

# Evaluation of empirical equations for scour downstream grade-control weirs

Dawei Guan

*College of Harbour, Coastal and Offshore Engineering, Hohai University*

Bruce Melville & Lu Wang

*Department of Civil and Environmental Engineering, The University of Auckland*

**ABSTRACT:** The grade control structures are frequently adopted in the Mountain Rivers and streams to prevent severe bed degradation and channel incision. These structures are constructed as a staircase-like sequence of transverse weirs or bed sills. Due to the plunging jets over structure crests, scour holes are generated downstream of the weirs. Once the local scour depth exceeds the embedded depth of the structure foundation, the structure itself and the function of grade-control system will fail. Therefore, accurate estimation of maximum (equilibrium) scour depth downstream of grade-control structures is necessary to ensure the safety of the structures. In this study, the related studies of local scour downstream of staircase-like weirs are reviewed, 5 important empirical scour equations for predicting equilibrium scour depth downstream of grade-control weirs are selected and evaluated. The evaluation process were established using the database formulated based on the equilibrium scour depth values (308 laboratory experimental data) reported in the previous literature. Several performance indicators, i.e. coefficient of determination, mean relative deviation, and under-prediction / over-prediction errors, were adopted and calculated to establish the ranking of the equations. The results show that the method proposed by Marion et al. (2006) was found to be the most accurate of the equations evaluated and is recommended for design use.

## 1 INTRODUCTION

Weirs or sills are low head hydraulic structures that are constructed in the channel for accomplishing grade-control, dissipation of excess energy, protection from bank erosion, division of the flow and improvement of navigation conditions (Guan et al., 2016). The weirs can be constructed using riprap (boulders), concrete, logs, soil-cement, sheet piling, gabions or other materials, in many different configurations. Conventional grade-control weirs usually have a sloping or vertical drop, and are sometimes built with various appurtenances (such as baffles or end sills). Often, in sloping channels, these structures are consecutively installed by excavating trenches in the stream bed at right angles to the flow and placing the structures into the trenches with the initial crest elevation protruding from the bed or at bed elevation. Neilson et al (1991) provided a comprehensive review on grade-control weirs with an annotated bibliography. The objectives of grade-control weirs are to control river bed slope and elevation, and to stabilize the bed. They are also frequently used to raise the bed

level downstream of a culvert, where serious bed incision occurs. Recently, the grade-control weirs have been used to prevent scouring at bridge piers by placing them downstream of the bridge (Deng and Cai, 2010, Grimaldi et al., 2009b, Grimaldi et al., 2009a, Tafarojnoruz et al., 2010, Wang et al., 2018).

Due to the plunging jets over structure crests, scour holes are generated downstream of the grade-control weirs, forming a step-and-pool profile (Figure 1). In Figure 1,  $d_s$  = scour depth;  $l_s$  = scour length;  $L$  = weir space;  $S_0$  = initial bed slope;  $S_{eq}$  = equilibrium bed slope. Once the local scour depth exceeds the embedded depth of the structure foundation, the structure itself and the function of grade-control system will fail. Therefore, accurate estimation of maximum (equilibrium) scour depth downstream of grade-control weirs is necessary to ensure the safety of the structures. In this study, the related studies of local scour downstream of staircase-like weirs are reviewed, 5 important empirical scour equations for predicting equilibrium scour depth downstream of grade-control weirs are selected and evaluated.

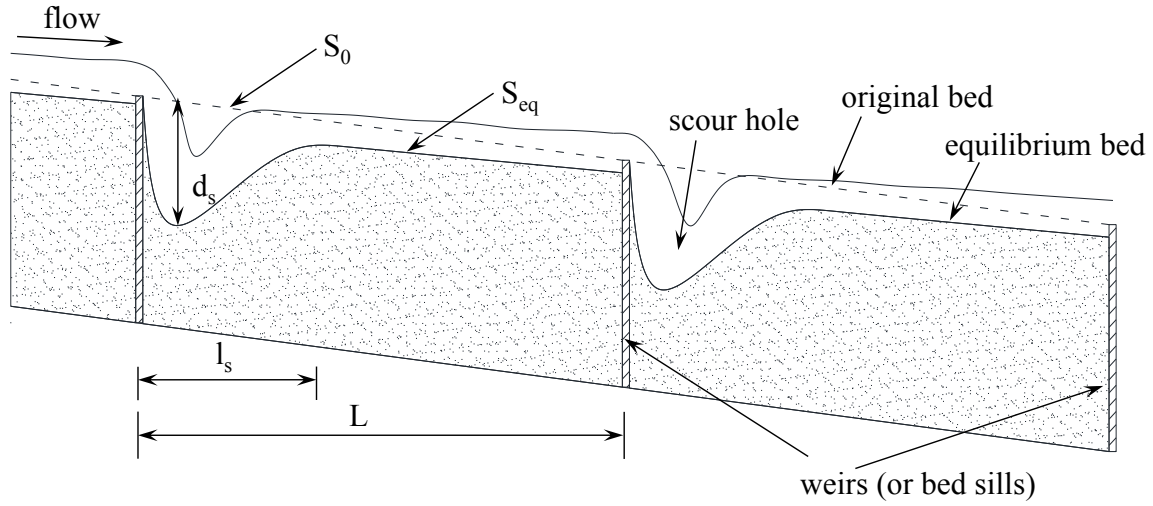


Figure 1. Scour model sketch for a sequence of weirs in mountain streams with steep slope

## 2 EMPIRICAL SCOUR EQUATIONS

In the past 20 years, many empirical scour equations have been developed for the prediction of equilibrium scour depth downstream of consecutive grade-control weirs (Gaudio et al., 2000, Lenzi et al., 2002, Gaudio and Marion, 2003, Lenzi et al., 2003a, Lenzi et al., 2003b, Marion et al., 2004, Lenzi, 2004, Comiti et al., 2005, Ben Meftah and Mossa, 2006, Marion et al., 2006, Tregnaghi et al., 2007, Lin et al., 2008, Martín-Vide and Andreatta, 2009, Tregnaghi, 2010). This section will chronologically review several selected important studies in this field.

### 2.1 Gaudio et al. (2000) and Lenzi et al. (2002)

Scouring downstream of consecutive rectangular weirs in sloping streams (Figure 1) was first studied by Gaudio et al. (2000). They defined the dependence of equilibrium scour depth downstream of bed sills as follows:

$$d_s = f(g, \nu, \rho, \rho_s, q, q_s, h_u, d_{50}, a_1) \quad (1)$$

where  $g$  = acceleration of gravity;  $\nu$  = kinematic viscosity of the fluid;  $\rho$  = water density;  $\rho_s$  = sediment density;  $q$  = flowrate per unit width;  $q_s$  = initial volumetric sediment discharge per unit width;  $d_{50}$  ( $d_x$ ) = sediment size for which 50% ( $x\%$ ) by weight is finer;  $h_u$  = normal flow depth on the equilibrium bed slope. The variable  $a_1$ , termed the 'morphological jump', was defined as:

$$a_1 = (S_0 - S_{eq})L \quad (2)$$

Using the Buckingham theorem and regression analysis, Gaudio et al. (2000) obtained the following predictive equations:

$$\frac{d_s}{H_s} = 0.189 \frac{a_1}{\Delta d_{95}} + 0.266 \quad (3)$$

where  $H_s$  = critical specific energy and calculated as  $1.5(q^2/g)^{1/3}$ ; and  $\Delta$  = relative submerged particle density =  $(\rho_s/\rho - 1)$ . It should be noted that Eq. (3) is applicable only to low-gradient gravel beds, initial bed slopes ranging from 0.0085 to 0.0160, and relatively large sill spacing.

Later, Lenzi et al. (2002) studied scouring at bed sills in steep streams (initial bed slope ranging from 0.078 to 0.148). Using the data from Gaudio et al. (2000) and their own studies, they adopted the equation proposed by Gaudio et al. (2000), to give:

$$\frac{d_s}{H_s} = 0.4359 + 1.4525 \left( \frac{a_1}{H_s} \right)^{0.8626} + 0.0599 \left( \frac{a_1}{\Delta d_{95}} \right)^{1.4908} \quad (4)$$

Eq. (4) is valid for  $0.161 \leq a_1/(\Delta d_{95}) \leq 1.15$  and  $0.225 \leq a_1/H_s \leq 1.872$ , and are applicable to both low and high gradient streams. The use of  $d_{95}$  in Eq. (4) is because it yields the best correlation in the multiple regression of the experimental data.

### 2.2 Lenzi et al. (2003a) and Marion et al. (2004)

Lenzi et al. (2003a) studied the effect of sill spacing on scouring downstream of rectangular bed sills. Their study indicates that the scouring dynamics become heavily affected by the presence of the downstream sill when the ratio between the critical water depth and the sill spacing rises above a certain threshold. Following this finding, Marion et al. (2004) conducted a series of experiments with different sill spacing and grain size distributions. In their study, the sill spacing  $L$  and the sorting index  $SI$  were introduced into the previous dimensional analysis:

$$d_s = f(g, \nu, \rho, \rho_s, q, q_s, h_u, d_{50}, a_1, L, SI) \quad (5)$$

where the sorting index  $SI$ , which relates to sediment size grading, can be defined as:

$$SI = \frac{1}{2} \left( \frac{d_{84}}{d_{50}} + \frac{d_{50}}{d_{16}} \right) \quad (6)$$

On the basis of the existing experimental data sets (Lenzi et al., 2002), available field data (Lunardi, 2002) and a new set of experiments, Marion et al. (2004) derived a new equation for the prediction of the equilibrium scour hole depth, which can be written as follows:

$$\frac{d_s}{H_s} = 2.68 \left( \frac{a_1}{H_s} \right)^{0.43} SI^{-0.19} \left( 1 - e^{-0.14 \frac{L}{H_s}} \right) \quad (7)$$

As can be seen in Eq. (7), scour depth is significantly reduced in a sediment with less uniform grading, but with no significant influence of the average sediment size. Both sill “interfering” and “non-interfering” conditions are considered in Eq. (7) by the inclusion of the new non-dimensional parameter  $L/H_s$ .

### 2.3 Marion et al. (2006)

Marion et al. (2006) attempted to evaluate the effect of upstream sediment supply on the scour hole dimensions. They carried out experiments to simulate conditions of a steady upstream sediment supply in a sloping flume. For their experiments, it was assumed that the imposed sediment transport affects the final conditions only by changing the equilibrium bed slope, which was already considered in the calculation of the “morphological jump”  $a_1$ . Therefore, they proposed that a new parameter is not required in Eq. (7). Their assumption was verified by the results of their study and the revised equations for equilibrium scour depth are as follows:

$$\frac{d_s}{H_s} = 3.0 \left( \frac{a_1}{H_s} \right)^{0.60} SI^{-0.19} \left( 1 - e^{-0.25 \frac{L}{H_s}} \right) \quad (8)$$

Eq. (8) is applicable to various conditions: high and low bed slopes, uniform and mixed grain sizes, clear water and live-bed conditions. They are valid for  $0.07 \leq a_1/H_s \leq 1.87$ . The study of Marion et al. (2006) is significant and the results are more applicable in

practice due to the consideration of the effects of upstream sediment supply.

### 2.4 Ben Meftah and Mossa (2006)

Ben Meftah and Mossa (2006) conducted an experimental study on scour downstream of bed sills in low-gradient channels under clear water scour conditions. They adopted the theoretical analysis developed by Gaudio et al. (2000), i.e. Eq. 1, and proposed the following scour equations:

$$\frac{d_s}{H_s} = 0.59 \frac{\Delta L S_0}{H_s} + 1.74 \quad (9)$$

Eq. (9) was obtained with  $\Delta d_{50}/H_s$  in the range of 0.074-0.154 and does not consider the sediment size effect. It also should be noted that Eq. (9) is applicable only to low-gradient gravel beds, and relatively large sill spacing (greater than 1 m).

### 2.5 Martin-Vide and Andreatta (2006)

Martin-Vide and Andreatta (2006) found that when a sequence of grade-control structures is placed on a sloping stream bed, there is a “slope reduction” in the equilibrium bed slope that existed before the structures were placed. They gave an empirical equation for the calculation of “slope reduction” as follows:

$$S_{eq} = 0.08 S_0 + 0.92 S'_{eq} - 0.01/L \quad (10)$$

where  $S'_{eq}$  = slope at equilibrium stage without grade-control structures; and  $S_{eq}$  = slope at equilibrium stage with grade-control structures. In the use of the Eqs. (1-8), the equilibrium slope,  $S_{eq}$ , is needed for the calculation of the ‘morphological jump’  $a_1$ . In previous studies (Gaudio et al., 2000, Lenzi et al., 2002, Marion et al., 2004, Marion et al., 2006), the equilibrium slope was calculated from  $q_s$  using an appropriate sediment transport equation, which in fact ignores the disturbance caused by grade-control structures on the equilibrium slopes. The equilibrium slope they calculated is actually the slope at equilibrium stage without grade-control structures,  $S'_{eq}$ . Therefore, the Eq. (10) should be considered when using Eqs. (1-8).

Table 1. Data sources for evaluation of scour equations

Data sources	runs	$d_{50}$ (mm)	$d_{90}$ (mm)	$q$ (m <sup>2</sup> /s)	$S_0$ (%)	$L$ (m)	$a_1$ (m)
Guadio et al. (2000)	19	4.1-8.5	5.5-11.5	0.018-0.033	0.9-1.7	2.0-6.5	0.018-0.060
Lenzi et al. (2002)	13	8.0	40.0	0.007-0.029	7.9-14.8	1.05	0.011-0.079
Lenzi et al. (2003a)	13	8.0	40.0	0.011-0.048	7.9-14.8	0.53	0.030-0.077
Gaudio and Marion (2003)	12	1.8	2.3	0.020-0.032	0.6-1.1	2.5	0.009-0.023
Marion et al. (2004)	88	8.7	13.9	0.017-0.061	4.5-8.0	0.5-1.5	0.007-0.015
Marion et al. (2006)	144	8.7	13.9	0.036-0.052	4.2-7.4	1.5-2.0	0.004-0.103
Ben Meftah and Mossa (2006)	19	1.8	2.3	0.017-0.047	0.9	1.0-4.0	0.004-0.026*

Note: \* = in the calculation of  $a_1$ , the equilibrium slope is calculated using Manning's equation

Table 2. Summarized results of evaluation for equations

Equations	MRD	R <sup>2</sup>	Underprediction (%)	Overprediction (%)	Overprediction >100% (%)
Gaudio et al. (2000)	0.40	0.58	89.0	11.0	0.3
Lenzi et al. (2002)	0.29	0.78	76.0	24.0	1.0
Marion et al. (2004)	0.27	0.82	43.8	56.2	2.3
Marion et al. (2006)	0.21	0.87	41.6	58.4	1.3
Ben Meftah and Mossa (2006)	1.04	0.54	0.3	99.7	38.0

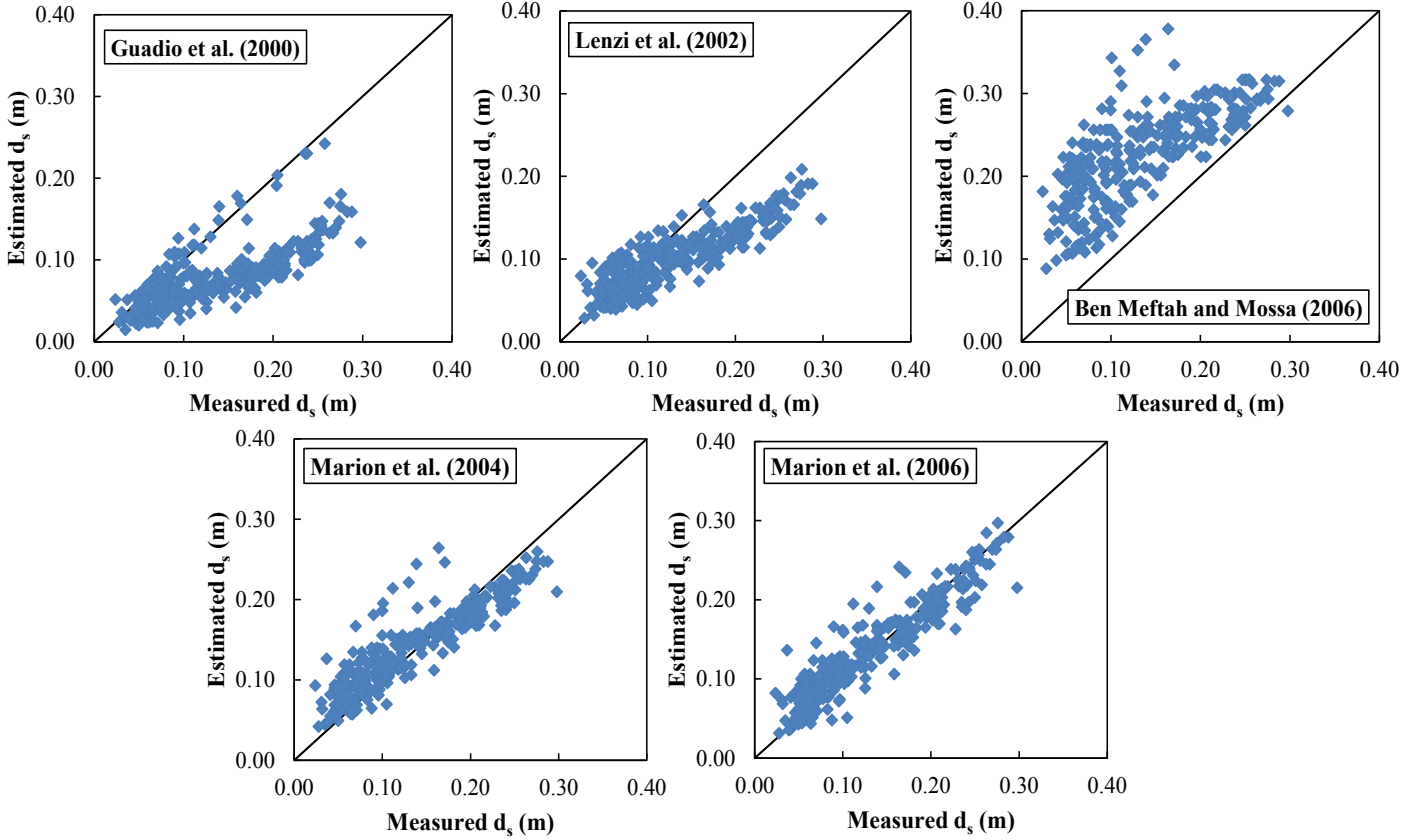


Figure 2. Comparisons of measured and estimated scour depths

### 3 EVALUATION OF SCOUR EQUATIONS

#### 3.1 Database

In total, 308 laboratory experimental data reported in the previous literature were used in the evaluation of the aforementioned empirical scour equations. The detailed information about the data sets used for evaluation is listed in Table 1.

#### 3.2 Evaluation results

The results of evaluation of Eqs. (3), (4), (7), (8) and (9), are shown in Table 2, in which the line of perfect fit is also plotted. The measured and estimated scour depths are compared in Figure 2. In Table 2,  $R^2$  = correlation of determination,  $MRD$  (mean relative deviation) is calculated as:

$$MRD = \frac{1}{n} \sum_{i=1}^n \left| \frac{d_{s\_estimated,i} - d_{s\_measured,i}}{d_{s\_measured,i}} \right| \quad (11)$$

As seen in Figure. 2 and Table 2, the equation proposed by Marion et al. (2006) has the best performance ( $MRD=0.21$  and  $R^2=0.87$ ) in the evaluation. The equations proposed by Gaudio et al. (2006) and Lenzi et al. (2002) underestimate most of the data points with small weir spacing. The equation proposed by Ben Meftah and Mossa (2006) gives conservative estimations (overprediction = 99.7%) for all data sets. This indicates that Eq. (9) may be useful for the conservative design of consecutive grade-control weirs.

## 4 CONCLUSIONS

Scour equations for predicting scour hole depth downstream of consecutive grade-control weirs were reviewed in this paper. The scour depth downstream of the weir is found to depend on the flow conditions, sediment characteristics, weir patterns and weir spacing. Although many researchers have spent much effort studying local scour at grade-control weirs, many unanswered questions remain. As reported in this paper, most of the previously proposed methods are found to be restricted to rectangular weirs and clear water scour conditions. Very limited investigations of local scour at sloped grade-control weirs have been conducted. The “slope reduction” effect [see Eq. (10)] was ignored in the derivation of Eqs. (1-8).

A comprehensive database of experimental equilibrium scour data for grade-control weirs was compiled. These data were used to evaluate the 5 selected equations for scour depth downstream of grade-control weirs. Generally, the estimation methods have improved over the years. The method proposed by Marion et al. (2006) was found to be the most accurate of the equations evaluated and is recommended for design use.

## 5 ACKNOWLEDGEMENT

This research was supported by the Young Scientists Fund of the National Natural Science Foundation of China (51709082) and the Fundamental Research Funds for the Central Universities (2018B13014).

## 6 REFERENCES

- BEN MEFTAH, M. & MOSSA, M. 2006. Scour holes downstream of bed sills in low-gradient channels. *Journal of Hydraulic Research*, 44, 497-509.
- COMITI, F., ANDREOLI, A. & LENZI, M. A. 2005. Morphological effects of local scouring in step-pool streams. *Earth Surface Processes & Landforms*, 30, 1567-1581.
- DENG, L. & CAI, C. 2010. Bridge scour: prediction, modeling, monitoring, and countermeasures—review. *Practice Periodical on Structural Design and Construction*, 15, 125-134.
- GAUDIO, R. & MARION, A. 2003. Time evolution of scouring downstream of bed sills. *Journal of Hydraulic Research*, 41, 271-284.
- GAUDIO, R., MARION, A. & BOVOLIN, V. 2000. Morphological effects of bed sills in degrading rivers. *Journal of Hydraulic Research*, 38, 89-96.
- GRIMALDI, C., GAUDIO, R., CALOMINO, F. & CARDOSO, A. H. 2009a. Control of scour at bridge piers by a downstream bed sill. *Journal of Hydraulic Engineering*, 135, 13-21.
- GRIMALDI, C., GAUDIO, R., CALOMINO, F. & CARDOSO, A. H. 2009b. Countermeasures against Local Scouring at Bridge Piers: Slot and Combined System of Slot and Bed Sill. *Journal of Hydraulic Engineering*, 135, 425-431.
- GUAN, D., MELVILLE, B. & FRIEDRICH, H. 2016. Local scour at submerged weirs in sand-bed channels. *Journal of Hydraulic Research*, 54, 172-184.
- LENZI, M. 2004. Local Scouring at Bed Sills in a Mountain River: Plima River, Italian Alps. *Journal of Hydraulic Engineering*, 130, 267.
- LENZI, M. A., MARION, A. & COMITI, F. 2003a. Interference processes on scouring at bed sills. *Earth Surface Processes & Landforms*, 28, 99-110.
- LENZI, M. A., MARION, A. & COMITI, F. 2003b. Local scouring at grade-control structures in alluvial mountain rivers. *Water Resources Research*, 39, 1176.
- LENZI, M. A., MARION, A., COMITI, F. & GAUDIO, R. 2002. Local scouring in low and high gradient streams at bed sills. *Journal of Hydraulic Research*, 40, 731-739.
- LIN, B.-S., YEH, C.-H. & LIEN, H.-P. 2008. The experimental study for the allocation of ground-sills downstream of check dams. *International Journal of Sediment Research*, 23, 28-43.
- LUNARDI, S. 2002. Indagine sperimentale presso il torrente Cordevole (Arabba, BL) sullo scavo a valle di soglie di fondo: aspetti sedimentologici e geometrici. Master Thesis, University of Padova.
- MARION, A., LENZI, M. A. & COMITI, F. 2004. Effect of sill spacing and sediment size grading on scouring at grade-control structures. *Earth Surface Processes & Landforms*, 29, 983-993.
- MARION, A., TREGNAGHI, M. & TAIT, S. 2006. Sediment supply and local scouring at bed sills in high-gradient streams. *Water Resources Research*, 42, W06416.
- MART N-VIDE, J. P. & ANDREATTA, A. 2006. Disturbance caused by bed sills on the slopes of steep streams. *Journal of Hydraulic Engineering*, 132, 1186-1194.
- MART N-VIDE, J. P. & ANDREATTA, A. 2009. Channel degradation and slope adjustment in steep streams controlled through bed sills. *Earth Surface Processes & Landforms*, 34, 38-47.
- NEILSON, F. M., WALLER, T. N. & KENNEDY, K. M. 1991. Annotated bibliography on grade control structures. Vicksburg, Mississippi U.S. Army Waterways Experiment Station.
- TAFAROJNORUZ, A., GAUDIO, R. & DEY, S. 2010. Flow-altering countermeasures against scour at bridge piers: a review. *Journal of Hydraulic Research*, 48, 441-452.
- TREGNAGHI, M. 2010. Effect of flood recession on scouring at bed sills. *Journal of Hydraulic Engineering*, 136, 204-213.
- TREGNAGHI, M., MARION, A. & GAUDIO, R. 2007. Affinity and similarity of local scour holes at bed sills. *Water Resources Research*, 43, W11417.
- WANG, L., MELVILLE, B. W., WHITTAKER, C. N. & GUAN, D. 2018. Effects of a downstream submerged weir on local scour at bridge piers. *Journal of Hydro-environment Research*, 20, 101-109.