EXTENDED-PERIOD MODELING OF WATER PIPE NETWORKS – A NEW APPROACH

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ABSTRACT

The extended-period (time-varying or dynamic) equations describing incompressible flow in pipe networks can be classified mathematically as a set of first-order, non-homogenous, nonlinear differential equations. Since this set of equations cannot normally be solved analytically, numerical integration or regression methods are typically used. In this paper, a new method for extended-period simulation, called the Explicit Integration method, is proposed for water pipe networks without demands. The method is based on the premise that a complex water pipe network can be represented by a number of simple base networks. The simple base networks are selected in such a way that their dynamic equations can be solved through explicit integration. In this paper a simple base network consisting of a fixed-head reservoir feeding a tank through a single pipeline is analyzed. It is then illustrated how a complex water pipe network can be decoupled into its constituent simple base networks and its dynamic behavior simulated using a step-wise procedure. The Explicit Integration method is then compared to the commonly used Euler numerical integration method. It is shown that the accuracy of the Explicit Integration method is considerably better than that of the Euler method for the same computational effort.

INTRODUCTION

Hydraulic models are important for the analysis and design of water pipe networks. This is due to the complex topology, frequent changes and sheer size of water distribution networks[1]. Hydraulic models are used for various important tasks by engineers, such as the design of new and analysis of existing distribution networks, long-term master planning and operational planning.

A pipe network consists of an interconnected grid of node-bounded hydraulic links. Nodes are generally associated with potential in the form of hydraulic head, while links are associated with flux in the form of flowrate. Nodes handle end conditions, such as known hydraulic heads and outflows from the network. Links are characterized by a unique relationship between their flowrates and the head differential between their start and end nodes, which is commonly known as the link's headloss relationship.

Under normal operating conditions, water can be assumed to be an incompressible fluid, which allows simple mass and energy balance equations to be used to develop a mathematical model of a network. Transient analysis, which takes the compressibility of a fluid into account, is normally only performed for special cases, such as sudden valve closures or pump switches.

Two types of simulations are used to calculate the network state (heads at the nodes and flows in the links) of a network under incompressible flow conditions, namely snapshot and dynamic simulations. Snapshot (also called static or steady-state) simulation is used to determine the network state at a given time instance. Mass or energy balance is used to draw up a system of non-linear equations, which are then solved using an iterative procedure, such as the Newton-Raphson [2] or Linear [3] methods. The number of equations is equal to the number of links or non-storage nodes in the system and can run into thousands. The linearised equations obtained from each iteration of the Newton-Raphson or Linear methods are solved using sparse matrix techniques and the Cholesky method [4, 5].

In certain cases, snapshot simulation of a network does not provide sufficient insight, and an analysis of the network's hydraulic behavior with time is required. This type of simulation is called dynamic or extended-period simulation in this paper. The applications of dynamic simulation were recognized as early as 1968 by Shamir and Howard [6], although very little had been published on the subject by 1977 compared to the progress in snapshot analysis [2]. A review of published papers shows that this is still the case today.

The applications of extended-period simulation are numerous [7], and include the evaluation of source extraction and tank heads, evaluation of levels of service for demonstrating adherence to legal requirements, operational optimization, evaluation of the network's dynamic response to events such as failures or emergencies, and long term stochastic analysis for evaluating network reliability.

In this paper, the basic theory of extended-period simulation of incompressible fluids is addressed first: the dynamic behavior of pipe networks is discussed, followed by an overview of the different methods currently used to solve the dynamic equations. A proposed new method for solving the dynamic equations, called the Explicit Integration method, is then developed for pipe networks without demands. The application of the Explicit Integration method is illustrated on an example problem, and its computational efficiency and accuracy compared to that of the commonly used Euler method.

DYNAMIC NETWORK EQUATIONS

Certain network parameters vary with time. These are called dynamic parameters in this paper and include tank heads, control schedules and certain end conditions. Dynamic end conditions typically include user demands, source heads controlled externally and events such as element failures or emergency conditions occurring in the network.

Certain dynamic parameters are not known in advance and have to be calculated. The most common of these dynamic variables is the tank head. Control schedules, failure and emergency events are considerably more difficult to calculate directly and are thus often determined indirectly, possibly in an optimization procedure, and evaluated using extended-period simulation. Once the values of all dynamic variables in the network are known, the network state can be calculated at any moment in time by performing a snapshot analysis.

Dynamic equations

The law of conservation of mass for incompressible flow, applied to a node with storage, dictates that the fluid mass entering a node minus the fluid mass leaving the node and the change in the node's storage volume must equal zero:

$$\sum_{i=1}^{N} Q_i - Q_d - \frac{dV}{dt} = 0 \qquad (1)$$

With: Q_i the flow towards the node in link i,

N the number of links connected to the node,

Q_d the demand at the node,

V the node's storage volume, and

t time.

If all the flows to, and the demand from the node are lumped together as a net inflow Q, and the tank is assumed to have a head H and a constant cross-sectional area A, the equation can be written as:

$$A\frac{dH}{dt} = Q \qquad (2)$$

This equation is called the dynamic tank equation. For large reservoirs, the tank area is much larger than the net inflow so that the term $\frac{dH}{dt}$ can be assumed to be zero, i.e. the tank head is a constant.

Now consider a network consisting of a number of tanks. The dynamic behavior of the network is described by the set of differential equations:

$$\mathbf{A}\frac{d\mathbf{H}}{dt} = \mathbf{Q} \qquad (3)$$

With: **A** the vector of tank cross-sectional areas,

H the vector of tank heads, and

Q the vector of net tank inflows.

The net tank inflows in a pipe network can normally not be determined analytically, but require an iterative solution of the static network equations (snapshot simulation). As a result, the dynamic tank equations can also not be solved analytically. This factor is the main complication in solving the dynamic equations of pipe networks. Another complicating factor is that the function that is being integrated (net tank inflow) is a function of the integration result (tank heads). The interdependence of net inflows and tank heads makes solving these equations much more difficult than ordinary numerical integration, where the function being integrated is known at future points.

The problem is further complicated by a number of factors commonly found in pipe networks, such as dynamic variations in network demands and end conditions, control rules based on the dynamically varying network state, imposed time-varying changes to the network configuration, and varying tank cross-sectional areas.

Classification of the dynamic equations

It is important to note that in most cases the dynamic parameters, including the tanks' net inflow rates and cross-sectional areas, are either known or are functions of the tank heads. This allows for the dynamic equations to be written in a general form as:

$$F(\mathbf{H}, \mathbf{H}', t) = 0 \tag{4}$$

With: $\mathbf{H}'$ the vector of first derivatives of $\mathbf{H}$ with respect to t.

Since the first derivatives of tank heads are not required for doing a snapshot simulation of the network, this equation may be written explicitly as:

$$\mathbf{H}' = f(\mathbf{H}, t) \tag{5}$$

The set of equations is now in a suitable form for application of numerical integration techniques such as the Euler and Improved Euler methods. To find the value of $\mathbf{H}'$ for any specific values of t and $\mathbf{H}$, the function value (right hand side of the equation) needs to be calculated. This involves (a) performing a snapshot simulation of the network to obtain the net tank inflows, and (b) converting these flow values to $\mathbf{H}'$ by taking the tanks' depth-area relationships into consideration (for tanks with constant cross-sectional areas, this simply means dividing the net inflow by the cross-sectional area).

This general set of tank differential equations may be classified mathematically as:

- Ordinary differential equations.
- First-order, since the highest order of a derivative in the equation is one.
- Nonlinear, since the order of H in F is not one. With pipes, for instance, the order of the relationship of flow as a function of differential head is close to 0.5.
- An initial value problem, since the vector of tank heads at the start of the analysis $\mathbf{H_0}$ is known, or $\mathbf{H}(t_0) = \mathbf{H_0}$.
- Non-homogeneous, since some terms on the right hand side of the equation will include only *t* and others only *H* [8].

SOLVING THE DYNAMIC EQUATIONS

There is no standard analytical solution for differential equations in the form of Equation 5, and also no solution for the specific case of pipe networks [9, 10]. This implies that in order to solve the dynamic equations, simplifying assumptions are required. Such simplifications can involve discretizing the differential equation, as is done in most numerical integration and numerical regression techniques. Alternatively, assumptions can be used to simplifying the equations themselves to a form that can be solved analytically. Techniques that are currently applied to solve the dynamic equations involve the former simplification, i.e. using numerical integration or regression techniques. These techniques are discussed in this section. A new method for solving the dynamic equations, based on simplifying the equations and then applying explicit integration to solve them, is proposed further on.

Numerical Integration Methods

Numerical methods are commonly used to solve problems for which no analytical solution is available. Numerical integration techniques can be reliable and accurate, but often require a high number of function evaluations. A trade-off between computational cost and accuracy exists with an increased number of function evaluations (and thus higher computational cost) required for increased accuracy. For pipe networks, each function evaluation requires a snapshot simulation of the network, resulting in a computationally expensive procedure.

In practice the two simplest, and least accurate, numerical integration methods are used to solve the dynamic tank equations. These methods are known as the Euler [11, 4, 12] and Improved Euler [10, 13] methods. Both the Euler and Improved Euler methods are applied by dividing the simulation period into a number of time steps, with the start and end times of a time step i given by (t_i, t_{i+1}); $t_{i+1} > t_i$, and the length of the time step $\Delta t = t_{i+1} - t_i$. The length of the time step depends largely on the network dynamics and input functions [10]. The hydraulic state of the network, and thus the net tank inflows, are assumed to stay constant for each time step.

The Euler method is the simpler of the two techniques and only requires one function evaluation at the start of each time step. The Improved Euler method is more accurate, but requires at least two function evaluations per time step. Firstly, the tank heads at the end of the time step are calculated using the Euler method. Another snapshot simulation is then performed at the end of the time step, and the tank heads at the end of the time step are then 'corrected' by using the average inflows of the two snapshot simulations. The process is repeated until convergence is achieved.

The main disadvantage of both the Euler and Improved Euler methods is the sensitivity of their approximation errors to time step length [14]. The accuracy of numerical integration methods decreases with increasing time step length. As a result, the time step length has to be restricted. In practice, time step lengths of one hour or less are often used [1]. A 24-hour simulation thus requires at least 24 snapshot simulations, which makes extended period simulations computationally expensive. Although the simulation time is not normally a problem, it can be an important factor when large numbers of extended-period simulations are required (for example in operational optimization runs), or long simulation periods are used.

The question can be raised as to why higher order, and thus more accurate, numerical integration techniques are not employed to solve the dynamic tank equations. A number of possible reasons can be listed:

- Historically the main emphasis of research, development and application of distribution models has been directed towards snapshot simulations, and good research on dynamic analysis has been lacking.
- There are normally large uncertainties present in information on demands and demand variations in water distribution systems. Improved accuracy in the simulation method will add little if it is overshadowed by inaccuracies and uncertainties in the input data.
- Snapshot simulations are in themselves iterative and computationally expensive procedures [9]. Higher order numerical integration techniques require more snapshot simulations and would thus further increase the computational cost and simulation times.
- Pipe networks are in a constant state of change due to variations in demands and operational changes in the network configuration, which require intermediate time steps to be used. The increase in the computational burden is directly proportional to the number of snapshot simulations a solution method uses per time step. For instance, each intermediate step will require one extra snapshot simulation if the Euler method is used, two if the Improved Euler method is used and four if the fourth order Runge-Kutta method is used. The effect of intermediate steps on the computational burden will thus be more pronounced for higher order methods.

It is doubtful whether the reasons listed above still make a convincing case for using the Euler and Improved Euler methods in the dynamic simulation of water distribution systems: few improvements are still made to snapshot simulation methods and computer simulation time is not a significant factor in most modeling applications. Finally, the accuracy of the most commonly used Euler method is far from good, as shown in the example application later in this paper, and it is not recommended for any practical use in literature [15]. It is thus important that research is conducted to find more appropriate methods for solving the dynamic equations of water distribution systems. While this paper proposes one such method, various standard numerical methods, such as the Runge-Kutta and Multi-step methods [16], exist which can improve the accuracy of dynamic simulations without introducing excessive computational burdens.

Euler Method

The Euler method is the simplest numerical integration technique available, and it is also the most commonly used for extended-period analysis [12, 17]. It involves the evaluation of the function at the start of the time step. The tank heads at the end of the time step are then estimated by assuming that the function values remain constant for the duration of the step. Walski [18] found that, when applying the Euler method, longer time steps results in fluctuations in predicted flows and pressures lagging behind the actual fluctuations in the network.

In the Euler method, the net tank inflows at the start of the time step are determined by performing a snapshot simulation. For each tank, the change in head for the time step is then estimated using the equation (from Equation 3):

$$\Delta H = \frac{Q_i \Delta t}{A} \tag{6}$$

The tank head at the end of the time step, H_{i+1} is then calculated by adding ΔH to the tank head at the start of the time step, H_i:

$$H_{i+1} = H_i + \Delta H \tag{7}$$

Improved Euler Method

In essence, the Improved Euler method applies the same strategy as the Euler method by assuming that the flowrate in a given time step remains constant. The main difference between the two methods is that, while the Euler method only does a single forward estimation for each time step, the Improved Euler method employs a predictor-corrector scheme to improve this estimate.

The first step in the Improved Euler method is to make an initial estimate of the tank heads at the end of the time step using the Euler method (Equations 6 and 7). Another snapshot simulation is then done at the end of the time step. The average tank inflow between the start and end of the time step is then used in Equations 6 and 7 to 'correct' the tank head at the end of the time step. The process is repeated until the change in tank head is within the required accuracy.

Rao and Bree [2] and Rao et al [19] were the first to suggest a systematic procedure for doing dynamic analyses of water pipe networks [20] by applying the Improved Euler method. Rao and Bree considered demand to be a continuous function, and used the instantaneous demands in the Improved Euler function evaluations. They recognized that, although the snapshot solution is only calculated at specific time instances, the user demand function is known in advance for the full time step. Combining this with the mass balance principle, which prescribes that the total net outflow from all the tanks must be equal to the total demand, they added an adjustment step in which the demand error is assigned to the different tanks in proportion to their net outflow rates .

It is simple to demonstrate that distributing the mass error between the tanks in proportion to their net outflow rates cannot always be justified: consider a pipe network consisting of a pumping line feeding a tank from a bulk storage reservoir. The full network demand is taken from the tank and is equal to the pumping flowrate. Under these conditions the net outflow from the tank will be zero and the full demand error will thus be assigned to the bulk storage reservoir, which has no demand and thus no associated mass balance error. The mass error allocation proposed by Rao and Bree can in this way increase the simulation error rather than compensate for it.

Bhave [20] proposed a way to incorporate Rao and Bree's error equation into the snapshot network equations and thus obtain both the net tank outflow and the corrected tank heads in a single calculation step. Bhave suggested that this method would remove the requirement for a predictor-corrector iterative procedure, since the error adjustment is incorporated directly into the solution process.

However, Bhave’s method for distributing mass balance errors between the tanks suffers from the same inherent flaw as that of Rao and Bree, namely that the net outflows from the tanks are not necessarily an indication of the demand (and thus of the mass balance error) attributable to the respective tanks.

The methods proposed by Rao et al and Bhave are relatively complex compared to the solution procedures used today [17]. The demand mass balance problem identified by Rao and Bree is generally overcome by performing the snapshot simulations with the average demand over a time step rather than the instantaneous demands at the start and ends of the time step. This eliminates the mass imbalance correction factor, but will have an effect on the flow distribution in the network due to the nonlinearity of the network hydraulics. However, compared to the complications introduced by a mass balance error and the inaccuracies introduced by numerical integration, the effect of this assumption can be considered negligible.

Hybrid Transient Approach

Filion and Karney [17] proposed a method that combines a numerical integration method with a transient simulation model to improve the accuracy and capabilities of extended-period simulations. Their method analyses a pipe network for short time periods near the start and end of a time step using a transient model, and then uses the insight gained to predict the behavior of the network using a modified Improved Euler approach. Their method shows a substantial increase in simulation accuracy, but at an increased computational cost making it impractical for the analysis of large networks.

Regression methods

In regression methods, empirical or semi-empirical functions are fitted to calculated data on the behavior of pipe networks under different operating conditions, such as relative tank heads, pump configurations and network demands. Lansey and Awumah [21], for instance, used the least square approach to fit polynomial functions to the rate of change of tank heads as a function of the tank heads.

The biggest drawback of regression methods is that a new curve needs to be calculated for each possible combination of dynamic variables for each tank, increasing the number of curves exponentially with the number of tanks and possible network states, and thus limiting the size of network that can be modeled. For instance, Lansey and Awumah [21] found that their methodology only works well for one or two tanks and a limited number of pumps.

A further disadvantage of the method is that it cannot be applied to any general network, but requires a new set of regression curves whenever a change is made to the network configuration or a new network is analyzed.

Another type of regression method was used by Coulbeck et al [9], who noted that the inefficiency of the dynamic solution procedure often derives from the accurate calculation of many intermediate values that are of less importance, when only the dynamic behavior of the tanks themselves is of interest. They suggested improving the method by reducing the number of equations to include only the important variables.

Coulbeck et al's simplified dynamic model describes the basic network operation in terms of control and demand action on tank heads. The model is expressed as a set of linear dynamic equations that can be solved explicitly for the dependent variables in terms of the known operating conditions. Independent variables are taken as tank heads, demands and pump and valve control settings; and the dependent variables as node pressure and tank outflow. The dynamic equation for tank outflows, for instance, is then written as the sum of its response to changes in the independent variables.

$$d\mathbf{Q}(t) = \frac{\partial \mathbf{Q}}{\partial \mathbf{H}} dH + \frac{\partial \mathbf{Q}}{\partial \mathbf{Q_d}} dQ_d + \frac{\partial \mathbf{Q}}{\partial \mathbf{Z_{Cp}}} dZ_{Cp} + \frac{\partial \mathbf{Q}}{\partial \mathbf{Z_{Cv}}} dZ_{Cv} \tag{8}$$

With: $\mathbf{Q}$ a vector of tank net inflows,

$\mathbf{H}$ a vector of source and tank heads in the network,

$\mathbf{Q_d}$ a vector of the demands in the network,

$\mathbf{Z_{Cp}}$ a vector of pump controls, and

$\mathbf{Z_{Cv}}$ a vector of valve controls.

The sensitivity coefficients for a given network state may be determined from further manipulation of the standard snapshot analysis results. For a network with various possible states (as is required in dynamic analysis), snapshot analyses are performed for perturbations of the independent variables for all possible states of tank heads and control pressure values to determine state variant coefficients. Average values over all states or different values for different states may be used, depending on the required accuracy. For on-line application of the model, the authors suggest that the coefficient values are updated continuously to compensate for changes in the network state or configuration.

Essentially the method proposed by Coulbeck et al is still a regression model, although it is more general in that the coefficients may be calculated automatically using a hydraulic model of the network. However, the number of coefficient values will also increase exponentially with increases in the numbers of variables and control rules. A trade-off would thus normally be required between complexity of the model (the number of coefficient values calculated) and accuracy.

EXPLICIT INTEGRATION APPROACH

In this section a new method for extended period modeling of pipe networks, called the Explicit Integration method, is developed. The Explicit Integration method is based on the assumption that a network can be represented by a number of simple base networks for which analytical solutions exist.

This paper is restricted to the Explicit Integration modeling of pipe networks without demands. The more complicated problem of pipe networks with demands falls outside the scope of this paper.

The Explicit Integration method is developed by first considering a simple base network, which consists of a fixed-head reservoir (source) which feeds a tank through a single pipe. The dynamic differential equation for the network is drawn up and solved using explicit integration. This is followed by a discussion of how complex networks can be separated into a number of simple base networks and their dynamic equations then solved (for the purpose of this discussion, pipe networks are considered complex if their dynamic equations cannot be solved analytically). The Explicit Integration method is then illustrated using an example problem.

Simple Base Network

For simple gravity pipe networks such as a fixed-head reservoir feeding a tank through a single pipe (Figure 1), it is possible to explicitly solve the differential equations describing the network dynamics.

Ideal position for Figure 1.

The headloss h in the pipe is in the form [20]:

$$h = KQ|Q|^{n-1} \tag{9}$$

With: K a constant headloss coefficient.
Q the flowrate in the pipe.
n a constant headloss exponent.

This equation may be rearranged to give the pipe flowrate as a function of the headloss:

$$Q = K'h|h|^{\frac{1-n}{n}} \tag{10}$$

With

$$K' = \left(\frac{1}{K}\right)^{\frac{1}{n}} \tag{11}$$

a constant flow coefficient.

Writing this equation to give the pipe flowrate in the simple base network, for the case where the fixed-head reservoir has a higher head than the tank, results in:

$$Q = K'(H_P - H)^{\frac{1}{n}}; \quad H \le H_P \tag{12}$$

With: H the tank head, and
H_P the fixed-head reservoir's head.

Substituting into Equation 2 and rearranging, results in a differential equation describing the dynamic behavior of the simple base network:

$$\frac{dH}{(H_P - H)^{\frac{1}{n}}} = \frac{K'}{A} dt \tag{13}$$

Although the equation is still nonlinear and non-homogenous, it is a special case for which an analytical solution exists in the form [14]:

$$\frac{n}{1-n}(H_P - H)^{\frac{n-1}{n}} = \frac{K'}{A} t + C \tag{14}$$

With C a constant term obtained from the initial condition $H = H_0$ at $t = t_0$. Substituting C into the equation and rearranging results in a dynamic equation for the tank head:

$$H = H_P - \left[(H_P - H_0)^{\frac{n-1}{n}} - \frac{(n-1)K'}{nA}(t - t_0) \right]^{\frac{n}{n-1}} \tag{15}$$

An expression for the flowrate as a function of time can be obtained by substituting this equation back into Equation 12:

$$Q = K' \left[(H_P - H_0)^{\frac{n-1}{n}} - \frac{(n-1)K'}{nA}(t - t_0) \right]^{\frac{1}{n-1}} \tag{16}$$

Similarly, the dynamic equation for the tank can be obtained for the case where $H \geq H_P$ as:

$$H = H_P + \left[(H_0 - H_P)^{\frac{n-1}{n}} - \frac{(n-1)K'}{nA}(t - t_0) \right]^{\frac{n}{n-1}} \tag{17}$$

Complex networks

Dynamic behavior of water pipe networks

The mechanical energy per unit weight at a point in a hydraulic network above a datum level is given by the Bernoulli equation:

$$E = Z + \frac{p}{\rho g} + \frac{v^2}{2g} \tag{18}$$

With: E the Bernoulli energy,

Z the elevation of the point above the datum level,

p gauge pressure,

ρ the fluid density, and

v the flow velocity.

The Bernoulli energy can be calculated at any point in a balanced pipe network. At reservoir and tank surfaces, both gauge pressure and velocity are zero (or negligibly small) and the Bernoulli energy is represented by the elevation of the water surface above the datum level (tank head).

Water always flows from a higher to a lower energy state. For gravity networks this implies that the heads of the higher tanks will tend to fall, while the heads of the lower tanks will tend to rise. The tank heads are linked by a collection of energy lines, corresponding to the energy levels at different points in the network. The energy lines can be visualized as a three-dimensional grid attached to the tank water surfaces and 'floating' on the network. The energy grid will move with the tank surfaces so that a given point on the energy grid could either be rising, falling or invariant with time. This observation leads to the definition of a pivot point.

Pivot Point

Consider the simple pipe network shown in Figure 2(a), consisting of two tanks connected by a single uniform pipe. The Bernoulli energy in the network is shown as a line with uniform slope that runs above the pipe and connects the two water surfaces. As time passes, the slope of the energy line reduces as its ends drop or rise with the tank heads. Since the water surfaces move in opposite directions, it is possible to identify a point on the energy line at which the energy remains invariant with time. This point is defined as the network's pivot point, since the energy line can be viewed as rotating about this point.

Ideal position for Figure 2.

At the pivot point the network energy remains invariant with time. The importance of this observation is that the pivot point can be replaced by a fixed-head reservoir (with head equal to the energy at the pivot point) without changing the network's dynamic behavior. The network is effectively decoupled into two simple networks as shown in Figure 2(b). Each decoupled sub-network resembles a simple base network as shown in Figure 1 and can thus be solved explicitly.

Unless the two tanks have fixed cross-sectional areas, the position of the pivot point will change as the tank heads rise or fall. However, for small changes in tank heads, the pivot point can be assumed to remain stationary.

Pivot Point Position

The existence of a pivot point in more complex networks is not immediately obvious. Consider, for instance, the network shown in Figure 3(a) that consists of a source (fixed-head reservoir) and three tanks. The tanks are interconnected with a network of pipes.

Ideal position for Figure 3.

The first step in determining a pivot point for the network is to simplify the pipe network to one in which each tank is connected to a central junction as shown in Figure 3(b). This is done by systematic application of three simplification operators; the first to remove pipes (e.g. dead end pipes) from the network, and the second and third to reduce pipes in parallel and series to single hydraulically equivalent pipes. Since no demands exist in the network, the simplification does not significantly affect the accuracy of the model.

In the second step, the tanks are rearranged in such a way that the energy at the junction will be invariant with time, and thus act as a pivot point. It is possible to move tanks in the horizontal plane without changing the hydraulic behavior of the network - a change in the pipe length can, for instance, be compensated for by adjusting its diameter. To find the new tank positions, a horizontal axis of rotation is defined through the proposed pivot point head. Tanks are now moved normally to the axis of rotation to distances directly proportional to the rate of change in their heads. Fixed-head reservoirs are moved to positions directly above or below the pivot point. For small changes in tank heads, the energy lines will now rotate around the pivot point. The network is then decoupled into its constituent base networks by replacing the pivot point with a fixed-head reservoir, and the sub-networks are solved using explicit integration. The position of the pivot point will change with time, meaning that the process will have to be repeated at regular intervals to ensure acceptable accuracy of the results.

The simplification procedure described above is complicated and time-consuming to apply. A simpler method was developed to estimate the position of the pivot point. A time step is selected and a snapshot simulation performed at the start of the step. From the results, the net inflow rates (negative for outflows) are determined for all the tanks in the network. These results already provide a valid range for the pivot point head, since the pivot point has to be lower than all tanks with net outflows, and higher than all tanks with net inflows to adhere to the observed flow directions.

An initial pivot point H_P is now chosen in the valid range. Using the assumed pivot point, the network is decoupled into its constituent simple base networks. The flow coefficient of each simplified sub-network is calculated from the results by rearranging Equation 10 to obtain:

$$K' = \frac{Q_0}{\left(H_P - H_0\right)\left|H_P - H_0\right|^{\frac{1-n}{n}}} \tag{19}$$

With: K' the pipe's flow coefficient,

Q_0 the flowrate into the tank, determined from the snapshot simulation,

H_0 the tank head, and

n the known headloss exponent.

Placing the flow coefficient estimates back into Equation 10 results in a flowrate in the sub-network as a function of the tank head, given by the equation:

$$Q = \frac{(H_P - H)}{(H_P - H_0)}\left|\frac{H_P - H}{H_P - H_0}\right|^{\frac{1-n}{n}} Q_0 \qquad (20)$$

The tank heads in the middle of the time step are now estimated using the standard Euler method, and another snapshot simulation is performed with the updated tank heads. This results in a new set of tank inflow rates, which are used to test the accuracy of the initial pivot point head selection. If the true pivot point head was chosen, the sub-networks should predict tank inflow rates matching those of the second snapshot simulation. However, it is more than likely that the initial choice of pivot point head was inaccurate and that significant differences in flowrates will exist. The true pivot point head is now found by minimizing the error between the flowrates calculated using the pivot point head and the results of the second snapshot simulation.

Equation 12 is used to predict the flowrates for the tank heads in the second snapshot simulation. The error in the flows predicted by the snapshot simulation (Q_1) and the Explicit Integration models provides a measure for the accuracy of the pivot point head:

$$Error = \sum_{\text{All simple base systems}} \left[\frac{(H_P - H)}{(H_P - H_0)}\left|\frac{H_P - H}{H_P - H_0}\right|^{\frac{1-n}{n}} Q_0 - Q_1\right]^2 \qquad (21)$$

To find the best pivot point, the function is minimized using a standard gradient search method.

It is necessary to expand further on two aspects of the above methodology:

- The Euler method is used to calculate the tank heads used in the second snapshot simulation. This is done despite the fact that the Euler method may introduce significant errors in the tank heads when relatively long time steps are used. However, the Explicit Integration variables are determined from the response of tank inflows to changes in the tank heads, and not from the actual tank head values. Even if the tank heads are not determined very accurately, the corresponding inflows are accurate since they are obtained from a full snapshot simulation of the network. Application of the method to various pipe networks confirmed that the Explicit Integration method is not very sensitive to the time step length and that much larger time steps than those used with the Euler method can thus be used.

- The second snapshot simulation is performed in the middle of the time step and not at the end of the time step as might be expected. It was found in various test applications that the estimation error made by the Euler method increases with time between the first and the second snapshot simulation. However, when the Explicit Integration method is used to predict the tank behavior beyond the second snapshot simulation, the estimation error reduces again. The estimation error is reduced to zero in this way at a point in time roughly double the difference between the two snapshot simulations. The reason for this behavior most probably lies in the way the pivot point shifts with time.

Methodology

A methodology is proposed for simulating gravity networks without demands using the Explicit Integration method. A flow diagram of the method is given in Figure 4. First the simulation period is divided into a number of time steps. The time step length is determined by the accuracy requirements and other factors such as operational controls in the network. Two snapshot simulations are performed in each time step, one at the start of the time step and the other in the middle of the time step. The tank head and flow results are used to determine the position of the pivot point by minimizing the error between the flow results of the Explicit Integration model and the second snapshot simulation. The tank heads at the end of the time step are then estimated and the process repeated for the next time step.

Ideal position for Figure 4

EXAMPLE

The Explicit Integration method for networks without demands is illustrated using the example network shown in Figure 3(a). The example network consists of a source with a fixed head of 180 m, which feeds three tanks, A, B and C, with initial heads of 116 m, 157.2 m and 53.8 m respectively. The Hazen-Williams pipe headloss equation was used, so that n in Equation 10 equals 1.852. The source and tanks are connected by a network of pipes.

For the purpose of the example, the maximum tank heads were not restricted, thereby allowing the tanks to fill throughout the 24-hour simulation period. As a consequence, large changes occurred in the tank heads, with tank A's head increasing by almost 40 m. It is unlikely that this scale of change will occur in real networks. However, the large head variations do provide a good basis for evaluating the accuracy of the simulation methods.

To determine the reference or true behavior of the network, an Euler simulation was performed using a small time step. The accuracy of the Euler method increases with decreasing time step length, and a time step of one minute was considered to have sufficient accuracy to provide the basis for calculating the simulation errors. The hydraulic grade variations of the three tanks are shown in Figure 5.

Ideal position for Figure 5

Another simulation was then performed using the Euler method, but this time with a time step length of one hour, which is in line with normal simulation practice.

Finally, the Explicit Integration method was applied to the example network. Two time steps of twelve hours each were used. A snapshot simulation was done at the start of the first time step, and another snapshot simulation in the middle of the step. The tank heads and corresponding net inflows are given in rows two to five of Table 1.

Ideal position for Table 1

A pivot point head for the network must now be determined, which will allow the network to be decoupled into its constituent simple base networks. The valid range for the pivot point head is determined by the flow patterns in the snapshot simulations. It is clear from rows two to five of Table 1 that the pivot point head has to be between 162.2 m (H_1 for tank B) and 180 m (the head of the fixed-head reservoir). The sum-of-square error (Equation 21) is shown for different pivot point head values in Figure 6. The minimum sum-of-square error shown in Figure 6 was found using a standard gradient search method to be 179.308 m. The corresponding K' values for the pivot point head are given in row seven of Table 1.

Ideal position for Figure 6

Replacing the values into Equation 17 provides a description of the tank heads as functions of time for the first 12-hour time step ($0 \leq t \leq 43200$):

$$\begin{aligned} H_A &= 179.307 - \left(6.7413 - 2.7411 \times 10^{-5}\, t\right)^{2.1737} \\ H_B &= 179.307 - \left(4.1547 - 2.1108 \times 10^{-5}\, t\right)^{2.1737} \\ H_C &= 179.307 - \left(9.2359 - 1.9072 \times 10^{-5}\, t\right)^{2.1737} \end{aligned} \qquad (22)$$

The process was now repeated to find the tank head function for the second 12-hour time step. The full 24-hour simulation thus only required four snapshot simulations.

The simulation errors introduced by the Euler and Explicit Integration methods were determined by comparing their results to the reference simulation results. This comparison showed that both the Explicit Integration and Euler methods performed well on Tank C, with final errors less than 2 mm. However, for Tanks A and B large differences in the performance of the two methods were observed as shown in Figures 7 and 8. The graphs clearly show that the Explicit Integration method achieved better accuracy than the Euler method for both tanks A and B. The Explicit Integration method also used only four snapshot simulations, compared to 24 snapshot simulations by the Euler method.

Ideal position for Figures 7 and 8

The Euler method’s error is greatest for tank A (305 mm), followed by tank B (149 mm) and then tank C (1.6 mm). This sequence corresponds to the changes in tank heads shown in Figure 5. During the simulation the flowrates into the tanks continually decline as the tanks fill up. The Euler method, however, does not take this decrease in flowrate into consideration, but estimates the tank head solely from the flowrate at the start of each time step. This results in an overestimation of the flowrate, and thus also of the tank head. The greater the change in flowrate over the time step, the greater the resulting error will be. This explains the correlation between change in tank head and simulation error.

The Explicit Integration method's error differs from the Euler method's error in that it is greatest for tank B (-69.0 mm) followed by tanks C (0.7 mm) and A (0.2 mm). This sequence corresponds to the sequence of tank flowrates (see Table 1). The pivot point head is determined by minimizing the sum-of-square flowrate error. This measure results in greater fractional errors for the sub-networks with smaller flowrates, such as tank B.

DISCUSSION

The most important reason for the Explicit Integration method's improved accuracy over the Euler method is that it uses considerably more network information. The Explicit Integration method specifically includes information on the network layout and pipe hydraulic behavior. The Euler method, on the other hand, only uses the tank inflows at the start of a time step to estimate the tank head behavior.

It should be pointed out that the Euler method error would not always continue to grow throughout a simulation as occurred in the example application. The reason for this is that the tank heads at the end of a 24-hour simulation will normally be similar to the tank heads at the start of the simulation. In such cases the Euler method error will first increase, but will then decrease again as the tank head returns to its initial value.

Another point of note is that the number of snapshot simulations required is also influenced by the number of operational events such as pumps switching, tanks filling up and valves changing status. When many such control events occur in a simulation run, the advantage of the reduced computational effort by Explicit Integration will be less. However, in most cases the Explicit Integration method should still provide significant improvements in modeling accuracy.

An Explicit Integration simulation results in a set of equations describing the tanks' dynamic behaviors. These equations provide detailed information on the tank behavior between simulation times and not only for the times at which snapshot simulations were performed. The Euler method, on the other hand, only provides tank head values at times when snapshot simulations are performed. To obtain tank heads at intermediate times linear or step-wise interpolation is typically used. Both these interpolation methods can result in significant increases in the simulation errors at intermediate times.

The Explicit Integration method requires little additional information compared to the Euler method, and most of the additional information can be obtained from the standard network data using automated procedures.

CONCLUSIONS

In this paper, three categories of methods for solving the dynamic tank equations were discussed, namely regression, numerical integration and explicit integration methods.

Regression methods are problem specific and their accuracy dependent on how well the regression equations represent the actual network behavior.

Two numerical integration methods are typically used for solving the dynamic tank equations, namely the Euler and Improved Euler methods. The most common of the two, the Euler method, does not have good accuracy and should not be used for practical results. It is important that other numerical integration methods are also investigated to find a more accurate solution method with reasonable computational effort.

An new method for extended period simulation of pipe networks without demands, called the Explicit Integration method, is proposed in this paper. In the Explicit Integration method, a pipe network is decoupled into a number of constituent simple base networks. The dynamic behaviors of the simple base networks are determined by integrating their dynamic tank equations explicitly, and are then used to estimate the dynamic behavior of the full pipe network. The accuracy of the Explicit Integration method is influenced by movements in the network's 'pivot point', a point on the energy line that is used to decouple the network into its simple base networks.

Using an example network, it was shown that the Explicit Integration method is able to model the dynamic behavior of tanks with improved accuracy while using substantially fewer snapshot simulations than the Euler method. In the example, the computational cost of dynamic modeling was reduced by almost 80 % from 24 snapshot simulations to just four.

LIST OF SYMBOLS

A = tank cross-sectional area;
$\mathbf{A}$ = vector of tank cross-sectional areas;
E = Bernoulli energy;
h = headloss;
H = tank head;
H_P = reservoir (fixed) head;
$\mathbf{H}$ = vector of tank heads;
$\mathbf{H}'$ = vector of first derivates of tank heads with respect to t;
K = headloss coefficient;
K' = flow coefficient = $\left(\frac{1}{K}\right)^{\frac{1}{n}}$;
n = headloss exponent;
N = number of links connected to a node;
p = pressure;
Q = net tank inflow;
Q_d = demand;
Q_i = link flowrate;
$\mathbf{Q}$ = vector of net tank inflows;
$\mathbf{Q_d}$ = vector of the demands in the network;
t = time;
v = velocity;
V = storage volume;
Z = elevation;
$\mathbf{Z_{Cp}}$ = vector of pump controls;

$\mathbf{Z}_{\mathbf{Cv}}$ = vector of valve controls;

Δ = a change;

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TABLES

Table 1 Calculation of the first pivot point for the example network

Row	Item	Tank A	Tank B	Tank C	Source
1	**Diameter (m)**	30	15.2	50	-
2	H_0 (m)	116.00	157.20	53.80	180
3	Q_0 (l/s)	395.0	45.50	110.4	-550.9
4	H_1 (m)	128.07	162.62	55.01	180
5	Q_1 (l/s)	352.8	38.06	110.0	-500.9
6	H_P (m)	179.307			
7	K'	0.042117	0.008326	0.008140	0.6111

FIGURES

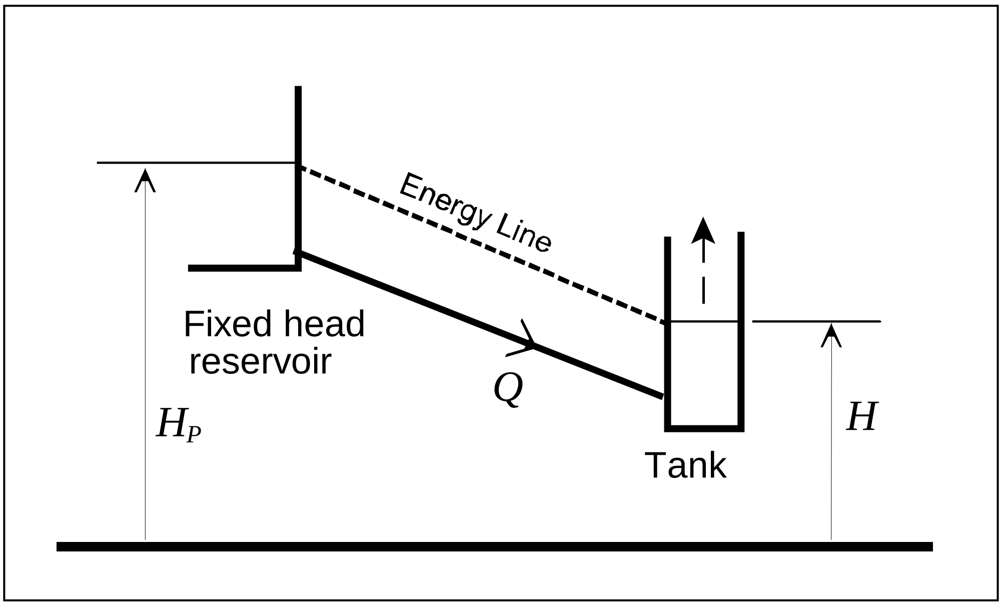

Figure 1 A simple base network consisting of a fixed-head reservoir feeding a tank.

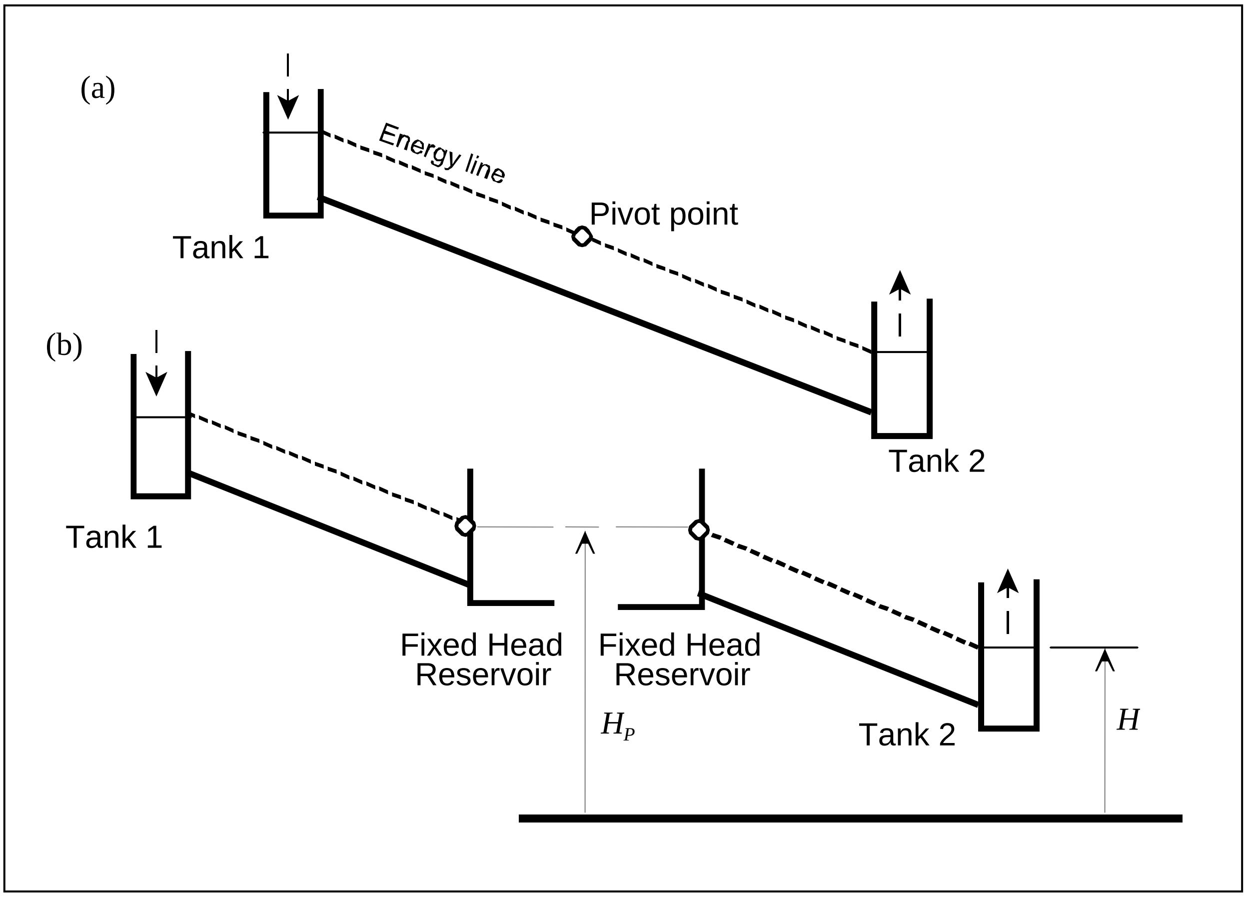

Figure 2 (a) A pipe network consisting of two tanks connected by a single pipe. (b) An equivalent network with the pivot point replaced by a fixed-head reservoir.

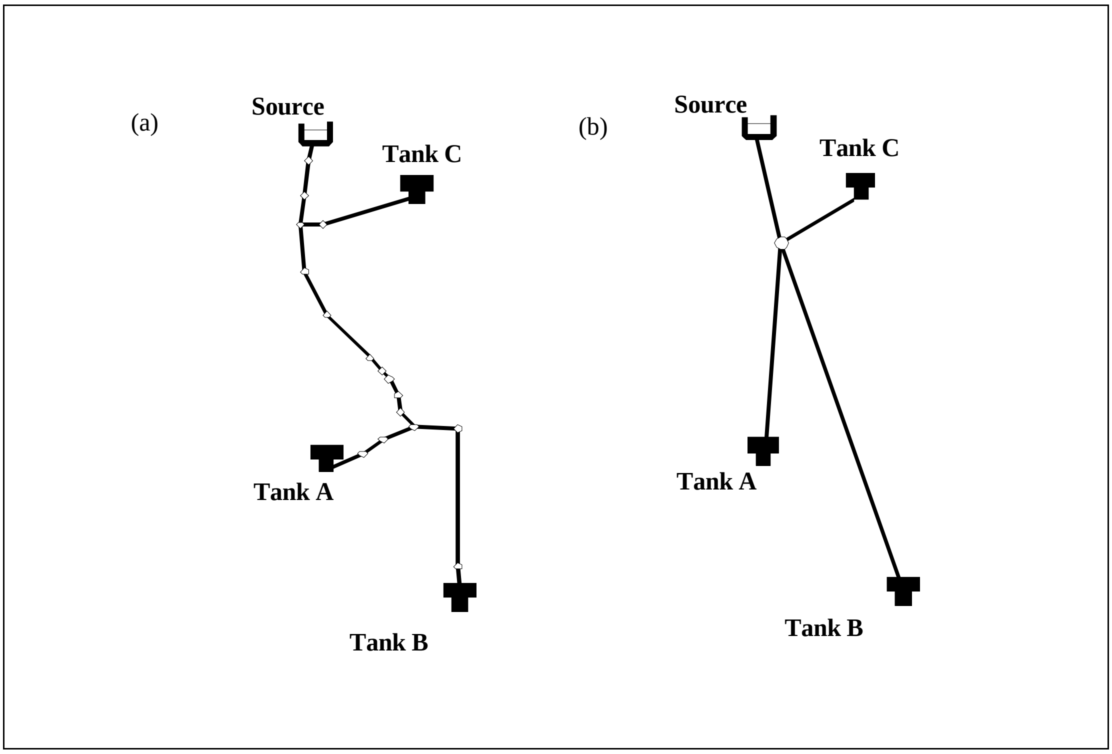

Figure 3 (a) An example pipe network without demands. (b) An equivalent simplified network.

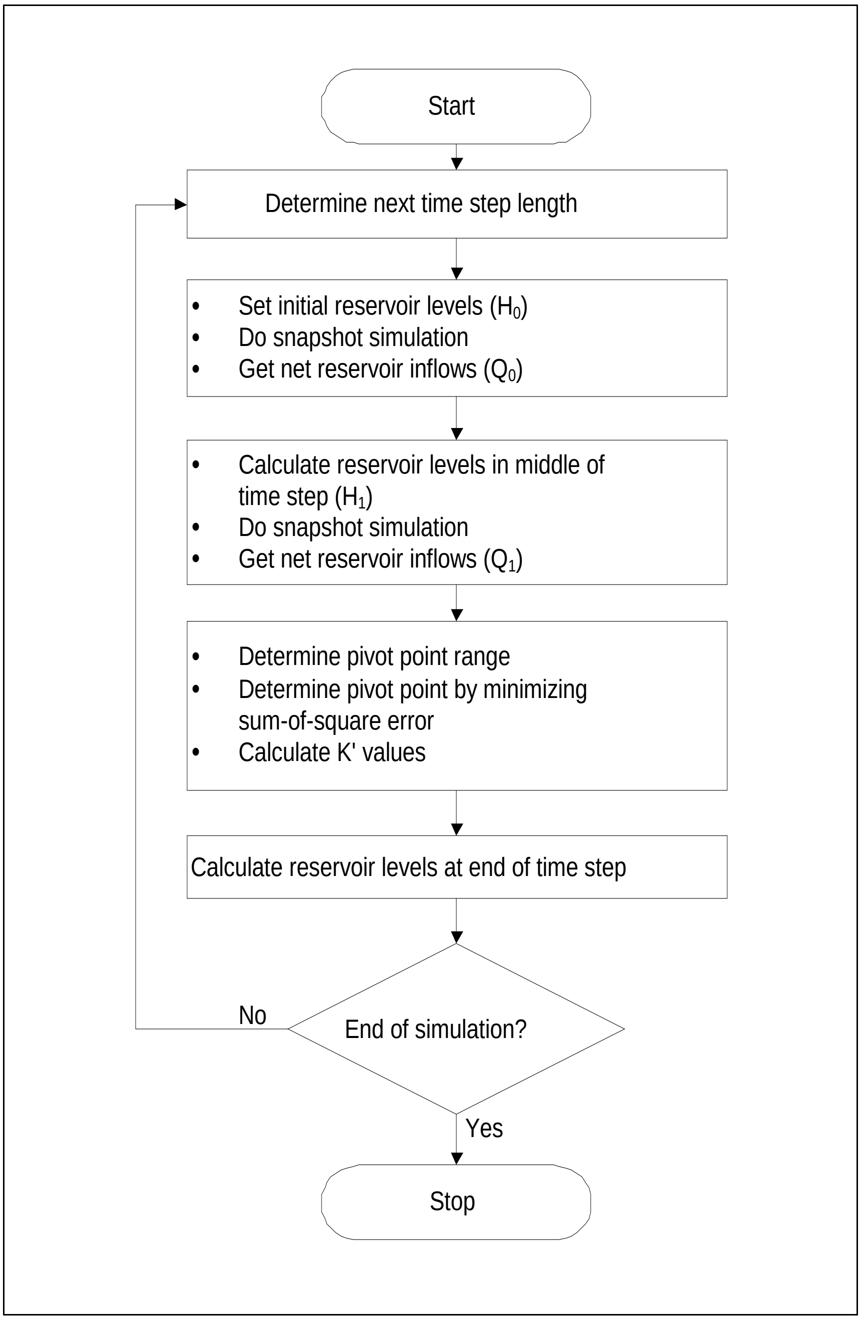

Figure 4 Flow diagram of the Explicit Integration method for gravity pipe networks without demands

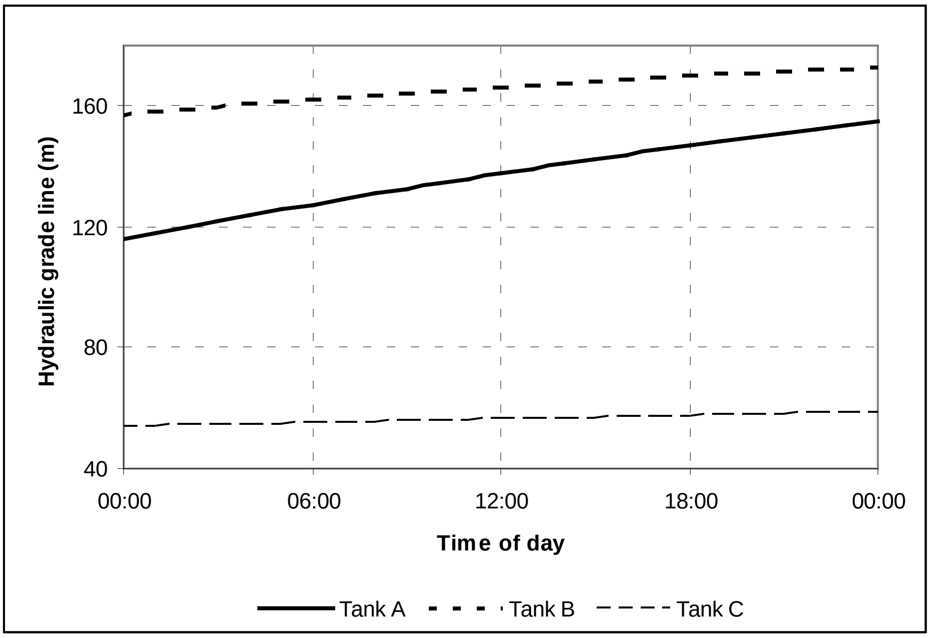

Figure 5 Changes in the tank hydraulic grades with time

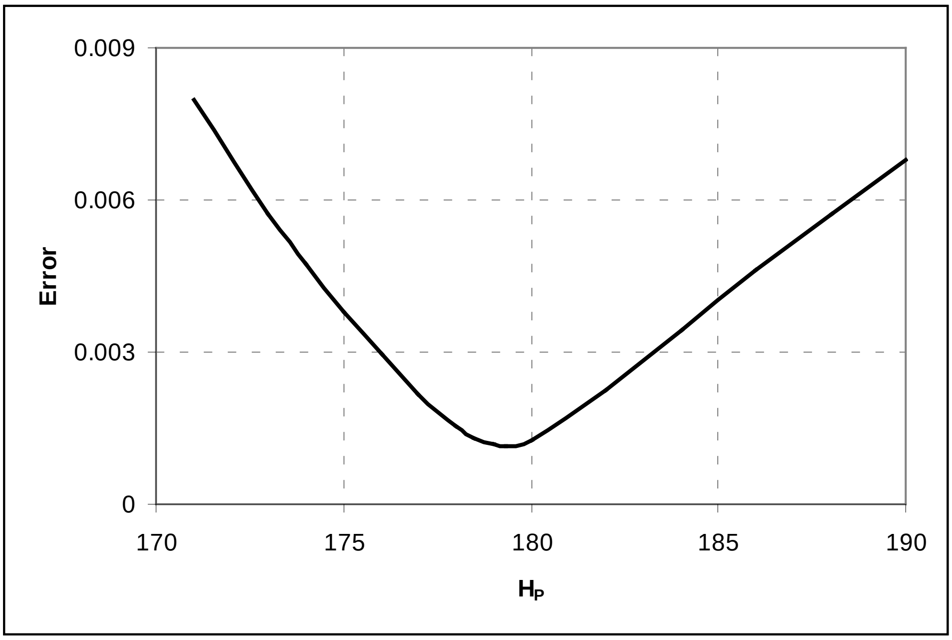

Figure 6 The sum-of-square error as a function of the pivot point (H_P)

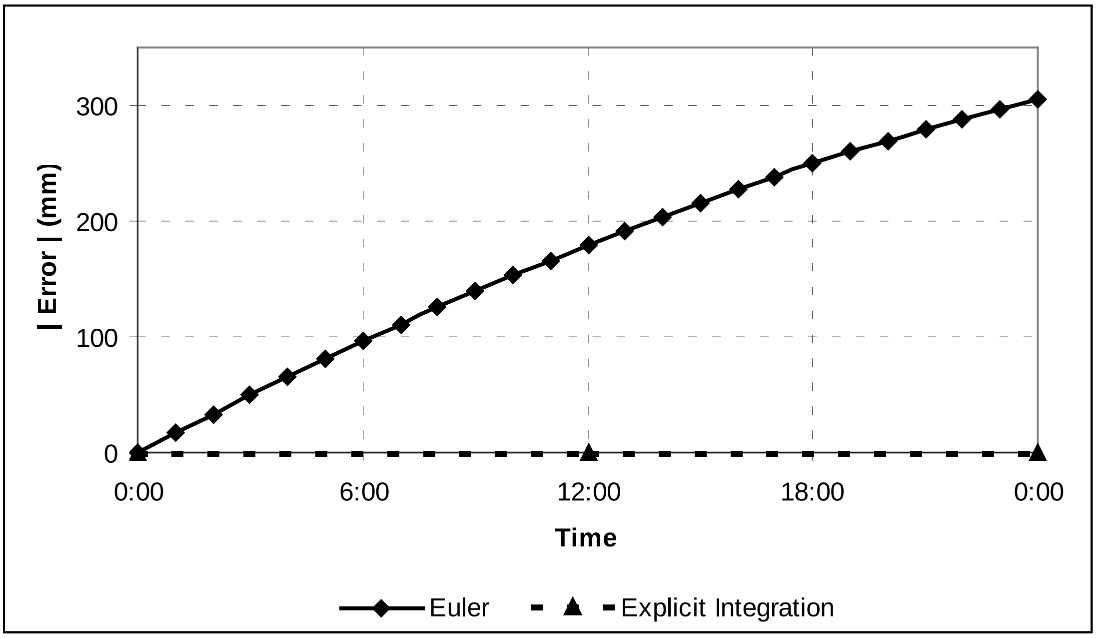

Figure 7 Simulation errors for tank A

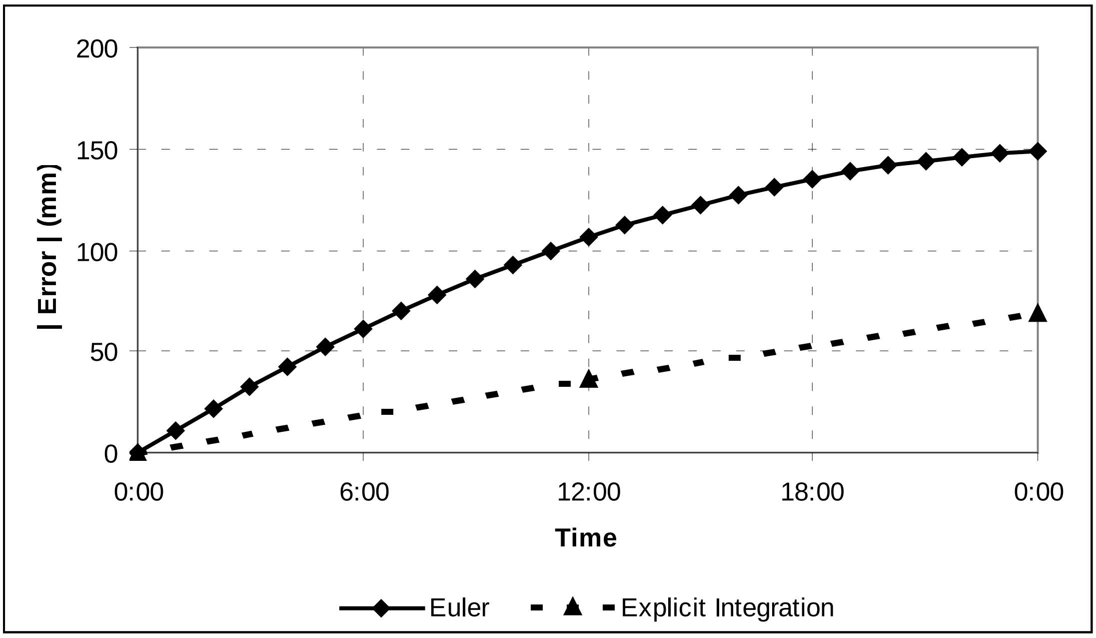

Figure 8 Simulation errors for tank B