A NUMERICAL INVESTIGATION INTO THE EFFECT OF PRESSURE ON HOLES AND CRACKS IN WATER SUPPLY PIPES

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Short Title: The Effect of Pressure on Holes and Cracks in Water Supply Pipes

Abstract

The effect of water pressure in a pipe on the rate of leakage from leak openings in the pipe is one of the main factors influencing leakage that is still not understood sufficiently. In this study, the behaviours of different types of leak openings (round holes and longitudinal and circumferential cracks) on pressurized pipes were investigated for different pipe materials (uPVC, steel, cast iron and asbestos cement) using finite element analysis. Linear elastic behaviour was assumed. The study found that 1) pipe stresses are significantly affected by a leak opening, and can easily exceed the material's yield strength in the vicinity of the opening; 2) round holes show the smallest expansion with pressure, followed by circumferential cracks and then longitudinal cracks; 3) the areas of all leak openings increase linearly with pressure; 4) longitudinal pipe stresses affect the behaviours of round holes and circumferential cracks, but not that of longitudinal cracks; and 5) the effect of pressure on a leak opening increases exponentially with increasing hole diameter or crack length. An equation is proposed for modelling the effect of pressure on individual leaks.

Keywords

Pipe failures, leakage, pressurized systems, stress concentration, finite element analysis, water distribution networks

NOMENCLATURE

a	crack length, distance
A	area of an orifice
b, n, p, q	dimensions, distances
c	leakage coefficient
C_d	coefficient of discharge
E	modulus of elasticity
F	geometric factor
g	acceleration due to gravity
h	pressure head
k	stiffness
K	stress-concentration factor
K_I	stress intensity factor
m	dimension, distance, slope
N1	leakage exponent
P	pressure
Q	leakage flow rate
r	internal radius of a pipe
S	stress, mean stress
t	thickness of a pipe
y	yield strength
θ	angle
v	Poisson's ratio
σ	allowable stress
σ_θ	stress
σ_{circ}	circumferential stress
σ_l	longitudinal stress
σ_{max}	maximum stress
σ_{nom}	nominal stress

INTRODUCTION

Pressure management is widely recognised as an important technique to control leakage in water distribution systems. While pressure management has been in use for over 20 years in some countries, there is still a lack of understanding of its importance in some sections of the water supply industry (Thornton, 2003). Key to the importance of pressure management is the high sensitivity of leakage to pressure in the distribution system, as found in several field and laboratory studies (Farley and Trow 2003; Greyvenstein and Van Zyl 2007).

The relationship between pressure head and leakage is normally described with the equation:

$$Q = ch^{N1} \tag{1}$$

Where *Q* is the leakage flow rate, *h* the pressure head in the system, *c* the leakage coefficient and *N1* the leakage exponent. A leak can be compared to an orifice, for which the well known Torricelli equation (see for example Chadwick, 1998) describes the relationship between flow rate and pressure head:

$$Q = C_d A\sqrt{2gh} \tag{2}$$

Where C_d the discharge coefficient, *A* the area of the orifice, and *g* acceleration due to gravity. Under conditions where a leak resembles an orifice, the leakage exponent *N1* may thus be determined from equation 2 as 0.5. However, field studies have shown that *N1*, and thus the sensitivity of the leakage rate to pressure, can be significantly larger than 0.5 and typically varies between 0.5 and 2.79, with a mean value of 1.15 (Farley and Trow 2003). Thus it is clear that leak behaviour is more complex than that of an orifice, and that better models are needed to describe the observed behaviour.

Van Zyl and Clayton (2007) proposed four mechanisms that may be responsible for the range of leakage exponents observed, i.e. leak hydraulics, pipe material behaviour, soil hydraulics and water demand. They concluded that material behaviour is likely to be the main mechanism, and can explain the range of exponents observed in the field. This was confirmed by Greyvenstein and Van Zyl (2007) in an experimental study of failed pipes taken from the field.

The objective of this study was to investigate the behaviour of water pipes with different types of leak openings under different internal pressure conditions using the finite element method. Four aspects of the pipe material behaviour were investigated: the stress distribution around the leak openings, the relationship between pressure and leak area, the effect of the opening size (hole diameter or crack length) on the leak behaviour, and the implications of the above on the observed leakage exponent. Three failure types, circular holes, and longitudinal and circumferential cracks; and four pipe materials, uPVC, steel, cast iron (CI) and asbestos-cement (AC) were included in the study. The study focused only on the elastic deformation induced by internal pressure on round holes and longitudinal and circumferential cracks in a pipe. External forces (e.g. soil, traffic, etc.), plastic deformation and propagation of a leak opening (such as a crack) were not considered. The following section introduces previous work on the effect of leak openings on the localised pipe behaviour. The methodology used in the current investigation is then explained, followed by discussions of the results for local deformations, leak area variations with pressure, effect of leak size, and the implications for the leakage exponent. Finally, the main conclusions of the study are summarised.

EFFECT OF LEAK OPENINGS ON LOCALISED PIPE BEHAVIOUR

It is important to recognise that a discontinuity in a pipe wall, such as a leak opening, will affect the stress distribution in the wall. The maximum stress around a leak opening in a pipe can be significantly larger than the stresses in the rest of the pipe. This can be illustrated by considering a flat plate with a small circular hole, subjected to a uniform tensile stress of magnitude *S* along one of its axes as shown in Figure 1 (Timoshenko 1951).

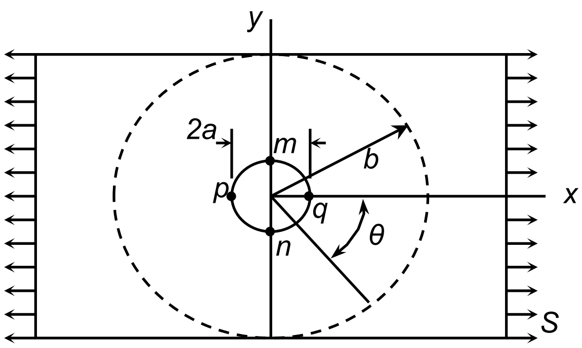

Figure 1 | Plate with hole subjected to uniform tension

It can be concluded from the Saint-Venant principle that any effect of the hole on the stress concentration will be negligible at distances that are large in comparison with the radius of the hole (defined as *a*). Thus, if a concentric circle of radius *b* is considered, with *b* large compared to *a*, the stresses at the larger circle are effectively the same as those in a plate without a hole.

At the edge of the hole, no radial principle stresses or shear stresses exist, and thus the only stresses present are in the circumferential direction. It can be shown theoretically that the circumferential stress at the edge of the hole is given by the equation (Timoshenko 1951):

$$\sigma_\theta = S - 2S\cos 2\theta \tag{3}$$

From this equation it is evident that the maximum tensile stress has a value of 3*S* and occurs at points *m* and *n*. Points *p* and *q* experience compressive stresses of size *S*. The effect of the hole is localised and as the distance from the hole edge increases, the stress approaches *S* very quickly. The distribution of stresses across the centre of the plate is shown in Figure 2.

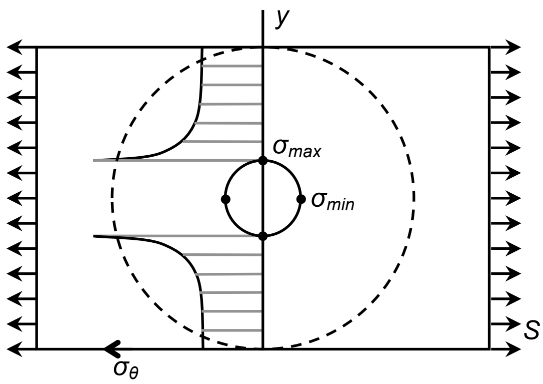

Figure 2 | Stress distribution around a hole in a flat plate (adapted from Timoshenko 1951)

While stresses in pipe walls can vary greatly and are affected by various factors (such as internal fluid pressures and external soil loads), only the wall stresses induced by internal fluid pressure were considered in this paper. It is possible to derive equations for the circumferential and longitudinal stresses in a pipe wall as a result of fluid pressure *p* (for example see Gere 2001):

$$\sigma_{long} = \frac{pr}{4t} \tag{4}$$

$$\sigma_{circ} = \frac{pr}{2t} \quad (5)$$

With *r* the inner radius of the pipe, *p* internal pressure, *t* thickness of the pipe wall and *σ* stress. It can be observed from the equations that the circumferential stresses are double the longitudinal stresses. Two loading states were thus considered in this paper: the bi-axial loading state in which longitudinal and circumferential stresses are present as described by Equations 4 and 5, and the uni-axial loading state where longitudinal stresses were assumed to be zero, and thus only circumferential stresses exist.

The case of a pipe with a round hole under uni-axial loading can be compared to the flat plate discussed above, and thus the highest stresses are likely to occur at the hole edges along the length of the pipe. The effect of a hole or other discontinuity on the material behaviour can be expressed using a stress concentration factor *K*, defined as the ratio of the maximum stress to the nominal stress (Gere 2001):

$$K = \frac{\sigma_{max}}{\sigma_{nom}} \quad (6)$$

Theoretical value of *K* for a flat plate with a round hole would be 3, but this is likely to be different for a pipe due to the curvature in the materials, and the presence of longitudinal stresses.

If the hole in Figure 1 is replaced by a crack transverse to the loading direction, fracture mechanics theory predicts that the highest stress concentrations will be found at the tips of the cracks. Linear elasticity then predicts a discontinuity (infinite stress) for an ideally sharp crack tip. Because of plastic deformation and subsequent load redistribution, this condition does not exist in practice. In fracture mechanics the severity of a crack is therefore described by the introduction of the stress intensity factor, K_I (Dieter 1988):

$$K_I = FS\sqrt{\pi a} \quad (7)$$

Where *F* is a geometric factor, *S* is the mean stress and *a* is the appropriate crack length. This expression is used in conjunction with the fracture toughness of the material to predict the susceptibility of a crack to sudden propagation.

METHODOLOGY

The finite element method is a numerical analysis technique for obtaining approximate solutions to a wide range of engineering problems. This method was originally developed to study stresses in complex airframe structures, but has since been adapted to a wide field of continuum mechanics (Huebner et al, 2001). Finite elements are used by creating different geometric regions, establishing separate approximating functions in each region and then joining them together. A finite element model of a problem therefore gives a piecewise approximation to the governing equations. This shows that a model can be split into a number of solution regions which can be analytically modelled by replacing these solution regions with a group of discrete elements. Since these elements can be put together in a variety of ways, they can be used to represent exceedingly complex shapes. In this study the geometric models were built using Solidworks Computer Aided Design software (Dassault Systèmes Solidworks Corp, 2009). Abaqus Standard Finite Element Analysis software (Hibbitt et al, 2004) was used to conduct the finite element analysis.

Three different leak openings were investigated in this study: circular holes and longitudinal and circumferential cracks. The study was based on a 110 mm diameter class 6 uPVC pipe with a wall thickness of 3 mm, where the class refers to the working pressure of the pipe i.e. class 6 has a working pressure of 600 kPa (61 m) (Myles) . In order to compare the behaviour of other pipe materials (steel, CI and AC), the properties of these materials were used to determine the wall thickness for a pipe with the same internal diameter and design pressure as the uPVC pipe, subject to the safety factors normally used for the different materials. It was not convenient to use commercial pipe properties for such a comparison, since internal diameters differ and parameters are often affected by other considerations, such as a minimum allowable wall thickness. The working pressure of the pipes was assumed to be 600 kPa (61 m).

The uPVC pipe was analysed in accordance with the South African National Standards (SANS1223:2003, SANS62-1:2003, SANS966-1:2004) code of practice, which specifies an allowable stress of 10.4 MPa (SANS 966-1, 2004). Along with the yield strength of 50 MPa, this implies a design safety factor of 4.8.

The equivalent steel pipe was calculated in accordance with the SANS 62-1 code of practice, which specifies an allowable stress of 99 MPa and a yield strength of 200 MPa. This implies a design safety factor of 2.1. For the equivalent CI pipe, an allowable stress of 25% of the ultimate tensile strength of 207 MPa was used in accordance with Molnar et al. (2004). The allowable stress was thus determined as 8.4 MPa, which implies a safety factor of 4. SANS 1223 was used to determine the equivalent AC pipe. The code specifies an allowable stress of 37.5 % of the ultimate tensile strength of the material of 22.5 MPa, which implies a safety factor of 2.67.

A summary of the properties of the materials and the dimensions of the equivalent pipes used in the analysis are given in Table 1.

Table 1 | Material properties and dimensions of equivalent pipes used in the analysis

Property	**uPVC**	**Steel**	**CI**	**AC**
Modulus of elasticity, *E* (GPa)	3	200	100	24
Poisson's ratio, ***v***	0.4	0.29	0.21	0. 17
Yield strength, *y* (MPa)	50	200	207	22.5
Allowable stress, *σ* (MPa)	10.4	99	52	8.4
Safety factor	4.8	2	4	2.67
Internal diameter (mm)	104	104	104	104
Wall thickness (mm)	3	0.314	0.603	3.7

Ten-noded quadratic tetrahedron elements were used throughout in the finite element analyses. A sensitivity analysis was done to ensure that the pipe section was long enough to avoid end effects affecting the results around the leak openings. Another sensitivity analysis was done for each pipe configuration to determine the optimal finite element sizes to use around the leak openings. Too large elements would not provide the required accuracy, while too small elements could reduce the computational speed with little accuracy gain. Generally the sizes of the elements around the circular holes were 0.5 mm for uPVC and AC and 0.3 and 0.4 mm for steel and CI pipes. For longitudinal and circumferential cracks, elements of 0.5, 1.0, 0.4 and 0.5 mm were used for uPVC, AC, steel and CI pipes respectively. Elements of 5 mm were used for areas of the pipe away from the leak openings. In all cases the material was assumed to be linearly elastic.

The pipe was clamped in the *x* and *y* directions along a longitudinal internal line furthest from the leak opening, as well as a point (clamped in *x*, *y* and *z* directions) on the outside of the pipe adjacent to the internal line. The loadings applied to the pipe consisted of a pressure applied to the internal surface of the pipe for a uni-axial loading state, and longitudinal stresses were applied on the ends of the pipes, which were calculated using Equation 4, for a bi-axial loading state. Finite element analyses were run for three pressures, 200, 400 and 600 kPa (or 20, 41 and 61 m), for each leak opening and material. After each analysis, the area of the leak opening was estimated and Equation 2 applied to calculate the leak discharge, assuming a constant discharge coefficient of 0.67.

LOCAL DEFORMATION AROUND LEAK OPENINGS

The effect of circular holes and longitudinal and circumferential cracks on the stress distributions in pressurised pipes of different materials are considered in this section.

Circular Holes

The results of the analysis for the Von Mises stresses in a class 6 uPVC pipe section with a 12 mm hole and under a pressure head of 600 kPa (61 m) are shown graphically in Figure 3. The expanded view shows the detail around the hole as seen from the inside of the pipe.

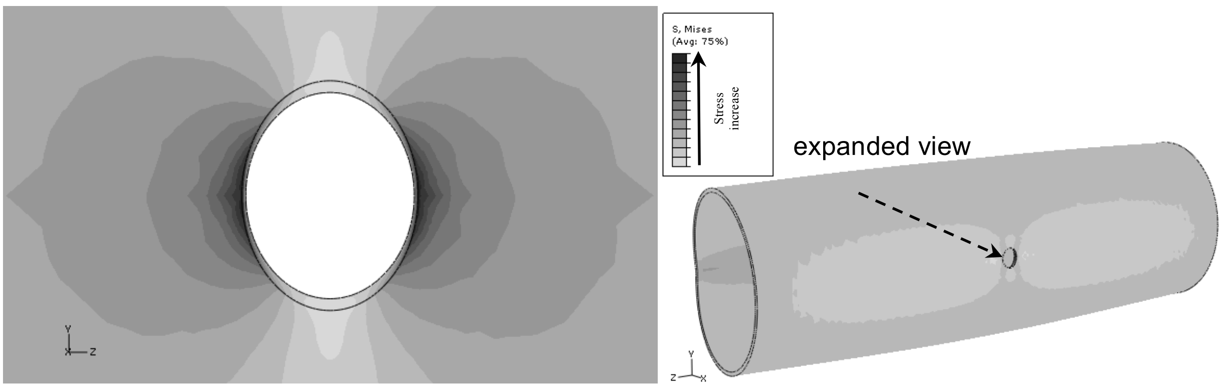

Figure 3 | Stresses and scaled up deformations (30 times) around a round hole in a pipe

The highest stresses occurred at the inside lip of the hole at the furthest edges along the length of the pipe as suggested in the theoretical discussion earlier. The lowest stresses occurred at the furthest edges in a transverse direction. Deformations were very small, but scaling them up revealed that the area around the hole expands outward, while the hole itself is pulled slightly into the pipe. The shape of the round hole is changed to an ellipse due to the uneven stresses in the pipe material. All pipe materials investigated displayed similar stress distribution patterns.

Stress concentrations were determined for different diameters of round holes for the pipes listed in Table 1, and the results are shown in Figure 4. The figure shows that the stress concentrations increase in a roughly linear fashion with increasing hole size. Steel and CI pipes display similar origins, with the stress concentration approaching 1 when the hole diameter approaches zero. While their slopes are similar, steel has higher stress concentrations than CI. uPVC and AC also displayed similar slopes with the stress concentration approaching a value of roughly 1.6 as the hole diameter approaches zero. uPVC has higher stress concentrations than AC for all hole diameters. uPVC and AC pipes have the highest stress concentrations for smaller holes (< 3.4 mm), while steel and CI have the highest stress concentrations for larger holes (> 10.2 mm). The theoretical stress concentration of 3 for a flat plate is found in the 6 to 11 mm hole diameter range.

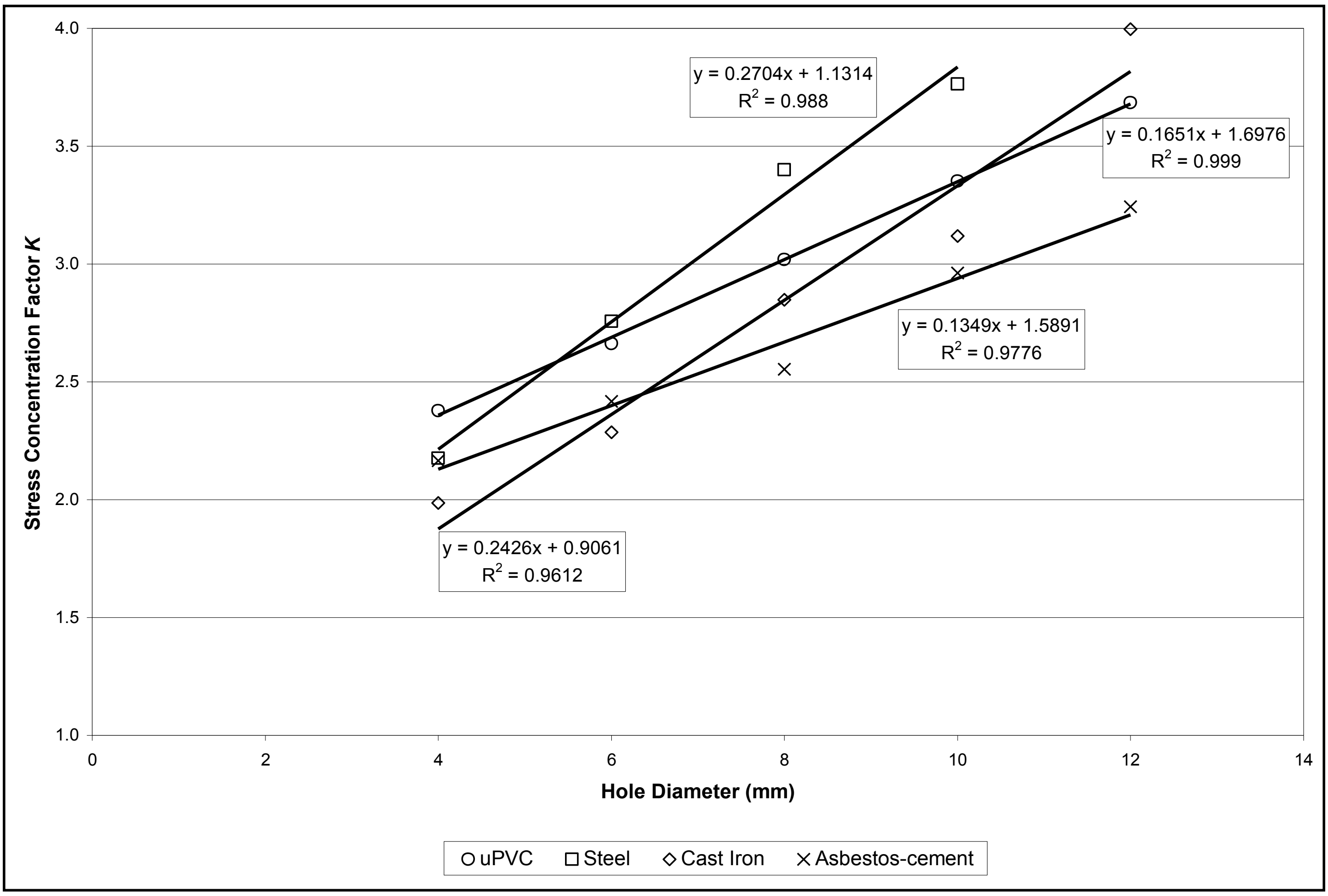

Figure 4 | Stress concentration factor, K versus circular hole size for a class 6 uPVC pipe

The analysis shows that the maximum stresses at a hole in a pipe can be substantially higher than the nominal pipe stresses. With the safety factors for the pipes studied varying between 2 and 4.8, the implication is that the allowable stresses in the pipe materials can easily be exceeded when a leak is present. For round holes, the yield strength of uPVC, steel, CI and AC pipes are exceeded when the hole diameters exceed 19, 3, 13 and 38 mm respectively.

Longitudinal Cracks

Figure 5 shows the stress distribution in a class 6 uPVC pipe section under a pressure head of 600 kPa (61 m) and with a longitudinal crack, initially 40 mm long and 1 mm wide. The crack tips were modelled with a constant radius of 0.5 mm. It is clear from the figure that the crack affects the stress distribution in the pipe, with the highest stresses occurring at the ends of the crack. The extent and shape of the deformation occurring around the crack is shown in Figure 5. The opening of the crack due to internal pressure is clearly visible.

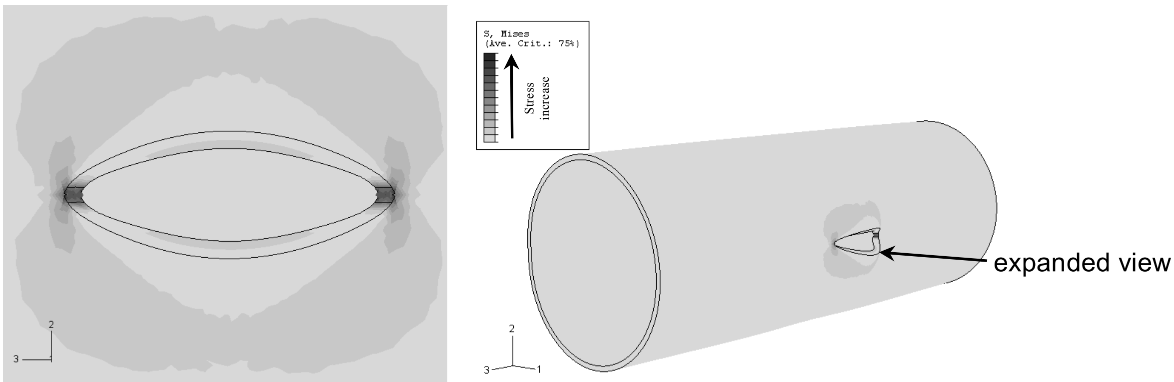

Figure 5 | Stresses and scaled up deformations (15 times) around a longitudinal crack in a pipe

Circumferential Cracks

Figure 6 shows the stress distribution in a class 6 uPVC pipe section under a pressure head of 600 kPa (61 m) and with a circumferential crack, initially 40 mm long and 1 mm wide. The crack tips were again modelled with a constant radius of 0.5 mm. As in the case of longitudinal cracks, it is clear that the crack affects the stress distribution in the pipe, with the highest stresses occurring at the ends of the crack. The deformation of the pipe is shown scaled up in Figure 6. Again the maximum deformation occurs at and around the crack and the figure clearly illustrates how the crack opens up due to the pressure in the pipe.

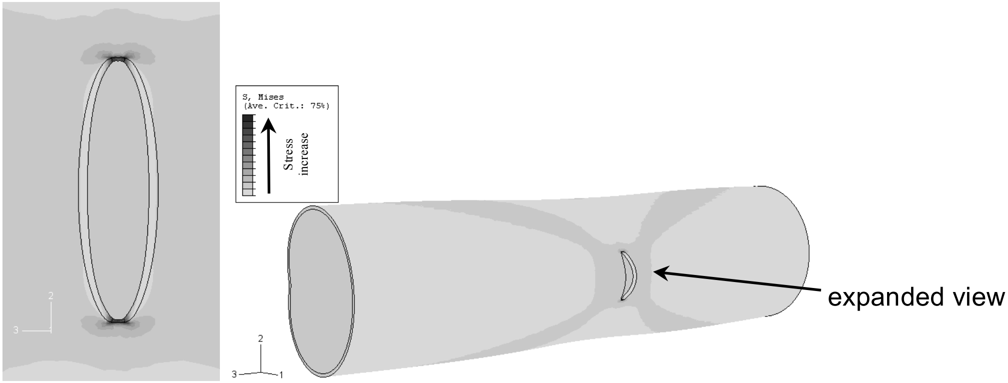

Figure 6 | Stresses and scaled up deformations (50 times) around a circumferential crack in a pipe

Discussion

It is clear from the analysis that pipe stresses are significantly affected by leak openings. Stresses are increased in certain regions of a leak opening, especially in cracks tips, or at the edges along the length of the pipe in round holes. At these stresses, the material will deform plastically to accommodate the stresses, or may fail catastrophically to create a burst when the stress intensity factor exceeds the fracture toughness of the material. According to fracture mechanics, round holes are the most stable. However, if the hole deforms into an elliptical shape and eventually starts to resemble a crack, the severity of the stress intensity increases and the possibility of sudden crack propagation increases.

The stress intensity of a leak opening subjected to a mean stress is a function of the characteristic length of the opening and its orientation relative to the applied mean stress. This implies that even a sharp crack of appreciable length may have a low stress intensity if orientated appropriately.

EFFECT OF PRESSURE ON LEAK AREA

To determine the increase in leak area with increasing pressure, different sizes of leak openings were modelled in each pipe material at pressure heads of 200, 400 and 600 kPa (or 20, 41 and 61 m). The maximum pressure of 600 kPa (61 m) represents the maximum operating pressure of the class 6 uPVC and equivalent pipes. At each pressure, the area of the leak opening was calculated from the positions of edge nodes in the finite element model.

The behaviour of leak areas for different leak sizes as functions of pressure are shown in Figures 7, 8 and 9 for round holes, longitudinal cracks and circumferential cracks respectively. While only uPVC pipe behaviour is shown in the figures, other pipe materials displayed similar behaviour. Relative area increases were used to allow the different leak sizes to be shown on the same figures. It was observed that the leak areas can be modelled as linear functions of pressure for all pipe materials, load states, leak shapes and leak sizes. The square Pearson product moment correlation coefficient (R^2) was found to be larger than 0.999 for the vast majority of cases.

The main assumption made in the finite element model was that all deformations were assumed to be elastic. While pipes in the field will experience dynamic changes in leaks' shapes and sizes due to plastic deformation and material fracturing, it is likely that the observed linear behaviour will occur in most leaks.

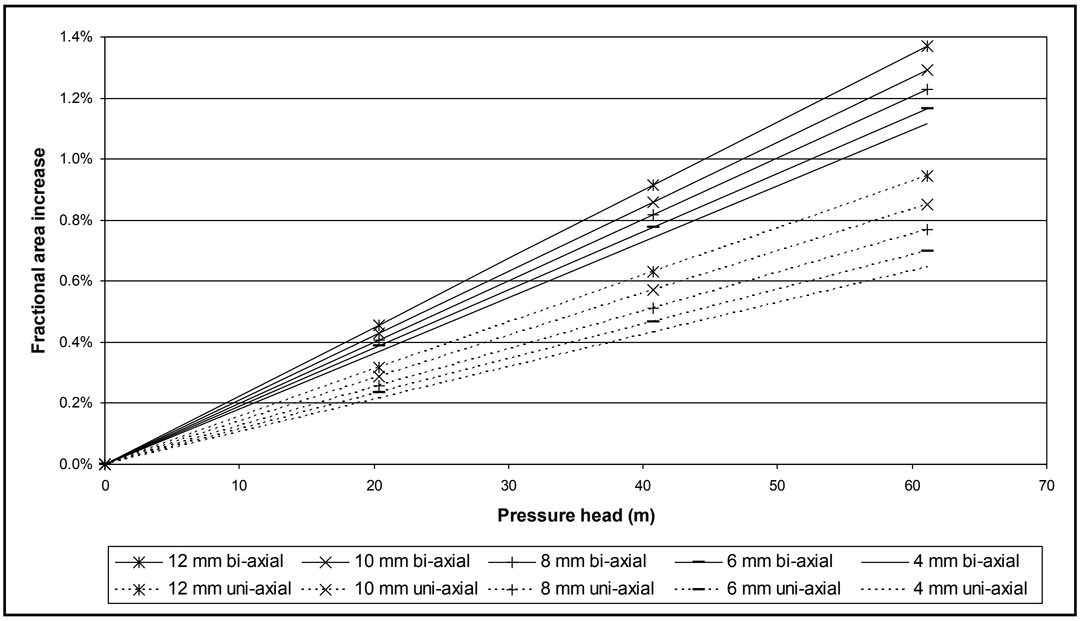

Figure 7 | Fractional increase in area of different diameter round holes as a function of pressure in uPVC pipes

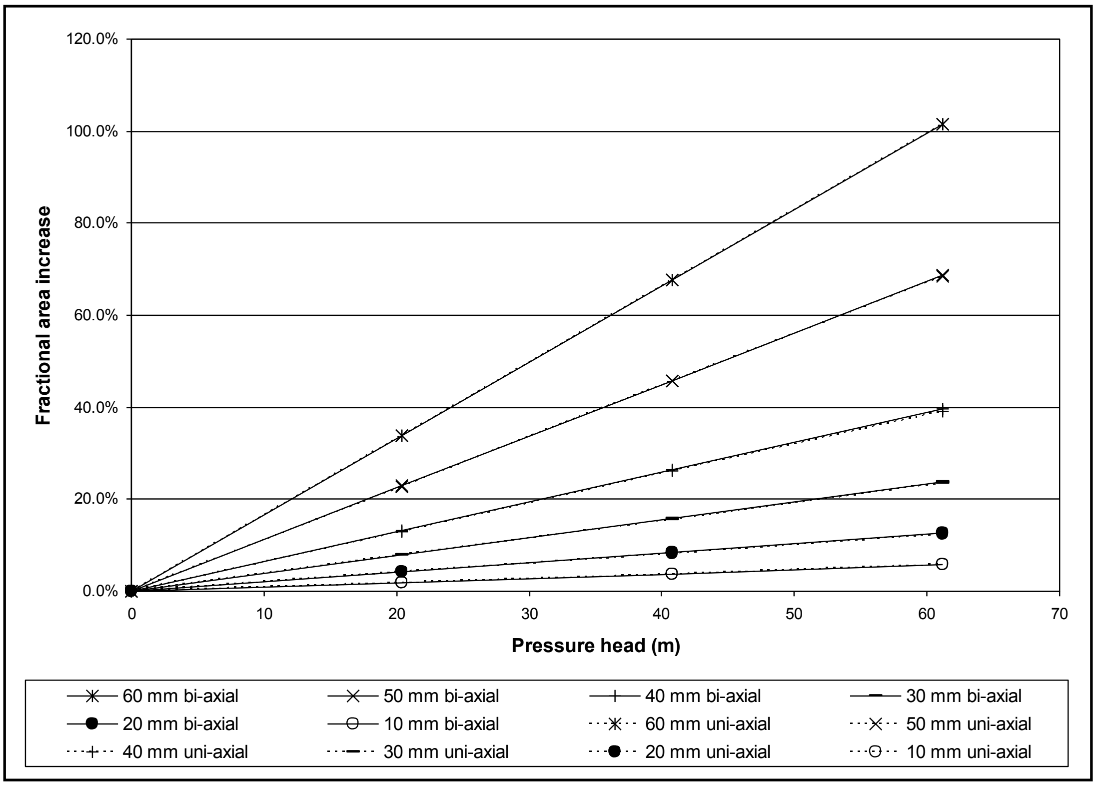

Figure 8 | Fractional increase in area for different longitudinal crack lengths as a function of pressure in uPVC pipes

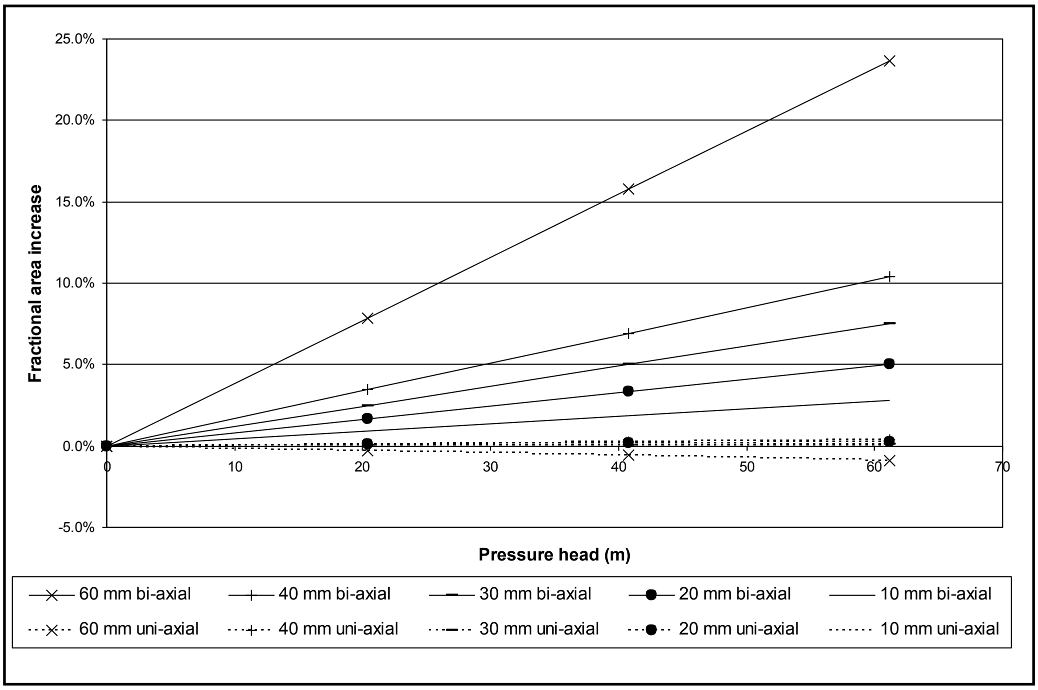

Figure 9 | Fractional increase in area for different circumferential crack lengths as a function of pressure in uPVC pipes

It is also clear from the results that the shape of the leak opening plays an important role in the behaviour of the leak: round holes display very little expansion with pressure compared to cracks, and circumferential cracks show less expansion than longitudinal cracks. In all cases, the impact of pressure on the leak area was greater for larger (or longer) leak openings.

The loading state affected the behaviour of round holes and circumferential cracks, but had no influence on the behaviour of longitudinal cracks. In round holes, the bi-axial loading state caused a significantly greater area increase than the uni-axial loading state. Circumferential cracks increased in area under the bi-axial loading state, but actually reduced in area under the uni-axial loading state. This behaviour was also observed experimentally by Greyvenstein and Van Zyl (2007). The area reduction is caused by an elongation of the crack in the circumferential direction, which is accompanied by a reduction in the width in the longitudinal direction due to Poisson's ratio effects.

Implications for Leakage Modelling

The fact that all leak types displayed proportional increases in leak area with pressure means that the behaviour of leak area with pressure head may now thus be described with a linear equation:

$$A(h) = mh + A_0 \tag{8}$$

With h the pressure head, A the leak area, A_0 the initial leak opening without any pressure in the pipe, and m the slope of the pressure-area line. Replacing this in Equation 2 results in the following equation for the leakage rate as a function of pressure:

$$Q = C_d\sqrt{2g}\left(A_0h^{0.5} + mh^{1.5}\right) \tag{9}$$

The form of the equation differs significantly from the commonly used Equation 1, and should provide a better description for the leakage behaviour of any system where leak areas increase linearly with pressure.

Equation 9 is similar to the one proposed by May (1994), who first proposed the concept of fixed and variable leaks. The main difference between the two equations is that May suggested that some leaks have fixed areas (with an exponent of 0.5), while others have variable areas (with an exponent of 1.5). Equation 9 assumes that all leaks have areas that vary linearly with pressure, and that it is only the extent of the variations that differs.

EFFECT OF OPENING SIZE ON LEAK BEHAVIOUR

The observation that leak areas increase linearly with pressure allows the behaviour of a leak opening to be characterised by the slope of the area vs. pressure line (called the pressure-area slope or *m* in the rest of this paper). This pressure-area slope was determined for different size leak openings in different materials, and the results are shown in Figures 10 to 12.

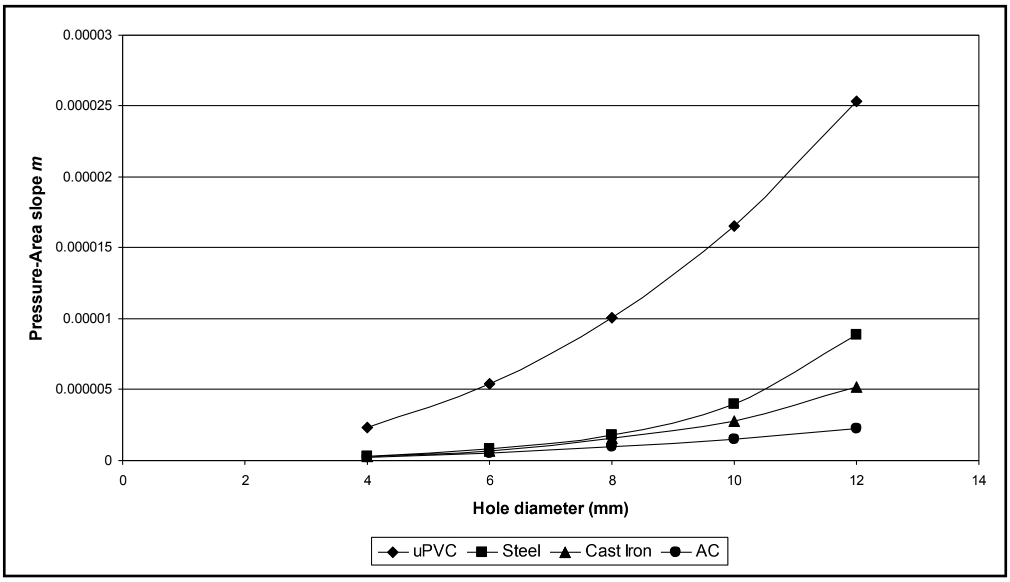

Figure 10 | Effect of pipe material and round hole diameter on the slope of the pressure head coefficient

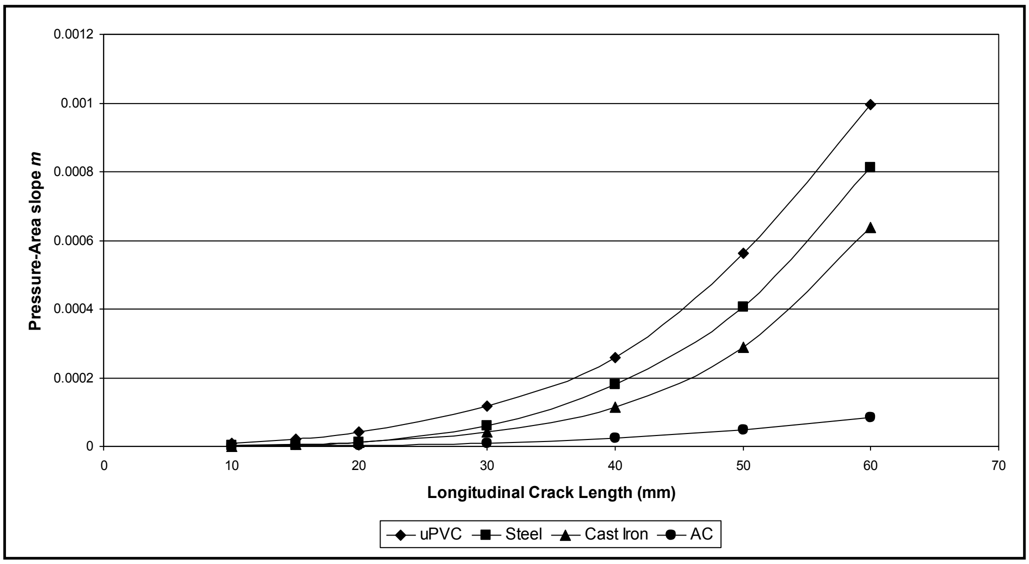

Figure 11 | Effect of pipe material and longitudinal crack length on the slope of the pressure head coefficient

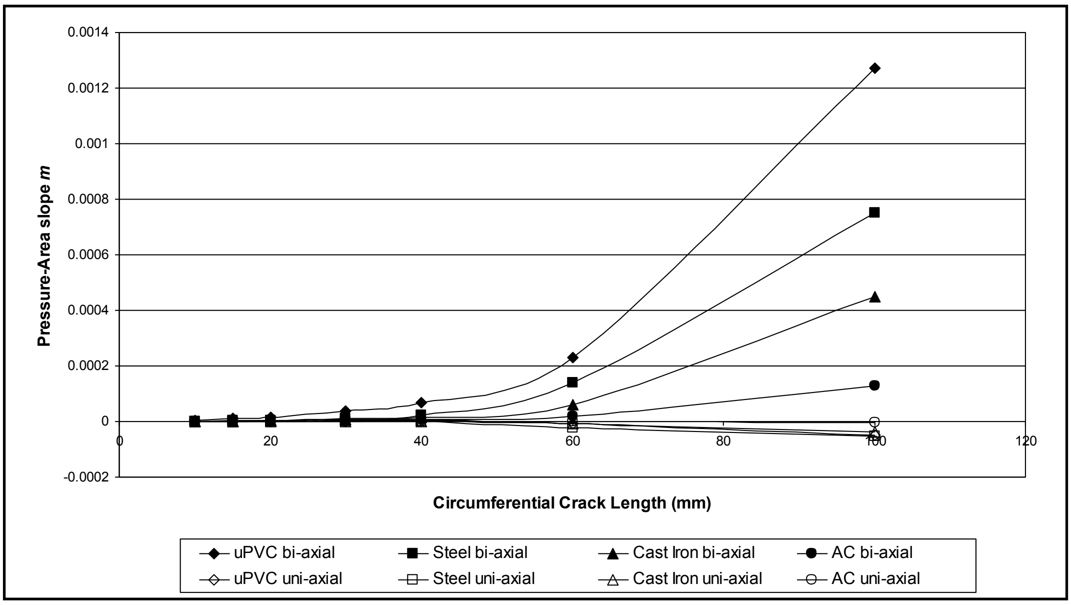

Figure 12 | Effect of pipe material and circumferential crack length on the slope of the pressure head coefficient

The results show that the pressure-area slope increases in an exponential fashion with increasing hole diameter or crack length, except for circumferential cracks under uni-axial loading, which displays decreasing negative slopes with increasing crack length. The effect of pressure on large leak openings will thus be substantially greater than the effect on small openings.

In all cases, the uPVC pipe displayed the greatest slopes, followed by the equivalent steel, CI and AC pipes. The reason for this is likely to be a combination of material and sectional properties, and local stiffness, and falls outside the scope of this study.

LEAKAGE EXPONENT N1

The finding that the areas of all types of leaks increases linearly with pressure has significant implications for the way the pressure-leakage relationship is modelled. In particular, Equation 9 provides a better description of the leakage behaviour than the commonly used Equation 1 for the behaviour of a single leak. However, the applicability of this equation to a whole distribution system with many leaks needs to be investigated further. Equation 9 predicts a maximum N1 of 1.5, and thus does not explain higher leakage exponents. Research on plastic deformation of leaks and other factors affecting the pressure-leakage relationship may throw more light on this problem in future.

Since the forms of Equations 1 and 9 differ, it is not simple to convert the leakage slope to an N1 coefficient. Such a conversion will also produce different N1 values when determined at different pressures. To allow some comparison, Figures 13 to 15 presents the equivalent N1 coefficients determined for the pressure-area slopes reported in Figures 10 to 12 at a pressure of 50 m.

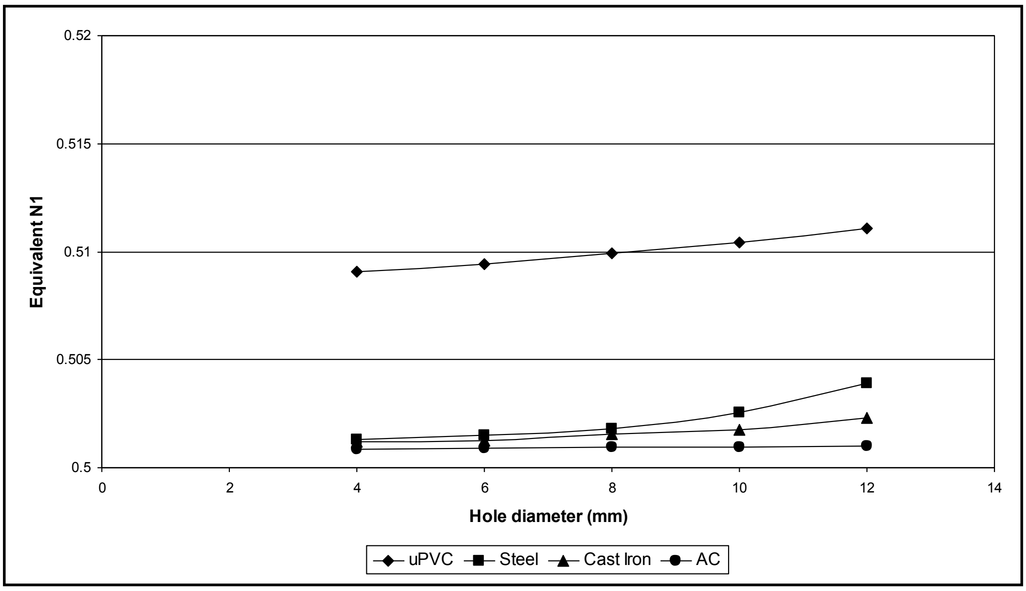

Figure 13 | The equivalent N1 for round holes at 50 m pressure head.

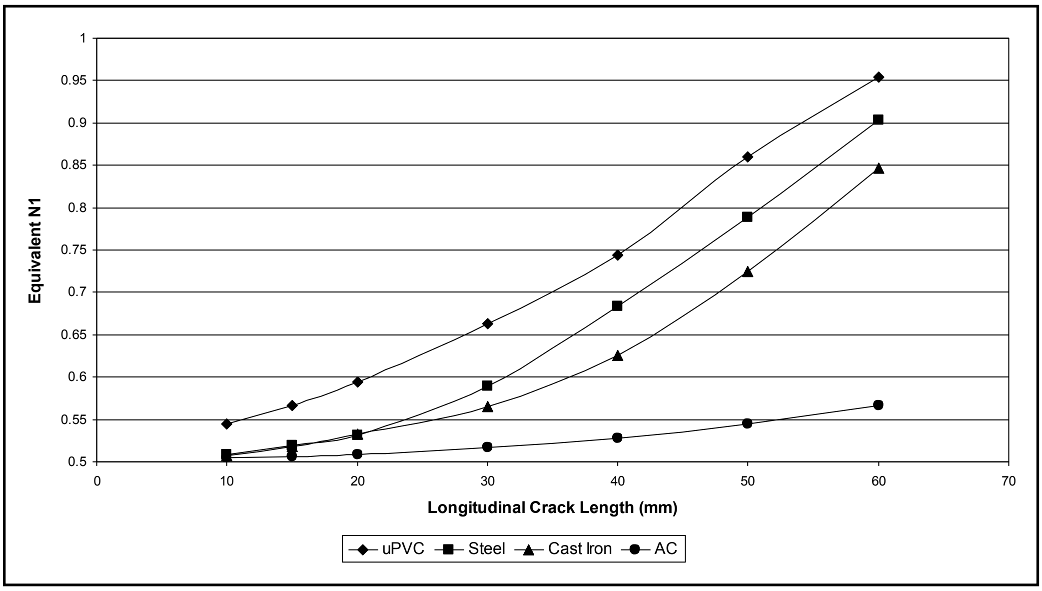

Figure 14 | The equivalent N1 for longitudinal cracks at 50 m pressure head.

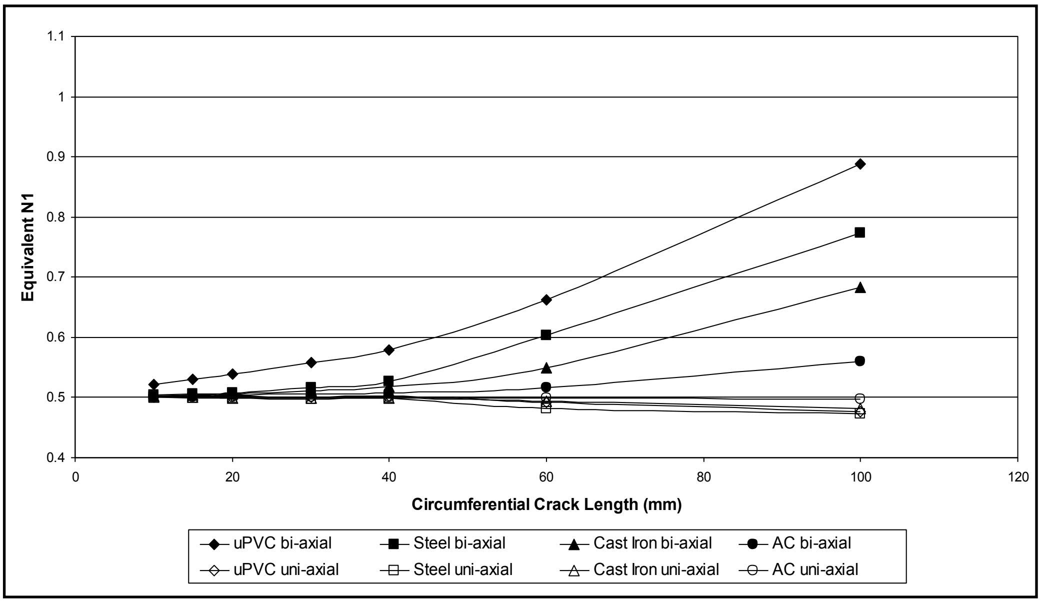

Figure 15 | The equivalent N1 for circumferential cracks at 50 m pressure head.

CONCLUSIONS

This study investigated the behaviour of various leak openings in pipes under pressure using finite element analysis. The pipes analysed were limited to an internal diameter of 104 mm and a 600kPa (61 m) working pressure, as well as being restricted to two loading states (bi-axial and uni-axial). These pipes assumed orifice hydraulics, i.e. Equation 2, and elastic behaviour of the pipe materials. Three failure types (circular holes, and longitudinal and circumferential cracks) and four pipe materials (uPVC, steel, CI and AC) were included in the study. Normal safety factors were used to calculate the properties of the steel, CI and AC pipes to be equivalent and comparable to the uPVC pipe. Linear elastic material behaviour was assumed in all analyses.

The main findings of the study can be summarised as follows.

- Pipe stresses are significantly affected by leak openings. Stresses are increased in certain regions of a leak opening, especially in cracks tips, or at the edges along the length of the pipe in round holes. Local stresses can easily exceed the material's yield strength, causing plastic deformation to take place. The resultant permanent changes in the leak opening can affect the leak area through deforming the opening, increasing the length of the opening, or even triggering catastrophic failure of the pipe.

- The areas of all three leak types were found to increase linearly with pressure for both loading states in all the materials tested. Substitution of this relationship into Equation 2 results in the following equation for the leakage rate as a function of pressure:

$$Q = C_d \sqrt{2g}\left(A_0 h^{0.5} + m h^{1.5}\right)$$

  With *h* the pressure head, A_0 the initial leak area at zero pressure, *m* the pressure-area slope, C_d the discharge coefficient and *g* acceleration due to gravity. This equation provides a better description of the leakage behaviour than the commonly used Equation 2. It predicts a maximum leakage exponent of 1.5.

- Round holes expanded little with pressure. Longitudinal cracks displayed the largest increases, followed by circumferential cracks.

- Round holes displayed significantly greater expansions under the bi-axial loading state than the uni-axial loading state. The areas of circumferential cracks expanded under the bi-axial loading state, but actually decreased in the uni-axial loading state. The loading state did not affect the behaviour of longitudinal cracks.

- The effect of pressure on large leak openings (described by the pressure-area slope) increases in an exponential fashion with increasing hole diameter or crack length.

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