Numerical investigation of the influence of viscoelastic deformation on the pressure-leakage behavior of plastic pipes

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ABSTRACT

It has been well established that leakage from pipes is more sensitive to changes in pressure than the square root relationship predicted by the orifice equation. The main reason for this is that leak areas are not static, but vary with changes in system pressure. While previous studies have shown that the leak area is a linear function of pressure for elastic materials, the effect of the viscoelastic behavior of plastic pipes on the pressure-leakage response is not yet well understood. In this study, finite element analysis was used to investigate the effect of viscoelastic behavior on round holes and longitudinal cracks in both High Density Polyethylene (HDPE) and Polyvinylchloride (PVC) pipes. The standard differential equation for linear viscoelastic deformation was then calibrated to the finite element results, resulting in equations that accurately describe the viscoelastic variation in leak areas as functions of time. The results also showed that the time-dependent behaviors of both pipe materials are proportional to their elastic behavior, and stabilize after about 12 hours. The total

deformations of HDPE and PVC pipes were found to be approximately 220% and 122% of their respective elastic deformations.

Author Keywords: Leakage; Pressure; Leakage exponent; Mathematical models; PVC, HDPE, Viscoelasticity, Creep

INTRODUCTION

Pressure management is internationally recognized as an effective technique for controlling losses from water distribution systems (Lambert et al. 2013). The rate of leakage from pressure zones is generally found to be significantly more sensitive to pressure than suggested by the orifice equation. This is mainly due to the fact that leak areas are not fixed, but vary with system pressure (Van Zyl & Clayton 2007; Greyvenstein & Van Zyl 2007). Besides leakage control, pressure management has also been shown to lower pipe failure rates, extend pipe service life and reduce wasteful consumption (Lambert et al. 2013).

Water pressure in distribution pipes is taken up as internal stresses in the pipe walls, which may be expressed in terms of their principle components in the longitudinal and circumferential directions. In the case of an unsupported cylindrical pressure vessel, the longitudinal and circumferential stresses are described by Equations 1 and 2 respectively (for instance see Griffel (1966)):

$$\sigma_{circ} = \frac{PD}{2t^*} \quad (1)$$

$$\sigma_{long} = \frac{PD}{4t^*} \quad (2)$$

where σ_{circ} and σ_{long} are the circumferential and longitudinal stresses respectively, P the water pressure, t^* the wall thickness and D the inner diameter of the pipe.

The actual stress state of a water distribution system pipe can vary greatly due to external forces on the pipe, and thus Cassa et al. (2010) defined two loading states as the basis for a study of leak behavior under linear elastic conditions: the biaxial load state with circumferential and longitudinal stresses adhering to Equations 1 and 2 respectively, and the uniaxial load state where only circumferential stresses are considered.

Cassa et al. (2010) and Cassa & Van Zyl (2014) conducted finite element method (FEM) analyses to investigate the behavior of leak areas under linear elastic conditions. These studies concluded that leak areas are linear functions of the system pressure for all pipe materials, pipe dimensions, leak types and loading conditions. The latter study also proposed equations to predict the slope of the linear relationship for longitudinal, circumferential and spiral cracks as functions of the pipe, material and leak properties.

Plastic pipes, mainly Polyvinylchloride (PVC) and High Density Polyethylene (HDPE), are commonly used in water distribution systems today. Unlike metallic pipes, plastic pipes display viscoelastic deformation and hysteresis in addition to elastic deformation (Fung 1993; Andrews 1968). A series of experimental studies (Ferrante et al. 2011; Ferrante 2012; Massari et al. 2012; Ferrante et al. 2013) have demonstrated that viscoelasticity and hysteresis influence the behavior of leaks in both PVC and PE pipes.

The aim of this study was to investigate the time-dependent behavior of leaks in plastic pipes considering both their elastic and viscoelastic properties using FEM. The behavior of round holes and longitudinal and circumferential cracks was investigated in PVC and HDPE pipes.

The pipe materials were assumed to experience relatively small strains during deformation and were therefore modeled as linear viscoelastic materials.

The next section describes the equations used to estimate the flow rate from leaks as a function of pressure. This is followed by an overview of viscoelastic modeling theory and a description of the method used to investigate the behavior of leaks in HDPE and PVC pipe materials. The results of the leak area investigations are then presented for the case of a single stepped increase in pressure. An equation for the leak area behavior is developed from the standard differential equation for creep and calibrated against the FEM results. The study is then expanded to cyclical changes in pressure before the impact of viscoelasticity on the pressure leakage relationship is discussed.

MODELING PRESSURE AND LEAKAGE

Leaks can be considered as orifices, for which the flow rate is described as a function of pressure by the orifice equation (Equation 3):

$$Q = C_d A \sqrt{2gh} \tag{3}$$

where Q is the orifice flow rate, C_d the discharge coefficient, A the leak area, g acceleration due to gravity and h the pressure head. Equation 3 is obtained by applying the principle of energy conservation to the orifice and incorporating a discharge coefficient to compensate for local effects and energy losses.

Pressure management field studies showed that the sensitivity of the leakage flow rate is often significantly higher than described by the orifice equation, and this led practitioners to adopt a more general power equation to model the observed behavior (for instance, see Thornton (2003)):

$$Q = Ch^{N1} \tag{4}$$

where $N1$ is the leakage exponent and C the leakage coefficient.

While orifice flow will result in a leakage exponent of 0.5, field studies have reported leakage exponents ranging from 0.5 to 2.79 (Farley and Trow, 2003). While none of these field studies have been published in peer-reviewed literature, several researchers such as Ávila (2003), Greyvenstein & Van Zyl (2007), Cassa et al. (2010), Ferrante (2012) and Ferrante et al. (2013) have investigated individual leaks and have shown that high leakage exponents can be caused by leak areas varying with pressure. Schwaller and van Zyl (2014) and Ferrante et al. (2014) have shown how individual leakage exponents may influence the behavior of a system with many leaks.

Cassa et al. (2010) have shown that leak areas increase linearly with pressure under elastic conditions. The higher leakage exponents may therefore be explained to some extent by replacing the area term in the Orifice equation (Equation 3) with a linear equation of leak area as a function of pressure head:

$$A = A_0 + mh \tag{5}$$

where A is the pressure-dependent leak area, A_0 the leak area under zero pressure conditions and m the area-head slope. This results in the FAVAD (Fixed and Variable Area Discharges) equation proposed by May (1994):

$$Q = C_d \sqrt{2g}\left(A_0 h^{0.5} + mh^{1.5}\right) \tag{6}$$

Van Zyl & Cassa (2014) proposed a dimensionless number, called the leakage number (L_N), which describes the ratio of expanding area flow (second term in Equation 6) to fixed area flow (first term in Equation 6) as:

$$L_N = \frac{mh}{A_0} \tag{7}$$

The leakage number can be used to convert between the parameters of the FAVAD (Equation 6) and power (Equation 4) equations using the following relationship (Van Zyl & Cassa, 2014):

$$N1 = \frac{1.5L_N + 0.5}{L_N + 1} \tag{8}$$

VISCOELASTICITY

Viscoelasticity may be defined as a combination of viscous and elastic behavior (Moore & Kline 1984; Banks et al. 2011). Elasticity is characterized by the stress in a material being a function of strain, but independent of the strain rate and load history. Viscosity is characterized by the stress in a material being a function of the strain rate.

Viscoelasticity results in the occurrence of creep, stress relaxation and hysteresis. Creep is defined as the time-dependent strain that occurs in a material subjected to a constant stress (Andrews 1968). A time dependent creep modulus may be calculated for materials undergoing creep by dividing the stress by the strain.

Stress relaxation is defined as the decrease in stress in a material when a constant strain is applied (Andrews 1968). When a strain is applied to a viscoelastic material, the stress required to maintain that strain decreases with time. A time dependent relaxation modulus may be calculated by dividing the stress by the strain.

Hysteresis is the phenomenon where different stress-strain relationships are observed when a material is subjected to cyclic loading (Fung 1993). Strains in materials due to hysteresis are often recovered after a period of time, which differentiates it from plastic deformation.

Viscoelastic material properties are temperature and loading rate dependent (Ward 1971; Farshad 2006) and therefore materials may behave differently under different conditions. An increase in temperature or low strain rate results in a low stress-strain modulus and more ductile behavior, while a decrease in temperature and high strain rate results in a higher stress-strain modulus and brittle behavior.

Viscoelastic behavior may be characterized as linear or nonlinear. Linear viscoelasticity is a good approximation for small strains and is characterized by the stress in a material being linearly proportional to its strain history (Banks et al. 2011). Nonlinear viscoelasticity is applicable for large strains and means that the stress is not linearly proportional to strain history.

Linear viscoelastic behavior was assumed in this study. It can be described with the help of a mechanical model known as the Standard Linear Model. This model consists of a spring and dashpot connected in parallel with another spring as illustrated in Figure 1 (for instance see Fung (1993)).

In Figure 1, E is the elastic modulus of the respective springs, F the force, u the displacement of the respective spring or dashpot and η is the viscosity of the dashpot.

Starting from conditions of equilibrium, a force F applied to the Standard Linear Model results in the following constitutive equation for response of a linear viscoelastic material:

$$F + \tau_{\varepsilon} \frac{dF}{dt} = E_R \left(u + \tau_{\sigma} \frac{du}{dt} \right) \tag{9}$$

where

$$\tau_{\varepsilon} = \frac{\eta_1}{E_1},\ \tau_{\sigma} = \frac{\eta_1}{E_0}\left(1 + \frac{E_0}{E_1}\right) \text{ and } E_R = E_0 \tag{10}$$

In Equations 9 and 10, τ_{ε} is the relaxation time of the load during constant strain, τ_{σ} is the relaxation time of strain during a constant load, t time and E_R the relaxation modulus of the model. When a load is applied to the setup in Figure 1 for an infinite time, the dash pot completely relaxes and the relationship between stress and strain is determined by spring (0). Hence, the relaxation modulus E_R of the model becomes the spring constant E_0.

The response of viscoelastic materials to applied loads or strains can thus be modeled with Equation 9. The value of the constants in Equation 9 may be obtained from creep data and a Prony series. Creep data includes stress and strain values recorded over various time periods. A Prony series is based on a method developed by De Prony in 1795 and is used for solving non-linear systems of equations that arise from exponential approximation functions (Hildebrand 1974). Equation 9 results in an exponential function when rearranged to find u at a given time t when a force F is applied. Prony series are commonly used for modeling viscoelastic parameters in the form:

$$f(t) = \sum_{i=1}^{n} R_i e^{\lambda_i t} \tag{11}$$

where $f(t)$ is an exponential function, n the number of terms in the series, i the ith term in the series and R and λ material constants. To solve for R and λ, experimental relaxation data is collected for a selection of time periods. The challenge in determining an appropriate series is that the appropriate number of terms n is unknown. Therefore, n has to be chosen beforehand and relies on intuition and understanding of the material.

In Abaqus, the FEM software used in this study, the viscoelastic Prony series is modeled as an exponential approximation of the relaxation modulus similar to Equation 11:

$$G(t) = G_\infty + \sum_{i=1}^{n} G_i e^{-t/\tau_i} \tag{12}$$

where $G(t)$ is the time dependent shear relaxation modulus, G_∞ the long term shear modulus and τ the relaxation time. A single term series was assumed appropriate for this investigation.

METHODOLOGY

For this project, pipe models with individual leaks were represented and analyzed in the FEM software Abaqus. The pipe models were constructed as deformable solids, extruded from the inner and outer diameter cross sections, with the longitudinal axis of the pipe placed along the z-axis. The pipe models were based on the stress properties of a Class 6 uPVC pipe (working pressure of 600 kPa), with an inner diameter of 104 mm, wall thickness of 3mm and length of 500 mm.

An equivalent Class 6 HDPE pipe was defined by assuming the same inner diameter of 104 mm. The wall thickness of the HDPE pipe was determined as 3.9 mm in accordance with an allowable pipe stress of 8 MPa (SANS, 2004). The individual leak openings in the pipes included round holes and longitudinal cracks and were modeled as cut extrusions centered at mid length of the pipe models. The sections for the cut extrusions were constructed in the y-z plane, and extruded through the pipe thickness in the x direction. Round holes of 1 and 12 mm in diameter, and longitudinal cracks of 10, 40 and 80 mm in length were investigated. Each crack had a width of 1 mm and a crack tip radius of 0.5 mm.

The elastic and viscoelastic material parameters used in this study are summarized in Table 1. The viscoelastic parameters were defined by the direct specification of a Prony series. These parameters were determined from the standard linear model for creep (Equation 9) using creep test data obtained from Janson (2003). Thereafter, the model was converted to a single term Prony series based on the method used by Jasinoski & Reddy (2012) to determine Abaqus viscoelastic inputs g_1^p and τ_ε. g_1^p is a dimensionless form of the shear relaxation modulus given by:

$$g_1^p = \frac{G_1}{G_0} \qquad (13)$$

From Equation 12 it is seen that G_1 is the shear modulus when $i = 1$ (at the first time interval) and G_0 is the instantaneous shear modulus at $t = 0s$.

The pipe models were meshed using a hex mesh technique, that is, a mesh comprised of six sided elements. The final mesh consisted of three dimensional brick elements with quadratic functions and therefore 20 nodes. The appropriate size of the elements for the global pipe was determined using a sensitivity analysis, which involved gradually reducing the element size of the mesh while measuring the stress at a specific point referred to as the “set out” until the stress converged. A global element size of 3 mm was selected for both pipe materials. Sensitivity analysis results for the global element size in HDPE are illustrated in Figure 2. For elements in close vicinity to the leaks, the sensitivity analysis in conjunction with the inbuilt mesh verification tool in Abaqus was applied to ensure that poor quality elements were eliminated. The size of the elements around the leak were smaller than the global element size.

Two boundary conditions adapted from Cassa et al. (2010) were applied to prevent rotation and translation of the pipe. These boundary conditions were located as far as possible from the leak to minimize their effect on the leak deformation. They included fixing an internal longitudinal line on the pipe directly opposite the hole in the x and y directions and fixing a point adjacent to the internal line on the exterior of the pipe in the x, y and z directions.

The study simulated the viscoelastic behavior of each pressurized pipe with an individual leak for a total time period of 100 000 s (27.8 h). Uniaxial and biaxial load states were investigated separately for each pipe. Pressures of 200 kPa, 400 kPa and 600 kPa were applied respectively for each pipe and load state.

The deformed leak area was calculated as a y-z projection of the area bound by the inner surface of the leak from the deformed mesh coordinates. The leak areas were determined at times 0 s (elastic deformation), 10 s, 100 s, 1000 s, 5000 s, 10 000 s, 50 000 s and 100 000 s.

In addition to the single load simulations, cyclic loading simulations were also carried out by increasing and decreasing the pressure to values of 200 kPa, 400 kPa and 600 kPa. Two loading and two unloading cycles were carried out and each loading step had a duration of 100 000 s. The cyclic loading pattern is illustrated in Figure 7.

LEAK AREA RESPONSE TO PRESSURE INCREASE

The first set of analyses was done by increasing the pressure in the simulated pipe from zero to a fixed positive value and holding constant for a time period of 100 000 s. The leaks all showed an initial expansion when the loading was applied, followed by a further time dependent increase. The initial expansions corresponded to the expected elastic deformation, as seen from Equation 9 at $t = 0s$. The results for the 1 mm diameter hole in HDPE, shown in Figure 3, are typical.

Figure 3 shows that the viscoelastic deformation becomes incrementally smaller and stabilizes in practical terms after approximately 12 hours. In the case of round holes, biaxial loading resulted in greater area expansion than the equivalent uniaxial loading as shown in Figure 3. The 12 mm diameter hole also displayed consistently greater proportional expansion than the 1 mm hole.

Longitudinal cracks displayed substantially higher proportional expansion compared with round holes, with the expansion also increasing with increasing crack length. However, unlike round holes the deformed areas of the longitudinal cracks were virtually identical for the uniaxial and biaxial load states. This was also observed by Cassa et al. (2010) for elastically deforming longitudinal cracks.

The relationship between leak area and pressure was observed to be linear at any given time after loading, as shown in Figure 4 for the typical case of an 80 mm longitudinal crack in HDPE subjected to pressures of 200 kPa, 400 kPa and 600 kPa. The elastic relationship between leak area and pressure (Equation 5) is shown as the bottom dotted line. The other lines give the crack area as a function of the pressure head at different times after loading, showing an increase in the gradients with time.

Further evaluation of the results showed that the total (elastic plus viscoelastic) deformation of all leak openings in the same material is proportional to the elastic deformation at any point in time. To illustrate this, a viscoelastic factor k_v was defined as the ratio of the total area deformation to the elastic-only area deformation ΔA_e of a leak opening:

$$k_v = \frac{A(t) - A_0}{A_e - A_0} = \frac{\Delta A(t)}{\Delta A_e} \quad (14)$$

Where $A(t)$ is the deformed leak area at time t, A_e is the deformed leak area due to elastic deformation only and $\Delta A(t)$ is the change in leak area due to both elastic and viscoelastic deformation.

k_v was plotted against time and this resulted in reasonably consistent trends for each material, irrespective of the leak type, loading state or size of the load. The results for k_v are shown in Figures 5 and 6 for HDPE and PVC respectively. The figures show that the total area deformation of leaks is more than double the elastic deformation in HDPE pipes, but only about 22 % greater than the elastic deformation in PVC pipes.

MODEL FOR LEAK AREA DEFORMATION

It is known that stresses (and hence deformations) in are linear functions of pressure (see Equations 1 and 2) and that leak areas also vary linearly with pressure. Since this study showed that at any given time after loading, the viscoelastic area deformation is also a linear function of pressure, it follows that the viscoelastic model in Equation 9 could be used to model the leak area deformation due to viscoelastic behavior.

Assuming that the force in Equation 9 is applied through a change in pressure ΔP at time $t = 0s$ and that the effective deformation force is proportional to the change in pressure $(F = a\Delta P)$. Equation 9 holds for the viscoelastic deflection u, which in this case is the viscoelastic change in leak area (ΔA_v) that occurs due to a change in pressure. Equation 9 may be written for the viscoelastic change in leak area ΔA_v (ΔA_v replaces u in equation 9):

$$\frac{d(\Delta A_v)}{dt} + \frac{\Delta A_v}{\tau_\sigma} = \frac{a\Delta P}{E_R \tau_\sigma} \tag{15}$$

where a is a constant. Equation 15 is a first order linear equation which may be solved to obtain an expression for the time dependent change in area and the result is:

$$\Delta A_v(t) = \left(1 - e^{-t/\tau_\sigma}\right)\frac{a\Delta P}{E_R} \tag{16}$$

where the term $\frac{a\Delta P}{E_R}$ describes the ultimate viscoelastic deformation (i.e. at time τ_σ) and the term $\left(1 - e^{-t/\tau_\sigma}\right)$ is a factor describing the development of the viscoelastic deformation with time.

The total leak area can now be described with the function:

$$A(t) = A_0 + \Delta A_e + \Delta A_v(t) \tag{17}$$

Equation 17 was applied to all the leaks included in this study in the following way:

- The initial area A_0 for each leak was calculated from its dimensions.
- The elastic leak area deformations ΔA_e were obtained from the FEM results and are summarized in Table 2. Alternatively the elastic deformation may be estimated using equations proposed by Cassa & Van Zyl (2013) or Tada et al (2000).
- The viscoelastic leak area deformation ΔA_v was described as a function of time using Equation 16. The value of the coefficient a was determined by fitting Equation 16 to the

modeling results. The ultimate viscoelastic deformation at 100 000 s was estimated from Equation 16. The ultimate deformed area was then calculated using Equation 17.

- The ultimate ratio of the total area deformation to the elastic-only area deformation k_{vu} (k_v at $t = 100000s$) was then calculated from Equation 14.

The values of a and k_{vu} for each leak in this study are given in Table 2. It is clear from the table that the values of a and k_{vu} are affected by the pipe material and leak type, but are relatively insensitive to pressure.

Replacing these parameters in Equations 14 and 17 for any material and leak type results in an equation for the deformation of leak area with time. These equations are plotted for a 1 mm diameter hole in HDPE as the lines in Figure 3. The figure shows that the equations fit the model data very well.

An important implication of these results is that the viscoelastic behavior of any leak area with time can be accurately described if the viscoelastic deformation is measured or modeled at only one point in time (to estimate a).

Another notable finding is that the ultimate values of k_v are similar for different leak types in the same material as is evident from Table 2 and Figures 4 and 5. For the HDPE leaks investigated the ultimate value of k_v varies between 2.00 and 2.22, and for PVC between 1.21 and 1.24. Adopting values of 2.1 and 1.22 for HDPE and PVC respectively should provide reasonable estimates of the ultimate deformed area as a fraction of the elastically deformed area.

It means that HDPE will experience substantially higher viscoelastic deformation (approximately 110 % of elastic deformation) than PVC (approximately 22 % of elastic deformation). This corresponds to findings by Covas et al. (2004) and (Covas et al. 2005) for polyethylene pipes where the dynamic modulus was approximately 2.0 times the static modulus. Soares et al. (2008) and (Soares et al. 2011) also found that for PVC pipes, the dynamic modulus was approximately 1.2 times the static modulus during transient analysis. The values of k_v may also be explained by analyzing the creep function of the material as reported in Covas et al. (2004). From Figures 3 to 6 it is evident that the ultimate deformation will be achieved in practical terms after about 12 hours.

LEAK AREA RESPONSE TO CYCLIC PRESSURE VARIATIONS

In this part of the study, pressure was increased and decreased in a cyclic pattern to investigate the impact of cyclic loading on the area of the 1 mm hole and 10 mm and 80 mm cracks in both HDPE and PVC. After each change in pressure, the system was simulated for a period of 100 000 s (27.8 hours), which allowed the viscoelastic deformations to stabilize. Pressure changes were made in steps of 200 kPa.

The cyclic loading pattern is shown in Figure 7, and the resulting behavior of the 80mm longitudinal crack in HDPE (biaxial load state) in Figure 8. The line in Figure 8 is a plot of Equation 17. Figure 8 also shows the ultimate areas (after 100 000 s) found in part one of the study.

The results show that the total expansion converges on the same ultimate value, irrespective of the loading path that was followed. Thus no hysteresis is evident, although it is clear from the results, and also investigations by Ferrante et al. (2011) and Ferrante (2012), that hysteresis will occur if the variation in pressure occurs at short intervals (i.e. not allowing the viscoelastic deformation to stabilize). Therefore, if at any time the pressure is fixed the total deformation will converge on the ultimate value for that pressure, irrespective of the loading history. However, it is difficult to obtain general results from practical investigations due to the effect of loading history dependent behavior in moduli of viscoelastic materials as observed by Soares et al. (2011) and Pezzinga (2014).

The hysteresis observed in HDPE pipe leaks by Ferrante et al. (2011) and Ferrante (2012) may, at least be partly explained by the continuous variation in pressure before the viscoelastic deformation stabilized. Other factors that may play a role are the occurrence of plastic deformation and that the pipe materials did not exhibit perfect linear viscoelastic behavior.

IMPLICATIONS FOR LEAKAGE BEHAVIOUR

It has been well established that leak areas vary linearly with pressure under elastic conditions (Cassa et al. 2010; Cassa & Van Zyl 2014) resulting in the FAVAD equation (Equation 6). The results of this study show that the leak area in linear viscoelastic materials can also be estimated as a linear function of pressure head at any given time (see Figure 4). The slope m of the linear area-head relationship can therefore be obtained for each time period. As can be seen from Figure 4, the slope of the line increases with time but stabilizes after about 12 hours.

The slope of the area-head relationship under elastic conditions is determined by Equation 19 below:

$$m_e = \frac{A_2 - A_1}{h_2 - h_1} \tag{19}$$

where m_e is the gradient of the elastic leak area-head relationship, A_1 is the initial leak area at pressure head h_1 and A_2 is the leak area after elastic deformation at pressure head h_2. Since the ultimate leak area under viscoelastic conditions can be estimated by multiplying the elastic area by the factor k_{vu}, the area-head slopes for a viscoelastic material can be estimated by Equation 20 below:

$$m_{vu} = k_{vu} m_e = \frac{k_{vu} A_2 - k_{vu} A_1}{h_2 - h_1} \tag{20}$$

where m_{vu} is the ultimate area-head slope for viscoelastic materials. The values of the ultimate expansion ratios (k_{vu}) are included in Table 2.

Note that Equation 20 assumes that the ultimate leak area had been reached both before and after the change in pressure. The impact of viscoelasticity on the area-head slope before the leak has stabilized is more difficult to determine, since it will depend on the extent of viscoelastic deformation both before and after the change in pressure. However, the slope may be estimated for a given situation using Figures 5 and 6.

Finally, the effect of viscoelastic behavior on the leakage exponent ($N1$ in Equation 4) can be estimated from Equation 8 after obtaining the viscoelastic slope and corresponding leakage number. The relationship between the leakage number and the leakage exponent is nonlinear and therefore the effect of viscoelasticity on the leakage exponent will depend on the elastic leakage number as shown for HDPE and PVC in Figure 9. It is clear that the impact of viscoelasticity on a leakage exponent of 0.5 and 1.5 will be small, and will increase for intermediate leakage exponents. The maximum increases in the leakage exponent occurs at an elastic leakage exponent of approximately 0.80, increasing the leakage exponent by 23 % and 5 % for HDPE and PVC respectively.

CONCLUSION

This study investigated the impact of viscoelasticity on the pressure-leakage relationship of leaks in PVC and HDPE pipes using finite element analysis. Different pressures were applied to pipes with variety of individual leak types up to the pipe's design pressure of 600 kPa.

All leaks investigated displayed an instantaneous elastic response when a change in pipe pressure occurred, followed by a time-dependent viscoelastic creep response. The rate of the creep response reduces with time, and the leak area stabilized in practical terms after approximately 12 hours.

It was found that expressing the total (elastic plus viscoelastic) deformation as a proportion of the elastic deformation results in reasonably consistent trends over time. HDPE pipes showed ultimate area expansion rates between 2.00 and 2.22 of the elastic leak area expansion. PVC pipes showed significantly lower expansion rates, varying between 1.21 and 1.24 of elastic deformation. These ratios are independent of the initial pressure, size of the pressure variation implemented, or the number of loading steps used to get to the current pressure.

Based on these findings it is recommended that the effects of viscoelastic deformation of leaks in HDPE and PVC pipes are modeled by increasing their elastic area-head slopes by factors of 2.1 and 1.22 respectively. The elastic area-head slopes may be estimated by the equations proposed by Van Zyl & Cassa (2014) or Tada et al. (2000).

Pressure in real distribution systems varies continuously due to transients and the effect of diurnal water demand patterns. As a result, leak areas will be in a constant state of creep, the rate of which will depend on the amplitude and timing of the system pressure variations. However, it can reasonably be expected that the creep effect at a given time of the day will be similar from day to day under consistent operational conditions, particularly during the minimum night flow period when pressures tend to be stable. It should be noted that discrepancies between modeled and experimental results may occur as a result of fluid friction losses and mechanical dumping as well as the effects of surrounding soils on buried pipes (Stephens et al. 2011). Should changes in operational conditions occur, such as the implementation of pressure management, isolation of sections of the network for repairs or changes in pump schedules, a period of at least 12 hours should be allowed for the system to stabilize before the new leakage rate is measured.

The impact of the observed increases in leak area-head slope on the leakage exponent will be a function of the leakage exponent at which the viscoelastic deformation takes place. However, it is possible to estimate the maximum increase in the leakage exponent to be 23% and 5% for HDPE and PVC pipes respectively.

NOTATION

The following symbols are used in this paper:

A -Leak Area;

A_0 -Initial leak area (under zero pressure conditions);

A_e -Elastically deformed leak area;

$A(t)$ -Leak area at a time t(s);

C -Leakage coefficient;

C_d -Discharge coefficient;

D -Inner diameter;

E -Young’s Modulus or elastic modulus;

E_R -Relaxed Young’s Modulus;

F -Force;

G -Shear modulus;

L_N -Leakage number;

$N1$ -Leakage exponent;

P -Pressure;

Q -Discharge;

R -Material constant for Prony series;

a -Constant for converting pressure to equivalent force;

g -Gravitational constant;

g_p^1 -Dimensionless shear modulus;

h -Pressure head;

i -i[th] term in series;

k_1 -Leak discharge-pressure relationship parameter;

k_v -Viscoelastic factor;

k_{vu} -Ultimate viscoelastic factor;

m -Leak area –pressure head gradient;

m_e -Elastic leak area – pressure head gradient;

m_{vu} -Ultimate leak area – pressure head gradient;

n -Number of terms in series;

t -Time;

$t*$ -Wall thickness;

u -Displacement;

η -Viscosity;

λ -Material constant for Prony series;

ν -Poisson's ratio;

σ -Stress;

τ -Relaxation time;

τ_ε -Relaxation time during constant strain;

τ_{σ} -Relaxation time during constant stress;

ΔA_e -Change in leak area due to elastic deformation;

ΔA_v - Change in leak area due to viscoelastic deformation;

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TABLES

Table 1. Material parameters used in the model

Elastic Properties		
Property	**PVC**	**HDPE**
Elastic Modulus (MPa)	3421.143	1126.760
Poisson's ratio (v)	0.4	0.4
Allowable stress (MPa)	10.4	8.0
Safety Factor	4.80	1.25
Prony series parameters for Viscoelasticity		
g_1^p	0.208	0.564
τ_ε **(s)**	3382.788	4348.761

Table 2. Values of elastic area, a and ultimate expansion ratios for the leaks in both HDPE and PVC

Test	Pressure (kPa)	HDPE			PVC		
		A_e (m^2)	a	k_{vu}	A_e (m^2)	a	k_{vu}
1mm hole uniaxial	200	7.864E-07	8.093E-06	2.0287	7.845E-07	4.059E-06	1.2306
	400	7.896E-07	8.044E-06	2.0224	7.859E-07	3.860E-06	1.2111
	600	7.928E-07	7.949E-06	2.0104	7.873E-07	4.028E-06	1.2177
1mm hole biaxial	200	7.879E-07	1.180E-05	2.0214	7.852E-07	5.417E-06	1.2000
	400	7.926E-07	1.188E-05	2.0281	7.872E-07	6.074E-06	1.2243
	600	7.974E-07	1.180E-05	2.0142	7.892E-07	5.871E-06	1.2168
12mm hole uniaxial	200	1.137E-04	2.170E-03	2.2027	1.133E-04	5.390E-04	1.1194
	400	1.144E-04	2.140E-03	2.1916	1.136E-04	5.380E-04	1.1196
	600	1.152E-04	2.120E-03	2.1854	1.140E-04	5.360E-04	1.1194
12mm hole biaxial	200	1.140E-04	2.950E-03	2.2116	1.134E-04	5.460E-04	1.0915
	400	1.150E-04	2.960E-03	2.2152	1.139E-04	5.460E-04	1.0916
	600	1.160E-04	2.970E-03	2.2186	1.143E-04	5.450E-04	1.0915
10mm longitudinal crack uniaxial	200	1.016E-05	1.110E-03	2.1616	9.940E-06	5.390E-04	1.2340
	400	1.054E-05	1.100E-03	2.1584	1.011E-05	5.380E-04	1.2358
	600	1.092E-05	1.090E-03	2.1523	1.027E-05	5.360E-04	1.2359
10mm longitudinal crack biaxial	200	1.016E-05	1.130E-03	2.1666	9.942E-06	5.460E-04	1.2342
	400	1.055E-05	1.120E-03	2.1617	1.011E-05	5.460E-04	1.2363
	600	1.094E-05	1.120E-03	2.1643	1.028E-05	5.450E-04	1.2367
40mm longitudinal crack uniaxial	200	5.100E-05	3.129E-02	2.1351	4.574E-05	1.892E-02	1.2347
	400	6.205E-05	3.059E-02	2.1174	5.165E-05	1.877E-02	1.2337
	600	7.296E-05	2.988E-02	2.0991	5.751E-05	1.861E-02	1.2326
40mm longitudinal crack biaxial	200	5.099E-05	3.138E-02	2.1387	4.573E-05	1.891E-02	1.2349
	400	6.208E-05	3.081E-02	2.1243	5.164E-05	1.878E-02	1.2340
	600	7.304E-05	3.024E-02	2.1097	5.751E-05	1.866E-02	1.2332
80mm longitudinal crack uniaxial	200	1.913E-04	3.139E-01	2.1448	1.396E-04	1.935E-01	1.2389
	400	2.998E-04	2.998E-01	2.1084	1.986E-04	1.905E-01	1.2368
	600	4.052E-04	2.857E-01	2.0711	2.567E-04	1.875E-01	1.2348
80mm longitudinal crack biaxial	200	1.913E-04	3.149E-01	2.1482	1.395E-04	1.935E-01	1.2391
	400	3.001E-04	3.021E-01	2.1154	1.985E-04	1.908E-01	1.2372
	600	4.061E-04	2.893E-01	2.0817	2.568E-04	1.880E-01	1.2353