STUDENTS' PERCEPTIONS OF ERRORS IN MATHEMATICS LEARNING IN TANZANIAN SECONDARY SCHOOLS

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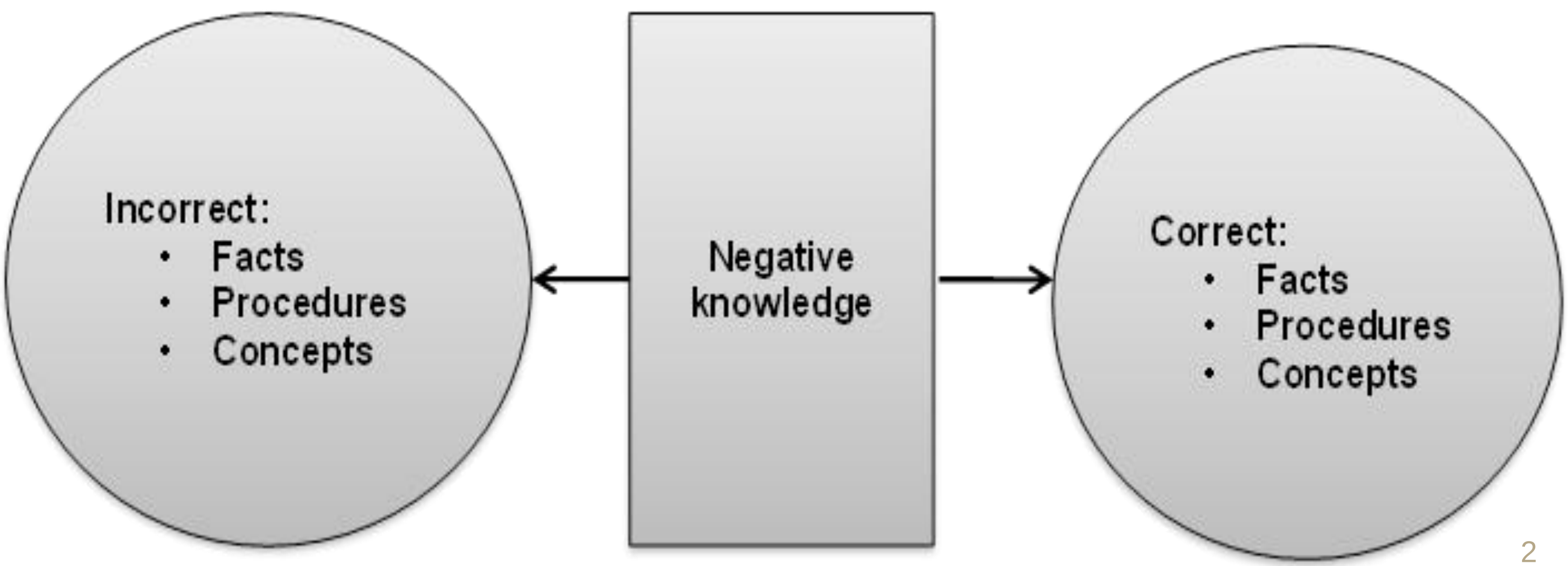
PME 43, Pretoria

Background theory

- Errors in mathematics may be a powerful instructional tool.
- Errors distinguishes between consistent and inconsistent practices of doing mathematics (Sfard, 2007).
- Errors emanates from a lack of *negative knowledge* that helps to identify and distinguish incorrect facts and procedures (Minsky, 1994).



Background theory

- Reflection on errors promotes procedural and conceptual mathematics knowledge (Heemsoth & Heinze, 2016).
- Effective use of errors are likely to promote student learning and motivation (Kapur, 2014; Käfer, Kuger, Klieme & Kunter, in press).
- Productive failure: reflection on unsuccessful proofs arguments and ability to do proof related tasks (Tsujiyama & Yui, 2018).
- Students do not use errors spontaneously (Heinze, Ufer & Reiss, 2013).
- Anxiety in error situations may reduce the chances of learning from them (Heinze et al 2013).
- Specific error handling strategies can stimulate learning from errors (Heemsoth & Heinze, 2016).
- Rach et al (2013) used a two-stage "train-the-trainer" approach.

Purpose and objectives

Role of a direct teacher error handling professional development (PD) training on teachers' cognitive support and student's individual use of errors in learning.

1) What is the effect of the PD training on students' perception of their teacher's support in error situations?

2) What is the effect of the PD training on (a) students' perception of individual use of errors in learning and (b) students' anxiety in error situations?

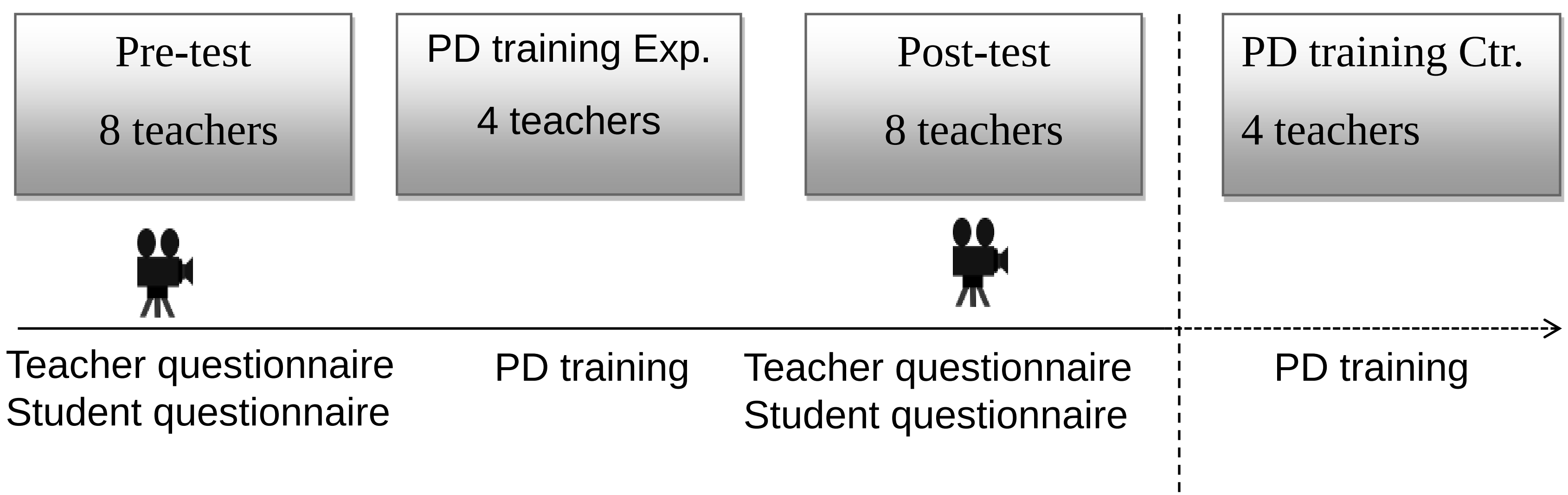
3) What error handling strategies are practiced by teachers before and after the PD training?

Method

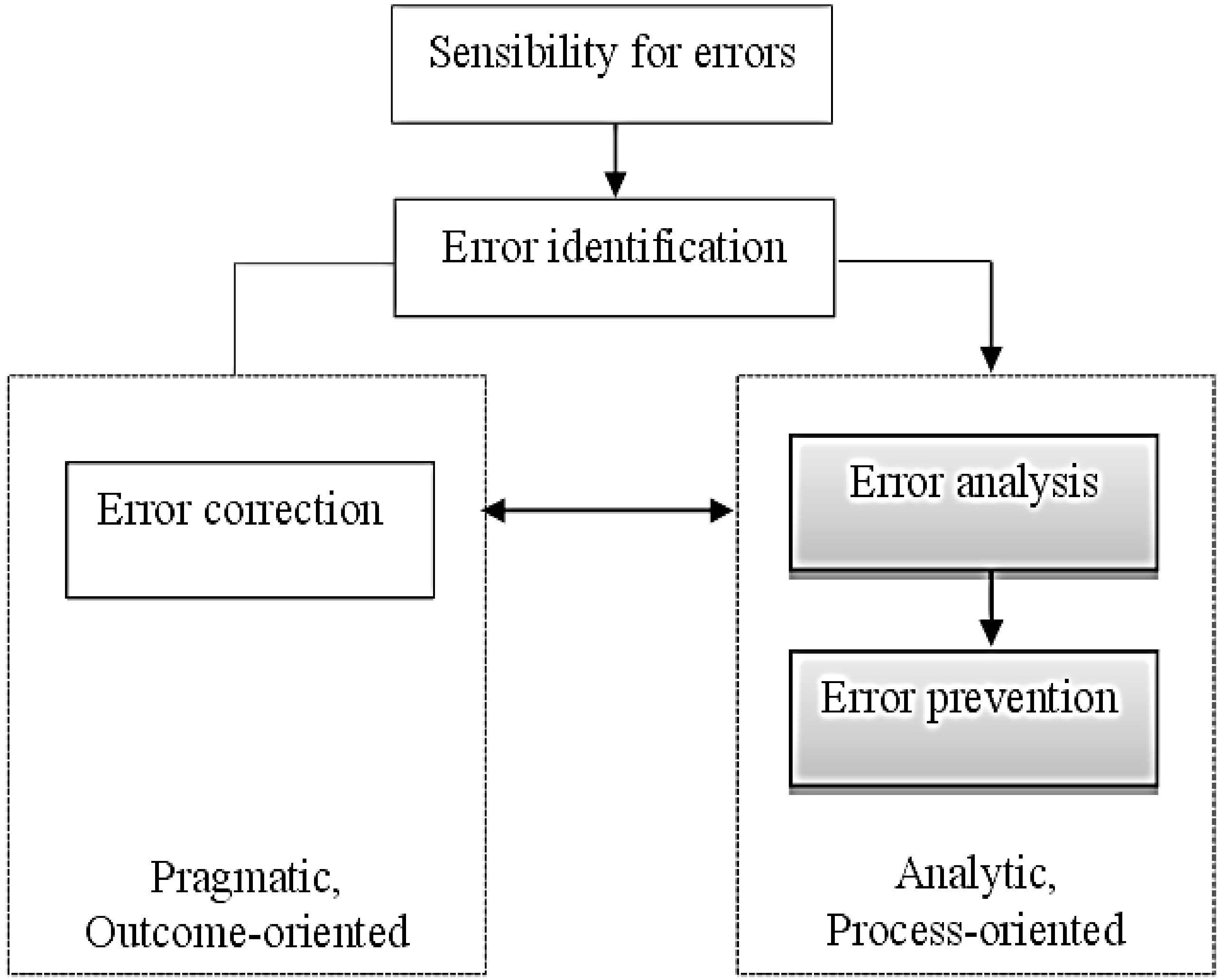
Eight (8) secondary schools in the Dar es Salaam region.

Form 3 (Grade 11) students (N = 251) & mathematics teachers (N = 8).

Design: Quasi-experimental, video-taped feedback plenary discussion, Control (N = 121) and Experimental (N = 130) groups.



Intervention



Cognitive strategies for learning from errors (Rach et al., 2013).

Instruments

Scale	k	Sample item	Cronbach's α		
			Original Study	*Present Study*	
				Pre-test	*Post-test*
Learning orientation (Student use of errors)	8	If I do something wrong in mathematics class I perceive this as an opportunity to learn.	.71	.75	.76
Anxiety in errors	5	I feel ashamed when I make a mistake in front of the class in mathematics.	.78	.49	.54
Teacher support in error situations	7	If I make a mistake in mathematics class, my teacher discusses it with me in a way that I really learn from it.	.79	.65	.56

(1) 'Error Culture in Secondary Education Questionnaire (Spychiger, Küster, & Oser, 2006).
(2) Evaluation of the authenticity of teacher's implementation of the feedback plenary discussion (Jacobs et al., 2003).

Analyses

- Applied the SEM-methods with latent mean analysis and took care to control all potential issues.

- Measurement invariances between groups (Cheung & Rensvold, 2002).

- Latent mean differences analyses (Marsh et al., 2017).

- The Wald χ^2 test of parameter constraints – whether differences in latent means among the groups were significant or not.

Results (RQ1 & RQ2)

Scales	**Manifest means (*SD*)**			
	Control		**Experimental**	
	Pre	*Post*	*Pre*	*Post*
1. Teacher support in error situations	4.87 (1.13)	4.95 (0.98)	4.60 (1.39)	5.07** (1.06)

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3. Anxiety in error situations	2.19 (1.21)	2.13 (1.05)	2.08 (1.04)	2.14 (1.28)

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3. Anxiety in error situations	2.19 (1.21)	2.13 (1.05)	2.08 (1.04)	2.14 (1.28)
4. Authenticity of feedback plenary discussions	5.01 (1.05)	4.93 (1.04)	4.88 (1.16)	4.98 (1.05)
5. Maths performance	53.62 (24.23)	53.35 (27.41)	44.51 (22.40)	41.28 (21.72)

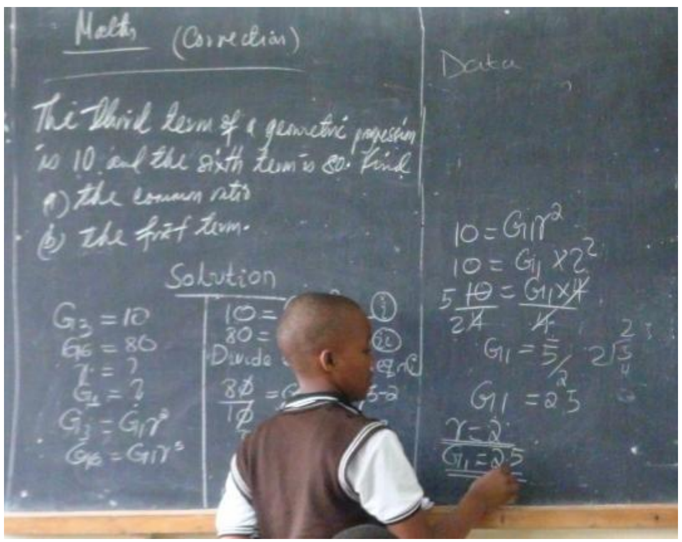
Results (RQ1 & RQ2)

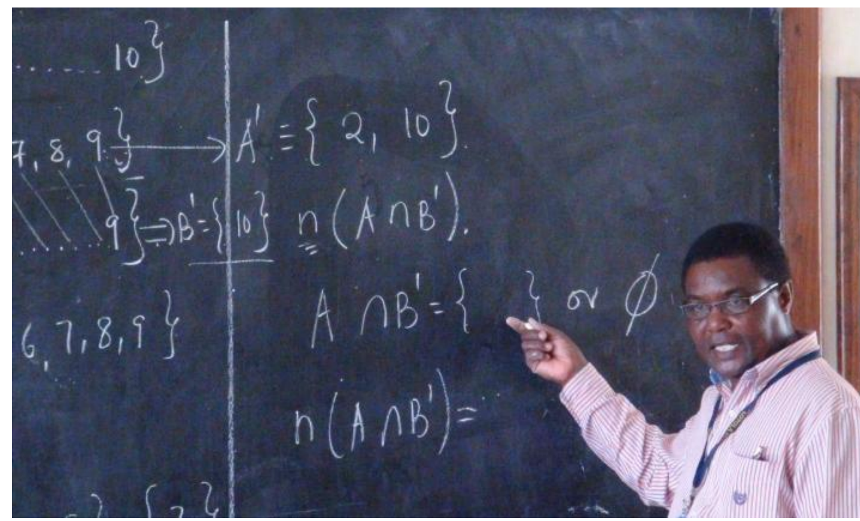
Error handling practices (RQ3)

Coding agreement 87.5% with a Krippendorff's alpha of .72

Student centered
50% (8 lessons)

Teacher centered
50% (8 lessons)





Error handling: mainly pragmatic (error correction), rarely analytic (analysis & prevention).

Discussion

- Students' perceptions of teacher support in error situations significantly increased in the experimental group at posttest.

- Unlike (Rach et al, 2013) the intervention had no effect on students' perceptions of anxiety in error situations.
  - *The social impact or consequences of failing a test.*

- Similar to prior student individual use of errors for learning were not affected by the intervention.

- Exploratory video case studies: somehow mathematics teachers in the experimental group were error friendly at the posttest.

Results

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$R = \{(x, y): y \leq x+1, 0 \leq y \leq 2, x \leq 4\}$. Find (a) R^{-1} (inverse of R), (b) Draw the graph of R.

T: Do you remember the principle? (The teacher explained: *If you want to find the inverse of R, first, interchange x and y, then make y the subject. Don't alter the inequalities the inequalities remain as they are*

T: The teacher writes: $R^{-1} = \{(x, y): x-1 \leq y, 0 \leq x \leq 2, y \leq 4\}$.

Outlook

- A detailed error management strategy that involves development of a mind-set on how to deal with errors (Frese & Keith, 2015).
- Longer intervention for improving student use of their own errors.

Thank you very much!

Asante sana!

Vielen Dank!

References

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Back-up

Errors in mathematics

Identify misconceptions
(Minsky, 1994).

Behaviourism

Cognitivism

Negative knowledge

Avoided and not discussed
(Skinner, 1961).

Useful for learning
(Borasi, 1994; VanLehn, 1999).

Analyses: Appendix A: Invariances

Models	Comparison	SRMR	RMSEA	X^2	Df	ΔX^2	Δdf	p-value	CFI	ΔCFI
1. Student perceptions of learning from errors										
Unconstrained (configural)-A	B vs. A	.089	0.071	408.74	204	25.70	27	**0.535**	0.87	
Measurement weights (metric)-B	C vs. A	.084	0.081	434.44	231	67.10	63	**0.338**	0.87	**-.001**
Measurement intercepts (scalar)-C	B vs. C	.079	0.091	475.84	267	41.39	36	**0.247**	0.86	**.002**
2. Authenticity and perception of feedback plenary										
Unconstrained (configural)-A	B vs. A	.075	0.143	185.79	52	26.78	15	**0.031**	0.91	
Measurement weights (metric)-B	C vs. A	.107	0.132	212.58	67	46.89	36	**0.106**	0.90	**.008**
Measurement intercepts (scalar)-C	B vs. C	.112	0.114	232.69	88	20.11	21	**0.514**	0.90	**.007**