available to a child, his mind will have a radically different structure.” (Vygotsky, 1978, p.126)

3.1 INTRODUCTION

The two key research questions posed in Chapter One are predicated on the assumption that an Integrated Technology Mathematics Curriculum (ITMC) is indeed an effective and desirable curriculum design. It is thus appropriate here to survey the literature for sufficient support to justify this assumption. Many studies in the literature make similar assumptions. Galbraith (2005, p. 12), for example, suggests that we should take it as axiomatic that technology in the hands of expert users greatly enhances their capabilities to solve problems, and that instead of questioning this assumption, we should focus on questions that emerge when technology is used for teaching and learning purposes. Lynch (2006), on the other hand, questions the common assumption that appropriate technologies, the requisite infrastructures, and ready access to the technology, are all available for those teachers and students who wish to use them:

Popular discourses tend to ascribe more agency to the technological artefact than is warranted, and government policies tend to focus on the ‘kitting out’ of schools and classrooms as sufficient action for effecting improved learning outcomes.

(Lynch, 2006, p.31)

Berger (1998) agrees that mere exposure to technology is insufficient to promote effective learning. Her study found little evidence of cognitive benefits of the graphics calculator. She contends this was due to the “add-on” nature of the way in which the tool was used. Berger concludes that the manner in which the technology is employed can profoundly affect the influence of this technology on the students. A proper
integration (however this is defined) of technology into the course and curriculum would significantly alter the traditional patterns of (knowledge) privileging, and hence the potential relationship between the user and the technology (Berger, 1998, p. 19).

The quote from Vygotsky that prefaces this chapter reintroduces theoretical aspects of technology, such as how various learning theories may be used to interpret learning from a technological perspective (e.g. constructivism and APOS theory). Other theoretical aspects which require further examination from a technological perspective include the concept of instrumentation which considers technology as a tool from a psychological perspective (Guin & Trouche, 1999; Artigue, 2002); the affordances and constraints of the technology (Gibson, 1977; Doerr, 2001; Brown, Stillman & Herbert, 2004); cognitive issues such as the formation of schemas (Skemp, 1979; Tall & Thomas, 1991); formalism (Tall, 1991; Balacheff & Kaput, 1996); and reflective abstraction (Dubinsky, 1991; Skovsmose, 1992). Considerations of curriculum goals, teachers’ and students’ beliefs, and decisions about which technology to use have all also been identified as significant issues in previous discussions.

The subsequent discussions in this chapter will be framed using three key questions that have emerged from the issues identified in the previous discussions:

- Why should we use technology in mathematics education?
- Which technology should we use?
- How may technology be integrated into the curriculum?

### 3.2 WHY TECHNOLOGY?

There are many perspectives that may be adopted when trying to justify the use of technology in mathematics education. Some studies begin by acknowledging the global social context of technology as a basis for justifying technology in the mathematics curriculum, espousing the view that we live in a knowledge or information society where technology is essential to develop, manipulate, store, access, analyse and communicate knowledge. For example, technological changes in industry and mathematics research require graduates to be familiar with a range of modern technology applications (Clayton, 1999; Anguelov, Engelbrecht & Harding, 2001; Steen, 2001, 2003), while commercially, there are huge sums of money involved in
technology companies convincing educational institutions to use their products. Enticing levels of ‘support’ are frequently offered to encourage use of their individual products (Kissane, 2000a; Butler, Cnop, Isoda et al., 2003). While government policies (and to a lesser extent funding) support attention to the infra-structure of technology, the implications for teaching and learning come much later, if at all (Leigh-Lancaster, 2002). At the tertiary level, one of the most important perspectives, in addition to preparation for a technological society, comes from the preparation of undergraduate students for other service courses. The skills that we as mathematicians deem critical to the mathematical development of our students may in fact be less important, given the significant changes in many academic and professional fields which use increasingly sophisticated technologies (Engelbrecht, 1998; Ganter & Jiroutek, 2000; Steen, 2003).

While such external pressures may lie behind the general consensus that technology cannot be avoided in mathematics education, other perspectives must be considered if technology is to be fully justified, and effectively adopted. If the ability to operate efficiently in a technological society was the only perspective, then there is a danger that the resulting education would produce only functional users of black-boxes, unaware of, and unable to judge or question the frozen mathematics implicit in the machine’s operation (Keitel, Kotzmann & Skovsmose, 1993). Lynch (2006, p. 32) believes there is a fundamental misunderstanding or failure to comprehend the nature of technology and its role in learning. Technology is seen as a finished product that can be inserted into an educational setting to create a particular effect. She labels this misconception an input → output approach, where learning effects result from the simple addition of technology into classrooms.

We must continue to question the social justifications, investigate the pedagogical perspectives of developing mathematical knowledge, and examine outcomes in knowledge, belief, attitude and habit that may be attributable to technology (Keitel, Kotzmann & Skovsmose, 1993; Penglase & Arnold, 1996; Lagrange, Artigue, Laborde & Trouche, 2003). As Lynch (2006, p. 32) concludes:

Much of the rhetoric that surrounds new technologies and schooling overestimates the degree of agency that a technological artefact may have and fails to account for the agency of human actors, the effects of existing systems and institutions, and the complexity of interactions that influence how new technologies are used.
This view is supported by Lagrange, Artigue, Laborde and Trouche (2003), who conducted an extensive review of 662 published works on technology integration between the years 1994 to 1998. The second stage of their analysis rejected all but 79 of these papers as lacking sufficient detail or in-depth analysis, with the great majority of studies (62%) looking predominantly at technical approaches for possible uses of technology, or classroom innovations. From the remaining papers, they distinguished two types of didactical research. One examines theories of learning and investigated theoretical and practical changes that technology may induce in learning situations. The other looks at evidence of better learning and associated factors that may promote or inhibit this (Lagrange, Artigue, Laborde & Trouche, 2003, pp. 255-256). These directions are used to structure the following discussions.

3.2.1 Learning Theories and Technology

Technology necessitates a re-examination and possibly a reconstruction of educational theories as they apply to mathematics education (Lagrange, 1999; Lynch, 2006). Heid (2003) believes that it is essential that all prospective secondary teachers of mathematics acquire an awareness of the development and refinement of theories related to the use of technological tools. Many writers also signal differences between different technologies. Berger (1998) argues that there are several crucial differences between computers and calculators, including the respective status of each technology as a cultural artefact; and the nature of the relationship that a learner forms with the artefact, based on the different interactive capabilities and physical natures of the two technologies. Heid (2003, pp. 34-37) postulates that CAS technology is much more than just another calculator or computer. She refers to it as a “cognitive” technology, and offers at least three ways in which CAS can operate in this manner. This includes its role as a reorganiser of the curriculum (see Kilpatrick & Davis, 1993).

However, there is some criticism of the state of theoretical studies in the use of technology. There is a disproportionate focus on the technical aspects of technology use (Lagrange, Artigue, Laborde & Trouche, 2003), or a preoccupation with the input→output view of technology, which ignores the complexity of ways in which technology affects learning (Lynch, 2006). Despite such criticism, there is common agreement that attention to the theoretical relationship between technology and learning
is critical to the successful integration of technology (e.g. Arnold, 1996; Lagrange, 1999; Wong, 2003; Heid, 2003; Thomas & Holton, 2003).

**Psychological Theories and Mathematical Cognition**

Many studies in the cognitive domain, particularly at the secondary level, adopt a constructivist approach to theorising about technology use (Lagrange, 1999; Lagrange, Artigue, Laborde & Trouche, 2003). These are grounded in the psychological theories of constructivism (von Glasersfeld, 1989; 1991), and the extensions of this theory to the individual and sociocultural perspectives based on the works of Piaget (see Devries, 2000) and Vygotsky (1978, 1987). Constructivist theory suggests that learning is not achieved through the transmission of knowledge from teacher to students; rather, each student constructs his or her own meaning from the learning experiences encountered; meaning that undergoes a process of personal and social negotiation before internalisation (Arnold, 1994). Piaget and Vygotsky view students as cognising individuals, who are continually interacting with each other and their environment (including technology), and adapting their own views through processes of interactive accommodation (Devries, 2000; Bowers & Doerr, 2001). However, like Bowers and Doerr (2001), Steffe and Thompson (2000, p. 204) recommend that “Researchers should not apply general models like von Glasersfeld’s or Vygotsky’s directly to the practice of mathematics education”. Likewise, Steffe and D’Ambrosio (1995) and Simon (1995) warn against extending constructivist principles for learning to the implication that there exists some legitimate corresponding model of “constructivist teaching.” Despite such cautions, many researchers believe there is value in these perspectives when examining students’ and teachers’ interactions with technology (e.g. Jones & Mercer, 1993; Smith, 1998; Bowers & Doerr, 2001).

Given that the theories that underpin the sociocultural studies of Piaget and Vygotsky are largely concerned with child development, they may seem less applicable at the tertiary level. Smith (1998) however dispels this inference. Cognitive psychology research contains many competing models of learning, and he believes it is critical that such models of learning are considered in undergraduate mathematics: “Research in cognitive psychology has been sending us consistent messages (about good practice in undergraduate education) for a half-century, but few mathematicians were listening until the current decade” (Smith, 1998, p. 781). He personally employs the Kolb
learning cycle, as a way to think about the relationship between university students and lecturers, observing that technology particularly supports the Kolb cycle (Kolb et al., 1984, in Smith, 1998, pp. 781-784).

There are many examples of the influence of constructivism in the tertiary sector. The standard lecture delivery method used in many universities does not appear particularly supportive of constructivist learning, and this may be one of the underlying reasons behind many of the calls to change traditional instruction methods (Smith, 1998; Holton, 2005). Frielick (2001) develops a model for higher learning in general (see Figure 2.6), while APOS theory (*Action-Process-Object-Schema*) is seen as a significant contributor to higher mathematical reasoning (Tall, 1999).

APOS theory extends Piaget’s mechanism of *reflective abstraction*, in the development of children’s logical thinking, to more advanced mathematical concepts. It is based on the constructivist-influenced hypothesis that mathematical knowledge consists of an individual’s tendency to deal with perceived mathematical problem situations by constructing mental *actions, processes, and objects*, and organising them in *schemas* to make sense of the situations and solve the problems (Dubinsky & McDonald, 2002, p. 2). This approach views the growth of mathematical understanding as highly non-linear. Students develop partial understandings that are repeatedly returned to, which is highly suited to the iterative qualities of technology (Selden & Selden, 1999, p. 7). As a process of instructional design, APOS theory involves an iterative process of *genetic decomposition*, a theoretical analysis that breaks down a concept into a set of mental constructions that a student might make, in order to understand the given concept (Dubinsky & McDonald, 2002, p. 5).

Tall (1999) is critical of some elements of APOS theory. He discounts the suggestion that physical or perceptual actions may be encapsulated in a permanent mental object, and notes the absence of any consideration of the place of symbols in the APOS model. Nonetheless, the application of new work practices (e.g. group learning), and useful learning sequences (e.g. genetic decomposition), to a wide range of advanced mathematical areas, including discrete mathematics, logic, calculus, linear algebra and group theory, are still seen as valuable (Tall, 1999). Dubinsky and McDonald (2002, pp. 5-6) provide a detailed example, of APOS theory using computers applied to the formulation of group axioms, as an aid to student’s conceptualisation of cosets.
Constructivist theory is also consistent with Skemp’s (1979) characterisation of the learning process, which extends Piaget’s concepts of assimilation and accommodation, and culminates in the re-construction of knowledge. Reconstruction involves the breakdown of existing schemas and reassembling them, to form new ones which accommodate the new knowledge. Learning is therefore seen as a spiral of equilibrium → disequilibrium → re-equilibrium, and the role of the teacher is to provide learning experiences that induce perturbations or dissonance in students’ schemas.

The teacher must intentionally cause enough chaos to motivate the student to reorganise. Obviously this is a tricky task. Too much chaos will lead to disruption (Bruner, 1973, Chapter 4), while too little chaos will produce no reorganisation. (Doll, 1986, p. 15)

Technology is seen as a useful means of providing opportunities for students to encounter such problematic situations, independent of the teacher (Arnold, 1994; Lagrange, 1999). In their study, Bowers and Doerr (2001) created novel, computer-based activities that were designed to challenge their student teachers’ expectations, and initiate dissonance in their views of mathematical change. They acknowledge the value of the technology in promoting fruitful discussions stemming from the participants efforts to reconcile what they found with what they anticipated (Bowers & Doerr, 2001, p. 134).

At the tertiary level in particular, Smith (1998) sees technology as one means of confronting the standard transmission style of teaching and learning which he views as inappropriate. He again refers to the Kolb learning cycle, which encapsulates aspects of personal knowledge construction in its four stage cycle. Smith observes that computers and calculators can be used as tools to facilitate the four stages of learning: concrete experience (CE); active experimentation (AE); reflection/observation (RO), and abstract conceptualisation (AC). He provides an example in differential calculus, using calculators to investigate population growth, which provides students with opportunities for concrete experience with data plots, reflective observation about what the plots mean, abstraction in the symbolic models and their solutions, and active testing of the symbolic solutions against the reality of the data (Smith, 1998, pp. 784-785). Smith (1998) concludes that, while few of those involved in early calculus reform projects with which he was involved knew much about the various psychological learning
theories, they have discovered empirically through their experiences many theoretical principles of good learning, and now share a general belief in the cognitive value of constructivist principles.

**Sociocultural Perspectives: Technology as Mediator and Linguistic Tool**

There are limitations in using solely constructivist approaches to theorise about technology use. Lagrange (1999, p. 56) believes that a purely constructivist view assumes that the technology settings will provide the means for a predictable and meaningful interaction, but he notes that such a perspective proved insufficient to analyse the interactions in their study. While grounded in sociocultural perspectives, several investigations extend the conceptualisation of technology beyond the constructivist framework. Such studies emphasise the situated nature of mathematical learning (e.g. Bowers & Doerr, 2001; Dubinsky & McDonald, 2002; Galbraith & Goos, 2003; Geiger, 2006), as evidenced in the highly specific undergraduate setting of this thesis. Noss and Hoyles (1996, p. 21) for example believe that problem-solving using computers is very contextualised, and note that it is unclear whether such knowledge helps with tasks outside the computer context.

Jones and Mercer (1993) consider Vygotsky’s emphasis on the role of language in cognitive development, and his conceptualisation of learning in the *zone of proximal development* (ZPD). They believe both have significant implications for our conceptions of the role of the computer in the learning process, and the role of the teacher when using computers in the classroom. The ZPD is often viewed as the gap between a learner’s present capabilities, and the higher level of performance that could be achieved with appropriate assistance. The computer and the teacher are considered as providing both a medium and mediator for such assistance (Jones & Mercer, 1993). This role of technology as a mediator reflects the central tenet of sociocultural studies; that human action is mediated by cultural tools, and is fundamentally transformed in the process (Wertsch, 1985, in Geiger, 2006, p. 183). The chief feature of this conceptualisation of technology is that in addition to its contribution in addressing mathematical concepts and processes, it also encompasses:

Interactions between teachers and students, amongst students themselves, and between people and technology, in order to investigate how different participation patterns offer opportunities for students to engage constructively and critically with
mathematical ideas. That is, while technology may be regarded as a mathematical tool (amplifies capacity), or as a transforming tool (reorganises thinking), it may also be regarded as a cultural tool (changes the relationship between people, and between people and tasks). (Galbraith, 2002, p. 14)

Technology as a cognitive amplifier and reorganiser was first suggested by Pea and Roy (1985, 1987). An amplifier is when technology is used to extend the existing curriculum, while the role of reorganiser is to facilitate changes in the balance, sequence, and priorities assigned to concepts and procedures in the mathematics curriculum (Pea & Roy, 1985, 1987). Heid (2003) observes that an instructor who views technology as an amplifier may give students access to the technology in class, but not in exams, or may insist on prior mastery of algorithms by hand prior to using the technology. The conceptualisation of technology as a cultural tool places less emphasis on the relationship an individual learner has with the technology (viewed as an impersonal tool), and more on the technology as a medium through which a teacher and learner can communicate (Jones & Mercer, 1993). Noss and Hoyles (1996) agree with this representation of the computer as an additional medium for expression between two people (e.g. student-student; student-teacher):

In the non-computer setting, (they) have a choice between natural language or mathematical language: the former good for communication in general, but badly tuned to rigorous and precise discourses like mathematics; the latter precisely the reverse. The computer affords a half-world in which effective communication and precision can be approached, where articulation and rigour can be made to converge. (Noss & Hoyles, 1996; p. 6)

Geiger (2006), however, believes that technology plays a far more important role than that of a presentation tool that mediates discussion and interaction. He suggests that technology can be regarded as a quasi-peer, and extends Vygotsky’s notion of a ZPD to include technology as a contributing member to a group of learners. He concludes that this characterisation has implications for teacher training and professional development. Berger (1998) too questions the generalisation of technology as a communicative medium, and draws a distinction between computers and calculators in this regard. She suggests that students may be more likely to form individual relationships with personalised technology such as calculators. This would seemingly negate some of the social interactions inherent in this communicative model.
Lagrange (1999) agrees and notes how the conceptualisation of the computer as a mental instrument in the Vygotskian sense (e.g. Noss & Hoyles, 1996) may prove less effective with complex calculators such as the TI-92. Lagrange believes that as more of the language and logic of a calculator becomes internalised, users may fail to distinguish between the interface through which they access the capabilities of the calculator, and the internal logic. He therefore believes that the relationship is far more complex than the two-dimensional reduction to a neutral interface and an internal algebraic language as such conceptualisations suggest.

Winslow (2000) examines the role of technology as a communicative medium from the linguistic perspective of a register, which views mathematical knowledge as dependent on linguistic competency. Winslow links this to the use of CAS, which he believes requires a fundamental distinction between symbol language as used by the technology, and natural language. He sees this distinction as an explicit organising principle for communication in this medium, namely ‘text’ and ‘symbol’ strings have entirely different status that may be viewed as two different channels of communication. While he is critical on linguistic grounds of the ways in which many CAS systems handle the interaction of natural and symbolic language, he nevertheless sees this role as a useful means for promoting the learning of advanced mathematics (Winslow, 2000, pp. 284-286). Balacheff and Kaput (1996, p. 475) relate such difficulties to our inability as instructors to enable students to use their cognitive and linguistic powers, to build meaningful connections between their everyday experience and the world of formal mathematics. They are critical of technology which employs symbolic and linguistic systems which further complicate the formal arena of advanced mathematics, a view shared by many (e.g. Tall, 1991; Heid, 2003). After describing a complex example of the use of a CAS template (TI-89 or TI-92) to solve a limit problem, Heid asks: “How can students who are operating at the informal stages of algebra cope with such a substantial demand in using CAS to communicate in a strictly formal language…theories of learning highlight the importance of, and difficulties associated with, students’ movements towards formal symbolic systems?” (Heid, 2003, pp. 44-45).

Chinnappan and Thomas (2000), and Doerr (2001), develop the role of technology as a medium in an epistemological sense, using the view of computers as a dynamic linkage between students’ actions and the different representations of mathematical concepts (Kaput, 1992; Noss & Hoyles, 1996). Doerr (2001) describes
how the microworld used by the students in her study prompted a significant shift in students’ algebraic reasoning, by allowing them to explore the relationship between graphical objects, and the simultaneously linked position and velocity graphs, using the language of graphical objects rather than equations. Chinnappan and Thomas (2000) suggest that technology, used appropriately by experienced teachers, can be effective in building conceptual understanding, in establishing links between concepts and across representations. They provide a tentative model that depicts the way in which technology may act as a catalyst in the linking of function representations (see Figure 3.1), which reflects the rule of three as espoused by Hughes-Hallet (1991, p. 121). This rule later became the rule of four with the addition of verbal representation. It states that wherever possible, topics should be taught graphically, numerically, analytically and verbally (Tall, 1999, p. 3). Chinnappan and Thomas (2000) suggest that one way technology may act as such a catalyst is as a prompt to accessing prior knowledge, which may otherwise remain dormant. However, they caution that their results suggest effective use is predicated on the experience of the teacher with the chosen technology. “Teachers need to build sound knowledge about the menus and procedures that are incorporated into the software or the calculator” (Chinnappan & Thomas, 2000, p. 178).

![Figure 3.1: Technology as a catalyst in the linking of function representations.](Chinnappan & Thomas, 2000, p. 173)

The role of the teacher within a sociocultural framework is considered further by Goos, Galbraith, Geiger and others (e.g. Goos, Galbraith, Renshaw & Geiger, 2000; Galbraith & Goos, 2003; Brown, 2006; Geiger, 2006; Goos, 2006). They collectively
develop an adaptation of Valsiner’s (1997) zone theory, to help explain the interactions between teachers, students and technology in a technology rich learning environment. Valsiner’s framework extends Vygotsky’s ZPD to incorporate the social setting and the goals and actions of participants. Valsiner (1997) describes two additional zones: the Zone of Free Movement (ZFM) and Zone of Promoted Action (ZPA). The ZFM structures an individual’s access to different areas of the environment, the availability of different objects within an accessible area, and the ways the individual is permitted or enabled to act with accessible objects in accessible areas. The ZPA represents the efforts of a more experienced or knowledgeable person to promote the development of new skills. For learning to be possible the ZPA must be consistent with the individual’s potential (ZPD) and must promote actions that are feasible within a given ZFM (Galbraith & Goos, 2003). Zone theory provides an effective framework for analysing the dynamic relationships between a teacher’s contextual setting, actions and beliefs, and how these may change over time (Goos, 2006). This framework is summarised in Table 3.1:

Table 3.1: Factors Affecting Technology Use (Goos, 2006, p. 192)

<table>
<thead>
<tr>
<th>Valsiner’s Zones</th>
<th>Elements of the Zones</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone of Proximal Development</td>
<td>Skill/experience in working with technology</td>
</tr>
<tr>
<td></td>
<td>Pedagogical knowledge (technology integration)</td>
</tr>
<tr>
<td></td>
<td>General pedagogical beliefs</td>
</tr>
<tr>
<td>Zone of Free Movement</td>
<td>Access to hardware, software, teaching materials</td>
</tr>
<tr>
<td></td>
<td>Support from colleagues (including technical support)</td>
</tr>
<tr>
<td></td>
<td>Curriculum &amp; assessment requirements</td>
</tr>
<tr>
<td></td>
<td>Students (perceived abilities, motivation, behaviour)</td>
</tr>
<tr>
<td>Zone of Promoted Action</td>
<td>Pre-service education (university program)</td>
</tr>
<tr>
<td></td>
<td>Practicum and beginning teaching experience</td>
</tr>
<tr>
<td></td>
<td>Professional development</td>
</tr>
</tbody>
</table>

Goos (2006, p. 196) concludes that zone theory provides a way of interpreting teachers’ actions in mathematics classrooms, that may generate informed discussion about conditions that support or inhibit teachers’ learning and adoption of new
technologies. Such a mechanism was identified as significant when discussing curriculum change and professional development in Chapter Two.

**Advanced Mathematical Thinking and Technology**

Thomas and Holton (2003) consider further the theoretical relationships between technology and advanced mathematical thinking developed in the essentially constructivist APOS theory described earlier. They highlight several areas that they consider important in undergraduate mathematics, especially those studies that investigate the process of the construction of mathematical knowledge through the development of conceptual schemas (see e.g. Tall & Thomas, 1991; Gray & Tall, 1993; 2001; Graham & Thomas, 2000; Tall, Thomas, Davis, Gray & Simpson, 2000), and studies that examine the representational systems and modes of thinking that enable interaction with the formed concepts (e.g. Noss & Hoyles, 1996; Kaput, 1998; Tall, 2000, Thomas, 2007). Tall and Meija-Ramos (2006, p. 3) describe the development of a framework, refined over many years (See Tall, 1999, 2000, 2003, 2004, 2005), which comprises three distinct mental worlds of mathematics:

- the *conceptual-embodied* (based on perception of and reflection on properties of objects);
- the *proceptual-symbolic* that grows out of the embodied world through actions (such as counting) and symbolization into thinkable concepts such as number, developing symbols that function both as processes to do and concepts to think about (called procepts);
- the *axiomatic-formal* (based on formal definitions and proof) which reverses the sequence of construction of meaning from definitions based on known concepts to formal concepts based on set-theoretic definitions.

Central to theorising about concept formation in these worlds is the focus on mathematical objects, as fundamental elements of advanced mathematical thinking (Thomas & Holton, 2003, p. 354). These objects are distinguished as perceived objects, procepts, axiomatic objects and embodied objects, with the second of these, procepts, being the subject of most discussion in the literature. The term “procept” is used to describe the duality, especially in algebraic and calculus concepts, between procedures and objects. While the procept notion has strong links to APOS theory, there are significant differences. These include the specific focus on the cognitive structure, as
opposed to the breakdown of a process and its encapsulation, as prescribed by APOS theory (Tall, 1999).

In Figure 3.2, Tall (1999, p. 116) portrays how students confront an increasingly complex succession of processes and objects in symbolic mathematics, culminating in the advanced, formal world of axiomatic mathematics.

![Figure 3.2: Development of process/concept in symbolic mathematics.](image)

Tall (1999, 2000, 2003, 2004, 2005) provides numerous examples from the fields of arithmetic, algebra and calculus to demonstrate the increasing complexity of mathematical thinking described in Figure 3.2, and the considerable difficulties students encounter in the transition between the mental worlds in this framework. He describes for example how procepts may be used to understand the conceptual difficulties that students experience when introduced to limits, and suggests that many computer approaches fail to address the issues that this conceptualisation of limits provides. The iterative capabilities of computers to compute increasingly accurate approximations creates the illusion that the calculations may go on and on without end, thus reinforcing the idea that the limit does not actually exist (Tall, 2000, p. 219).

Heid (2001, 2003) considers the concept of scaffolding from a cognitive science perspective, as a means of examining and remedying common sources of student error (Kutzler, 2000b; Heid, 2003; Lagrange, Artigue, Laborde & Trouche, 2003). Davis (1984, in Heid, 2003, p. 39) labels the causes of such errors as mathematical superprocedures and subprocedures, and he points out that errors often come not from mistakes in executing subprocedures, but in the incorrect choice of superprocedure.
Heid (2003) adapts this conceptualisation to technology, describing Davis’s categorisations respectively as macroprocedures (menu-driven), and microprocedures (steps taken in using the macroprocedure, that may themselves be lower-level menu-driven commands). Heid explains the significance of these for students, observing that:

Students who understand the macroprocedures seem to have a framework within which to place the microprocedures when and if they learn them ... (they) learned the macroprocedures long before they had mastered the component microprocedures. For example, the algebra students solved quadratic equations before they had learned to factor, and the calculus students maximised functions of one variable before they learned to take derivatives by hand. When, at a later date, these same students learned procedures for factorising and derivatives, their exposure to scaffolding provided them with something on which to “hang their skills”...Consequently the students’ mathematical development seemed to proceed more rapidly. (Heid, 2003, p. 40)

One such example is the choice to solve a quadratic equation using the macroprocedure Solve command from the CAS menu, versus adopting a step-by-step method using the algebraic functions of the CAS (e.g. expand, factorise). In this way, technology may scaffold students to avoid such errors as described by Davis, by giving students the responsibility for selecting the appropriate macroprocedures, but using CAS to execute the microprocedures. The advantages of technology to reduce the cognitive strain on students and scaffold their learning in such a way are frequently reported (e.g. Hillel, 1993; Dubinsky, 1995; Tall, 2000; Winslow, 2000).

Previous discussions have signalled the criticism computer routines receive for their “black-box” nature, performing tasks where the user has no concept of what is being carried out. There is no great pedagogical value in the mere performance of technical tasks, except perhaps in saving time (Dana-Picard & Kidron, 2006, p. 128). However, the black-box phenomena can be used in a positive manner to provide scaffolding for students. Initially, complicated procedures maybe carried out by the technology, with only the input and output understood by the student, followed by a more theoretical treatment later (Hillel, 1993, p. 36). Tall (2000) describes several examples of this in his formulation of the Principle of Selective Construction:
This principle leads to the design of software that allows the learner to focus on part of the theory whilst the computer invisibly carries out the underlying processes...For instance, the approach to differential equations focuses on providing a manipulable visual representation in which the computer carries out the algorithms and draws pictures while the student focuses on getting a global sense of the theory. At another time, the student might implement the algorithms using a simple computer program in order to focus on the details of a numeric approach, or on a study of available symbolic methods for solution. (Tall, 2000, p. 222)

Winslow (2000) observes that such benefits may be even further enhanced in technologies where programming is available. Rather than just using the built-in routines, programming may be a way to develop a greater structured understanding of concepts. Such a belief underlies the development of the ISETL mathematical programming language (Dubinsky, 1995).

However, many tertiary mathematicians remain unconvinced of the value of such practices, continuing to believe that students must learn certain procedures first, if they are to fully understand mathematical concepts (Keynes & Olson, 2001). Further, while such theoretical considerations as this, and others described in this section, may indeed be useful and necessary in the successful integration of technology, more than just an awareness of such issues is needed to change strongly entrenched views of mathematics and learning (see Section 2.3.1). We will now look at studies that consider the more specific personal relationship between the user and the technology, as opposed to the theoretical framework within which that technology is used.

**Instrumented Activity: Technology and Staff-Student Interactions**

Artigue (2002) and Lynch (2006) view the critique of constructivist, theoretical frameworks of technology research in terms of the relationship between technical and conceptual learning in mathematics. Keitel, Kotzman and Skovsmose (1993) believe it is essential, from an epistemological perspective, to seek a balance between the development of mathematical and technological knowledge. Artigue and Lynch however perceive a privileging of the conceptual over the technical domain in mathematics education research, which (Artigue, 2002, p. 247) describes as the *technical-conceptual cut*. Dissatisfaction with the way in which most studies from a constructivist approach seem to accentuate this technical-conceptual divide led Artigue
(2002), and others (e.g. Lagrange, 1999, 2003; Guin & Trouche, 1999), to explore an anthropological approach to their work, an approach that shares the sociocultural vision of mathematics as a product of human activity. From this view, mathematical productions and thinking modes are seen as dependent on the social and cultural contexts within which they develop. The term artefact is used in these studies to describe the material or symbolic object or tool (in the Vygotskian sense, Lynch 2006), as distinct from the instrument, seen as “…a mixed entity, part artefact, part cognitive schemes which make it an instrument” (Artigue, 2002, p. 250). This view of instrumentation draws on the work in cognitive ergonomics by Vérillon and Rabardel (1995), who have examined professional learning processes in complex technological environments. In this view, an instrument is seen as a psychological construct: “The instrument does not exist in itself; it becomes an instrument when the subject has been able to appropriate it for himself and has integrated it with his activity” (Vérillon & Rabardel, 1995, p. 84).

Instrumented activity, i.e. activity that employs and is shaped by the use of instruments, has a twofold outcome. There is a process of instrumentalisation, in which the subject shapes the artefact for specific uses, and simultaneously a process of instrumentation, in which the subject is shaped by interactions with the artefact. This dialectic by which learner and artefact are mutually constituted in action is described as instrumental genesis (Artigue, 2002; Hoyles, Noss & Kent, 2004). The process of instrumental genesis leads to the development or appropriation of schemes of instrumental action, which progressively take shape as techniques, to facilitate an effective response to given tasks (Lagrange, 1999; 2003, Trouche, 2000). Until relatively recently, the complexity of instrumental genesis was widely underestimated (Artigue, 2002), but the emerging body of studies based on this anthropological approach is now seen as offering a increasingly sophisticated means of analysing the relationships between users and technology (Lynch, 2006, p. 40). Indeed, many researchers now see the attainment of instrumental genesis by both students and teachers as an essential element of successful integration of technology (e.g. Hoyles, Noss & Kent, 2004; Artigue, 2006). An example of the complex issues is provided by Lagrange (1999), in his formulation of schemes and techniques for technology use. He cautions that the mathematical schemes students develop using technology may be less productive than the schemes they may have used previously. He gives an example of
obtaining a derivative using the CAS-software ‘Derive,’ where the technology-based scheme generates a restricted mathematical meaning. In this sense, schemes are described as internal adaptive mental constructions that a learner develops to assimilate knowledge, and can’t be taught. Techniques are distinguished as rational elaborations that are taught as an official means of achieving a task (Lagrange, 1999, p. 61). The formation of appropriate schemes and techniques are both seen as pivotal for instrumental genesis (Andresen, 2006, p. 19).

There is some diversity in the approaches and terminologies used for what seems to be similar conceptualisations of instrumented activity and interactions between students, teachers, and technology (Brown, Stillman & Herbert, 2004; Thomas & Chinnappan, 2008). One example is in the analysis of affordances and constraints offered by the technology. This theoretical perspective is based on the work of Gibson (1977), who coined the term affordance to describe the attributes of the environment that contribute to the potential for an interaction to occur, while constraints are seen as the characteristics of the affordance that provide structure and guidance for the interaction (Thomas & Chinnappan, 2008). Brown, Stillman and Herbert (2004) observe that while the term ‘affordance’ is becoming increasingly prominent in technology literature, there is a diversity of explicit definitions and implied meanings. In particular, there seems to be some confusion between the concepts of constraints, obstacles and negative affordances. While constraints may pose limits on learning, they should not necessarily be viewed as obstacles or negative affordances; they are imposed by the setting, and are therefore complementary to the affordances (Brown, Stillman & Herbert, 2004, p. 122). Forgasz, Griffith and Tan (2006) use similar concepts but different terminologies when they talk about encouraging and inhibiting factors, while Drijvers (2000, 2002) and Thomas (2006) both use the terminology of obstacles to technology use, again in seemingly different senses. Drijvers appears to use the term obstacle to describe barriers to instrumentation, almost in the sense of negative affordances (as opposed to constraints), while Thomas reserves the notion of obstacles to describe something that prevents the presence of the affordance. For this thesis, the distinction between affordances and constraints is provided in the following summary of the history and development of this theoretical perspective:
Interactions between learners and technological devices necessarily involve both the ability of the learner and the affordance of the technology. These combine to determine the potential of the interaction in any given situation. Some constraints are artefacts of the technological environment. For instance, in a graphing calculator learning environment one is constrained in terms of representational control to directly altering only the algebraic or numerical representations of a function. The graphical representation cannot be so controlled. Other constraints may be imposed by the teacher, the student, or derive from the general learning environment. (Brown, Stillman & Herbert, 2004, p. 122)

Regardless of terminology, there is agreement on the existence of affordances and constraints with respect to technology, and the critical need for attention to these, if the learning benefits of a specific technology are to be realised in the classroom (Brown, Stillman & Herbert, 2004). “[Such benefits] depend not only on the affordances of the technological tool, but [also] on the exploitation of these affordances embedded in the educational context and managed by the teacher” (Drijvers, 2003, p. 78). Doerr and Zangor (2000, p. 80), for example, note that some teachers in their study did not change their methods or approaches at all to reflect their use of graphing calculators.

With respect to constraints associated with the artefact itself, Guin and Trouche’s (1999) categorisation (for CAS-calculators) identifies significant elements of the transposition of mathematical knowledge. They describe three types of constraints for symbolic calculators: the internal constraints intrinsically linked to the hardware, i.e. internal representation of objects and their calculation processing; the command constraints linked to the possibilities of action given the user (choice of implemented commands and the syntax used for these commands); and the organisation constraints linked to the commands access and the organisation of the keyboard and the screen. Internal constraints are further sub-divided into three types: those linked with limitations of the symbols available in memory; those associated with discrete traces on a screen composed of finite pixels; and the coexistence of several modes of calculation within the same tools (Guin & Trouche, 1999, pp. 203-204).

Some specific examples of the affordances and constraints of different technologies, particularly in the tertiary environment, are considered later in the discussion of technology choices (Section 3.3). A lack of awareness of such theoretical considerations may be a significant contributing factor to the negative views of CAS,
and scepticism towards calculators in particular, held by many tertiary educators as suggested in section 2.4.3 (Keynes & Olson, 2001). Given this, ensuring staff are sufficiently familiar with the technology to allow them to realise the appropriate affordances and constraints seems both critical and problematic.

**Student Instrumentation**

Artigue (2006) distinguishes between instrumentation for students and teachers, and several studies examine the technical-conceptual view of technology specifically from the student perspective. Building on their earlier study in 2002, which examined technical and personal aspects of technology use, Pierce and Stacey (2004) identify four major aspects that contribute towards how well a student uses CAS for learning and doing mathematics (three cognitive and one affective). The first of these, the *machine aspect*, while seen as influential in driving the choice of which technology to use, is regarded by Pierce and Stacey (2004, p. 63) as pedagogically uninteresting. This aspect will be pursued in Section 3.3, as technology design factors seem more critical in the choice of technology at the tertiary level (Wong, 2003; Healy, 2006). While the *mathematical aspect* is seen as extremely important with respect to considerations of the effect of CAS on curriculum content and pedagogical practices (see sections 2.3.3, 2.3.4), they concentrate in the 2004 study on developing further the *technical* and *personal aspects*. Technical aspects refer to the knowledge and skills related to the software, as opposed to the hardware or machine, which is described by the machine aspect. An example of personal aspects is in the value students attach to the availability of CAS. Students with a negative attitude may be less persistent in the face of technical difficulties, and more likely to avoid its use, a view supported by Stewart, Thomas and Hannah (2005).

The framework developed by Pierce and Stacey (2004) to examine technical and personal aspects of student’s technology use is reproduced in Table 3.2. As an example of element 1.2, Pierce and Stacey note that while the ability to move quickly between algebraic, graphical and tabular representations of functions is often said to be one of the strengths of CAS (see e.g. Tall, 1996b; Chinnappan & Thomas, 2000), Lagrange (1999) and others have noted that swapping representations can be a source of difficulty. Students need to develop a mental map of the architecture of the system, to
understand how the symbolic, graphical, table and home modules are connected and not connected (Pierce & Stacey, 2004, p. 68).

**Table 3.2:** Framework for the Effective Use of CAS (Pierce & Stacey, 2004, p. 65)

<table>
<thead>
<tr>
<th>Aspects</th>
<th>Elements</th>
<th>Common Instances</th>
</tr>
</thead>
</table>
| 1. Technical     | 1.1 Fluent use of program syntax         | 1.1.1 Enter syntax correctly  
|                  |                                         | 1.1.2 Use a sequence of commands and menus proficiently  
|                  |                                         | 1.2 Ability to systematically change representation  
|                  |                                         | 1.2.1 CAS plot a graph from a rule and vice versa  
|                  |                                         | 1.2.2 CAS plot a graph from a table and vice versa  
|                  |                                         | 1.2.3 Create table from a rule or vice versa  
|                  | 1.3 Ability to interpret CAS output     | 1.3.1 Locate required results  
|                  |                                         | 1.3.2 Interpret symbolic CAS output as conventional mathematics  
|                  |                                         | 1.3.3 Sketch graphs from CAS plots  
|                  |                                         | 1.3.4 Interpret CAS non-response  
| 2. Personal      | 2.1 Positive attitude                   | 2.1.1 Value CAS ability for doing mathematics  
|                  |                                         | 2.1.2 Value CAS ability for learning mathematics  
|                  | 2.2 Judicious use of CAS                | 2.2.1 Use CAS in a strategic manner  
|                  |                                         | 2.2.2 Discriminate in functional use of CAS  
|                  |                                         | 2.2.3 Undertake pedagogical use of CAS  

As an example of 2.2, judicious use of CAS, Pierce and Stacey cite Guin and Trouche (1999), who observed that students used various avoidance strategies which resulted in unthinking forms of CAS use. Further examples of the technical aspect are displayed in Figure 3.3. Pierce and Stacey (2004) consider the knowledge and skills at all positions on the continuum depicted in Figure 3.3 as essential for doing mathematics, and they view studies on instrumental genesis, such as those of Lagrange (1999), Guin and Trouche (1999), and Artigue (2002), as one well-theorised approach to examining these. The nature of the examples they choose to illustrate this continuum illustrates its relevance to undergraduate mathematics: “One can no more use a calculator with an unreadable screen (machine extreme) than one can solve a differential equation without understanding the importance of boundary conditions (mathematical extreme) or
without being able to interpret the output from the machine in familiar mathematical notation (a technical aspect)” (Pierce & Stacey, 2004, p. 62). Unfortunately, while this framework may provide a valuable means of examining interactions with technology, it is unlikely that many undergraduate mathematics teachers (especially inexperienced or technology sceptical ones) will be aware of such considerations.

**Figure 3.3:** The continuum of knowledge and skills required for using CAS. (Pierce & Stacey, 2004, p. 62)

Further developments of instrumentation can be related to elements in Pierce and Stacey’s (2004) framework. Pierce and Stacey themselves offer four classifications: passive (just observing others); random (trying anything); responsive (using CAS according to directions); or strategic (following their own reasoned plan). Others offer similar conceptualisations using different terminologies (e.g. Guin & Trouche, 1999, 2000; Goos, Galbraith, Renshaw & Geiger; 2000; Stewart, Thomas & Hannah, 2005; Dana-Picard & Kidron, 2006). For example, Guin and Trouche (2000) identify a sufficient mathematical background as a significant factor in students achieving sophisticated levels of instrumental genesis (strategic use). They describe a critical stage, where the student’s mathematical knowledge and degree of instrumentation is
sufficient to allow them to accept the algebraic register as taking priority over the graphical register.

The group of studies by Goos, Galbraith, Renshaw and Geiger (e.g. Goos, Galbraith, Renshaw & Geiger, 2000; Galbraith, 2002; Geiger, 2006), and the study by Stewart, Thomas, and Hannah (2005), warrant further mention here. The studies by Goos, Galbraith, Renshaw and Geiger are important, because while they do differentiate between teachers’ and students’ interactions with technology, they use the same general categorisations to examine each, albeit with slightly different descriptors of the identifying features. They collectively describe a descriptive taxonomy of sophistication (instrumental genesis), expressed in four metaphors: technology as master (the user is limited by knowledge of the technology, may be characterised by blind acceptance of output); technology as servant (used to replace time-consuming or complicated procedures, e.g. replacement for pen and paper); technology as partner (creative and directed use, seen as valuable assistant); and the highest level of functioning which again has parallels to the strategic level of Pierce and Stacey (2004), technology as extension of self, where technology is used to support mathematical argumentation and is an integral part of a teachers or students’ mathematical repertoire (Goos, Galbraith, Renshaw & Geiger, 2000, pp. 307-308; Galbraith, 2002, p. 15).

A similar study by Stewart, Thomas and Hannah (2005) was conducted at the tertiary level. They identified four primary classes of student-instrumentation, which can be seen to reflect various elements of the technical and personal aspects described by Pierce and Stacey (2004), and are analogous in some ways to the categorisations of Guin and Trouche (1999) and Galbraith (2002). The first type of student believes strongly in the superiority of by-hand techniques for doing and understanding mathematics, and opposes the use of technology, the second type is happy to use the computer, but their instrumentation is limited, they see the value only as a tool for checking answers, avoiding careless errors or expediting tedious algorithms (c.f. technology as servant, Galbraith, 2002). A third category is the student who relies too much on the computer, usually a weaker student who would find the mathematics too hard without the technological support (see Hong, Thomas & Kwon, 2000). The final group comprises students who are positive towards the technology and who try different methods. Students in this group are seen to be the most advanced in their level of instrumentation (Stewart, Thomas & Hannah, 2005, pp. 747-748).
Hannah (2005, p. 748) conclude that if the application of CAS to the learning of tertiary mathematics is indeed desirable, then it remains essential that we find ways of encouraging instrumental genesis in our students, and that an awareness of the different groups of students as described in their study may prove helpful in achieving this objective.

**Instrumentation: Teacher Perspectives**

The overriding implication from the preceding discussion is that while the theoretical aspects of instrumentation (for both teachers and students) are extremely complex, an ignorance of such issues, or disregarding ways of encouraging students’ instrumental genesis, may have serious and unforeseen negative consequences for student learning, and the effective integration of technology. As Thomas, Monaghan and Pierce (2004, p. 156) conclude, there are multiple factors affecting the ultimate effectiveness of technology use, and “while CAS has great potential, not all students will attain the (necessary) level of instrumentation…or indeed will see the need to”.

There is a need therefore to examine the role of the teacher in students’ instrumental genesis, as in the work of Kendal and Stacey (1999) on teacher privileging (see p. 67 this thesis). Goos (2005, 2006) extends the work done by her and colleagues on the roles of students and teachers’ modes of working with technology, using the metaphors of technology described earlier. She describes how a teacher epitomised by the highest categorisation (which is seldom achieved), *technology as extension of self*, may effect the use of technology in their classroom. This occurs when technology is “seamlessly incorporated into a teacher’s pedagogical and mathematical repertoire such as through the integration of a variety of technology resources into course planning and the everyday practices of the mathematics classroom” (Goos, 2005, p. 40). Pierce and Stacey (2004) suggest specific teaching strategies to develop the personal and technical aspects of student’s instrumentation described in Table 3.2. They suggest that for any mathematical software tool, it is essential that teachers explicitly teach specific technical skills, and overtly signal the value of the technology, in their own use and the design of suitable tasks, to promote positive attitudes towards technology among their students (Pierce & Stacey, 2004, pp. 88-90).

Another recent study also looks to address the way teachers organise the conditions for instrumental genesis, and to what extent they foster mathematics learning
through an instrumental genesis (Laborde, Assude, Grugeon & Soury-Lavergne, 2006). This study seeks to analyse the degree of instrumental integration, defined as “to what extent mathematical knowledge and knowledge of handling a tool are intertwined in the organisation of the instrumental genesis by the teacher” (Laborde, Assude, Grugeon & Soury-Lavergne, 2006, p. 319). Several indicators are used to measure this, including: focus of the task (focus on tool or mathematics); techniques for solving (mainly instrumental abilities (IA), or mathematical knowledge (MK)); and the extent to which the two abilities are intertwined (IA/MK). From their study, they distinguish four modes of integration:

*Instrumental initiation:* the teacher’s aim is mainly that the pupils learn how to use the technology…The relation between IA and MK is minimal.

*Instrumental exploration:* the teacher aims at improving both some IA and MK. Pupils explore technology through mathematical tasks.

*Instrumental reinforcement:* pupils are faced with instrumental difficulties when solving a mathematical task. The teacher provides elements of information about how to use a specific item of the artefact, to help them overcome the technical difficulties. The teacher’s aim is improving mathematical knowledge.

*Instrumental symbiosis:* pupils are faced with mathematical tasks that allow them to improve both their IA and MK because these are connected. The relationship between IA and MK is maximal: each one allows the other to increase and the connection between paper-pencil work and (technology) is strong. (Laborde, Assude, Grugeon & Soury-Lavergne, 2006, p. 319)

The authors of this study suggest that it provides a useful means of evaluating the degree of technology integration into teaching practice, through the explicit expression of the constraints and conditions for such integration, and the provision of theoretical tools through which to develop suitable indicators for integration (Laborde, Assude, Grugeon & Soury-Lavergne, 2006, p. 317). Although the definition of technology integration in their study is narrower than the curricular focus of this thesis, their framework certainly provides a means of measuring at least one aspect of the curriculum, i.e. teaching practice, and their approach may prove useful in other areas. Their study reinforces the findings of Lynch (2006), who concludes that the complexity of issues arising from considering the technical-conceptual interface mean that inputting
the same artefact at different sites or at different times does not necessarily result in the implementation of the same technology. The context-specific nature of the current investigation, within the undergraduate mathematics domain, suggests such factors may well prove significant.

The discussions in Section 3.2.1 suggest that an appreciation of learning theories and their relationship to technology is critical in designing an effective ITMC. This is especially so, given the assertion that a considerable number of tertiary mathematics educators remain generally unaware of such theoretical considerations (Keynes & Olson, 2001). Keynes and Olson believe that one key means of engaging sceptical or reluctant colleagues to at least experiment with new pedagogies, including the use of new technologies, is to provide studies that demonstrate distinct learning benefits for students in such use. Examples of this, along with some studies that give contrasting views, are considered in the next section.

### 3.2.2 Benefits and Disadvantages of Using Technology

The previous discussions have theorised the learning benefits of technology in mathematics education. However, as Keitel, Kotzmann and Skovsmose (1993, p. 278) emphasise, even when the reasons for use are justified convincingly and integration pursued effectively, if the extensive range of possible effects are ignored, then the implementation risks being superficial or non-sustainable. The use of technology should acknowledge both the effects which are of importance, e.g. the realisation of appropriate mathematical knowledge, as well as those effects that may be less desirable or otherwise neglected as random or inconsequential side-effects. However, such effects are very difficult to measure, and hence few studies demonstrate measurably improved learning associated with the uses of technology they describe, especially at the tertiary level. The majority of studies report interesting uses and potential benefits of technology, particularly for specific content areas. Examples include the use of CAS in solving linear equations and applications to vector spaces (Latorre, 1993); computers and calculator use with functions and calculus (Tall, 1996b); the use of dynamic geometry software with mechanical linkages and developing mathematical proof (Vincent & McCrae, 2000); using CAS to promote dynamic representational links in teaching derivatives (delos Santos & Thomas, 2002, 2005); and
CAS in teaching beginning differential equations (King, Hillel & Artigue, 2001; Fay & Joubert, 2005). The meta-analysis of handheld technology commissioned by Texas Instruments (Burrill, Allison, Breaux et al., 2002) reported specific benefits in the 43 studies they examined, but these studies were focused at the secondary level. While a later meta-analysis of the effect of technology on student achievement and attitude levels by Ellington (2006) did include some cases from undergraduate mathematics, this study was limited to non-CAS graphing calculators. The discussions in this section will therefore examine studies at the tertiary level where this thesis is located, in which demonstrable benefits or disadvantages of technology have been identified, as well as some upper-secondary level studies where the findings may be reasonably generalised to the tertiary level. Ganter (1999) draws a distinction between the studies in the calculus reform movement that evaluate the benefits of technology use in learning programmes (e.g. increased participation rates and improved student attitudes), and those that evaluate actual student learning. This distinction will be used to help frame the following discussion.

**General Achievement, Instrumentation, and Affective Factors**

Thomas and Holton (2003) describe a number of studies in undergraduate mathematics that show positive benefits in confidence and attitude, along with general improved mathematical performance. For example, after some initial student resistance to using computers in undergraduate courses at Roger Williams University, Silverberg (1999, pp. 245-246) found increased student confidence in their mathematical skills, and an analysis of final examination results showed a dramatic improvement in mathematical performance for students using ISETL and Maple, compared with students on a ‘traditional’ course. Another report by Schwingendorf (1999) similarly found that reform students often received higher calculus grades and did just as well in courses beyond calculus. Other studies have collectively found that technology leads to improved attitudes to calculus; better problem solving skills in the sense of formulating problems and interpreting results; improved understanding about computational processes and proof, and an increase in conceptual understanding (e.g. Park & Travers, 1996; Demana, 1998; Bookman & Friedman, 1999). While not at the tertiary level, Slavit (1996) noted that the use of graphics calculators was associated with higher levels of discourse in the classroom, and an increase in the number of analytic questions and
student initiations during periods of such use. Similar results may be achieved at the undergraduate level, although this may be difficult to achieve in large lecture-style classrooms.

Many of the studies and reports in the tertiary sector adopt a technology-assumed stance, and investigate the effects of specific technologies used in their courses. One large scale study (687 students) compared the use of the TI-92 to other types of calculators in a first year calculus course (Keller, Russell & Thompson, 1999). This study found that, independent of the instructional format, the students using the TI-92 performed significantly better than students using other calculators:

The TI-92 students more successfully completed one-step problems and put combinations of steps together to solve intermediate problems. In particular, on problems with multiple steps, TI-92 students across all groups performed at a greater success rate … (and in one problem), the TI-92 students scored ten percentage points higher than (other) students, even though the TI-92 should not have influenced the students’ explanation.

(Keller, Russell & Thompson, 1999, p. 203)

The findings of this study led the authors to conclude that regardless of equity factors, their students should be required to purchase a TI-92 calculator over other calculators. Not all studies are as positive or conclusive, and often it is not clear exactly how much of the observed improvements may be attributed to technology use, and how much to other reform initiatives such as group learning and changes to course content and delivery. For example, Meel (1998), although in an admittedly small study, found more equivocal results. While he too found that reform students (n=16) performed better at solving real-world and complex multi-step problems, traditional students (n=10) did significantly better on limit tasks and conceptual items. Based on a wide variety of data gathered from university records, questionnaires, and a problem solving interview, Martin (2000) concluded that the impact of graphing technologies (GT) was less than optimal. “GT students in this study did not display the expected enhanced conceptual knowledge of important calculus concepts. The course appeared to have a lasting influence, but neither to the extent, nor in the conceptual domain that one might have hoped for” (Martin, 2000, p. 109).

In another comparative study, Palmiter (1991) concluded that while the computer-based students significantly outscores the traditional group on both the conceptual and
computational exams, several other underlying factors may have contributed to the positive results. Chief amongst these may have been the high levels of enthusiasm shown by the students in the computer group. While improved attitudes is in itself a worthwhile result, Palmiter (1991, p. 155) observes that some of the improvements may be attributed to increased time on task as a result of improved interest, rather than any learning benefits offered specifically by the technology. Thomas (2001) suggests CAS calculators may have beneficial uses as both procedural and conceptual representation tools, with each requiring qualitatively different perspectives on the mathematical objects; Doerr and Zangor (2000) list five different ways in which students may make use of technology to realise such benefits. However, the evidence is that many students initially use CAS only for checking working or procedural calculations, with little understanding of conceptual use (Thomas, Monaghan & Pierce, 2004, p. 155).

Other factors which contribute to less than expected benefits include issues of student facility, such as instrumentation described in earlier discussions (Guin & Trouche, 2000), and the additional cognitive overload placed on students learning work in a new medium (Norton & Cooper, 2001a; Norton, Cooper & Baturo, 2001). Technology is not a pedagogical panacea that automatically induces a more questioning and reflective mathematical attitude for all students. Guin and Trouche (2000, p. 13) note that weaker students often give up trying to understand the calculator functions, and employ avoidance strategies such as random trials and switching to other commands. Difficulties with manipulating the calculator commands also revealed conceptual difficulties (such as recognising two equivalent algebraic expressions), even though the calculator provided relief from the technical tasks (Guin & Trouche, 2000). Others note similar difficulties posed by the syntactical and command structures of computer algebra software packages. For example, Goos and Cretchley (2004, p. 159) perceive the relative syntax complexity of Mathematica as a potential barrier for learning, and Pierce (1999, p.173) found that while most students had little trouble with the command structure, the rigour of the syntax in DERIVE caused problems for many. Like Guin and Trouche (2000) for calculators, Galbraith and Pemberton (2002) note a significant effect of syntax and functionality with the use of Maple on students’ success rate, especially for mathematically weaker students.
Our experience suggests that symbolic manipulator software, far from simplifying demands, throws into relief learning issues that add substantially both to our understanding of student problems, and to the challenge of meeting them. On the other hand for those students who possess sound understanding…. the software (is used) as a power tool rather than a learning tool. They are able to use its capacity to extend the boundaries of their capability (Galbraith & Pemberton, 2002, p. 291).

Blyth and Labovic (2005) observed similar differences in the use of Maple between high achieving and low achieving students, with the bottom 10% of students struggling with the software commands. For the higher achievers, any confusion that arose between the various representations and different syntactical notations was seen as largely beneficial, in that it stimulated discussion, and prompted resolution of the resultant cognitive conflict. Cretchley and Galbraith (2003, p. 44) suggest that the evidence of significant advantages for students with high levels of technology competence is a serious equity concern, a case of “the rich getting richer”.

The ability to interact effectively with the technology is also closely related to student attitudes towards technology, and confidence in using it (Pierce & Stacey, 2001b; Goos & Cretchley, 2004). While some recent studies suggest that previous use of technology seems to improve students’ effectiveness and attitudes towards its use (e.g. Stewart, Thomas & Hannah, 2005; van der Hoff & Harding, 2007), others have found no significant differences in technology performance or attitude to technology in learning mathematics between those with previous experience and those without (e.g. Blyth & Labovic, 2005; Cretchley, 2007). For example, Stewart, Thomas and Hannah (2005) found that students with prior use of technology (second-year engineering students using Maple and Matlab) were making greater movement towards CAS-based instrumented techniques, and displayed greater preferences to use, and better attitudes towards computers than less-experienced users. By contrast, Cretchley (2007) found no such correlation between performance and attitude for 87 undergraduate students with the use of Matlab. This ambiguity suggests that perhaps concerns about which technology to adopt in undergraduate mathematics, based on students familiarity from prior experiences, may not be a significant factor (for example opting for graphics calculators because student have used them in school). However, there may well be other reasons for such considerations, for example equity or portability (see the next section).
Some students remain resistant to the use of technology, exhibiting preferences for by-hand techniques, and a lack of trust in the capabilities of technology (see Galbraith & Haines, 2000; Goos & Cretchley, 2004; Stewart, Thomas & Hannah, 2005). These attitudes often reflect the students’ previous learning experiences and their teachers’ attitudes towards technology, and may be seen as a manifestation of teacher-privileging against technology (Kendal & Stacey, 2001a). There is also evidence of gender differences in attitudes towards technology at early undergraduate level, in opposite directions with relation to attitudes to mathematics, and performance. While female students exhibit significantly more positive attitudes with respect to scales measuring mathematics engagement and mathematics motivation, a different picture emerges with respect to computers, where males scored significantly more positively on scales measuring computer confidence and computer motivation (Galbraith & Haines, 2000, p. 31). As Goos and Cretchley (2004, p. 159) conclude, such attitudinal findings highlight the importance of considering learners’ differences when implementing technologies in an educational setting.

**Mathematical Content, Reasoning and Skills**

This part of the discussion will consider studies that examine learning issues associated with specific content areas within the broader areas of calculus, algebra and geometry. Benefits in calculus are commonly associated with the ability of technology to provide connections between the algebraic, numeric and graphical representations of functions, and facilities such as “zooming-in”, which permit students to examine in-depth the graphs of functions and observe features of local linearity, continuity and differentiability with relative ease. (e.g. Wilson & Krapfel, 1994; Tall, 1996b, Pierce, 1999; Thomas & Holton, 2003). Ruthven (1992, p. 96) for example found that students using graphics calculators significantly outperformed a control group in two main respects. Firstly they were better at recognising particular types of graphs, and secondly, they were more successful in forming symbolic descriptions by exploiting salient information such as orientation, extreme values, zeroes, and asymptotes.

Arnold (1992) suggests that by examining multiple examples of a variety of functions using different representations, secondary school students using graphics calculators were better able to conceive of functions as a process, rather than objects, and that these students performed better. In seeming contrast to this, other studies
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Theoretical Issues In Technology

indicate that it is important for students to be able to view a function as an object in its own right (Tall, 1996b). Hong and Thomas (1997, 1998) describe the ability of technology to improve students’ limited conceptualisation of Riemann integration. They found that first year university calculus students had a tendency to view integral calculus as a series of procedures and associated algorithms, and were unable to operate successfully when there was no obvious algorithm to employ.

By relating the algebraic and graphical representations, studies by Witt (1997) and Brauer (2000) demonstrate increased insight into the concept of stability when working with ordinary- and partial differential equations (ODEs & PDEs). Few studies were found at the tertiary level investigating differentiation specifically, but the study by Lagrange (1999) does show benefits of technology in linking the theoretical concept of limits to those of tangents and instantaneous change, while Ferrini-Mundy and Graham (1994) found a more enactive conceptualisation of limits in general. Thomas and Holton (2003, p. 370) also suggest that the results of studies by Kendal and Stacey (2000, 2001b) and Delos Santos and Thomas (2002), which demonstrate the importance of inter-representational links for senior secondary students in learning differentiation, may be transferable to the tertiary level for first-year calculus courses.

The benefits of technology in calculus for establishing inter-representational links and encouraging inter-representational thinking are seen as equally important by the majority of studies in algebra (e.g. at the secondary level, see Hong, Thomas & Kwong, 2000; Doerr, 2001; Kendal & Stacey, 2001b; Pierce & Stacey, 2001b). For example, with respect to specific representations, several studies have described benefits of technology on secondary students’ concepts of a variable (e.g. Tall & Thomas, 1991; Boers, 1992; Graham & Thomas, 2000). In addition, there is the power of CAS-technology to enable new representations, and facilitate new interactions within specific representations themselves (e.g. Thomas, Monaghan & Pierce, 2004; Thomas, 2007).

At the tertiary level, Stewart and Thomas (2003) describe how linear algebra in particular remains one of the most difficult conceptual challenges facing undergraduate mathematics students. While technology is certainly not regarded as a panacea, there are studies that do demonstrate the benefits of technology in both linear and abstract algebra. These include investigative approaches to algebra which facilitate discovery learning, and develop students’ learning strategies (Dreyfus & Hillel, 1998). Hollar and Norwood (1999) concluded that students using graphing calculators in an intermediate
algebra course performed significantly better than traditional students (without
technology) on all four components measured in their study (modelling, interpreting,
translating and reifying). In addition, they found no difference between the two groups
in traditional algebraic skills, thus contradicting concerns that technology causes
deterioration in students’ skill levels (e.g. Koblitz, 1996). In addition to the work on
APOS theory by Dubinsky and others described earlier, that demonstrated benefits of
technology for group theory and abstract algebra, Thomas and Holton (2003, p. 371)
ote the extensive body of research by Hillel, Sierpinska, Dreyfus and others on the use
of technology in early linear algebra. Hillel (2001b), for example, provides three
eamples designed to facilitate the learning of matrices using CAS. An earlier study by
Dreyfus and Hillel (1998) found that the use of Maple enabled students to dispel several
misconceptions about approximations and other aspects of inner product spaces.
Sierpinska (2000) found that students enjoyed the practical approach of using Cabri
Géomètre in a linear algebra course, and concluded that this can be beneficial, leading
to new approaches and insights.

Surprisingly, given the importance attached to the visualisation benefits provide
by technology, the case is much less clear for geometry, certainly in the many countries
(e.g. Australia, New Zealand, UK, USA) where relatively little geometry is now taught
(Thomas & Holton, 2003, p. 377). There are many activities and specific software
packages provided to assist in the teaching of geometry (see e.g. Laborde, J-M. 1990;
2004; Vincent & McCrae, 2000), but most of these are not evaluative studies that
demonstrate confirmed benefits. One study that did so is that of Stevenson and Noss
(1999). They looked at the case of non-Euclidean geometry, and found that students
were better able to visualise hyperbolic surfaces through the window, in the sense of
Noss and Hoyles (1996), provided by the computer microworld they were using.

Notwithstanding such benefits, the studies discussed above all note problems with
the use of technology. These are often expressed as obstacles to learning, commonly
associated with instrumentation and technical aspects of the technology as described in
the earlier theoretical discussions (e.g. Ruthven, 1996; Cavanagh & Mitchelmore,
2000a, 2000b; Trouche, 2000; Drijvers, 2000, 2002), or provisos and limitations to the
observed benefits (e.g. Tall, 1996b; Thomas, 2001, Kidron & Dana-Picard, 2006). For
example, students are reported to experience difficulties integrating various
representations from one register to another (for example graphic to algebraic), even
though this is the precise ability that others identify as a major benefit of technology (Slavit, 1998; Norton & Cooper, 2001a). Studies at the secondary level have shown that deficiencies in software and calculator graphing displays and other features of the operating systems create problems for students, and there is evidence that these factors remain an issue for tertiary students (Boers & Jones, 1992a). Low screen resolution and poor pixel discrimination; problems with scale associated with zooming and the absence of numeric values on axes; poor understanding of the procedures adopted by the software in displaying solutions and graphs; and difficulties in entering and interpreting expressions, have all been identified as significant problems for students (Ruthven, 1996; Cavanagh & Mitchelmore, 2000a, 2000b; Pierce, 2000; Trouche, 2000). There are also examples of specific mathematical problems for which certain technologies encounter difficulties (e.g. Postel & Zimmermann, 1996; Lehning, 1998; Wester, 1999; Kawski, 2003). Verwoerd (2003) provides an example, using Mathematica, of a common problem arising from applying symbolic software to real world examples, where the explosion of complexity makes an explicit solution as bad as or worse than a numeric one.

Ball, Pierce and Stacey (2003) found that senior secondary students experience major difficulties in recognising the various equivalent algebraic expressions that CAS-systems employ, and concluded that more emphasis must be given to algebraic expectation, and addressing the tensions between by-hand and CAS techniques. Studies by Drijvers and others (e.g. Drijvers, 2000, 2002; Kieran & Drijvers, 2006) identify other obstacles to learning in a CAS environment. These include global obstacles, such as the inability to decide when and how CAS can be useful; the black box nature of the CAS, and the difficulty in interpreting the CAS output. Local obstacles include the preference for numerical over algebraic solutions; the flexible conception of variables and parameters required by CAS, and problems with recognition of equivalence and expectation in algebraic representations (Drijvers, 2002, p. 227). While such obstacles may be potentially significant barriers to learning, Drijvers prefers to make them the explicit subject of classroom discussion, in which the meaning of the techniques and the conceptions are developed. “This approach turns the obstacles of computer algebra use into opportunities for learning, and enriches mathematical discourse in the classroom” (Drijvers, 2002, p. 228).
Several studies at the tertiary level concur with these results. Dreyfus and Hillel (1998) found that students had difficulty identifying where, when and how to use Maple to help them, and frequently found the graphs inadequate and disappointing, while Sierpinska (2000) noted that while Cabri helped students in linking representations, they still had problems relating these to matrix arithmetic. Kidron and Dana-Picard (2006) found many students have an inability to differentiate between errors due to mathematical meaning and those due to the instrument. Students frequently attributed errors inappropriately to the technology, citing rounding or reproduction errors as the cause, instead of the limitations of the numerical method they were using, such as approximations. Technology also seemed to reinforce misconceptions that gradual causes have gradual effects (Kidron & Dana-Picard, 2006, pp. 274-275).

The discussions in this section have demonstrated a number of benefits associated with the use of technology in mathematics teaching and learning to support the underlying assumption of this thesis about the appropriateness of an ITMC. While a number of problems have also been identified, much of the evidence suggests that many of the learning difficulties associated with the use of technology described above become less significant with experience (Trouche, 2000; Stewart, Thomas & Hannah, 2005; Kidron & Dana-Picard, 2006), and further, that as long as they are explicitly identified and adequately managed by teachers, the problems may be seen as assisting rather than obstructing learning. Addressing such issues thus seems critical when implementing technology, and suggests an important area of investigation for this thesis. As Kieran and Drijvers (2006, p. 286) conclude:

Even if such complications in applying CAS techniques may seem to be hindrances (see Drijvers, 2002) to students’ progress, in fact our experience suggests that they should be considered occasions for learning rather than as obstacles. However, a precondition for these complications to foster learning is their appropriate management in the classroom by the teacher. (Kieran & Drijvers, 2006, p. 286)

3.3 WHICH TECHNOLOGY?

The range of possible technologies that may be employed by departments and instructors continues to proliferate, and the choices presented are often difficult for those faced with making an appropriate selection. Factors that may influence choices
include familiarity, cost, student access, the complexity of the syntax, availability of help with learning to use it, CAS or non-CAS, and specific domain-oriented demands such as statistical analysis or numerical analysis capabilities (Goos & Cretchley, 2004; Brown & Leigh-Lancaster, 2004; Bottino, Artigue, Cerulli et al., 2006). At the tertiary level, educational forums such as the Delta undergraduate mathematics symposium series provide numerous examples of interesting and challenging applications of specific technologies. These suggest that familiarity with specific technology packages in mathematicians’ personal research domains may be a definite factor in their resultant choice of technology for teaching (e.g. experiences with MATLAB in a scientific computation course and first year mathematics, Colgan, 1998, Broughton, 2003; functions and operators in MAPLE and Matlab, Kawski, 2003; using Maple in first year courses, Blyth & Labovic, 2005). There is however little reference in the literature to the influence of familiarity and domain-specificity on technology choice, which suggests one worthwhile area for this thesis to pursue.

Dissatisfaction or uncertainty with available products has in many cases led research mathematicians with an educational interest to develop their own technological learning tools, some of which have become commercially produced (e.g. Cabri, Laborde, J-M., 1990, 2004), while others have been freely available (e.g. ISETL, Dubinsky, 1995). There has also been an explosion of supplementary programs, applications and task-specific applets devised for existing technologies, many of which are freely available to download on the internet (Brown & Leigh-Lancaster, 2004). The functionality provided by the provision of formerly computer-based software products such as Derive and Cabri on the TI-92 calculator, or the CAS-program Symbolic that may be added to the non-CAS graphics calculator TI-83 Plus, has caused a blurring of the common distinctions between different technologies such as graphics calculators, CAS-calculators, and computers (Guin & Trouche, 1999; Brown & Leigh-Lancaster, 2004). Recent studies have even begun to consider theoretical issues of design and functionality for different technology enhanced learning systems, suggesting theoretical perspectives that may facilitate the development of more effective learning tools (e.g. Bottino, Artigue, Cerulli et al., 2006; Healy, 2006). One example of this is the development of 3DMath, a dynamic visualisation software system that seeks to incorporate theories about the construction of mathematical knowledge and semiotics into the technological design. “This is so that the pedagogy is fully integrated as a basis
for technological design, rather than the pedagogy, as is often the case, being based on what the technology appears to permit” (Jones, Christou, Pittalis & Mousoulides, 2006). More recently, Texas Instruments have released the latest generation graphing calculator, the TI-Nspire, which addresses many of the previous technological distinctions. This maths tool combines a wide range of applications including a CAS and dynamic geometry software, and is offered both as a hand-held calculator, and as a computer software package (PC and Mac), for use with data-shows, electronic whiteboards, or in computer laboratories. In addition, it offers integrated data collection in both the graphing and analysis environments. The influence of pedagogical theory in its design can be seen in many of its features, including:

- Dynamic linking of different representations (algebraic, graphical, geometric, numeric and written), to see how changes to one affect others.
- Drop-down menus (similar to MS-Windows) for easier, more intuitive operation, and the ability to enter and review functions as written in textbooks (syntactical conformity).
- Identical functionality between software and hand-held versions, and copy and paste functionality with Word and Excel.
- Ability to save all work to allow for critical review.

These combined features may help address concerns about the inferior nature of graphics calculators amongst research mathematicians, if they could be made aware of them. The difficulty of choice is not helped by the fact that many of the studies examining learning theories and benefits as described in previous discussions are specific to one technology (e.g. CAS-calculators, Matlab, or dynamic geometry software), and do not contrast the different technologies available. This is further exacerbated in the entrenched debate over calculators versus computers, and the more recent arguments about CAS. The conclusion by Goos and Cretchley (2004, p. 159), that the “appropriate choice of software for early undergraduates, and level of use for computer algebra, are clearly issues that need to be considered very carefully” suggests a critical element for consideration in this thesis.

When talking about technology, it is also important to distinguish between the varieties of ways it may be used (Kenelly, 1996; Wong, 2003). In many cases, the use is determined by the design of the technology, rather than through the user’s choice. Dick
(1996) distinguishes between mathematical toolkits (machines designed as general computational/graphical tools for which educators devise pedagogical activities, e.g. Matlab), and computer software pedagogically designed for students to learn a particular area of mathematics (e.g. microworlds, Cabri). Love (1995, p. 111) suggests a greater degree of differentiation, observing that mathematical software may be devised for several different purposes:

- specifically for use by professional mathematicians
- for ‘users’ of mathematics
- for learning purposes
- for purposes not primarily recognised as ‘mathematical’

Love believes that mathematicians should take special care to distinguish between the needs of learners and users. Software such as Maple and Mathematica were created essentially to help professional mathematicians explore mathematical problems and examine behaviours of functions and algebraic structures. In using such software, students encounter many difficulties, with the result that the technology detracts from, rather than adds to their learning (Love, 1995, pp. 114-115). Wiest (2001, pp. 46-51) gives a similar but more detailed description of four categorisations of computer-assisted mathematics instruction, which she labels as Tool Software (e.g. Microsoft Excel, Matlab), Instructional Software (e.g. 3DMath), The Internet, and Programming. Wong (2003) suggests a further differentiation, distinguishing between mathematically educational games; delivery mechanisms such as data-shows and the interactive whiteboard; calculators; and computer software such as graphing software, computer algebra systems, mathematical word processors, and dynamic geometry.

Love (1995) emphasises that the design intentions he suggests are by no means mutually exclusive. For example, while Matlab may have been initially designed as a powerful computational tool for professional mathematicians, its use as a learning tool has developed greatly in recent years (see e.g. Broughton, 2003; Kawski, 2003). Similarly, while the microworld of LOGO was clearly designed for instructional purposes, it is also a programming language, for which the essentially geometric uses it was initially intended have expanded greatly, to now encompass a wide variety of educational uses. The TI-Nspire described earlier clearly intentionally blurs the
boundaries between categorisations, purposely designed to function across a wide range of uses and applications.

The boundaries between the various non-mathematically specific technologies are also becoming less distinct. While the Internet was not developed primarily as a mathematical learning tool, there are many studies and reports describing new and effective ways that it may be used for just this purpose. Engelbrecht and Harding (2001) note that for most web-based courses, the internet serves as a management tool containing little mathematics. However, some do provide comprehensive problems and demonstrations, content notes, and even make use of computer algebra systems, such as in the ‘Calculus & *Mathematica*’ course at the University of Illinois (Davis, Porta & Uhl, 2006). Very few such courses offer continuous guidance and support from lecturers, partly due to cost and resource constraints (Crowe & Zand, 2000). Engelbrecht and Harding (2001) describe the design of an internet-based calculus course at the University of Pretoria, which offers a dynamic, structured approach to guiding students’ learning. This course extends the more common administrative functions of the internet to include day-to-day scheduling of learning objectives and problems, and adopts novel learning approaches (for on-line courses) such as group activities. They also report positively on their extensive use of combined on-line and written assessment techniques, which they elaborate on further in later studies (Engelbrecht & Harding, 2003; 2004). Two later papers by Engelbrecht and Harding (2005a; 2005b) examine the wider use of internet education in mathematics. The first of these in some ways parallels the current investigation, examining the types and extent of internet use in a number of courses, to arrive at a taxonomy for the variety of usages they describe. The second paper explores the pedagogical and structural attributes of internet courses, discusses some of the implications of these, and speculates about future possibilities. Thomas and Chinnappan (2008) describe a number of ways in which the internet is now being used by teachers, including for remediation, drill and practice, and for engaging in mathematical discourse. Wiest (2001) too lists several common uses under her category of *Instructional Software*, including assessment, remediation and tutorials, simulations, and games.

Other non-mathematically specific uses include the use of digital photos and movies to bring real world situations into the mathematics classroom (Pierce, Stacey & Ball, 2004), and the use of many different technologies such as PowerPoint, videos and
data-projectors in the delivery of mathematical courses (e.g. Tobin, 1993; Meyinsse, 2001). PowerPoint slide-shows have been used in lectures to provide step-by-step development of mathematical problems, through the use of animated effects and written narrative, to model how solutions are arrived at, although the pre-recorded, impersonal nature of these are seen as less effective if students view them removed from the lecturer’s commentary (Bulmer & Rodd, 2005; Budgett, Cumming & Miller, 2007). While the use of data-shows and computer tablets to deliver and record lectures may seem essentially generic to all subjects, there is a growing awareness that such technologies may provide special mathematical benefits (see for example Dekkers & Shepherd, 2005; Loch, 2005). There is evidence that recorded lectures using tablet-PC’s and audio-capture allows students the opportunity to review the step-by-step construction of a mathematical problem, and revisit concepts in contrast to the linear order in which they are often presented in a mathematics course (Bonnington et al., 2007).

Despite such a proliferation of uses, this thesis focuses on technologies designed primarily for mathematical purposes. The main distinction in the literature in this regard is between calculators and computers, and within this distinction, the capabilities of specific calculators and types of software, especially with respect to CAS and non-CAS facilities (Schultz, 2003). While Kenelly (1996) argues that the essential element of the debate about calculators versus computers should be about the underlying purposes and activities for which the technology is used, rather than the technological capabilities, there are some particular arguments that distinguish between the two. Laughbaum (1999) found that a large proportion (85.1%) of the 600 respondents to a survey of 2330 colleges and universities in the USA reported using graphing calculators, and 34% using CAS. Given that there are fewer differences between the mathematical capabilities of computers and calculators (for example the inclusion of software packages such as Derive and Cabri on the TI-92, and the functional compatibility of the TI-Nspire), one of the main advantages of hand-held technologies (calculators) is their portability, and the ease of personal access to technology that they provide students (Kissane, 1995; Arnold, 2004). Wood describes a study by Bradley and Kemp (2002, in Wood, 2004, p. 229) that showed a clear preference for calculators over computer software packages in a first-year university statistics course, with portability a key issue.
While the supposed ease of access to calculators is often cited as an advantage, the cost of increasingly complex models may still be prohibitive for many students, and in any case, the growing provision of sizeable computer laboratories appears to provide similar ease of access for many students (Galbraith & Pemberton, 2000). Concerns about equity caused by huge differences in access to technology through financial circumstances, often labelled as the “digital divide”, are frequently raised (Kissane, Bradley & Kemp, 1994; Zevenbergen, 1999, 2001; Forgasz, 2002, Berger & Cretchley, 2005). Van der Hoff and Harding (2007) suggest that the simple, widely available spreadsheet software Excel is a comparatively equitable technological solution for many South African university students. It requires relatively little prior knowledge for students to reach an effective level of familiarity, and may be used to teach even quite advanced concepts such as differential equations. Other studies suggest that some students do not like graphics calculators (Kissane, Bradley & Kemp, 1994; Faragher, 1999), and that gender may play a role in the preferences for different types of technology and provide further sources of potential inequity (Forgasz, Griffith & Tan, 2006). At a more theoretical level, some studies compare differences in the affordances and constraints offered by calculators and computers, although the descriptions of these do not specifically favour one technology over another (Doerr, 2001; Brown, Stillman & Herbert, 2004).

Notwithstanding the findings of Laughbaum (1999) for the high levels of calculator use in USA universities and colleges, and despite studies such as that of Keller, Russell and Thompson (1999) cited earlier which provide evidence for the benefits of CAS-calculators, the disregard shown by many research mathematicians for calculators (Keynes & Olson, 2001) still often leads to preferences for computers when making technological choices. While many of the available microworlds and integrated learning systems (ITLs) are focused primarily on graphs and beginning algebra at the secondary level (e.g. Logo, Cricket Graph, Stewart, 1991; The Learning Equation, Norton & Cooper, 2001a; Aplusix, E-Slate, ARI-LAB 2, Bottino, Artigue, Cerulli et al., 2006), there is a considerable number of available CAS products more suited to undergraduate and advanced mathematics. A number of studies and reports examine the virtues of individual products (e.g. Maple: Bergsten, 1996, Hannah, 2003; Mathematica: Fearnley-Sander, (2001); Matlab: Cretchley, Harman, Ellerton & Fogarty, 1999, Broughton, 2003). However, while many others also compare various
elements of the more common CAS, the conclusions of these are somewhat ambiguous, and may not be of great assistance in making a final choice.

For example, Meagher (2006) contrasts Derive and Mathematica, and seems to favour Derive for learning mathematics, with its menu-driven functions which rely less on the learning of a specific syntax. While Dana-Picard and Kidron (2006) agree with the step-by-step advantages of the Derive menus, they see this structured approach as somewhat restricting. They believe that the greater variety of options provided by Maple and Mathematica provides students with freedom of choice, and hence an opportunity for deeper conceptual understanding (Dana-Picard & Kidron, 2006, pp. 132-133). Two substantial comparisons of the capabilities of seven major general purpose CAS systems (Axiom, Derive, Macsyma, Maple, Mathematica, MuPAD & REDUCE) are provided by Postel and Zimmermann (1996), and Wester (1999). The first of these used a wide set of more than 50 ODE’s to provide an insight into the capabilities of the various programmes in solving differential equations. The second study, from an admittedly applied mathematics bias, examined the results of 542 short problems from a broad range of symbolic mathematics, and summarised these in a detailed table. Neither study endorses any one product over another, with both noting shortcomings and special strengths for all systems. Postel and Zimmermann (1996) conclude that the best answer is to try all of them, and use the best one for a given occasion, as several peculiarities exist. Kawski (2003) concurs with the need to consider the particular strengths and weaknesses of individual products in his comparison of aspects of Matlab to those of Maple and Mathematica. He worries that the powerful convenient functions that make Matlab so effective for advanced users often cause severe overload for students, while he describes the way in which Maple and Mathematica distinguish between pure functions and expressions as a potential nightmare for the unprepared user.

However, few mathematical educators charged with choosing technologies for their department or course are likely to be familiar with all such aspects of all available products, and the table provided by Wester (1999) would quite probably prove too complex to be useful to many. A recognition of the difficulty of choice, costs of individual commercially available products, and the inevitable variation in selections arrived at by individual departments and instructors, have led to the development of some learning resources that are not specific to any one technology. At the secondary
level, the Victorian Curriculum and Assessment Authority recognises this variation by stipulating no particular type of technology, although some evaluative reports do demonstrate privileging of some technologies over others (e.g. Evans, Norton & Leigh-Lancaster, 2004), and this may affect technology choices in future. However, since they also note that these differences may have as much to do with the nature of the questions as the particular technology, the changes may equally well occur in the assessment practices as in the choice of technology (Evans, Norton & Leigh-Lancaster, 2004, p. 229). At the undergraduate level, both the SimCalc Project developed by the Mathematics Education Research Group at the University of Massachusetts (Kaput, 1998), and Project CALC at Duke University (Smith & Moore, 2001), incorporate generic technologies. Project CALC is especially generic, with laboratory manuals and supporting computer files published for Maple, Mathcad, Mathematica, Derive, and TI and HP calculators, as well as a web version for Mathwright, and a separate web-based version developed by the Connected Curriculum Project. The developers emphasise that additional hardware and software platforms are continually under consideration.

As Day (1993, p. 36) advocates, “An eclectic approach may work best, picking and choosing from general tool software and topic-specific software as your needs and desires dictate, and your time and budget permit”. However, as the preceding discussions have shown, the complexities involved in making such decisions have significant repercussions for the intellectual and economic resources of staff and departments, and for students in learning to use the different technologies. Perhaps the most significant conclusion that can be reached from these discussions about technology choices is provided by Kawski (2003, p. 168), who suggests that rather than favouring one technology over the other, it is essential to be aware of the consequences of choosing a specific technology, both in the potential hazards for curricular integrity, and the exciting opportunities for curricular improvement.

3.4 HOW TECHNOLOGY? ISSUES OF IMPLEMENTATION AND INTEGRATION

While the previous section has described important considerations with respect to technology selection, the meta-analysis of calculus reform on some 110 tertiary institutions led Ganter (1999) to conclude that “the impact of reform is perhaps
not so dependent on what is implemented, but rather the educational environment that is created in which to implement it” (Ganter, 1999, p. 235). Leigh-Lancaster (2002) describes a hierarchy for the level of technology use (ranging from none at all through to universal use), as a starting point for making implementation decisions, but even within this hierarchy, the form of computer-based instruction varies widely. Galbraith and Pemberton (2000, p. 267) illustrate this with two extreme examples, which may both be regarded as exhibiting extensive or universal use of technology: on one-hand a gigantic computer emporium at Virginia Tech designed largely for institutional efficiency, where hundreds of students sit at computer booths working on their own, assisted by roving tutors; on the other hand they describe an expensive model where all the teaching is done in a small electronic classroom, incorporating full computer and multi-media facilities, and using designer courseware.

Such variations suggest a need to discriminate between the types of technology use as opposed to just the extent of use, and examine the reasons for, and effects of, the differences. Some studies involving moderate use of technology recognise that this usage is less than optimal, suggesting they would use it more if possible. For example, Barling (1993) notes that when introducing graphics calculators in the Applied Science Faculty at Swinburne University, they had to limit themselves to an implementation that was achievable in the short term. While calculators were incorporated at every relevant point, no serious rewriting of the curriculum was attempted. Bloom, Forster and Mueller (2001) describe a similarly constrained introduction of CAS-calculators into their first-year elementary calculus course at Edith Cowan University in Perth. The course was designed around the reform calculus text by Hughes-Hallet, Gleason and McCallum (1998), with the systematic integration of technology into the teaching, learning and viewing of each topic from as many as possible of the analytic, geometric, numeric and word-description aspects. The calculators were available for use in all class-work, tests and the final examination. The implementation was not particularly successful however. Many of the students did not have their own calculator, accessing the technology solely through the class-set provided in lectures, with a few machines available in the library out of class. This created inequities, as well as potential problems for students progressing to courses without calculators provided. In addition, they also encountered initial setbacks with the specific technology (HP49G), such as difficulties in handling one-sided limits or singularities. While many of these were ultimately resolved, they
recommend that attention to the limitations of the technology is an important consideration, with clear articulation of the skills of visual interpretation (Bloom, Forster & Mueller, 2001, p. 9).

Practical limitations seem to underlie many technology programmes, even those involving more extensive use of technology, and programmes which are regarded as otherwise successful. For example, Galbraith and Pemberton (2000) attribute the design of the programme at Virginia Tech to economic rationalism, while both the programmes described by Henderson (2002) and Pemberton (2002) suggest that decisions about the use of Matlab and Maple, respectively, in their courses depended to a large extent on limits of access to computers in the laboratory, as opposed to pedagogical reasons. While both these studies describe technology as integrated in their courses, students’ interaction with the technology in a supervised teaching environment was limited to one hour a week in a computer laboratory. In contrast to Barling (1993) and Bloom, Forster and Mueller (2001), however, neither of these studies reported significant disadvantages of this seemingly limited approach, with Henderson (2002) finding that students still appreciated the well-defined structure of lectures and tutorials to support the laboratory component. Pemberton (2002) provides a caveat, stressing the importance of supporting the use of the technology in lectures. Another study also argues for a deliberately limited approach, finding that students introduced to MATLAB as a supplementary aid to the first year mathematics course at the University of Southern Queensland displayed general enthusiasm for this add-on approach. The researchers concluded that deep immersion in technology may be too sudden a change in learning experience for many students (Cretchley, Harman & Ellerton, 1999, p. 86). Despite this, it seems limited approaches to technology are more usually responses to insufficient funding and limited resources, than they are measured pedagogical choices (Hockman, 2005).

A number of studies report success with more extensive approaches. At the level of individual courses, Balas, Goulet and Smith (2002) describe a process of continual evolution, where over a period of ten years, their programme evolved from a traditional course of four one hour lectures, through to one involving a mixture of lectures, tutorials and computer labs, ultimately leading to the complete remodelling of their classrooms to incorporate computers in all their teaching. Pierce, Turville and Giri (2003) also describe the complete revision of their curriculum in response to dissatisfaction from education students with the dominance of functions and calculus in
the traditional course, and declining numbers of such students taking the course. The course incorporated a variety of technologies (e.g. spreadsheets, CAS calculators and dynamic geometry packages), to support new pedagogical methods, such as portfolios and projects as assessment tools, with a focus on mathematical thinking and communication. Staff and students’ reactions to the new curriculum were very encouraging (Pierce, Turville & Giri, 2003).

Quesada (1999) similarly describes the redesign of the traditional linear algebra course at The University of Akron, Ohio, over a two-semester period. Their changes were built around the use of the TI-92, changing the existing theoretical course to one oriented around matrices with relevant applications. Other considerations included reducing arithmetic calculations, and encouraging geometric thinking and mathematical writing. The students were required to have a TI-92, and a new textbook was selected more in keeping with the focus on applications and matrices. While use of the technology was modelled by instructors, no specific handouts on the use of the technology was provided. However, worksheets using the *scripts* function of the TI-92 were provided to facilitate the introduction of new topics and emphasise key algorithmic steps. Calculators were used in all course assessment, the scope and nature of which exceeded that of previous years. Exam questions were more complex and conceptual, with a new oral presentation component to the final grades.

The use of technology generally increases the number of possible approaches (to a question)…test questions (must) include not only what the student must calculate but also specific approaches…students need to be constantly reminded that they should express symbolically or in plain English what they are doing and show the key intermediate results. (Quesada, 1999, p. 324)

On a wider departmental level, Muller (2001) and Buteau and Muller (2006) describe a similar but more prolonged evolution of integrating technology into courses at Brock University in Canada. They describe how over a twenty-five year period, initial favourable experiences in Maple laboratories have inspired increasing levels of technology interaction, leading to the current vision in which the whole learning programme has been redesigned. The time taken for staff to gain the necessary experience and confidence with technologies was seen as an essential pre-requisite in the design of a new core curriculum, and the subsequent success of the programme:
Although one can point to certain years when major changes were implemented, the reality is that evolution and innovation in university mathematics education is a slow process. One reason for this is that few mathematics doctoral programmes require teamwork or provide opportunities for reflection on teaching and learning of mathematics. Yet these experiences are necessary for faculty in a department to work together as a team and for its faculty to critically redesign a mathematics program. There is much evidence that technological innovations that are instituted in a course by a single faculty member rarely survive when the course is taken over by another colleague...for changes to permeate beyond a set of courses, a consensus needs to be built. (Buteau & Muller, 2006, p. 75).

The Project Calc and SimCalc projects (see section 3.3) demonstrate similar programmes of implementing technology within long-term curricular development. Brown (1996) describes a more immediate plan to effect change in their programmes at the University of Michigan. In 1992, they set about implementing an entirely new course that introduced graphics calculators into the coursework, along with other pedagogical changes such as group-work, and team homework projects. The project involved extensive planning, was based around a reform text, and included training for instructors in the use and pedagogical opportunities of the calculator. Brown notes several problems with their approach, particularly that the amount of planning and work required was far greater than anticipated, and that it actually took some two years just to get the logistics of the course resolved (Brown, 1996, p. 57).

Regardless of the variations in technology use described so far in this discussion, several common factors that affect the success of reform projects (and by inference technology implementation) are identified in most evaluative studies (e.g. Rochowicz, 1996; Ganter, 1999; Darken, Wynegar & Kuhn, 2000). Hoyles (1998) identifies several obstacles to successful implementation of technology, which she lists as:

- Limitations on access to technology
- Lack of time for students and staff to familiarise themselves with the technology
- Being technology centred instead of mathematics centred
- Delivery by technology as if by a lecturer
- Emphasis on calculation versus explanation and reflection

(Hoyles, 1998, p. 39)
Lagrange, Artigue, Laborde and Trouche (2003) conducted a meta-analysis of more than 600 such evaluative studies, and from this, they developed a framework from which to examine the integration of technology from a largely theoretical perspective. They identify seven dimensions in their framework, namely the: General approach of the integration; epistemological and semiotic dimension; cognitive dimension; institutional dimension; instrumental dimension; situational dimension; and the teacher dimension (Lagrange, Artigue, Laborde & Trouche, 2003, p. 254). These dimensions can be related to elements of the general curriculum model of Valero-Duenas (2002, see Figure 2.3), and the model for a technology rich learning environment depicted in Figure 2.10 (Arnold, 1998). For example, the epistemological and semiotic dimensions (the influence of technology on mathematical knowledge and practices, Lagrange, Artigue, Laborde & Trouche, 2003), may be aligned to elements of knowledge and methodology at the intermediate and micro-levels of the Valero-Duennas (2002) curriculum model. This relationship is now examined under a series of broad headings that emerge from the models in Figures 2.3 and 2.10, and such issues as those identified by Hoyles (1998).

**Time and Planning**

Time issues cover a broad spectrum, including time to plan and execute technology implementations, time for staff and students to familiarise themselves with the technology, time to change often deep-seated negative attitudes towards technology use, sufficient time using the technology to privilege its use, and sufficient time for staff to change their practice and undertake professional development. Implementing technology should be recognised as a gradual evolutionary process, which allows time for all involved to comfortably adapt to the necessarily new learning and teaching styles (Balas, Goulet & Smith, 2002, p. 6). More than 84% of respondents to Rochowicz’s (1996) survey believed that using technology in calculus instruction requires significantly more time from the teacher, while insufficient time to change their practice was identified as a critical barrier to technology use by 60% of respondents to another large survey of tertiary mathematics educators in the United States (Kersaint, Horton, Stohl & Garofalo, 2003).

Several studies suggest that the time required to become familiar with the technology is frequently underestimated. Coupland (2000) found that one semester’s
work involving one hour per week in a computer lab was insufficient time to convince most students that Mathematica was a productive tool for their own use, and Cretchley, Harman, Ellerton and Fogarty (1999) agree that more time is required to create and maintain the motivating effect of technology. The concept of integrating the technology into students’ and teachers learning and teaching processes, described earlier as ‘instrumental genesis’ (Rabardel, 1995; Artigue, 2002; Hoyles, Noss & Kent, 2004), requires careful and attentive planning and presents a great challenge for mathematics educators (Mariotti, 2006). The complex syntax and functionality of modern CAS-machines take considerable time for students to master, and learning the capabilities of the technology and discovering new pedagogical and mathematical potentials involves continual time commitment for staff (Galbraith & Pemberton, 2000; Kersaint, Horton, Stohl & Garofalo, 2003). Thomas and Hong (2004, p. 303) conclude that the results of their study “are consistent with the view that instrumentation (of CAS) is not a short, easy process, but rather its development takes time.”

While not exclusively a time issue, redevelopment of course structures (for example considering new, or deleting past topics; changing the focus from skills to conceptual understanding) is also included here, because it is often omitted due to time and resource demands (Barling, 1993; Rochowicz, 1996, Brown, 1996). Several studies describe the importance of such considerations (for example in examining the pedagogical, epistemological and pragmatic value of curriculum topics, Artigue, 2002; Stacey, 2003), and some propose ways of approaching them (e.g. Rochowicz, 1996; Lauten, Graham & Ferrini-Mundy, 1999). Decisions about these are seen as complex and demanding of significant discussion, and consequently of time (Stacey, 2003).

These discussions suggest that the time necessary to effectively implement technology clearly involves significant investment from staff and students. Such a commitment requires recognition on their part that their efforts will result in a worthwhile outcome (Keynes & Olson, 2001; Pierce & Stacey, 2004). This is considered next.

**Teachers and Students**

The discussions in Chapter Two described a variety of issues that influence curricular change in general and technology specifically, including students and teachers attitudes and beliefs about technology, and their familiarity and ability to
interact effectively with it. Attention to, or at the very least an awareness of such factors, is seen as critical for successful technology implementation. For students, Galbraith, Pemberton and Cretchley (2001) emphasise that the complexity of issues affecting them can induce very different outcomes in seemingly similar situations. For teachers, Healy (2006) recognises the critical need to understand the role of the teacher in every step of the implementation process, and Anguelov, Engelbrecht and Harding (2001, p. 152) support this, when they state that:

Any change in mathematics teaching is implemented through the initiative, participation, support and involvement of the mathematics teachers. Teachers’ ideas about mathematics and mathematics education strongly affect their motivation for adopting one or another approach.

At the tertiary level, the ideas of lecturers and professors are seen as even more relevant to effecting change, given their relative independence over teaching methods and course content (Anguelov, Engelbrecht & Harding, 2001). Keynes and Olson (2001, p. 124) identify five key professional development issues for teaching staff that need addressing to effect successful technological change, summarised as follows:

1. An understanding of beliefs and values with respect to mathematical knowledge, computational techniques and what students are expected to learn.
2. Knowledge of modern pedagogical approaches.
3. An awareness of the significant impact teachers have on their undergraduate students’ learning.
4. The value of team instructional approaches and continued mentoring and professional development of teaching staff.
5. Appreciation of the pedagogical potential of different technologies as opposed to just the computational or research capabilities.

Some of these issues are reflected in the study by Anguelov, Engelbrecht and Harding (2001). They emphasise the need to recognise the different status and attitudes towards mathematics and mathematics teaching, between mathematics teachers, mathematics educators and research mathematicians. They suggest that as long as mathematics courses centre on teaching theories unrelated to practical experience, the impact of technology will not be significant. They believe successful exploitation of technology depends on the educator’s attitude towards applications, and they question whether much has changed since the findings of an older survey. This 1990 survey
sought the opinions of staff in non-mathematics departments about mathematics courses, and found that:

The mathematics faculty does not know or appreciate application … Mathematics departments have become so abstract oriented that their courses are not given any applied context … The content of most mathematics courses focuses on theoretical development.

(Garfunkel & Young, 1990, in Anguelov, Engelbrecht & Harding, 2001, p. 153)

Another study at the secondary level by Hong, Thomas and Kiernan (2000, p. 334) concludes that “teachers should be made aware that, when they teach using technology such as the TI-92, there are a number of possible pitfalls and disadvantages as well as benefits. The initiation of a programme of training to educate teachers in the classroom is recommended”. Tertiary mathematics educators clearly agree with this conclusion, with 68% of respondents in one survey indicating that limited personal knowledge about technology was a significant barrier to the use of technology in their mathematics teaching (Kersaint, Horton, Stohl & Garofalo, 2003, p. 570). Heid (1997, p. 9) stresses that teachers must be aware of the redefinition of epistemological authority that occurs when students have constant access to technology, and be prepared to relinquish some of the responsibilities of their former authority to the students.

A series of papers in a study by Kendal and Stacey (1999, 2001a, b) collectively demonstrate the significant effects of teachers’ beliefs and values about mathematical knowledge and technology on what students learn. They consider the concept of teacher privileging, which examines the effects on students, of choices made by teachers about teaching approach, content, and use of technology (Kendal & Stacey, 2001b).

Kendall and Stacey (1999) found that teachers will choose to highlight attributes of CAS which support their own beliefs and values about mathematics. Teachers who value routine procedures can find on a CAS a plethora of routine procedures to teach students; teachers who value insight can find many ways in which they can demonstrate links between ideas better than ever before.

(Stacey, Asp & McCrae, 2000, p. 248)

Cretchley, Harman, Ellerton and Fogarty (1999), for example, emphasise that students must perceive that teachers themselves value the technology, while Zbiek (2003, p. 205) stresses that teachers need to emphasise more than final answers and
monitor students’ methods of using CAS. The *RIPA* rubric (Reasons, Information, Plan, Answers) developed by Ball and Stacey (2003, pp. 294-295) is one example of how students may record their solutions to problems in a CAS environment. Such examples are necessary at the undergraduate level, where staff often have limited awareness of alternative assessment practices (Keynes & Olson, 2001). Teacher privileging has an impact on the beliefs and values of students towards technology and mathematics, which in turn has been shown to significantly affect technology implementation. There are a number of validated scales that examine these issues (e.g. Galbraith & Haines, 2000; Fogarty, Cretchley, Harman et al., 2001), and while it may not be practical for all those implementing new technologies to administer such questionnaires themselves, the factors listed by Keynes and Olson (2001) suggest that at least an awareness of the findings of studies using these scales is advisable. Such studies suggest that the disposition of students towards technology in learning mathematics, and their confidence in using it, can have considerable influences on the strategic choices they make when using it (Galbraith, Goos, Renshaw & Geiger, 2001; Galbraith, Pemberton & Cretchley, 2001, Cretchley & Galbraith, 2002).

Muller (2001) illustrates the fifth issue in Keynes and Olson’s (2001) list, when he observes that research mathematicians must appreciate the complexities of the technologies they use, and explicitly demonstrate the potential benefits to students through their teaching and the design of specific resources. The fact that the research interests of many such researchers are in mathematical areas far removed from undergraduate mathematics does not make this task any easier (Anguelov, Engelbrecht & Harding 2001). Muller (2001, p. 386) concludes, with respect to the inter-representational benefits offered to students by Maple, that “These are not evident (to students) because Maple was developed by mathematicians for mathematicians. Thus to integrate Maple into the learning and teaching of mathematics…students need to be guided through activities in which the various representations were highlighted” (Muller, 2001, p. 386).

While attention to student and teacher factors is clearly a significant issue in successful implementation, no interactions can occur without effective and equitable access to the technology. These issues are considered next.
Access and Equity

Technology is sometimes seen as providing more equitable access to mathematics, for example by allowing students with weaker skills to attain higher levels of achievement (Zevenbergen, 1999). However, such a position assumes students have regular access to technology tools (Heid, 1997), and this assumption is often not a reality for many students, for whom personal access to technology is financially impossible. Access may still be problematic, even where access to technology is provided for all students, through mechanisms such as class sets of calculators, or regulated times in a computer laboratory. Zevenbergen (1999, 2001) challenges the belief that unrestricted access to common laboratories addresses equity concerns over access. Economically disadvantaged students often have less previous exposure to technology, which limits their ability to take advantage of the new opportunities. Further, work commitments for these students, or parenting and family responsibilities for mature students, may also limit chances for these groups to access the laboratories during available times (Zevenbergen, 2001, p. 24).

Other equity concerns include gender and cultural issues, and the impact of factors such as socio-economic status, language background and geographic location on access to, and attitudes towards, computers (Forgasz, Griffith & Tan, 2006). Several contend that greater emphasis on technologies may impact negatively on girls (Hoyles, 1998; Forgasz, 2002; Vale, 2003; Forgasz, Griffith & Tan, 2006), while Vale (2003, p. 298) stresses that the use of computers may exacerbate cultural inequalities. Hoyles (1998, p. 40) further acknowledges that “there is a real danger that innovators focus on the potential of technology for the brightest and ignore the others.” Although a study at tertiary level found little evidence of gender-based differences in students’ interactions with technology, the researchers do conclude that other factors, such as group-work (reportedly favoured by women), may have contributed to this apparent discrepancy (Wood, Viskie & Petocz, 2003, p. 274). Regardless of their findings, they stress that it is vitally important that women and men have equitable experiences of learning using technology at the tertiary level. As Vale (2003, p. 298) concludes:

Unless the teacher adopts deliberate strategies in computer-based mathematics learning environments to enhance gender equity, the classroom tends to become a masculinised domain that threatens the principle of achieving social justice.
Assessment and Congruency

The discussions in section 2.3.4 described Leigh-Lancaster’s (2000) concept of congruency for technology use between curriculum, pedagogy and assessment, and suggested that such congruency was a potential indicator of integrated technology. This notion of congruency is reflected in the assertion by Dugdale, Thompson, Harvey and Demana (1995, p. 351) that “it makes little sense to use calculators … to express algebraic concepts graphically, then examine the students with tests that do not include graphical representations.” Regardless of the level of technology use, there is common agreement that technology use at least requires serious consideration in assessment practices, and at best significant changes (e.g. Kissane, Kemp & Bradley, 1996; Stacey, McCrae, Chick et al., 2000; Garner & Leigh-Lancaster, 2003). Such changes should extend not just to the type of questions asked, but to the forms of assessment used:

If the student is going to learn calculus in an environment using computers and calculators, then evaluation of student learning will encompass more than multiple choice questions on a closed format examination. Assessment must involve evaluating clearly written results of laboratory discoveries, multiple approaches and representations of concepts, and group as well as individual learning. (Rochowicz, 1996, p. 429)

Engelbrecht and Harding (2003; 2004) agree that technology offers opportunities for multiple assessment formats, and suggest the use of online assessment as one example of this. Online assessment opens up new possibilities, and should be embraced for its advantages, and the multiple formats it offers. It provides an effective and appropriate complement to standard paper-assessment (Engelbrecht & Harding, 2003, pp. 63-64).

One way of considering assessment in a technological environment is to consider assessment items according to whether they are technology neutral, technology active (or positive), or technology free (Kissane, Kemp & Bradley, 1996; Stephens & Leigh-Lancaster, 1997). The conceptions of integrated technology posited by this thesis to date seem to argue for a technology-active environment, where technology is assumed and required in all areas of assessment, and certainly this position is advocated by many (e.g. Kutzler, 2000a; Stacey, McCrae, Chick et al, 2000; Leigh-Lancaster & Stephens, 2001). However, while many assessment practices now assume a technology-active
stance, changes in the types of questions asked, to reflect technology, particularly CAS, are slow to eventuate (e.g. Kiernan, Oates & Thomas, 1998; Brown, 2001). This has implications for the concerns of equity raised earlier (Zevenbergen, 1999, 2001). Two studies in particular suggest that proficient use of CAS trivialises many traditional skills-based questions, and affords significant advantages for students using CAS, over those who don’t. The first of these examined questions from university entrance examinations in New Zealand over the period 1992 to 1997, and concluded that users of CAS such as the TI-92 could gain an advantage of approximately 10% over those without (Hong, Thomas & Kiernan, 2000). The second study examined questions from end of year examinations in two 2000 Mathematical Methods papers for the Victorian Certificate of Education (Flynn & McCrae, 2001). This study used a sophisticated classification system to analyse the impact of CAS on the questions, and concluded that about 40% of the questions would need to be changed to minimise advantages offered by CAS.

At the tertiary level, Pierce and Stacey (2001a) describe a key change in the examination questions at the University of Ballarat, in response to the introduction of the CAS into an undergraduate course’s assessment during the mid-1990’s. More questions became interpretive in nature, as opposed to recall of facts and by-hand skills (6 out of 12 interpretive questions in 1997 compared to 2 in 1996).

Laughbaum (1998) too supports a technology-active approach, when he suggests that the easiest way to encourage the use of technology is to ask questions that require it (for examples of assessment tasks designed to do this, see Boyd & Cutler-Ross, 1996; Kissane, Kemp & Bradley, 1996; Tobin, 2002). Stacey, McCrae, Chick et al. (2000) however note that not all questions need be designed to require technology, while others (e.g. Boers & Jones, 1994; Kemp, Bradley & Kissane, 1996; Flynn & McCrae, 2001) describe different ways that questions may elicit technology use in students’ question-solving practices, in for example:

1. a source of graphical information when this is specifically requested;
2. an aid to analytic work by helping guide the solution or as an alternative means of arriving at a solution;
3. a device to check the accuracy of algebraically derived solutions.

(Boers & Jones, 1994, p. 513)
Concerns that extended use of CAS may affect students’ ‘by-hand’ skills have been raised in previous discussions. Pierce and Stacey (2001a) reported mixed views about this from both staff and students in their study, and they and others (e.g. Herget, Heugl, Kutzler & Lehmann, 2001; Ball & Stacey, 2004) note that more research is needed on this. Ball (2003) compared responses of CAS and non-CAS students \( (n=78 \) for each group) to a common, final secondary-year, external examination question in Victoria, Australia. She found evidence that CAS-students were developing different practices for writing mathematical solutions. A later study by Ball and Stacey (2004, p. 87, 93) examined two further common questions from the examination, and concluded that while changes did occur in mathematical notation, there need be no fear that students will replace standard notation with incomprehensible machine-speak:

Students who learn mathematics with CAS calculators are likely to develop new practices for doing and recording mathematics. Students who had learned with CAS wrote generally shorter answers, used more ordinary words and used function notation more frequently but they did not over-use non-standard calculator syntax.

Ball and Stacey (2004) suggest that the findings of this study have important implications for teachers of CAS-active courses, in the appropriate setting of questions, in the need to actively guide students’ solving and recording practices, and in the appropriate interpretation and assessment of less traditional student solutions. An earlier study, that examined students’ responses in examination scripts using graphics calculators, led Boers and Jones (1992b) to conclude that the common fear that graphics calculators will deskill students is misplaced. Indeed, rather than over-simplifying examinations, Boers and Jones (1994, p. 514) suggest that “somewhat paradoxically, it would appear that giving students access to a graphics calculator in an examination can make calculus testing more difficult for students than if they did not have access to the technology”. This may be due to multiple factors, including ineffective use of the technology and difficulties in integrating algebraically and graphically derived knowledge; limited instrumental genesis; or the increased difficulty of interpretive-style questions over instrumental, skills-based ones (Boers & Jones, 1992a, b; Pierce & Stacey, 2001a). Forster and Mueller’s (2002, p. 35) conclusions with respect to graphics calculators summarises these arguments:
Implications of our enquiry for assessment are that an awareness of the demands of graphical interpretation and of likely misinterpretation needs to be brought to the setting of questions. In addition, a balance between opportunities for visual, empirical approaches and analytic methods needs to be built into examination papers.

While not proffered necessarily as an ideal solution, if calculator-neutral or calculator-free examinations are considered as a means of addressing concerns about technology in assessment, then several options are suggested. Hong, Thomas and Kiernan (2000) conclude that their study shows it is possible to set examinations where supercalculators such as the TI-92 have minimal influence, and they provide several examples of CAS-neutral questions for use in elementary calculus tests. Some, such as Kutzler (2000a) and Leigh-Lancaster and Stephens (2001) suggest the idea of two-tier examinations, where students are examined in one part on their manual calculation skills with no technology permitted. This practice is being adopted in the external components of New Zealand’s final-year secondary school assessment (New Zealand Qualifications Authority, 2008), where CAS will be permitted in three of the standards, but not the fourth.

However, despite the concerns outlined, the concept of congruency that began this part of the discussion remains a convincing argument (Leigh-Lancaster, 2000). If technology is used in other areas of learning, then technology-active assessment seems the most appropriate position. As Stacey, McCrae, Chick et al. (2000, p. 75) conclude, “We believe that assessment should be aligned with teaching as closely as possible…a technology-free component of assessment would endanger mathematics remaining a sensible subject where students learn to use up-to-date methods”.

**Section 3.4 Summary**

This section has demonstrated the complexity of issues confronting those wanting to introduce technology into undergraduate mathematics courses. Figure 3.4 reproduces the concept map, constructed by Norton and Cooper (2001b), to summarise the findings of their study. While their study was focused at the senior secondary level, this map provides a useful visual summary of many of the issues identified in the preceding discussions.
Norton and Cooper (2001b) surveyed senior mathematics teachers in two technology-rich secondary schools, and concluded that teachers believe about mathematics, beliefs about students, pedagogical knowledge, knowledge about using technology in their teaching, cultural press and perceptions of assessment are all critical factors affecting the decisions teachers make about technology use.

As Laborde (2002, p. 31) concludes, “…the process of integrating technology into mathematics teaching is a long and complex process – technology is not just an additional element of the system, since it interacts with all components of the system, which are (all themselves) subject to change”.

**Figure 3.4:** Factors influencing teachers’ responses to the use of computers in their mathematics teaching. (Norton & Cooper, 2001b, p. 388)
3.5 CHAPTER THREE SUMMARY

This chapter has considered the relationship between technology and contemporary theories of learning. It has reviewed studies that demonstrate positive benefits for student learning with appropriate use of technology, to support the underlying assumptions of the research questions framed by this thesis. It has also examined studies that identify possible disadvantages that may impact on the success of the technology implementation programmes, and discussed the questions raised by the wide variety of possible technologies that may be adopted by departments. It concluded by examining the implications for these issues on the implementation of technology.

The studies reviewed in this and the preceding chapter have described the extensive range of theoretical and practical issues which confront those responsible for the implementation of technology. The next chapter will outline the methodological strategy that this thesis will employ to investigate these issues further.
4.1 INTRODUCTION

This chapter outlines the overall design of the investigation, including the methods and procedures used to collect evidence. There are few comparable studies from which to draw suitable methodological exemplars, and in such cases, Ocean (2002, p. 101) suggests that a qualitative, exploratory study is the most appropriate. While this suggestion is well supported in the literature (e.g. Denzin & Lincoln, 1994; Robson, 2002), Bishop (1992) warns against research being method-driven, and both he and Romberg (1992) suggest that the chosen research strategy should be problem-led. Romberg provides a commonly accepted model of ten essential activities for research, as listed in Table 4.1 (For a fuller description of each of the activities, see Romberg, 1992, pp. 51-53). The first four activities in this model are seen as the most important when developing the methodology. While the absolute order of activities may vary, Romberg observes that the decisions about what methods to use and the specific procedures employed (activities five to eight) should follow directly from the questions one selects, from the world view in which those questions are situated, from the tentative model one has built in order to explain the phenomenon of interest, and from the conjectures one makes about the needed evidence. Thus the methodology should be seen as driving the final choice of methods (Romberg, 1992, p. 52).

Activities one, three and four from the table have been largely addressed in preceding discussions. The phenomena of interest and the research questions were described in the introductory chapter, while Chapters Two and Three have placed this thesis within the existing literature. This chapter will describe the two surveys that first
confirmed the phenomena of interest, and then led to the construction of an early model, with respect to the second activity.

**Table 4.1: A Research Model (Romberg, 1992, p. 51).**

<p>| | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>1.</td>
<td>Identify a phenomenon of interest</td>
</tr>
<tr>
<td>2.</td>
<td>Build a tentative model</td>
</tr>
<tr>
<td>3.</td>
<td>Relate the phenomenon and model to others’ ideas</td>
</tr>
<tr>
<td>4.</td>
<td>Ask specific questions or make a reasoned conjecture</td>
</tr>
<tr>
<td>5.</td>
<td>Select a general research strategy for gathering evidence</td>
</tr>
<tr>
<td>6.</td>
<td>Select specific procedures</td>
</tr>
<tr>
<td>7.</td>
<td>Collect the information</td>
</tr>
<tr>
<td>8.</td>
<td>Interpret the information collected</td>
</tr>
<tr>
<td>9.</td>
<td>Transmit the result to others</td>
</tr>
<tr>
<td>10.</td>
<td>Anticipate the action of others</td>
</tr>
</tbody>
</table>

This chapter will then consider activities five through to seven from Table 4.1, building on the information gained from the preceding activities, as Romberg suggests. The final three activities will be addressed in later chapters.

**4.2 PILOT STUDIES: INFORMING THE RESEARCH**

Chapter One outlined the researcher’s personal interest in the use of technology in the teaching of undergraduate mathematics courses, and especially the implementation of graphics calculators in early 2001. The complexity of implementing technology was identified as the main reason for undertaking the current study. This next section describes how the two preliminary studies helped to identify the phenomenon of interest, and to generate a tentative model.

**4.2.1 Pilot Study One: A Survey of Students’ Use of CAS-Calculators**

Two reports by Oates and Thomas (2001, 2002) expand on the history, briefly described in the introductory chapter that lay behind the initial use of graphics calculators in two foundation courses, and the subsequent extension of technology to the introduction of CAS-calculators into the principle undergraduate courses at The
University of Auckland. Oates and Thomas (2002) describe how the use of calculators in the main entry-level undergraduate mathematics course MATHS 151 underwent a rapid transformation, with no permitted use of even elementary scientific calculators prior to 2000, to the fully encouraged use of algebraically-capable graphics calculators (CAS) in semester one 2001. The speed at which this implementation occurred generated much discussion about a number of issues, such as inconsistencies in use between different staff members, and how to provide adequate support for students. Oates and Thomas (2001) note that in the planning meetings held at the end of 2000 and early 2001, it was recognised that many of the decisions made about the implementation were less than optimal. However, the course coordinators felt that, given the generally favourable atmosphere within the department for proceeding at the time compared to earlier opposition to such moves, it was more important to get the calculators in use than it was to perfect all aspects of their implementation immediately. Oates and Thomas did voice some concerns in regard to this debate, noting that

The inherent danger in this implementation approach is that the whole process takes on the nature of an experiment, with both staff and students in the role of guinea pigs. There is an element of risk that some of the less than optimal facets of such an experiment may lead to the ultimate sabotaging of the entire project.

Certainly a comprehensive review surveying both staff and students will need to be carried out at the end of the first semester in all facets of the use of graphics calculators in Maths 151. (Oates & Thomas, 2001, p. 82)

While a full review was not possible at the time, especially with respect to a debriefing of teaching staff, a survey was constructed and distributed to students. Given that this survey was primarily intended to provide information for the department, rather than a formal research study, its design was predominantly rooted in the experiences of staff and students involved with the project, as opposed to being embedded in the literature. However, it generally employed acceptable survey techniques (see e.g. Groves, Fowler, Couper et al., 2004; Czaja & Blair, 2005), and a variety of sources including feedback from the planning meetings’ minutes, and consultation with teaching staff and tutors, was used to inform the design. For example, the survey included a mixture of open- and closed-response questions, to balance the increased richness of information gained from open-ended responses (Groves, Fowler, Couper et al., 2004, p. 222), against the potential for increased item-response from the closed-responses that
are frequently perceived as less burdensome (Groves, Fowler, Couper et al., 2004; Czaja & Blair, 2005, p. 24). The survey questions were also trialled with a small group of twelve students from a MATHS 151 tutorial, before the final survey was administered. The pilot survey is provided in Appendix A2.

The surveys were administered in class lectures during the last week of lectures. While attendances at small-class tutorials were higher and may have gained a better response rate, the assessment nature of tutorials made it ethically unacceptable to demand time from students to complete the surveys. The survey received 215 replies from the total of 320 students who sat the final examination (a response rate of 67%). The responses to the questions were coded according to the schedule provided in Appendix A2(ii). Care was taken in the coding process to ensure that the answer categories developed were mutually exclusive and exhaustive (Czaja & Blair, 2005, p. 26), and discussions were held with several teaching staff in arriving at the final categorisations. A sample of the coding of twelve survey responses was given to two colleagues from the MATHS 151 teaching team, to test the robustness of the coding schedule, and the researcher’s interpretations (Groves, Fowler, Couper et al., 2004, pp. 306-310).

Oates and Thomas (2002) describe two other sources of data that were available to illuminate students’ experiences and views about the new technology. The first of these was the standardised departmental course survey of students conducted at the end of the semester one 2000, prior to the formal introduction of CAS-calculators, which asked a range of questions about teaching quality, the suitability of the course text, assignment relevance and fairness, and course workload. In addition, the University Students’ Association scheduled semester one 2001 to survey the MATHS 151 course as part of the routine, rotational sampling of courses for the student-centred Alternative Calendar. While the design of both of these surveys lies outside the methodological control of this thesis, they do comprise part of the formal “University Framework for Quality Assurance of Teaching and Learning”, and as such, may be considered as sufficiently sound to allow for confident use in this thesis.

Commentary and analysis of data from these three sources is elaborated on in later chapters. Nevertheless, the responses received to the preliminary survey confirmed that concerns about the technology implementation process were reasonable, and supported
continued investigation of these issues as the phenomena of interest. Further investigation was thus called for, to help in the formulation of a tentative model.

4.2.2 Pilot Study Two: A Survey of Technology Use in Tertiary Institutions

Following the preliminary study, a subsequent survey was proposed, as a means of addressing Romberg’s second research activity of building a tentative model (Table 4.1). This second survey gained further momentum during the presentation of a conference paper describing the early issues arising from the technology implementation initiative at Auckland (Oates & Thomas, 2001). Members of the audience voiced support for such a survey of technology usage in undergraduate mathematics. This interest prompted the impromptu formation of an informal focus group of delegates from the conference, moderated by the researcher, to discuss possible questions for the survey. Czaja and Blair (2005, p. 20) suggest that the use of focus groups in this way is an important procedure in questionnaire development, to maximise coverage of important issues.

On return from the Delta conference, a draft survey was constructed, using feedback from the focus group, information gained from the first preliminary studies, and issues identified in the literature reviews conducted for this and a subsequent conference paper (Oates & Thomas, 2001; 2002). For example, discussions in the focus group questioned the different ways in which students may make use of technology (e.g. exploratory, supportive, active), the first pilot study identified concerns from the students about the fairness of a course using technology that not all students had access to, and issues of assessment, equity, and professional development and training for staff and students were identified in the pilot study and subsequent literature review (Oates & Thomas, 2001, p. 132). Such issues were reflected in the draft questionnaire, and are still clearly visible in the final survey questions (see Appendix A3).

Issues from the field of survey methodology were considered in more detail in the construction of this second pilot study, helping to inform the questionnaire design, the data-collection, and the coding of the data (see Robson, 2002; Groves, Fowler, Couper et al., 2004; Czaja & Blair, 2005). For example, Czaja and Blair (2005, p. 64) provide a model of the survey data collection process that was closely followed for this study.
They suggest that the three most fundamental characteristics of a good questionnaire are:

1. [It should be] a valid measure of the factors of interest. This requires a clear specification of what the factors are, with measurement along appropriate dimensions.
2. It [should] convince respondents to cooperate.
3. It [should] elicit acceptably accurate (reliable) information. Respondents must understand the question as the researcher intends, have the necessary information, and be willing to provide an answer in the question requires.

(Czaja & Blair, 2005, p. 65)

Formulation of the questions followed the guidelines suggested by Groves, Fowler, Couper et al. (2004, pp. 226-236). These include the provision of all reasonable possibilities in closed-response questions, the use of uncomplicated and unambiguous language, and the avoidance of double-barrelled questions when asking about respondent’s attitudes. Czaja and Blair (2005) also suggest that a successful questionnaire may require several drafts, with various levels of testing, ranging from informal testing with colleagues, to testing with focus groups from the targeted sample population (in this case mathematics teachers using and implementing technology at the tertiary level). An opportunity for testing the draft questionnaire with a second focus-group was presented at the Auckland Bridging Mathematics Conference in 2002. The draft questionnaire was distributed to nine volunteer delegates attending this presentation, from the target population for the survey. They were first given time to respond to the questionnaire, and then discussions were held about their experiences, including their ease in answering, and suggestions for improvement. Feedback from this focus group was then used to further refine the questionnaire prior to final distribution (see Appendix A3).

The second pilot survey was finally distributed in late 2003 and early 2004. The initial distribution was conducted following the 2003 Delta symposium in Queenstown New Zealand, at which permission was gained to email the questionnaire to 102 of the delegates attending this conference. The selective nature of this group is acknowledged, with delegates to this conference obviously possessing a special interest in tertiary mathematics education just by virtue of their attendance (Oates, 2004a). The process of
distribution used in this survey also raises the issue of Sample Frame Bias (Czaja & Blair, 2005, pp. 14-18), with the exclusion of tertiary teachers who did not attend the conference. However, the widespread international representation of delegates (12 countries across both hemispheres), all involved in the teaching of undergraduate mathematics, suggests the sample may be considered sufficiently representative for such an exploratory study. Another concern about the survey design is that other than the location and nature of the courses and the institutions, the survey did not seek specific demographic information about the respondents, such as their age, gender, teaching experience, or their technological expertise or views towards technology. There was hence a suspicion that those who chose to respond may exhibit a more than usually favourable predisposition towards the use of technology, raising concerns about potential self-interest bias (Czaja & Blair, 2005, pp. 14-18). Such concerns were somewhat allayed however, when nearly 20% of those who ultimately responded to the survey described little or no experience in the use of technology. This suggests it is unlikely that the selected sample may be considered as an extraordinary techno-centric capture group. When coding and analysing responses, evidence was also sought that age and gender may have been a factor. However, the overall demographics of the conference delegates (fairly even spread of gender and age) suggested that in most instances the sample should be reasonably representative across most variables, despite the lack of specific information gathered in the responses.

The survey received 31 responses from tertiary mathematics and statistics teachers, representing 21 different institutions from 5 countries. Although the response rate of approximately 30% was better than initially anticipated, and the responses sufficiently detailed as to provide a useful range of data for exploratory analysis, the rate is sufficiently low to suggest the potential for non-response bias. Czaja and Blair (2005, p. 89) observe that it is usually difficult to measure this bias, but in general, a low response rate will reduce confidence in the survey’s results.

Czaja and Blair (2005, p. 38) suggest that in cases where there are concerns about low response rate, we need to know as much as possible about the non-respondents, in order to assess whether they differ in some way from those who did reply. They describe several factors commonly affecting response rates in postal and internet surveys which have been shown to be more significant in respect of bias. These include an inability to answer the survey (e.g. lack of technological access or computer...
illiteracy, low educational levels, failure to understand the questions), and a lack of interest in the topic (Czaja & Blair, 2005, pp. 38-44). These factors seem less likely in this study, given the educated, technology-proficient and motivated nature of the target sample in this survey, and given that the researcher is familiar with most of the delegates in the sample, it seems reasonable to assume that the non-response was largely due to lack of time, rather than any significant difference in their views with respect to the survey. Because of professional and ethical concerns, and time constraints, it was deemed inappropriate to use methods such as incentives and resubmission of the questionnaires, which are commonly used as a means of increasing response rates in such cases (Czaja & Blair, 2005, p. 40).

Following the collection of the data, an initial analysis was carried out to construct a draft coding schedule for the responses. Examples of this coding are provided with the discussion of results in the next chapter, while the complete coding schedule constructed for this survey data can be seen in Appendix A3(ii). It was decided not to quantify responses in this initial coding, but to assign responses to defined categories, informed by both the responses themselves, and using factors of interest identified in the preliminary literature reviews. These categorisations were entered into a table (see Table 5.1 for an example of this), using the matrix system of presenting and analysing qualitative data (Miles & Huberman, 1994b, pp. 240-241). As with the earlier pilot study, a random sample of survey responses (six in this case) was then given to two colleagues for moderation of the coding schedule. They were asked to test both the robustness of the categories in the schedule (i.e. mutually exclusive and complete), and the researcher’s decisions about the appropriate categorisation for the given data. This process resulted in some changes to the coding schedule, most significantly in some changes to the wording where it was felt that the descriptors were not sufficiently clear or exclusive.

This section has discussed the methodological underpinnings of the second pilot study. The implications of this (and the earlier preliminary study) for the subsequent methodology of the thesis are discussed next, while a more in-depth presentation and examination of the findings of the pilot studies themselves is left for the next chapter.
4.3 METHODOLOGICAL FRAMEWORK

The previous discussions have described how the preliminary studies and a preliminary review of the literature were used to satisfy the first four research activities proposed by Romberg (1992, see Table 4.1). This next section discusses the subsequent design of this thesis, in addressing activities five to seven of this research model. Issues considered include the manner in which the researcher’s teaching experience informed the approach to the problem, methodological perspectives that suggested the research and the paradigm within which the thesis is located, the structure of the thesis, and the methods of data collection. The discussion reflects on the personal values and experiences that influenced the decision to locate the thesis in a post-positivist naturalistic paradigm, and what assumptions underlie this approach. Then the choice of methods appropriate to this thesis within the selected paradigm are described and justified, along with the specific procedures chosen to gather evidence, and the ways in which that evidence was collected. The chapter concludes by considering the quality criteria for this thesis. The trustworthiness of the various data collection methods, the analysis of that data, and conclusions arising from it are examined.

4.3.1 Methodological Perspectives: Formulating a Research Strategy

The theoretical framework that underpins a research project provides the overarching framework for the project in its conceptualisation, analysis and writing (Zevenbergen & Begg, 1999, p. 170). The introductory chapter outlined how this thesis has its roots in the researcher’s personal experiences with technology, firstly as a secondary mathematics teacher, and subsequently an undergraduate mathematics teacher and mathematics education researcher at The University of Auckland. This personal discussion reflects characteristics of Peshkin’s (2000) observation that we are not indifferent to the subject matter of our enquiries. The questions we ask and the manner of seeking evidence are a function of our starting points and values. In this thesis, the researcher has stated a clear belief in the potential benefits of effectively used technology in tertiary mathematics, while his experiences with the technology implementation initiative have certainly affected the ways in which the research questions have been framed. The framing of the research questions reflects his interest
in improving the outcomes of technology implementations, and an underlying assumption that it is both desirable and possible to achieve an *Integrated Technology Mathematics Curriculum*.

The belief system and world view within which a researcher operates is commonly referred to as their paradigm or interpretive framework (Guba & Lincoln, 1994, p. 105). Denzin and Lincoln (1994) believe it is essential that a researcher be explicitly aware of the paradigm which guides and influences their research. The researcher must be aware of the underlying assumptions the paradigm within which they operate makes, about the three fundamental questions of ontology (the nature of reality), epistemology (the relationship between the enquirer and the object of enquiry), and methodology (Guba & Lincoln, 1994, p. 108-109).

The positioning of this thesis within one specific paradigm is however somewhat problematic. Lincoln and Guba (1985) provide a contrast between the traditional positivist paradigm and one form of post-positivist approaches that they describe as naturalistic inquiry, but which is more recently referred to as interpretive or constructivist (Noddings, 1990; Guba & Lincoln, 1994). Lincoln and Guba (1985, pp. 37-38) list seven axioms or beliefs contrasting these two paradigms, and a comparison of these against the aims of this thesis argue effectively against the positivist approach in most respects. Essentially, positivism, which has historically predominated in science research, assumes that science quantitatively measures independent facts about a single apprehensible reality (Healy & Perry, 2000). In this thesis, the researcher is clearly unable to act independently of the object of enquiry (Axiom 2: epistemology). He is intimately involved with the technology implementation project that forms the primary basis of the thesis. Additionally, personal relationships established in the collegial tertiary mathematics education research networks within which he operates clearly also exert an influence on the object of enquiry. Equally, the complexity of responses generated by the second pilot study suggests strongly that one apprehensible reality does not exist (Axiom 1: ontology). The values and beliefs about technology that the researcher brings to this study, as described earlier (see pp. 7-8), also clearly demonstrate that the thesis is not value-free as is assumed within this paradigm (Axiom 5), and as such, the employment of an objective methodology to guarantee valid and reliable results is neither dictated nor possible (Lincoln & Guba, 1985, p. 38).
However, it is not obvious in which of the contrasting interpretive paradigms to situate this thesis. While all paradigms following the positivist may be collectively regarded as post-positivist, the term *post-positivist* is usually used to describe the specific paradigm that seeks to address criticisms of the positivist paradigm whilst still maintaining a degree of objectivity (Robson, 2002, p. 27). To avoid confusion, this paradigm is sometimes described as *critical realism* in recognition of its ontological perspective (Healy & Perry, 2000, pp. 118-119), and this is how it will be described in this thesis. The ontological, epistemological and methodological dimensions listed by Guba and Lincoln (1994, p. 109) suggest that this thesis contains elements of both the critical realism and constructivist (naturalistic) paradigms. Table 4.2 shows a comparison of the relative positions for this thesis within each of these two, modified from Guba and Lincoln (1994, p. 109).

### Table 4.2 Positioning the Thesis within a Paradigm

<table>
<thead>
<tr>
<th>Dimension</th>
<th>Constructivism (Naturalism)</th>
<th>Critical Realism (Post-positivism)</th>
<th>Which Paradigm?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ontology</td>
<td>Relativist: Multiple realities: suggested by complexity of pilot survey responses, and literature.</td>
<td>One reality, but not perfectly apprehensible. As suggested by first research question to describe ITMC.</td>
<td>Mixture.</td>
</tr>
<tr>
<td>Epistemology</td>
<td>Subjectivist: researcher intrinsically linked to findings.</td>
<td>Objectivist: findings probably true.</td>
<td>Largely constructivist, but survey quantifiable.</td>
</tr>
<tr>
<td>Methodology</td>
<td>Largely qualitative</td>
<td>Multiplism: Mix of quantitative and qualitative.</td>
<td>Critical Realism</td>
</tr>
</tbody>
</table>

The philosophical positions described in Table 4.2 have significant consequences for conducting an enquiry (Guba & Lincoln, 1994), and the following two examples of this further demonstrate the ambiguity of this thesis’ position within the two paradigms. For the issue of *inquiry aim*, the constructivist paradigm seeks understanding and reconstruction, while the realist position is one of prediction and control. With respect to the *nature of knowledge*, the positions are described as “individual reconstruction coalescing around consensus” and “non-falsified hypotheses that are probable facts or
laws” respectively (Guba & Lincoln, 1994, p. 112). The first research question of this thesis aims to identify the elements of an Integrated Technology Mathematics Curriculum, and looks to make predictions based on this framework. This direction of enquiry tends towards the hypothesis-building, predictive nature of the realistic paradigm. However, the complexity of issues as evidenced by the responses to the second pilot study, and identified in the literature reviews in chapters two and three, suggest that one identifiable reality is an unlikely objective in this thesis, with the consensus of a constructivist position being a far more realistic position. Such difficulty in positioning a study exactly is relatively common, and it is now often accepted that is largely unnecessary and often impossible to distinguish so explicitly between the different paradigms (Begg, 2007). For example, Guba and Lincoln (1994, p. 105) observe that the question of paradigm should not be confused with the choice of quantitative or qualitative methods, arguing that both may be used appropriately within any paradigm. Keeves and Sowden (1997) note that while some philosophers would disagree, it is now generally acknowledged that true objectivity is not a plausible position, even within the positivist paradigm. Bishop (1992) observes that it is extremely difficult to isolate the relationships a researcher establishes through their beliefs, values and experiences with all aspects of the study (e.g. views about knowledge, the research problem, and the subjects); what is truly important is that the assumptions upon which the study is founded are made explicit.

Robson (2002) provides a useful solution to the apparent dichotomy displayed in Table 4.2. He proffers the philosophical position of pragmatism, or the search for truth that works, as a feasible approach. The beliefs espoused by the pragmatic position, as summarised in the list below, closely resembles a synthesis of the constructivist and realist positions (Robson, 2002, p. 43). This suggests that the pragmatic paradigm provides an appropriate philosophical position for this thesis, and is hence the position that may be assumed in the following discussions.

- The value-ladeness of enquiry;
- The theory-ladeness of facts;
- Reality is multiple, complex, constructed and stratified; and
- Theory is undetermined by fact (i.e. any particular set of data is explicable by more than a single theory).
As Robson states, this list bears a remarkable similarity to the view of science research taken by the realist position, but the recognition of multiple realities allows for compatibility with the relativistic stance taken by the constructivist philosophy (p. 44).

**Strategies of Enquiry**

Denzin and Lincoln (1994) describe how the paradigm within which a particular study is positioned shapes the selection of a suitable research strategy, which is in turn used to connect the given interpretive paradigm to the specific methods of collecting and analysing evidence to be used in the study (see activity five in Table 4.1). Janesick (1994, p. 212) provides an extensive but not exhaustive list of potential research strategies, and consideration was given to many of the outlined strategies in arriving at a suitable strategy for this thesis.

The strategy ultimately selected for this thesis is that of an instrumental case study, as opposed to other particular types of case study, e.g. intrinsic, comparative, or community (Stake, 1994, p. 237; Robson, 2002, p. 181). In this sense, the term case study is being used to describe an approach to the research, as opposed to the methods that may be employed within the case-study strategy (Denzin & Lincoln, 1994; Robson, 2002). While Stake (1994) associates the use of case studies more strongly with the constructivist paradigm using essentially qualitative methods, Perry, Riege and Brown (1998) see it as consistent with the critical realist paradigm, in which quantitative methods are also accommodated. They state that that the “primary objective (of a case study) is to understand the (reality of the) phenomenon under research, and interpret the respondents’ experiences and beliefs in their own terms” (Perry, Riege & Brown, 1998, p. 9). Thus a case study approach may be seen to satisfy the synthesis of the constructivist and realist paradigms within the adopted pragmatic approach. Other characteristics of a case-study strategy that support the selected pragmatic paradigm include investigation of the phenomenon within its real-life context, typically in situations where the boundary between the phenomenon and its context are not clear, and the use of multiple methods of evidence or data collection (Robson, 2002, p. 179).

Three other strategies (Janesick, 1994, p. 212) were given serious consideration, *participant observer, ethnography, and grounded theory*, but these were all ultimately disregarded in favour of the case study approach. Each of these strategies has characteristics that are consistent with the pragmatic paradigm (see Denzin & Lincoln,
1994, pp. 202-205; Robson, 2002, pp. 186-195), and aspects of each suggest an appropriate means of addressing the research questions. However, the more useful aspects of each could be accommodated within a case study approach. For example, most case study accounts have the researcher involved in a central role that mirrors aspects of the participant observer strategy, as does an ethnography. However, while the fieldwork nature of an ethnography seems ideally suited to the process of observing and documenting the technology implementation project at Auckland University (comparable to a community case study, Robson, 2002, p. 181), canvassing the experiences and views of colleagues in the wider international undergraduate mathematics teachers’ community is difficult to accommodate within an ethnographic strategy. Likewise, the first research question for this thesis (to identify the characteristics of an integrated technology mathematics curriculum) suggests the generation of a new theory that may fit well within a grounded theory strategy. However, grounded theory studies should be inductive (theories are developed from the data), with minimal influence from pre-existing constructs (Strauss & Corbin, 1994; Robson, 2002, Dey, 2004). The influence of the literature on the design of this thesis makes such a premise difficult to maintain. Case studies provide a similar but less constrained opportunity for theory construction, as opposed to theory testing and verification. While also inductive, prior theory can have a pivotal function in the design of the study and the analysis of its data. This is especially so in the early stages when existing studies can be used to inform the theory, as has happened with the pilot studies in this case (Perry, Riege & Brown, 2000, p. 11).

In addition to offering the beneficial aspects of the three strategies as described, a case study approach provides an excellent means of categorising data and determining the relationship between the categories. “The details uncovered in a case can delve into the complexities and processes of people and organisations” (Perry, Riege & Brown, 2000, p. 11), an outcome consistent with the aim of this thesis to establish a framework for integrated technology. The choice of which particular style of case study to use was largely dictated by the research questions, and the site within which the case study was to be conducted (Miles & Huberman, 1994b, p. 27). The choice of an instrumental case study reflects the realistic characteristics of the pragmatic paradigm, and the construction of theory suggested by the research questions. Here the case is being studied as a way of interpreting phenomena outside the case itself, as opposed to an
intrinsic study, where the aim is to gain a deeper understanding of only the particular
case itself (Stake, 1994). It was considered that focusing primarily on the technology
implementation process in the Mathematics Department at The University of Auckland
would provide an opportunity for a more intensive examination than employing a
comparative case-study approach, and that this would also recognise the unique
contextual nature of the research site (Robson, 2002). However, the data from the pilot
studies do allow for some aspects of a comparative case-study approach, through the
comparison of interesting differences or similarities that may emerge between the
institutions in the survey, whilst still recognising and preserving the uniqueness of each
case (Bogdan & Biklen, 1982).

4.3.2 The Evidence: Data Collection Methods and Analysis

The specific procedures one employs in a study should follow from the design of
the study, and must be carefully selected to shed light on the research questions
(Table 4.1, activity six. Romberg, 1992, p. 52). The particular methods commonly
associated with the selected case-study approach that seem appropriate to this thesis are
observation (of the technology implementation project), interviews (with staff involved
in the project and from the wider tertiary mathematics community), questionnaires, and
document analysis (Denzin & Lincoln, 1994; Robson, 2002). The use of such mixed-
methods studies is well supported (e.g. Miles & Huberman, 1994b; Robson, 2002),
combining the quantifiable features of a survey (opportunity for statistical analysis and
possible generalisation of findings), with the close-up contextual analysis provided by
the qualitative procedures (Miles & Huberman, 1994b, p. 42). One of the main
advantages of using multiple methods is to provide triangulation of the data, or an
effective means of assessing the reliability and validity of the data and the findings
(Robson, 2002, p. 371). This aspect will be considered in section 4.4.

In addition to the actual procedures chosen, a decision needs to made about the
ways in which the procedures will interact (for example the order of use), to provide a
coherent and effective source of information. Miles and Huberman (1994b) propose a
model of several possible designs for combining qualitative and quantitative methods.
The way in which this thesis has already developed in the pilot studies suggests one of
their designs as especially appropriate for the selected case-study approach. This design,
depicted in Figure 4.1, shows the use of multiple data collection instruments, interacting with and informing each other, in a process of continuous observational fieldwork.

\[ \text{Figure 4.1: Design Two: Linking qualitative and quantitative data.} \]
\[(\text{from Miles & Huberman, 1994b, p. 41)}\]

For this thesis, the pilot studies can be seen to constitute the first quantitative waves, with subsequent waves formed by the third major survey and the interviews, while the observation of the technology implementation project and document analysis associated with this make up the essentially qualitative phase of continuous fieldwork.

\[ \text{Figure 4.2: Research structure informing the research questions.} \]
The model in Figure 4.1 may be adapted for this thesis, to demonstrate how it specifically aims to address the research questions as described in chapter one. Figure 4.2 demonstrates how the mixture of quantitative and qualitative methods selected for this thesis combine together, within the case study strategy, to inform the research. The structure of this thesis as described in Figure 4.2 can be seen to reflect the iterative process of interpretive inquiry: a continual matching of data and literature until a satisfactory explanation of both is achieved (Keeves & Sowden, 1997). Aspects of each of the specific selected procedures will now be considered.

**The Third Survey**

As shown in Figure 4.2, the third survey was specifically aimed at addressing the first research question, seeking to identify the characteristics of an integrated technology mathematics curriculum. However, in keeping with the iterative structure of this thesis, it was hoped that the survey may also shed some light on the second research question, which examines ways of implementing such a curriculum. For example question D1 asks about barriers or difficulties experienced with implementing technology (see Appendix A4). This final survey followed the progressively robust attention to survey methodology demonstrated in the development of the earlier pilot survey discussions. In addition to considerations about the design of the questionnaire and issues of data analysis, more attention was given with this survey to sampling issues and analysis of the data. The more extensive review of the literature and the insight gained from the pilot studies combined to better inform the wording of the questions, and the issues that the survey sought to investigate. For example, with respect to the design of the questionnaire, the six categories identified through the coding process in the second pilot survey suggested questions examining each of these issues (see Appendix A3ii, Table 5.2). Influences from the literature can be seen in questions that reflect issues such as the epistemic, pedagogical and pragmatic value of topics within the mathematical curriculum in a technological environment (Figure 2.8), and the obstacles to successful integration of technology, such as limitations on access to technology, identified by Hoyles (1998).

Czaja and Blair (2005) suggest that borrowing questions from other research studies is also to be encouraged, as it provides a possible means of testing the validity and reliability of items through the comparison of results. However, no existing
Chapter Four

Research Design

research studies with previously tested and validated items appropriate to this thesis were found. The studies that did provide previously tested and validated scales were predominantly aimed at students, and were largely associated with issues of computer confidence and attitudes towards computer use (e.g. Galbraith & Haines, 2000; Fogarty, Cretchley, Harman et al., 2001).

One deficiency identified in the earlier survey (see p. 149) was with respect to respondents’ beliefs and attitudes towards technology. However, measuring beliefs and attitudes can often be difficult using questionnaires, especially when the subject is of a sensitive nature (Czaja and Blair, 2005). They recommend asking as few such questions as strictly necessary, while Robson (2002) recommends interviews as the best means of examining such issues, particularly if there are no standardised scales available. Several studies were found that developed a validated scale for use in investigating secondary school teacher trainees’ mathematical and pedagogical beliefs about the nature of mathematics, mathematics teaching, and mathematics learning (Frid, 2000; Goos & Bennison, 2002; Goos, 2003). Whilst the latter two of these studies included questions specifically aimed at examining the student teacher’s beliefs about technology and mathematics learning, it was felt that differences between the subjects of this study and the current investigation (tertiary mathematics researchers and teachers, compared to secondary mathematics teacher trainees) would preclude any effective data matching. This difference reduced confidence that the questions may be equally valid within the different context, and indeed, the third study (Goos, 2003) specifically supports such a conclusion, when it references the significance of the context in its title …”A study of beliefs-in-context”. So while it was not considered appropriate to use the scales from these studies directly, they did help inform the phrasing of questions within the new survey, and suggested the inclusion of some questions examining some beliefs identified as significant.

Some studies were found containing survey questions about specific issues at the tertiary level of interest in this thesis. These were used to either inform the design of the questions, or in a few cases reproduced or adapted for use in the new survey. For example, question C2 is copied from the Calculus Questionnaire suggested by Lauten, Graham and Ferrini-Mundy (1999), while the effect of other items from their questionnaire can be seen in other questions in the survey (see Appendix C). However,
no comparable results were available to support validation of the items. The final structure of the questionnaire as administered can be seen in Appendix A4.

With respect to sampling, it was recognised that this survey needed to extend its scope, to target tertiary mathematics teachers in addition to the narrow range of Delta delegates used for the pilot survey. While the Delta network was used again, a wider sample was considered necessary to minimise the potential for Sample Frame Bias raised as a concern in the earlier pilot survey (Czaja & Blair, 2005, pp. 14-18). The Delta network was obviously still an appropriate choice as it maintains an email circulation list, from delegates who have attended previous symposia, in excess of one hundred international tertiary mathematics educators. Permission was gained to contact colleagues through the New Zealand Mathematics Society, the representative body of professional mathematicians in New Zealand with around 150 members, and MERGA, the Mathematics Education Research Group of Australasia. In the latter case however, while the membership totals some 570, members from the tertiary sector had to be selectively targeted using the 2007 membership list, as membership of the group extends to all levels of education, and mass emails seeking responses to research studies are considered inappropriate.

In total, survey requests were sent to 134 colleagues involved in the teaching of mathematics at the tertiary level from the three organisations described (note that many are members of all three organisations, which reduced the total potential sample pool). This sample included colleagues from 9 New Zealand tertiary institutions, and the following international institutions in order of frequency: 10 Australian institutions, 9 from the USA, 4 from South Africa, 3 from South America (Uruguay, Argentina and Brazil) and the United Kingdom, and one or two from each of Canada, France, Germany, Israel and Sweden. Thus the sample used can be seen to be much more extensive than the earlier surveys, with a greater opportunity for inclusiveness of mathematics teachers in the tertiary sector. However, not all tertiary mathematics teachers within the targeted institutions are members of one of the organisations used to initiate contact. While it is very difficult to assess what differences the beliefs and experiences of those excluded may make to this thesis, the existence of such a potential effect must be acknowledged in any analysis and subsequent conclusions (Czaja & Blair, 2005, pp. 14-18). Like the earlier survey, demographic information about age, gender and teaching experience was again not specifically sought. In retrospect, such
additional information may have been helpful, but in its absence, care was taken to look for any evidence that such factors may have been an influence when coding the data, especially in the questions in Part B which sought respondents’ experiences with technology, and again in Part C with respect to beliefs and attitudes (Appendix A4(i)).

The data analysis of the third survey followed similar procedures to that outlined for the earlier pilot surveys, in for example coding and categorisation of the data, again adapting the matrix system proposed by Miles and Huberman (1994b). However, a slightly more robust procedure was used to check the coding of the data. Once initial coding and categorisation had been conducted, four of the respondents were asked to review the resultant classification of their responses, and comment on the degree to which they agreed with the assigned categories. Czaja and Blair (2005, p. 118) recommend such a process, part of what they call respondent debriefing, as one means of reducing coding errors. In addition, two of the survey respondents were included in the interview part of the thesis, both as a further means of debriefing, but also to allow for possible comparison with the interview subjects from the observational part of the thesis. However, Czaja and Blair (2005, p. 253) suggest that the most important factor here is to report any difficulties in coding certain items, usually encountered with open-ended questions, so that any uncertainties are made explicit. This is reported in the results section. Coding of data was not necessary for the questions requiring numerical responses. These questions asked respondents to assess the level of emphasis that should be assigned to six different uses of technology, measured on a five-point Likert scale. Data for these questions was entered directly into a spreadsheet, with analysis of data again reported in the next chapter.

The Interviews

Figure 4.2 shows that the interviews were introduced to the methodology with two main purposes in mind, in addition to providing one of the means of triangulation. Firstly, they sought to examine the experiences, perceptions and beliefs of staff members involved in the technology implementation project. Secondly, they sought to provide a degree of respondent debriefing with respect to the surveys, and allow for potential comparison between survey respondents and those in the observational study. Robson (2002, p. 271) suggests that in such circumstances, a semi-structured qualitative, respondent-style interview should be conducted, where the interviewer leads
the interview based on a set of pre-determined questions or themes that may be modified according to individual circumstances.

Following Robson’s recommendations, a set of specific questions was drawn up, seeking to investigate many of the issues identified in the literature and the surveys, but especially those relating to beliefs and attitudes to technology and mathematics, issues to which interviews are ideally suited (Robson, 2005, p. 272). The relationship between the interview questions and the issues of interest can be seen in Appendix B.

However, one question theme developed for the final interview protocol is worth discussing further here. It emerged when trialling the draft interview questions with two colleagues (as per recommended practice, see Fontana & Fray, 1994; Robson, 2002). One of the draft interview questions was designed to follow up on the survey question, seeking opinions about the relative values of curriculum topics in a technological environment (Stacey, 2003). It became clear that the trial respondents had difficulty answering, so two examples were suggested as prompts. The first example was relatively straightforward, the discontinuation of teaching algorithms for the calculation of square roots with the introduction of simple numerical calculators. This occurred with very little discussion or fanfare either at the time, or subsequently. The second one was drawn from linear algebra; respondents were asked their opinions about teaching Gaussian elimination, now that CAS-calculators and computer software automatically calculate both row-reduced and inverse matrices. Even given the complexity involved in assessing the relative values of a curriculum topic described in the literature review (Stacey, 2003), the dissimilarity of responses to these questions from each of the trial respondents, and the intensity of emotion evoked by the questions, came as a surprise to the researcher. This prompted the inclusion of a specific example in the final interview protocol to explore this issue further (see Appendix B).

All interview candidates were volunteers, drawn from teachers involved in the technology implementation project, and from indicative responses to the third survey. Such purposeful sampling, while not statistically random, is often used in interpretive research, where the chief focus is to understand and explain certain phenomena or conjectures, as opposed to making wider generalisations (Patton, 1990). Purposeful sampling makes clear the assumptions underlying the choice of participants, and in this case, the choices were made using a mixture of extreme and theoretical sampling techniques. Theoretical sampling is used to sample people on the basis of their
manifestation of important theoretical constructs, while extreme sampling focuses on cases that are potentially rich in information because they are outstanding in some way (Patton, 1990).

In deciding on numbers of people to interview, a balance was struck between what was feasible and a suitable number to provide sufficient information. A sample of seven was finally chosen, a sample that provided an appropriate cross-section of those involved in the observational study, as well as including some from outside for possible comparison. While seemingly a small sample, Erlandson, Harris, Skipper and Allen (1993) argue that there are no rules for sample size in interpretive research, and Keeves and Sowden (1997, p. 296) warn that sampling too many cases can cause information to be cast aside in inappropriate data reduction. Five of the candidates were drawn from the observational study, based on the researcher’s knowledge of the roles they performed in the technology implementation project, as well as conversations and records elicited in observations. One of these candidates was a technology enthusiast who was one of the chief motivators behind the implementation project (Interview Respondent IR1, extreme sampling), another served an important role on the department’s technology committee (IR2, theoretical sampling), one was involved in teaching courses using technology throughout the entire period of the observation (IR3, theoretical), the fourth was included specifically because of strong views about the effects of technology on certain curriculum topics, witnessed during observations (IR4, extreme), and the fifth because of their limited knowledge of technology and the difficulties they experienced during the implementation (IR5, extreme). The two candidates selected from outside the observational study at The University of Auckland (IR6 and IR7, theoretical) were identified through their responses to the third survey, with their responses to the question of changing values of curriculum topics an important consideration in their selection. (See selected survey responses from these two interview candidates, H6 Australia; H10 USA; Appendix A4(iii))

The interviews took between 45 to 60 minutes. A list of points to be covered was kept at hand, and a record sheet was kept to identify critical moments during the interview. In keeping with the interpretive, semi-structured nature of the interviews, while most questions were asked of all participants, the depth and order of coverage did vary. The interviews were audio-taped, but full transcription was not conducted. Instead, immediately following the interviews, the tapes were reviewed, and selective
transcription was carried out, using the ‘critical moments’ notes made during the interviews. Again, as for the sample-size, selective transcription is consistent with the design of this thesis, where the focus of the interviews is to identify key beliefs and explain specific phenomena (Robson, 2002, p. 290). All tapes, interview record sheets and transcripts were kept for auditing purposes.

Many discussions about interviews emphasise the need to attend to practical aspects of the interview. These include the setting, understanding the language and culture of respondents, establishing rapport, and status differences between interviewer and respondents (Fontana & Frey, 1994; Robson, 2002). However, given the collegial relationship between the interviewer and most of the participants in this thesis, and his intimate knowledge of the thesis location and culture, these factors were considered of minimal significance. Participants were still given the opportunity for informed consent, and all interviews were conducted in places and at times of their choosing. It was apparent during the interviews that establishing rapport and trust were not an issue, with all participants being very forthcoming in their responses (Bogdan & Biklen, 1992).

It is necessary to explicitly acknowledge the subjectivity of data analysis arising from the pragmatic paradigm of this thesis (Fontana & Fray, 1994). Miles and Huberman (1994b) suggest three approaches that can be used to communicate the nature and rationale for decisions made by the researcher and to facilitate evaluation of the analysis by others. These are data-reduction (usually involving coding), data display (e.g. as in the matrices used earlier for the survey), and conclusion drawing and verification. This thesis makes use of all three approaches. They are described when presenting the interview results in later chapters, and in the next section examining the trustworthiness of the studies findings.

**Observational Study and Document Analysis**

This component was the most prolonged method employed in the thesis, monitoring the progress of technology implementation into undergraduate mathematics courses in the department over a considerable time frame (late 2000/early 2001, through to early 2008). Such an extended period made for the possibility of an inordinate amount of data, and also made it impossible to sustain continual observation. Such circumstances are common in semi-longitudinal observational studies (Adler & Adler, 1994; Robson, 2002), and Robson recommends the establishment of an observational
schedule, to make data-collection less demanding and potentially more useful. Another strategy is to target specific aspects of the object or group under observation, as opposed to a general, holistic view. This targeted strategy involves a systematic, formal approach to the observation, and is more ideally suited to the observer-as-participant style of observation adopted by this thesis. While this approach risks the loss of complexity and completeness, it is easier to sustain, simpler to apply from the external position of an observer-participant, and it is generally simpler to achieve better reliability and validity (Robson, 2002).

This thesis therefore adopted a structured approach to the observations, with the observer making observations using a specified schedule. The schedule becomes the primary instrument, as opposed to the observer as primary instrument in participant-observer studies, and as such, care must be taken in drawing up, and clearly describing and administering the schedule, to minimise bias and increase consistency of measurement. Robson (2002), for example, raises a concern about observer consistency (the ability to measure the same events with equal results), which becomes especially significant over an extended study. The issue of inter-observer consistency is less relevant in this study, given that the researcher was the primary observer, although colleagues’ assistance was inevitably sought in several instances where the researcher could not attend. Consistency issues are considered less significant in study designs where the observational component is supportive or supplementary to the study, as opposed to the primary instrument (Adler & Adler, 1994, p. 389; Robson, 2002, p. 312). The observational component in this thesis is informed by, and is used to support and test, the findings from the other instruments, “part of a methodological spectrum… (that provides) a powerful source of validation” (Adler & Adler, 1994, p. 389).

Hence the observational schedule was informed by the issues identified in the previous surveys and the literature review. The interviews performed a dual role in this part of the thesis, simultaneously informing the construction of the schedule, as well as comprising one of the later parts of the observational schedule itself. Although the latter stages (post-2004) of the technology implementation were extended across the majority of undergraduate courses in the department, and many of the documents collected were generic to the whole project, it was decided that the primary focus of the observations would be MATHS 108. This is the department’s largest first year course, with an annual enrolment of around 2000 students. Within this framework, it was decided to further
limit the observations to the semester one course, which has an enrolment of around 900-1000 students, on the basis that courses do not change during the year. Even this limiting of the observations presented challenges, with up to five streams (lecture times) per day, and some 30 tutorials per week, involving teaching teams of up to 10 lecturers and 15 tutors in a semester. A summary of the observational schedule is provided in Table 4.3.

The introduction of new technology was not the only change to the MATHS 108 course during the observational period. Other changes to the course may or may not have had an effect on the technology implementation, some decided within the department, and some imposed by the university. For example, lectures were reduced from four to three hours per week in 2004 in response to departmental initiatives, and then the course content was reduced again in 2006 by about 1/8 to reflect changes in the degree structure by the university. Between 2001 and 2004, tutorials also changed significantly, with the provision of sufficient tutorials for all students to attend in small classes (of about 25-30 students), and the introduction of marks awarded for coursework associated with tutorial attendance and performance. Another significant change was the move from a purpose-written course-book in the early years, to prescribed text-books in the latter stages. The observational schedule in Table 4.3 identifies many such factors as a target of interest, and the data-analysis in later chapters will attempt to identify where these factors may have impacted on the technology implementation project.

The data collection and analysis used selective attention techniques to identify significant moments, with codification and content analysis of collected documents (Robson, 2002, pp. 324-356). The codification system used similar categories to those established in the surveys. Again, the categories used are objective, explicitly defined, exhaustive and mutually exclusive (Robson, 2002, pp. 332, 355). While care was taken to ensure that no data was omitted that did not fit one of the stated categories, it was not possible to get an independent observer to check the extensive range of documents and data against these categories. However, all the documents and observation records are available for inspection and auditing purposes. Examples of the categories and data reflecting these are provided in later chapters.
Table 4.3: An Observational Schedule for MATHS 108 (2001-2008)

<table>
<thead>
<tr>
<th>Factor of Interest</th>
<th>Data Collection</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Departmental Meetings: Technology discussed or not featured?</td>
<td>Attendance with note taking, appointed observer on a few occasions, one audio-taped session, meeting minutes.</td>
</tr>
<tr>
<td>2. Technology Committee Meetings: Decisions reached about type of technology, evidence of beliefs etc.</td>
<td>Appointed observer with note-taking and debriefing, minutes of meetings.</td>
</tr>
<tr>
<td>3. Policy statements and changes.</td>
<td>Copies of statements, and dates/examples of public notification (e.g. emails).</td>
</tr>
<tr>
<td>4. Use of technology in lectures and tutorials. (Include changing nature of tutorials)</td>
<td>Noted different streams/lecturers &amp; tutors use of technology in their teaching, familiarity with technology etc. Identified by visit to lecture or tutorial or brief informal interview.</td>
</tr>
<tr>
<td>5. Changes to course assessment.</td>
<td>Examples of assignments, tests and examinations. Examination of questions style etc for evidence of technology influence.</td>
</tr>
<tr>
<td>6. Students’ use of technology.</td>
<td>Where and how students use technology: In class? Tutorials? In labs for assignments?</td>
</tr>
<tr>
<td>7. Technology assistance. (How is technology reflected in course-books? Texts?)</td>
<td>How is use of technology supported in course materials? Teaching? Practical assistance (e.g. help tutorials)?</td>
</tr>
<tr>
<td>8. Course Content (Technology considered when making structural changes?)</td>
<td>Any changes to curricula, especially in response to technology changes? Examples from course descriptions.</td>
</tr>
</tbody>
</table>

4.3.3 Criteria for Judgement: Issues of Trustworthiness

Given the pragmatic paradigm within which this thesis is conducted, the conventional standards of reliability, validity and generalisability used to judge studies grounded in the empirical positivist paradigm are difficult to apply (Husen & Postlethwaite, 1994; Healy & Perry, 2000). In positivist studies, reliability means repeatability, which is an unlikely possibility (Robson, 2002). While it may be possible to measure statistically the internal validity with some of the structured elements of this
thesis, the localised nature of the case-study approach allows for limited comparability, and an associated difficulty in assessing external validity, or making convincing generalisations (Lincoln & Guba, 1985, p. 290). There are several suggestions for standards of trustworthiness in a pragmatic paradigm, analogous to those of reliability and validity. These measures include credibility, transferability, dependability, and confirmability within the constructivist paradigm (Lincoln & Guba, 1985; Husen & Postlethwaite, 1994), and contingent validity, methodological trustworthiness, analytic generalisation and construct validity within the realist paradigm (Healy & Perry, 2000). Miles and Huberman (1994a, p. 438) further suggest that verification, or checking for common and insidious biases, is also an important part of establishing trustworthiness.

A principal means of addressing many of these measures is provided by the mixed-methods design employed in this thesis (see Figure 4.2), which allows for triangulation of the data (Miles & Huberman, 1994a; Robson, 2002). Triangulation should involve a complementary mix of quantitative and qualitative methods, to facilitate plausibility and validity (Robson, 2002, p. 373). Triangulation promotes credibility, dependability and confirmability, to the extent that the different methods produce complementary or convergent results (Husen & Postlethwaite, 1994, pp. 3488-3489). Triangulation of methods is also seen as an effective way to promote contingent and construct validity (Healy & Perry, 2000). “Triangulation makes explicit the multiple perceptions that underlie the reality of the case being studied, providing a benchmark by which the findings may be judged” (Healy & Perry, 2000, p. 123). In this thesis, the interviews provide an insight into the different perceptions of the seven participants, the two surveys provide multiple national and international perspectives, and the observational component provides a close-up examination of perspectives of colleagues and students through the window of the department (Miles & Huberman, 1994b). Triangulation also allows for greater confidence in the analytic generation of theories, as the data converges towards an emergent theory (Miles & Huberman, 1994a, p. 438; Healy & Perry, 2000, p. 125).

A second significant means of addressing several of the measures of trustworthiness is the provision of a clear audit trail, recording the processes used in the collection and analysis of data (Lincoln & Guba, 1985). Dependability and confirmability, or methodological trustworthiness demand explicit reporting of data and procedures so that the reader can verify reported conclusions, and secondary analysis of
the data is made possible (Miles & Huberman, 1994a, p. 439). This thesis has been transparent in its decisions, and methods of data collection and means of analysis have been clearly described.

In addition to clear reporting of procedures, the subjectivity of qualitative data must be made explicit during the study (Husen & Postlethwaite, 1994, p. 3488). Here, the value of tacit knowledge and prior experience is paramount. While being too familiar with an environment can render some things unseeable (Strauss & Corbin, 1994), some level of familiarity with the environment “enables the emergence of theory that could not otherwise have been articulated” (Lincoln & Guba, 1985, p. 208). The researcher’s experience and the foundations on which his assumptions are based have been articulated in support of this tacit knowledge, and also to make overt potential sources of bias that may arise from his subjective stance. A source of such bias lies in the potential to misinterpret negative data due to the researcher’s favourable stance towards technology (Miles & Huberman, 1994b).

Several measures were suggested to facilitate evaluation of this thesis by others and alleviate concerns about subjectivity, namely: data analysis, data reduction, data display, and conclusion drawing and verification (Miles & Huberman, 1994b). Where data reduction was employed in this thesis, the coding and assignation of data to categories were checked by colleagues, and against a selected sample of respondents. Samples of quotes from interview transcripts were checked with participants, to confirm the story they were being used to illustrate (Keeves & Sowden, 1997). Similarly, the categories used in the display matrices were checked by colleagues, and the categorisations were also subjected to cooperative review, through presentations of findings at conferences, presentations to colleagues at The University of Auckland and The University of Melbourne, and refinement through focus groups drawn from colleagues in tertiary mathematics teaching. Cooperative review is another strategy suggested by Ocean (2002, p. 127) to provide a check against the subjective judgements that may be applied by the researcher, and the values and assumptions that they bring to the research.

Keeves and Sowden (1997) suggest there are two important aspects to the last of these tactics recommended by Miles and Huberman (1994b), conclusion drawing and verification. The first aspect involves reporting any research based on qualitative data, in such a way that it can be readily comprehended and verified. This has been addressed
through the review of literature, and the provision of an adequate audit trail. The second aspect refers to minimising sources of error and bias, and Miles and Huberman (1994b) identify several common sources of error that should be considered in developing conclusions from qualitative data. Those especially relevant to this thesis include holistic fallacy, elite bias and going native. Holistic fallacy results from ignoring extreme or inconsistent data, so that the story seems more coherent than it really is. This is a possibility here, given the highly purposive sampling used in the choice of interview subjects. Elite bias describes the attachment of increased credence to respondents of perceived higher status. Going native describes the tendency to accept perceptions and explanations based on experience and familiarity, without adequate external or scholarly examination. This thesis uses triangulation to guard against holistic fallacy and elite bias. Husen and Postlethwaite (1994, p. 3489) suggest that this risk of purposive sampling is probably offset by the increased transferability such sampling provides, through the generation of richer data and a deeper understanding of the context within which the data is collected. The grounding of this thesis in the literature, and the checking mechanisms through the use of focus groups and collegial scrutiny, were used to guard against going native.

4.4 CHAPTER FOUR SUMMARY

This chapter has described how the methodological framework developed for this thesis has its roots in an interpretive paradigm, grounded in the experiences, beliefs and values of the researcher, and developed through a consideration of data gained from two preliminary studies and the review of the literature. The thesis combines aspects of post-positivist naturalistic or constructivist enquiry with those of a realistic nature, characterised as a pragmatic approach to research (Robson, 2002). The thesis employs a range of quantitative and qualitative methods within an instrumental case-study strategy, with some opportunity for comparison of data. The design of the thesis follows an iterative approach, allowing for various perspectives on the data informing each other in an interactive way, providing a means of triangulation to improve the trustworthiness of the findings. The next three chapters will detail, analyse and interpret the evidence so collected.
“Calculators and computers are neither good nor bad teaching tools, only using them makes them either” (Kutzler, 2000b, p. 11).

5.1 INTRODUCTION

Figures 4.1 and 4.2 presented the methodological structure of this thesis as an iterative process of observational fieldwork, informed by successive waves of data-collection. Figure 4.2 depicts the first wave as the construction of an initial model of integrated technology, formulated from an early review of the literature, and the preliminary studies. This preliminary model was used to inform the next wave, to investigate the first research question, “What are the characteristics of a tertiary Integrated Technology Mathematics Curriculum (ITMC), and how might we measure the nature of such technology integration?”

The structure of this chapter hence follows the iterative nature of the thesis. It first presents the results of the early pilot studies, and demonstrates how these were used to inform the construction of the second survey, which examined technology use in tertiary institutions. An initial taxonomy for integrated technology is proposed, derived from responses to the second survey. This taxonomy is used to measure technology use in selected courses, using an instrument developed for this thesis. Next, the formulation of the third and final survey is described, and the results used to refine the taxonomy. The chapter concludes by using the refined taxonomy to address the first research question.

5.2 A Preliminary Model

The first investigative look at calculator use in mathematics courses at The University of Auckland came in 1997, through a simple, informal questionnaire that sought students’ reactions to the introduction of graphics calculators in the bridging
mathematics course, *Mathematics Two* (see Appendix A1). The survey was administered in one of the final lectures for the course, and 143 replies were received (approximately 50% of enrolled students). In a report to the Department, Barton and Oates (1997) note several surprising results, and they make several recommendations based on findings from this survey.

The first surprise was the low-level of uptake, with only 15% of students owning a graphics calculator by the end of the course, despite provision for sale at discounted rates, active encouragement and modelling of the calculators by teaching staff, allowance for calculator use in all areas of assessment, and the provision of specific specially designed calculator-based examples in the course-book. It was anticipated that more students would have brought graphics calculators with them from school, and given that 44% of students without graphics calculators indicated that they felt disadvantaged by not having access to the technology, a higher take-up rate could have been expected. While there was no specific question to determine reasons for non-purchase, 8 responses made reference to cost, and 3 responses stated that it was not worth buying the calculator if they couldn’t use them in other courses. These responses suggested the need to further investigate the motivations for calculator access, and prompted one of the recommendations; to encourage calculator use in other mathematics courses, to provide an additional incentive for purchase (Barton & Oates, 1997).

The second surprise lay in the types of use that students reported. Limited use was made of the graphics calculator resources provided in the course-book (40%), even though over half reported difficulty in learning to use the calculators. Three quarters of those with calculators reported using it in tests, even though the test questions were supposedly designed to be graphics-calculator neutral (for example provision of graphs of functions in the test questions where appropriate), while less than half reported using them in lectures. This latter usage was a reverse of that expected by the teaching team. The authors concluded there was a need for further investigation into the types of use engaged in by students (Barton & Oates, 1997).

The influences of this preliminary study can be seen in the first pilot survey for this thesis. It followed the introduction of CAS-calculators into the main entry-level mathematics courses in 2001, as described in Chapter Four (see pp. 145-148, and Appendix A2). Student uptake of the calculators rose to 38% (81 out of 215 responses),
and there was an increase also in the number of students with graphics or CAS-calculators prior to starting the course (17%), suggesting that increased use in schools was filtering through to the university. The ease of purchase and the discount offered seemed a factor, with 63 of the 81 students buying their calculator through the department. Of the 134 students who did not purchase a calculator, 60% gave cost as the principal reason. Of the remainder, a significant number (19%) gave reasons which displayed similar beliefs about the nature of mathematics and technology to studies found in the literature (e.g. Cretchley & Galbraith, 2002; Stewart, Thomas & Hannah, 2005). For example, one student replied that “I like working materials manually, I learn more this way”, while another stated “I couldn’t afford it and I didn’t think I’d use it for I like to use my brain”. There was also evidence of the dislike of calculators found in some studies (Kissane, Bradley & Kemp, 1994; Faragher, 1999), with several students from both groups making comments to this effect. One student said they could not see the point of buying a “gimmicky toy”, while another with a CAS-calculator commented that “the department should get into the real world and use computers”.

There was a suggestion that decisions about buying a calculator may have been related to the use by lecturers in the course, and the signals this gave about the value placed on the calculators by the teaching staff. Almost half (45%) of the students without CAS-calculators came from the one lecture stream, out of three, in which the lecturer did not demonstrate CAS-calculator use. However, it is not possible to determine whether they opted for this stream because they did not have a calculator, or whether the lecturer’s stance influenced their decision after they were placed in the stream, as suggested by several studies (Cretchley, Harman, Ellerton & Fogarty, 1999; Kendall & Stacey, 2001a, b; Mueller, 2001). One student’s response at least supports the notion of teacher privileging influencing their decision, when they stated that “couldn’t see the point, the lecturer never used it”.

Equity concerns were evident in many responses. 53% of students with CAS-calculators felt that they enjoyed advantages over those without, although slightly less of those without calculators felt disadvantaged (46%). It is evident from many responses however that this belief was at least partly due to the way in which it was used, with the variable levels of lecturer use and student access seen as especially unfair or problematic. Several responses from both groups made specific reference to “unfairness” or “irrelevance” in this respect. For example, one student with a calculator
responded that “I was able to follow what the lecturer did, I think it gave me an advantage”; another student without one felt “I didn’t miss out, the lecturer didn’t use it, and the test didn’t need it.” 51% of those without calculators stated they would have preferred less use of the calculator by lecturers, seeing it as an unnecessary waste of time. These results are consistent with equity concerns in the literature (e.g. Zevenbergen, 1999, 2001; Berger & Cretchley, 2005), although no conclusions can be drawn from this survey about other equity concerns, such as gender or cultural effects raised by other studies (Hoyles, 1998; Vale, 2003; Forgasz, Griffith & Tan, 2006), as these dimensions were not measured.

By contrast, the large majority of students with calculators either supported the current usage by lecturers, or would have preferred greater use (35% and 43% respectively). Of this group, 60% also said they would recommend students taking the course in the future to buy a calculator (only 15% disagreed with this statement), and the comments suggested this figure would increase if it was used more widely in all lecture streams. Twenty-three responses made specific reference to the value attached to the use by the lecturer, in helping them to learn to use the technology, while another thirteen comments connected lecturer’s technology use to perceived improved conceptual understanding. These figures support the importance of modelling the technology in privileging student usage, signalled by several studies (e.g. Kendal & Stacey, 2001a, b; Mueller, 2001; Pemberton, 2002), especially since a reasonable proportion of those without calculators (31%) also liked calculator use by lecturers.

Fifteen percent of students with calculators argued against their continued use, for example the student who stated that the calculators are “…not needed to do maths. I think it creates a dependency on it when you clearly need to understand, not just push buttons.” Such responses match those found in the literature, for example Stewart, Thomas and Hannah (2005) found some quite hardened attitudes against technology for some tertiary students, and Geiger (2003, p. 386) concluded that despite generally positive student support for integration of technology, many exhibited fears that technology may become the master of their mathematical competence. The majority of the responses (10 of the 15%) were associated with frustrations over learning to use the technology, and the limitations on its effective use in a course where assessment was supposedly calculator-neutral. In the open-ended question soliciting comments on what students disliked about the calculators, there appeared comments like “Hard to learn
how to use, confusing, complicated, manual hard to follow, frustrating, couldn’t remember how to use, some functions hard to follow”.

Oates and Thomas (2002) note that this survey was conducted prior to the final examination, and it is possible that perceptions from both groups (with and without calculators) may have changed in response to their examination experiences. Some analysis of examination questions will be conducted in the next chapter, and this could help gauge any potential effect the exam may have had on the findings (see e.g. Kiernan, Oates & Thomas, 1998; Hong, Thomas & Kwon, 2000). Regardless, the overall results demonstrate strong student support for greater consistency in the use of calculators, which agrees with calls in the literature for congruency between curriculum, pedagogy and assessment (e.g. Dugdale, Thomson, Harvey et al., 1995; Leigh-Lancaster, 2000).

This survey also examined two aspects of the course with respect to student usage. Firstly, which components of the course (for example tests, lectures, assignments) elicited such use, and secondly, for which particular content and tasks (for example checking solutions, reducing matrices to echelon form) were they most likely to be used. Unfortunately these distinctions were not linked, so whilst 71% of students for example reported frequent use of the calculators in their assignments, the exact nature of this use was unclear. It was obvious however that those with calculators used them in some capacity in all aspects of the course, with at least 75% of students responding sometimes or often in each category (see Appendix A2).

With respect to particular use of the calculator, more than half the students reported using the calculators at least sometimes in most categories. The one exception was the 66% of students who reported they did not copy work done by lecturers in class, which appears to contradict the importance of teachers modelling technology as was highlighted earlier (see pp. 133-136). Most of the usage reflected the less sophisticated, more technical levels of use described in the literature (e.g. Galbraith, 2002; Thomas & Hong, 2004; Stewart, Thomas & Hannah, 2005). For example, students reported greatest use in checking solutions to problems (67% often, 21% sometimes), and carrying out routine matrix operations (78% total). Supporting comments included “...it was easier to check answers. Didn’t have to worry about simple algebraic mistakes.” This usage appears to match the technology as servant category described by Galbraith.
and others (e.g. Galbraith, Goos, Renshaw & Geiger, 2001; Galbraith, 2002; Geiger, 2006), and is consistent with the findings of Stewart, Thomas and Hannah (2005).

There was some evidence of more sophisticated use, such as the student who commented that “It made life easier….more focus can be directed into solving a problem rather than working out trivial answers”, where the technology is seen as a valuable assistant (c.f. technology as partner, Galbraith, 2002; or strategic use, Pierce & Stacey, 2004). However, there was little evidence of students progressing to the higher level of technology as extension of self, with 72% reporting that they had not experimented with any functions on the calculator other than those modelled in class, or those they were specifically directed towards by the teacher or the notes. Again, these findings are consistent with those in the literature, for example the conclusion by Stewart, Thomas and Hannah (2005), that students require considerable time to develop sufficient levels of instrumentation for effective use.

Two other general surveys were conducted at The University of Auckland around the same time, and were available for comparison. One of these was conducted in semester one 2001 by the University’s Students’ Association, for a publication known as the Alternative Calendar, which tends to elicit frank comments from students. This survey asked no specific questions about technology, but it is interesting given the change to CAS-calculators that semester, that the survey reported no comments about technology in open-ended responses, even though students did make specific reference to several other aspects of the course. The other survey, administered by the Mathematics Department at the end of semester one 2001, did feature technology, but not specifically the CAS-calculators. As described earlier, a precursor to the CAS-calculator initiative was a relaxing of the rules in 2000, to allow the use of scientific and graphics calculators in the course, something not previously permitted. Various other changes were enacted in semester one 2001, including dramatic changes to the tutorials, and a newly published lecture manual. Being allowed to use a calculator elicited the greatest level of student support in the survey; 82.7% of students liked being able to use calculators, only 2.1% disliking them. Such a positive response may lie behind the lack of comments in the Alternative Calendar survey, where responses to open-ended questions tend towards complaints. Such a positive general attitude towards the introduction of technology agrees with findings in the literature (e.g. Geiger, 2003).
Oates and Thomas (2002, p. 138) conclude with respect to the pilot survey that “two of the more critical findings that have emerged from this study are the need to ensure that all students have access to whatever technology is used, and that teaching staff are well trained and have a shared commitment to using that technology. Failing to meet these conditions clearly limits the effectiveness of the technology”. Other issues warranting further investigation include attention to student factors, and congruency of use in different components of the course. These are considered next, when examining the results of the second pilot survey, which investigated technology use in tertiary institutions.

5.2.1 A Definition and Taxonomy for Integrated Technology

The preliminary studies identified a number of issues from students’ perspectives that may be significant in technology implementation. However, the research questions for this thesis relate to the extent to which such factors are considered in undergraduate mathematics programmes. The second pilot survey, as described in Chapter Four (see pp. 148-151), was therefore directed at mathematics educators. The questions reflect the initial interest in graphics and CAS-calcators inspired by the early choice of technology at The University of Auckland (see Appendix A3). However, a preamble invited respondents to substitute “computer software” for “calculators” in the questions, and many responses confirmed the ultimately more general technology focus of this thesis.

Even given the range of 21 different institutions from 5 countries (Australia, New Zealand, South Africa, Uruguay, United States) represented in the 31 responses, there was a wide degree of variation in the reported use of technology, both within, and between countries and institutions. For example, one institution in the United States reported intensive use of technology in all its first year courses, an extremely high rate of individual ownership of calculators, and expected use in all assessment including examinations. Another institution, in New Zealand, reported optional use of CAS-calcators in its primary first year course, student ownership of about 40%, and calculator-neutral examinations. The type of technology used, and the courses for which technology was allowed, also varied greatly. The use of computer-based applications in applied courses such as engineering and statistics was commonly
accepted, but much less so in core mathematics courses. Others used graphics calculators, but not CAS capable machines such as the TI-89 or TI-92. Six respondents had no experience with the use of technology in courses they had taught or were currently teaching, and their responses provide insight into factors influencing technology implementation. One respondent stated that “… (my) view of graphics calculators is rather a negative one. I consider them to be obsolete now tools such as Excel are available”. Another indicated that they would like to use technology, but it was not permitted in the undergraduate courses in their department.

The complexity of the data presented a challenge to portray in a representative and comprehensible fashion. The coding process used to initiate this was described in Chapter Four, and an example of the coding for one question (Question 2i) is provided in Table 5.1 (see Appendix A3 for the complete coding schedule).

<table>
<thead>
<tr>
<th>Sample Survey Question: Q2(i)</th>
<th>Category of Response</th>
<th>Typical Response or Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Is technology permitted and/or expected in assessment?</td>
<td>All, assessment requires technology (technology-active)</td>
<td>“We write the exams assuming students have a graphics calculator”</td>
</tr>
<tr>
<td></td>
<td>All, but assessment is technology-neutral</td>
<td>“Can’t disadvantage the students without a graphics calculator”</td>
</tr>
<tr>
<td></td>
<td>Some, e.g., not exams.</td>
<td>“The final exam is not in a computer laboratory”</td>
</tr>
<tr>
<td></td>
<td>All except special technology free section (usually skills)</td>
<td>“We include a compulsory skills component in the exam where calculators are not permitted”</td>
</tr>
<tr>
<td></td>
<td>None</td>
<td>“Students are not allowed to use calculators in any assessment, although in practice, it is impossible to monitor this for assignments”</td>
</tr>
</tbody>
</table>

The coding of the data highlights the difficulty of arriving at consensus for a definition of integrated technology. The coding categories displayed in Table 5.1 and
Appendix A3(ii) present a complex picture of the possibilities. Many of the respondents used generic statements such as “the technology must be integral to the course”, or other general terms such as “essential”, in response to the question seeking their understanding of what an Integrated Technology Mathematics Curriculum (ITMC) might look like. Whilst these terms do conjure up a picture of what is meant, respondents clearly had difficulty explaining the concept. A more definite means of quantifying the degree of integration is required for valid comparison between courses, and if any inferences about the importance of technology integration are to be made.

A preliminary review of the literature proved useful in suggesting a means of consolidating the complex variety of responses reflected in the coded data. Suggested factors include curricular congruency (Dugdale, Thomson, Harvey et al., 1995; Leigh-Lancaster, 2000), obstacles to successful technology implementation (Hoyles, 1998), access to technology and issues of equity (Zevenbergen, 1999), pedagogical considerations (Kutzler, 2000b; Stacey, Asp & McCrae, 2000), and issues of staff and students’ technological proficiency (Kawski, 2003). The efforts of Engelbrecht and Harding (2005a), to categorise and develop a taxonomy for their investigation of web-based learning of mathematics, suggest a useful basis for considering the data in this study. Much of the process they used, and their resultant descriptors may be usefully adapted to the discussion of technology integration in general. They discuss the general classification of web-use in education, by Harmon and Jones (1999, in Engelbrecht & Harding, 2005a, p. 241), which includes the descriptors supplemental, immersive, and essential. Supplemental may be used to describe a course where the teachers use the technology solely as a demonstration tool, while a course where the students all possess and interact frequently with the technology is categorised as immersive. Essential describes a course where technology is compulsory, expected, and required to succeed in the course. Engelbrecht and Harding (2005a, pp. 244-245) develop these general descriptors, formulating six mathematics-specific categorisations for online mathematics courses. They identify these as: Dynamics and Access (What is the frequency of access necessary for success in the course?); Assessment (How much of the assessment is done on site?); Content (How much of the course content is on the site?); Communication (How much communication happens on-line?); Richness (How many enriching components does the site have?); and Independence (How independent from face-to-face contact is success in the course?). The descriptions of these categories
can be adapted to measurements of technology use in general. For example, their categorisation for Assessment directly reflects the question used to examine and code the data displayed in Table 5.1. Independence may be adapted to measure to what extent students are able to succeed in the course, with or without technology.

Using Engelbrecht and Harding’s (2005a) categories as a basis, six defining characteristics were developed to broadly describe the extent of technology use in a course. For example, issues concerned with how students accessed the technology, ranging from tutorials in a computer laboratory, through to compulsory purchase of a CAS-calculator, are broadly categorised under Access. These six characteristics, summarised in Table 5.2, are proposed as an initial taxonomy for an integrated-technology course.

**Table 5.2: A Taxonomy for Integrated Technology**

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Example of question asked to examine the degree of integration for each characteristic.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A Access</td>
<td>To what extent do students have access, e.g. is it compulsory? Do they own their own, or access it in computer labs?</td>
</tr>
<tr>
<td>B Student Facility</td>
<td>How proficient are students with the use of the technology, and what assistance is provided to help them?</td>
</tr>
<tr>
<td>C Assessment</td>
<td>Is technology expected and/or permitted in assessment?</td>
</tr>
<tr>
<td>D Pedagogy</td>
<td>How and when do the staff and students interact with the technology? For example, is it used mainly as a complex calculation device and demonstration tool, or to develop and explain concepts?</td>
</tr>
<tr>
<td>E Curriculum</td>
<td>Has the course curriculum, for example content, order of teaching, changed to reflect the use of technology?</td>
</tr>
<tr>
<td>F Staff Facility</td>
<td>Are staff familiar with the use and capabilities of the technology, both mathematically and pedagogically?</td>
</tr>
</tbody>
</table>

While all of the coded responses could be adequately placed within this framework, it did prove difficult to find mutually exclusive categorisations. There is some overlap in the categories, for example assessment, professional development and pedagogy may all be broadly considered as curriculum issues, while assessment and
student and staff facility are also pedagogical considerations. However, the survey data and the literature do suggest sufficient basis for categorising these issues separately, and the checks by colleagues described in the methodology support the inclusiveness of the developed categories. The taxonomy is used next to measure the extent of technology integration in courses selected from the survey responses, and an instrument is developed to facilitate comparisons between these courses.

5.2.2 An Instrument for Comparison of Curricula and Technology

The taxonomy provides a means of examining the degree of technology integration within a course. However, it does not give an absolute measure of this level of integration, as a basis for comparing two courses. Given the extreme variation in technology use found in the survey of tertiary institutions, finding a suitable means of comparing technology usage between courses presents a real challenge.

It is evident from the data that decisions about which technology to use reflect the availability of a given technology, and decisions about student access. These include minimising the cost to the department and students, or the decision to use graphics calculators because many students bring them with them from secondary school. Other times, the decisions are made on pedagogical grounds, for example the use of particular computer-based statistical packages because they are widely used in the commercial and research fields. Thus any description of integrated technology should consider the factors influencing decisions about the type of technology used, but still allow a comparison between courses that is independent of which technology is ultimately used. It may be that valid comparisons are limited to courses of a similar nature, for example comparing two first-year statistics courses, but not a second-year applied maths course with a first-year calculus course. Students in a second-year course may already have a sufficient level of facility with the technology from previous use in first year courses, so the student facility imperatives differ in such cases (Stewart, Thomas & Hannah, 2005). Similarly, an applied mechanics course may contain a pre-requisite for a particular software use, which makes it radically different to a more traditional pure calculus course. However, it would be useful to develop a strategy that transcended such imperatives.
Engelbrecht and Harding (2005a, p. 244) encountered similar difficulties in their attempt to characterise on-line mathematics courses. “It is difficult to compare the scope and extent of any two online courses, because both might be lacking in certain, not necessarily the same, aspects and exceed again in other, different, aspects”. They developed a strategy for comparing the components of the course by assigning each category in their taxonomy to the radial of a radar chart, with an associated quantifying measure. For example, in response to the question “What is the frequency of access expected for success in the course?” with respect to their category of Dynamics and Access (see p. 181 this thesis), Engelbrecht & Harding (2005a, p. 244) used the following quantifiers:

1-once per term
2-once per month
3-once per week
4-two to three times per week
5-daily

A similar quantification system is used for their other categories, for example in response to the question “How much of the assessment is done on the site? (Engelbrecht & Harding, 2005a, p. 244):

1-little
2-almost half of it
3-more than half of it
4-almost all of it
5-all of it

The area of the subsequent diagrams provides an effective visual means of comparing the extent of internet utilisation, although Engelbrecht and Harding (2005a) are careful to note that it does not necessarily mean that the bigger the area, the better the course. In fact in many instances the objectives of the course design mean that some radials are inappropriate or superfluous (Engelbrecht & Harding, 2005a, p. 245). However, their results suggest that this system may be equally useful when comparing the level of technology integration. This strategy is explored to compare several courses selected from the survey responses.

Courses from four institutions, each representing a different country, were purposely selected from the survey responses. Three first-year calculus courses were
chosen to allow for comparison of similar courses in the primary focus area of this study (Institutions X, Y and Z), while the fourth (Institution W) describes a third-year engineering course by way of contrast. Individual responses for each course were compared against the six characteristics identified in the taxonomy (Table 5.2), and a system of numerical values was developed to quantify each characteristic. These values were developed using the coding of responses for each question in the survey (See Appendix A3(ii)), adapted from the system developed by Engelbrecht and Harding (2005a), with reference to the Harmon and Jones’s (1999) characterisations of immersive, essential and supplemental (see p. 181 this thesis). Table 5.3 provides an example of the descriptors developed to quantify two taxonomy characteristics, Access and Assessment (Table 5.2). The complete list of numerical descriptors for the other taxonomy characteristics is summarised in Appendix A3(iii).

Table 5.3: Quantification of Technology Integration

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Quantifying Descriptors</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access</td>
<td>5 = Full access, e.g. compulsory ownership of calculators; classes taught in labs; equal and regular access to technology, e.g. computer labs/individual computers for all students.</td>
</tr>
<tr>
<td></td>
<td>4 = Not compulsory, but majority of students own (&gt;80%); Not all classes in labs, but technology modelled by staff; ready access to technology outside of class for all students.</td>
</tr>
<tr>
<td></td>
<td>3 = As for 4, but no formal classes in labs, Fewer students own (&lt;80%) Restricted access outside of class, e.g. class sets of calculators, or technology use for assignments, tutorials only.</td>
</tr>
<tr>
<td></td>
<td>2 = As for 3, but less access, e.g. only some students have own calculator (&lt; 50%); lab access restricted to specific tutorials, assignments.</td>
</tr>
<tr>
<td></td>
<td>1 = Very limited student ownership (&lt; 10%); restricted access to computer labs for specific tasks (e.g. computer project).</td>
</tr>
<tr>
<td></td>
<td>0 = No student use, limited to staff demonstration.</td>
</tr>
<tr>
<td>Assessment</td>
<td>5 = Technology Assumed/Active: Students may use at any appropriate time in all assessment.</td>
</tr>
<tr>
<td></td>
<td>4 = As for 5, but some specific technology-free component.</td>
</tr>
<tr>
<td></td>
<td>3 = Technology Specific: Some technology neutral or prohibited component (e.g. final exam); some specific technology targeted component, e.g. computer project.</td>
</tr>
<tr>
<td></td>
<td>2 = Technology Neutral: All assessment questions written so no supposed advantage from student access to technology.</td>
</tr>
<tr>
<td></td>
<td>1 = Technology allowed, but no specific allowances made for technology in assessment questions.</td>
</tr>
<tr>
<td></td>
<td>0 = Prohibited: No technology allowed in any evaluative assessment.</td>
</tr>
</tbody>
</table>
Each institutional response was assigned a value from zero (low) to five (high) for each taxonomy characteristic, measured against the corresponding numerical descriptors. Tables 5.4 and 5.5 display the values assigned to each taxonomy component for two of the sample institutions. The quantifications for each variable for this sample were initially conducted by the researcher, but were subsequently validated by colleagues, and in one case (Institution Y), the assigned values were checked by the respondent, to confirm the researcher’s conclusions. Similar assessments and checks were made for each of the other two selected institutions. As an example of how these values were reached, the access component of Institution X was not assigned a maximum value of 5, since although calculator ownership is expected and required in their courses, not quite all students owned their own graphics calculators. The value of 2 for Institution Y reflects the 40% student ownership of calculators.

Table 5.4: Quantification of Technology Integration, Institution X.

<table>
<thead>
<tr>
<th>Taxonomy Component</th>
<th>Radial</th>
<th>Value</th>
<th>Rationale for assignation of value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access</td>
<td>A</td>
<td>4.5</td>
<td>Students expected to have a graphics or CAS-calculator (i.e. compulsory). Most have already but some have to borrow for exam.</td>
</tr>
<tr>
<td>Student Facility</td>
<td>B</td>
<td>4</td>
<td>Manual provided, special tutorials, examples in course text. Modelled in class by staff.</td>
</tr>
<tr>
<td>Assessment</td>
<td>C</td>
<td>4.5</td>
<td>Technology assumed in all assessment, except for small skills-based technology-free pre-test (entrance test).</td>
</tr>
<tr>
<td>Pedagogy</td>
<td>D</td>
<td>4</td>
<td>Used as demo/modelled in class, examples chosen to help understanding, students expected to follow through examples.</td>
</tr>
<tr>
<td>Curriculum</td>
<td>E</td>
<td>3.5</td>
<td>Not designed with technology focus. Statement about technology in course description. Greater emphasis on some topics, changes in order, new questions in tests but not change in style of test or other assessment.</td>
</tr>
<tr>
<td>Staff Facility</td>
<td>F</td>
<td>4</td>
<td>All staff expected to use the technology. Most very experienced, already using and familiar, no formal training/ongoing professional development, but high level of collegial support.</td>
</tr>
</tbody>
</table>
Table 5.5: Quantification of Technology Integration, Institution Y.

<table>
<thead>
<tr>
<th>Taxonomy Component</th>
<th>Radial</th>
<th>Value</th>
<th>Rationale for assignment of value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access</td>
<td>A</td>
<td>2</td>
<td>Ownership of TI-89’s strongly encouraged, discounted sales provided, at time of survey, approximately 40% of students have their own.</td>
</tr>
<tr>
<td>Student Facility</td>
<td>B</td>
<td>2</td>
<td>Some examples in back of text, calculator manual, some tutorials. Modelled in class by some staff.</td>
</tr>
<tr>
<td>Assessment</td>
<td>C</td>
<td>3</td>
<td>Allowed to use everywhere, but exam &amp; test set as technology-neutral.</td>
</tr>
<tr>
<td>Pedagogy</td>
<td>D</td>
<td>2</td>
<td>Some teachers used as demo in class, some tutors also model use in tutorials.</td>
</tr>
<tr>
<td>Curriculum</td>
<td>E</td>
<td>2.5</td>
<td>Thought given to usage and design, some topics altered, some assessment changed, additional course resource provided.</td>
</tr>
<tr>
<td>Staff Facility</td>
<td>F</td>
<td>2</td>
<td>Most staff quite experienced and discussed and compared usage, but some opposed/unfamiliar and didn’t use, no explicit training at start or subsequently, but collegial support evident.</td>
</tr>
</tbody>
</table>

Some problems were encountered in the assignation of values. For example, it was difficult to assign a value for staff facility to one course, where although no explicit professional development was undertaken for technology, very high levels of staff facility were evident (Institution X). Despite the check on this process by colleagues, assigning values in such problematic cases is somewhat arbitrary, and suggests the development of a more robust relationship between each taxonomy component and its numerical measure. One means of addressing this could be to ask respondents to assign values themselves, using the descriptors as shown in Table 5.3 and Appendix A3(iii).

The information presented in Tables 5.4 and 5.5 is then displayed in radar diagrams similar to those developed by Engelbrecht and Harding (2005a), as shown in Figures 5.1 to 5.3. The shaded area in this case gives an indication of the extent to which technology is integrated into the course (as opposed to the extent of internet utilisation). The shape of the shaded area in the radar charts is also significant. The diagram for Institution X shows an even spread across all components, indicating that all aspects of the taxonomy are considered in that course with respect to the use of technology. Institution Y also displays a relatively even spread across the components,
but the smaller area suggests a lesser depth of technology integration than for Institution X. This comparison was much more difficult to discern directly from the raw data.

![Diagram of technology integration comparison between Institution X: United States and Institution Y: New Zealand.](image)

*Figure 5.1 Comparison of technology integration in two first-year Calculus courses (Institutions X and Y).*

The order in which the component radials are displayed in the diagrams is kept the same in each diagram, as changing the order can change the shape of the resultant area, and thus affect the visual impact and comparative accuracy provided by the diagrams. The order selected for the three comparisons shown here groups together student-oriented components on the right, and organisational and staff-oriented components on the left. The right-hand side radials (A, B, C) measure the ability of students to use the technology, that is: Are they able to access it? Do they know how to use it? Can they use it in assessment? The left-hand side radials (D, E, and F) measure the design and delivery of the course; namely pedagogical considerations, curriculum, and staff facility and professional development. There is overlap because assessment issues concern staff as well as students, and students are affected by pedagogical considerations, hence these characteristics are grouped together at C and D on the chart.

Figure 5.2 shows a high level of attention to student factors in the Australian course, with less depth of integration on the left-side institutional-based components. The survey responses suggest this may be attributed to one staff member’s enthusiasm for technology, but a lesser level of support from colleagues and the department as a whole. By comparison, the responses for Institution X revealed a high level of departmental and collegial support and commitment to the use of technology.
Figure 5.2 Comparison of technology integration in two first-year Calculus courses (X and Z).

The Uruguay course depicted in Figure 5.3 shows a clear weighting towards student factors, which is consistent with the applied focus of the course, and the expectation that by the third year, students are sufficiently familiar with the technology to require little assistance.

Figure 5.3 Comparison of technology integration between a first-year calculus course (X) and a third-year engineering course (W).

The value of five for access arises from the compulsory use of technology by students described in the responses, reflecting the essential nature of computer software in modern-day engineering. This is also evident in assessment, where technology is assumed in all areas, but especially a large computer-based project which comprises a
significant part of the student’s final grade. The lesser overall attention to staff factors reflects, in part, the assumed expertise on the part of staff with domain-specific software, and the incremental way in which technology has crept into applied mathematics fields, without much attention to changes in curriculum. This is evident in the responses that “all our teaching staff use Maple and Matlab regularly in our research,” and “the course is still the same, we just use computers a lot more for things like numerical analysis”. The diagrams make explicit how technological imperatives may differ greatly between courses, and can thus initiate an exploration of the underlying reasons for the displayed effects. However, one limitation of the radar-diagram strategy is the inevitable loss of individual information when data is grouped within a graded or coded schedule (Oates, 2004b). For example, while the quantification used for the curriculum component does display the level of attention given to such factors, exactly which of the individual aspects are present for a given course are lost. Two courses may appear visually the same, but differ in specifics. The radar diagrams thus tend to over-simplify the complexity of technology use revealed in the survey. It may prove necessary to refine the existing radials, or even establish further taxonomy components to provide a fuller description of integrated technology. For example, the instrument currently groups staff and student aspects of the pedagogical component together, but several survey responses distinguished between these.

The taxonomy might also provide greater differentiation within the pedagogical, and the staff and student facility components. It could, for example, consider the types of use made by staff and students with respect to the technology (cf. technical versus functional use, Table 3.2), and examine the resources and constraints of the technology, and issues of instrumentation and instrumental genesis, compared with just looking at levels of experience and familiarity. It could also consider the effects of staff and students’ beliefs about the nature of mathematics and technology. The next section describes the second methodological wave of the thesis, which includes the wider review of the literature that identified such additional factors, and a further survey of technology use in undergraduate mathematics courses. The results of this survey are used to refine the taxonomy of integrated technology, and further develop the instrument for measuring and comparing technology use between courses.
5.3 REFINING THE TAXONOMY

Figure 4.2 depicts the position of the third survey in the methodological structure of this thesis. The survey primarily addresses the first research question, to identify what an integrated technology mathematics curriculum (ITMC) might look like. It builds on the findings from the previous surveys, as well as the more extensive review of the literature conducted in Chapters Two and Three. Thus, questions were asked with respect to each of the six components in the taxonomy (Table 5.2), while the influence of additional factors identified in the literature can be seen throughout the questionnaire (see Appendix A4). For example, there are questions on beliefs about technology and mathematics, and the types of use to which technology is suited in mathematics (Norton & Cooper, 2001b; Stacey, 2003). The professional development required to effect successful technological change, identified by Keynes and Olson (2001), is also evident. The survey also partly addresses the second research question, with a group of questions examining implementation issues. Some results from these latter questions will be presented here, although discussion is mostly left for the next chapter.

5.3.1 Responses to the Third Survey

As described in Chapter Four, this survey was sent to 134 colleagues from 44 tertiary institutions involved in the teaching of undergraduate mathematics. Forty-one separate responses (31% response rate) were received, representing 31 tertiary institutions from 8 countries (6 institutions from New Zealand, 10 from Australia, 7 from the United States, 2 each from South Africa, the United Kingdom and Uruguay, and one each from Canada and France), and a total of 72 different courses (see Appendix A4 for a summary of these). The individual response rate was actually better than the 31% indicates, as 5 of the institutional replies represented a departmental response from a number of individual teaching staff (a total of 16 individual survey requests). In addition, another 9 colleagues excused themselves from the sample because they had no current experience of teaching mathematics with technology. While the latter group may appear as a form of non-response, the reasons stated suggest minimal effect on any results. They were no longer teaching undergraduate mathematics (with or without technology), instead, they had moved into other fields (postgraduate courses, administration, teacher education), so their inability to answer was unrelated to
the technological focus of the survey. Given these 9 responses, and the 16 individuals represented in the grouped responses, the actual response rate was considerably higher, at 42%. However, the reasons and effects of the non-response should still be considered.

The discussions in Chapter Four described several sources of non-response, and the possible effects of these (Groves, Fowler, Couper et al., 2004). Except for one question using a numerical-scale response, this survey required many wordy responses. Indeed, several respondents commented on this, in excusing delays for their response. Such question styles are commonly responsible for lower response rates (Bradburn & Sudman, 1991). It is therefore reasonable to assume that some of the non-response was due to lack of time and the demanding nature of the survey, rather than any significant difference in their views with respect to technology. Only two of the 41 responses were from colleagues teaching mathematics without technology, possibly because the third survey was disregarded by those not using technology. Unfortunately, it is impossible to assess the impact of this on the non-response rate. While responses from this group, especially to the questions in section D (see Appendix A4), are useful for the second research question examining factors influencing technology implementation, the non-response should have no effect on the results for the first research question examining actual use. Another potential source of non-response bias is item non-response, where respondents miss out particular parts of the survey, especially likely for wordy questions (Bradburn & Sudman, 1991). However, this was minimal in the responses received, and in all but one instance, reasons were provided that allow an assessment to be made.

Responses were initially compared against the taxonomy in Table 5.2, using the coding developed for the second survey. Any cases where placement within the taxonomy using this coding was problematic were noted, and these are documented in Appendix A4(ii). In particular, as the earlier discussion of the second survey revealed (see p. 190), the examination of the data looked for examples where the taxonomy over-generalised within categories, or failed to differentiate between different responses. Evidence of factors identified in the literature was also sought, especially for the pedagogical, and staff and student facility components. These included looking for responses which exhibited consideration of instrumental abilities compared to mathematical knowledge (IA and MK, Laborde, Assude, Grugeon & Soury-Lavergne, 2006; see p. 109 this thesis); examples of pedagogical technology knowledge (PTK, Hong & Thomas, 2006; see p. 69 this thesis); the value of guiding student learning and
teacher privileging (Kendal & Stacey, 2001; Muller, 2001); consideration of the curriculum value of topics using technology (Stacey, 2003; see Figure 2.8 this thesis); consideration of the capabilities and limitations in making choices about technology (Bloom, Forster & Mueller, 2001; see p. 128 this thesis), and attention to student factors such as technical or functional use, and instrumental genesis (Pierce & Stacey, 2004; Artigue, 2006). Responses to beliefs about the role of technology and its appropriate use proved especially difficult to locate within the existing taxonomy. For example, one response provided clear evidence of the differentiation between the technological needs of users and learners (Love, 1995; see p. 122 this thesis). The respondent teaches a first-year course in discrete mathematics for IT, Engineering and Business students in Australia. The course makes use of several software products (Matlab, J, and Java), for which the use is predominantly mathematical in nature, focusing on the algorithmic and programming functions and the numerical capabilities of the technology. The technology is regarded as having minimal pedagogical value, except for the ability to solve complex problems. Quite where this response might be positioned within the existing taxonomy is not obvious:

No special advantages (in technology) for students who don’t wish to learn ... Certainly beneficial when students are involved in the subject material and make the mathematics relevant to their computer studies. Technology per se has no significance for education ... (but) there is no doubt that the teaching of mathematics has been changed by the emergence of powerful numerical capabilities and computer algebra systems. Very few working mathematicians would now be struggling on without the help of a CAS. The field of differential equations has been completely revolutionized by the ability to obtain numerical solutions of spectacular accuracy on a desktop machine. (N2, Australia)

The overall responses to the third survey exhibited equally wide variation in technology use to that found earlier, with some real extremes shown. Five departmental responses displayed widespread use of a variety of technologies, and may be considered as well-integrated, as defined by the taxonomy. An additional nine responses also displayed highly integrated levels of technology for individual courses (see Figure 5.4). The slightly lower levels of integration for the “pedagogy” and “staff-facility” components in the second diagram are due to factors outside the control of the individual course. For example, the use of technology is explicit in the course syllabus,
but no changes to course content have been made, as this is determined by the department. For staff-facility, not all staff teaching the course are equally enthusiastic or experienced with the use of technology.

By way of contrast, other departmental responses indicated limitations on technology use, such as bans on calculators, and considerable variation in use between courses. One institution in the United Kingdom used a specially developed online assessment system for first-year calculus, but calculators were permitted in examinations only in statistics courses. The same respondent also noted big variations within team-taught courses with respect to technology use by staff, noting that “Some embrace technology. Many ignore it totally”. Another response described how their large first-year engineering mathematics course (450 students) had moved away from the use of technology. After several years of using Maple for student assignments, its use is now limited to occasional use to demonstrate particular topics in lectures.

The types of technology reported were more varied than the previous survey, with several products not encountered in the literature review, especially for applied courses. Examples include the commercial programming data-analysis software J, a powerful programming language best suited to mathematical and statistical data analysis, especially with matrices; the software package developed by Cornell University known as Interactive Differential Equations (IDE); and a variety of statistical analysis products

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**Figure 5.4** A sample departmental response and an individual course from the third survey showing high levels of technology integration.
such as Minitab, SAS, and R. A full list of reported technologies, with frequency and examples of use, is given in Appendix A4.

Reported CAS-usage was evenly distributed between CAS-calculators, Matlab and Maple, with Mathematica slightly less popular (12, 14, 11 and 5 responses respectively). Other products such as Scientific Notebook were reported less frequently. Maple and Matlab were most favoured in engineering and other applied mathematics courses (except for statistics), and second and third-year calculus and analysis courses. A common sentiment in this respect is reflected in the comment that “the more advanced the student, the more sophisticated the technology”. CAS-calculators, Matlab and Mathematica were more favoured in first-year courses, with calculator usage (both graphics and CAS) more popular in introductory and bridging courses, and teacher education. Scientific or general calculator use was common in many courses for numerical and computational support, but eight responses noted limitations on this use in formative assessment.

Question C2 in the survey examined respondents’ views on the use of calculators and computers in mathematics education. Responses, summarised in Table 5.6, demonstrate general support for technology use in undergraduate mathematics. Many responses viewed technology as essential, observing for example that “a student who completes a major in mathematics should be able to use technology effectively and efficiently”. Only one response gave a predominantly negative view of technology use (Value of 1 for all parts except a 5 for use of calculators for numerical or computational purposes):

For the most part I think technology in calculus courses should be confined to numerical methods...Its value lies in computation. It does not help with concepts...Technology should be used only when it is the only alternative or when students have demonstrated that they are able to demonstrate that they only need the technology to solve problems faster... Most of the courses I teach are better off, on the whole, with student access to technology being limited. (N1, USA)

Applied and advanced mathematics respondents, especially those teaching engineering mathematics, and programming-based courses, tended to favour computers over calculators. One respondent, now predominantly teaching higher-level courses, gave the use of graphics calculators in his department’s introductory-level courses as a
contributing reason for his lesser involvement, stating that he “can’t stand gc’s”, while another engineering mathematics teacher believed that graphics calculators should not even be used in schools: “Get rid of these ridiculous graphics calculators in schools and use Excel instead. All sorts of modelling and simulations may be done”.

Table 5.6 shows general acceptance of calculators for numerical, computational and graphing purposes. There is less support for the symbolic manipulation capabilities of CAS-enabled calculators, but conversely favourable attitudes towards the use of CAS-enabled software. The comparison between (iii) and (iv) is not conclusive, since the questions are worded differently, but comments to other questions for some respondents suggest this may be due to greater familiarity for many mathematicians with computer software. Some responses, especially for the applied courses, inferred that the use of Maple and Matlab was favoured more for their superior solving and computational abilities in applied and advanced courses. It is possible some of these replies may have been less favourable towards purely symbolic manipulation.

<table>
<thead>
<tr>
<th>C2</th>
<th>Please select the response that best represents your view about the ideal use of calculators and computers.</th>
<th>Number of Responses (1 = Little or No Emphasis, 5 = Heavy)</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>(i)</td>
<td>Calculators for numerical or computational purposes</td>
<td>4</td>
<td>1</td>
</tr>
<tr>
<td>(ii)</td>
<td>Calculators for graphing purposes</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>(iii)</td>
<td>Calculators for symbolic manipulation</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>(iv)</td>
<td>Computer software (e.g. Matlab, Maple)</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(v)</td>
<td>Modifying existing software or programming</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>(vi)</td>
<td>Spreadsheets or tables</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

The conflicting views towards calculators and computers are reinforced in the pattern of responses not revealed by inspecting the figures in Table 5.6. Five respondents gave values of 1 to all three views on calculator usage, and 5's for all three with computers, which presented a visually impressive pattern of the contrast in their...
views. The figures were greater for Questions (iii) and (iv) taken alone, with seven giving matching 1’s and 5’s respectively. All but one of these seven responses came from colleagues teaching applied or more advanced mathematics courses, which suggests that technology preferences are strongly influenced by mathematicians’ personal research domains (Love, 1995). It seems a lack of pedagogical technology knowledge with respect to calculators may be responsible for some of these attitudes (see Hong & Thomas, 2006), or at least a belief that calculators are an inferior product that privileges educational objectives over mathematical ones. This belief is implicit in the response from an Australian respondent, who strongly criticised calculators:

The best available software should be used, not something regarded as “good enough for education”. What is “best available” depends mainly on the design of the software and its notational features. (N2, Australia)

Some specific criticisms were similar to those described in the literature. The poor interfaces in CAS-calculators were cited as reasons against their use for symbolic manipulation (see Guin & Trouche, 1999; Cavanagh & Mitchelmore, 2001b), although recent developments in calculator technology are addressing this (e.g. the TI-Nspire, see p. 120). Most departmental responses advocated more balanced use of both calculators and computers. Some individual courses showed strong preferences for particular technologies (e.g. 7 out of 12 statistics respondents favoured spreadsheets and tables), while three respondents qualified their response to the use of computer software. One respondent for example noted that “it depends on the software, 1 or 2 for Matlab & Maple, 5 for dynamic geometry software”. Some favoured calculators for their portability, with greater opportunities for use in lectures and work outside of computer laboratories. However, many inferred that this may change in an ideal world with individual student access to computers. One respondent believed that student ownership of laptops will “enable us to break out of labs” and thus eventually spell the end of calculators, while another stated that, despite a calculator ban in his department:

I get students to use their calculators during lectures whenever we are talking about things like simple iterations. I hardly ever use computers in lectures (I have always found it rather boring watching someone else do this in lectures).

(N4, New Zealand)
The use of technologies non-specific to mathematics was high, with 49 responses reporting the use of technologies such as the internet, Data-Shows, PowerPoint and Tablet-PC’s in course delivery, and 10 for the use of computer-aided or online assessment (see Appendix A4). Online quizzes were especially common in large first-year courses (9 out of 16 responses), as were electronically submitted computer-based assignments in applied courses (15 of 23 responses). Four courses were delivered entirely online, and three of these required specific mathematical technology such as Matlab. This approach was described by one respondent as a “technologically consistent response to modern student realities”.

The new questions examining beliefs about technology use, and visions for future development, provided additional insight into the complexity of technology use found in the second survey, and helped to explain the variations in usage. The results suggest a refinement of the taxonomy presented in Table 5.2, which is considered next.

5.3.2 *A Refined Taxonomy of Integrated Technology*

The level of difficulty encountered in placing some responses from the third survey within the existing taxonomy (Table 5.2) can be gauged from large number of such responses listed in Appendix A4(iii). Questions in Part B of this survey were developed from the earlier study, so it is not surprising that fewer problems were found in categorising these responses. Even so, several new issues were identified that were difficult to place. Chief amongst these was the effect of departmental (and sometimes institutional) policies and organisational structures on technology outcomes, and the dictates of meeting service course needs. One United States Business College course required the use of a specific technology. The respondent noted that “the course is under constant discussion with the Business College. Learning the technology (Excel) was one of the central purposes of the course”. Another respondent, also from the United States, reported little use of technology in his own teaching, even though he personally favoured it, and the department’s organisational structure theoretically allowed it:

The department has no policy on technology use by faculty: any faculty member has (in theory) complete freedom in any course he or she teaches. In particular, this applies to any post-calculus courses, which are always taught by faculty. For non-faculty instructors in courses at calculus or pre-calculus level, the policy will
be set by the course co-ordinator. At present, no co-ordinator allows technology beyond numerical calculators. (H10, USA)

By contrast, a New Zealand respondent reported an entirely converse outcome, where he was actually able to use technology quite extensively in his first year calculus course, despite official policy against it:

Officially, calculators are banned in tests and exams, but as indicated above, we have the discretion to ignore this. In numerical papers in particular it is recognised that this rule may not be appropriate. The calculator ban came after a discussion in the department several years ago. People have come and gone in the meantime, but I suspect a department meeting would still endorse the ban. (N4, New Zealand)

In other instances, external policies either support the use of technology, or reinforce its consistent use. One Australian university mandates equivalence in delivery and assessment in subjects offered in more than one stream (or campus), thus requiring all staff to make use of technology when it is prescribed in their courses. In France, fifth-year university students must pass a national certificate on the use of technology, which the respondent views as an effective carrot for the inclusion of technology in undergraduate courses, and engagement with the technology by students. These examples suggest the taxonomy needs to acknowledge the influence of bureaucratic and organisational factors on technology integration.

Other responses in Part B identified a need for more differentiation within the Assessment component, to distinguish between mathematically specific assessment using technology, and assessment designed to take active advantage of technology, such as on-line assessment. The use of online and computer-aided assessment, while easy to locate in the taxonomy, was much more problematic to categorise using the quantification schedule developed earlier. Another response revealed the underlying complexity that would be hidden by simply classifying assessment as “active” on the basis of allowing CAS-calculators in examinations: “Students may use any hand held calculator, but in exams they must show full written working to reach the answer. So the calculator is often used (only) to check the result.” A few responses also proved difficult to categorise within the Pedagogy, and the Staff- and Student Facility components of the taxonomy. For example, the previous response about calculator use in examinations has pedagogical implications. Other examples include the effect on a mostly integrated
course by staff “resistant to the use of technology, [who] give little to no time or effort to the consideration of teaching with it”, and the observation from another course that “Some students avoid software and do not do that part of subject. Other subjects you cannot do this and so the students must use the labs.”

However, the new questions on beliefs, attitudes and implementation experiences (see Appendix A4, Parts C and D) proved more problematic to categorise in respect of these three taxonomy components. Responses to these questions collectively support the need for greater differentiation within the pedagogical, staff and student facility components, in order to distinguish between staff and student use, and the exact nature of such usage. Responses categorised under Staff Proficiency with Technology and Student Instrumentation (see question D1, Appendix 4(iii)) suggest that considerable information would be lost by grouping them collectively within the staff and student facility taxonomy components. For example, staff need to be aware of the impact of technology on students’ solutions (Ball & Stacey, 2003; Zbiek, 2003), as evidenced by the following observation:

[Its difficult] getting teachers familiar with the benefits of use. It requires more attention from the teacher to understand the solutions made by students, since the range of possible solutions (correct and incorrect) is larger than in paper and pencil.

(Teacher Education Course, H7, France).

Equally varied and complex references to student instrumentation are evident in many responses. One example is the belief that students have become too dependent on graphics calculators, which affects their ability to interact meaningfully with both the technology and the mathematics (cf. page 109 with respect to IA and MK, Laborde, Assude, Grugeon & Soury-Lavergne, 2006):

School kids coming in are too reliant on calculators for the smallest of calculations. Further, these days with graphics calculators, too reliant also on calculator for graph sketching and algebraic manipulation. So, it’s hard to change the mindset of students, to think of calculators/computers as a tool to further their understanding of concepts, rather than something that does all their work for them.

(First-year Calculus Course, G5, Australia)

The new categories of Staff and Student Attitudes and Beliefs (question D1, Appendix A4) are not currently specified in the taxonomy. The following examples,
from a tutor in a first-year general mathematics and statistics course in Australia, suggests that responses in these categories may actually cross over several components of the taxonomy, in this instance with implications for both the Pedagogy and Curriculum components of the taxonomy:

Quite a few of the staff seem to think it is cheating to use the calculator and using it limits mathematical ability. So when you are a tutor, if the unit controller does not like technology, you are teaching topics that are quite out-dated as it can easily be done on a computer or calculator.

I would like the reduction of many arithmetical processes in statistics and give more credit for the interpretation and application. I would like to see the image of mathematics and mathematics educators changed by teaching methods that reflect the creativity rather than the chore of endless calculations. (H6, Australia)

In addition to the inadequacies of the taxonomy components discussed so far, the review of the literature with respect to curriculum studies in Chapter Two suggests that “Curriculum” may not be an appropriate categorisation for the remaining component. The wider view of curriculum assumed by this thesis (see pp. 21-25) encompasses all aspects of the taxonomy, with the current “Curriculum” taxonomy component (Table 5.2) in reality addressing only the goals, mathematical knowledge and content parts of this wider curriculum. This limitation is clearly illustrated by comparing the elements of the existing taxonomy (Table 5.2) against the Valero-Duenas (2002) curriculum model adopted by this thesis (see Figures 2.1, 2.2, 2.3). Three elements of the taxonomy may be directly related to the elements of the Valero-Duenas model at the intermediate and micro-levels of the curriculum (Students, Teachers, and Assessment). Considering the descriptors previously applied to the taxonomy when measuring the level of technology integration (see Appendix A3(iii)), the “Pedagogy” component may be seen to match the element of “Methodology” in the curriculum model, while the “Curriculum” component has similarities to the “Knowledge”, “Contents” and “Aims” elements. This comparison thus supports maintaining the three global categories of Students, Teachers, and Assessment within the Taxonomy, albeit with a greater level of differentiation as demonstrated earlier. These components are therefore relabelled in the refined taxonomy as Student Factors, Staff Factors, and Assessment (see Table 5.7). The change acknowledges the need to include beliefs, attitudes, and types and degrees of use
within the staff and student categories, in addition to the more limited measure of familiarity previously examined under the *Facility* categorisation.

There is also support within the Valero-Duenas model for the inclusion of an *Organisational Factors* component, matching the “Institutional” element of the model. Goals, previously located in the “Curriculum” component, may also be included here, as course syllabi are frequently centrally formulated, or at least influenced by organisational policies. Other issues previously considered within the “Curriculum” component need to be re-categorised. The descriptors for this component in Appendix A3(iii) identify content, order of topics, technology specific-syllabi, and goals as a basis for measuring this component. The discussions in Chapter Two also identified the nature of mathematical knowledge, and beliefs about this, as critical influences on curriculum development, especially with respect to the topics that are seen as important in the curriculum, and the skills which are deemed necessary for students to learn (see sections 2.3.1, 2.3.2). Studies such as those by Stacey (2003), and Herget, Heugl, Kutzler and Lehmann (2001) question the relative value and order of topics and skills in a technological environment (see pp. 53, 64-67 this thesis), and there is considerable support for the inclusion of such issues in the taxonomy from survey responses. The following response from a first-year calculus course in the United States illustrates this:

The biggest change has been in the approach to integration. I now spend far more time on the definition of the integral as a limit of Riemann sums, and have students solve integration problems as soon as this definition is introduced, by setting up Riemann sums and either taking the limit, or just computing with a large number of subintervals. Conversely, I spend much less time on techniques of anti-differentiation, but stress to the students that any alleged anti-derivative must be checked by differentiation.  

(H10, USA)

Many respondents also perceive that technology has changed the fundamental nature of both mathematics and teaching:

The study of dynamical systems and chaos is a classical example of changes in the nature of mathematics which also led to changes in the way differential equations are taught. Calculus was also affected (see for example the latest editions of Calculus by Stewart).  

(H1, New Zealand)
While some were enthusiastic about the effects of technology on the teaching of mathematics, others were frustrated by the slowness with which teaching has adapted to the technological changes in mathematics:

Yes, it has (changed mathematics teaching) a lot. It is now much easier to have students

- Reason graphically
- Understand the power of numerical approximations
- Experiment and learn to conjecture from their results
- Work with real data and real models. (H4, USA)

Mathematics teaching has adopted technology very slowly (much more slowly than mathematicians) but has also been very conservative in its use once adopted. CAS have been widely available for at least 15 years but in Australia they are only being routinely introduced into schools and Universities this year. This has been married to virtually no change in the manner in which maths is taught. So no, technology has not really changed mathematics teaching up until now. (G7, Australia)

Such issues are collectively categorised within the taxonomy component of Mathematical Factors, collecting together the separate elements of “Knowledge” and “Contents” from the Valero-Duenas model (Valero-Duenas, 2002).

There is no direct connection to the Valero-Duenas model for the remaining “Access” component of the taxonomy. Responses from the survey suggest that this aspect is influenced by many factors, including the cost of the technology, in both licensing fees for the department, and ownership for students; institutional, departmental and service course imperatives that may dictate access, in, for example the provision of shared laboratories; adequate access to and support for the technology for staff; and questions of use in assessment. An Australian respondent noted that “the major barrier to (technology integration) is the cost of hardware and software and the fact that not all students can afford to have the latest and best”, while another department in the United States gave the financial burden on its students as a principal reason for its decision against the use of technology. Economic considerations were seen as especially significant in developing countries, for example Uruguay and South Africa, where one respondent noted that usage was mostly limited to infrequent staff demonstrations, with a little student use of Excel in inadequate computer laboratories.
“Other technologies such as graphing calculators, Maple, and Matlab are out of reach for our students”.

Access to technology for assessment also differed, with several responses advocating at least some technology-free aspect, while others favoured greater congruency, with technology “available and used at all appropriate opportunities including in class, assignments, tests, exams, and homework”. Such considerations cross over several other components of the taxonomy (Organisational, Staff, and Student Factors and Assessment), and given the importance of appropriate access to technology identified in the literature (see for example pp. 125, 137), the continued inclusion of Access as a separate component in the taxonomy is warranted.

The six components of the refined taxonomy, and the sub-categories for each, developed from these discussions, are summarised in Table 5.7. Justification for the structure of the taxonomy is provided, using examples from studies identified in the literature, and sample responses from the survey. As previous discussions have intimated, this taxonomy is considered most appropriate for the undergraduate mathematics courses that are the primary focus of this thesis. However, while some elements may have less relevance for other courses, a consideration of these is still seen as worthwhile, if only to subsequently discount them as not requiring attention. The next section uses the taxonomy to re-examine the use of technology in selected courses, to develop a definition of an ITMC.
Organisational Factors

Assessment: Technology Policy and Goals; Planning: Technology Policy and Goals;...
Table 5.7 continued: A Refined Taxonomy for Integrated Technology

<table>
<thead>
<tr>
<th>Component</th>
<th>Sub-Category</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Type of Use:</strong></td>
<td>Technical, Functional?</td>
</tr>
<tr>
<td><strong>Proficiency:</strong></td>
<td>Instrumental Genesis; Mathematical Reasoning; Solutions.</td>
</tr>
<tr>
<td><strong>Training &amp; Assistance:</strong></td>
<td>Tutorials, Manuals.</td>
</tr>
<tr>
<td><strong>Personal:</strong></td>
<td>Previous experience; Motivation &amp; Needs, e.g. major, service course; Technology ownership, Equity.</td>
</tr>
<tr>
<td><strong>Beliefs &amp; Attitudes:</strong></td>
<td>Teacher Privileging.</td>
</tr>
<tr>
<td><strong>Type of Use:</strong></td>
<td>Professional Domain; Modelling Technology; Teacher Privileging; Applications and/or Educational.</td>
</tr>
<tr>
<td><strong>Proficiency:</strong></td>
<td>Instrumental Genesis; PTK; Affordances &amp; Constraints. Interactions.</td>
</tr>
<tr>
<td><strong>Beliefs and Attitudes:</strong></td>
<td>Nature of Maths; Technology; Learning; Constructivism.</td>
</tr>
<tr>
<td><strong>Training &amp; Support:</strong></td>
<td>Professional Development, Consistency &amp; Sustainability.</td>
</tr>
<tr>
<td><strong>Time:</strong></td>
<td>Change, Resource Preparation.</td>
</tr>
<tr>
<td><strong>Content:</strong></td>
<td>Order and value of topics.</td>
</tr>
<tr>
<td><strong>Subject Imperatives:</strong></td>
<td>e.g. Algebra-CAS &amp; symbolic manipulation; Service, Applied, Pure courses; Domain-specific technology.</td>
</tr>
<tr>
<td><strong>Cognition, Reasoning and Skills:</strong></td>
<td>Technical versus Conceptual, IA and MK; Representational Versatility; APOS theory; Objects &amp; Procepts; Design Limitations.</td>
</tr>
<tr>
<td><strong>Mathematical Knowledge:</strong></td>
<td>Nature of Mathematics, Objectives and Goals; Needs of users verus learners.</td>
</tr>
</tbody>
</table>

**References:**

- Guin & Trouche, 1999; Tall, 1999-2002; Goos et al., 2000; Hong, Thomas & Kwong, 2000; Norton et al, 2001; Artigue, 2002; 2006; Heid, 2003; Lagrange et al., 2003; Goos & Cretchley, 2004; Hoyles et al., 2004; Pierce & Stacey, 2004; Thomas, Monaghan & Pierce, 2004; Stewart et al., 2005; Andresen, 2006; Van der Hoff & Harding, 2007.
- Hong & Thomas, 2006; Thomas & Chinnappan, 2008.
- Kendal & Stacey, 1999; 2001; Chinnappan & Thomas, 2000; Devries, 2000; Goos et al., 2000; Brown et al., 2004; Pierce & Stacey, 2004; Artigue, 2006; Laborde et al., 2006.
- Love, 1995; Cretchley et al. 1999; Hong, Thomas & Kiernan, 2000; Anguelov et al., 2001; Keynes & Olson, 2001; Muller, 2001; Norton & Cooper, 2001b; Kersaint et al., 2003; Zbiek, 2003; Buteau & Muller, 2006.
- Less emphasis on techniques, more powerful visualisation. H14, New Zealand.
- Less emphasis on techniques, more powerful visualisation. H12, USA.
- Less emphasis on techniques, more powerful visualisation. H14, New Zealand.
- A tool for accomplishing teaching objectives. H12, USA.
- A tool for accomplishing teaching objectives. H12, USA.
- The big problem is how much the majority of students do on calculators and how much is New Zealand.
- Some staff believe students will lose ability to do routine calculations. G3, New Zealand.
- "Some staff believe students will lose ability to do routine calculations." G3, New Zealand.
- A more positive attitude from students as to what it can contribute to their learning and development. Consensus & sustainability. H16, Refined Experience. H17, Needs & User Service Courses.
- Technology seems to have a negative effect on all involved. G7, Australia.
- Technology seems to have a negative effect on all involved. G7, Australia.
- It's difficult for students with strong opinions against the use of technology, especially those who had High School maths teachers with strong opinions against the use of technology. H2, Canada.
- A more positive attitude from students as to what it can contribute to their learning and development. Consensus & sustainability. H16, Refined Experience. H17, Needs & User Service Courses.
- Technology seems to have a negative effect on all involved. G7, Australia.
- Some staff believe students will lose ability to do routine calculations. G3, New Zealand.
- Technology should be integrated into teaching and learning as soon as possible, providing clear pathways to learning. G3, New Zealand.
- Some staff believe students will lose ability to do routine calculations. G3, New Zealand.
- A more positive attitude from students as to what it can contribute to their learning and development. Consensus & sustainability. H16, Refined Experience. H17, Needs & User Service Courses.
- Technology seems to have a negative effect on all involved. G7, Australia.
- Technology seems to have a negative effect on all involved. G7, Australia.
5.4 An Integrated Technology Mathematics Curriculum (ITMC)

The complexity of the refined taxonomy in Table 5.7 illustrates the difficulty in describing what an ITMC might resemble. As Figure 5.4 demonstrated, the radial diagrams may still be used to provide a visual measure of the overall level of integration for the six components of the refined taxonomy. However, new quantification codes would need to be developed to recognise the many new descriptors, and the added complexity would greatly increase the difficulties encountered earlier when using the radar diagram strategy with the simpler taxonomy model (see p. 189). As such, a rigorous examination to develop the necessary codes is not pursued here. Nevertheless, a visual comparison could still be useful for “bench-marking” purposes, especially for those wanting to make a case for greater levels of integration in their department or individual courses. It could also show the progression of technology integration over time, and help to identify specific areas within the curriculum where attention to technology issues may be needed. An example, using an approximate quantification schedule assessed against the taxonomy in Table 5.7, is shown in Figure 5.5. For this simple illustration, the values for each radial were estimated by the researcher. Despite the provisional nature of this assessment, this diagram still helps to identify the lack of organisational support in an otherwise integrated course. This respondent felt frustrated by the wider department’s lack of interest in technology, noting that “since I am almost the only instructor in our department who uses technology consistently, I have to use the same book as is used in other sections of the course”.

![Figure 5.5](image)

*Figure 5.5* Calculus course from the United States showing gap in technology integration for “Organisational Factors”.

*Integrated Technology in the Undergraduate Mathematics Curriculum*
The impression gained from Figure 5.5 should be sufficient to stimulate further investigation into the underlying causes and effects of the organisational issues identified as under-attended. In this case, the effects are frustrations for some staff as indicated by the earlier quote, and inconsistencies in technology use between courses, which resulted in complaints from students. Possible causes for the technology position that led to these effects can be seen later in the same response:

The department has discussed the possibility of introducing technology in its pre-calculus courses, although the last official discussion was about 10 years ago. The final decision has always been to maintain the status quo; i.e. no technology. The two reasons which seem to carry weight are: (a) our pre-calculus courses are precisely that: preparation for calculus; and students need to improve their (pencil-and-paper) algebraic skills in order to succeed at calculus; (b) purchase of technology, such as a graphing calculator or student copy of Maple, would be an additional financial burden on many of our students. (H10, USA)

The discussions in Section 2.3.4 and Section 3.4 described several possible models for integrated technology that may be considered against this taxonomy. Branson’s model (1990, see Figure 2.9) sees technology as a central, focal element of the curriculum, fulfilling a mediating role as an expert system between students and teachers. Several other descriptions evocative of the concept of integration were also discussed (see pp. 70-71), for example the concepts of “pedagogy-embedded technology” and CAS-intensive curricula proposed respectively by Dana-Picard and Kidron (2006) and Heid (2003). These general descriptions of integration support the consideration of technology over all aspects of the curriculum, as depicted in the taxonomy. Sub-categories within the Staff and Student Factors of the taxonomy also consider the alternative, more specific uses of the term “integration” described in Chapter Two (p. 73), to describe the specific integration of technology into their practice through the achievement of instrumental genesis by both staff and students (Hong & Thomas, 2006; Goos, 2006). However, while instrumental genesis for staff and students is viewed as a key challenge in the overall integration of technology in the classroom and the curriculum, it is too narrow a conceptualisation to serve as a definition of an ITMC (Hoyles, Noss & Kent, 2004).

Arnold’s model for a “technology-rich learning environment” (1998, see Figure 2.10) also has technology as a central focus, but provides a more detailed examination,
with the links in his model between students’ and teachers’ pedagogical and mathematical thinking closely matching the elements of the staff, student and mathematical factors represented in the taxonomy. Many survey respondents supported such an inclusive view, for example an Australian teacher who stated that:

I feel that the use of technology should be integrated into the main stream of many of the first year service units rather than being viewed as an optional extra…At the moment the version of the online quizzes bear little resemblance to the examination questions and cause problems due to errors in translating the answers to Maple. This is not to say that Maple, itself, is bad but students do not see the reason behind having to use Maple. (H6, Australia)

However, not all respondents supported such views, and any definition of an ITMC must account for these too, as the taxonomy does with the consideration of beliefs and attitudes within the Staff and Mathematical Factors components. For example, excerpts from responses cited in the taxonomy and earlier in section 5.3.1 warned against the imposition of technology (see G7, Australia, p. 206), and described limited use of technology only when there was no alternative (N1, USA, p. 195). Another respondent from the United Kingdom believes that we should just let the implementation of technology progress naturally, stating “let it roll on and find its own level without its introduction being forced.” Studies by Heid (2003) and Healy (2006) cited in Chapter Two (pp. 70-74) similarly warn against the forced “insertion” of technology, and the need to consider the cognitive and epistemological dimensions, as well as providing effective access to the technology. These add further weight to the wider scope of integration issues considered by the taxonomy.

While all the models discussed so far provide support for particular elements of the taxonomy, each has been shown to have certain inadequacies. Thomas and Holton’s (2003) extensive review of the use of technology in undergraduate mathematics was regarded earlier as a useful basis for an ITMC model, but it still requires an explicit description of what is meant by integrated technology (pp. 73-74). Kemp and Kissane’s (1995) proposal, that adequate integration of technology into undergraduate mathematics should consider all aspects of the course (i.e. the mathematical content, lectures, tutorials, workshops, assignments, tests and examinations), was described
earlier as a more inclusive possibility (p. 72), and there were many examples in the survey responses supporting this:

- Fully integrated technology in class/assessment/exam. Chosen by student. Part of learning is to use and choose appropriate tools. (H3, Australia)
- Tech should be available at all appropriate opportunities including in class test exams and homework. (H7, France)
- It should be available and used at all appropriate opportunities including in class, assignments, tests, exams, homework at least from year 2. (H14, New Zealand)
- Technology should be integrated into undergraduate courses wherever it assists understanding or allows for bigger &/or more complex data sets to be explored. (H15, Australia)

However, Kemp and Kissane’s (1995) model does still not consider some aspects, such as goals (see Table 2.3). Nor does it specifically reference organisational factors, or explicitly consider the role of beliefs and attitudes (see for example Figure 3.4). It was suggested earlier that Leigh-Lancaster’s (2000) concept of congruency between curriculum, pedagogy and assessment provides the best potential definition of integrated technology, although it too is seen by some studies as too narrow (e.g. Stewart, Thomas & Hannah, 2005), for example failing to explicitly consider such issues as student instrumentation (see e.g. pp. 68, 132). The wider use of Curriculum in this thesis also suggests some modification to Leigh-Lancaster’s description is required, given that assessment and pedagogy are both seen as curricular elements in the Valero-Duenas (2002) model adopted here, as opposed to competing elements as in his description.

The definition of integrated technology should therefore extend beyond the curricular components of assessment and pedagogy. Integration should involve a congruent approach across the wider curriculum, with attention, wherever possible, to all the elements identified in the taxonomy (Table 5.7). A failure to address any one component risks limiting the resultant level of technology integration. Not all elements will need equal attention in all cases, for example, some studies and survey responses describe the need for less attention to student factors for higher-level courses, as student instrumentation improves (e.g. Stewart, Thomas & Hannah, 2005, see p. 114). However, the instrumental genesis demonstrated by experienced students in such courses still suggests a level of congruency has been achieved, without the need for
specific consideration. Support for this concept of curricular congruency is provided in the extended explanations for many of the examples cited in the construction of the taxonomy. These earlier examples describe concerns about staff, student, mathematical, and organisational factors in support of the taxonomy components. The importance of considering a wider range of factors is further demonstrated in the following caveats to some respondents’ general support of integration:

Technology is mostly useful but one needs to design the course delivery and assessments carefully to make sure students develop critical thinking and do not rely on technology entirely. I like to think that technology helps students understand concepts as well as providing a computational tool. (H1, New Zealand)

My personal experience is that it is difficult to strike the right balance of using technology to aid student understanding and to avoid unnecessary hard algebraic work, but also being able to think and do mathematics without a heavy dependence on technology. (H13, New Zealand)

I consider it a valuable tool in the discovery and illustration of key concepts. My experience is that students view it initially as a convenient path to the right answer but, eventually, come to use it as a conceptual aid. Some of my colleagues feel it has a negative impact on students’ technical ability. (G7, Australia)

It should be noted that any such congruency should be positive, as it is also possible to achieve consistency across the elements of the taxonomy by completely ignoring the use of technology throughout. While consistent, this latter approach does not represent congruency with respect to integrated technology, but rather the reverse. Thus, with respect to the first research question that this thesis sought to address, this thesis proposes that an Integrated Technology Mathematics Curriculum (ITMC) is characterised by a consistently high level of attention to the six components, and their corresponding sub-characteristics, as identified in the taxonomy described in Table 5.7.
5.5 CHAPTER FIVE SUMMARY

This chapter has described the investigation of the first research question of this thesis, “What are the characteristics of a tertiary Integrated Technology Mathematics Curriculum (ITMC), and how might we measure the nature of such technology integration?” It has presented the results of successive surveys used to look at this question, and used these, along with factors identified in the literature, to progressively develop a taxonomy for an Integrated Technology Mathematics Curriculum. This taxonomy, summarised in Table 5.7, demonstrates the complexity of issues involved in integrating technology into the curriculum. This thesis proposes that effective integration of technology may be seen as achieving a consistently high level of congruency across all elements of this taxonomy. The taxonomy is provided as a basis for examining the level of technology integration within a specific course, a necessary starting point for studying the effects of such integration, and investigating the second research question, “How can we facilitate the effective and sustainable implementation of a tertiary Integrated Technology Mathematics Curriculum?”

While not an unequivocal condition, dissatisfaction with the use of technology was seen in several cases to correspond to a lack of attention or consideration to certain elements of the taxonomy, and this will be explored further in the next chapter, which examines the implementation of technology in one case study at The University of Auckland. The experiences from this observational study, along with selected interview transcripts, and responses to the third survey question with respect to barriers and difficulties encountered with technology integration, are compared against the taxonomy developed in this chapter, in order to investigate the second research question.
6.1 INTRODUCTION

Figure 4.2 depicts the observational study spanning the duration of this thesis, informed at successive stages by the surveys and interviews. This chapter will present and discuss the results of the observational schedule as depicted in Table 4.3, which identifies the particular elements to be examined in this case study. The experiences of the Mathematics Department at The University of Auckland, as documented through this schedule, are compared against the taxonomy developed in Chapter Five (Table 5.7), to address the second research question “How can we facilitate the effective and sustainable implementation of a tertiary Integrated Technology Mathematics Curriculum?” In particular, Assessment and Curriculum, identified as key elements of the taxonomy in Chapter Five, are more closely examined. This part of the study also includes selected commentary from the interviews, and examines the relative curriculum value of a specific topic, namely Gaussian Elimination, using the framework described in Figure 2.8 (Artigue, 2002; Stacey, 2003). The chapter concludes by summarising the key factors affecting the integration of technology, in order to inform and facilitate the technology implementation process.

6.2 TECHNOLOGY AT THE UNIVERSITY OF AUCKLAND: HISTORY AND DOCUMENTAL EVIDENCE

The main focus of the observational study is an examination of the use of technology in the Department’s core first-year mathematics course Maths 108, over the period 2001 to 2008. The study makes a distinction between pure and applied
mathematics courses, with less consideration given to technology use in applied courses. Previous discussions have also described technology use prior to the period detailed in Table 4.3, while the pilot studies earlier in the thesis examined technology use in courses other than Maths 108.

All first and second-year core courses teach a blend of Calculus and Linear Algebra topics. Although Maths 108 has always been the largest first-year course, it has not always been the principal entry-level course. Its design in earlier years (up to 2004) was largely as a service course with a focus on applications appropriate to Commerce, Biology, Computer Science and the Bachelor of Technology degree. Most students taking Maths 108 would not continue in mathematics, although a second-year paper Maths 208 does exist for students wishing to follow on. Changes to the course structure for the department in 2004 saw Maths 108 become the principal-entry course for most mathematics students. The two-course suite previously regarded as the main first-year continuing-mathematics courses (Maths 151 and Maths 152) were reconstituted as a first and second-level course for mathematics majors, now known as Maths 150 and Maths 250 respectively. While similar in content in many respects to Maths 108, Maths 150 has a much more abstract and proof-based focus, and Maths 250 is considerably more rigorous in this respect than Maths 208. Thus these discussions will also consider some aspects of technology use in other courses, especially in their relationship to Maths 108. Further course changes were made in 2006, when all undergraduate courses were reduced by 15%, to enable compulsory general-education courses to be included in all degrees. The Department of Mathematics saw this as an opportunity to reconsider the place of technology in our courses. Figure 6.1 presents a chronological summary of the organisational changes that occurred, and the relationship between these and the technology implementation.

Three distinct periods of technology use may be identified in the timeline in Figure 6.1: Pre-2001 when technology use was limited or banned in most courses; 2001-2005, when CAS-calculators were used in the core courses; and post-2005, when Matlab was adopted as the Department’s principal technology platform. This pattern is similar to those reported by survey respondents cited in Chapter Five, and it illustrates several elements of the ITMC taxonomy. The use of technology in a few individual courses prior to 2001 (e.g. graphics calculators in Maths 102), despite the otherwise limited use elsewhere, mirrors the responses recorded for H10 and N4 (see p. 199).
### Pre-1997
- Calculator use in all courses limited to Scientific-type.
- No use of any calculators in tests or exams for principal pure courses.
- Applied courses use Matlab, Maple for assignments, projects.
- Minimal use of Matlab and/or Maple by Maths 108 in assignments (matrix
  exam calcs only).

### 1996
- Graphics Calculator (TI-83 modelled in lectures) introduced in bridging
  Calculus course Maths 102. Permitted in tests and exams.
- GC (e.g. TI-83) use also allowed in first and second-year Statistics.
- Change to University calculator policy: Unrestricted Calculator (UC) to allow
  alpha-numeric and programmable.

### 2000
- Departmental Retreat: Technology not specifically included on agenda. Lack
  of consistency noted by working party on curriculum development.
- Relaxation of calculator rules. Use of graphics and scientific calculators
  permitted in exams for some courses.

### 2001
- CAS-Calculators adopted in Maths 108, 151. Permitted in tests and exams.
- TI-89 modelled in lectures, sold through department at discounted rate.
- Compulsory small-class tutorials introduced. Change from 5 hours
  lectures per week, to 4 hours + 1 hour tutorial.

### 2004
- On-line quizzes (Cecil) developed for Maths 108.
- June: Departmental forum to discuss policy on technology use. Working
  group established to formulate draft technology policy. Five meetings up to
  November 26.
- TI-92 removed from students by exam invigilators. University Policy revised
  to allow use.
- Changes to degree structure announced effective 2006. Technology to be
  considered in restructured courses.
- November Forum on Teaching: Draft Technology policy presented for
  discussion, 2006 restructuring process established.

### 2006
- February/March: Many teething problems. Matlab Tutorials established.
- Lecture recording with Tablet-PC trial.
- April: Review of Matlab in Maths 108, 150. Roll out to second-year courses
  recommended for semester two.
- August: E-lecturn training with Matlab for lecturers after problems
  encountered. Reminder that usage is now departmental policy.

### 2005
- February: Department meeting. Technology recommendations presented.
  Teams established to draft course changes, including technology.
- April to September: Email circulation of draft proposals and vigorous discussion.
- September: Departmental retreat. Technology Policy adopted. MuPad as
  computer-based CAS adopted as official technology, supported by CAS-calculator
  use. Lots of questions about features etc.
- October: Late change to Matlab, as MuPad no longer viable. Many
  organisational meetings. To be rolled out systematically over 2006 year, starting
  with first-year courses in semester one.

### 2007
- August: Departmental retreat. Review of technology use and
  policy, including Matlab and trial of tablet-lecture recordings.

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**Figure 6.1** Timeline of changes in technology use in the Department of Mathematics,
The University of Auckland (pre-2001 to 2008).
This isolated use illustrates the autonomy over personal course design given to teaching staff in many undergraduate departments, and is indicative of the *Organisational Factors* component of the taxonomy (see Figure 5.6). Other examples include the limited use of Matlab and Maple in Maths 108 at that time, mainly in response to service course client demands, and the problems encountered with the University’s examination calculator policy. The problems with the examination policy are a reflection of the growing power of hand-held technology, with the “Qwerty” keyboard of the TI-92 calculator responsible for its removal by exam invigilators on more than one occasion. The tension between departments such as mathematics and statistics, which desire open access to such technology, and other departments who require limits on dictionaries and the potential for stored data, has ultimately led to an unrestricted calculator category in the University’s examinations policy, with responsibility for policing left to individual course coordinators at examination time.

A close examination of the individual milestones on the timeline reveals a growing level of congruency in the use of technology between elements in the taxonomy over the observational period. Consider, for example, the *Organisational Factors* component, where the motivational effects of the University’s degree changes, and the departmental technology policy which mandates its consideration in all undergraduate courses, are both evident in current technology use. The earlier decision to adopt CAS-calculators in 2001 was reached by the Department’s core courses committee, and as such was not mandated by departmental policy. Thus decisions about their use in any particular course were ultimately left to course coordinators, and the impact of CAS calculators was subsequently mixed. The two core courses Maths 108 and Maths 151, along with the bridging mathematics course Maths 102 (which had previously used only graphics calculators), did implement CAS-calculator use, but this use never gathered sufficient momentum to impact much on higher-level courses. The applied mathematics courses which were already using computers saw no need for introducing the calculators.

By contrast, the technology policy adopted in September 2005 has had a more sustained and obvious impact on technology use. This policy states that “at least one computer algebra system (CAS) be appropriately integrated into the teaching of all stage one and two courses except the general education courses.” It also specifies particular CAS-software, and provides for the continued support of calculator-based CAS,
including acceptance in tests and examinations. Since 2006, most official departmental documents have reflected these technology imperatives. The undergraduate student handbook makes a specific statement about technology, which notes that while some courses may have restrictions on calculator use, all first and second-year courses are expected to make use of Matlab (general courses excepted). Further information about the computer laboratories and tutorial help is also provided in this document. Nearly all course descriptions on the Department’s webpage now include references to technology, with links to resources such as the Matlab Guide and computer laboratory information. All course Study Guides are required to include a statement about technology.

Statements from the study guides for the two main entry-level courses Maths 108 and Maths 150 (151 prior to 2004) for the three distinct periods of technology use are displayed in Table 6.1. These statements illustrate the effect of policy changes on the stance towards CAS-calculators. A departmental meeting in 2000 noted the lack of a policy, and the inconsistent approach to use of technology that existed within the department, as evidenced in the differences between the two study-guide statements for that period. There was considerable opposition from some staff to the introduction of CAS-calculators in 2001, particularly towards their use in examinations. However, a decision to adopt a consistent approach to the teaching of the two core offerings, Maths 108 and Maths 151, led to similar policies towards CAS-calculator use in both courses over the 2001 to 2005 period. With the adoption of Matlab as the principal technology in the 2006 policy, statements about technology in general for the two courses remain similarly explicit, although the policy now places less emphasis on calculators. This lesser role for calculators, especially in examinations, is evidenced in the 2006 study guide statement for Maths 150. Only Maths 102 and Maths 108 still provide explicit support for CAS-calculator use, for example in the link to the TI-89 resources on the Maths 108 site, although admittedly more limited than previously.

Other documents detailed in Table 4.3 demonstrate the effects of beliefs and other elements of the ITMC taxonomy on the process displayed in Figure 6.1. Assessment issues, course content from the Mathematical Factors component, and Access as it relates to congruency between assessment and pedagogy are considered separately in the next section. However, some Access factors such as equity and access to technology during class-time are considered here.
### Table 6.1 Technology Statements in Course Study Guides

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Maths 108</th>
<th>Maths 151/150</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-2001 (Before CAS-calculators)</td>
<td><strong>Calculators:</strong> Calculators may be used in the course. In examinations calculators are restricted to non-programmable non-graphical, non-alpha-numeric types.</td>
<td><strong>Calculators:</strong> Calculators will NOT be permitted in the final examination. A list of formulas will not be provided.</td>
</tr>
<tr>
<td>2001-2005 (CAS-calculators)</td>
<td><strong>CAS Calculators</strong> Calculators may be used in the course. In examinations there is no restriction on calculator type, provided that the memory is cleared before the exam starts. It is not compulsory for students to buy a graphics calculator. It will be possible to pass without one. However, we encourage all students to buy a graphics calculator and learn to use it. … Use of these calculators will mean that less time needs to be spent learning and practicing techniques.</td>
<td><strong>Calculator Usage</strong> The TI-89 is a personal calculator which is capable of abstract symbolic manipulation in both algebra and calculus as well as graphical and programming capacities. We have chosen to support this model specifically because it is capable of providing all the abstract manipulation required in courses through to Maths 253 and will enable us to use it as a teaching tool in lectures. It is also useful for Statistics courses. Lecturers will specifically use the features of the TI-89 to explain key mathematical ideas, and they can be used in the examination and test which permit “unrestricted calculators” as long as their memories are cleared.</td>
</tr>
<tr>
<td>2006 onwards (Matlab as principal technology, CAS-calculators permitted)</td>
<td><strong>Matlab</strong> The Department of Mathematics uses the software package Matlab for most undergraduate courses... The basics of Matlab are easy to understand and a valuable skill to gain. Matlab is an important part of this course; it is a component of most assessments, including the test and exam.</td>
<td><strong>Matlab and calculators:</strong> All mathematics courses have access to the programming package Matlab and we expect Maths 150 students to use Matlab in some assignments. Calculators, including programmable calculators like the Texas Instruments TI89, are allowed in tests and exams, with the proviso that their memories will be cleared before these. However full working must be shown in order to gain full marks in all written work.</td>
</tr>
<tr>
<td></td>
<td><strong>Calculators</strong> You are permitted to use a calculator in the Maths 108 test and examination. Any standard scientific calculator (e.g. Casio FX82) is sufficient, but many students prefer a programmable (CAS) calculator.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Quiz</strong> Fortnightly, online skills quizzes (on Cecil) alternate with the written assignments. They are worth 4% of your final grade.</td>
<td></td>
</tr>
</tbody>
</table>

Equity considerations for students, and costs to the department, were a major factor in the decision to opt for MuPad over Matlab or Maple as the CAS-technology of choice for all core courses in the initial technology policy in 2005 (applied courses
would continue to use Matlab), with the availability of a free alternative to the commercially available MuPad product called MuPad Lite. After extensive trials, the technology committee felt that this version satisfied all the requirements of the core courses, and this functionality was confirmed during extensive debate about MuPad’s capabilities at a departmental retreat in September that year. Minutes from the technology committee meetings at the time also acknowledge the relative simplicity of MuPad’s syntax and symbolic functionality as a major consideration, with the belief that MuPad would be simpler for students to learn than Matlab or Maple. There was however some concern expressed about the need for staff to learn another technology platform.

A significant organisational element is also demonstrated in the timeline with respect to the choice of technology. Following the demonstration of MuPad at the retreat, it was installed on the staff servers and in student laboratories, along with user-guides for staff. After the considerable time taken to select MuPad, a late change was then needed in October 2005, when MuPad Lite became unavailable. A rapid decision was made to adopt Matlab, with its Symbolic Math Toolbox, as the universal technology for the department. A significant element favouring the choice of Matlab at this late stage was its existing use by applied mathematics courses, which meant several organisational issues had already been addressed (e.g. licensing, installation in laboratories). The value of the symbolic toolbox, evaluated earlier by the technology committee, also influenced this decision. Considerable energy was then directed towards making quick changes to resources and access for staff and students, in time for the start of summer school in January 2006.

Another access issue arising from the decision to opt for computer-based CAS-technology was that, as lectures and tutorials continued to be held outside of computer laboratories, direct student access to the technology during classes was not possible unless they had their own laptops. This aspect was recognised in technology committee minutes as one reason for the continued acceptance of CAS-calculators. It is also recognisable in the technology policy, which regards staff demonstration of Matlab use in lectures as one of the minimum provisions of technology integration. Use in lectures was facilitated by the almost simultaneous but unrelated provision of computers and data display units (“e-Lecterns”) in most university lecture theatres. Concerns that the large numbers involved might swamp the computer laboratories led to a decision to
phase in the use of Matlab in courses gradually, starting with the core first-year courses in semester one 2006.

Email correspondence from the earlier period, when CAS-calculators were introduced, provides further insight into the part beliefs and research domains play in decisions about technology. As has been outlined previously (pp. 8-12, 146-147), the decision to adopt CAS-calculators in 2001 was a fairly rapid response to the enthusiasm of a small number of colleagues in the department for the pedagogical capabilities of the TI-89, and technology in general. A number of colleagues in the core courses remained unconvinced about the calculators’ worth, and there was considerable critical debate generated. Some criticisms were about calculators in general, others about specific features of the TI-89 model itself. These included the small screen-size, and doubts about the ability to compute large matrix calculations, or display direction fields. The following excerpt from an email response on the Department’s mailing list from one of the key supporters of the CAS-calculators illustrates the nature of this debate:

I checked out your immediate criticisms of the TI-89 and so far they don’t stack up:
1. Direction fields redraw in higher detail in blow-up so the resolution is essentially unlimited. Although the screen is small, you can move around and look at detail at any feature.
2. It also has a matrix editor…which makes for easier matrix input…the command line is still easier than Maple…
3. It inverted a 10 x 10 matrix with an unknown in it in less than a minute and can view the entire matrix on screen.

Of course the TI-89 is dinky, but this doesn’t mean it is a trivial toy.

The text of this email suggests that beliefs about calculators in particular are behind much of the criticisms, although these beliefs may be due to limitations of pedagogical technical knowledge (PTK, see p. 69). Since it was expected that applied courses would continue to use Matlab, some colleagues teaching applied mathematics in particular did not see the value in their introduction. The following challenge was issued in response to some concerns expressed about the calculators:

I invite anyone in applied math to provide five examples of problems in (their courses) which require a lab computer to do and which you think the TI-89 might not be able to solve satisfactorily.
One interview candidate [IR 2] gave the following response, which rather than displaying a limited awareness of PTK, supports the arguments for domain-specific technology in some, particularly applied, mathematics courses, and the beliefs about mathematical knowledge and technology typified by research interests in such domains:

We teach Matlab as an example of a high-level programming language – for future substantial programming the student will do in other courses. We also use Matlab as a language to specify algorithms – for example stochastic simulation – as if it were pseudo-code. We are not, fundamentally, teaching math – we are teaching modelling and programming – you might also say that the mathematics we teach is chosen to illustrate these topics.

However, it seems calculators are seen as essentially toys by some, with several colleagues at the technology forum held later in 2004 to consider future technology use expressing such opinions in their support for a computer-based technology policy. The persistent nature of beliefs against technology is also evident in the subtext to a concerned email sometime after the changes (in May 2008). A Physics’ colleague had overheard that Maths 150 may again become calculator-free. Such a consideration was never in fact contemplated by the department, and indeed is clearly against the current technology policy. However, the rumour does show a lingering belief against calculators by some staff, who voice such possibilities.

The timeline and study-guide excerpts reflect the growing attention to Staff and Student Factors from the start of the CAS-calculator introduction in 2001. Two resources prepared to help students with the TI-89 were added to the Maths 108 and Maths 150 course-books. Statements in study guides from this time forward signal the value of technology, and the importance the Department attaches to it. However, there was no specific tutorial support for students, except for highlighting the names of a few tutors able to assist with the calculators in the assistance room. While TI-89 calculators were provided to most teaching staff, this did not extend to tutors, and there was no explicit help for staff or tutors in learning to use the calculators. The PowerPoint slideshows for the Maths 108 course at that time provide no specific pointers to appropriate times for staff to demonstrate the use of the technology, so use in lectures depended on individual teacher’s experience. In semester one 2001, four of nine staff
responsible for Maths 108 made significant use of the TI-89 in lectures, three used it on occasion, and two did not use the calculator at all.

By contrast, the technology policy in 2005 emphasises the preparation of resources, including guides and manuals for students and staff, tutorials for students to help them learn to use Matlab, and support for staff with using Matlab in lectures. In-house guides and manuals for both Matlab and the TI-89 are available for students on the Maths 108 website (among others), and the course-book provides specific reference to Matlab code, and examples where it might be used, both for staff and students. There is a detailed guide for staff to access Matlab in lecture theatres, and training sessions have been organised to facilitate this use. The use of Matlab has moved primarily from its use to produce diagrams and graphs for printed material in course-books and lecture slides, to active modelling of the creation of these in lectures, although such use still varies between teachers. In semester one 2007, six of the eight staff made regular use of Matlab in lectures, and the dedicated lecture slides mean all staff now have to reference Matlab, even if they do not model its use as actively. However, the effect of the lesser emphasis on calculators is also evident. Compared to 2001, only two of eight staff in semester one 2007 used a calculator in lectures. One of these cited an informal “show-of-hands,” which revealed less than 20% of the 180 students with a calculator of any sort in the lecture, and less than half of these were CAS. This is despite students being allowed (and in some cases encouraged) to use calculators in all assessment.

6.3 THE TAXONOMY AND TECHNOLOGY IMPLEMENTATION

The previous section has presented several outcomes of technology implementation at The University of Auckland, and discussed how these may be interpreted with respect to the six components of ITMC taxonomy. Two of the taxonomy components, Assessment, and content issues from the Mathematical Factors, are next examined more closely over the observational period, focusing primarily on the core mathematics course Maths 108. Both these issues were selected for more in-depth inspection largely due to interesting responses from interview subjects, and the frequency of responses that demonstrated problems with respect to these issues in the surveys (see Appendix A3(iii)).
6.3.1 Assessment Issues: An Evaluation of Sample Examinations

The taxonomy in Table 5.7 identifies five main issues with respect to technology and assessment:

- Congruency between pedagogy and assessment;
- The nature of questions asked in formal assessments (e.g. tests, examinations);
- Fairness: Equity of access to, and experience with, the use of technology;
- The opportunity for alternative forms of assessment;
- Recognising different student solutions.

Evidence exists for each of these issues in all three periods depicted in the timeline. For example, Maths 102, which first introduced graphics calculators, has always allowed calculator use in all aspects of the course, and from 2001 onwards, the study guide statements shown in Table 6.1 encourage technology use in all areas of assessment. However, even during the height of the CAS-calculator period from 2001 to 2005, student ownership of CAS-calculators never exceeded 50% (often significantly less), which means that the intended congruency in final examinations was not realised for the majority of students. This result was one reason to support for the change to a computer-based technology policy for 2006, as explained in the interviews by one of the key proponents of CAS-calculators:

The aim of using the calculators was that they could be used across the board in lectures, assignments, tests and exams. The difficulty is that the entire medium is in rapid revolution, and we only had partial adoption, the number of students who had the TI-89’s was too small. The evolution of Matlab, with the symbolic toolbox, is cheaper for them to buy, and easily available in the labs, so lots more use it. [IR 1]

While the use of Matlab meant that students could no longer actively engage in a hands-on fashion with the technology in lectures or examinations, this was not actually a change for the majority of the students who did not possess calculators. The greater overall access to technology provided by the computer laboratories was seen by the above respondent as outweighing this particular drawback. The continued allowance by the technology policy of CAS-calculators in examinations also means that some students continued to enjoy any benefits they provide, although this has ethical considerations (see later). Non-availability in examinations is not necessarily seen as
incongruent in some courses, particularly in applied mathematics. One interview respondent’s explanation demonstrated the different ways technology is used by applied mathematics courses, with the emphasis much more on its value as a computational tool, as opposed to symbolic manipulation features, which are seen as less useful:

I don’t see that as a problem…we assess it [technology] in our assignments and tutorials [lab-based], and that’s a much more appropriate place…Maybe that’s where its different [from pure maths], it’s not there as a support for them. It’s different things we’re assessing, when we use technology in applied maths, it’s not for doing something they could equally well do in another way. The stuff we assess in exams is different material, or different aspects of the same material. [IR 2]

Maths 108 addresses the disadvantage of not having access to Matlab in examinations as best as possible. Matlab use is actively promoted in assignments, using investigations and questions that require Matlab to solve, and questions are included in tests and examinations that require students to interpret Matlab output. This requires students to engage with the technology, and helps signal that the use of technology is valued in the course. However, by necessity, the examination questions fall largely into two categories, those that directly test familiarity with Matlab use, for example recall-type questions of Matlab features, and static reproductions of Matlab output which students are asked to interpret. Two multiple-choice examples are provided in Figure 6.2 to demonstrate this, the first from the 2007 summer school test and the second from the 2007 semester two examination. The second question in Figure 6.2 does address two of the learning objectives commonly supported in the literature; it attaches value to the use of technology (pp. 104-110; 134-136), and it uses technology to provide inter-representational links and examine conceptual understanding (pp. 94-100; p. 115-117). There are several examples of similar questions in Maths 108 examinations that require students mathematically interpret Matlab output, and then apply this in solving problems. However, for most effective learning, the literature suggests that students should be actively engaged with the technology (pp. 91-92, 125). In addition, there is the danger that testing conceptual understanding at the same time as technological facility unnecessarily complicates the question, especially for weaker students (pp. 112-114). Two of the interview subjects involved with teaching Maths 108 see value in including questions such as those shown in Figure 6.2 in tests and examinations, but both admitted that it is less than ideal. One was optimistic that developments in cheaper
Chapter Six

An Observational Study of Technology Implementation

Laptop technology would soon see many more students with access to these in lectures, and potentially in examinations.

26. Which one of the following is a useful Matlab command for sketching an equation?
   (a) ezplot (b) drawit (c) plotit (d) ezdraw

39. The Matlab output tells us that \( f'(0.7967) = 0 \). Which one of the following is TRUE?
   (a) Since \( f(0.7967) = 0.5361 > 0 \), the function \( f \) has a relative minimum at \( x = 0.7967 \)
   (b) Since \( f'(0.7967) = 0 \), the function \( f \) is undefined at \( x = 0.7967 \)
   (c) Since \( f''(0.7967) = -1.4616 < 0 \), the function \( f \) has a relative maximum at \( x = 0.7967 \)
   (d) Since \( f'(0) > 0 \), the function \( f \) has a relative maximum at \( x = 0.7967 \)

Figure 6.2 Sample questions from Maths 108 formal assessment.

Maths 108 (and Maths 102) has also pursued alternative forms of assessment, introducing online quizzes which examine basic skills and count towards final grades. Students may repeat the quizzes, and are given feedback and suggestions for help in addressing any problems. Several smaller courses also conduct collaborative tutorials in the computer laboratories, where students are assessed using technology in a group environment. This is not a feasible option for the large Maths 108 course.

There is some evidence that alternative student solutions have been considered in the department, although not that widely. Maths 108 requires Matlab output for some
assignment questions, with 2% of the final grade specifically for Matlab worksheets. While the multiple-choice format it uses for much of the test and examination questions was principally introduced for practical reasons (e.g. ease of marking and security against cheating), this format also negates concerns about the need to recognise or accept alternative technology-inspired solutions. A Maths 102 mark-scheme from 2007 directs markers to accept correct solutions without working for many questions where technology may have been used to obtain the answer. However, there is evidence that solutions provided by students obtained using technology are discouraged, particularly in examinations. For example, the Maths 150 study guide from 2006 emphasises full working, even if technology is used, while one interview subject believes that more students are now giving “nonsense answers” in examinations based on CAS-output that shows little comprehension of either the problem or the solution.

The observational study suggests that questions asked in tests and examinations remain a significant issue in technology use: Are questions technology-active, -neutral, or -free; do the questions give unfair advantages for students with access to technology; and what are the implications of fewer instrumental and more conceptual questions? Such issues were of considerable concern when graphics calculators were introduced to Maths 102, as described in Kiernan, Oates and Thomas (1998). They provide several examples from early tests, where it was found that the use of graphics calculators trivialised the question by providing direct solutions. Other questions gave significant advantages to students with calculators, who were able to view graphs that greatly enhanced their ability to understand the problem. The authors describe how, largely in the interests of equity, questions were rephrased to neutralise such advantages as much as possible. This practice is maintained in Maths 102, as access to technology in examinations remains inequitably low.

While no formal analysis of Maths 108 questions has yet been conducted, minutes from technology-committee meetings in 2005, and statements from the interviews suggest these issues have been explored. For example, four interview subjects [IR1; IR 3; IR 4; IR 7] raised similar concerns about the nature of questions in a technological environment, although they differed in their responses. They all noted that CAS-availability in examinations trivialises many of the common “padding-type” instrumental questions. One respondent [IR 3] described how in recent years, they have endeavoured to assign such questions to the web-based quizzes, while trying to ensure
examination questions have a more conceptual basis. Another [IR 4] believes that routine skill-based questions are inappropriate in any case, so their removal due to technology would be a beneficial outcome. However, even with many routine questions removed, he still perceives that CAS-calculators offer an advantage, and he continues to emphasise them to students as essential in examinations.

Studies by Hong, Thomas, and Kiernan (2000) and Flynn and McCrae (2001) suggest a means of examining the nature of questions in Maths 108, and identifying any changes which may have occurred in response to technology developments over the three distinct periods of use. Flynn and McCrae compare three different classification schemes devised to assess the impact of CAS-calculators on examination questions. Here, impact is adapted from that defined in an earlier study (Jones & McCrae, 1996), to mean that “a CAS-user would have access to a more efficient solution strategy than a graphics calculator user, … not just a broader number of possible solution strategies” (Flynn & McCrae, 2001, p. 210). All three classifications exhibit some of the factors described in the taxonomy that affect students’ effective use of the calculators. The scheme by Kutzler (2000, in Flynn & McCrae, 2001, p. 211), for example, identifies three categories of questions: Primary (CAS-use is the major activity); Secondary (CAS-use plays a minor role), and No CAS use (CAS is of no assistance). The scheme further differentiates within the first two categories, between questions which require superficial knowledge of the tool, and those that require sophisticated knowledge. However, Flynn and McCrae found all these schemes problematic to use, especially when differentiating between questions that require both conceptual understanding and algebraic manipulation:

…with any classification scheme, there is no clear-cut dividing line between the categories, because the reality is continuous, not discrete. Hence, for some exam questions it may appear arbitrary to put them in one or the other category.

(Kokol-Voljc, 2000, in Flynn & McCrae, 2001, p. 212)

Hong, Thomas and Kiernan (2000) use a simpler classification, which distinguishes between calculator-positive and calculator-neutral questions, depending on whether the calculator provides any perceived advantage in answering the question. This assessment does not require a judgement to be made about the effectiveness of the strategy, or the level of advantage it offers. Given the problems Flynn and McCrae
(2001) encountered with the more elaborate schemes, and the fact that this examination of Maths 108 questions is concerned mainly with whether CAS-availability has had any effect on the nature of questions, as opposed to the degree of that advantage, this study uses the simpler classification suggested by Hong, Thomas and Kiernan.

Three sample semester one Maths 108 examinations were selected for inspection, one for each distinct period of technology use (1999, 2004 and 2007). Some elements of the comparison are not directly equivalent, for example the examination changed from three to two hours in 2006, multiple-choice questions were not introduced until 2004, and there were changes to content in 2006, such as moving eigenvalues and eigenvectors to the follow-on second-level course. The comparisons are therefore made using percentages of the total marks for which CAS was seen as advantageous. The consideration of whether CAS should provide an advantage was done largely from the perspective of the TI-89 used most often by students in this course. However, students are not limited in their choice of CAS; they can use higher-powered calculators such as the TI-92, which provides an additional advantage for some questions.

The analysis was checked by a colleague who has taught Maths 108 and is familiar with the TI-89 and the TI-92. Unlike the studies by Jones and McCrae (1996) and Flynn and McCrae (2001), providing an “advantage” was not limited necessarily to a more effective strategy, for example it includes here the checking of solutions, since this is a particularly useful strategy for answering multi-choice questions. For several long answer questions, it was difficult to decide the exact proportion of marks, as some parts were aided by CAS, and others were not. Where a consensus could not be reached, the questions were left as undecided. The results are summarised in Table 6.2, which demonstrates a dramatic drop in CAS-positive questions from the short answer section in 1999, to the multiple choice section in 2004. There remains a large difference from 1999, even with the subsequent increase in 2007. This clearly reflects a response to the use of CAS-technology, with a move away from the mostly instrumental skills-based questions that are directly solvable using CAS, to more conceptually based CAS-neutral questions, as demonstrated in the comparison between the first two examples shown later in Table 6.3. The drop in questions favoured by the TI-92 in 2007 seen in Table 6.2 is largely due to the removal of skills-based questions on eigenvalues and eigenvectors. This pattern, also reflected in the long answer questions, suggests that the
advantages afforded by the higher-powered CAS technologies, such as those described by Flynn (2003), may be of more concern in advanced courses.

**Table 6.2 Comparison of Maths 108 Examination Questions from 1999 to 2007**

<table>
<thead>
<tr>
<th>Year</th>
<th>Section of Exam: marks/total</th>
<th>Percentage of CAS-Positive Marks in this section.</th>
<th>TI-89</th>
<th>TI-92</th>
<th>Undecided</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999 (Before CAS-calculators)</td>
<td>Short Answers: 30/100</td>
<td>80</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Long Answers: 70/100</td>
<td>33</td>
<td>39</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2004 (CAS-calculators)</td>
<td>Multiple Choice: 54/180</td>
<td>26</td>
<td>37</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Long Answers: 126/180</td>
<td>42</td>
<td>56</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>2007 (Matlab as principal technology, CAS-calculators allowed)</td>
<td>Multiple Choice: 40/120</td>
<td>35</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>Long Answers: 80/120</td>
<td>40</td>
<td>0</td>
<td>0</td>
<td>8</td>
</tr>
</tbody>
</table>

The questions in the selected examination papers in Table 6.3 also demonstrate a change in the nature of technological advantages provided, with fewer multiple-choice questions in later years being directly solvable using CAS. This is hardly surprising given that not even scientific or graphics calculators were permitted in 1999. Consequently, there was no technological advantage, so no attention to this was necessary. The majority of the CAS-positive questions in latter years resemble the third and fourth examples in Table 6.3, which evoke a congruency between pedagogy and assessment. While CAS does provide an advantage, these questions require a level of competency with the use of the technology (instrumental genesis), combined with mathematical knowledge and conceptual understanding, in order for this advantage to be realised. A reduction in skills based questions, and greater attention to conceptual understanding, was confirmed as an explicit objective of the Maths 108 teaching team by one of the interview subjects [IR 3].

Example 5 in Table 6.3 illustrates the sophisticated CAS-knowledge required in some questions to realise an advantage (Flynn & McCrae, 2001). While the TI-89 will provide a solution, considerable work is needed by the student to recognise the solution as one of those presented in the choices provided. Example 6 is a question for which
CAS will provide a solution, but a competent student could solve this question much more efficiently without it. Such a question would not be considered CAS-positive using the scheme of Flynn and McCrae (2001). Example 7 shows that despite the big drop in numbers of such questions from 1999 to 2004, some questions are still trivial using CAS. These increased again from 2004 to 2007, although not to the 1999 levels, suggesting that less consideration has been given to this factor with the reduced emphasis on calculators in the Matlab period.

Table 6.3 Sample Examination Questions from Maths 108 (1999 to 2007)

<table>
<thead>
<tr>
<th>Example and Year</th>
<th>Question</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example 1: 1999</td>
<td>If ( f(x) = \frac{2x+5}{3x+1} ), find ( f'(x) )</td>
</tr>
<tr>
<td>Example 2: 2007</td>
<td>When differentiating the following functions, for which is the Chain Rule useful?</td>
</tr>
<tr>
<td></td>
<td>(a) ( f_1(x) = \tan x \cdot \ln x )</td>
</tr>
<tr>
<td></td>
<td>(c) ( f_3(x) = \tan x(\ln x) )</td>
</tr>
<tr>
<td></td>
<td>(b) ( f_2(x) = \frac{\tan x}{\ln x} )</td>
</tr>
<tr>
<td></td>
<td>(d) ( f_4(x) = e^x \tan x )</td>
</tr>
<tr>
<td>Example 3: 2007</td>
<td>Suppose it is known that ( \int f(x)dx = e^x + C ). Then ( \int f(x-1)dx = )</td>
</tr>
<tr>
<td></td>
<td>(a) ( e^x + C )</td>
</tr>
<tr>
<td></td>
<td>(b) ( e^{x-1} + C )</td>
</tr>
<tr>
<td></td>
<td>(c) ( e^x -1 + C )</td>
</tr>
<tr>
<td></td>
<td>(d) ( e^x(x-1)+C )</td>
</tr>
<tr>
<td>Example 4: 2004</td>
<td>The function ( f ), where ( f(x) = \ln(\ln(x)) ) has domain:</td>
</tr>
<tr>
<td></td>
<td>(a) ((0,1))</td>
</tr>
<tr>
<td></td>
<td>(b) ((-\infty,0))</td>
</tr>
<tr>
<td></td>
<td>(c) ((0,\infty))</td>
</tr>
<tr>
<td></td>
<td>(d) ((1,\infty))</td>
</tr>
<tr>
<td>Example 5: 2004</td>
<td>Given that ( x ) and ( y ) satisfy the equation ( x^2 - y^2 = 2xy+1 ). One takes differentials. Which of the following is true?</td>
</tr>
<tr>
<td></td>
<td>(a) The result is ( 2x-2y(dx+dy) = 2 )</td>
</tr>
<tr>
<td></td>
<td>(b) It is not possible to take differentials in this case.</td>
</tr>
<tr>
<td></td>
<td>(c) The result is ( 2xdx-2ydy=2ydx+2xdy ).</td>
</tr>
<tr>
<td></td>
<td>(d) The result is ( 2dx-2dy=2dxdy ).</td>
</tr>
<tr>
<td>Example 6: 2007</td>
<td>Let ( A = \begin{bmatrix} 1 &amp; 3 \ 4 &amp; 2 \end{bmatrix} ) and ( C = \begin{bmatrix} 4 &amp; 4 \ 6 &amp; 6 \end{bmatrix} ). If ( AB = C ), then which of the following represents the matrix ( B )?</td>
</tr>
<tr>
<td></td>
<td>(a) ( \begin{bmatrix} 4 &amp; 4 \ 0 &amp; 0 \end{bmatrix} )</td>
</tr>
<tr>
<td></td>
<td>(b) ( \begin{bmatrix} 1 &amp; 0 \ 0 &amp; 1 \end{bmatrix} )</td>
</tr>
<tr>
<td></td>
<td>(c) ( \begin{bmatrix} -2 &amp; -2 \ 2 &amp; 2 \end{bmatrix} )</td>
</tr>
<tr>
<td></td>
<td>(d) ( \begin{bmatrix} 1 &amp; 1 \ 1 &amp; 1 \end{bmatrix} )</td>
</tr>
<tr>
<td>Example 7: 2007</td>
<td>( \int 12x^3 , dx = )</td>
</tr>
<tr>
<td></td>
<td>(a) ( 3x^2 + C )</td>
</tr>
<tr>
<td></td>
<td>(b) ( 4x^2 + C )</td>
</tr>
<tr>
<td></td>
<td>(c) ( 3x^4 + C )</td>
</tr>
<tr>
<td></td>
<td>(d) ( 4x^4 + C )</td>
</tr>
</tbody>
</table>
Unlike the short-answer sections, the figures for the long answer questions in Table 6.2 show a small increase in CAS-positive questions, which is again consistent with considerations of congruency and the objectives of the Maths 108 teaching team. While this seems a reasoned response to the introduction of CAS-calculators in the 2001-2005 period, the low numbers of students with access to them raises concerns of fairness, and this is exacerbated in the new post-2005 Matlab era, since the percentage of CAS-positive questions has remained relatively constant, while the numbers of students with calculators has declined. Like the multiple-choice questions, the majority of these require a level of conceptual understanding in order to realise the technological advantages, but they do still provide an advantage to students with CAS-access. For example, a question in the 2007 examination required students to find the derivatives of three separate functions, all of which were directly solvable using CAS, with minimal competency. Given that the low proportion of students accessing the CAS-calculators was an important factor in the decision to change, the fact that examination questions continue to provide advantages for a minority of students should be of concern.

The long-answer questions which proved most difficult to categorise involved a combination of conceptual and technological objectives. Consider the example from the 2004 exam shown in Figure 6.3:

29.

The matrices $A = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 3 & -1 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 1 & 1 \\ -3 & 3 & 2 \\ 2 & -1 & -1 \end{bmatrix}$ are inverses of each other.

Use this fact to solve the following system of linear equations:

\[-x + y + z = 4 \]
\[-3x + 3y + 2z = 4 \]
\[2x - y - z = 1 \]

**Figure 6.3** Sample question from the 2004 Maths 108 examination.

Exactly how many of the five marks allocated to the above question could be considered CAS-positive could not be agreed upon, even after seeking a third opinion (another colleague in addition to the researcher and the colleague who originally checked the analysis). While all three colleagues agreed that CAS provides a definite advantage in this question, even after assigning a step-by step marking schedule for the question, it was not obvious to what extent students were afforded this advantage. One
felt that the main advantage lay in the ability to check the answers at each step, another felt that the question required such a depth of understanding to know what to do, that the advantage gained in using the technology was largely instrumental.

The examination of Assessment issues from the taxonomy conducted in this section demonstrates that the impact of CAS on examination questions in particular remains a complex issue. Questions require real care and attention to balance the examination of students’ skills and conceptual understanding in a fair and appropriate manner. Those responsible for setting examination questions should look more closely at the questions they ask. Such an inspection should also include a wider perspective than the simple CAS-positive measure used here. It should for example consider issues of equity associated with differences in the affordances provided between different forms of CAS, as certain CAS-products have been shown to provide significant benefits over others for some questions (Flynn, 2003). Even given the generally narrow technology choices for Maths 108 students (Matlab and predominantly TI-89 CAS-calculators), this issue is apparent in some examination questions (e.g. the TI-92 for eigenvalues).

6.3.2 Consideration of the Value of a Curriculum Topic

There are several examples of the effect of technology on course syllabi over the observational period. The recommendations of the technology committee at the February meeting in 2005 (Figure 6.1) identified four content issues, largely derived from the literature (see page 56), to be considered when restructuring courses. As a result, most course syllabi now include specific statements about technology in the descriptions for each topic, for example the following excerpt for the lectures on “Solving Systems of Linear Equations” from the revised 2006 Maths 108 Linear Algebra Syllabus:

Comments on Delivery: It is extremely important that students firmly know which matrix is in the reduced echelon form and which is not. They have to see lots of them.

Technology Exercises: Students have to learn how input matrices of systems of linear equations…They will have to learn how to interpret the output of the command linsolve (or similar) to find out if a system is consistent, if it has a unique solution or not and find that unique solution if it exists.
Such an explicit statement in the syllabus reinforces the value attached to technology and directs teachers to the use expected of it. The statement also reflects one of the major benefits of technology identified in the literature (presenting students with the opportunity for repeated exposure to multiple dynamic representations, see pp. 61-63), and it demonstrates the consideration given to the appropriate use of technology (conducting routine algorithmic processes).

There are also a few examples of the influence of technology on changes to topics. In a response to the need to reduce course material, a discussion document at the 2005 Department Retreat (see Figure 6.1) noted that the teaching of methods of integration had been cut from the Maths 250 syllabus, but this would be partly compensated for by the use of MuPad. The proposal for the revised lectures in “Differentiation” in the Maths 108 Calculus Syllabus in 2005 considered the use of technology to assist in the required reduction in lectures, noting that:

Curve sketching will be relegated to tutorials and exercises, supported with CAS.

In using CAS for curve sketching, it is particularly important to emphasise the need to consider the critical points to get an idea of what interval of the domain to graph the function over.

Chapter Two discussed potential changes to the value of a particular topic in a technological environment (see Figure 2.8), and also considered how a mathematicians’ beliefs may affect their views on this (pp. 52-59). The complexity of assessing any given topic’s value is demonstrated in a series of email exchanges in 2003 that were a forerunner to the course changes ultimately enacted in 2005. This exchange, which began as a response to a query from one colleague about students working together on assignments, sparked a passionate debate about the value of a particular technique and beliefs about the nature of mathematics, teaching and technology. The topic was an exercise on $LU$ factoring (to solve $Ax=b$):

1. (It’s a) bad idea to teach this obsolete, tedious $LU$ factorisation, which no one needs anymore…(while) it still has some applicability, … currently no one client department needs it…we should give preference to teaching ideas, not techniques.
2. The issues divide neatly between the importance of the technique and the extent to which it should be laboured as a teaching item. The topic is intrinsically important because it is at the centre of all practical scientific computation…Is it important as a teaching item? Sometimes a mathematical concept has to be introduced without a
directly practical application…I personally think that the opportunity to introduce $LU$ with pivoting is like finding a flower in the desert.

3. This flower unfortunately has to be uprooted, together with integration by trigonometric substitution and other techniques which have lost relevance for the wider audience. A regular person with regular needs will be much better off using Maple. Such techniques should be taught in specialised courses…It takes too much precious time which can be better spent on building understanding.

4. I believe that a good portion of (any) honest technique is useful for students learning mathematics, as a training of ability for prolonged logical concentration. Separating learning of ideas from learning of adequate technical support looks similar to learning by heart a French song without learning French language.

Questions about the pragmatic and epistemic value of $LU$ factorisation are clearly evident in this debate (see pp. 53-59). Some see little pragmatic value remaining in this topic, and are happy to see the responsibility assumed by technology, others believe strongly in the epistemic value. Several studies consider ways in which technology can be used to resolve such conflicting arguments (see p. 116-117). Hillel (2001b) illustrates this with an activity using CAS to perform elementary row reductions on the coefficient matrix of a relatively large homogeneous system with non-trivial solutions. The purpose of the activity is clarification of theory covered in class:

Grasping the relation between elementary row operations and equivalent systems is the key notion, not the actual procedure for row-reducing matrices. Once understood, I see no reason why students should not be given free rein to use CAS and go directly to row reduced echelon form of a matrix without actually performing the row operations (and later, to go directly from a system of equations to a CAS-generated solution). (Hillel, 2001b, p. 374)

LaTorre (1993, p. 306) describes a similar process using CAS to perform all the individual steps of solving a system of linear equations, observing that in this manner, students “control the whole process from start to finish by deciding when and where to pivot, which row interchanges to make, and then effect back substitution interactively. The calculators carry only the computational burden”. Both these examples suggest a reduction in the pragmatic value of Gaussian elimination with technology, but both clearly believe that the procedure retains its epistemic value in aiding students’ understanding. However, as witnessed in the email debate and the examples presented
by Stacey (2003), deciding exactly which elements of a particular topic may retain their pragmatic or epistemic value and which may be assigned to technology is a difficult task on which to achieve consensus. This was confirmed in the interviews, when colleagues were asked for examples of any topics for which they perceived technology had changed its value, as well as their specific beliefs about Gaussian elimination in this respect.

Many of the examples suggested in interviews were similar to those found in the surveys and the literature, for example the developments in Chaos Theory. However, questioning the relative value of specific topics revealed some new perspectives. One interview respondent who teaches applied mathematics suggested that the removal of techniques of solving differential equations from the syllabus is a direct result of technology changing the pragmatic value of a topic:

Before computers, there used to be a big emphasis on special techniques for special differential equations, …students had to recognise some 15 different types of differential equation, you had no options, you had to solve it explicitly, there was no numerical option. You had to know the technique, all that’s gone, if you don’t recognise a differential equation, you whack it on a computer. [IR 2]

The value of technology is seen here not just in its computational capabilities, but also in the opportunity it allows to investigate problems and develop flexible solving strategies, as opposed to learning a catalogue of instrumental techniques. While the transition from learning techniques to using technology to investigate and calculate solutions was a comparatively natural and uncontroversial progression at Auckland, this colleague noted that such an approach is by no means universal. Indeed, many textbooks still include specific techniques as an important part of a Differential Equations course, and these courses are still taught traditionally in many places.

Another respondent considered integration by substitution as a similarly specific technique with little pragmatic or epistemic value in a technological environment:

Most students can barely see how it fits, but they get used to a standard technique of putting the things in, … the question mathematicians need to answer now is, do such mechanistic techniques generalise to more general problem-solving type situations later which are going to be useful?, and I think the answer is no. [IR 4]
The effect of technology on calculating eigenvalues and eigenvectors is also seen as less clear by the applied mathematics respondent [IR 2]. While some pragmatic value has been lost with the ability of technology to compute these directly, they still perceive some pragmatic and epistemic value in teaching these procedures. Matlab does not easily find families of eigenvectors, and often presents the results of complex eigenvectors in an unusual structure that requires considerable understanding from students to become recognisable. Hence while students are encouraged to use technology for such calculations in tutorials and assignments, they are still taught and examined on the manual procedures.

The five interview subjects who commented specifically on Gaussian elimination [IR 1; IR 3; IR 4; IR 5; IR 7] also varied considerably in their views, and for some, these were intricately linked to their beliefs about the teaching of mathematics. All five reflected similar views to the examples above (LaTorre, 1993; Hillel, 2001b), believing there is a continued epistemic value in students going through the process, especially in understanding the nature of solutions and the effects of transformations on the rows and columns of a matrix. All agreed that there is significant reduction in the pragmatic value, although one respondent [IR 4] believes that some pragmatic value is retained when the system has none or infinite solutions, as many CAS platforms do not distinguish these. While most respondents appeared very assured of their views in this respect, one colleague [IR 5] changed his mind several times as he thought through the issue, finally settling with reasonable certainty on this position. Initially he was quick to discount any pragmatic or epistemic value in teaching the procedure itself, comfortable that students could use technology to directly calculate row-reduced matrices, using an analogy of driving a car without knowing how it works. The technological “black-box that provides a solution to a set of linear equations is probably all most people need”. However, he later reconsidered this, and decided that students really do need to understand the process first, before using the black-box. He also noted that he, like many colleagues and students, enjoys Gaussian elimination as a process, “there’s an intrinsic enjoyment that makes it worth doing” [IR 5].

However, there was also some agreement among all interview subjects that technology was not always of benefit, especially for weaker students for whom learning is often reduced to a mechanical process. They believed that for such students, technology may exacerbate the problem as they become reliant on it, or be of little use if
they do not know when its use is appropriate. One interview subject gave an example from his marking of Maths 108 examinations, observing with respect to Gaussian elimination that:

The main problem doesn’t seem to be that they can’t do the operations (for which the calculator can help them); it’s that they don’t know what operations to do. They’ll do three pages of working and still won’t have any zeros in their matrix…the students don’t understand what the goal is. I’m not sure how technology can help with this. [IR 3]

In this respect, all the interview subjects revealed a greater emphasis on the technical and computational capabilities of the technology, and a lesser awareness of the pedagogical value, although one colleague [IR 1] did see great educational value in the symbolic manipulation features of the TI-89 and Matlab's symbolic toolbox. Two interview respondents [IR 1; IR 4] took a slightly different perspective, believing that the problem goes beyond deciding on whether teaching Gaussian elimination is worthwhile. They believe that the teaching of mathematics has become too focused on mechanical and routine procedures, and we first need to redefine our goals of teaching mathematics, before we can answer the question about Gaussian elimination. One of these respondents observed that:

When you talk about epistemological and pragmatic, there’s another kind of question I think mathematics is really about, and that is training the human mind. If you take Gaussian elimination, there is an argument for teaching this, since if students don’t master (such topics), they don’t even develop mathematical structures that are relevant for making mathematical assessments about any other problems they meet. [IR 1]

However, while sharing a similar perspective, these colleagues differed on the implications for technology arising from their views. The colleague quoted above did not see this position as contradicting the use of technology; indeed he views the use of technology in achieving this goal as an essential and equally relevant element of modern, professional mathematics. The other colleague [IR 4] believes that as long as we insist on teaching techniques and routines, calculators do indeed provide a significant technical advantage. However, he sees little educational value in technology
for the more important role of developing critical thinking and effective problem-solving strategies:

Depends what one wants, I can’t see how a student can understand the process by pushing a button, it may be OK for an engineer who just needs a seriously good program to provide the numbers at the end, and you don’t need to know anything about Gaussian elimination, … but most students haven’t got a clue what their answer means, they know nothing more about their solutions than they are a result of what they do. [IR 4]

He sees little pragmatic value for Gaussian elimination in such an environment, regardless of whether technology is used. Students who have learnt to row-reduce flawlessly without technology are frequently no better off than those who perform the operations using a calculator, so technology is seen as having no significant learning advantage other than to save time, check on working or avoid arithmetic errors. Notwithstanding the importance attributed by Stacey (2003) to examining the relative value of topics in a CAS-environment (see p. 59), the disparity of views between the five interview subjects for just one topic demonstrates the difficulty faced in making decisions about any given topic.

This section has shown that, as for Assessment, elements of the Mathematical Factors component of the taxonomy played a critical part in the technology implementation at The University of Auckland. In particular, departmental policy led to explicit statements about technology in course syllabi, and a consideration of content issues. As emphasised in the literature, staff beliefs have been seen to affect considerations of content value (see Figure 3.4), and decisions about the relative value of particular topics are seen as complex and difficult subjects on which to achieve consensus. The examples have also illustrated the need to reconsider the goals of mathematics education in a technological environment (pp. 46-51).

### 6.4 DEVELOPING AN INTEGRATED TECHNOLOGY MATHEMATICS CURRICULUM

A quote from Oates and Thomas (2001) in Chapter Four (see p. 146) cautioned that implementing technology without due consideration may be risky. While the team responsible for implementing CAS-calculators was aware of some of the factors
described in Figure 5.6, the actual implementation was some way short of what they considered optimal in addressing these issues. This was largely due to the desire to get the technology into the course in the face of opposition from some colleagues (Oates & Thomas, 2001, p. 82). The comparative success of the later Matlab initiative suggests that while such experiments may not be totally effective, they may well be a necessary part of the progressive development of technology, as described by many studies in the literature (see pp. 129-133). Responses to the third survey also support the evolutionary nature of technology implementation, for example, one American respondent who noted that he “has worked steadily at bringing technology into all of my courses...not very successfully in the early days, but much more so with time”, and several comments reflected the findings in the literature, that staff and students need time to adjust to technology (see pp. 75, 80-81, and Appendix A4).

While this has yet to be confirmed from the students’ perspective, most colleagues, including all five interview subjects, from The University of Auckland believe that the adoption of Matlab has been generally successful from the department’s perspective. Indeed, one colleague commented they were surprised at how smooth and unexceptional the final transition has been, while one interview respondent [IR 2] commented that many of their international colleagues were envious of the current position in our department. The increase in integration levels is evidenced in the contrast between the CAS-calculator period and the later period of Matlab use, as depicted in Figure 6.4:

![Figure 6.4: A comparison of technology integration in the Department of Mathematics at The University of Auckland between the periods 2001-2005, and post 2005.](image-url)
Figure 6.4 supports the general impression of success for the most recent implementation. Radial values for this comparison were estimated in similar fashion to the system previously described (see pp. 183-187; 207). The interview respondent from applied mathematics [IR 2] attributes some of the resultant success to past experiences with progressively implementing Matlab in applied mathematics courses, which provided a lot of guidance for the final roll-out across all courses in the department. Many potential difficulties with installation, licensing, demonstrator training, and providing access to staff in lecture rooms and on personal computers had already been resolved. However, the indication from the preceding discussions in this chapter is that attention to many other factors from the taxonomy has also contributed to the comparative success of the latest Matlab initiative. Despite the general feeling of success, the evidence suggests that effective, sustained integration requires more than just time and experience. Taxonomy components identified as significantly affecting the Matlab implementation in the discussions so far are summarised in Table 6.4:

<table>
<thead>
<tr>
<th>Taxonomy Component</th>
<th>Elements Identified as Affecting Technology Implementation in the Auckland Observation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Organisational Factors</strong></td>
<td>Planning; Policy; Infrastructure; Syllabi.</td>
</tr>
<tr>
<td><strong>Mathematical Factors</strong></td>
<td>Content; Subject imperatives; Research domain effects.</td>
</tr>
<tr>
<td><strong>Access</strong></td>
<td>Cost; Easy availability in computer laboratories; Value of technology explicitly signalled to staff and students.</td>
</tr>
<tr>
<td><strong>Assessment</strong></td>
<td>Changes to questions – technology free versus technology positive; Equity issues associated with access in examinations; Alternative assessment practices; Accepting different student solutions.</td>
</tr>
<tr>
<td><strong>Staff Factors</strong></td>
<td>Teacher Privileging; Staff beliefs about mathematics, teaching and technology; PTK; Resource preparation.</td>
</tr>
<tr>
<td><strong>Student Factors</strong></td>
<td>Training and Assistance: Tutorials, Manuals; Personal ownership; Equity.</td>
</tr>
</tbody>
</table>

The evidence suggests that there are intricate links between the elements of the taxonomy. This supports the conclusion from Chapter Five, that an effective ITMC
requires an integrated, consistent approach, which recognises the interdependence between the elements in the taxonomy. For example, while low levels of student access to CAS-calculators during the 2001 to 2005 period was cited as an important factor in the decision to change to Matlab, it seems that lack of support for the initiative may have contributed to the less than satisfactory access. Ownership rates at the start of the CAS-calculator introduction reached almost 50% (viewed as still too low by interview respondents), but there was a steady decline in the figures for student purchases from 2002 onwards (230 calculators were sold to students in 2002, but this had dropped to 105 in 2005, when sales through the department virtually ceased). Staff use over the same period showed a similar decline. It is possible that greater interest in the calculators may have been generated if other factors supporting the later Matlab initiative had been similarly addressed for the calculators. These include greater consistency in technology use between courses and staff within a particular course, more assistance for staff and students (e.g. resources and tutorials), and greater modelling of the technology to signal its value by staff.

Comments from interviews support this premise, with three Maths 108 lecturers describing a variety of reasons for their non-use of calculators [IR 3; IR 4; IR 5]. All three described an ignorance of the use of CAS-calculators in general, despite being shown some of the capabilities in demonstrations, suggesting a lack of both pedagogical technical knowledge and functional competency. It was difficult to demonstrate the calculator effectively in lectures prior to the e-lecterns being installed, since the system of placing a view-screen placed on an OHP is cumbersome, and while it displays the graphics screen, it does not enable the use of the calculator functions to be modelled. Linking the calculator use to lecture notes was also an obstacle, since connecting the TI-89 to a computer is not straightforward (this has been improved greatly with the TI-NSpire). One colleague [IR 4] noted that using the CAS-calculator in lectures seemed a bit pointless, since so few students had them, which makes it unclear which comes first, while another [IR 3] noted that it depended partly on the lecture stream (time) you were assigned to, none of the lecturers in his stream used it, while all those in another did.

Some of these issues were not confined to the calculators. An applied mathematician [IR 2] described a similar organisational obstacle when they wanted to demonstrate Matlab in large lectures prior to 2006. They had to carry a data-projector and lap-top with her to lectures, often in remote parts of the university, and this proved
too discouraging for many lecturers to attempt. While this particular issue has not entirely disappeared, for example one lecturer [IR 4] described how accessing Matlab on some e-lecterns is somewhat convoluted and this has limited his personal efforts to use it, most current Maths 108 lecturers describe the use of Matlab in lectures as a straightforward exercise. The level of planning and the establishment of a technology policy have been especially critical to the comparative success of the Matlab initiative, and these two factors seem to have had a disproportionate impact on many other elements of the taxonomy. Certainly the difference in planning between the CAS-calculators and the Matlab initiatives is marked. The use of CAS-calculators was first raised in late 2000, and meetings were still being held as late as February 2001 to discuss the implementation process in time for the first-semesters’ teaching in March that year. By contrast, the discussions to reconsider technology use which ultimately led to Matlab as the technology of choice included the establishment of the technology committee in 2004, and a considerable amount of planning continued through 2005, before the first wider-departmental use of Matlab in 2006 (see Figure 6.1). The planning even extended to a progressive roll-out to 2006, to allow time to assess the success and make changes in response to any difficulties as described earlier.

The establishment of a departmental technology policy has had an obvious effect on the Matlab initiative, and was clearly influential in many other components of the taxonomy. Previous discussions have described for example how it has impacted on course descriptions, study guides, resource preparation, and staff usage. Two interview subjects [IR 4; IR 5] attributed the policy as a major reason for their use of Matlab in lectures, and one of these saw this as a major distinction between this and the CAS-calculators:

Yes, I’m sure it [the policy] did really [have an effect], I was told I had to use Matlab starting summer school in 2006, and I made an effort to use it then, and in semester one, … I never really had to use the calculators, and even though I quite liked what I saw, I was unsure how to use them, and so never really did. [IR 5]

This response supports the view of another interview subject, who believes the mandate to use technology provided by the policy helped to overcome beliefs in staff who were initially against the technology use, or who were afraid about using it in front of students [IR 2].
Some comments from the interviews signalled the importance of seemingly unrelated organisational changes on the success of the Matlab initiative. For example, as part of the course changes in 2006, the department established “teaching teams,” responsible for overseeing the changes for each course. Two of the interview subjects [IR 3; IR 5] regard the team approach as a significant factor in the success of the Matlab initiative for Maths 108 in particular, contributing to the consistency of approach between the many streams, and the sizeable teaching staff. Maths 108 Team meetings consider such issues as examination questions, discuss problems with using Matlab (e.g. lecture access, manuals, staff training etc), and help provide solutions to these problems. One colleague [IR 3] believes that this will contribute greatly to the sustainability of Matlab within the course, providing a link between successive semesters and years, to facilitate and develop consistent and effective practices. While this was one of the objectives of establishing the teaching teams, technology use was not the primary focus at the time.

Similar effects to those described for the CAS-calculators appear to be happening again with the use of the Tablet-PC in lectures, and to a certain extent the use of on-line quizzes in courses. Like the CAS-calculators, the tablet initiative began largely as an experiment fired by the enthusiasm of one colleague with a particular interest in technology, who perceived the potential in delivering lectures using this medium, and started using it in his third-year analysis course (see Bonnington, Oates, Parnell et al., 2007). Other colleagues became interested, and use of the Tablet-PC spread to several first-year courses, including Maths 108. The initiative has been well received by students, and after a number of initial teething problems, the staff who have used it are mostly equally enthusiastic. However, tablet-use has not been mandated in policy, and many colleagues within the department remain unaware of its potential. Its use has not spread widely to other courses, and has been inconsistent or even declined in some courses that participated in the trial.

Bonnington, Oates, Parnell et al. (2007) report many potential benefits of the tablet for mathematics teaching, but they also note problems with its use in the Auckland trial. There are organisational difficulties, as currently, those wanting to use the tablet have to share a few tablets between them, which can be problematic in courses such as Maths 108 with multiple streams and lecturers. This mirrors the experiences with data-shows and OHP view screens for the CAS-calculators and Matlab.
use in lectures, prior to the e-lecterns. Other effects include those within the \textit{Staff} and \textit{Student Factors} components of the taxonomy. Teachers using the tablet have to restructure their courses, requiring additional resource preparation, and there is a whole new teaching paradigm that they need to adapt to, including the physical restrictions posed by teaching on the small tablet screen, and the need to be connected by a microphone to the tablet, which restricts the teacher’s movements. There are also concerns about their lectures being viewed by other staff. Students have to adopt new strategies to use the recorded lectures effectively. Some staff voiced concerns about an apparent drop-off in lecture attendance, and felt that a reliance on the recorded lectures at the expense of attending lectures may adversely affect weaker students in particular (Bonnington, Oates, Parnell et al., 2007).

Both the tablet initiative and the on-line quizzes may benefit from a more unified approach stimulated by the establishment of a departmental policy. There is little data currently available for their use at Auckland, but on-line quizzes are well-supported in the literature (see p. 123), and there is considerable anecdotal data, supported by interview comments, that students like the quizzes and view them as an effective assessment and learning strategy. However, until 2008, this use had not spread beyond the two courses that first adopted them (Maths 108 and Maths 102), and few colleagues in the department still seem aware of their potential. The teaching-teams also do not seem to have had the same effect on consistency and continuity of tablet use within courses as they have for Matlab use. One course that helped pioneer their use in 2006 (Maths 101) currently has no lecturers using the tablet, and use of the tablet varies within Maths 108, although there are competing forces at work here. It is unclear whether the benefits gained from recording all four lecture streams individually would outweigh the organisational and pedagogical concerns described previously. One of the principal users at the start of the tablet project is still using the tablet extensively in another course, but its use appears to have stalled elsewhere. The current use of quizzes and the tablet-PC reflect the evolutionary nature of technology described earlier, with the quizzes due to be extended to the follow-up course to Maths 108, and a newer development for pre-entry diagnostic testing to be introduced in 2009. However, the inconsistency in their use to date does support the need for congruency across the ITMC taxonomy. It is difficult to achieve effective, sustainable technology integration by addressing particular issues in isolation.
6.5 CHAPTER SIX SUMMARY

This chapter has documented the experiences in implementing technology within the Department of Mathematics at The University of Auckland over a ten-year observational period. Three distinct periods of technology use were identified, and these were examined against the taxonomy developed in the previous chapter. The evidence from these observations, supported by interview responses, suggests that several factors from the taxonomy (see Table 6.4), played a critical role in technology integration. This is especially evident in the contrast between the CAS-calculator use in 2001 to 2005, and the comparative success of the Matlab period that followed in 2006, as demonstrated in Figure 6.4. The effect of the technology policy established in 2005 in particular was seen to be intricately linked to the effects of many other elements of the taxonomy, with a significant effect on the overall level of integration. It has affected many organisational components (for example statements in official documents, resource preparation, access to technology), positively influenced staff and students’ engagement with technology, and resulted generally in much greater consistency of use across the department.

A closer examination of two specific elements of the taxonomy, Assessment and Mathematical Factors reveals that they indeed have a critical role in establishing effective integration of technology. Examination questions continue to offer advantages to students with access to technology, and the issue of congruency between pedagogy and assessment is not ideal when the primary technology platform is accessed through computer laboratories, divorced from major assessment and teaching practices. While this has been shown to be a complex task, there is a pressing need to re-examine course content, with consideration of the changing pragmatic and epistemic values of specific topics, and the goals of mathematics education, within a rapidly evolving technological environment. Raising the awareness of pedagogical technology knowledge is also seen as a critical factor in influencing staff beliefs about the value of technology in their teaching, and their subsequent acceptance of, and engagement with, the technology.

The findings presented in this chapter address the second research question of the thesis, “How can we facilitate the effective and sustainable implementation of a tertiary
An Observational Study of Technology Implementation

Integrated Technology Mathematics Curriculum? The discussions suggest that while there is an evolutionary nature inherent in technology developments, it is essential to recognise the interdependent relationship between the elements identified in the taxonomy (Table 5.7). An integrated, holistic approach, with consistent attention to all elements of the taxonomy, offers a more effective means of achieving a sustainable ITMC.
REVIEW AND IMPLICATIONS

“Technology integrated intelligently with curriculum and pedagogy produces measurable learning gains” (Tall, Smith & Piez, 2008).

7.1 OVERVIEW

This thesis began with a number of observations about technology use in undergraduate mathematics courses in the Department of Mathematics at The University of Auckland. The apparent mismatch between students’ use of technology in upper-secondary school mathematics and first-year courses stimulated an investigation into the underlying reasons for the inconsistent and minimal use observed at undergraduate level. The study sought ways of facilitating more effective responses to technological developments. A pilot survey of students’ use of graphics calculators in one foundation calculus course confirmed the concerns raised in the initial observations, and identified further issues. A preliminary review of the literature exposed the variety of ways the term integrated is used with respect to technology, and the lack of a satisfactory definition of integrated technology. Two research questions emerged:

1. What are the characteristics of a tertiary Integrated Technology Mathematics Curriculum, and how might we measure the nature of such technology integration?
2. How can we facilitate the effective and sustainable implementation of a tertiary Integrated Technology Mathematics Curriculum?

Figure 4.2 illustrates the iterative design used to investigate these questions. The thesis has been constructed to reflect the different phases of the study, and discussion has taken place with the reporting of each phase. This chapter summarises the results of
this study. It will highlight the key findings, consider the implications and significance of these, and suggest some directions for further investigation.

7.2 SUMMARY

Chapters Two and Three carried out a review of the literature with respect to curriculum studies, and the place of technology within a mathematics curriculum. Chapter Two identified in particular the variety of ways in which the term curriculum is used in the literature, and nominated a specific model to be used as a basis for this thesis. The model, by Valero-Duenas (2002), is useful when considering curriculum issues at the tertiary level. It positions the elements of the curriculum at three levels, which allows for the examination of technology at the localised institutional or departmental level (the intermediate and micro-levels, see Figures 2.1–2.3). Chapter Three provided support for an underlying assumption of the research questions, that there is educational value in an integrated approach to technology use in mathematics education. The studies reviewed here reinforce the learning benefits of technology, when appropriately used and effectively integrated. Chapters Two and Three both confirmed the variety of ways in which integrated technology is used in the literature, and considered a number of possible interpretations and descriptions as a basis for a definition (e.g. Hoyles, 1998; Lagrange, Artigue, Laborde & Trouche, 2003; Thomas & Holton, 2003). A number of studies also reported on technology implementation issues.

The literature review for this thesis revealed a critical need to examine the effects of university mathematicians’ beliefs about mathematical knowledge, content, pedagogy, and technology on the use of technology in their teaching. While most undergraduate lecturers use technology in their research domains, and many make use of the computational capabilities of software packages such as Matlab and Maple in their courses, many also remain unaware of the theoretical and pedagogical issues associated with technology and student learning. A number of specific issues need addressing if they are to be convinced of the value of using technology in their teaching and hence engage effectively with that technology. In particular, the review suggests that more studies are required that demonstrate a clear relationship between the use of the technology, and improved student learning (Anguelov, Engelbrecht & Harding, 2001; Keynes & Olson, 2001; Hoyles & Lagrange, 2005). The studies reviewed in
section 3.2.2 (pp. 110-119) provide evidence of demonstrable learning benefits, and hence assist in addressing this pressing need.

Chapter Four presented the methodological framework of the study, and outlined the methods used in this investigation. It described the construction of the series of surveys used to gather the data reported in Chapter Five, to address the first research question, and outlined the observational study and the interview process used to investigate the second research question in Chapter Six. The discussions concluded by considering the trustworthiness of the study’s findings. Triangulation of the data using multiple methods of investigation was used as the principal means of meeting this issue.

Chapter Five describes the phase of the study used to address the first research question, and discusses the findings from this. The key outcomes of this phase are the establishment of a taxonomy of technology integration (see Table 5.7), the definition advanced for an *Integrated Technology Mathematics Curriculum* based on this taxonomy (see p. 212), and the radar diagrams used to measure and compare technology use within and between courses and departments (see Figures 5.1-5.5; Figure 6.4). The development of the taxonomy reflects the iterative nature of this study, with a tentative model generated from the results of the pilot studies, a preliminary review of the literature, and the first survey of undergraduate mathematics educators (see Table 5.2). These results were used to inform the third major survey, the responses to which, along with issues identified in a further review of the literature, resulted in the refined taxonomy as presented in Table 5.7.

The taxonomy identifies six main categories, each with a number of associated issues, which may be used to assess the degree of integration in a particular undergraduate mathematics course. Table 5.7 demonstrates the complexity of issues associated with technology integration, and reinforces the difficulty of formulating a definition. The definition of an *Integrated Technology Mathematics Curriculum* forwarded by this study is that which achieves a comprehensive level of attention to all the elements identified in the taxonomy. Chapter Five also explores ways in which the taxonomy may be used to compare technology use between specific courses. The radar diagrams provide a visual measure of the overall level of integration (see Figure 5.4). However, the complexity of issues presented in Table 5.7 limits the effectiveness of the diagrams, as specific information is concealed in grouping data. The radar diagrams remain useful for bench-marking purposes, and for identifying general areas within the
curriculum where attention to technology issues may be lacking (see Figure 5.5). They also proved helpful in Chapter Six, to identify differences in the levels of technology integration between different periods of technology use at Auckland University. While not identifying the underlying reasons, Figure 6.4 demonstrates the contrast in technology integration for the two periods, and the elements of the taxonomy where these differences were most evident.

Chapter Six presents and discusses the evidence from the observational study, examined against the taxonomy elements in Table 5.7, including a closer examination of two particular elements, course content and assessment. Documental evidence, supported by interview responses, identified a number of factors critical to successful technology integration, as summarised in Table 7.1:

<table>
<thead>
<tr>
<th>Taxonomy Component</th>
<th>Critical Elements and Imperatives</th>
</tr>
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</table>
| Organisational Factors | • Planning: Recognise time required, and evolutionary nature of technology developments.  
• Policy: Signals value of technology to staff and students; helps overcome fears and resistance; facilitates consistency between and within courses.  
• Statements in official documents reaffirm the value of technology, and reinforce policy directives.  
• Ease of access and technical support for staff. |
| Mathematical Factors | • Goals: Need to reconsider these in a technological environment.  
• Content: Reassess the relative values of topics in the curriculum (e.g. pragmatic and epistemic values).  
• Technology consistent with subject and staff research domains. |
| Assessment | • Fairness: Address the advantages afforded by unequal access to technology in formative testing, between those with and without technology, and between different forms of CAS.  
• Validity: Instrumental questions trivialised by technology.  
• Congruency: Address discrepancies between use of technology in teaching, learning and assessment. |
| Access | • Cost: Critical factor in students’ engagement.  
• Congruency: Availability in class outside of computer labs. |
| Staff Factors | • Pedagogical Technical Knowledge: Educate staff to the pedagogical value of technology, to promote effective use in teaching.  
• Beliefs: Significant in staff engagement with, and choice of technology.  
• Training and Support: Fears from unfamiliarity, technical problems. |

Integrated Technology in the Undergraduate Mathematics Curriculum
Chapter Seven

Review and Implications

The influence of the factors identified in Table 7.1 is highlighted in the contrast between the CAS-calculator initiative and the generally successful transition to Matlab. Through more consistent attention to the elements of the taxonomy, mathematics courses at The University of Auckland have made significant progress towards an ITMC (see Figure 6.4) following the move to Matlab.

In addition to identifying the critical elements detailed in Table 7.1, the key finding from this part of the study is in the greater complexity of the taxonomy than is suggested by the Chapter Five discussions. The definition of an ITMC proffered in Chapter Five emphasises the need to address the elements of the taxonomy in a comprehensive, holistic fashion, to achieve successful technology integration. In addition to the influence of the individual taxonomy elements on the implementation process, the evidence from the observational study and the interviews identifies a significant inter-relationship between the elements. This is particularly evident in the influence that the establishment of a Departmental technology policy had on many other elements; the effect that including technology in examination questions has had on lecturers and students’ engagement with the technology; and the relationship between access to technology and assessment. The mandated policy has resulted in much greater consistency of use of technology by staff, and the inclusion of Matlab questions in the Maths 108 examination has ensured lecturer’s must at least reference technology use, a significant difference from the earlier CAS-calculator period. An integrated approach in implementing technology is essential; addressing elements in isolation results in inconsistencies between and within courses, and limits the sustainability and effectiveness of the technology.

7.3 SIGNIFICANCE AND IMPLICATIONS FOR CURRICULUM DESIGN AND TECHNOLOGY IMPLEMENTATION.

Chapter One identified how studies investigating technology use at the tertiary level are limited, particularly with a curricular focus (see pp. 9-12), so the synthesis of studies in this area provided by the review in Chapters Two and Three is timely. This study has emphasised how the literature is characterised by variations in the conceptualisation of integrated technology, and the ill-defined manner in which the term integration is used. This is compounded by the extensive range of technologies
now available, and the variety of ways in which technology itself is used in undergraduate mathematics. The uses identified in the literature include (but are not restricted to):

- Technology as a learning tool. Benefits for CAS-technologies in particular include inter-representational versatility; scaffolding and support for students with weaker skills; improved problem-solving and analytical skills, and saving time on repetitive calculations to concentrate on developing understanding.
- Technology as a computational tool: Has facilitated access to new areas of mathematics, saves time and allows more complex problems to be investigated. Has replaced the use of specialist techniques in some case.
- Technology as a social imperative: Students must be familiar and proficient with the appropriate use of contemporary technologies.
- Technology for course delivery: Includes internet use and on-line assessment; data-shows and tablet-PC’s using a variety of presentation-software.

The complexity of these and other issues identified in the review suggests it may be unreasonable to expect all those charged with developing and implementing new curricula to consider every issue when planning such a programme. However, the taxonomy of integrated technology developed in Table 5.7 encapsulates the many imperatives identified in this investigation, and draws them together in an explicit, and inter-connected manner. This taxonomy hence makes significant progress in identifying the characteristics of an Integrated Technology Mathematics Curriculum, and provides a necessary basis for further examination of the effects of integrated technology on student learning.

The evidence from Chapter Six extends the value of this taxonomy. The results from the observational study and the interviews suggest that technology implementation must recognise the inter-related structure of the taxonomy. Attendance to the factors in a comprehensive fashion results in higher and more sustainable levels of technology integration. Attendance to some elements in isolation may stimulate changes, but is unlikely to lead to sustained and effective technology integration. Some of the evidence for this finding is somewhat counter-intuitive, and disagrees with some claims in the literature. For example, the report on the pilot study for this thesis (Oates & Thomas, 2002) attributes much of the change to CAS-calculators in 2001 to the enthusiasm of a
few key staff, and Flashman (1996) regards the presence of a “champion for reform” as a key ingredient. However, evidence from the latter stages of this study, in particular the contrast between the CAS-calculator and Matlab periods, demonstrates that such a factor was insufficient to sustain and develop CAS-calculator use, and has had little role in the latter success. The support generated by the mandated policy, and the more consistent approach to other elements of the taxonomy has had a far more significant effect on the integration of Matlab. Table 7.2 summarises the key implications of this study for future technology initiatives:

Table 7.2: Implications for Technology Implementation

1. Recognition and comprehensive attendance to the interdependence of the taxonomy components for integrated technology results in improved learning, of even more mathematics, using appropriate technological tools, in a contemporary, efficient and sustainable manner.

2. Attention to individual elements will likely prove less successful, and have minimal sustainable impact on technology use.

3. The establishment of a mandated technology policy affects many elements of the taxonomy. It has a critical role to play in staff engagement with technology, signalling the value of technology, and facilitating consistency of use. This policy must be adaptable to new developments in technology.

4. Staff awareness of pedagogical technical knowledge, especially with respect to CAS, remains limited in undergraduate mathematics.

5. Specific elements of the taxonomy require greater attention than may be feasible at the local departmental or course level. In particular, the consideration of the relative values of many fundamental curriculum topics, in an integrated technology environment, warrants further examination.

6. Assessment issues remain problematic, even in an otherwise integrated environment. Continued consideration of the inequitable advantages afforded by unequal access to technology is required. This should include the different advantages afforded by alternative forms of CAS (see e.g. Flynn & McCrae, 2001; Flynn, 2003).
7.4 LIMITATIONS AND DIRECTIONS FOR FUTURE RESEARCH

The taxonomy developed in this study lays the foundations for several future areas of investigation suggested by the findings. One noticeable omission from the critical elements identified in Table 7.1 is the Student Factors component of the taxonomy, apart from those aspects linked to access and teacher privileging within other components. This is not surprising, given the methodological focus of the latter part of the study. Indeed, it is explicitly stated that the impact on student learning is not a focus of this study, when framing the research questions in the introductory chapter (see p. 15). While this thesis has assumed that an ITMC has positive benefits for student learning, further examination of this issue is warranted, following the establishment of the taxonomy achieved in this study. The effects of integration on student learning is identified as a critical question in the aims of the ICMI-17 study (Hoyles & Lagrange, 2005), and is certainly needed at The University of Auckland, where no formal evaluation of the change to Matlab has been conducted from a student’s perspective.

The radar diagram strategy developed in Chapter Five provides a useful visual impression of technology integration. However, determining the radial values proved problematic, and it is worth investigating new measurement tools to assist in the assigning of values. Improved validity of the scales would greatly enhance the value of the radar instrument for future studies.

The observational study could also be extended to investigate case studies from other departments and courses. Some comparison was possible using literature studies, and survey responses. However, the strategy selected for this study was an in-depth observational study of one case, an approach chosen to facilitate a more intensive interrogation of the integrated technology model. Further observations would allow confirmation of the findings of this study (or not), and may identify additional critical factors that were not evident in the Auckland case.

Directions for further study are also suggested in the rapidly evolving nature of technology itself. The literature, survey and interview responses all signal the importance of adequate preparation for impending developments in the digital revolution. This study has largely focused on mathematics-specific technologies, particularly CAS. However, developments in non-specific technologies such as
interactive class-rooms, the growing use of web-based course delivery, and tablet-technology to deliver and record lectures have all been shown to have increasing benefits specific to mathematics education, and this suggests a rich area for further study. The study by Bonnington, Oates, Parnell et al. (2007), for example, suggests that teaching and recording lectures using tablet-PC’s challenges the hierarchal, linear view of the mathematics curriculum. Several survey and interview respondents also identified the prospect of rapidly increasing student access to personal lap-tops, on wireless campuses, as presenting a real challenge for course design, and the potential effects of this development should be assessed against the taxonomy. While the taxonomy was developed with a wide variety of technologies in mind, it may well require further refinement to recognise new technologies and new technological environments.

7.5 FINAL COMMENT

This chapter begins with a quote emphasising the potential advantages of intelligently integrated technology in mathematics education (Tall, Smith & Piez, 2008). The taxonomy developed in Chapter Five identifies the essential elements of such an integrated approach. The observational study examines the elements of this taxonomy, and highlights issues found to be critical in establishing an Integrated Technology Mathematics Curriculum at The University of Auckland. The thesis concludes that effective integration of technology in undergraduate mathematics requires comprehensive attention to the interdependence of the taxonomy components, or curricular congruency with respect to technology, as measured across the taxonomy. A quote from one Australian survey respondent provides a fitting conclusion to these discussions:

Technology has the potential to change the nature of mathematics teaching, but the uptake of its use is dependant on the willingness of teaching staff to embrace new/better ways of teaching current and future curriculum. Current curriculum is challenged by technology, but only the brave take up the challenge.

This thesis suggests that bravery alone is insufficient for sustained and effective integration. The taxonomy and the other findings of this thesis provide a means by which departments may meet the challenge collectively, to realise the full potential of an Integrated Technology Mathematics Curriculum.


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References


**PRELIMINARY SURVEY:** Graphics Calculator Use in Mathematics Two (1997).

<table>
<thead>
<tr>
<th>Question</th>
<th>YES</th>
<th>NO</th>
</tr>
</thead>
<tbody>
<tr>
<td>Do you own a graphics calculator?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If YES, did you buy it this year?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>If YES, do you feel disadvantaged?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Did you like the lecturer using the graphics calculator?</td>
<td></td>
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</tr>
<tr>
<td>Do you think the graphics calculator should have been used MORE or LESS?</td>
<td></td>
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</table>

**STUDENTS WITH A GRAPHICS CALCULATOR**

<table>
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<tr>
<th>Question</th>
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<th>NO</th>
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<tbody>
<tr>
<td>Did you find the graphics calculator useful for this paper?</td>
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<td></td>
</tr>
<tr>
<td>Did you use the graphics calculator in lectures?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Did you use the graphics calculator in the Terms Test?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Did you try the graphics calculator questions in the text?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Were the graphics calculator exercises useful for learning mathematics?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Did you have difficulty learning to use the graphics calculator?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Did the graphics calculator help you DO mathematics, or UNDERSTAND mathematics or BOTH?</td>
<td></td>
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</table>

**DO Maths**    **UNDERSTAND Maths**    **BOTH**
## Pilot Survey One: Students Use of CAS-Calculators Semester One 2001

MATHS 151 Graphics Calculator (GC) Survey: Semester One 2001

Please Answer Questions 1 to 4, even if you do not own/use a graphics calculator.

1. Did you buy a TI 89 for this course? **YES** □
   **NO** □ Why not? ……………………………

   Owned a GC Before 151 Course □
   Please State model ……………………………

2. Do you think students without a graphics calculator were disadvantaged in this course? **YES** □ **NO** □
   Please explain…………………………………………………………………………………………

3. Did you like the lecturer using the graphics calculator? **YES** □ **NO** □ **Lecturer Didn’t Use** □
   If YES or NO, please explain…………………………………………………………………………

4. Do you think the lecturer should use the graphics calculator MORE, SAME or LESS in this course? **MORE** □ **SAME** □ **LESS** □
   Please explain…………………………………………………………………………………………

You need only answer the next questions if you own a graphics calculator.

5. How much do you estimate you used the graphics calculator?
   (a) In 151 lectures **Often** □ **Sometimes** □ **Not much or never** □
   (b) In 151 tutorials **Often** □ **Sometimes** □ **Not much or never** □
   (c) In 151 Assignments **Often** □ **Sometimes** □ **Not much or never** □
   (d) In the Test **Often** □ **Sometimes** □ **Not much or never** □
   (e) Working at home **Often** □ **Sometimes** □ **Not much or never** □
   (f) In other courses **Often** □ **Sometimes** □ **Not much or never** □
   Please state other courses (eg STATS 102, MATHS 162 etc)……………………………………
6. Would you like to have used the graphics calculator MORE, SAME or LESS in this course?  

   MORE □     SAME □     LESS □

Please 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### 13. While working on MATHS 151, did you use your graphics calculator (GC) to:

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<th>Question</th>
<th>Often</th>
<th>Sometimes</th>
<th>Never</th>
</tr>
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<tbody>
<tr>
<td>a  Copy GC work done by the lecturer during class?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b  Checking the solution to a problem using a method available on the GC?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c  Exploring ideas when starting to solve a problem?</td>
<td></td>
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<tr>
<td>d  Graph a function on the GC to help understand the problem?</td>
<td></td>
<td></td>
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<tr>
<td>e  Do an arithmetic calculation?</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>f  Solve a problem using the table feature?</td>
<td></td>
<td></td>
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<tr>
<td>g  Solve a problem graphically?</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>h  Write a program?</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>i  Use any of the matrix operations?</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>j  Reduce a matrix to echelon form?</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>k  Evaluate trigonometric functions?</td>
<td></td>
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<tr>
<td>l  Evaluate logarithmic and exponential functions?</td>
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<tr>
<td>m  Evaluate the quadratic formula?</td>
<td></td>
<td></td>
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<tr>
<td>n  Do extra problems suggested by the lecturer?</td>
<td></td>
<td></td>
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<tr>
<td>o  Use the statistics functions?</td>
<td></td>
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<tr>
<td>p  Experiment with any other special functions on the GC? (Please state which functions in the space below this table)</td>
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</table>

Please note any other uses which you made of the graphics calculator which you see as useful and/or important that are not mentioned in the table above.

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### 14. What if anything did you particularly like about the graphics calculators?

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### 15. What if anything did you particularly dislike about the graphics calculators?

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THANK YOU FOR YOUR TIME IN FILLING OUT THIS SURVEY
Pilot Survey One: Coding Schedule for Responses

Note: Numbers for coded responses are not hierarchal, except for Likert-Scale responses to Questions 7-12. The numerical responses for other questions are for categorisation purposes only.

**MATHS 151 Not GC Survey: Semester One 2001 Codings**

1. Yes = 0, No = 1, Owned Previously = 2

**Codes:**
- 0 = Cost related, eg couldn’t afford, too expensive
- 1 = Forgot, didn’t get around to it, may buy soon, Didn’t know about etc
- 2 = Don’t want one, not useful to course, not necessary etc
- 3 = Only doing one maths paper, not continuing
- 4 = Prefer manual methods, shouldn’t use calculators etc
- 5 = Couldn’t use, didn’t know how, too complicated etc.
- 6 = Other

For Other Model: Enter model eg Casio FX 2200

2. Disadvantaged? Yes = 0, No = 1

Please explain your answer.

**Codes:**
- Yes
  - 0 = Speed issues: eg Quicker with, slower without etc
  - 1 = Checking answers, right or wrong etc
  - 2 = Graphs of functions, picture or graph helps
  - 3 = Helps understanding
  - 4 = Used a lot in eg tests. assignments
  - 5 = Other uses of GC eg solves equations, matrices etc
  - 6 = Confidence
  - 7 = Other reason why disadvantaged

- No
  - 8 = Doesn’t show working, methods etc
  - 9 = Too hard to use, takes too much time to learn/use etc
  - 10 = Lecturer or course tried to avoid/course did not allow etc.
  - 11 = Other reason why not disadvantaged

**Note:** Some may give some negatives & some positives.

3. Lecturer Use Yes = 0, No = 1, Didn’t Use = 2

Please explain your answer.

**Codes:**
- 0 = Not all students have a calculator
- 1 = Wastes time, not relevant
- 2 = Couldn’t see properly, hard to read, follow, confusing, too quick.
- 3 = Don’t like calculators, prefer to do by hand etc
- 4 = Different model
- 5 = Liked not using
- 6 = Helped understanding, illustrates concepts etc
- 7 = Helps to learn how to use GC
- 8 = Interesting
- 9 = Other
Not GC Survey Codings: Continued

4. Lecturer Use  More = 0, Same = 1, Less = 2

Please explain your answer.

Codes:  
0 = Not all students have a calculator, not relevant for those without etc
1 = Time Issues: eg Waste of time, takes too much time, not enough time etc.
2 = Not needed, not useful, not essential
3 = Don’t like calculators, prefer to do by hand, should learn maths without etc
4 = Not lecturers job, students should learn how to use themselves etc,
5 = Ok as is, eg good balance, Liked not using
6 = Speeds up learning how to use, learn more functions etc.
7 = More effective use, should use more if part of course
8 = Speeds up work, useful for checking etc.
9 = Depends, other reason

6. Liked to use themselves? More = 0, Same = 1, Less = 2

Please explain your answer.

Codes:  
0 = Useful for other courses
1 = Would buy if could afford it.
2 = Not needed/necessary, not useful, not essential
3 = Other

MATHS 151 Have GC Survey: Semester One 2001 Codings

1. Yes = 0, No = 1, Owned Previously = 2

Codes:  
0 = Cost related, eg couldn’t afford, too expensive
1 = Forgot, didn’t get around to it, may buy soon, Didn’t know about etc
2 = Don’t want one, not useful to course, not necessary etc
3 = Only doing one maths paper, not continuing
4 = Prefer manual methods, shouldn’t use calculators etc
5 = Couldn’t use, didn’t know how, too complicated etc.
6 = Other

For Other Model: Enter model eg Casio FX 2200

2. Disadvantaged?  Yes = 0, No = 1

Please explain your answer.

Codes:  
Yes 0 = Speed issues: eg Quicker with, slower without etc
1 = Checking answers, right or wrong etc
2 = Graphs of functions, picture or graph helps
3 = Helps understanding
4 = Used a lot in eg tests. assignments
5 = Other uses of GC eg solves equations, matrices etc
6 = Confidence
7 = Other reason why disadvantaged
Appendix A2  

H ave GC Survey Codings: Continued

2. No  
   8 = Doesn’t show working, methods etc  
   9 = Too hard to use, takes too much time to learn/use etc  
  10 = Lecturer or course tried to avoid/course did not allow etc.  
  11 = Other reason why not disadvantaged  

Note: Some may give some negatives & some positives.

3. Lecturer Use  
   Yes = 0, No = 1, Didn’t Use = 2  

Please explain your answer.  
Codes:  
  0 = Not all students have a calculator.  
  1 = Wastes time, not relevant.  
  2 = Couldn’t see properly, hard to read, follow, confusing, too quick.  
  3 = Don’t like calculators, prefer to do by hand etc  
  4 = Different model  
  5 = Liked not using  
  6 = Helped understanding, illustrates concepts etc  
  7 = Helps to learn how to use GC  
  8 = Interesting  
  9 = Other  

4. Lecturer Use  
   More = 0, Same = 1, Less = 2  

Please explain your answer.  
Codes:  
  0 = Not all students have a calculator, not relevant for those without etc  
  1 = Time Issues: eg Waste of time, takes too much time, not enough time etc.  
  2 = Not needed, not useful, not essential  
  3 = Don’t like calculators, prefer to do by hand, should learn maths without etc  
  4 = Not lecturers job, students should learn how to use themselves etc,  
  5 = Ok as is, eg good balance, Liked not using  
  6 = Speeds up learning how to use, learn more functions etc.  
  7 = More effective use, should use more if part of course  
  8 = Speeds up work, useful for checking etc.  
  9 = Depends, other reason  

5. How much do you estimate you used the graphics calculator?  
   (a) to (f) Often = 0, Sometimes = 1, Not much or Never = 2  

Other Courses?  
Codes:  
  0 = Statistics Courses  
  1 = Maths 162  
  2 = Computer Science Courses  
  3 = Physics  
  4 = Accounting  
  5 = Other
### Have GC Survey Codings: Continued

**6. Liked to use themselves?** More = 0, Same = 1, Less = 2

Please explain your answer.

**Codes:**
- 0 = Useful for other courses
- 1 = Would buy if could afford it.
- 2 = Not needed/necessary, not useful, not essential, too complicated
- 3 = Prefer manual methods, don’t like calculators, will forget how to do without calculator etc
- 4 = Not fair, everyone should own etc.
- 5 = Allow more use eg own programming, allow to use more without showing working
- 6 = Saves time, quicker.
- 7 = Aids learning, increases confidence
- 8 = Ok as is, no difference, don’t want to over-use.
- 9 = Need to learn how to use, practice, get better use of/benefits of owning
- 10 = Other

**7. The graphics calculator helped me with written assignments**
Enter value circled from 1, 2, 3, 4, 5

**8. The graphics calculator helped me with the test** Enter value circled from 1, 2, 3, 4, 5

**9. The graphics calculator helped me understand some of the mathematical content of MATHS 151** Enter value circled from 1, 2, 3, 4, 5

**10. Learning how to use the graphics calculator was too difficult**
Enter value circled from 1, 2, 3, 4, 5

**11. I now feel confident about using the graphics calculator**
Enter value circled from 1, 2, 3, 4, 5

**12. I would recommend students taking MATHS 151 in the future to buy a TI 89**
Enter value circled from 1, 2, 3, 4, 5

**13. While working on MATHS 151, did you use your graphics calculator (GC) to:**

(a) to (p) Often = 0, Sometimes = 1, Never = 2

**Other Uses:**

**Codes:**
- 0 = Differentiation and/or Integration
- 1 = Algebraic Operations eg. Solve, factor, expand, Substitution
- 2 = 3D Graphs
- 3 = Differential Equations
- 4 = Relation Graphing
- 5 = Other
### Have GC Survey Codings: Continued

14. What if anything did you particularly **like** about the graphics calculators?

<table>
<thead>
<tr>
<th>Codes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Everything/lots</td>
</tr>
<tr>
<td>1</td>
<td>Nothing, hate it etc</td>
</tr>
<tr>
<td>2</td>
<td>Checking answers, making sure answer is correct, eliminating errors/mistakes</td>
</tr>
<tr>
<td>3</td>
<td>Drawing graphs, graphing mode etc</td>
</tr>
<tr>
<td>4</td>
<td>Useful/handy/helpful/reliable, Wide range of functions, does so much, solves problems, functions other than graphing eg limits, equations etc</td>
</tr>
<tr>
<td>5</td>
<td>Easy to use, user friendly, portable, price etc</td>
</tr>
<tr>
<td>6</td>
<td>Saves time, quicker,</td>
</tr>
<tr>
<td>7</td>
<td>Can see lot of work at once eg many lines, previous working</td>
</tr>
<tr>
<td>8</td>
<td>Other</td>
</tr>
</tbody>
</table>

15. What if anything did you particularly **dislike** about the graphics calculators?

<table>
<thead>
<tr>
<th>Codes</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Nothing, loved it etc</td>
</tr>
<tr>
<td>1</td>
<td>Everything, hated it etc.</td>
</tr>
<tr>
<td>2</td>
<td>Hard to learn how to use, confusing, complicated, Manual hard to follow, frustrating, couldn’t remember how to use, Some functions hard to follow</td>
</tr>
<tr>
<td>3</td>
<td>Price</td>
</tr>
<tr>
<td>4</td>
<td>Slow to use, eg some function slow to use (programming, formulas etc)</td>
</tr>
<tr>
<td>5</td>
<td>Time issues eg. too much time spent in lectures, learning how to use</td>
</tr>
<tr>
<td>6</td>
<td>Not as clear/fast as computers, small screen, hard to read</td>
</tr>
<tr>
<td>7</td>
<td>Still have to show working, doesn’t show working etc</td>
</tr>
<tr>
<td>8</td>
<td>Calculators wreck brain etc</td>
</tr>
<tr>
<td>9</td>
<td>Not allowed to use fully eg for tests/exams in other courses</td>
</tr>
<tr>
<td>10</td>
<td>Other</td>
</tr>
</tbody>
</table>
### (i) Survey Two: Technology Use in Tertiary Institutions

**Note:** I am particularly interested in this study with graphics calculator technology (including especially CAS systems), but also welcome your responses if you have used other computer software packages instead/as well, and the following questions seem equally applicable, especially if for example you considered the calculators but opted for the software instead. So please feel free to substitute "computer software" for "graphics calculator" in the questions. Please indicate where this has been done.

**Question 1** Have you been involved in the teaching or design of a tertiary mathematics and/or statistics course that makes use of graphing calculator technology, including CAS systems?

If no, then please ignore remainder of questions.
If yes, then please answer:

(i) Briefly describe the nature of the course (e.g. first year calculus; 3rd year engineering etc), the number of students in a typical course, and name of the tertiary institution.

(ii) Type of technology used, e.g. CAS systems such as TI-89, TI-92, or simple graphics calculators (non-CAS) such as TI-83, computer-based CAS software.

**Brief background to following questions:**
Much of the literature in support of technology use and in particular graphics calculators supports their integration into curricula for maximum effectiveness. I am trying to identify what are the features of an Integrated Technology Curriculum may be.

**Question 2** (i) Are the calculators permitted in course assessment: e.g. Assignments? Tests? Exams?

(ii) Is it compulsory for students to have a graphics calculator? If yes, are they provided (e.g. class set), or do they have to buy themselves? If not, how are they encouraged to purchase one?

(iii) Do the teachers of the course use the calculators themselves in class (e.g. lectures, tutorials)? A little? Some? A lot? Briefly describe how they are used, e.g. as a demonstration tool; to actively take students through specific examples where the calculator is useful?

(iv) Is there any active teaching of how to use the calculators: For students, e.g. tutorials? Show specific examples in lectures? Hand-out or course-book materials? For staff, e.g. professional development?

(v) Is the use of the technology specifically described in any course material such as a curriculum document, or course description? Were any changes made to the curriculum when the technology was introduced? E.g. Change in content (new material introduced, other material omitted)? Order of teaching of topics? Style of assessment?
(i) Survey Two Continued: Technology Use in Tertiary Institutions

**Question 3** Can you please briefly describe what you understand a tertiary mathematics course with an *Integrated Technology Curriculum* might look like? E.g. What features may the course have to have in order to be so described?

(ii) Survey Two: Coding Schedule for Responses (Questions 2 and 3)

<table>
<thead>
<tr>
<th>Question 2</th>
<th>Category of Response (coding)</th>
<th>Example of Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Are the calculators permitted in course assessment?</td>
<td>All: Assessment assumes technology (technology-active)</td>
<td>“We write the exams assuming students have a graphics calculator”</td>
</tr>
<tr>
<td></td>
<td>All, but assessment is technology-neutral</td>
<td>“Can’t disadvantage the students without a graphics calculator”</td>
</tr>
<tr>
<td></td>
<td>Some: E.g. assignments, but not exams.</td>
<td>“The final exam is not in a computer laboratory”</td>
</tr>
<tr>
<td></td>
<td>All, except special technology free section (usually skills)</td>
<td>“We include a compulsory skills component in the exam where calculators are not permitted”</td>
</tr>
<tr>
<td></td>
<td>None</td>
<td>“Students are not allowed to use calculators in any assessment, although in practice, it is impossible to monitor this for assignments”</td>
</tr>
<tr>
<td>(ii) Is it compulsory for students to have a graphics calculator? How do they access it? Purchase or class set (laboratory) etc?</td>
<td>Compulsory &amp;/or all students have equal &amp; regular access.</td>
<td>“All students have easy access to well-resourced computer labs where all classes are held”</td>
</tr>
<tr>
<td></td>
<td>Not compulsory, but most have, strongly encouraged and/or expected</td>
<td>“Most have GC’s anyway, and if they don’t, we suggest they borrow, especially for exams”</td>
</tr>
<tr>
<td></td>
<td>Not compulsory, encouraged, but only some have (&gt; 20%)</td>
<td>“It is not an essential purchase, students buy most suitable to them, may not be GC”</td>
</tr>
<tr>
<td></td>
<td>Very few students have/use (&lt; 20%)</td>
<td>“GC’s encouraged, but not compulsory, and only about 10% get one, must have general scientific one though.”</td>
</tr>
</tbody>
</table>
### (ii) Survey Two Coding Schedule Continued:

<table>
<thead>
<tr>
<th>Question 2</th>
<th>Category of Response (coding)</th>
<th>Example of Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>(iii) How do instructors (lecturers, tutors) use the technology? Extent of use? Type of use (demonstration tool, modelling use for students)?</td>
<td>Extensive (integrated?): Used to demonstrate, model for students, staff expected to use.</td>
<td>“All lessons are in the computer labs, so that theory and practical are combined. There is a data-projector in every lab and lecturers know how to use it and are expected to do so.”</td>
</tr>
<tr>
<td></td>
<td>Specific: Used to demonstrate examples where appropriate, for certain uses, e.g. complicated numbers, calculations.</td>
<td>“The primary use is for demonstrations and getting answers when real world data is involved.”</td>
</tr>
<tr>
<td></td>
<td>Variable: Used on occasions, depends on instructor.</td>
<td>“This depends utterly on the faculty member. We have classroom display units, and most of us use them some of the time. Some probably never do though.”</td>
</tr>
<tr>
<td></td>
<td>Minimal: Examples provided in notes, some reference in lectures, but not demonstrated or modelled.</td>
<td>“I don’t use Maple during actual lecture, although I do show slides of Maple graphs where they seem helpful.”</td>
</tr>
<tr>
<td>(iv) What assistance is there for students and staff in the use of the calculators? Any professional development? Tutorials, hand-out, or course-book materials for students?</td>
<td>Immersive: Familiarity, confidence and versatility developed through continual use.</td>
<td>“All teaching done in labs with computers for all students. Detailed instructions and supervision of computer lab work using Maple.”</td>
</tr>
<tr>
<td></td>
<td>Targeted: Tutorials at start of course, worksheets or lab manual provided.</td>
<td>“I spend one or two lectures going over introductory worksheets with the students…We have not done much in the way of professional development.”</td>
</tr>
<tr>
<td></td>
<td>Assumed: Staff and students require little assistance, as mostly familiar with, or quick to catch on.</td>
<td>“We’ve always assumed students can use their own calculator….After some early short staff workshops, no further need since enough people are familiar with to provide assistance for instructors who need it.”</td>
</tr>
<tr>
<td></td>
<td>Minimal: Little or no assistance provided. Implication that more required.</td>
<td>“We teach them to use Matlab in one hour! … No professional development”</td>
</tr>
</tbody>
</table>
### (ii) Survey Two Coding Schedule Continued:

<table>
<thead>
<tr>
<th>Question 2</th>
<th>Category of Response (coding)</th>
<th>Example of Response</th>
</tr>
</thead>
</table>
| (v) Were any changes made to the curriculum to reflect technology? Goals,  | Technology Design:  
Technology is considered from outset. Reflected at all or most levels of the course.  
Evolutionary: Changes have been made over time, in response to technology developments, new pedagogical realisations.  
Variable: Changes, often considerable, but lack of congruency, e.g. Included in course description, but no change to topics; Changes to course materials and topics, but not assessment.  
Little Significant Change:  
"Specified in course handbook. Course was designed with technology in mind...most appropriate technology selected for course...assessments written to force students to integrate technology.”  
“Maple labs in calculus have added project components to the course assessment...software has led naturally to the inclusion of more extensive assignments and more realistic data sets.”  
“No change to course prescription...We did adapt the course materials—for example in calculus. Topics changed (eg more numerical methods of integration...more on slope fields for DEs). Test questions changed correspondingly (but not style of test).”  
“Maple is mentioned in the course description. No the curriculum was not changed. Assessment only changed to the extent that a Maple-based assignment replaced a ‘pen and paper-based assignment.” |                                                                                                                                                                                                 |
| course descriptions, content (new material added, some deleted), order of topics changed, types of assessment? |                                                                                                                                                                                                                                 |                                                                                                                                                                                                                     |
(ii) Survey Two Coding Schedule Continued:

<table>
<thead>
<tr>
<th>Question 3</th>
<th>Category of Response (coding)</th>
<th>Example of Response</th>
</tr>
</thead>
<tbody>
<tr>
<td>What do you understand is meant by an <em>Integrated Technology Mathematics Curriculum</em>? What might such a curriculum look like?</td>
<td>Integral: Essential part of teaching and learning. Not an add-on. All aspects of curriculum considered.</td>
<td>“Technology is a sufficient part of the course that a student could not avoid it and still pass…assessment would require the technology …curriculum designed to take advantages of the opportunities provided by technology.”</td>
</tr>
<tr>
<td></td>
<td>Appropriate: Technology included where it aids mathematical understanding.</td>
<td>“The technology would enhance the presentation of the core mathematical ideas. It would not dominate the curriculum and/or appear to the student to be the main idea of the course.”</td>
</tr>
<tr>
<td></td>
<td>Congruent: Technology links assessment to class. Used in most aspects.</td>
<td>“Lab reports, worksheets, mini projects, and even an end-of-semester project are possible with technology. So the students should be evaluated on these, and they should be made aware that in the class there will be multiple methods of assessment.”</td>
</tr>
<tr>
<td></td>
<td>Selective: Use technology for specific (usually applied) purposes.</td>
<td>“Use traditional ways to teach concepts and use technology to study applications.”</td>
</tr>
</tbody>
</table>
### (iii) Survey Two Responses: Quantification of Technology Integration
(For numerical assignment of radar diagram radials)

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Quantification Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Access</strong></td>
<td></td>
</tr>
<tr>
<td>5 = Access</td>
<td>Full access, e.g. compulsory ownership of calculators; classes taught in labs; equal and regular access to technology, e.g. computer labs/individual computers for all students.</td>
</tr>
<tr>
<td>4 = Not compulsory</td>
<td>Not compulsory, but majority of students own (&gt;80%); Not all classes in labs, but technology modelled by staff; ready access to technology outside of class for all students.</td>
</tr>
<tr>
<td>3 = Access</td>
<td>As for 4, but no formal classes in labs, Fewer students own (&lt;80%) Restricted access outside of class, e.g. class sets of calculators, or technology use for assignments, tutorials only.</td>
</tr>
<tr>
<td>2 = Access</td>
<td>As for 3, but less access, e.g. only some students have own calculator (&lt; 50%); lab access restricted to specific tutorials, assignments.</td>
</tr>
<tr>
<td>1 = Access</td>
<td>Very limited student ownership (&lt; 10%); restricted access to computer labs for specific tasks (e.g. computer project).</td>
</tr>
<tr>
<td>0 = Access</td>
<td>No student use, limited to staff demonstration</td>
</tr>
<tr>
<td><strong>Student Facility</strong></td>
<td></td>
</tr>
<tr>
<td>5 = Facility</td>
<td>High level of facility. Students have considerable previous experience and/or comprehensive training provided, e.g. manuals, examples in course notes, tutorials; modelled in class.</td>
</tr>
<tr>
<td>4 = Facility</td>
<td>Some targeted assistance, e.g. early tutorials; manual or examples in course notes; modelled by staff.</td>
</tr>
<tr>
<td>3 = Facility</td>
<td>No specific targeted assistance; Reasonable manual, examples or modelling by staff.</td>
</tr>
<tr>
<td>2 = Facility</td>
<td>Some specific examples, manual, or modelling done by staff.</td>
</tr>
<tr>
<td>1 = Facility</td>
<td>No specific assistance provided, e.g. may approach tutors or individual staff.</td>
</tr>
<tr>
<td>0 = Facility</td>
<td>No help provided, students assumed to know, or expected to learn themselves.</td>
</tr>
<tr>
<td><strong>Assessment</strong></td>
<td></td>
</tr>
<tr>
<td>5 = Assessment</td>
<td>Technology Assumed/Active: Students may use at any appropriate time in all assessment.</td>
</tr>
<tr>
<td>4 = Assessment</td>
<td>As for 5, but some specific technology-free component.</td>
</tr>
<tr>
<td>3 = Assessment</td>
<td>Technology Specific: Some technology neutral or prohibited component (e.g. final exam); some specific technology targeted component, e.g. computer project.</td>
</tr>
<tr>
<td>2 = Assessment</td>
<td>Technology Neutral: All assessment questions written so no supposed advantage from student access to technology.</td>
</tr>
<tr>
<td>1 = Assessment</td>
<td>Technology allowed, but no specific allowances made for technology in assessment questions.</td>
</tr>
<tr>
<td>0 = Assessment</td>
<td>Prohibited: No technology allowed in any evaluative assessment.</td>
</tr>
</tbody>
</table>
(iii) Survey Two Quantification of Technology Integration Continued:

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Quantification Descriptor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedagogy</td>
<td><strong>5 = Fully Integrated</strong>: All staff and students expected to use technology whenever appropriate. Staff model use of technology in class, with simultaneous student interaction, e.g. computer lab with data-show, or linked calculators.</td>
</tr>
<tr>
<td></td>
<td><strong>4 = Staff as for 5, but student interaction may be more limited</strong>, e.g. via data-show in lectures where not all students access technology (but most do).</td>
</tr>
<tr>
<td></td>
<td><strong>3 = Targeted</strong>: Staff model for specific examples, but not used all the time, student use follows for specific examples or assessment components, e.g. computer project.</td>
</tr>
<tr>
<td></td>
<td><strong>2 = As for previous, but use by staff not expected, some may not use at all.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>1 = Technology not modelled by staff, but reference to specific examples in course-notes, or lectures. Specific use required by students, e.g. assignments.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>0 = No specific use of technology by staff, expect perhaps reference to course materials. No specific use required or expected of students, but may use when they want.</strong></td>
</tr>
<tr>
<td>Curriculum</td>
<td>One point for each factor considered, maximum of five. Consider for example curriculum design, course description, goals, inclusion in course materials, content changed (new, deleted topics), order of topics, changes to assessment.</td>
</tr>
<tr>
<td>Staff Facility</td>
<td><strong>5 = High level of facility</strong>: Training for all staff (lecturers, tutors), and ongoing professional development; Staff confident with use, and aware of pedagogical as well as mathematical applications of technology.</td>
</tr>
<tr>
<td></td>
<td><strong>4 = As for 5, but lesser emphasis and/or awareness of pedagogical values, more domain-specific mathematical functions.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>3 = Some initial training provided, some provision for periodic review of use, or formal professional development.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>2 = As for 3, but no formal mechanisms, ongoing largely informal, word-of-mouth, collegial support.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>1 = As for 2, but no obvious systems for training or support.</strong></td>
</tr>
<tr>
<td></td>
<td><strong>0 = No assistance or professional development for staff. Largely domain-specific computational use where staff facility is assumed by virtue of their mathematical expertise.</strong></td>
</tr>
</tbody>
</table>
(i) Final Survey: Technology Use in Tertiary Institutions

Technology Integration Questionnaire 2007

Brief Background Statement:
This survey seeks to determine the extent to which technology is Integrated into the teaching and learning of undergraduate mathematics in your department or individual mathematics courses (some institutions refer to papers, programmes instead of courses), and the process through which the adoption of new technologies has taken place. You may choose to answer broadly on behalf of the department (or group of courses), or just on a specific course that you teach or are responsible for. There are 25 questions, please answer as many as you can and please state where you feel the question is not applicable or you feel unable or unwilling to answer.

**Part A**  Department, Course and Technology Details.

A1 Are you replying on behalf of a department (or group of courses), or one specific course that you teach or for which you are responsible? Please specify.
- If department, state e.g. department of mathematics, engineering, statistics etc.
- If specific course, state e.g. first year calculus course, third year analysis course; number of students in typical course; is the course a “maths major”, or “bridging” or “service course”; your role in the course.

A2 Describe the type of technology used in your course(s) (e.g. CAS-calculators, Matlab etc). Please note any variations in usage between courses if more than one, e.g. first-year Calculus uses Matlab, third-year course uses Maple etc. Are there some courses for which technology is not used/permitted?

**Part B**  Nature of Technology Usage

B1 (i) Does your department have a global or consistent policy or guidelines with respect to technology use in its courses, or is this the decision of individual courses or teaching staff?

Please either briefly describe the policy, or if possible, give the policy details or statement, e.g. does the policy have specific goals with respect to technology use, does it specify the type of technology, describe how it may be used?

(ii) How was the policy established (e.g. committee, grew from existing practice) and long has it been in place and/or how long since it was changed or updated? (If you know)
Appendix A4

Final Survey: Technology Use in Tertiary Institutions

B2 (i) Has there been any review of your curriculum that has considered the way in which technology will be used or impacts on the course(s)? Please specify, e.g. no specific considerations of technology; we have considered technology in the process of our normal course revision/development; we designed a new course incorporating technology.

(ii) Have there been any changes to the content of the course because of the technology you use? Please specify, e.g. have some topics been dropped? Have any new topics been added? Has the order of topics been changed?

(iii) Do you use new textbooks that were chosen at least partly because of technology, or have you written specific technology sections in course-books?

B3 (i) Have there been any changes in assessment because of technology? e.g. assignment questions that require technology; laboratory project.

(ii) Is technology permitted, required or not allowed in assignments, tests or examinations? Please specify, e.g. allowed or required in assignments; not used in tests/exams as not in laboratory; can use in exam (e.g. CAS or graphics calculator); question in exam that requires knowledge of technology (e.g. Matlab printout); tests or exams held in laboratory.

B4 (i) How do you (or other teachers) use technology in course delivery? Please specify, e.g. demonstrate usage regularly/occasionally in classes; teach in a computer laboratory where it is used all the time.

(ii) Do all teachers in courses where more than one teacher use the technology equally? Are they all equally familiar/enthusiastic with its use?

(iii) To what extent is training and/or professional development in the use of technology provided for teaching staff. e.g. they are given a manual; some training provided when they start teaching; some ongoing professional development support is in place.

B5 (i) What levels of access do students have to the technology? Please specify, e.g. are expected to buy their own calculator; must visit the computer laboratory; may purchase their own version of the software; use personal laptops in class.

(ii) How do the students use the technology? e.g. they have their own calculator which they can use in all classes, tutorials etc; classes are taught in computer laboratory and students can have their own software for use at home; students visit the computer laboratory for tutorials and/or assignments; students use laptops in class.

(iii) What training and/or support is provided for students in the use of technology? e.g. they get a manual or textbook that provides support; help tutorials are provided; demonstrations are given in class; they are familiar with the technology from previous courses.

B6 Has any evaluation been conducted on the use of technology in your course(s)? Please specify, e.g. discussion at department meeting; questions included in course evaluations; specific surveys or feedback from students.
Part C: Beliefs about technology use

C1 (i) Please state your personal experience and your feelings and/or beliefs about the role of technology in undergraduate mathematics. Consider any benefits or disadvantages that you are aware of.

(ii) How do you perceive the usage of technology is regarded in your course(s) by you, your colleagues and/or your students? Is its value mainly as an analytical or computational tool, or does its use provide students with ways of helping their understanding of concepts?

(iii) To what extent do you believe technology should be integrated into undergraduate mathematics courses? Please specify, e.g. it should be available and used at all appropriate opportunities including in class, assignments, tests, exams, homework; should be some elements of “technology-free” assessment in the course; should be limited to specific examples where it is useful, chosen by teaching staff (give example if possible).

C2 Please select the response that best represents your view about the ideal use of calculators or computers

<table>
<thead>
<tr>
<th>Amount of emphasis</th>
<th>Little or None</th>
<th>Heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) Calculators for numerical/computational purposes</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>(ii) Calculators for graphing purposes</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>(iii) Calculators for symbolic manipulation</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>(iv) Computer software (e.g. Matlab, Maple)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>(v) Modifying existing software or programming</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>(vi) Spreadsheets or tables</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
</tbody>
</table>

Comments?

C3 Do you think technology has changed the nature of mathematics or mathematics teaching? Please give any examples you can.

Part D: General

D1 What barriers/difficulties have you encountered or observed with regard to technology use and/or implementation in your course(s)?

D2 What changes would you like to see with respect to the use of technology in your course(s)?

D3 What changes do you anticipate in your course(s) in future because of technological developments?

D4 Any additional comments?

Thankyou for your participation! 😊
## (ii) Final Survey: Responses

### Part A. A1: Course Details

<table>
<thead>
<tr>
<th>Type of Course</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pure Mathematics</td>
<td></td>
</tr>
<tr>
<td>1st Year Calculus and/or</td>
<td>16</td>
</tr>
<tr>
<td>Linear Algebra (many not</td>
<td></td>
</tr>
<tr>
<td>differentiated)</td>
<td></td>
</tr>
<tr>
<td>Geometry</td>
<td>4</td>
</tr>
<tr>
<td>2nd and 3rd year courses: e.g. multivariable calculus</td>
<td>10</td>
</tr>
<tr>
<td>Differential Equations (taught in both pure and applied courses)</td>
<td>5</td>
</tr>
<tr>
<td>Applied Mathematics</td>
<td></td>
</tr>
<tr>
<td>Business</td>
<td>6</td>
</tr>
<tr>
<td>Discrete Mathematics/Computing</td>
<td>6</td>
</tr>
<tr>
<td>Engineering</td>
<td>6</td>
</tr>
<tr>
<td>Statistics</td>
<td>9</td>
</tr>
<tr>
<td>Bridging Courses</td>
<td></td>
</tr>
<tr>
<td>Elementary calculus/algebra</td>
<td>4</td>
</tr>
<tr>
<td>Teacher Education</td>
<td></td>
</tr>
<tr>
<td>Secondary Mathematics</td>
<td>3</td>
</tr>
<tr>
<td>Primary Mathematics</td>
<td>1</td>
</tr>
<tr>
<td>Unspecified Courses</td>
<td></td>
</tr>
<tr>
<td>General Departmental Response</td>
<td>2</td>
</tr>
</tbody>
</table>

### A2: Type of Technology

<table>
<thead>
<tr>
<th>Type of Technology</th>
<th>Product</th>
<th>Frequency Reported</th>
<th>Where and How Product Used? Type of Course, Restrictions etc.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Assessment:</td>
<td>Locally developed, e.g. CAA, Cecil</td>
<td>3</td>
<td>Online access to learning management systems, e.g.</td>
</tr>
<tr>
<td>Computer-Aided or</td>
<td>Online quizzes</td>
<td>6</td>
<td>WebCT, Cecil, ALEKS for quizzes, or lab-based assignments and tests using course-software, electronic assignment submission.</td>
</tr>
<tr>
<td>online</td>
<td>Lab-based assignments, tests</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>Calculators:</td>
<td>Scientific</td>
<td>9</td>
<td>Usually permitted in support of other technologies, but often not allowed in tests/exams.</td>
</tr>
<tr>
<td>Graphics</td>
<td>5</td>
<td>Permitted, but as support, not as main technology source.</td>
<td></td>
</tr>
<tr>
<td>CAS</td>
<td>3</td>
<td>Principal technology in course</td>
<td></td>
</tr>
<tr>
<td>CAS-enabled,</td>
<td>Journey through Calculus Maple</td>
<td>10</td>
<td>Student and staff use</td>
</tr>
<tr>
<td>graphing and analysis</td>
<td></td>
<td>4</td>
<td>Staff-only demonstration Favoured in engineering, higher-level mathematics.</td>
</tr>
<tr>
<td>software</td>
<td>Matlab</td>
<td>9</td>
<td>Student and staff use</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Staff-only demonstration More first-year than Maple.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Mathematica</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Scientific Notebook</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Scilab</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
A2: Type of Technology (continued):

<table>
<thead>
<tr>
<th>Type of Technology</th>
<th>Product</th>
<th>Frequency Reported</th>
<th>Where and How Product Used?</th>
</tr>
</thead>
<tbody>
<tr>
<td>Course Delivery</td>
<td>CD/DVD’s</td>
<td>3</td>
<td>Usually as complement to other products.</td>
</tr>
<tr>
<td></td>
<td>Datashows</td>
<td>19</td>
<td>Modelling of course technology</td>
</tr>
<tr>
<td></td>
<td>Interactive White-Board</td>
<td>2</td>
<td>Used as demonstration for specific examples, topics.</td>
</tr>
<tr>
<td></td>
<td>PowerPoint</td>
<td>9</td>
<td>Presentations, sometimes made available to students.</td>
</tr>
<tr>
<td></td>
<td>Tablets and screen-recording</td>
<td>4</td>
<td>Lectures delivery and recording for student playback.</td>
</tr>
<tr>
<td>Geometric software &amp; other drawing.</td>
<td>Cabri Geometers</td>
<td>2</td>
<td>2nd year Linear algebra, Geometry (non)-Euclidean Geometry</td>
</tr>
<tr>
<td></td>
<td>SketchPad</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>MathCad</td>
<td>1</td>
<td>Engineering</td>
</tr>
<tr>
<td>Internet: Delivery and web-based products</td>
<td>Applets</td>
<td>3</td>
<td>Source of shareware for specific topics, examples.</td>
</tr>
<tr>
<td></td>
<td>Web-based courses (ALEKS, WebCT)</td>
<td>4</td>
<td>Distance learning. Whole course delivered online.</td>
</tr>
<tr>
<td>Programming analysis, solving packages.</td>
<td>C++</td>
<td>1</td>
<td>Discrete mathematics courses taught in conjunction with computing, numerical analysis, algorithmic programming and differential equations courses.</td>
</tr>
<tr>
<td></td>
<td>Excel &amp; other spreadsheets</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>IDE (Interactive Differential Equations)</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>J</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Java</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>LogicWorks 4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>VisualBasic.Net</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>Statistical Analysis:</td>
<td>Excel</td>
<td>7</td>
<td>Used because of greater ease of student access, and familiarity. Only as demonstration.</td>
</tr>
<tr>
<td></td>
<td>Minitab</td>
<td>4</td>
<td>Variety of products here. Some indicated multiple use. No clear indicators for preference. Frequent mention of combination with calculator use. Usually not differentiated between scientific or CAS, mostly for computational use.</td>
</tr>
<tr>
<td></td>
<td>QuickSTAT</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td></td>
<td>R</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SAS (JMP)</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>SPSS</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Stata</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>WebSTAT</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>
(iii) Part B and Part C: Responses Difficult to Categorise Using Taxonomy in Table 5.2

Responses are identified by anonymous code, according to country of origin.

**B1**

N4: NZ Officially, calculators are banned in tests and exams, but as indicated above, we have the discretion to ignore this. In numerical papers in particular it is recognised that this rule may not be appropriate. The calculator ban came after a discussion in the department several years ago. People have come and gone in the meantime, but I suspect a department meeting would still endorse the ban.

There are university teaching goals which say we are going to incorporate new technologies into our teaching, but these are vague (referring possibly to distance education or possibly to the day-to-day workings of each course) and it’s yet to be seen how these goals will be implemented.

Curriculum review has never involved deciding how a course should be taught. That is left to the professional judgement of the teaching team. (This is why technology has been able to be used, even though the majority sentiment seems still to be that technology hinders rather than helps mathematical skills and possibly even understanding.)

H1: NZ The use of Matlab is part of the syllabus. The use of calculators and WebCT is a decision of an individual.

H10: USA The department has no policy on technology use by faculty: any faculty member has (in theory) complete freedom in any course he or she teaches. In particular, this applies to any post-calculus courses, which are always taught by faculty. For non-faculty instructors in courses at calculus or pre-calculus level, the policy will be set by the course co-ordinator. At present, no co-ordinator allows technology beyond numerical calculators.

**B2**

N6: Australia The move away from using Maple was two fold.

1. Students were coming into engineering with very poor backgrounds and the time used in teaching and using Maple was put into building up algebraic and functional skills
2. Maple was used for assessment. There was no way of knowing that the assignments being submitted were the work of the individual or not. With over 450 students it was too difficult to monitor.

G8: Sth Africa Internet tests, and on-line formative assessment.

H4: USA Business: yes, the course is under constant discussion with the Business College. Learning the technology was one of the central purposes of the course. It is expected that students of public administration will be able to use real data in the way that it is used in government----which means Excel.

H6: Australia There has been changes in order of topics; some topics have been easier as the arithmetical procedures and such are eliminated with the use of graphic calculators. i.e. Regression Analysis, curve sketching etc Though some staff insist that the students still complete these calculations by hand.

H7: France The students of 5th university year should now pass a national certification on the use of technology called “C2I”. Each student must be able to show that he was
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able to use technology in a given number of courses. It means that he or she asks
the corresponding teacher to certify that s/he did have a satisfactory use of tech in
the course.

H10  The department has discussed the possibility of introducing technology in its pre-
calculus courses, although the last official discussion was about 10 years ago. The
final decision has always been to maintain the status quo; i.e. no technology. The
two reasons which seem to carry weight are: (a) our pre-calculus courses are
precisely that: preparation for calculus; and students need to improve their (pencil-
and-paper) algebraic skills in order to succeed at calculus; (b) purchase of
technology, such as a graphing calculator or student copy of Maple, would be an
additional financial burden on many of our students.

The biggest change has been in the approach to integration. I now spend far more
time on the definition of the integral as a limit of Riemann sums, and have
students solve integration problems as soon as this definition is introduced, by
setting up Riemann sums and either taking the limit, or just computing with a
large number of subintervals. Conversely, I spend much less time on techniques
of anti-differentiation, but stress to the students that any alleged anti-derivative
must be checked by differentiation.

Since I am almost the only instructor in our department who uses technology
consistently, I have to use the same book as is used in other sections of the course
(currently Stewart’s “Essential Calculus”).

H12: USA  For us, courses drive technology rather than technology driving courses.
Technology is seen as a tool for accomplishing teaching objectives

B3  N6  Students may use any hand held calculator, but in exams they must show full
written working to reach the answer. So the calculator is often used to check the
result.

B4  G7: Australia  No. Some staff are resistant to the use of technology and give little to no time or
effort to the consideration of teaching with it.

H13: Australia  Yes. At our university there must be equivalence in delivery and assessment in
subjects offered in more than one stream and/or campus.

B5  H3: Australia  Some students avoid software and do not do that part of subject (may lose 10%).
Other subjects you cannot do this and so the students use the labs.

C1  N1: USA:  For pre-college mathematics, computer adaptive testing is the only practical way
to implement mastery learning, which I believe is essential for these students. For
the most part I think technology in calculus courses should be confined to
numerical methods, which is where I have my students use it. Since most of our
students in calculus one have been contaminated by poorly taught secondary
school courses it is not possible to use technology for discovery in calculus.

Its value lies in computation. It does not help with concepts. A case in point is
graphing. For example, if you graph by hand you can feel inflection points.
Furthermore students learn to ignore the inessential and focus on the essential
features of a graph.
Technology should be used only when it is the only alternative or when students have demonstrated that they are able to demonstrate that they only need the technology to solve problems faster.

N2: Australia No special advantages for students who don’t wish to learn. In fact, they may even think less about the content, because having pressed buttons B, C and D they feel their job is done. Certainly beneficial when students are involved in the subject material and makes the mathematics relevant to their computer studies.

Technology per se has no significance for education. One can learn about multiplication and addition without an abacus, but if the student is thinking about what he is doing then an abacus can highlight the significance of a place valued numeral system.

N6: I believe that students should have a working knowledge of the principles of mathematics without the use of calculators. That tedious calculations should be done on calculator, that real world problems be informed by technology. In maths assessment I would have a non-calculator part and another part where technology is expected and the questions of such a type to demand the use of technology.

G2: NZ Great for many of the practical things we do with large data sets. Sometimes though students are putting their focus on doing the right thing with the software rather than thinking about the process.

G3: NZ Technology is part of maths world; ∴ I believe students should be encouraged to use it. I believe it is great for developing intuition & practice etc. Some staff believe students will lose ability to do routine calculations.

Should be an integral part of most courses. Some courses don’t yet have extensive material on-line or through software. These are mainly Pure Maths topics…Its harder to use technology in these areas.

The big problem is how much in the way of skills do students need to have and how much is OK if they rely on technology to do algebraic manipulation etc.

G4: UK Technology is still continuing to evolve but it is now reasonable to expect students to have full access even using their own kit.

This is the 64,000$ question. The best and most proven use of technology is in quick and automated feedback via e/mail and website. We have a resident e/mail help site to which students can write with questions. It is rarely abused and feedbacks on lecture queries, tutorial questions, and general admin come very quickly.

I have been involved with computerised assessment for many years. It works very well at a formative level. A computerised formal test or exam can impose an additional stress on students and must only be introduced when students are fully experienced in its use from formative assessment experience.

G5: Australia I think technology can be very advantageous in tertiary maths teaching. However, students first need to get over the mindset of technology as simply something to save them thinking and understanding, and view it instead as a means of enhancing their thinking and understanding, eg through visual/graphical facilities.

Should be used where it can enhance understanding, but limited elsewhere. Maths is after all about thinking and trying to gain understanding of concepts. There
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should be technology free components of assessment to assess thinking/reasoning skills.

G6: Uruguay It has many benefits if all the students can reach almost the same technology...otherwise it creates important differences between them. If technology is reachable to all the students, it should be used in all appropriate opportunities.

G7: Australia Used well, technology can be a valuable tool in teaching undergraduate mathematics. It can allow for the consideration of patterns and concepts which are often obscured beneath tedious or difficult calculations. It can also allow the students to attack non-trivial applied problems and motivate their desire for abstract solutions rather than a single specific solution.

I consider it a valuable tool in the discovery and illustration of key concepts. My experience is that students view it initially as a convenient path to the right answer but, eventually, come to use it as a conceptual aid. Some of my colleagues feel it has a negative impact on students’ technical ability.

Technology should be integrated only by staff who believe it is useful. Imposition of technology seems to have a negative effect on all involved. Once adopted, I think it should be integrated as much as possible. To achieve the best results, students should be able to conceptualise through the use of technology whenever they feel it is necessary. I don’t think any assessment need be "technology free”. It is certainly possible to test all key concepts in such a way that technology is simply an adjunct.

G8 It is essential for more than one reason, e.g. visual possibilities, symbolic features, assessment. Should be integrated into a blended course but not entirely replace teaching.

H1 Technology is mostly useful but one needs to design the course delivery and assessments carefully to make sure students develop critical thinking and do not rely on technology entirely. I like to think that technology helps students understand concepts as well as provides a computational tool.

I think that any mathematics course should have elements of “technology-free” assessment. There is no substitute to human contact and human thinking. Critical thinking and ingenuity will still be important even when computers could eventually exhibit “common sense”. For this reason keeping some of the mathematics teaching “technology free” is important.

H2: Canada We think that we also provide students with ways of helping their learning and understanding of maths. For students, sometimes at the beginning it is difficult to make sense of the use of technology, especially those who had High School maths teachers with strong opinions against the use of technology. For some students, they simply feel it is the way to go.

At appropriate opportunities - I agree in the way it is integrated in (our) program.

H3 Transition to professional work: Must use professional tools. Integrated in curriculum: technology used when appropriate. Cost of licence means students mostly using at uni, need home use.

Fully integrated technology in class/assessment/exam. Chosen by student. Part of learning is to use and choose appropriate tools.
Its part of the way the world works and its part of the way mathematics is done professionally. Thus it should be part of courses too.

I feel that the use of technology should be integrated into the main stream of many of the first year service units rather than being viewed as an optional extra. The technology has to be carefully planned so that it does not create more difficulties than it is suppose to solve. At the moment the version of the online quizzes bear little resemblance to the examination questions and cause problems due to errors in translating the answers to Maple. This is not to say that Maple, itself, is bad but students do not see the reason behind having to use Maple.

Unfortunately because I am only a tutor I have little control of the use of technology. As I have stated before, the online quiz is used to save time mainly for the staff though some students spend 5-6 hours on questions worth only 0.2 %. They would never do that in an exam or test. I have tried to incorporate the use of graphic calculators in the realm of understanding concepts, mainly with the graphing component, though it is limited only to tutorial and greatly limited by time. I feel it the graphics calculator is used only as computational device in many units by my colleagues (those who will let students use them) and unfortunately by me when I am a tutor (I have to complete the allotted questions or go over theory).

When students go out and work in the real world, they are expected often to be able to use many different types of software to perform mathematical and statistical operations. So I feel all types of technology should be embraced and used realistically. CAS-calculators are not allowed in first year service units at all which is a pity as I found them interesting when shown a demonstration of their use. I had not used them before or after as it happens. I work in a mainly anti-calculator department on the whole.

Quite a few of the staff seem to think it is cheating to use the calculator and using it limits mathematical ability. So when you are a tutor, if the unit controller does not like technology, you are teaching topics that are quite out-dated as it can easily be done on a computer or calculator.

benefits: maths become more experimental, students can try, receive feedback, try other strategies, visualisation enhanced giving a meaning to formal aspects of maths.

Disadvantages: tech may favour trial and error strategy without reflection, time needed for learning how to use the tech.

Technology should be available at all appropriate opportunities including in class test exams and home work. It would be fruitful to ask students to be able to use different technologies for solving the same problem. I also consider that it would be good not to ask students to justify every result given by tech but to use the results given by tech in order to go further in the problem.

I think that tech changes the nature of maths and of maths teaching, in particular in the case of dynamic geometry that I know better than other kinds of software. In DG (dynamic geometry) the diagram is a variable diagram, a class of drawings and not at all of the same nature as a paper and pencil diagram. It behaves and reacts ‘mathematically’ when dragged. So it offers to students an environment reifying geometrical objects and properties. This does not mean that students will immediately become genius in geometry as they still have to interpret what they see. But the teacher can help them progressing in this interpretation process.
H9: NZ As long as it doesn’t require lots of technical expertise to get to grips with the technology then I believe it can enhance the learning experience. Lots of standard reasons here. E.g. more realistic examples can be explored. (We consider street networks with up to 20 unknowns!) It frees the student from the drudgery of calculation to concentrate on the maths. You can try guess and check techniques. Patterns can be seen easily and conjectures made etc. In math 242 mainly computational but definitely helps understanding in many instances. Particularly good in iterative type situations eg dynamical systems where you are predicting future growth etc.

Only where it is appropriate and enhances the learning experience. For example in the modern algebra course I teach I have not introduced the GAP package (which I use in my own research) because it is not particularly user friendly and I don’t see it being that helpful at an introductory course.

H11: USA I have worked steadily at bringing technology into all of my courses…not very successfully in the early days, but much more so as it became more portable. Benefits: Current students have never known a world without technology, and it is criminal for education to ignore that fact. Disadvantages: Temptation toward over-reliance on technology without understanding fundamental ideas. Conclusion: Education must focus on understanding.

H12 Technology is a useful tool for exploring mathematics & statistics – in particular, exploring more complicated examples & illustrations — and more of them. Too many students see the lab exercises as just one more set of hoops to jump through, instead of seeing them as more sophisticated opportunities for cementing their understanding the concepts.

Technology should be used as a tool for understanding, exploration, and (potentially) for assessment. There must be some technology-free assessment in most courses.

H13 My personal experience is that it is difficult to strike the right balance of using technology to aid student understanding and to avoid unnecessary hard algebraic work, but also being able to think and do mathematics without a heavy dependence on technology.

It depends on the objectives of the subject. In some technology would be more integrated than in others. But a student who completes a major in mathematics should be able to use technology effectively and efficiently.

H14: NZ It should be available and used at all appropriate opportunities including in class, assignments, tests, exams, homework at least from year 2.

H15: Australia Technology has potential to facilitate conceptual understanding as opposed/in addition to procedural knowledge. Some students find it very difficult to change over from graphics calculator to CAS calculator.

Students value the ease of calculation and preparation of graphs. Few use calculators to investigate concepts. Students in the probability unit are impressed with the use of Excel for simulations and some say it helps them to understand better. Eg. some were surprised to see so graphically that a gambler will lose money in the long run.

Technology should be integrated into undergraduate courses wherever it assists understanding or allows for bigger &/or more complex data sets to be explored.
The best available software should be used, not something regarded as “good enough for education”. What is “best available” depends mainly on the design of the software and its notational features.

I don’t have a uniform view about any of these issues. For each course I think about how best to get across the key ideas. If technology can help then I use it to whatever degree seems appropriate, if not then I don’t use it.

Let it roll on and find its own level without its introduction being forced.

Yes, but not always in a desirable way. The pernicious use of calculators, in particular decimal approximations of rational numbers, in secondary and primary school, is producing generations of student with no number sense.

There is no doubt that the teaching of mathematics has been changed by the emergence of powerful numerical capabilities and computer algebra systems. Very few working mathematicians would now be struggling on without the help of a CAS. The field of differential equations has been completely revolutionized by the ability to obtain numerical solutions of spectacular accuracy on a desktop machine.

Get rid of these ridiculous graphics calculators in schools and use Excel instead. All sorts of modelling and simulations may be done. The students enjoy it – I have done it for 12 years or more at Bond. I can tell you that they all like Excel.

Absolutely, yes. I would use a totally different approach in my AT subject if I did not have Excel and LogicWorks, plus the web. I think it is crazy to ignore it nowadays. We are already perceived as almost irrelevant by large sections of the community (mathematicians, that is), and even by university administrations. If we bury our heads in the sand and say that we don’t need technology, then this will only distance us further from others who are taking advantage of computers. We need to use technology to show the beauty and power of maths, and that maths is ubiquitous. Far from needing less maths, we now need it more than ever, as new IT applications continue to emerge.

I don’t think it has changed mathematics, but in some situations it has broadened the ways you can use to approach maths/stats and hence probably made those subjects more accessible to more students.

I have become very disconnected from our intro level courses, many of which soon will not be taught by our new school of math anymore at all (e.g. anything w/ graphing calculators), and I really don't know about their current policies, anyhow I can't stand gc.

I think it should, but our students are so far behind in understanding the basics that calculators often interfere with there learning. They think that pushing buttons on a calculator is maths.

Technology is changing mathematics. Numerical solutions, e.g. to P.D.E’s and Gröber Basis work. Teaching is catching up.

Don’t think it’s changed the nature but certainly has opened up new areas. Has changed teaching in that it’s so easy to look at graphs of complicated functions and examine different scenarios.
Yes. Maths is now a laboratory subject. This is good. There have been notable failures and frustrations in misguided overzealousness in introducing some computerised ventures.

Yes, it has. It enables computations to be done with ease, so that rather than spending time on tedious calculations, more time can be spent on understanding the concepts/processes. It can also enhance understanding through visual means.

Yes, it has changed a lot of things: the way of teaching (in the computer lab, etc.), in the assessment (projects where Maths software is widely used) and even in the syllabus...I remember when I was a teenager the algorithm for the square root used to be taught at Secondary School...fortunately this was changed long time ago!

Mathematics has moved forward in leaps and bounds through the advent of computers. Take, for example, the area of optimization. Only with the ability to do large scale computations has this branch of mathematics really taken off, but it is now one the most dynamic growth areas in Mathematics. Mathematics teaching has adopted technology very slowly (much more slowly than mathematicians) but has also been very conservative in its use once adopted. CAS have been widely available for at least 15 years but in Australia they are only being routinely introduced into schools and Universities this year. This has been married to virtually no change in the manner in which maths is taught. So no, technology has not really changed mathematics teaching up until now.

Yes. The study of dynamical systems and chaos is a classical example of changes in the nature of mathematics which also led to changes in the way differential equations is taught. Calculus was also affected (see for example the latest editions of Calculus by Stewart).

It certainly did: see (our) program, e.g. the courses that provide very unique mathematics experience to students (1st year students conjecturing and designing and implementing an exploratory plate-form to test their conjectures!). It did change the practices of the mathematicians (e.g. my colleague working in Number Theory uses Maple to check the plausibility of some of his conjectures before attacking the proofs).

Absolutely. Notation is different. Less time in teaching procedures, more visual. Able to do more difficult tasks earlier. Games and imaginative tasks, e.g. use Mathematica to draw a cat in 3D.

Yes, it has. It is now much easier to have students
- Reason graphically
- Understand the power of numerical approximations
- Experiment and learn to conjecture from their results
- Work with real data and real models.

To some extent, but not as much as predicted when computer algebra systems were introduced. We changed the balance since we started using CAS. At the beginning we emphasized more conjecturing and experimentation, but noted that students were not learning as much as we hoped, and their symbol manipulations abilities diminished, so we put back in more formal algebraic manipulations.

Less emphasis on techniques, more powerful visualisation
H15 I think it has potential to change the nature of mathematics teaching, but the uptake of its use is dependant on the willingness of teaching staff to embrace new/better ways of teaching current and future curriculum. Current curriculum is challenged by technology, but only the brave take up the challenge.

Part D: Coding Schedule for Responses.

<table>
<thead>
<tr>
<th>Question</th>
<th>Category of Response (coding)</th>
<th>Frequency and Sample Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>D1: What barriers/difficulties have you encountered or observed with regard to technology use and/or implementation in your course(s)?</td>
<td>Cost of hardware, software, licensing fees for departments and/or students, especially developing countries.</td>
<td>7 The major barrier is the cost of hardware and software and the fact that not all students can afford to have the latest and best.</td>
</tr>
<tr>
<td></td>
<td>Functionality: Technical problems, inadequate IT support</td>
<td>4 You are often reliant on others (IT people etc) to set up labs and maintain them. Equipment often fails at the least opportune time.</td>
</tr>
<tr>
<td></td>
<td>Infrastructure: Laboratories too small, limited access; Have to share equipment, e.g. datashows, laptops for demonstration.</td>
<td>4 The lab space is limited so only so many classes in one semester can have access to software available in the lab.</td>
</tr>
<tr>
<td></td>
<td>Departmental or Institutional policies or structures, e.g. imposed technology, existing curricula etc.</td>
<td>3 Bureaucracy slow to change. Use often isolated to single course.</td>
</tr>
<tr>
<td></td>
<td>Resources: Non-availability and cost of new textbooks, support material for technology.</td>
<td>3 You no sooner adopt a book and set up a course based on the text when a new edition comes out with “fully revised problems”!</td>
</tr>
<tr>
<td></td>
<td>Staff Attitudes and Beliefs: e.g. resistance to change or negative beliefs about technology.</td>
<td>7 Quite a few of the staff seem to think it is cheating to use the calculator and using it limits mathematical ability. So when you are a tutor, if the unit controller does not like technology, you are teaching topics that are quite out-dated as it can easily be done on a computer or calculator.</td>
</tr>
</tbody>
</table>
### Part D: Coding Schedule for Responses continued.

<table>
<thead>
<tr>
<th>Question</th>
<th>Category of Response (coding)</th>
<th>Frequency and Sample Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D1: continued</strong></td>
<td>Staff: Time and energy to prepare new resources, keep up to date with new technologies, curricular changes etc</td>
<td>10 Generally it requires more time to prepare a course making use of tech, all files must be ready.</td>
</tr>
<tr>
<td></td>
<td>Staff Proficiency with technology: Lack of knowledge, training, not aware of new pedagogical implications for students (inadequate PTK).</td>
<td>4 Getting teachers familiar with the benefits of use. It requires more attention from the teacher to understand the solutions made by students, since the range of possible solutions (correct and incorrect) is larger than in paper and pencil.</td>
</tr>
<tr>
<td></td>
<td>Students: Attitudes and Beliefs.</td>
<td>5 A more positive attitude from students as to what it can contribute to their learning and applications of mathematics.</td>
</tr>
<tr>
<td></td>
<td>Student Instrumentation: Too reliant on technology, don’t know when mathematically appropriate, software hard to learn, too much time spent on technology. None, or no long-term ones. In some cases due to limited use anyway.</td>
<td>7 Bringing the students to make sense of the use of technology for their learning and doing mathematics.</td>
</tr>
<tr>
<td></td>
<td>Early on, the barriers were cost, resistance from colleagues, and unsuitable classroom configurations. There are very few barriers now.</td>
<td>5 Early on, the barriers were cost, resistance from colleagues, and unsuitable classroom configurations. There are very few barriers now.</td>
</tr>
</tbody>
</table>
### Part D: Coding Schedule for Responses continued.

<table>
<thead>
<tr>
<th>Question</th>
<th>Category of Response (coding)</th>
<th>Frequency and Sample Responses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>D2:</strong> What changes would you like to see with respect to the use of technology in your course(s)?</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>More and better use generally. In some cases just some use!</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>More integrated, consistent, planned approach.</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Specific Technologies: e.g. student laptops, more CAS, more Excel, J.</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>Better Access, ease of use for staff and/or students, e.g. Virtual labs, better IT support.</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>Better Use: Recognise value of technologies students bring with them (e.g. calculators); more computer-aided assessment. More time to teach technology skills.</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Improved Technology: Maximise mathematical benefits, better user interfaces.</td>
<td>3</td>
</tr>
<tr>
<td></td>
<td>Student factors: e.g. attitudes, school exposure to technology.</td>
<td>3</td>
</tr>
</tbody>
</table>
Part D: Coding Schedule for Responses continued.

| Question                                                                 | Category of Response (coding)                                                                 | Frequency and Sample Responses                                                                                                                                                                                                 |
|--------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------|                                                                                                                                                                                                                             |
| **D3:** What changes do you anticipate in your course(s) in future because of technological developments? | Greater Use: Communication Technology, Internet, distance learning; self-determined, less traditional lectures etc; greater use in demonstrating technology use, online resources. | 5 More distance teaching and interactive learning, where students learn at their own pace, instead of the large lecture streams we have at the moment. 4 (More) Students will have laptops. This will break us out of labs. Students of next years should be more familiar with technology and techno phobia should disappear. 4 I think the new "manipulate" control function from Mathematica will bring significant changes in our service courses (we are waiting for its equivalent in Maple). My university is currently piloting the use of wireless tablets and interactive lectures by interconnecting all students’ tablets. I am still figuring out how this technology could be used to improve maths teaching and learning. Currently looking at incorporating TI-Nspire in calculus units. 5 More integration of tech across subjects in a degree. More use to help understanding, not just for computational and applied purposes! Courses will be more dynamic, with no long calculations, centred in the concepts more than in algebraic manipulations. 5 The traditional three-hour written exam has been with us for decades if not centuries. Computers will become increasingly better for the testing of key mastery skills in all subjects. They are complementary. 4 Not in the foreseen future – Matlab is very powerful. |
|                                                                          | Better Student Access, familiarity.                                                            |                                                                                                                                                                                                                             |
|                                                                          | Better Technologies: more mathematically appropriate, e.g. decision-making, TI-Nspire; More interactive. |                                                                                                                                                                                                                             |
|                                                                          | Better Use: Better integrated, more appropriate use, more acceptance of CAS, changes to courses, e.g. less procedural. |                                                                                                                                                                                                                             |
|                                                                          | Assessment: More computer-aided assessment, changes to dominance of exams, traditional assessment measures. |                                                                                                                                                                                                                             |
|                                                                          | Little change: Resistance, status quo OK, too costly.                                          |                                                                                                                                                                                                                             |
Interview Protocol: Sample Questions

1. Please describe your personal experiences in using technology for:
   - mathematics research
   - teaching mathematics
   What differences do you see in technology usage between these two different domains?

2. What are your personal beliefs and/or philosophy about the nature of mathematics, mathematics teaching, and the role of technology in these?

3. What benefits and/or disadvantages do you see in the use of technology in teaching mathematics?

4. Can you think of any examples of mathematics that we have taught in the past which have been taken over by technology? Is this a good or bad thing? Has this been considered, or has it just happened by de facto because technology can do it?

5. Can you think of any examples of mathematics that we currently teach, that we should reconsider (either in content, or sequencing), in light of technological developments?

6. What problems have you encountered with using or implementing technology into courses you have been involved with teaching or developing?

7. What research (if any) are you aware of with respect to the use of technology in teaching mathematics, especially at the tertiary level?

8. Do you have a preference for the type of technology that should be used in tertiary mathematics courses, e.g. calculators? Computer software packages? Please explain.

9. Most CAS-systems calculate row-reduced matrices, determinants, inverses, and directly calculate solutions to sets of linear equations. In light of that, do you think we still need to teach Gaussian elimination? Consider whether technology has changed its pragmatic or epistemic value (interview subject to be shown Figure 2.8).
CALCULUS QUESTIONNAIRE (Lauten, Graham and Ferrini-Mundy (1999))

Please answer all questions and comment as often as you wish.

1. CONTENT:

1A. Please indicate the level of emphasis that you feel should be placed on the following topics in a first-year calculus course.

<table>
<thead>
<tr>
<th>Amount of emphasis</th>
<th>little or none</th>
<th>heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Preliminaries (functions, absolute value, etc.)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>b. Limits (lengthy treatment, rate “heavy”)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>c. Derivative as a rate, slope of tangent line, etc.</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>d. Using definition to find derivative</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>e. Techniques of differentiation</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>f. Applications of the derivative (max/min, related rates, etc.)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>g. Techniques of integration</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>h. Fundamental Theorem of Calculus (lengthy treatment, rate, “heavy”)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>i. Techniques of integration</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>j. Applications of integration (arc length, volumes of solids, surface area, etc.)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>k. Applications of exponential/logarithmic functions (growth, decay, etc.)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>l. Solving differential equations</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>m. Applications of differential equations</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>n. Series (lengthy treatment, rate “heavy”)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>o. Series – techniques to determine convergence</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>p. Taylor series (lengthy treatment, rate “heavy”)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>q. Applications of Taylor Series</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>r. Parameterisations, Vectors</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
</tbody>
</table>

1B. Please add any other topics that you feel should be included, and any other remarks concerning content. Technology will be addressed separately.
2. THEORETICAL METHODOLOGY:

2A. Please indicate the level of emphasis that you feel should be placed on the following methods in a first-year calculus course. It might be helpful to think about how much emphasis you place on these items in your assessment of students. Technology will be addressed separately.

<table>
<thead>
<tr>
<th>Amount of emphasis</th>
<th>little or none</th>
<th>heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Formal definitions</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>b. Statements of theorems, counterexamples, etc.</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>c. Proofs of significant theorems</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>d. Historical themes in mathematics</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>e. Writing assignments</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>f. Student practice of routine procedures</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>g. Applications of real world problems</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>h. The analysis and solution of non-routine problems</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
</tbody>
</table>

2B. Please state your own definition of the phrase “mathematical rigor.”

2C. Based on your definition, please describe what level is appropriate in first-year calculus and how you would assess grade, for example, at that level.

3. TECHNOLOGY FOR LEARNING:

3A. Please state your feelings about the role of technology in the classroom, in promoting or hindering learning in a first year calculus course.

3B. To what extent do you feel technology should be integrated throughout the course versus for special projects, if at all.

3C. Please select the response that best represents your views about the ideal use of calculators or computers in the classroom.

<table>
<thead>
<tr>
<th>Amount of emphasis</th>
<th>little or none</th>
<th>heavy</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Calculators for numerical purposes</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>b. Calculators for graphing purposes</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>c. Calculators for symbolic manipulation</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>d. Computer courseware (Maple, Mathematica, etc.)</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>e. Modifying existing programs/Programming</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>f. Spreadsheets or tables</td>
<td>1 2 3 4 5</td>
<td></td>
</tr>
</tbody>
</table>
4. CASSROOM TEACHING APPROACHES

4A. Please respond: In an ideal calculus course, how frequently would your students use the following instructional systems?

<table>
<thead>
<tr>
<th>Instructional System</th>
<th>Amount of emphasis</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Use lecture notes as basis for learning</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>b. Participate in a specially designed calculus laboratory</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>c. Use concrete materials/equipment to explore calculus ideas</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>d. Work in small groups on mathematics problems</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>e. Work in small groups on projects that take several class meetings to complete</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>f. Practice calculus procedures in the classroom</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>g. Make conjectures, explore more than one possible method to solve a calculus problem</td>
<td>1 2 3 4 5</td>
</tr>
</tbody>
</table>

4B. Please add any additional comments. Perhaps you would like to address the phrase “ideal classroom.”

What types of support might your department in an ideal world provide you with to help you accomplish your teaching goals?

5. STUDENT ASSESSMENT/EVALUATION

5A. In your calculus course, what importance to course grade do you assign to each of the following items? (1) Please rate on the scale. (2) Please circle the methods of assessment that you would like to discuss.

<table>
<thead>
<tr>
<th>Assessment Item</th>
<th>Amount of emphasis</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Quizzes, tests, or examinations that measure individual mastery of content material</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>b. A final examination that measures individual mastery of content material</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>c. Individual tests of mastery of content material</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>d. Small group tests of mastery of content material</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>e. Lab reports – individual grades</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>f. Lab reports – group grades</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>g. Quizzes, tests, or examinations of material learned in labs</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>h. Homework exercises – individual grades</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>i. Projects – individual grades</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>j. Projects – group grades</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>k. Journals</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>l. Class participation</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>m. Portfolios</td>
<td>1 2 3 4 5</td>
</tr>
<tr>
<td>n. Other: Please describe:</td>
<td>1 2 3 4 5</td>
</tr>
</tbody>
</table>
6. PERSPECTIVES ON CALCULUS REFORM

6A. How aware are you of “calculus reform” issues and efforts? Please explain.

6B. What do you find encouraging about the directions of calculus reform?

6C. What are your concerns about the directions of calculus reform?

7. PERSPECTIVES ON THE CURRENT TEXT: Please write your comments about the text in use in your department? Do you want to change the current text? Please explain.

8. PERSPECTIVES ON THE CURRENT STUDENTS: It is important to share perspectives about the students we teach. Please give your impressions of the latest Calculus class you have taught, and share any comments from the students that you would like to pass on to members of the department.