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MODELING ELASTICALLY DEFORMING LEAKS IN WATER DISTRIBUTION PIPES

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Abstract

In this study, the relationship between the conventional power equation and the FAVAD (Fixed and Variable Area Discharges) equation for modeling leakage as a function of pressure was investigated. It is shown that different leakage exponent (or N1) values are obtained for the same leak when measured at different pressures. An analytical exploration of the two equations shows that the leakage exponent tends to 0.5 when the system pressure tends to zero, and to 1.5 when the system pressure tends to infinity. A dimensionless Leakage Number L_N is defined as the ratio between the variable and fixed portions of a leak, and it is shown that a single function can be used to describe the relationship between the Leakage Number and leakage exponent. This model was combined with previous research to accurately predict the leakage exponents of several published studies in cases where elastic deformation occurred.

Keywords
Leakage, pressure, leakage exponent, mathematical models

INTRODUCTION

Pressure management has been established as an effective and frequently used technique to reduce the leakage rate from and prolong the service life of pipes in distribution systems (Thornton & Lambert, 2005). Pressure management has proved to be more effective for leakage management than conventionally believed based on the Torricelli equation, which predicts the flow through an orifice to be proportional to the square root of the pressure head upstream of the orifice. It is now known through various field studies done in different parts of the world that the leakage rate from a distribution system can be much more sensitive to pressure than the Torricelli equation predicts. This relationship is most likely to be close to a linear relationship, but findings of exponents in the range 0.5 to 2.8 have been reported (Farley &Trow, 2003).

Since system leakage is made up a large number of individual leaks, an understanding of the behavior of individual leaks is fundamental to explaining the observed system behavior. The aims of this paper are to propose a method for modeling the pressure-leakage behavior of individual leaks based on a new dimensionless Leakage Number, and then to illustrate how this model can be combined with previous work by Cassa and Van Zyl (in press) to predict the behavior of various types of individual cracks.

An overview of previous research experiments and models on the relationship between pressure and leakage is presented in the next section. This is followed by an exploration of the properties of and links between the two models commonly used to model the relationship between pressure and leakage of individual leaks. Finally, the ability of the proposed model to predict the pressure-leakage behavior of individual leaks is illustrated and verified on previously published data.

The models and recommendations of this paper are limited to leaks that undergo elastic deformations only. Thus conditions under which leaks will undergo creep, plastic deformation or fracture are excluded. Previous research and the results of this study shows that this is a reasonable assumption, except for plastic pipes where plastic deformation has been shown to occur in many cases.

BACKGROUND

The hydraulics of orifices is well understood and a great deal of research has been conducted on different orifice shapes and conditions (Cassa & Van Zyl, 2011). Orifice hydraulics is based on the Torricelli equation, which states that the velocity of flow through an orifice is given by:

$$v = \sqrt{2gh} \tag{1}$$

Where v is velocity, g acceleration due to gravity and h the pressure head at the orifice.

The Torricelli equation describes the conversion of potential pressure energy to kinetic energy under conditions of zero energy loss. In a real orifice the effective area of the orifice is reduced by the fluid jet contracting downstream of the orifice (called the vena contracta) and frictional energy losses. To account for these factors, a discharge coefficient C_d is introduced. The following resulting equation describes the flow rate Q through the orifice:

$$Q = C_d A\sqrt{2gh} \tag{2}$$

Many studies have been published on the behavior of orifice discharge coefficients and are available in sources such as Idelchik (1994). Experimental results have shown that the square root relationship between flow rate and pressure head in Equation 2 is only valid for turbulent flow, thus for an orifice Reynolds number greater than 4000 to 5000. For laminar flow through orifices (Reynolds numbers less than 10), the discharge coefficient also becomes a function of the pressure head, effectively resulting in linear relationship between flow rate and pressure head (e.g. see Idelchik, 1994).

Leak openings in pipes can be considered orifices and thus should adhere to Equation 2. However, in practice it has been shown that this equation does not provide a satisfactory model for the behavior of system leakage with pressure. As a result, a more general leakage equation was adopted by leakage practitioners, even as far back as 1881 (Lambert, 2001), in the form of a power equation:

$$Q = Ch^{N1} \tag{3}$$

Where N1 is the leakage exponent and C the leakage coefficient. Efforts to characterize the behavior of system leaks with pressure have focused mainly on the leakage exponent and field tests have found system leakage exponents substantially higher than 0.5 (Gebhardt, 1975; Lambert, 2001; Al-Ghamdi, 2011; Farley &Trow, 2003). Van Zyl and Clayton (2007) proposed four factors that may be responsible for the higher leakage exponents, i.e. leak hydraulics, pipe material behavior, soil hydraulics and water demand. Of these factors, pipe material behavior, meaning that the leak area increases with increasing pressure, is the most important. A further factor that should be considered is the way that individual leaks in a water distribution system will respond in different ways to changes in pressure, and how these changes in combination affect the way the total system leakage responds.

One of the earliest published field studies on the relationship between pressure and leakage was conducted in Japan by Ogura (1979), who isolated sections of a network using network valves, and then used a pump and measuring equipment to estimate the pressure-leakage relationship for each section. Leakage exponents found for the eight successful experiments varied between 0.65 and 2.1, with an average of 1.15. The leakage exponent of 1.15 is often referred to as a typical exponent, even though this conclusion cannot be made from Ogura's paper due to the small number of points and large range of the results.

Ogura suggested that soil hydraulics may be responsible for the higher exponents, but this was later shown to be improbable by Hiki (1981) and Walski et al (2006). The interaction between a leaking pipe and its surrounding soil is complex, and flow rates are unlikely to be a linear function of pressure, as a result of interaction of soil particles with the jet and the orifice, turbulent flow in the soil, the changing geometry of the unconfined flow regime, hydraulic fracturing and piping (Van Zyl & Clayton, 2007).

Several laboratory-based tests have been done on individual leaks in pipes showing that for round holes the leakage exponent is typically close to 0.5 irrespective of the pipe material or hole size (Hiki, 1981, Greyvenstein & Van Zyl, 2007). However, the leakage exponent can be substantially higher for cracks (Ávila, 2003; Greyvenstein and Van Zyl, 2007; Walski et al, 2009; Ferrante, 2012). The relationship is complicated for plastic

pipe through hysteresis and plastic deformation as documented by Ferrante et al. (2011), Ferrante et al (in press) and Massari et al. (2012).

May (1994) proposed that leaks in flexible materials have leakage exponents of 1.5, and combining this with the orifice equation, he proposed a combined leakage equation in the form:

$$Q = k_1 h^{0.5} + k_2 h^{1.5} \tag{4}$$

May's paper became highly influential as it explained the wide range of pressure:leak flow relationships measured in the UK and internationally, and led to the development of the FAVAD (Fixed and Variable Area Discharges) concept, which was later adopted and recommended for international use by the IWA Water Losses Task Force. In 2009 it was found that Ledochowski (1956) had earlier used the same concept for identifying whether leaks on newly laid trunk mains were located at joints (exponent 1.5) or on the metal pipes (exponent 0.5).

Cassa et al (2010) and Cassa and Van Zyl (2011) used finite element modeling under the assumption of linear elastic behavior to show that the areas of various types of leak openings (round holes and longitudinal, circumferential and spiral cracks) varied linearly with pressure irrespective of the pipe dimensions, material and loading conditions. Thus the area of any leak undergoing elastic deformation can be described as a function of pressure with the equation:

$$A = A_0 + mh \tag{5}$$

Where A_0 is the initial leak area (under zero pressure conditions) and m the head-area slope. Replacing this relationship into Equation 2 results in:

$$Q = C_d\sqrt{2g}(A_0 h^{0.5} + mh^{1.5}) \tag{6}$$

While this equation is identical in form to the equation proposed by May (1994), it has an important interpretive difference in that leaks are not considered either fixed or variable, but that all leaks are considered variable. In other words, all leaks will increase in area with increasing pressure. For leaks with small head-area slopes, the first term of Equation 6 is likely to be dominant, resulting in an effective leakage exponent of 0.5. Conversely, for flexible leaks with high head-area slopes, the second term of the equation will be dominant, resulting in leakage exponents of 1.5. From Equation 6 it is evident that, under elastic conditions, the pressure response of a leak can be fully characterized by knowing its initial area A_0 and head-area slope m.

POWER AND FAVAD EQUATIONS

As discussed above, the flow rate of a leak undergoing elastic deformation can be described as a function of pressure head by the FAVAD equation (Equation 6). However, the power leakage equation (Equation 3) is commonly used to model leakage, and is likely to remain so. Thus it was considered important to investigate the accuracy with which the power equation can model a leak known to be undergoing elastic deformation. In addition, the link between the power and FAVAD equations was investigated leading to the formulation of a dimensionless Leakage Number, which is able to characterize the pressure response of elastic leaks better than the power model. The Leakage Number also allows conversion between the power and FAVAD models.

Modeling elastic leaks with the Power Equation

To investigate the performance of the power equation in modeling elastic leaks, it was tested on three 60 mm long leaks in a 110 mm class 6 uPVC pipe. The cracks were oriented in longitudinal, circumferential and spiral directions respectively, and were modeled using finite elements as described in Cassa and Van Zyl (2011). First, the leak areas were determined at different pressures and plotted against the pressure head as shown in Figure 1. It can be seen from the figure that straight lines fit the behavior very well, and the head-area slopes of the three cracks can be determined from these lines as $1.195\text{x}10^{-6}$, $8.800\text{x}10^{-7}$ and $2.446\text{x}10^{-7}$ m respectively for the longitudinal, spiral and circumferential cracks.

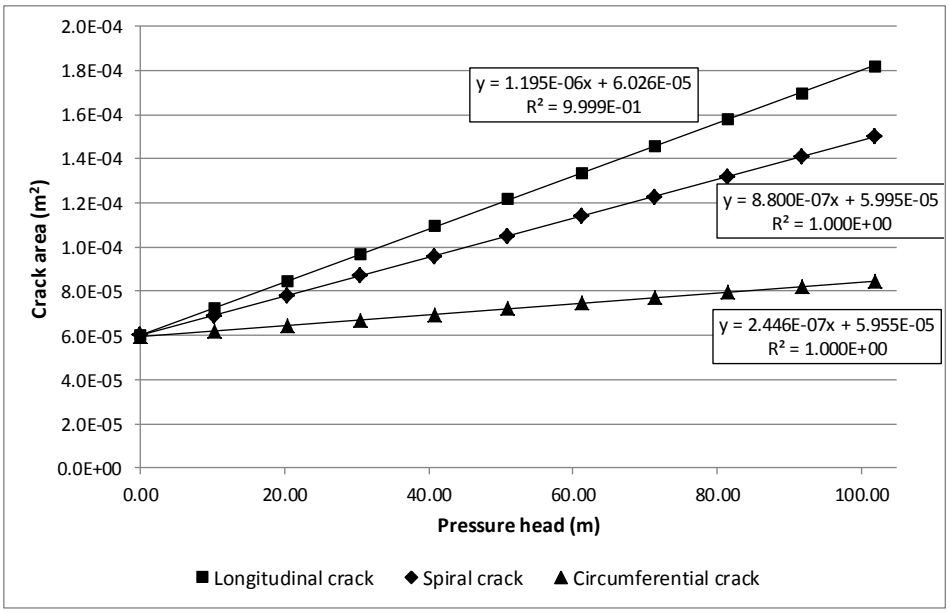

Figure 1. The areas of 60 mm long cracks in a class 6 uPVC pipe as a function of the pressure head as determined by finite element analysis.

From these areas, the leakage flow rate can be estimated by applying the FAVAD equation (Equation 6) (or alternatively by applying the orifice equation (Equation 2) for each measured area) to obtain the behavior of the leak flow rates as a function of pressure as shown in Figure 2. The conventional approach is to fit the power leakage equation to the flow to obtain leakage exponents of 0.91, 0.84 and 0.63 respectively. While the power curves in Figure 2 fit reasonably well, they clearly don't follow the trend of the data for the higher pressures. In addition, the quality of the fit reduces significantly if more low pressure data points are added.

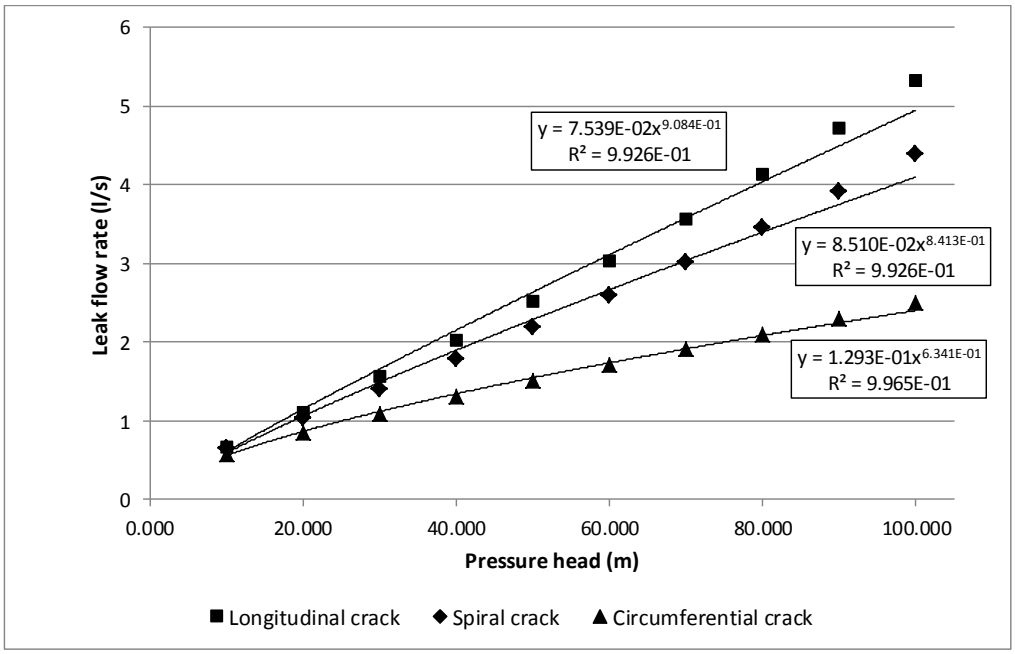

Figure 2. The flow through 60 mm long cracks in a class 6 uPVC pipe as a function of the pressure head

The limitations of the power equation are exposed by fitting it not through all the points, but at each point by calculating the change in leakage flow for a small change in pressure. The results are shown in Figures 3 and 4 for the leakage exponent and coefficient respectively. Figure 3 shows that the leakage exponent of a given leak is not fixed, but is higher at higher pressures and lower at lower pressures. The inverse is true for the leakage coefficient. The variations in the parameters are substantial, indicating the unsuitability of the leakage exponent to characterize the pressure response of leaks. For example, the leakage exponent obtained for the 60 mm longitudinal crack increases from 0.67 at a pressure head of 10 m to 1.16 at a pressure head of 100 m, an increase of more than 73 %.

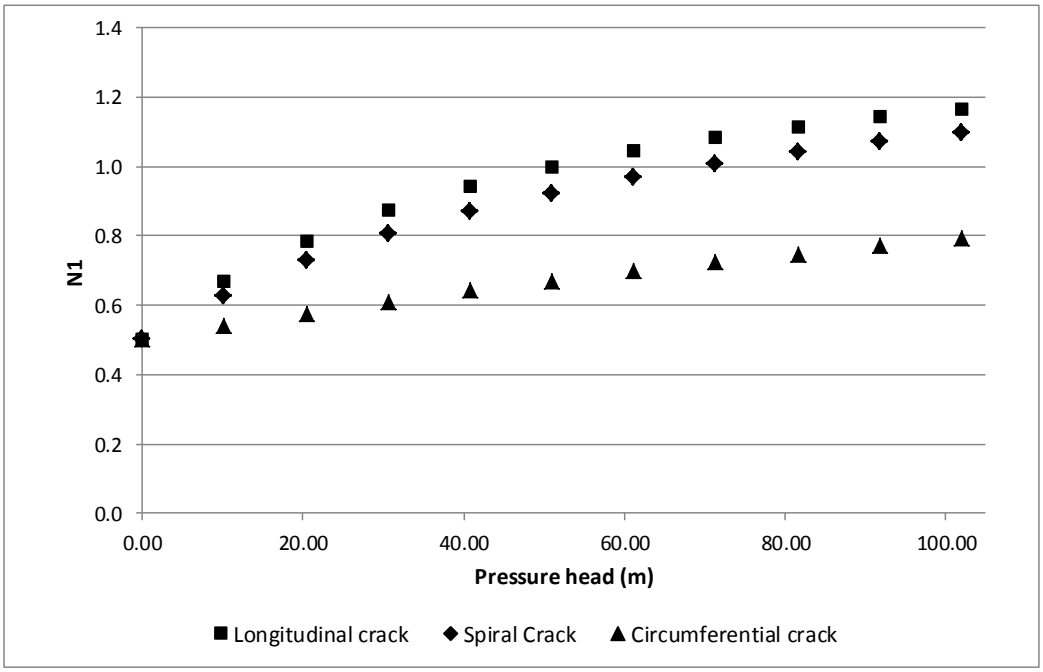

Figure 3**.** Leakage exponents obtained at different pressures for the h-Q curves in Figure 2

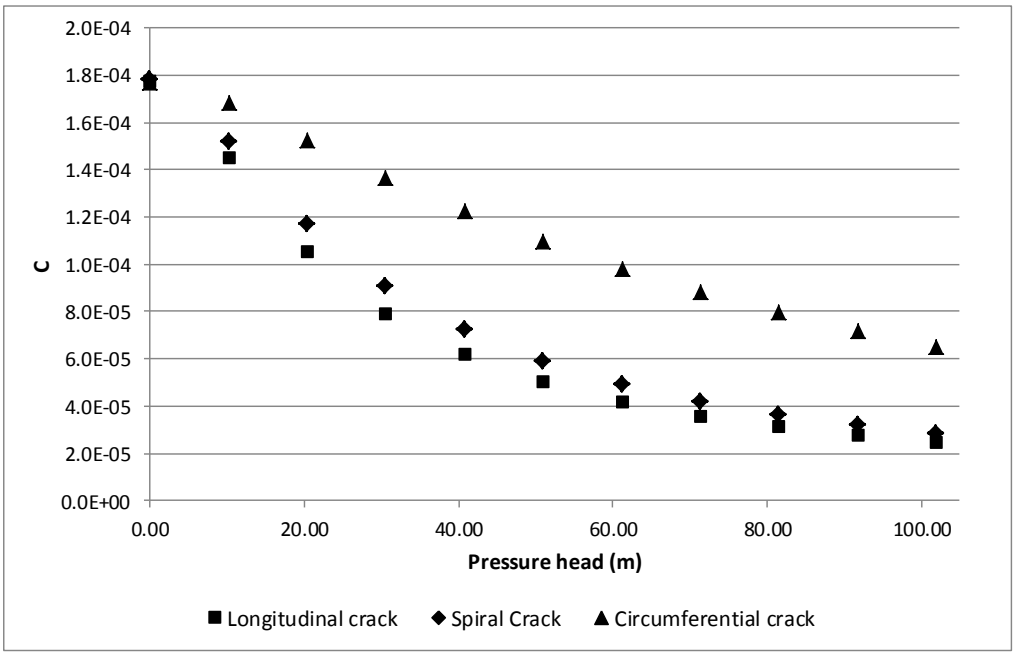

Figure 4**.** Leakage coefficients obtained at different pressures for the h-Q curves in Figure 2

Analytical exploration

To investigate the relationship between the power and FAVAD equations further, an analytical exploration may be done by first equating Equations 3 and 6:

$$Ch^{N1} = C_d\sqrt{2g}(A_0h^{0.5} + mh^{1.5}) \tag{7}$$

Dividing both sides by the orifice equation results in:

$$C'h^{N1-0.5} = 1 + \frac{mh}{A_0}; \text{ with } C' = \frac{C}{C_d\sqrt{2g}A_0} \tag{8}$$

The term mh/A_0 on the very right of the equation represents the ratio between the variable and fixed portions of the leakage, and is now defined as the dimensionless Leakage Number L_N:

$$L_N = \frac{mh}{A_0} \tag{9}$$

Through further manipulation, an expression can be found for *N1* in the form:

$$N1 = \frac{\ln(L_N+1)-\ln(C')}{\ln(h)} + \frac{1}{2} \tag{10}$$

This equation confirms that the leakage exponent is a function of *h*. In addition, by exploring the limits of *h* it can be shown that:

- In the limit as *h* reduces to zero, the leakage exponent *N1* = 0.5 and $C = C_dA_0\sqrt{2g}$. Thus, at a pressure of zero the leak behavior can be described by the orifice equation.
- In the limit as *h* increases to infinity, the leakage exponent N1 = 1.5 and $C = C_dm\sqrt{2g}$. Thus, if the pressure is sufficiently high the leak behavior can be described by the variable area part of Equation 6.

Leakage Number

Further exploration of the relationship between the power and FAVAD equations revealed that plotting the leakage exponent against the Leakage Number always results in the same line, irrespective of the values that A_0, *m* and *h*. To illustrate this, the leakage exponent and Leakage Numbers were calculated for cracks with m/A_0 ratios of 0.0001, 0.001, 0.01, 0.1, 1 and 10 pressures between 0.00001 and 200 m. These curves all plot the same line as shown in Figure 5.

Based on this curve, it may be concluded that the relationship between *N1* and L_N display the following characteristics:

- *N1* = 1 when $L_N = 1$
- *N1* > 1 when $L_N > 1$
- *N1* < 1 when $L_N < 1$
- *N1* is practically 0.5 for all $L_N < 0.01$
- *N1* is practically 1.5 for all $L_N > 100$

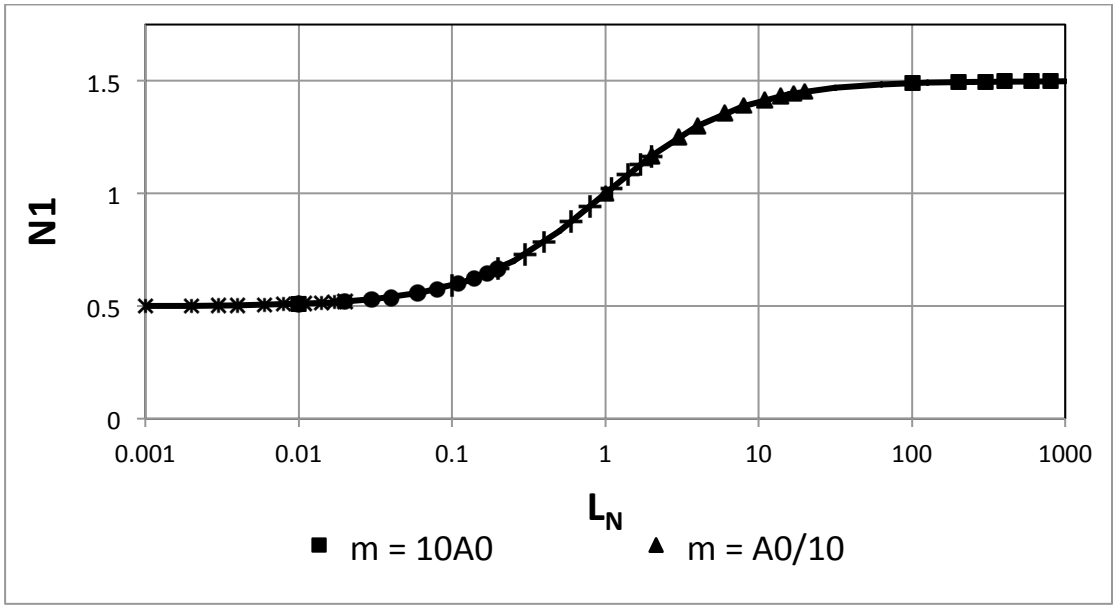

Figure 5. The relationship between the leakage exponent *N1* and Leakage Number L_N for different m/A_0 ratios

In addition, it was possible to find the following expression that provides an exact formula for the conversion between *N1* and L_N (also plotted on Figure 5):

$$L_N = \frac{N1-0.5}{1.5-N1} \tag{11}$$

$$\text{or } N1 = \frac{1.5L_N+0.5}{L_N+1} \tag{12}$$

Finally, Equations 9, 11 and 12 may be used to convert the pressure-leakage response of a leak between the conventional and FAVAD leakage models. At the same time, the Leakage Number allows the range of leakage exponents to be calculated for any pressure range that a leak may be subjected to. Thus the Leakage Number can be used to characterize the pressure-response of elastic leaks.

For example, the longitudinal crack plotted in Figure 1 has A_0 = 60 mm^2 and m = 1.195x10^{-6} m. The Leakage Number of this leak can be estimated at a pressure of (say) 30 m from Equation 9 as L_N = 0.60, and then the leakage exponent from Equation 12 as *N1* = 0.88. This value corresponds exactly with the leakage exponent shown in Figure 3. The Leakage Number and exponent for other pressures can also be determined from Equations 9 and 12 without the need for further measurements.

PREDICTING LEAK RESPONSE TO CHANGES IN PRESSURE

The model developed above can be used to predict the leakage exponent of any leak at different pressures if the initial area A_0 and head-area slope m of the leak are known. Since it is known from both experimental and finite element results that round holes have leakage exponents close to 0.5, it may reasonably be assumed in that they have fixed areas (i.e. negligibly small head-area slopes) and are thus excluded from the rest of this discussion.

The initial area of a crack is simple to determine if its length and width are known. The head-area slopes of cracks were investigated by Cassa & Van Zyl (under review) using finite element modeling. They proposed equations to predict the head-area slopes of cracks as a function of the length of the crack L_c, internal diameter d and wall thickness t of the pipe, elasticity modulus E of the pipe material, and longitudinal stress σ in the pipe (for spiral and circumferential cracks):

$$m_{long} = \frac{2.93157.d^{0.3379}.L_c{}^{4.80}.10^{0.5997(\log L_c)^2}.\rho.g}{E.t^{1.746}} \tag{13}$$

$$m_{spiral} = \frac{3.7714.d^{0.178569}.L_c{}^{6.051}.\sigma_l^{0.0928}.10^{1.05(\log L_c)^2}.\rho.g}{E.t^{1.6795}} \quad (14)$$

$$m_{circ} = \frac{1.64802\times10^{-5}.L_c{}^{4.87992662}.\sigma_l^{1.09182555}.10^{0.82763163(\log L_c)^2}.\rho.g}{E.t^{0.33824224}.d^{0.186376316}} \quad (15)$$

Interestingly the crack width was found to have negligible impact on the head-area slope. However, the width of the crack is a major determinant of its initial area, and thus from Equation 9 of its Leakage Number. This means that narrow cracks will have higher leakage exponents than wider cracks of the same length, and that cracks that have close to zero initial width will likely have leakage exponents of 1.5. Since most experimental work to date has been done on wider cracks that those commonly found in real pipes (due to the removal of pipe material to create the artificial crack), it is likely that leakage exponents of cracks in the field will be higher than those reported from most laboratory tests.

Based on the work described above, it is now possible to plot the combinations of crack lengths and widths that will produce different ranges of leakage exponent for a given crack orientation, pipe diameter and pipe material: for each crack the head-area slope is obtained from Equations 13 to 15, the Leakage Number from Equation 9 and then the leakage exponent from Equation 12. For example, Figure 6 gives the ranges of the leakage exponent for longitudinal cracks in 100 mm nominal diameter uPVC, asbestos cement and cast iron pipes. The graph is drawn for a pressure of 50 m, and will vary slightly if the pressure is changed. The properties used in the calculations are given in Table 1.

Table 1 Properties of Class 9 equivalent 100 mm nominal diameter uPVC, asbestos cement and cast iron pipes

Parameter	**uPVC**	**Asbestos cement**	**Cast iron**
Internal diameter (mm)	104	102	103
Wall thickness (mm)	3	10	12
Elasticity modulus (GPa)	3	24	100

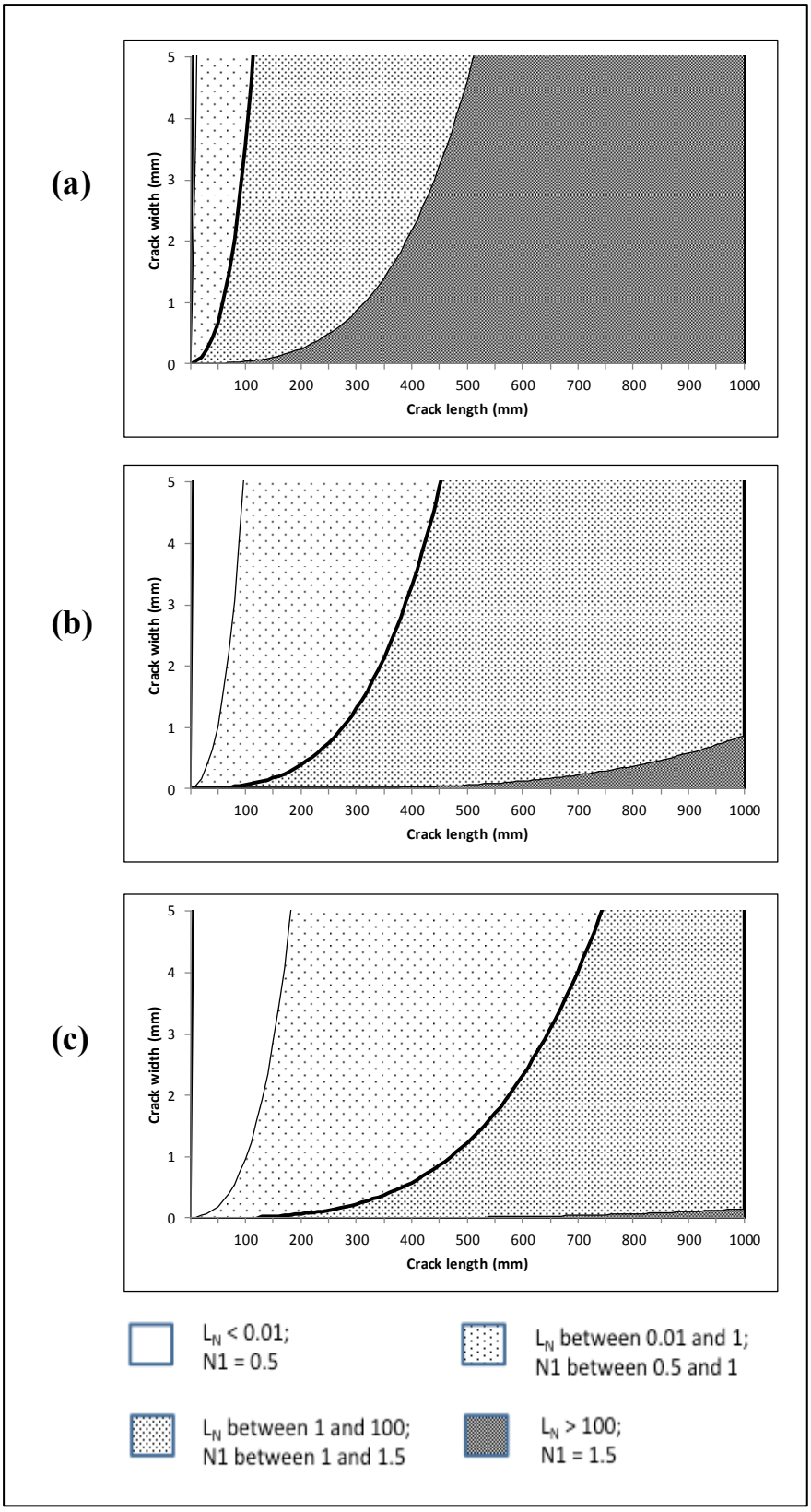

Figure 6 Leakage exponent diagrams for longitudinal cracks in Class 9 (or equivalent) a) PVC, b) asbestos cement and c) cast iron pipes

It can be seen from the graphs that the leakage exponent will tend to 1.5 as the width of the crack reduces to zero (the scale of the graphs don't show this in full detail, but it should be clear from the theory discussed earlier). Figure 6 also shows that the same crack will have a significantly higher leakage exponent in uPVC than in the other two materials, and that asbestos cement will have slightly higher leakage exponents than cast iron. For instance, a crack with a length of 300 mm and an initial width of 1 mm will display leakage exponents of 1.5, 0.9 and 0.6 respectively in uPVC, asbestos cement and cast iron pipes.

Finally, to test the proposed method, the results of experimental tests on longitudinal cracks in pipes were collected from various sources (too few circumferential and spiral crack tests have been published to evaluate these). Four relevant sources were identified, i.e. Ávila (2003), Ferrante (2012), Ferrante (2013) and Greyvenstein (2007), and the relevant results from these sources are reproduced in the first nine columns of Table 2. The crack, section and material properties of the pipes tested were obtained from the sources, but since the elasticity moduli were not reported, values in Table 2 were used. Where the data was available, the average pressures of all the data points were recorded in Column 8. Where not, the average pressure was estimated as the average of the pressure range used in the tests.

The head-area slope of each leak opening was estimated using Equation 13 and is given in Column 10 of Table 2. The Leakage Number of each opening was then estimated using Equation 9 as listed in Column 11. Finally, the Leakage Numbers were used to predict the leakage exponent of the crack using Equation 12 and are shown in in Column 12. The predicted leakage exponents were then compared to the measured exponents (Column 9) and are shown in Figure 7.

Figure 7 shows a close match between the predicted and measured leakage exponents for certain cracks, but less for others. Looking at the data in more detail it is clear that the model is able to predict the leakage exponent well (within 10 %, and shown in bold in Table 2) for cracks shorter than 100 mm, which were less likely to have experienced plastic deformation. The only exceptions are points 20 and 27 (the experimenters reported plastic deformation for point 27).

The comparison shows a clear trend of greater underestimation of the leakage exponent with increasing crack length in uPVC pipes. Ferrante et al. (2011) and Massari et al. (2012) convincingly showed the occurrence of plastic deformation in plastic pipe materials, which can reasonably be expected to be more pronounced for longer crack lengths and in cracks with sharp edges, such as those induced in pipes without removing any pipe materials (points 4 to 23).

In summary, the model is able to predict the measured leakage exponents well for pipes likely undergoing elastic deformation, while it underestimates the leakage exponent for cracks undergoing significant plastic deformation. It is clear that plastic pipes in particular do not conform to the assumption of elastic deformation, and that further work is required to understand this behavior and its impact on leakage hydraulics.

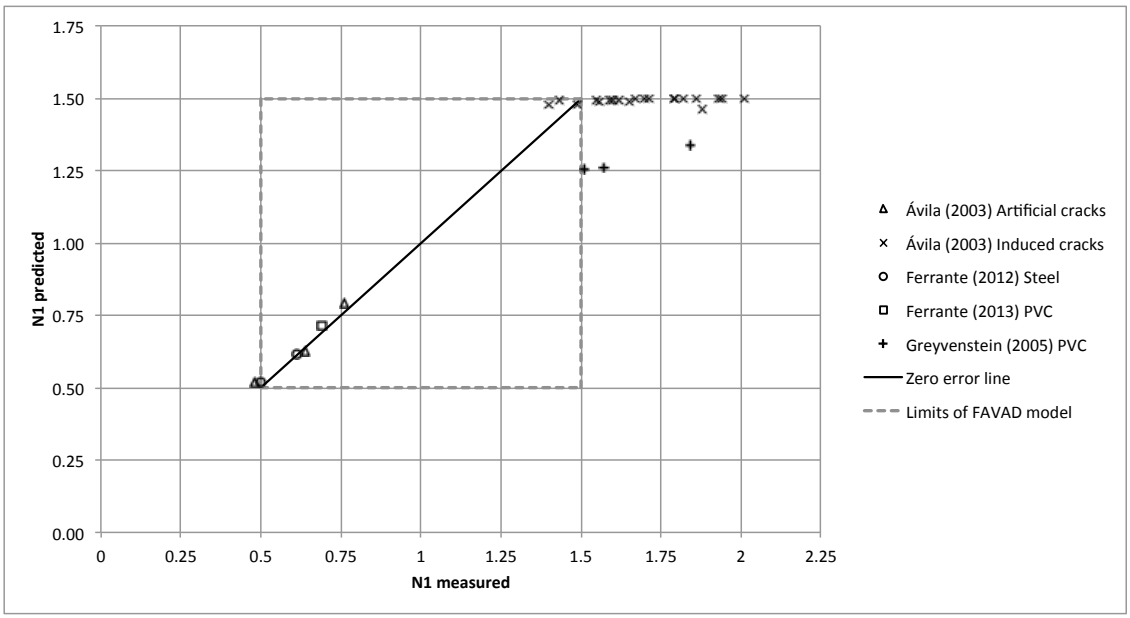

Figure 7 Comparision between predicted and measured leakage exponents

Table 2 Experimental results of longitudinal cracks compared to model predictions. Predictions with errors less than 10% are shown in bold.

1	2	3	4	5	6	7	8	9	10	11	12	13
No	**Source**	**Pipe**			**Crack**		**Experimental results**		**Modeled values**			
		Material	**Internal diameter (mm)**	**Wall thickness (mm)**	**Length (mm)**	**Width (mm)**	**Average pressure (m)**	**N1**	**Head-area slope (/m) (Eq. 13)**	**Leakage Number (Eq. 9)**	**N1 (Eq. 12)**	**Predic-tion error**
1	**Ávila (2003)**	**PVC**	**76.2**	**3.63**	**12.25**	**1***	**32.0**	**0.48**	**7.57E-09**	**1.98E-02**	**0.52**	**8.3%**
2	**Ávila (2003)**	**PVC**	**76.2**	**3.63**	**37.2**	**1***	**32.0**	**0.64**	**1.69E-07**	**1.46E-01**	**0.63**	**-1.6%**
3	**Ávila (2003)**	**PVC**	**76.2**	**3.63**	**60**	**1***	**31.5**	**0.76**	**7.85E-07**	**4.12E-01**	**0.79**	**3.9%**
4	**Ávila (2003)**	**PVC**	**50.8**	**2.42**	**50**	**0.01#**	**30.0**	**1.49**	**7.63E-07**	**4.58E+01**	**1.48**	**-0.7%**
5	**Ávila (2003)**	**PVC**	**50.8**	**2.42**	**70**	**0.01#**	**30.0**	**1.55**	**2.34E-06**	**1.00E+02**	**1.49**	**-3.9%**
6	**Ávila (2003)**	**PVC**	**50.8**	**2.42**	**90**	**0.01#**	**30.0**	**1.6**	**5.60E-06**	**1.87E+02**	**1.49**	**-6.9%**
7	Ávila (2003)	PVC	50.8	2.42	100	0.01#	30.0	1.7	8.16E-06	2.45E+02	1.50	-11.8%
8	Ávila (2003)	PVC	50.8	2.42	130	0.01#	30.0	1.67	2.14E-05	4.93E+02	1.50	-10.2%
9	Ávila (2003)	PVC	50.8	2.42	160	0.01#	30.0	1.86	4.70E-05	8.81E+02	1.50	-19.3%
10	Ávila (2003)	PVC	50.8	2.42	167	0.01#	30.0	1.94	5.54E-05	9.95E+02	1.50	-22.7%
11	**Ávila (2003)**	**PVC**	**63.5**	**3.2**	**60**	**0.01#**	**30.0**	**1.4**	**9.19E-07**	**4.60E+01**	**1.48**	**5.7%**
12	**Ávila (2003)**	**PVC**	**63.5**	**3.2**	**90**	**0.01#**	**30.0**	**1.43**	**3.71E-06**	**1.24E+02**	**1.49**	**4.2%**
13	Ávila (2003)	PVC	63.5	3.2	120	0.01#	30.0	1.71	1.05E-05	2.63E+02	1.50	-12.4%
14	Ávila (2003)	PVC	63.5	3.2	150	0.01#	30.0	1.82	2.43E-05	4.86E+02	1.50	-17.6%
15	**Ávila (2003)**	**PVC**	**76.2**	**3.63**	**80**	**0.01#**	**30.0**	**1.56**	**2.09E-06**	**7.85E+01**	**1.49**	**-4.5%**
16	**Ávila (2003)**	**PVC**	**76.2**	**3.63**	**110**	**0.01#**	**30.0**	**1.59**	**6.52E-06**	**1.78E+02**	**1.49**	**-6.2%**
17	Ávila (2003)	PVC	76.2	3.63	150	0.01#	30.0	1.79	2.07E-05	4.15E+02	1.50	-16.2%
18	Ávila (2003)	PVC	76.2	3.63	221	0.01#	30.0	1.93	9.44E-05	1.28E+03	1.50	-28.7%
19	Ávila (2003)	PVC	76.2	3.63	230	0.01#	30.0	2.01	1.11E-04	1.45E+03	1.50	-24.9%
20	Ávila (2003)	PVC	101.6	4.84	60	0.01#	30.0	1.88	5.23E-07	2.62E+01	1.46	-22.3%
21	**Ávila (2003)**	**PVC**	**101.6**	**4.84**	**90**	**0.01#**	**30.0**	**1.65**	**2.11E-06**	**7.04E+01**	**1.49**	**-9.7%**
22	**Ávila (2003)**	**PVC**	**101.6**	**4.84**	**131**	**0.01#**	**30.0**	**1.62**	**8.29E-06**	**1.90E+02**	**1.49**	**-8.0%**

23	Ávila (2003)	PVC	101.6	4.84	170	0.01[#]	30.0	1.79	2.24E-05	3.95E+02	1.50	-16.2%
24	**Ferrante (2012)**	**Steel**	**93**	**1.5**	**90**	**2[*]**	**15.0**	**0.5**	**2.34E-07**	**1.96E-02**	**0.52**	**4.0%**
25	**Ferrante (2012)**	**Steel**	**93**	**0.5**	**90**	**2[*]**	**15.0**	**0.61**	**1.59E-06**	**1.33E-01**	**0.62**	**1.6%**
26	**Ferrante (in press)**	**uPVC**	**96.8**	**6.6**	**90**	**2[*]**	**40.0**	**0.69**	**1.21E-06**	**2.70E-01**	**0.71**	**2.9%**
27	Greyvenstein (2005)	uPVC	104	3	86	0.3[*]	19.0	1.51	4.18E-06	3.08E+00	1.25	-17.2%
28	Greyvenstein (2005)	uPVC	104	3	100	0.3[*]	13.4	1.57	7.15E-06	3.20E+00	1.26	-19.7%
29	Greyvenstein (2005)	uPVC	104	3	150	0.3[*]	7.1	1.84	3.21E-05	5.06E+00	1.34	-27.2%

Notes: [*] Cracks cut or machined into the pipe
[#] Cracks induced without removing any pipe material. A width of 0.01 mm was assumed.

CONCLUSION

This paper investigated the behavior of leaks in pipes undergoing elastic deformation due to variations in pressure. Previous studies showed that the area of such leaks expand linearly with pressure, which leads to the FAVAD formulation of the leak flow rate as a function of pressure.

It was found that the conventional power leakage equation does not provide a satisfactory characterization of elastic leaks, and a more consistent characterization based on a dimensionless Leakage Number is proposed. It is shown how the leakage exponent of a leak can be estimated at any pressure once the Leakage Number is known.

Finally, the equations proposed by Cassa and Van Zyl (in press) to predict the head-area slope of a crack based on the properties of the crack, pipe section and pipe material were used to illustrate how the leakage exponent of cracks can be predicted. A comparison of predicted values and the results of several studies on longitudinal cracks showed that the model gives accurate predictions for cracks undergoing elastic deformation. The model underestimates the leakage exponents of plastic pipes with longer cracks that are more likely to undergo plastic deformation.

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