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Random Effects Models for Ordinal Data

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Abstract

One of the most frequently encountered types of data is where the response variables are measured on an ordinal scale. Although there have been substantial developments in the statistical techniques for the analysis of ordinal data, methods appropriate for repeatedly assessed ordinal data collected from field experiments are limited.

A series of biennial field screening trials for evaluating cultivar resistance of potato to the disease, late blight, caused by the fungus *Phytophthora infestans* (Mont.) de Bary has been conducted by the New Zealand Institute of Crop and Food Research since 1983. In each trial, the progression of late blight was visually assessed several times during the planting season using a nine-point ordinal scale based on the percentage of necrotic tissues. As for many other agricultural field experiments, spatial differences between the experimental units is one of the major concerns in the analysis of data from the potato late blight trial.

The aim of this thesis is to construct a statistical model which can be used to analyse the data collected from the series of potato late blight trials. We review existing methodologies for analysing ordinal data with mixed effects particularly those methods in the Bayesian framework. Using data collected from the potato late blight trials we develop a Bayesian hierarchical model for the analyses of repeatedly assessed ordinal scores with spatial effects, in particular the time dependence of the scores assessed on the same experimental units was modelled by a sigmoid logistic curve.

Data collected from the potato late blight trials demonstrated the importance of spatial effects in agricultural field trials. These effects cannot be neglected when analysing such data. Although statistical methods can be refined to account for the complexity of the data, appropriate trial design still plays a central role in field experiments.
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# Table of Contents

Abstract .......................................................................................................................... i

Acknowledgements ........................................................................................................ ii

List of Figures ................................................................................................................... v

List of Tables ..................................................................................................................... x

Chapter 1. Introduction ................................................................................................... 1
  1.1. Ordinal Data ...................................................................................................... 2
  1.2. The Potato Late Blight Trials ......................................................................... 2
  1.3. Thesis Outline .............................................................................................. 4

Chapter 2. Literature Review ......................................................................................... 5
  2.1. Modelling the Distribution of Ordinal Response Variables ......................... 5
  2.2. Bayesian Modelling of Ordinal Data and Crop Variety Trials ....................... 16
  2.3. Conclusion ..................................................................................................... 22

Chapter 3. The Potato Late Blight Trial ....................................................................... 24
  3.1. Description of the Trial and the Data .............................................................. 24
  3.2. Traditional Analysis ...................................................................................... 29
  3.3. New Initiatives .............................................................................................. 35

Chapter 4. Preliminary Approaches ............................................................................. 36
  4.1. General Formulation for the Mixed Effects Models ...................................... 36
  4.2. Ante-dependence Model .............................................................................. 38
  4.3. Jansen’s Method ......................................................................................... 43
  4.4. Bayesian Approaches for Experimental Designs ........................................... 53
  4.5. Conclusion ..................................................................................................... 65

Chapter 5. Single Trial with Single Assessment ............................................................ 66
  5.1. Sum of the Repeated Scores ......................................................................... 66
  5.2. Ordinal Scores from a Single Assessment ..................................................... 70

Chapter 6. Single Trial with Repeated Scores – Continuous Outcomes ....................... 75
  6.1. Nonlinear Sigmoid Decline model in GenStat ................................................ 76
  6.2. A Bayesian Sigmoid Model for Continuous Outcomes .................................. 84
  6.3. Conclusion ..................................................................................................... 93

Chapter 7. A Bayesian Hierarchical Sigmoid Model for Repeated Ordinal Outcomes .... 94
  7.1. Model Specification ...................................................................................... 94
  7.2. Posterior Result ........................................................................................... 99
List of Figures

Figure 1 The distribution of a latent variable, cutpoints and the ordinal response variable. .......... 11
Figure 2 Effect of the shifting in the distribution of latent variable. ........................................... 12
Figure 3 Estimated mean disease scores with 95% least significance intervals using Hannah-Quigley method for presenting the result from ordinal regression in the original ordinal scale.... 13
Figure 4 Late blight progression during the 2003-2004 season trial ................................................. 28
Figure 5 Late blight progression during the 1999 trial ................................................................. 30
Figure 6 Ratios of variance components over time ........................................................................... 34
Figure 7 Distribution of ranks. WinBUGS ranks the smallest tensile strength to the largest tensile strength from 1 to 4. For the four mixing method, method 2 with rank 4 is the best mixing method.......................................................................................................................... 63
Figure 8 Histogram for the posterior samples of tensile strength obtained from WinBUGS. Dashed lines indicate sample means of tensile strength from ANOVA........................................... 64
Figure 9 Layout for the 2003 potato late blight trial. ................................................................. 66
Figure 10 Plot of the estimated variety means, the REML technique using GenStat vs. a Bayesian hierarchical using WinBUGS................................................................. 68
Figure 11 Estimated (a) row and (b) column random effects, REML using GenStat vs. Bayesian hierarchical model using WinBUGS with $y = x$ reference line........................................... 70
Figure 12 Estimated (a) row (b) column random effects from Bayesian hierarchical model using WinBUGS with lowess smooth curve........................................................................... 70
Figure 13 Plot of the estimated scores vs. the observed scores from a Bayesian hierarchical ordinal logistic model fitted to the fourth score of the 2003 potato late blight trial using WinBUGS with $y = x$ reference line................................................................. 73
Figure 14 Estimated (a) row (b) column random effects from a Bayesian hierarchical ordinal logistic model fitted to the fourth score of the 2003 potato late blight trial using WinBUGS with lowess smooth curve......................................................................................... 74
Figure 15 Observed mean blight scores over time for some selected varieties from the 2003 trial..... 76
Figure 16: The fitted sigmoid curves for the 2003 blight data using the GenStat FITNONLINEAR directive. The individual plots of land were modelled as homogeneous units ignoring cultivar, row and column effects. Note that the nine-points ordinal scores were transformed with a minimum and maximum bound of 0 and 1......................................................................................... 81
Figure 17 Fitted curves of the first 20 cultivars for the standard logistic model with cultivar effect using GenStat FITCURVE directive......................................................................................... 82
Figure 18 The observed scores (in the transformed scale) for cultivar 2, 3, 25, 48, 70 and 76. These 6 cultivars were problematic when sigmoid model were fitted to the data. ........................................ 83

Figure 19 Fitted standard logistic curves for the 2003 trial data using the transformed scores. ........ 86

Figure 20 History plots of the parameters of the standard logistic sigmoid model fitted to Ilam Hardy data from 2003 trial............................................................................................................................................. 88

Figure 21 History plots of the standard logistic model fitted to the 20 replicates simulated data. .... 89

Figure 22 History plots of the standard logistic model fitted to the 40 replicates simulated data. .... 89

Figure 23 History plots of the standard logistic model fitted to the 60 replicates simulated data. .... 90

Figure 24 Fitted curves for Ilam Hardy with 60 replicates simulated data in the transformed scale using standard logistic model using WinBUGS. ................................................................. 90

Figure 25 Observed cultivar means vs. The posterior sample means from fitting the standard logistic model using WinBUGS to the 2003 trial data with cultivar effect at each assessment. ...... 92

Figure 26 Two cultivars which were unable to reach convergence using GenStat FITNONLINEAR and FITCURVE directives. The red circles are the observed scores with jittering. For cultivar 70, the black line is the fitted curve at the final 1000th iteration. ............................. 93

Figure 27 The same two cultivars which WinBUGS provided good fit to the data. The red circles are the observed scores with jittering. The black lines are the fitted curves using the posterior sample means of the parameters from the standard logistic model. ........................................ 93

Figure 28 Flowchart for the development of informative prior distributions and initial values. ........ 97

Figure 29 Observed (in blue, jittered) and posterior sample means (in red) of the disease scores over time using a Bayesian hierarchical standard logistic model with row and column spatial effect for the first 20 cultivars of the 2003 potato late blight trial. ........................................... 102

Figure 30 Cultivar means of the observed scores vs. the posterior sample means of the cultivars for each scoring assessment with means and modes displayed at observed mean values, displayed with the reference line $y = x$. Lines in yellow indicated minimum and maximum score. ........................................................................................................................................... 103

Figure 31 Boxplots of the posterior samples of the row and column spatial effects displayed on the left and right respectively and overlaid with lowess smooth curves ($f=0.2$ and $0.5$). ............. 104

Figure 32 Extrapolated ideal cut-points vs. posterior sample means of the cut-points estimates from the Bayesian standard logistic model with regions indicated with the corresponding ordinal scores. ........................................................................................................................................ 106

Figure 33 Plots of observed scores vs. the posterior sample means of the latent scores with density overlaid as the red lines. The posterior sample means of the cut-points and the region of ordinal scales were also displayed as dash lines and numerals in blue respectively. (a) 2003 late blight trial. (b) 1985 late blight trial. ....................................................................................................................................... 107
Figure 34 Boxplots of the posterior predictive distributions for the first 25 plots of land with the observed scores displayed as red dots................................................................. 111

Figure 35 Boxplots for the distribution of the mean scores for each cultivar at each scoring occasion from posterior predictive simulation with red dots indicated the observed cultivar mean scores.................................................................................................................................. 113

Figure 36 Histogram of the posterior predictive p-values. ............................................................................. 114

Figure 37 Observed scores (jittered) and posterior sample means for cultivar 24, C. Royal from the 1987 trial using logistic model for ordinal outcomes (a) using result from the logistic model with continuous outcomes as initial values (b) using result from cultivar 29, Tekau, in 1985 trial as initial values. Fit of the model in (b) increased dramatically compared to (a). ...... 115

Figure 38 Five types of disease profile using a subjective visual method................................................. 117

Figure 39 Posterior sample means (in red) and observed scores (in black) for cultivar 21 to 35 for the 1987 late blight trial. The posterior sample means for cultivar 24, C. Royal, showed a significant improvement over the original model, see Figure 37. .............................................. 119

Figure 40 Histograms of posterior predictive samples of the row and column standard deviations, $\sigma_\kappa$ and $\sigma_\rho$, of the 1991 late blight data from models with four different prior distributions: (a) and (e) uniform prior distribution on $\sigma_\kappa$ and $\sigma_\rho$, (b) and (f) inverse-gamma(1, 1) on $\sigma_\kappa^2$ and $\sigma_\rho^2$, (c) and (g) inverse-gamma(0.01, 100) on $\sigma_\kappa^2$ and $\sigma_\rho^2$, (d) and (h) half-Cauchy prior on $\sigma_\kappa$ and $\sigma_\rho$. The inverse gamma distributions were defined by the shape and rate parameters......................................................... 125

Figure 41 Posterior sample means of the row and column effects..................................................... 126

Figure 42 Posterior sample means of the estimated blight score at each scoring occasion for each cultivar, labelled by the scoring occasion, with a reference line................................................. 127

Figure 43 Estimated late blight progression using posterior sample means of cultivar scores from individual trial analyses.................................................................................................................. 132

Figure 44 Overall means of posterior sample means over time for a selection of cultivars............... 133

Figure 45 Posterior distributions of the row effects from fitting a Bayesian hierarchical standard logistic sigmoid model, year 1983 to 2005. ........................................................................... 135

Figure 46 Posterior distributions of the column effects from fitting a Bayesian hierarchical standard logistic sigmoid model, year 1983 to 2005. ........................................................................... 136

Figure 47 Observed scores (jittered) and the posterior sample means for cultivar one to twenty, from a model with common cultivar effect and included the first seven assessments only. Green dots represent observed scores from the 1987 trial and the red dots represent the observed scores from the 1985 trial; where the black dots are the cultivar specific posterior sample means................................................................................................................. 140
Figure 48 Cultivar specific posterior sample means of the first seven assessments obtained from the model which included the first seven assessments vs. the model which included all eight assessments, with the reference line with intercept at zero and a gradient of one. 141

Figure 49 Observed scores (jittered) for cultivar 7 ‘1461.3’. 144

Figure 50 Observed scores (jittered) and posterior sample means by cultivar and year for cultivar one to twenty. Posterior sample means were obtained from the model including cultivar and year effects for the parameter $B$ and $M$ in the sigmoid decay curve. Pink and light green dots represent the observed scores (jittered); red and dark green dots represent the posterior sample means for the 1985 and 1987 trial respectively. 146

Figure 51 Observed scores (jittered) and posterior sample means for cultivar number 7, ‘1463.1’. Pink and light green dots represent the observed scores for the 1985 and 1987 trial respectively. (a) The model with cultivar effect only. (b) The model with cultivar and year effects allowing for varying slopes and points of inflections between different trial years but of the same cultivar. (c) and (d) The model with cultivar and year effects allowing for varying slopes, maximum, minimum bounds and points of inflections between different trial years but of the same cultivar. For figure (a), black dots represent the posterior sample means for this cultivar. For figures (b) to (d) red and dark green dots represent the posterior sample means for the 1985 and 1987 trial respectively. 149

Figure 52 Observed scores (jittered) and posterior sample means for cultivar number 7, 19, 47 and 48. Pink and light green dots represent the observed scores for the 1985 and 1987 trial respectively. Pink and light green dots represent the observed scores (jittered); red and dark green dots represent the posterior sample means for the 1985 and 1987 trial respectively. 152

Figure 53 The posterior distributions for the row and column effects with lowess smooth curves... 153

Figure 54 Boxplots of the posterior predictive distributions for the first 25 plots of land with the observed scores displayed as red dots. 154

Figure 55 Boxplots of the mean scores of the posterior predictive distribution for the first 25 cultivars at each scoring occasion from the 1985 trial with red dots indicated the observed cultivar mean scores. 155

Figure 56 Histogram of the posterior predictive p-values. 156

Figure 57 Estimated mean disease scores with 95% least significance intervals using Hannah-Quigley method for presenting the result from ordinal regression in the original ordinal scale..... 167

Figure 58 $DIC$ and its components obtained from a Bayesian standard logistic model for ordinal data with different precisions for the priors plotted against levels of the precisions for the highly significant parameters. 183

Figure 59 Observed vs posterior sample means at cultivar level, Model 1. 185

Figure 60 Observed vs. posterior sample means at plot level, Model 1. 186

Figure 61 Observed vs. posterior means at cultivar level, Model 3. 186
Figure 62 Observed vs. posterior means at plot level, Model 3................................. 187
Figure 63 Observed vs. posterior means at cultivar level, Model 5............................. 187
Figure 64 Observed vs. posterior means at plot level, Model 5................................. 188
Figure 65 Observed vs. posterior means at cultivar level, Model 6............................. 188
Figure 66 Observed vs. posterior means at plot level, Model 6................................. 189
Figure 67 Plots of posterior sample means, model 1 vs. model 6, cultivar level............ 189
Figure 68 Plots of posterior sample means, model 1 vs. model 6, plot level................. 190
List of Tables

Table 1 Number of potato cultivars assessed in the late blight trials. Diagonal cells are the number of cultivars in a single trial; other cells are the number of common cultivars in a pair of trials. 27
Table 2 Late blight scoring system. 28
Table 3 The 1999 potato late blight trial data. 30
Table 4 Estimated cumulative scores for each variety using REML for the 1999 late blight data. Note that the average LSD is displayed here as every pair of cultivar comparison has a different SED due to the unbalanced layout of the trial. 32
Table 5 Estimated variance components from the REML analysis for the combined data. 35
Table 6 ANOVA for the strawberry example. 56
Table 7 Data for tensile strength of cement example. 58
Table 8 Estimated variance components from the REML technique and a Bayesian hierarchical model. 69
Table 9 The estimates and standard errors for the parameters of the five sigmoid models using GenStat FITNONLINEAR directive. 80
Table 10 Residual sum of squares for the full blight dataset and the subset which excluded cultivars 2, 3, 25, 48, 70 and 76. RSS of the full dataset unavailable for model 2, 3 and 4 because parameter estimates were not available for some of the problematic cultivars. 84
Table 11 Parameter estimates from GenStat FITNONLINEAR directive and posterior sample means and sample standard deviations from WinBUGS, standard logistic sigmoid model fitted to the 2003 trial data using the transformed scores. 86
Table 12 Posterior sample means of the estimated late blight scores for the 2003 trial using a Bayesian standard logistic sigmoid model with row and column effects. 101
Table 13 Number of cultivars in each year by type of profiles. 117
Table 14 Means of the posterior sample means for the parameters of the sigmoid logistic model for each of the five types of disease profiles. 118
Table 15 Number of cultivars, columns, rows, replicated plots and assessments of the potato late blight trials. 130
Table 16 Number of potato cultivars assessed in the late blight trials. Diagonal cells are the number of cultivars in a single trial; other cells are the number of common cultivars in a pair of trials. There were 17 common cultivars between 1985 and 1987 trials. In total 48 cultivars were assessed in the 1985 and 1987 trials. 138
Table 17 List of combinations of the treatment factors. 162
Table 18 Red core disease in strawberries data. The data recorded is the number of plants in each of the categories of the grade in a plot. ................................................................. 171

Table 19 Estimated cut-points and treatment effects for the red core strawberries data assuming no random blocking effects. ................................................................. 174

Table 20 Estimated cut-points and parameter estimates for the red core disease in strawberries data using Jansen's method for randomised complete block design. ......................... 178

Table 21 Model specification. .................................................................................................................. 180

Table 22 DIC obtained for 2003/2004 blight data using standard logistic model with 5 points ordinal scale. .................................................................................................................. 181

Table 23 Summary of linear regression with A, B, C and M at level 0 as the reference level. ........ 182