Manuscript No: HY23465

EXPLICIT INTEGRATION METHOD FOR EXTENDED-PERIOD SIMULATION OF WATER DISTRIBUTION SYSTEMS

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Keywords: Water distribution; Simulation models; Dynamic analysis

Abstract: Extended-period simulation of incompressible and inertialess flow in water distribution systems is normally done using numerical integration techniques, although regression methods are also sometimes employed. A new method for extended-period simulation, called the Explicit Integration (EI) method, is proposed. The method is based on the premise that a complex water distribution system can be represented by a number of simple base systems. The simple base systems are selected in such a way that their dynamic equations can be solved through explicit integration. In this paper a simple base system consisting of a fixed-head reservoir feeding a tank through a single pipeline is analyzed. It is then illustrated how a complex water distribution system can be decoupled into simple base systems and its dynamic behavior simulated using a step-wise procedure. The EI method is compared to the commonly used Euler numerical integration method using two example networks. It is shown that the accuracy of the EI method is considerably better than that of the Euler method for the same computational effort.

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INTRODUCTION

Extended-period simulation tracks the dynamic behavior of a water distribution system over a period of time under incompressible and inertialess flow conditions. Extended-period simulations allow engineers to evaluate the service levels provided by a distribution system throughout a day, including minimum and maximum pressures, tank levels and water quality parameters. The applications of extended-period simulation were recognized as early as 1968 by Shamir and Howard, although little related work had been published by 1977 compared to the activity in snapshot analysis (Rao and Bree 1977). A review of published papers shows that this is still the case today.

The output of extended period simulation is normally the behavior of tank heads with time, which then allows the state of the system at any particular time to be determined by through a snapshot simulation. The set of equations describing the dynamic behavior of tanks in a water distribution system is complex and can generally not be solved analytically (Coulbeck 1980). Two approaches have traditionally been employed to solve the dynamic equations, namely numerical integration and regression methods. Numerical methods are reliable and accurate, but require a large number of snapshot simulations to achieve acceptable accuracy. The simplest two numerical integration methods available, the Euler (Wood 1980; Rossman 1993; Ulanicka et al. 1998) and Improved Euler (Brdys and Ulanicki 1994; Water Research Centre 1994) methods are commonly applied in extended-period simulation of water distribution systems.

The main disadvantage of both the Euler and Improved Euler methods is the sensitivity of their approximation errors to the length of the time step used (Kreyszig 1999). Since the accuracy of the two methods decreases with increasing time step length, the time step is typically restricted to one hour or less (Walski et al 2001). As a result, a 24-hour simulation requires at least 24 snapshot simulations, which makes expended period simulations computationally expensive and time consuming. Although the simulation time is not normally a problem on modern high-speed computers, it can be an important factor when large numbers of extended-period simulations are required (for example in operational optimization runs), or long simulation periods are used (Van Zyl et al 2004).

In regression methods, empirical or semi-empirical functions are fitted to calculated data on the behavior of distribution systems under different operating conditions. The equations can then be used to simulate the system's dynamic behavior efficiently. However, a new set of curves has to be calculated for each possible combination of dynamic variables for each tank, increasing the number of curves exponentially with the number of tanks and possible system states, and thus limiting the size of network that can be modeled. Additionally, a new set of curves has to be generated whenever a change is made to the system configuration, or when a new system is analyzed.

Filion and Karney (2003) noted that rapid transients and the associated fluctuations in pressures are often present in a water distribution system and that equilibrium, an implicit assumption of extended period simulation, is thus never really achieved. They proposed a new method (Filion and Karney 2000) that combines a numerical integration method with a transient simulation model to improve the accuracy and capabilities of extended-period simulations. Their method analyses a water distribution system for short time periods near the start and end of a time step using a transient or water hammer model, and then uses the insights gained to predict the behavior of the system with a modified Improved Euler approach. Their method shows a significant increase in simulation accuracy, but requires substantially more system information and computational effort.

In this paper, a new method for solving the incompressible dynamic equations of water distribution systems is introduced (Van Zyl 2001). The method is called the Explicit Integration (EI) method and is based on the premise that a complex water distribution system can be represented by a number of simplified systems (called simple base systems) with calculable dynamic behavior

SIMPLE BASE SYSTEM

Basic Dynamic Equations

Consider a simple base system consisting of a fixed-head reservoir feeding a tank through a single pipe, with a demand taken directly from the tank as shown in figure 1.

Ideal position for figure 1

Since mass is conserved, the dynamic behavior of the tank in can be described by the differential equation:

$$A\frac{dH}{dt} = Q_i - Q_d = Q \qquad (1)$$

Where Q_i the flowrate in the pipe, Q_d the demand at the tank, Q the net tank inflow, A the tank cross-sectional area, H the tank head, and t time.

The headloss h in the pipe is given by (Bhave 1988) (resistance form):

$$h = rQ_i \left| Q_i \right|^{n-1} = H_P - H \tag{2}$$

Where r a pipe resistance coefficient, H_P the fixed upstream reservoir head, and n a headloss exponent.

This equation can be rearranged to give the pipe flowrate as a function of the head differential between the reservoir and tank (conductance form):

$$Q_i = r'\left(H_P - H\right)\left|H_P - H\right|^{\frac{1-n}{n}} \tag{3}$$

Where
$$r' = \left(\frac{1}{r}\right)^{\frac{1}{n}} \tag{4}$$

It is normal modeling practice to group demands and apply them as known end conditions to the nodes of a hydraulic model. Strictly speaking, demand flowrates are not only functions of time since the node outflows occur via orifices (e.g. open taps or valves) and are thus dependent on the pressure in the system. However, this relationship is generally ignored in hydraulic analysis (Reddy and Elango 1989) and demand values are imposed on the system as functions of time only. This assumption simplifies the modeling process considerably but has been shown to be invalid only in cases where the system is operating under failure or abnormally low pressure conditions (Germanopoulos 1985). The demand function is given by $Q_d = Q_d(t)$. Substituting this and (3) into (1) results in a differential equation for the dynamic behavior of the tank head:

$$A\frac{dH}{dt} = r'(H_P - H)\left|H_P - H\right|^{\frac{1-n}{n}} - Q_d(t) \tag{5}$$

This equation is both non-homogeneous and nonlinear, and can seldom be solved explicitly. However, the equation can be solved if demands are ignored (Van Zyl et al 2005) or the system hydraulics is linearized.

Linear approximation of hydraulics

General formulation

The nonlinearity in (5) can be removed by approximating the pipe flowrate Q_i with a linearized flowrate function Q_{il}:

$$Q_{il} = aH + b \tag{6}$$

Where a and b the coefficients of the linear equation. The value of n in (2) depends on the type of flow experienced in the pipe. Under laminar (low Reynolds number) flow conditions, n equals one and the linear approximation would thus fit the headloss relationship exactly. At the other extreme, n equals two under rough turbulent (high Reynolds number) flow conditions. Although laminar flow conditions does occur in distribution systems, especially under low demand conditions (McKenna et al 2004), flow in commercial pipes usually occurs in the transitional turbulent zone (Brater and King 1976). The worst case when linearizing the pipe hydraulic behavior will clearly be the extreme where n equals two, and is thus considered further.

Given any two points, (H_0, Q_0) and (H_1, Q_1) with $H_0 < H_1 < H_P$, on a pipe's headloss curve, the coefficients of the linear approximation can be estimated. Substituting Q_{il} for Q_i in (5) results in a linear differential equation, which can be integrated explicitly to obtain:

$$H(t) = Ce^{\frac{a}{A}t} - \frac{b}{a} - \frac{1}{A}e^{\frac{a}{A}t}\int e^{-\frac{a}{A}t}Q_d(t)dt; \quad a \neq 0 \tag{7}$$

Where C an unknown constant term.

Formulation for constant pipe flow

Equation 5 is not valid for the special case where $a = 0$ and thus where Q_{il} is constant. This represents the case where the flowrate in the pipe is independent of the tank head, or in practice, where the effect of a change in tank head on the pipe flow is small. This case is considered separately by substituting $Q_{il} = b$ for Q_i in (5) and integrating the equation explicitly to obtain:

$$H = \frac{b}{A}t - \frac{1}{A}\int Q_d(t)dt \tag{8}$$

Approximation error

It is simple to find the linear coefficients a and b from any two points on the pipe's headloss curve. The approximation error E is defined as the difference between Q_{il} and Q_i, and a typical case is shown in figure 2. The error is zero at the two interpolation points ($H = H_0$ and $H = H_1$). The figure shows that linear approximation underestimates the true flowrate when interpolating between the data points and overestimates the true flowrate when extrapolating.

Ideal position for figure 2

Furthermore, the approximation error increases rapidly when extrapolating in the direction of the decreasing head differential ($H > H_1$), but much more gradually when extrapolating in the direction of increasing head differential ($H < H_0$). As a result extrapolation in the direction of decreasing head differential should be avoided. However, extrapolation in the direction of increasing head differential can be used to balance the interpolation error. For a uniform decrease in head from H_1 it was found that the extrapolation error equals the interpolation error at a distance roughly double the distance between H_0 and H_1.

Polynomial Demand Function

To make the method more specific, a polynomial demand function is now assumed in the form:

$$Q_d = \sum_{n=0}^{N} d_n t^n \tag{9}$$

Where d_n the coefficient of the n^{th} term, and N the order of the polynomial function.

Substituting the polynomial demand function into (7) and simplifying results in:

$$H(t) = Ce^{\frac{a}{A}t} - \frac{b}{a} - \frac{1}{A} e^{\frac{a}{A}t} \sum_{n=0}^{N} d_n \int t^n e^{-\frac{a}{A}t} dt; \quad a \neq 0 \tag{10}$$

Integrating this equation now results in an expression for the tank head as a function of time:

$$H(t) = Ce^{\frac{a}{A}t} - \frac{b}{a} + \sum_{n=0}^{N} \left[d_n \sum_{m=0}^{n} \left(\frac{A^m}{a^{m+1}} \frac{n!}{(n-m)!} t^{n-m} \right) \right]; \quad a \neq 0 \tag{11}$$

The constant C is obtained by substituting the boundary condition $H = H_0$ at $t = t_0$ as:

$$C = e^{-\frac{a}{A}t_0} \left\{ H_0 + \frac{b}{a} - \sum_{n=0}^{N} \left[d_n \sum_{m=0}^{n} \left(\frac{A^m}{a^{m+1}} \frac{n!}{(n-m)!} t_0^{n-m} \right) \right] \right\} \tag{12}$$

An expression for the average tank head $\overline{H}$ in time interval (t_i, t_{i+1}) may be calculated from (11) as:

$$\overline{H} = \frac{AC}{a(t_{i+1} - t_i)}\left(e^{\frac{a}{A}t_{i+1}} - e^{\frac{a}{A}t_i}\right) - \frac{b}{a} + \frac{1}{(t_{i+1} - t_i)}\sum_{n=0}^{N}\left[d_n \sum_{m=0}^{n}\left(\frac{A^m}{a^{m+1}}\frac{n!}{(n-m+1)!}\left(t_{i+1}^{n-m+1} - t_i^{n-m+1}\right)\right)\right] \tag{13}$$

The average demand $\overline{Q}_d$ may be calculated from (9) as:

$$\overline{Q}_d = \frac{1}{(t_{i+1} - t_i)}\sum_{n=0}^{N}\left[\frac{d_n}{n+1}\left(t_{i+1}^{n+1} - t_i^{n+1}\right)\right] \tag{14}$$

It is also necessary to consider the special case where $a = 0$ with a polynomial demand function. Replacing the polynomial demand function (9) into (8) and integrating results in:

$$H(t) = \frac{b}{A}t - \frac{1}{A}\sum_{n=0}^{N}\left[\frac{d_n}{n+1}t^{n+1}\right] + C \tag{15}$$

With the constant C obtained by substituting the end condition $H = H_0$ at $t = t_0$ as:

$$C = H_0 - \frac{b}{A}t_0 + \frac{1}{A}\sum_{n=0}^{N}\left[\frac{d_n}{n+1}t_0^{n+1}\right] \tag{16}$$

A general expression for the average tank head over a given time interval (t_i, t_{i+1}) may be determined as:

$$\overline{H} = \frac{b}{2A}(t_{i+1} + t_i) - \frac{1}{A(t_{i+1} - t_i)}\sum_{n=0}^{N}\left[\frac{d_n}{(n+1)(n+2)}\left(t_{i+1}^{n+2} - t_i^{n+2}\right)\right] + C \tag{17}$$

Although linearizing the pipe hydraulics inevitably reduces the accuracy of the EI method, it has significant advantages over numerical integration techniques:

- The demand function is included explicitly, which gives the method a significant advantage where demands play an important role in the system.

- A polynomial representation of the demand gives flexibility in representing the demand, depending on the level of detail available in the demand data.

- The system hydraulics is taken into consideration. Even though this is done using a linear function, it is still an improvement on the Euler method, which assumes constant flows throughout a time step.

COMPLEX SYSTEMS

Complex systems

This paper considers water distribution systems to be complex if they cannot be modeled directly with the equations developed in the previous section. Complex systems are handled by first decoupling them into sets of equivalent simple base systems, and then solving the simple base systems individually. The parameters of the simple base systems are updated at regular intervals to ensure that the required level of accuracy is maintained.

The simple base system used in the analysis (figure 1) takes all its demands directly from the tank. The flow in the pipe that links the fixed-head reservoir and the tank is not directly affected by the demand, but only indirectly through the demand's effect on the tank head. The net inflow into the tank is thus made up of two distinct components: a hydraulic component (the flow in the pipe, a function of the tank head and time) and a demand component (a function of time only).

In some water distribution systems a clear distinction between the bulk and reticulation water supply systems can be made, with demands taken from the reticulation system only. Such a system fits the structure of the simple base system well and is relatively easy to model. However, when demands in the system are connected to more than one tank, such shared demands have to be distributed between appropriate simple base systems.

Isolated Demands

An isolated demand is defined as a demand that is hydraulically connected to only one tank. An example of a system with only isolated demands is shown in figure 3(a). An isolated demand can consist of a single user or a collection of users such as a pressure zone. There is thus a convenient separation between the hydraulic and demand components in the system, which fits the structure of the simple base system well.

Ideal position for figure 3

If isolated demands are known as functions of time, they can be applied directly to the relevant tanks. Under normal operating conditions, no cognizance has to be given to the local hydraulics of an isolated demand (for example the flows and pressures inside a pressure zone) since they will not affect the total demand taken from the tank (and thus the dynamic analysis).

Systems with only isolated demands are modeled by doing two snapshot simulations at different tank heads. The snapshot results are then used to approximate the hydraulic component of each tank with a linear function. The linear hydraulic and demand coefficients are then replaced in (11) [or (15) for constant hydraulics] to provide an expression for the dynamic tank head behavior. These equations can then be used to predict the tank head changes with time. To maintain accuracy, the process needs to be repeated at regular intervals.

Shared Demands

Demands from networks connected to more than one tank are shared between the tanks. Even if the water for a shared demand is fully supplied from only one of the tanks, the demand will affect the flow between the tanks and thus the behavior of all the connected tanks. Systems with shared demands are more difficult to model since they do not match the simple base system (figure 1) as well as systems with only isolated demands. Figure 3(b) shows a system in which the demands are shared among all four reservoirs and tanks.

One way of separating the hydraulic and demand components is to perform two snapshot simulations of the system: one with and one without demands. The hydraulic component is taken as the tank inflow under zero demand conditions. The demand component is then calculated as the difference between the tank inflow with and without demands, or:

$$Q_d = Q_i - Q \tag{18}$$

The hydraulic flow components are thus only functions of the head differences between the tanks in the system, and are not affected by demands in the system. In most water distribution systems tank heads change gradually and thus have little effect on the distribution of demands between the tanks. The distribution of demands between tanks is determined by the hydraulic resistances of the routes linking the demand to the tanks, rather than the relatively small variations in the tank levels.

Source Groups

Water distribution systems are often more complex than the system shown in figure 3(b). Instead of a single network of pipes connecting all the tanks to each other, real systems often have more than one distinct network of pipes between different sets of tanks. To handle such distribution systems, the concept of a source group is introduced. A source group is defined as a collection of reservoirs and tanks that are interconnected by a single network of links. Isolated demands are considered as separate networks connected to a single tank.

Source groups are not connected directly to each other, but are separated by tanks. The interaction between the source groups thus only occurs indirectly via the changes in tank heads with time. In snapshot simulations the source groups do not affect each other. Since the network is connected to each tank via a separate pipe it is possible to calculate each source group's contribution to the net tank inflows. A separate EI model is then set up to describe the behavior of each group a tank is connected to. The tank dynamic equations are set up by aggregating all its associated EI models.

Non-pipe links

Non-pipe links, such as control valves and pumps, are commonly used in water distribution systems. When such elements create sudden changes in a distribution system, for example when a valve closes or a pump starts, a new time step is used to model the altered system. New dynamic equations are generated and the modeling process is continued.

In many cases, however, changes in system elements occur gradually, for example when pressure-reducing valves adjust to compensate for gradual changes in system pressure. These changes will be reflected in the tank inflows obtained from the snapshot simulations. The snapshot simulation results are, in turn, used to set up the EI model of the system. In this way the EI model handles gradual changes in valve settings implicitly. Pumps are handled in the same way as pipes, by linearizing the pump hydraulics.

METHODOLOGY

A methodology was developed for applying the EI method and is shown in figure 4.

Ideal position for figure 4

First, the demands should be described as polynomial functions of time. The EI method is initialized by calculating the hydraulic and demand flow components of each tank under initial ($t = 0$) conditions. If only isolated demands are present in the system, a single snapshot simulation is adequate to estimate both hydraulic and demand flow components. However, if shared demands are present, the two flow components are estimated from the results of two snapshot simulations: one with and one without demands.

A single point on each hydraulic flow component's headloss curve has now been determined. By assuming a constant hydraulic flow component, the tank's head behavior can be estimated using (15).

Tank levels are set to the estimated average values for the time step and another set of snapshot simulations is performed to determine the associated hydraulic and demand flow components. Since two points on the hydraulic flow component's headloss curves are now available, the hydraulic component can be estimated with a linear function. EI models for each tank's dynamic head behavior can now be set up using (11).

The accuracy of the method can be improved by repeating the calculations until the change in average head value is sufficiently small. However, one iteration is normally adequate.

The next time step length is now determined and the process is repeated. However, instead of using the constant hydraulics assumption to estimate the average head in the new time step, the previous time step's linear coefficients are used. This reduces the number of snapshot simulations required in each time step and speeds up the simulation.

EXAMPLES

The application and accuracy of the EI method are illustrated using two example problems. To determine the true behavior of the example systems, Euler simulations were performed using a small time step. A time step of one minute was considered to have sufficient accuracy to serve as basis for calculating the simulation errors. The time steps used in the EI method were chosen arbitrarily to illustrate its performance compared to that of the Euler method. It might be possible in the future to develop an algorithm to determine the optimal time step length for the EI method.

Both example problems use higher order polynomial functions to describe their demand patterns: Example 1 uses four parabolic functions, and Example 2 a single eighth-order polynomial function. However, any polynomial demand function can be used in the EI method, including constant and linear demand functions. Any number of polynomial functions can be used in succession to describe the demand pattern, and the periods covered by these functions do not have to correspond to the time steps used for estimating the hydraulic components of the system. It is thus easy to incorporate detailed measured data into the EI method. To apply the demand patterns in the Euler method, the average demand was calculated for each time step.

Example 1: Isolated Demands

The example distribution system in figure 5 consists of a source with a fixed head of 180 m, which feeds three tanks, A, B and C, with initial heads of 116 m, 157.2 m and 53.8 m respectively.

Ideal position for figure 5

Each supply area in the system has a demand that is made up of a fixed and a varying component. The fixed demand components are 371 l/s, 21 l/s and 17 l/s, and the average varying demand components 74 l/s, 13 l/s and 98 l/s respectively for tanks A, B and C. The varying demand components all follow the same demand pattern consisting of four parabolic functions. The true head behavior of the three tanks were estimated using the Euler method with a one minute time step and is shown in figure 6.

Ideal position for figure 6

The EI method was applied in four time steps corresponding to the periods covered by the parabolic demand functions. The resulting dynamic equations for the tanks were written in the form:

$$H = c_0 + c_1 t + c_2 t^2 + Ce^{C' t} \qquad (19)$$

Where c_0, c_1, and c_2, C and C' constant coefficients determined for each time step.

The simulation results showed that both the EI and Euler methods (using a 1 hour time step) performed well on Tank C, with final errors less than 1 mm. However, for Tanks A and B large differences in the performance of the two methods were observed as shown in figure 7.

Ideal position for figure 7

The EI method errors are considerably smaller than the Euler method errors for all three tanks. This is despite the fact that the EI method used only five snapshot simulations, compared to 24 snapshot simulations in the Euler method. The computational effort involved in an extended-period simulation can be considered directly proportional to the number of snapshot simulations, since the computational effort involved a snapshot simulation is much greater than that of the dynamic calculations. The computational effort using the EI method is thus reduced by approximately 80 %.

The results show that the EI method is able to give substantially better performance both in accuracy and computational effort than the Euler method.

Example 2: Shared demands and pumps

In the second example, the EI method is applied to a problem with both shared demands and pumps. Time step lengths for the Euler and EI methods were selected so that an equal number of snapshot simulations were required. In this way the performance of the two methods could be compared based on similar computational efforts. Since the EI method requires two snapshot simulations in each time step (one with demands and one without demands), its time step length was selected as double that of the Euler method. Time steps of one hour and half an hour were used for the EI and Euler methods respectively.

As is shown in figure 8, the example system consists of two interconnected tanks with an initial head difference of 10 m. The tanks are supplied from a source by a single pump and a pump line that splits to the respective tanks. The system demand (average 150 l/s) is taken from two nodes on a separate gravity system between the two tanks. The demand variation is described by the single eighth-order polynomial function. Under low demand conditions a net flow exists from the upper to the lower tank. However, under peak demand conditions both tanks have net outflows.

Ideal position for figure 8

The true head behavior of the three tanks were estimated using the Euler method with a one minute time step and is shown in figure 9.

Ideal position for figure 9

The simulation errors of the Euler and EI methods were calculated from the base simulation results and are shown in figure 10.

Ideal position for figure 10

The EI method's errors remain considerably smaller than those of the Euler method for both tanks throughout the simulation period. The results clearly show that, for the same computational effort, the EI method has substantially better accuracy for the example problem than the Euler method.

CONCLUSIONS

In this paper, the new EI method was developed for extended-period modeling of water distribution systems. In the EI method, a water distribution system is decoupled into a number of constituent simple base systems. The dynamic behaviors of the simple base systems are determined by integrating their linearized dynamic tank equations explicitly, and are then used to estimate the dynamic behavior of the full water distribution system.

The EI method is illustrated using two example problems: one with only isolated demands and another with both shared demands and pumps. The accuracy and efficiency of the EI method is shown to be superior to that of the standard Euler method for both example systems.

ACKNOWLEDGEMENTS

The authors would like to acknowledge the Commonwealth Scholarship Commission in the UK for financial support of the research.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = tank cross-sectional area;

a = coefficient of linear flowrate function;

b = coefficient of linear flowrate function;

C = constant;

C′ = constant;

c = constant coefficient;

d_n = coefficient of the nth term of a polynomial demand function;

e = base of the natural logarithms;

H = tank head;

$\overline{H}$ = average tank head;

H_P = fixed reservoir head;

m = counter variable;

N = order of a polynomial demand function;

n = headloss exponent;

n = exponent of the nth term of a polynomial demand function;

n = counter variable;

r = headloss coefficient;

r' = flow coefficient = $(1/r)^{\frac{1}{n}}$;

t = time;

Q = net tank inflow;

Q_d = demand;

$\overline{Q}_d$ = average demand;

Q_i = pipe flowrate;

Q_{il} = linearized pipe flowrate;

FIGURES

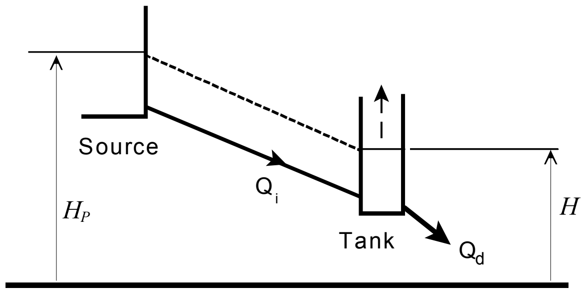

Figure 1 A simple base system consisting of a fixed-head reservoir feeding a tank, with a demand taken directly from the tank.

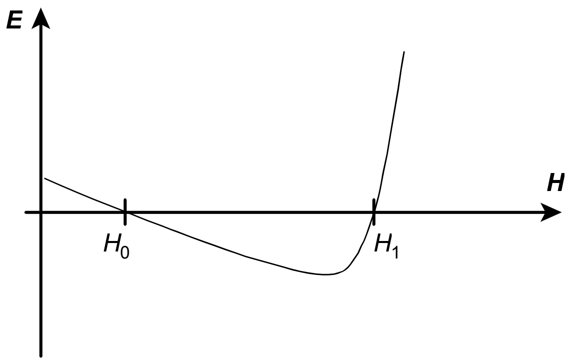

Figure 2 Linear approximation error

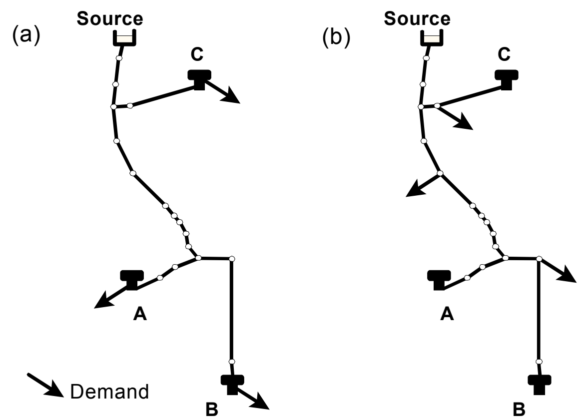

Figure 3 An example distribution system with a) isolated demands and b) shared demands. Demands are shown as arrows.

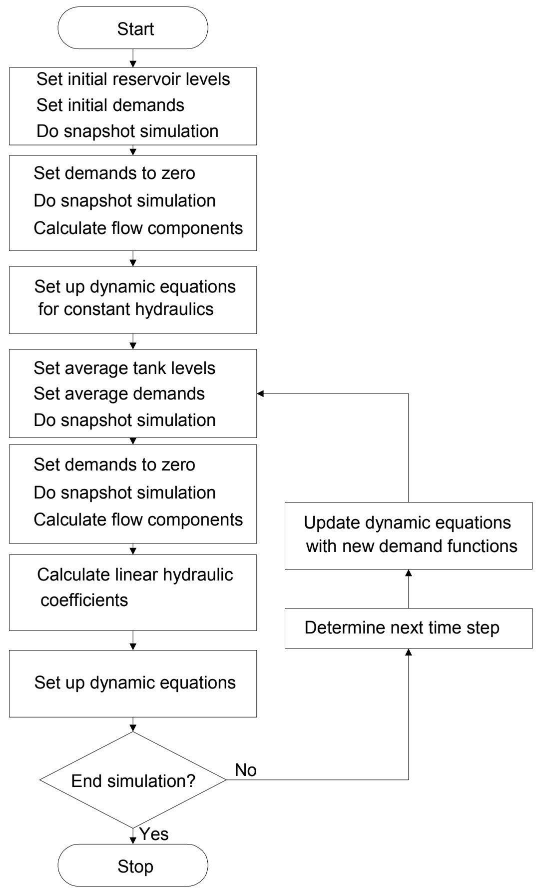

Figure 4 Flow diagram of the Explicit Integration (EI) method.

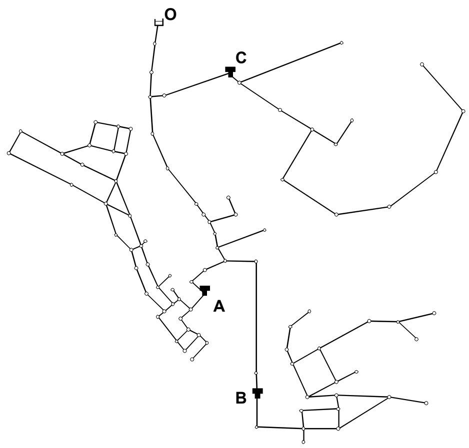

Figure 5 Layout of the distribution system used in Example 1

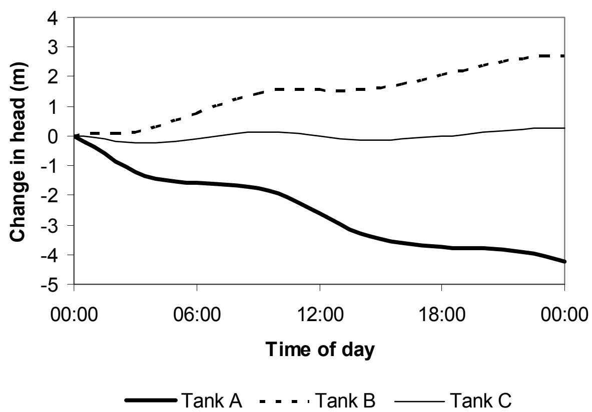

Figure 6 Example 1 changes in tank heads with time

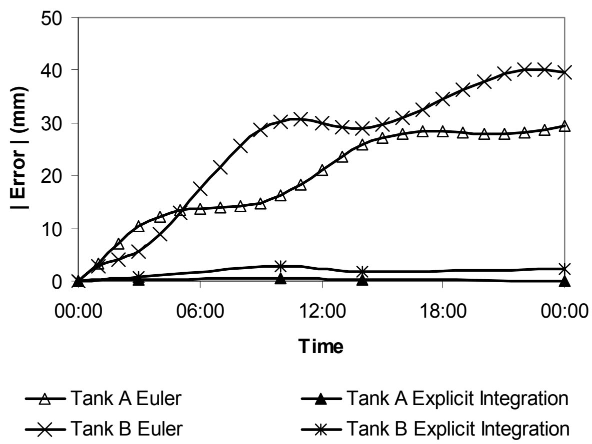

Figure 7 Example 1 simulation errors

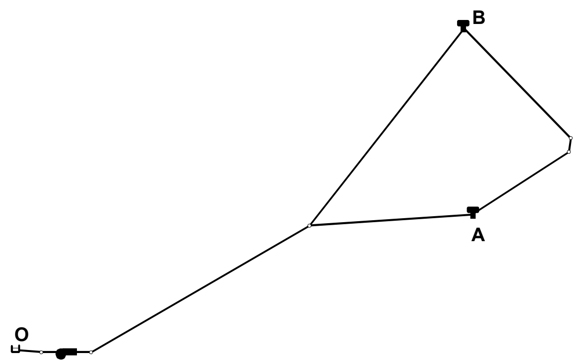

Figure 8 Layout of the distribution system used in Example 2

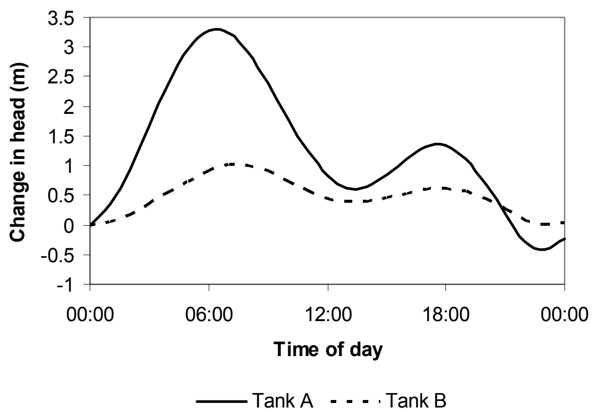

Figure 9 Example 2 changes in tank heads with time

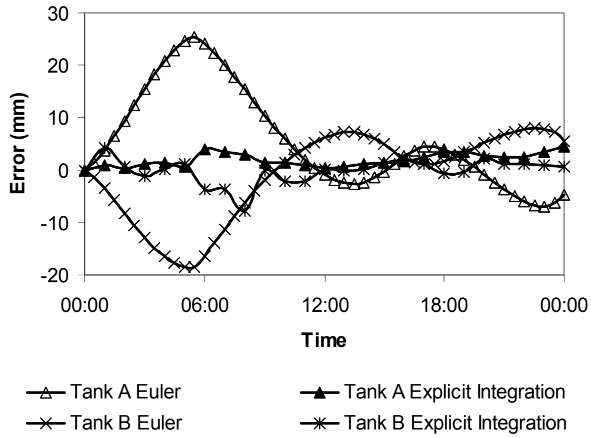

Figure 10 Example 2 simulation errors