

Retardation-induced anomalous population trapping in a multiply-excited, multiple-emitter waveguide-QED system

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We study in a numerically-exact manner a multiple-emitter, multiple-excitation waveguide quantum-electrodynamics (waveguide-QED) system including propagation time delay. In particular, we demonstrate anomalous population trapping as a result of the retardation in the excitation exchange between the waveguide and three initially excited emitters. Allowing for local phases in the emitter-waveguide coupling, this population trapping cannot be recovered using a Markovian treatment, proving the essential role of non-Markovian dynamics in the process. Furthermore, this time-delayed excitation exchange allows for a novel steady state, in which one emitter decays entirely to its ground state while the other two remain partially excited.

Introduction.— One-dimensional (1D) waveguide-QED systems are attractive platforms for engineering light-matter interactions and studying collective behavior in the ongoing efforts to construct scalable quantum networks [1–12]. Such systems are realized in photonic-like systems including photonic crystal waveguides [13–19], optical fibers [20–24], or metal and graphene plasmonic waveguides [25–28]. Due to their one-dimensional structure, long-distance interactions become significant [3, 5, 29]. As a result of these interactions mediated by left- and right-moving quantized electromagnetic fields, strongly entangled dynamics and collective, cooperative effects related to Dicke sub- and superradiance emerge [1, 6, 12, 17, 22–24, 30–37].

These systems are widely explored in the Marko-

vian limit, or in the non-Markovian single-excitation and multiple-emitter, or multiple-excitation and single-emitter regimes [9, 31, 38–42]. Such limits can be described by a variety of theoretical methods including real-space approaches [5, 38, 43, 44], a Green’s function approach [45–48], Lindblad master equations [49, 50], input-output theory [51–55], and the Lippmann-Schwinger equation [56–58]. Already in these regimes, exciting features have been predicted. For example, strong photon-photon interactions can in principle be engineered, allowing for quantum computation protocols using flying qubits (propagating photons) and multilevel atoms [5, 11, 57, 59, 60]. Furthermore, bound states in the continuum are addressed via a joint two-photon pulse, showing that excitation trapping via multiple-photon scattering can occur without band-edge effects or cavities [7, 44, 56, 61].

In this work, we study the non-Markovian multiple-excitation, multiple-emitter regime. We focus, in particular, on the three-emitter and three-photon case, treating the emitters as two-level systems, which couple to the left- and right-moving photons and thereby interact with each other, subject to time delays associated with the propagation time of photons between emitters [59, 62, 63]. We choose throughout the paper the triply-excited state as the initial state and compare the relaxation dynamics in the Markovian and non-Markovian cases. To compare both scenarios on the same footing, we employ the quantum stochastic Schrödinger equation approach [64–66] and numerically solve the model using a matrix-product-state algorithm [60, 67–71]. We report on striking differences between the Markovian and non-Markovian description. First, we find that in the case of non-Markovian excitation exchange, the triply-excited initial state allows for population trapping, in strong contrast to the Markovian description. Second, time-delayed excitation exchange allows for anomalous population trapping, in which one emitter relaxes completely into its ground state while the two other emit-

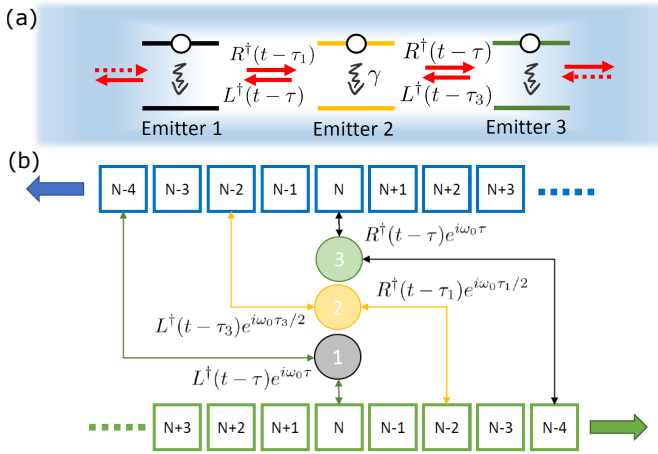


FIG. 1. Scheme of the simulated waveguide QED system. (a) The system consists of three identical emitters with transition frequency ω_0 which couple to left- and right-moving quantized light fields via the decay constant $\sqrt{\gamma}$. (b) Due to the delay (in the scheme two time steps $\tau_1 = \tau_3 = 2\Delta = 2\Gamma^{-1}/10$) a closed loop is formed between the first and third emitter interacting with their respective past bins. The interaction strongly depends on the phases $\omega_0\tau_1$ and $\omega_0\tau_3$.

ters form a singly-excited dark state together with the waveguide field in between. No local phase combination in the Markovian case allows for such anomalous population trapping, rendering the non-Markovian description qualitatively and quantitatively different from a Markovian treatment.

Model.— We are interested in a system consisting of three identical emitters with transition frequency ω_0 . All three emitters interact with left- (l_ω^\dagger) and right-moving photons (r_ω^\dagger) in a one-dimensional waveguide, as depicted in Fig. 1(a). The Hamiltonian governing the free evolution of the combined, one-dimensional waveguide photon-emitter system reads:

$$H_0/\hbar = \omega_0 \sum_{i=1}^3 \sigma_i^{22} + \int d\omega \omega (r_\omega^\dagger r_\omega + l_\omega^\dagger l_\omega), \quad (1)$$

where the emitters are treated as two-level systems, with $|1\rangle$ as the ground state and $|2\rangle$ as the excited state, and with $\sigma_n^{ij} := |i\rangle_{nn}\langle j|$, the flip operator of the n -th emitter. The interaction Hamiltonian describes the emitters interacting with right and left moving photons at the emitters' positions:

$$H_I = \hbar g_0 \sum_{i=1}^3 \sigma_i^{12} \int d\omega \left(r_\omega^\dagger e^{i\omega x_i/c} + l_\omega^\dagger e^{-i\omega x_i/c} \right) + \text{H.c.},$$

where we have assumed a frequency-independent coupling of the emitters to the quantized light field. The position of the second emitter is chosen as $x_2 = 0$, leading to $x_1 = -d_1/2 = -c\tau_1/2$ for the first and $x_3 = d_2/2 = c\tau_3/2$ for the third emitter, with c the speed of light in the waveguide. After transforming into the interaction picture with respect to the free evolution Hamiltonian, and applying a time-independent phase shift to the left- and right-moving photonic field, the transformed Hamiltonian reads $H_I(t) = \hbar g_0 \int (H_I^{r,\omega} + H_I^{l,\omega}) d\omega$, where

$$\begin{aligned} H_I^{r,\omega} &= r_\omega^\dagger(t) (\sigma_1^{12} + \sigma_2^{12} e^{-i\frac{\omega}{2}\tau_1} + \sigma_3^{12} e^{-i\frac{\omega}{2}\tau}) + \text{H.c.}, \\ H_I^{l,\omega} &= l_\omega^\dagger(t) (\sigma_1^{12} e^{-i\frac{\omega}{2}\tau} + \sigma_2^{12} e^{-i\frac{\omega}{2}\tau_3} + \sigma_3^{12}) + \text{H.c.}, \end{aligned} \quad (2)$$

with $r_\omega^\dagger(t) = r_\omega^\dagger(0) \exp[i(\omega - \omega_0)t]$ and $l_\omega^\dagger(t) = l_\omega^\dagger(0) \exp[i(\omega - \omega_0)t]$, and $\tau = (\tau_1 + \tau_3)/2$ [72]. In the following, the left- and right-moving excitations are treated collectively:

$$R^\dagger(t) = \int d\omega r_\omega^\dagger(t), \quad L^\dagger(t) = \int d\omega l_\omega^\dagger(t). \quad (3)$$

Given these definitions, the non-Markovian interaction Hamiltonian reads:

$$\begin{aligned} H_I^{\text{NM}}(t)/\hbar &= g_0 (\sigma_1^{12} (R^\dagger(t) + e^{i\omega_0\tau} L^\dagger(t - \tau)) + \text{H.c.}) \\ &+ g_0 (\sigma_2^{12} R^\dagger(t - \tau_1/2) e^{i\frac{\omega_0}{2}\tau_1} + \text{H.c.}) \\ &+ g_0 (\sigma_2^{12} L^\dagger(t - \tau_3/2) e^{i\frac{\omega_0}{2}\tau_3} + \text{H.c.}) \\ &+ g_0 (\sigma_3^{12} (R^\dagger(t - \tau) e^{i\omega_0\tau} + L^\dagger(t)) + \text{H.c.}). \end{aligned} \quad (4)$$

In the following, we compare the Markovian with the non-Markovian case [42, 73, 74]. The Markovian case neglects retardation effects between the excitation exchange, therefore in the Markovian approximation we set $R^{(\dagger)}(t - t') \approx R^{(\dagger)}(t)$ and $L^{(\dagger)}(t - t') \approx L^{(\dagger)}(t)$. In this approximation, only the local phases but not the retardation in the amplitude are taken into account. Consequently, the Markovian interaction Hamiltonian reads:

$$\begin{aligned} H_I^{\text{M}}(t)/\hbar &= g_0 \left[L^\dagger(t) (\sigma_1^{12} e^{i\omega_0\tau} + \sigma_2^{12} e^{i\frac{\omega_0}{2}\tau_3} + \sigma_3^{12}) + \text{H.c.} \right] \\ &+ g_0 \left[R^\dagger(t) (\sigma_1^{12} + \sigma_2^{12} e^{i\frac{\omega_0}{2}\tau_1} + e^{i\omega_0\tau} \sigma_3^{12}) + \text{H.c.} \right]. \end{aligned} \quad (5)$$

We solve for the system's dynamics in both cases via the time-discrete Schrödinger equation with the time-step size Δ :

$$\begin{aligned} |\psi(n)\rangle &= U_{\text{NM/M}}(n, n-1) |\psi(n-1)\rangle \\ &= \exp \left[-\frac{i}{\hbar} \int_{(n-1)\Delta}^{n\Delta} H_I^{\text{NM/M}}(t') dt' \right] |\psi(n-1)\rangle, \end{aligned} \quad (6)$$

where Δ is small enough to minimize the error in the Suzuki-Trotter expansion [60, 67–71], and the evolution is taken either in the Markovian (M) or in the non-Markovian limit (NM). To efficiently simulate this multiple-emitter, multiple-excitation case, we employ the matrix-product-state formalism [60, 67–71], and choose a collective basis for the flip operators of the emitters to allow for entangled initial states: $|ijk\rangle = |i2^2 + j2^1 + k2^0\rangle$, which leads to, e.g., $\sigma_1^{12} \equiv |0\rangle\langle 4| + |2\rangle\langle 6| + |1\rangle\langle 5| + |3\rangle\langle 7|$. In the collective picture, we have a combined system of three emitters which interact with different time-bins, but subsequently interact, via the left- and right moving photon time bins, with each other, as shown in Fig. 1(b) [75]. In the following, we choose the initial state as $|7\rangle$, with all emitters in the excited state, and the left- and right-moving time bins in the vacuum state.

Markovian limit - no time delay.— We start our investigation in the Markovian limit, and calculate the system's dynamics with U_{M} and the initial state $|\psi(0)\rangle = |111\rangle = |7\rangle$ until the steady state is reached. The Markovian case allows for a master-equation treatment with $g_0 = \sqrt{2\pi\gamma}$ [76]. Tracing out the left- and right-moving photons leads to a collective jump operator, $J := \sqrt{\gamma} (\sigma_1^{12} \exp[i\varphi_1] + \sigma_2^{12} + \sigma_3^{12} \exp[i\varphi_3])$. The phases φ_i can be chosen individually via local unitary transformations, or they arise from the spatial position without taking the finite distance into account in the evolution [6, 63]. In the following, we nevertheless solve the dynamics using the quantum stochastic Schrödinger equation ignoring time-delay effects to give a Markovian evolution.

In Fig. 2, the phase dependence of the integrated reservoir excitation in the steady state is plotted for different initial states: $I = \sum_{n=0}^{N-f} \langle R^\dagger(n)R(n) \rangle + \langle L^\dagger(n)L(n) \rangle$ with N as the number of time-steps to reach the steady

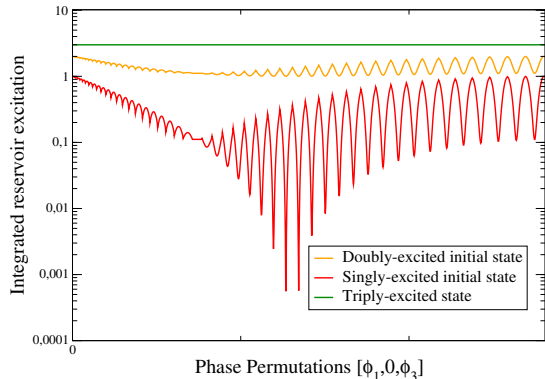


FIG. 2. The impact of different phase choices $\varphi := (\varphi_1, \varphi_2, \varphi_3) = ([0, 2\pi], 0, [0, 2\pi])$ in the atom-waveguide couplings in the Markovian limit with photon operators: $R(t-t') = R(t)$ and $L(t-t') = L(t)$, on the integrated reservoir population in the steady state. If the system is initialized in the triply-excited state (green line), for all choices of phases, all excitation is radiated into the reservoir. If the system is initialized in a superposition of doubly- (orange line) or singly-excited states (red line), the only case where all excitation is radiated into the reservoir is when all phases are a multiple of 2π .

state and $f = (\tau_1 + \tau_3)/\Delta$. The phases are permuted by changing φ_1 from 0 to 2π while φ_3 is increased from 0 to φ_1 for every value of φ_1 . If the system is initialized in the triply-excited state $|\psi(0)\rangle = |7\rangle$, the phases have no impact at all on the steady-state values and all excitation will eventually be radiated into the reservoirs on the left of emitter one and on the right of emitter three, leading for all phase permutations to the integrated reservoir occupation of 3, cf. Fig. 2 (green line). In contrast to the triply-excited case, the steady states of the emitters initially in a superposition of singly- ($|1\rangle + |2\rangle + |4\rangle$, red line) and doubly-excited states ($|3\rangle + |5\rangle + |6\rangle$, orange line) are strongly influenced by the choice of phases. For those initial states, only if the phase difference vanishes, $\varphi_1 = 2\pi = \varphi_3$, is all radiation emitted into the reservoir. For all other phase combinations, population trapping occurs, and all emitters have a finite probability to be found in the excited state [3, 6, 23, 45, 67, 77].

We conclude that, within the Markovian treatment, we find that either all emitters relax to their ground state or none. For systems initialized in the triply-excited state, excitation trapping cannot be achieved. And for the superradiant, symmetric singly-excited and doubly-excited initial states, the emitters undergo complete decay only in the case of vanishing phase difference. We show now that including retardation and back-action effects changes this picture completely.

Symmetric time delay.— We initialize the emitters in the triply-excited state and assume symmetric delays:

$\tau_1 = \tau_3$. In case of quantum coherent feedback, the delay between the excitation exchanges introduces a corresponding phase [60, 67–69, 78–83]. In the following, we assume a transition frequency of the emitters to yield: $\omega_0\tau/2 = 2\pi$, i.e. $\exp[\pm i\omega_0\tau] = 1 = \exp[\pm i\omega_0\tau/2]$. We show now that the evolution under influence of a finite delay in between the emission events, and subsequent back-action from previous emissions of each emitter, leads to population trapping in strong contrast to the Markovian case.

In Fig. 3, the emitter populations in the presence of coherent quantum feedback are shown, e.g., $\langle \sigma_1^{22} \rangle = \sum_{i=1}^4 |\langle 2i-1|\psi(t)\rangle|^2$. Even for short feedback in comparison to the decay time, i.e., $\gamma\tau = 0.5$, population trapping is observed (upper panel). Emitter one (black dotted) and three (orange solid line) exhibit the same dynamics due to the symmetry of the system. Both start to decay exponentially as expected before the first excitation with a neighboring emitter takes place $t \in [0, \tau/2]$. From this moment on (indicated with a dashed line), the decay is slowed down considerably due to the re-excitation and re-emission dynamics. For longer times, the emitter starts to partially interact with its own "past" and after several round trips the population in the emitter is stabilized and a dark state is formed for this particular chosen phase. We emphasize that this anomalous population trapping depends on the presence of left- and right-moving photons and not solely on the finite time between the excitation exchanges. Emitter two (green line) has, for longer delays (middle and lower panel), a slightly higher population than emitters one and three due to excitations from both the left and right emitters (black and orange line). For very long feedback, i.e., $\gamma\tau = 6.25$, a regime where the feedback phase ceases to have a strong influence, we observe an interesting oscillatory behavior in the emitter populations due to the feedback and finite excitation, which settles eventually to a small but finite steady-state value.

This example shows that allowing even for a short retardation and back-action time, the dynamics of the emitter populations changes qualitatively and quantitatively. Population trapping from an initial triply-excited emitter state cannot be recovered just with local phases in the Hamiltonian. This impossibility is lifted due to a time-delayed coherent feedback mechanism. We emphasize that for very long delay the need for a particular choice of $\omega_0\tau/2 = 2\pi$ is partially lifted, and it takes divergently long for the emitter population to decay. However, for long delay times $\gamma\tau \gg 1$, the population is also trapped in the reservoir between the emitters and the absolute stored population in the emitter system is exponentially small.

Asymmetric time delay.— Until now, we have discussed symmetric time delay $\tau_1 = \tau_2$, or $|x_1| = |x_3|$. Asymmetric time delay provides a further example how

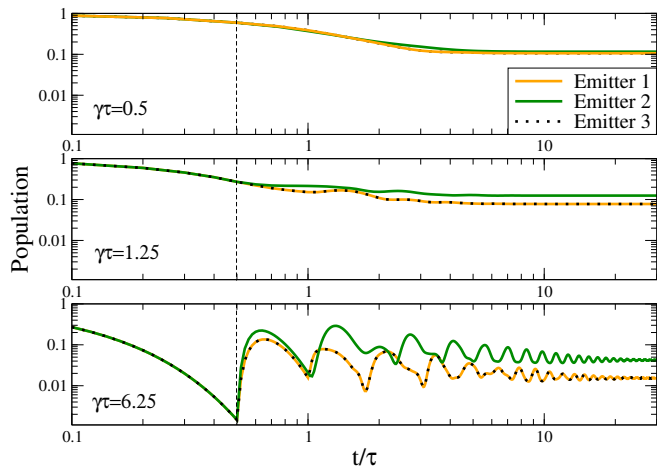


FIG. 3. The dynamics of the emitter populations for different feedback lengths with phase $\omega_m \tau / 2 = 2\pi$ for a system initially in the triply-excited state. Already short feedback times ($\tau = 2\text{ns}$, i.e. $\gamma\tau = 0.5$) lead to population trapping in contrast to the Markovian case. For longer feedback ($\tau = 25\text{ns}$, i.e. $\gamma\tau = 6.25$), slowly decaying oscillations become visible.

the full non-Markovian and quantized description of many-excitation dynamics in waveguide-QED deviates qualitatively from the Markovian treatment. As shown above, in the Markovian treatment either all emitters remain partially excited or none of them do. For the triply-excited state, no population trapping occurs, and for other initial states the system is not able to reach a steady state with only one emitter in the ground state and the other emitters partially excited. We show now that the non-Markovian description with asymmetric time delay allows for another example of anomalous population trapping, where one emitter decays completely into its ground state whereas the other emitters have a finite probability to be found in the excited state.

In Fig. 4, we choose a phase $\omega_0 \tau = 3\pi$ and delay times $\gamma\tau_1 = 1$ between the left (1) and middle emitter (2), and $\gamma\tau_3 = 0.5$ between the middle and the right emitter (3). Excitingly, this setup allows for the right emitter (green line) to decay entirely to its ground state while both the left (black line) and middle emitter (orange line) form a dark state together with the waveguide field in between and exhibit population trapping. This effect results from the asymmetric delay between left- and right emission events. For $t < \tau_3$, all emitters radiate unperturbed into the reservoir. For $\tau_3 < t < \tau_1$, the left emitter (black line) continues to radiate unperturbed whereas the middle and right emitters start to interact with the emitted photons. Due to the symmetry, both the right and middle emitters exhibit the same decay behavior for $t < \tau_1$. This picture changes for larger times, as now the middle emitter's field starts to constructively interfere with the right-moving photons from emitter one. Emitter three interacts with its own past emission and decays faster,

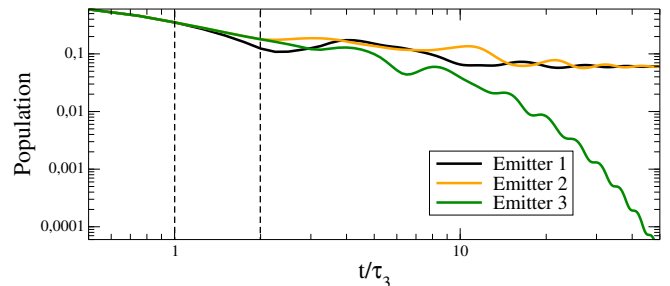


FIG. 4. The dynamics of the emitter populations if all emitters are initially in their excited state: $|\psi(0)\rangle = |7\rangle$ for a phase choice of $\omega_0(\tau_1 + \tau_3) = 3\pi$, and $\tau_1 = 2\tau_3$. Emitter 3 (green line) decays completely into its ground state while emitter 1 (black line) and 2 (orange line) remain in a partially excited state. This steady state is impossible to reach in the Markovian treatment if no additional interactions are included.

while emitters one and two start to form a superposition state. After several roundtrip times, $t \gtrsim 15$, emitter three has decayed, and no emission takes place.

Interestingly, a necessary condition for this feature to happen is asymmetric feedback. A symmetric feedback $\tau_1 = \tau_3$ exhibits, as in the Markovian case, only finite population in all emitters, or none. This effect depends only on the destructive and constructive interference between left- and right-moving photons. For different $\varphi = \omega_0 \tau$, a different positioning needs to be chosen. Quantity $\gamma\tau$ determines the extent of population trapping between emitter one and two, but not the qualitative effect.

Conclusion.— We have investigated a waveguide-QED system consisting of three emitters initialized in the triply-excited state, which interact via left- and right-moving photons. We compared the Markovian and the non-Markovian case, i.e., without and with time delay in propagation between them. In the Markovian case, only a local phase is taken into account but no delayed amplitude in the re-emission events. We recovered the well-known results, that the triply-excited state decays, independent of phase choice, while the doubly- and singly-superradiant superposition state shows population trapping for any non-vanishing phase differences. In strong contrast, a non-Markovian excitation exchange results in population trapping even if the system is initialized in the triply-excited state. Furthermore, quantum feedback allows for states in which two emitters form a superposition state together with a part of the reservoir whereas the third emitter relaxes entirely into the ground state; a state that is not possible to realize in a Markovian setup if only local phases in the jump operators, and no additional interactions, are assumed. These examples prove the significance of time delay in many-emitter, many-excitation systems and the possibility of entirely new physics beyond the Markovian regime.

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