



<http://researchspace.auckland.ac.nz>

ResearchSpace@Auckland

Copyright Statement

The digital copy of this thesis is protected by the Copyright Act 1994 (New Zealand).

This thesis may be consulted by you, provided you comply with the provisions of the Act and the following conditions of use:

- Any use you make of these documents or images must be for research or private study purposes only, and you may not make them available to any other person.
- Authors control the copyright of their thesis. You will recognise the author's right to be identified as the author of this thesis, and due acknowledgement will be made to the author where appropriate.
- You will obtain the author's permission before publishing any material from their thesis.

To request permissions please use the Feedback form on our webpage.

<http://researchspace.auckland.ac.nz/feedback>

General copyright and disclaimer

In addition to the above conditions, authors give their consent for the digital copy of their work to be used subject to the conditions specified on the Library Thesis Consent Form.

Random Effects Models for Ordinal Data

Arier Chi-Lun Lee

A thesis

submitted in partial fulfilment

of the requirements for the degree of

Doctor of Philosophy

The University of Auckland

June 2009

Abstract

One of the most frequently encountered types of data is where the response variables are measured on an ordinal scale. Although there have been substantial developments in the statistical techniques for the analysis of ordinal data, methods appropriate for repeatedly assessed ordinal data collected from field experiments are limited.

A series of biennial field screening trials for evaluating cultivar resistance of potato to the disease, late blight, caused by the fungus *Phytophthora infestans* (Mont.) de Bary has been conducted by the New Zealand Institute of Crop and Food Research since 1983. In each trial, the progression of late blight was visually assessed several times during the planting season using a nine-point ordinal scale based on the percentage of necrotic tissues. As for many other agricultural field experiments, spatial differences between the experimental units is one of the major concerns in the analysis of data from the potato late blight trial.

The aim of this thesis is to construct a statistical model which can be used to analyse the data collected from the series of potato late blight trials. We review existing methodologies for analysing ordinal data with mixed effects particularly those methods in the Bayesian framework. Using data collected from the potato late blight trials we develop a Bayesian hierarchical model for the analyses of repeatedly assessed ordinal scores with spatial effects, in particular the time dependence of the scores assessed on the same experimental units was modelled by a sigmoid logistic curve.

Data collected from the potato late blight trials demonstrated the importance of spatial effects in agricultural field trials. These effects cannot be neglected when analysing such data. Although statistical methods can be refined to account for the complexity of the data, appropriate trial design still plays a central role in field experiments.

Acknowledgements

This thesis only exists because of the support of many people. First I would like to express my special thanks to my supervisor, Professor Chris Triggs. It has been a very long road and I could not have stayed the course without his continuous support, encouragement and understanding. I would also like to thank my advisor John Anderson, Crop & Food Research, for providing me the potato late blight datasets and the helpful comments and suggestions. My research was funded by a FORST Bright Futures Doctoral Scholarship and a University of Auckland Doctoral Scholarship.

A huge thank you must go to the entire Department of Statistics especially the Tamaki team who always supported me with kindness and encouragement during my ups and downs. A special thank you to Bronwyn and Angela, whose warm conversation and friendship always kept me going.

I would also like to acknowledge the huge amount of support and encouragement from my wonderful loving family. Especially to my mum – thank you for making all this possible. No words can express my gratitude to you. To my little boy, William, your happy face always lightens up my day.

Finally, I must thank all the friends I met along the way. This work would have been impossible without the support and guidance of a great number of people.

Table of Contents

Abstract	i
Acknowledgements	ii
List of Figures	v
List of Tables	x
Chapter 1. Introduction	1
1.1. Ordinal Data	2
1.2. The Potato Late Blight Trials	2
1.3. Thesis Outline.....	4
Chapter 2. Literature Review	5
2.1. Modelling the Distribution of Ordinal Response Variables	5
2.2. Bayesian Modelling of Ordinal Data and Crop Variety Trials	16
2.3. Conclusion.....	22
Chapter 3. The Potato Late Blight Trial	24
3.1. Description of the Trial and the Data.....	24
3.2. Traditional Analysis.....	29
3.3. New Initiatives.....	35
Chapter 4. Preliminary Approaches	36
4.1. General Formulation for the Mixed Effects Models	36
4.2. Ante-dependence Model	38
4.3. Jansen’s Method	43
4.4. Bayesian Approaches for Experimental Designs.....	53
4.5. Conclusion.....	65
Chapter 5. Single Trial with Single Assessment	66
5.1. Sum of the Repeated Scores	66
5.2. Ordinal Scores from a Single Assessment	70
Chapter 6. Single Trial with Repeated Scores – Continuous Outcomes	75
6.1. Nonlinear Sigmoid Decline model in GenStat	76
6.2. A Bayesian Sigmoid Model for Continuous Outcomes	84
6.3. Conclusion.....	93
Chapter 7. A Bayesian Hierarchical Sigmoid Model for Repeated Ordinal Outcomes	94
7.1. Model Specification	94
7.2. Posterior Result.....	99

7.3. Posterior Predictive Assessment.....	107
7.4. Systematic Priors and Initial Values of the Sigmoid Parameters.....	114
7.5. Selection of Prior Distributions for the Row and Column Variance Parameters.....	119
7.6. Discussion on the Effect of the Precisions of the Informative Priors	127
7.7. Conclusion.....	129
Chapter 8. Results from the Analysis of the individual Potato Late Blight Trials	130
8.1. Late Blight Resistance in Selected Cultivars.....	131
8.2. Row and Column Spatial Effects.....	133
Chapter 9. Analysis of Potato Late Blight Trials Combined Over Multiple Years	137
9.1. Introduction.....	137
9.2. A Model with Cultivar Effects Only.....	138
9.3. A Model with both Cultivar and Year Effects.....	141
9.4. A Model with Cultivar, Year, and Spatial Effects	149
9.5. Conclusion.....	156
Chapter 10. Conclusions.....	158
Appendix A. Hannah and Quigley Method	160
A.1. Method	160
A.2. Soft rot of calla tubers example	162
A.3. Conclusion.....	168
Appendix B. A Nine-Point Ordinal Scale by Cruickshank	169
Appendix C. Jansen’s Method for the Analysis of Data from Ordinal Regression Models with Random Effects	170
C.1. Red Core Disease in Strawberries Example	170
C.2. McCullagh’s Ordinal Regression Model for data from a randomized block design.....	171
C.3. Jansen’s Method for the Randomised Complete Block Design.....	174
Appendix D. Effect of Priors and Their Precisions	179
D.1. Method	179
D.2. Result	181
D.3. Conclusion.....	185
References.....	191

List of Figures

Figure 1 The distribution of a latent variable, cutpoints and the ordinal response variable.	11
Figure 2 Effect of the shifting in the distribution of latent variable.	12
Figure 3 Estimated mean disease scores with 95% least significance intervals using Hannah-Quigley method for presenting the result from ordinal regression in the original ordinal scale.....	13
Figure 4 Late blight progression during the 2003-2004 season trial	28
Figure 5 Late blight progression during the 1999 trial	30
Figure 6 Ratios of variance components over time	34
Figure 7 Distribution of ranks. WinBUGS ranks the smallest tensile strength to the largest tensile strength from 1 to 4. For the four mixing method, method 2 with rank 4 is the best mixing method.....	63
Figure 8 Histogram for the posterior samples of tensile strength obtained from WinBUGS. Dashed lines indicate sample means of tensile strength from ANOVA.....	64
Figure 9 Layout for the 2003 potato late blight trial.	66
Figure 10 Plot of the estimated variety means, the REML technique using GenStat vs. a Bayesian hierarchical using WinBUGS.....	68
Figure 11 Estimated (a) row and (b) column random effects, REML using GenStat vs. Bayesian hierarchical model using WinBUGS with $y = x$ reference line.....	70
Figure 12 Estimated (a) row (b) column random effects from Bayesian hierarchical model using WinBUGS with lowess smooth curve.....	70
Figure 13 Plot of the estimated scores vs. the observed scores from a Bayesian hierarchical ordinal logistic model fitted to the fourth score of the 2003 potato late blight trial using WinBUGS with $y = x$ reference line.	73
Figure 14 Estimated (a) row (b) column random effects from a Bayesian hierarchical ordinal logistic model fitted to the fourth score of the 2003 potato late blight trial using WinBUGS with lowess smooth curve.....	74
Figure 15 Observed mean blight scores over time for some selected varieties from the 2003 trial.....	76
Figure 16: The fitted sigmoid curves for the 2003 blight data using the GenStat FITNONLINEAR directive. The individual plots of land were modelled as homogeneous units ignoring cultivar, row and column effects. Note that the nine-points ordinal scores were transformed with a minimum and maximum bound of 0 and 1	81
Figure 17 Fitted curves of the first 20 cultivars for the standard logistic model with cultivar effect using GenStat FITCURVE directive.....	82

Figure 18 The observed scores (in the transformed scale) for cultivar 2, 3, 25, 48, 70 and 76. These 6 cultivars were problematic when sigmoid model were fitted to the data.	83
Figure 19 Fitted standard logistic curves for the 2003 trial data using the transformed scores.	86
Figure 20 History plots of the parameters of the standard logistic sigmoid model fitted to Ilam Hardy data from 2003 trial.....	88
Figure 21 History plots of the standard logistic model fitted to the 20 replicates simulated data.....	89
Figure 22 History plots of the standard logistic model fitted to the 40 replicates simulated data.....	89
Figure 23 History plots of the standard logistic model fitted to the 60 replicates simulated data.....	90
Figure 24 Fitted curves for Ilam Hardy with 60 replicates simulated data in the transformed scale using standard logistic model using WinBUGS.	90
Figure 25 Observed cultivar means vs. The posterior sample means from fitting the standard logistic model using WinBUGS to the 2003 trial data with cultivar effect at each assessment.	92
Figure 26 Two cultivars which were unable to reach convergence using GenStat FITNONLINEAR and FITCURVE directives. The red circles are the observed scores with jittering. For cultivar 70, the black line is the fitted curve at the final 1000th iteration.	93
Figure 27 The same two cultivars which WinBUGS provided good fit to the data. The red circles are the observed scores with jittering. The black lines are the fitted curves using the posterior sample means of the parameters from the standard logistic model.	93
Figure 28 Flowchart for the development of informative prior distributions and initial values.	97
Figure 29 Observed (in blue, jittered) and posterior sample means (in red) of the disease scores over time using a Bayesian hierarchical standard logistic model with row and column spatial effect for the first 20 cultivars of the 2003 potato late blight trial.....	102
Figure 30 Cultivar means of the observed scores vs. the posterior sample means of the cultivars for each scoring assessment with means and modes displayed at observed mean values, displayed with the reference line $y = x$. Lines in yellow indicated minimum and maximum score.	103
Figure 31 Boxplots of the posterior samples of the row and column spatial effects displayed on the left and right respectively and overlaid with lowess smooth curves ($f=0.2$ and 0.5).	104
Figure 32 Extrapolated ideal cut-points vs. posterior sample means of the cut-points estimates from the Bayesian standard logistic model with regions indicated with the corresponding ordinal scores.	106
Figure 33 Plots of observed scores vs. the posterior sample means of the latent scores with density overlaid as the red lines. The posterior sample means of the cut-points and the region of ordinal scales were also displayed as dash lines and numerals in blue respectively. (a) 2003 late blight trial. (b) 1985 late blight trial.	107

Figure 34	Boxplots of the posterior predictive distributions for the first 25 plots of land with the observed scores displayed as red dots.	111
Figure 35	Boxplots for the distribution of the mean scores for each cultivar at each scoring occasion from posterior predictive simulation with red dots indicated the observed cultivar mean scores.	113
Figure 36	Histogram of the posterior predictive p -values.	114
Figure 37	Observed scores (jittered) and posterior sample means for cultivar 24, C. Royal from the 1987 trial using logistic model for ordinal outcomes (a) using result from the logistic model with continuous outcomes as initial values (b) using result from cultivar 29, Tekau, in 1985 trial as initial values. Fit of the model in (b) increased dramatically compares to (a).	115
Figure 38	Five types of disease profile using a subjective visual method.	117
Figure 39	Posterior sample means (in red) and observed scores (in black) for cultivar 21 to 35 for the 1987 late blight trial. The posterior sample means for cultivar 24, C. Royal, showed a significant improvement over the original model, see Figure 37.	119
Figure 40	Histograms of posterior predictive samples of the row and column standard deviations, σ_κ and σ_ρ , of the 1991 late blight data from models with four different prior distributions: (a) and (e) uniform prior distribution on σ_κ and σ_ρ , (b) and (f) inverse-gamma(1, 1) on σ_κ^2 and σ_ρ^2 , (c) and (g) inverse-gamma(0.01, 100) on σ_κ^2 and σ_ρ^2 , (d) and (h) half-Cauchy prior on σ_κ and σ_ρ . The inverse gamma distributions were defined by the shape and rate parameters.	125
Figure 41	Posterior sample means of the row and column effects.	126
Figure 42	Posterior sample means of the estimated blight score at each scoring occasion for each cultivar, labelled by the scoring occasion, with a reference line.	127
Figure 43	Estimated late blight progression using posterior sample means of cultivar scores from individual trial analyses.	132
Figure 44	Overall means of posterior sample means over time for a selection of cultivars.	133
Figure 45	Posterior distributions of the row effects from fitting a Bayesian hierarchical standard logistic sigmoid model, year 1983 to 2005.	135
Figure 46	Posterior distributions of the column effects from fitting a Bayesian hierarchical standard logistic sigmoid model, year 1983 to 2005.	136
Figure 47	Observed scores (jittered) and the posterior sample means for cultivar one to twenty, from a model with common cultivar effect and included the first seven assessments only. Green dots represent observed scores from the 1987 trial and the red dots represent the observed scores from the 1985 trial; where the black dots are the cultivar specific posterior sample means.	140

Figure 48 Cultivar specific posterior sample means of the first seven assessments obtained from the model which included the first seven assessments vs. the model which included all eight assessments, with the reference line with intercept at zero and a gradient of one.....	141
Figure 49 Observed scores (jittered) for cultivar 7 '1461.3'.....	144
Figure 50 Observed scores (jittered) and posterior sample means by cultivar and year for cultivar one to twenty. Posterior sample means were obtained from the model including cultivar and year effects for the parameter B and M in the sigmoid decay curve. Pink and light green dots represent the observed scores (jittered); red and dark green dots represent the posterior sample means for the 1985 and 1987 trial respectively.	146
Figure 51 Observed scores (jittered) and posterior sample means for cultivar number 7, '1463.1'. Pink and light green dots represent the observed scores for the 1985 and 1987 trial respectively. (a) The model with cultivar effect only. (b) The model with cultivar and year effects allowing for varying slopes and points of inflections between different trial years but of the same cultivar. (c) and (d) The model with cultivar and year effects allowing for varying slopes, maximum, minimum bounds and points of inflections between different trial years but of the same cultivar. For figure (a), black dots represent the posterior sample means for this cultivar. For figures (b) to (d) red and dark green dots represent the posterior sample means for the 1985 and 1987 trial respectively.	149
Figure 52 Observed scores (jittered) and posterior sample means for cultivar number 7, 19, 47 and 48. Pink and light green dots represent the observed scores for the 1985 and 1987 trial respectively. Pink and light green dots represent the observed scores (jittered); red and dark green dots represent the posterior sample means for the 1985 and 1987 trial respectively.	152
Figure 53 The posterior distributions for the row and column effects with lowess smooth curves. ..	153
Figure 54 Boxplots of the posterior predictive distributions for the first 25 plots of land with the observed scores displayed as red dots.	154
Figure 55 Boxplots of the mean scores of the posterior predictive distribution for the first 25 cultivars at each scoring occasion from the 1985 trial with red dots indicated the observed cultivar mean scores.	155
Figure 56 Histogram of the posterior predictive p -values.	156
Figure 57 Estimated mean disease scores with 95% least significance intervals using Hannah-Quigley method for presenting the result from ordinal regression in the original ordinal scale.....	167
Figure 58 DIC and its components obtained from a Bayesian standard logistic model for ordinal data with different precisions for the priors plotted against levels of the precisions for the highly significant parameters.	183
Figure 59 Observed vs posterior sample means at cultivar level, Model 1.	185
Figure 60 Observed vs. posterior sample means at plot level, Model 1.	186
Figure 61 Observed vs. posterior means at cultivar level, Model 3.	186

Figure 62 Observed vs. posterior means at plot level, Model 3.....	187
Figure 63 Observed vs. posterior means at cultivar level, Model 5.....	187
Figure 64 Observed vs. posterior means at plot level, Model 5.....	188
Figure 65 Observed vs. posterior means at cultivar level, Model 6.....	188
Figure 66 Observed vs. posterior means at plot level, Model 6.....	189
Figure 67 Plots of posterior sample means, model 1 vs. model 6, cultivar level.....	189
Figure 68 Plots of posterior sample means, model 1 vs. model 6, plot level.....	190

List of Tables

Table 1 Number of potato cultivars assessed in the late blight trials. Diagonal cells are the number of cultivars in a single trial; other cells are the number of common cultivars in a pair of trials..	27
Table 2 Late blight scoring system.....	28
Table 3 The 1999 potato late blight trial data	30
Table 4 Estimated cumulative scores for each variety using REML for the 1999 late blight data. Note that the average LSD is displayed here as every pair of cultivar comparison has a different SED due to the unbalanced layout of the trial	32
Table 5 Estimated variance components from the REML analysis for the combined data	35
Table 6 ANOVA for the strawberry example.....	56
Table 7 Data for tensile strength of cement example.....	58
Table 8 Estimated variance components from the REML technique and a Bayesian hierarchical model.....	69
Table 9 The estimates and standard errors for the parameters of the five sigmoid models using GenStat FITNONLINEAR directive.....	80
Table 10 Residual sum of squares for the full blight dataset and the subset which excluded cultivar 2, 3, 25, 48, 70 and 76. RSS of the full dataset unavailable for model 2, 3 and 4 because parameter estimates were not available for some of the problematic cultivars.	84
Table 11 Parameter estimates from GenStat FITNONLINEAR directive and posterior sample means and sample standard deviations from WinBUGS, standard logistic sigmoid model fitted to the 2003 trial data using the transformed scores.	86
Table 12 Posterior sample means of the estimated late blight scores for the 2003 trial using a Bayesian standard logistic sigmoid model with row and column effects.....	101
Table 13 Number of cultivars in each year by type of profiles.	117
Table 14 Means of the posterior sample means for the parameters of the sigmoid logistic model for each of the five types of disease profiles.....	118
Table 15 Number of cultivars, columns, rows, replicated plots and assessments of the potato late blight trials.....	130
Table 16: Number of potato cultivars assessed in the late blight trials. Diagonal cells are the number of cultivars in a single trial; other cells are the number of common cultivars in a pair of trials. There were 17 common cultivars between 1985 and 1987 trials. In total 48 cultivars were assessed in the 1985 and 1987 trials.	138
Table17 List of combinations of the treatment factors.....	162

Table 18 Red core disease in strawberries data. The data recorded is the number of plants in each of the categories of the grade in a plot.	171
Table 19 Estimated cut-points and treatment effects for the red core strawberries data assuming no random blocking effects.	174
Table 20 Estimated cut-points and parameter estimates for the red core disease in strawberries data using Jansen's method for randomised complete block design.....	178
Table 21 Model specification.....	180
Table 22 DIC obtained for 2003/2004 blight data using standard logistic model with 5 points ordinal scale.	181
Table 23 Summary of linear regression with A, B, C and M at level 0 as the reference level.	182