Sizing Municipal Storage Tanks Based on Reliability Criteria

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ABSTRACT

Municipal storage tanks are used to balance differences in supply and demand. Tanks have traditionally been sized based on deterministic criteria for balancing, fire and emergency storage components. In this paper a stochastic analysis method is proposed to model both the deterministic and probabilistic components of consumer demand, fire demand and pipe failures in water distribution systems. The method estimates a number of tank reliability criteria as functions of tank capacity, which provide a site-specific way of determining the required tank capacity based on user-defined reliability criteria. The method is illustrated by applying it to a 'typical' water supply system. It was found that the tank failure duration follows a Weibull distribution. The tank failure rate was found to be very sensitive to tank capacity and can be described with an exponential distribution. It is proposed that one failure in ten years under seasonal peak conditions is used as a design criterion for tank sizing. In many cases this will result in substantially smaller tanks than is currently specified by design guidelines.

Keywords: Water distribution systems, Storage tanks, Reliability, Stochastic models, Design criteria

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INTRODUCTION

Bulk supply to a storage tank is generally best supplied at a fixed flow rate, which allows capital costs to be minimized and purification plants and pumps to operate at maximum efficiency. Water demand, on the other hand, is highly variable and is influenced by a large number of factors. The main function of a municipal storage tank is to balance differences between supply and demand in order to provide users with a reliable water supply in the most economic way.

A storage tank can be considered to have failed if it runs dry, and thus it is possible to describe the reliability (or lack thereof) of a tank through its failure behavior. Increasing tank capacity can reduce pumping costs and will invariably improve tank reliability, but this comes at an increased capital cost and longer water retention times, which can affect water quality negatively (Clark et al. 1996). It is thus necessary to find the optimal balance between these opposing objectives, both in determining the required capacity and operational policies of a storage tank.

Tanks have traditionally been designed and analyzed according to deterministic design guidelines, specifying tank components for balancing, fire and emergency storage. Typical design parameters for low-density residential areas in the USA (Boulos et al. 2004), France (Brière 2000) and South Africa (CSIR 2000) are provided in Table 1.

In practice water distribution systems do not display deterministic behavior, but are subjected to continuously varying conditions that have both deterministic and probabilistic components. A more realistic view of a network's performance can thus be obtained through a stochastic analysis that includes both deterministic and probabilistic parameters (Aly and Wanakule

2004; Homwongs et al. 1994; Kretzmann and Van Zyl 2004).

In this paper, a stochastic analysis method is presented for analyzing the reliability of municipal storage tanks. The proposed method incorporates generic unit models that describe both the deterministic and probabilistic components of consumer demand, fire demand and pipe failure events. The method is applied to a simple water supply system with 'typical' model parameters. The results of the analysis are characterized and compared to a number of tank design guidelines. It is illustrated how stochastic analysis can allow the designer to determine the required tank capacity based on reliability criteria rather than general and conservative design guidelines, and can thus lead to more appropriate tank sizing for site-specific conditions.

METHODOLOGY

Introduction

To model the behavior of a tank, it is necessary to understand the factors that can affect either the supply to, or demand from the tank. These factors can be classified into three groups:

- Scheduled events including rehabilitation, testing, cleaning and maintenance of network elements.
- Operational factors, including consumer demand, pipe failures and fires.
- Disasters, such as earthquakes, tornados and terrorist attacks.

Scheduled events are within the control of the municipality and can be performed at times when there is minimum risk to the system. It is thus reasonable to assume that the potential impact of scheduled events on tank reliability should be considered in the planning of these events, and not when sizing tanks. Disasters, on the other hand, are extreme events outside the control of a municipality. It is possible that areas under threat of disasters may want to increase tank size requirements, but such considerations have to be part of a site-specific disaster response plan. Based on the above, this study specifically excluded scheduled events and disasters and only considered operational factors.

Operational factors can be further classified as having long, medium or short term effects:

- Long term effects cause sustained growth in demand and occur over the scale of several years.
- Medium term effects occur within a year, but are significantly longer than the tank retention time, for instance seasonal variations in demand.
- Short term effects occur within a day or a week, and affect the immediate behavior of a storage tank.

How these categories are included in a stochastic model depends on the purpose of the model. The reliability analysis of a tank can only realistically be done for a particular time in the design horizon of the system. To model long term effects, different analyses are thus required at specific times in the design horizon. Medium term effects will cause the reliability of a tank to vary throughout the year and can be included in a stochastic model, but this will require very long simulation runs. A more practical approach is to model the behavior of a tank at the most critical time in a year to estimate its minimum reliability. Analyses can be performed at different times of the year if the seasonal variation in tank reliability is required.

Stochastic Analysis

A stochastic model generates outputs that are predictable only in a statistical sense. Repeated use of a given set of model inputs produces outputs that are not the same but follow certain statistical patterns (Lewis 1996). Stochastic modeling is frequently used in the analysis of complex systems where risk and uncertainty play important roles (Yang et al. 1996).

Stochastic analysis has been applied to water distribution systems in a number of previous studies (Damelin et al. 1972; Wagner et al. 1988; Yang et al. 1996), although these studies generally focused only on link failures. Work by Nel and Haarhoff (1996) and Haarhoff and Van Zyl (2002) also incorporated limited stochastic models for water demand and fire events.

The proposed stochastic analysis method is based on Monte Carlo simulation (Fishman 1995), which entails the repeated calculation of the system performance, each time with a different combination of input parameters. The stochastic analysis method was implemented using the public domain software Epanet (Rossman 2000) and Ooten (Van Zyl et al. 2003), an object oriented programmers toolkit for Epanet.

The method can be applied to any water distribution network defined in Epanet. An Epanet Reservoir is placed in the network where the tank reliability has to be analyzed. Epanet Reservoirs have fixed hydraulic heads although the user can vary the water level using a time pattern. The user specifies the tank sizes that need to be considered in the reliability analysis. These tanks are not hydraulically modeled, but the Reservoir in and outflows are used to adjust the volume of water in each of these ‘dummy’ tanks at each simulation time step. When a ‘dummy’ tank runs dry, the duration of the failure is determined and logged.

At the start of each modeling day, stochastic unit models for water demand, fire demand and pipe failures are used to calculate the projected behavior of network components for that day. The calculated water and fire demands are converted to standard Epanet Demand Patterns linked to the stochastic demand nodes. If pipe failures are generated in a day, the affected pipes are closed and opened according to their failure times using standard Epanet Controls. Once the parameters are set, Epanet is used to model the system for the day, typically using hourly time steps and intermediate time steps when required. At each simulation step, various performance criteria, including tank failures, are logged. This process is repeated for a user-defined number simulation days before the logged data is statistically analyzed and the results presented.

STOCHASTIC UNIT MODELS

Stochastic unit models were developed for consumer demands, fire demands and pipe failures. In each case generic components for the unit model were identified and used to build the model.

Consumer Demand

Municipal water demand is a highly variable process due to the large range of possible user types and numerous influencing factors. Influencing factors can be categorized as socio-economic (household size, income, stand size, social status, number of household appliances, social patterns, public and school holidays, tourism and water price), climatic (temperature, rainfall, humidity, time since last rainfall and the number of preceding hot days) and structural (number of users, water metering, plumbing fitting properties, pressure and network capacity). As a result it is impossible to predict water demand with certainty and any prediction model will invariably include significant errors.

Aly and Wanakule (2004) classified water demand models into three categories: end-use, econometric and time series models, although many modelers have found it useful to combine different models. Some notable examples of demand models include the Poisson rectangular pulse (PRP) method (Buchberger and Wu 1995), Auto Regressive Integrated Moving-Average (ARIMA) (Bougadis et al. 2005), state-space (Billings and Agthe 1999) multiple regression (Maidment et al. 1985), adaptive smoothing-filtering approach (Homwongs et al. 1994), pattern recognition (Shvartser et al. 1993) and artificial neural networks (Jain et al. 2001).

Based on a review of water demand literature, it was possible to identify four generic components for the water demand unit model: average demand, cyclic patterns, persistence and randomness. The average demand is the average consumption over the period modeled. This value may vary, for instance the annual average demands of most areas increase from year to year.

A number of cyclic patterns can be identified within a given year, including seasonal, day-of-the-week and diurnal patterns. Seasonal variations in demand are slow and can be adequately described with weekly or monthly demand factors.

Persistence describes how much the demand of a current period is affected by previous periods and may be observed on different temporal scales (Alvisi et al. 2003; Homwongs et al. 1994). Aly and Wanakule (2004) concluded that persistence is more pronounced in water demand than correlations with weather parameters. Persistence can be described through a series of autocorrelation coefficients that is defined by the number of elements in the autocorrelation series, and their values. In a water demand pattern, autocorrelation can be identified on a daily or hourly level and both levels were included in this study.

Random effects are common in water distribution systems due to the many influencing factors that cannot be predicted with certainty. After the deterministic factors have been identified and removed from data, it is possible to characterize the remaining white noise component using a statistical distribution function. In a good model the remaining white-noise components should have a zero mean and constant variance (Homwongs et al. 1994). Several examples of demand models using a normal distribution are present in the literature, for instance see Xu and Goulter (1998) and Aly and Wanakule (2004).

The demand model adopted in this study consisted of two distinct steps: the first to model daily and the second to model hourly water demand variations. In each step the cyclical patterns are modeled using a multiplicative model and the remainder as an auto-correlated random process. The model for daily demand is as follows:

$$D_d = D_{ave} \cdot C_{dow} \cdot \upsilon_d \tag{1}$$

Where D_d is the average demand in day d, D_{ave} is the average demand for the period studied, C_{dow} is the day-of-week demand factor and υ_d is the daily demand residual function, described by:

$$\ln \upsilon_d = \sum_{i=1}^{m} \phi_i \ln \upsilon_{d-i} + \ln \varepsilon_d ; \quad \ln \varepsilon_d \sim \mathrm{IN}\left(0, \sigma_D^2\right) \tag{2}$$

Were i is a lag counter, m the number of daily autocorrelation lags, ϕ_i the daily auto regression coefficient for lag i and $\ln\varepsilon_d$ a white-noise process. The notation $\ln \varepsilon_d \sim \mathrm{IN}\left(0, \sigma_D^2\right)$ indicates that the natural logarithms of the residuals are normally and independently distributed with mean 0 and variance σ_D^2.

The hourly demand variation is modeled with a similar model:

$$D_h = D_d \cdot C_h \cdot \upsilon_h \tag{3}$$

Where D_h is the average demand for hour h, D_d is the average demand for the current day, C_h is the hourly demand factor and υ_h is the hourly demand residual function, described with a similar equation as the daily demand residuals, i.e. (2).

Fire demand

The municipal water distribution system is generally the main source of water for fire fighting. There generic components of fire demand were identified: occurrence, duration and fire flow.

European countries often have substantially lower fire water requirements than the USA, mainly due to water quality concerns caused by large pipes and long retention times in the system (Van Zyl and Haarhoff 1997). In the USA fire water requirements are normally calculated for individual buildings using the ISO method (Walski et al. 2001). This method yields Needed Fire Flow (NFF) that can be used for design and evaluation of water distribution systems. The NFF fire flow requirements vary between 32 and 757 ℓ/s, and fire durations between 2 hours for small fires and 4 hours for large fires. In a study on fire water consumption, Van Zyl and Haarhoff (1997) analyzed water used to fight large fires (requiring more than 5000 ℓ of water) in Johannesburg, South Africa for a period spanning 12 consecutive years. The study found that both fire duration and fire demand follow log-normal distributions.

We assumed that the occurrence of fires events, like many other similar random events, can be modeled with a Poisson process. A Poisson process is characterized by a rate parameter λ such that the number of events in a time interval with length τ follows a Poisson distribution with parameter $\lambda\tau$. The Poisson process with counting process *N(t)* is described by:

$$P[N(t+\tau)-N(t))=k]=\frac{e^{-\lambda\tau}(\lambda\tau)^k}{k!};\quad k=0,1,... \tag{4}$$

Where *P* is the probability of occurrence, *t* is time, $N(t+\tau)-N(t)$ describes the number of events in time period τ, *k* is the number of occurrences, and *e* is the base of the natural logarithm. The expected number of events in time interval $(t, t+\tau]$ is given by:

$$E[N(t+\tau)-N(t)]=\lambda\tau \tag{5}$$

For practical reasons fire events were modeled by simulating the times between successive fires which, for a Poisson process, follow an exponential distribution. The probability density function of an exponential distribution is given by:

$$f(\Delta t;\lambda)=\lambda e^{-\lambda\Delta t};\quad \Delta t\geq 0 \tag{6}$$

Where Δt is the time between successive fire events. Fires at the same node that occur before the end of the current fire were ignored.

Once a fire has occurred, both the fire duration and fire flow were estimated using log-normal distributions. The probability density function for a log-normal distribution, written in this case for the fire duration, is given by:

$$f(T;\mu,\sigma) = \frac{1}{T\sigma\sqrt{2\pi}} e^{-(\ln T - \mu)^2 / 2\sigma^2}; \quad T \geq 0 \tag{7}$$

Where T is the duration of the fire event, μ the mean of the logs of the durations and σ the standard deviation of the logs of the durations.

Supply system

The reliability of a storage tank is affected by both the capacity and reliability of the system supplying it. Issues of economy and practicality often dictate that a tank is supplied at a constant flow rate over an extended part of the day. Assuming a known and stable demand pattern (i.e. no long term growth) and ignoring interruptions in the supply, it is possible to estimate extreme values for the combination of supply and tank capacities: if the supply capacity is equal to the maximum system demand there is no need for balancing storage. On the other hand, if the supply capacity is less than the average demand, no tank will be large enough to provide a sustained service. Neither of the two extremes provides a practical solution, and it is thus necessary to find an appropriate balance between supply and tank capacities.

The reliability of a supply system can be defined in terms of its ability to provide a constant and uninterrupted supply to the tank. Emergency interruptions to the supply system can result from failure of any link in the chain of supply, including the water source, purification plant, pumps, pipes or another storage tank. In the literature on water distribution systems, pipe failures are most commonly dealt with and were thus used as basis for modeling supply failures in this study. The effects of other elements in the supply chain can be incorporated by using dummy pipes to simulate their failure behaviors.

Many factors influence the pipe failure rate, including the pipe properties (diameter, pipe material, wall thickness, protection, handling damage, jointing, bedding, age, failure history), environmental (soil properties, heaving soils, ground water, water temperature, external loads, stray currents, land development), and service conditions (pressure, water hammer, water quality, maintenance practices). Clark, Stafford et al. (1982) found that a minority of the pipes were responsible for a majority of the failures, and that the time interval between successive failures in a pipe became increasingly shorter. Other studies have noted correlations of failure rate with the number of previous failures (LeGat and Eisenbeis 2000; Pelletier and Townsend 1996), period installed, pipe diameter (Ciottony 1983; Kettler and Goulter 1985; O'Day 1983; Sullivan 1982; Walski and Pelliccia 1982) and age (Kettler and Goulter 1985; Walski and Pelliccia 1982). The correlation with age was not observed in all studies and O'Day (1983) suggested that age is a poor or even misleading indication of failure rate. This seems to be particularly true when studying datasets with a notable proportion of old pipes and observations restricted to a relatively narrow observational window. This is due to an infant mortality characteristic in the data, i.e. the most robust among old pipes are the most likely to have survived until the observational window, and the data thus reflects a selective survival bias that hides the effect of age.

Three categories of models have been used to evaluate the structural state of water pipe networks (Pelletier and Townsend 1996): aggregate models, which are exponential or linear models of the number of failures as a function of the pipe age; regression models, in which various factors that affect pipe degradation are modeled; and probabilistic models, which uses statistical techniques such as survival analysis. Notable recent advances have been made in probabilistic modeling of pipe failures, including a mix of Weibull and exponential distributions (Eisenbeis 1994; Pelletier and Townsend 1996), and using a non-homogenous Poisson process with covariates (Røstum 2000).

For the purpose of this study, four generic components of pipe failures were identified: occurrence, duration, clustering and severity. As with fires, a Poisson process was used to model the occurrence of pipe failures. This is similar to the approach followed in various other pipe failure studies (Guercio and Xu 1997; Mays 2004; Shinstine et al. 2002). The Poisson model is described by (4) to (6). Modeled failures on the same pipe that occur before the end of the current failure were ignored.

No published data on shutdown durations for pipe repairs could be found. The duration of a supply system failure represents the time that the supply is interrupted and will normally correspond to the time that a pipe section is isolated by the repair team. Haarhoff and Van Zyl (2002) used a lognormal distribution and this approach was also adopted in this study.

Various authors (Clark et al. 1982; Goulter and Coals 1986; Goulter et al. 1993) have observed that pipe failures are not independent events, but display both spatial and temporal clustering. For the purpose of this study, it is not important where on the supply pipeline failures take place and spatial clustering could thus be ignored. Also, since the stochastic analysis is done for a particular point in time, the failure rate will include the effects of temporal clustering and no additional allowance for temporal clustering is required.

The severity of a failure event refers to the impact of this event on the supply flow rate. While closure of a single supply pipeline will completely stop the tank inflow, this is not the case if more than one parallel pipe is used.

APPLICATION

The stochastic analysis methodology is illustrated through application on a simple, but frequently used supply system configuration. While we recognize that each water supply system is unique and it is impossible to generalize, an attempt was made to define a 'typical' system by selecting a layout and modeling parameters that are commonly found in real systems.

System Description and Methodology

The example system (Fig. 1) consists of a source feeding a tank via a single pipeline. Users are connected to the tank via a separate pipe. It was assumed that the tank will be analyzed and sized for seasonal peak conditions, and thus for the minimum rather than the annual average tank reliability. For this reason the seasonal pattern was not included in the stochastic model, but the simulation was run for a day representing the seasonal peak in the network. A peak demand of 80 ℓ/s and a 20 % higher supply pipe capacity (96 ℓ/s) were assumed for the system. Using a seasonal peak factor of 1.5, this represents an average annual demand of 4.6 Ml/day, which is equivalent to a low density (suburban) residential area of 3 to 5 thousand stands.

The other parameters of the demand model were based on the measured demand of three small residential towns located in the area of Moselle in the east of France. The data consisted of hourly demands measured between September 1993 and December 1996. The data set has a number of gaps and, after removing all days with incomplete data, 65 % of the above period was covered. The average daily demands were collated and analyzed first. The demand displayed a typical seasonal demand pattern with minimum and maximum seasonal factors of 0.75 and 1.5 respectively. The seasonal pattern was removed from the data by dividing each day's demand by its average weekly demand factor.

The average day-of-week demand factors were then identified and the residuals analyzed to estimate the daily autocorrelation and white noise components. Day-of-the-week factors varied between 0.93 and 1.14 with weekend days displaying the highest demands. Daily autocorrelation were found to start with a positive coefficient at lag one, and then to be negative for several days. For the purpose of the base model it was decided to only use the lag-one autocorrelation coefficient with value 0.12. The natural logarithms of the daily white noise component displayed a normal distribution with a standard deviation of 0.068.

Diurnal demand variations were then analyzed by first identifying and extracting the average hourly patterns, and then determining the hourly autocorrelation and white noise components. No distinction was made between the behavior of week and weekend days as these displayed similar diurnal patterns. The hourly factors follow a classical residential demand pattern and vary between 0.38 and 1.49. Significant hourly autocorrelation coefficients go back as far as 50 hours (the limit of the analysis), but are dominated by the lag-one coefficient with a value of 0.70. Only this one autocorrelation coefficient was used in the analysis. The natural logarithms of the hourly white noise component displayed a normal distribution with a standard deviation 0.13.

For a low-density residential area, it was assumed that 95 % of the fire flows and durations will fall within the lowest NFF values. Based on the Johannesburg fire water study (Van Zyl and Haarhoff 1997), it was further assumed that both fire flow and fire duration will follow lognormal distributions with standard deviations of 1.31 and 0.66 respectively. From this the average of the natural logarithms of the fire flow and duration were calculated as 1.31 and -0.393 respectively. This corresponds to geometric means of 3.8 ℓ/s and 0.68 h respectively and indicates that most fires will be small, with a scattering of large fires in between. It was also assumed that the fire brigade will be required to extinguish a fire once every two months, i.e. at a rate of 6 fires/a.

What is important for the tank reliability is the number of failures that the supply pipe will experience. The pipe fail frequency is a function of the pipe length, which may vary from a few meters to tens of kilometers. For the purposes of this study, it was assumed that the supply pipe will experience an average of two failures per year. It was assumed that the pipe outage time follows a lognormal distribution with an expected duration of 5 hours and that 95 % of repairs will be done within 12 hours. The average and standard deviation of the logs could then be estimated as 1.49 and 0.48 respectively. No pipe failures were modeled on the demand pipe. This is considered conservative since failures on the demand side will reduce or discontinue the demand from the tank for the duration of the failure.

The number of days to simulate was determined by running the base model for different numbers of days, varying between 1 000 and 10 000 000 days, and observing at what duration the results stabilize. The repeatability of the results was also tested by running the simulation from ten different random seeds. It was found that the tank failure properties are consistently within 5 % of the ultimate values when the number of tank failures exceed 2 000. In this study all results were thus based on a minimum of 2 000 failure events.

Results and Discussion

The stochastic simulation calculates the failure characteristics for various user-specified tank sizes and can be expressed in terms of the average number of failures per year (Fig. 2) or the average tank failure duration (Fig. 3).

As shown in Fig. 2, it was found that the average annual number of failures can be described with by an exponential function of the tank size. The goodness of the fit allows the function to be extrapolated to estimate the behavior of large tanks that fail very infrequently. The results show that the average annual number of failures is very sensitive to the tank capacity. For instance, a tank with 13.3 hours storage will fail once a year on average. To reduce the failure rate to one in 10 and one in 100 years respectively, the tank capacity has to be increased to 17.9 and 22.6 hours. Thus, increasing the tank capacity by 35 % and 70 %, each increases the tank reliability by an order of magnitude.

The average failure duration (Fig. 3) is at a minimum of 3.1 h for a storage capacity of 12 hours. For smaller and larger tanks the average failure duration increases, but does not exceed 4.2 hours. On the whole the average failure duration does not vary much and is thus not a suitable parameter for determining the required tank capacity.

To get a better picture of the distribution of tank failure durations, 51 700 failures were generated for a 12 h capacity tank and statistically analyzed. A Weibull distribution was found to provide a good description of the failure duration distribution. The Weibull survival function of failure duration data T with position parameter β_0 and scale parameter σ is written as:

$$S(t;\beta_0,\sigma) = P\{T > t\} = \exp\left[-\exp\left(\frac{\ln t - \beta_0}{\sigma}\right)\right] \tag{8}$$

With t a failure duration. The distribution function is given by $1 - S(t, \beta_0, \sigma)$ and is shown in Fig. 4 data. The estimated values of β_0 and σ were found to be 1.2163 and 0.7525 respectively. While the average failure duration is 3.1 h, it can also be observed from Fig. 4 that 90 % of failures will be shorter than 6.5 hours and 98 % shorter than 9.5 h. If the maximum failure duration of a tank is a critical design parameter, a Weibull distribution can be used to determine the required tank size based on an allowed exceedance probability of a given maximum duration. This may be expressed in terms of a return period, similar to the approach followed in designing hydraulic structures for storm events.

The fact that the data follows a Weibull distribution can now be used to determine an expression for the minimum number of tank failures required to ensure reliable results. Such an expression will be faster and more accurate than the simulation based approach described earlier.

Consider an exceedance probability $\alpha \in [0,1]$, and the related t_α such that $S(t_\alpha; \beta_0; \sigma) = \alpha$. From (8):

$$t_\alpha = \exp\left(\sigma \ln(-\ln \alpha) + \beta_0\right) \tag{9}$$

By assuming that the estimates of the Weibull parameters, and thus also of t_α, are asymptotically normally distributed, it is possible to apply the Delta method to determine the variance of t_α *as:*

$$\mathrm{Var}(t_\alpha) = t_\alpha^2 \ \mathrm{Var}(\beta_0) + [\ln(-\ln \alpha) t_\alpha]^2 \ \mathrm{Var}(\sigma) + 2t_\alpha^2 \ln(-\ln \alpha) \mathrm{Cov}(\beta_0, \sigma) \tag{10}$$

With Var and Cov indicating variance and covariance respectively. If the variance and covariance were estimated from a random sample of size n, and n is sufficiently large that estimates are consistent, one can estimate the variance values that would have been obtained with a different sample size *n'*:

$$\mathrm{Var}(t_\alpha ; n') \approx \frac{n}{n'} \mathrm{Var}(t_\alpha ; n) \qquad (11)$$

This makes it possible to calculate a minimum sample size *n'* to ensure that the estimate of t_α is within the 95 % confidence interval ±ρ t_α as:

$$n' = \left(\frac{1.96\sqrt{nVar(t_\alpha ; n)}}{\rho t_\alpha} \right)^2 \qquad (12)$$

For example, it is possible to determine the minimum number of data points required to calculate the median tank failure duration (i.e. $\alpha = 0.5$) within 5 % of its true value (i.e. $\rho = 0.05$). From the data analysis, Var(β_0), Var(σ), Cov(β_0, σ) were determined as 1.2131E-5, 6.530E-6 and -2.773E-6 respectively. From (9) the median failure duration $t_{0.50}$ = 2.5613 h, from (10) Var($t_{0.50}$) = 9.867E-5 and from (12) the minimum number of data points *n'* = 1 194. This is lower than the 2 000 data points estimated using the earlier simulation approach, and thus the results satisfy this measure.

Comparison to International Standards

The tank design parameters in Table 1 were used to estimate the required tank capacities for the example system. The results are given in Table 2 and indicate that the USA, France and South Africa have sizing requirements that vary between 49 and 59 hours of annual average demand, or between 32 and 40 hours of seasonal peak demand for the example system.

Applying these storage requirements to the results of the stochastic analysis (Fig.2) indicate that under normal operational circumstances the average frequency of failures for the example system will be one in 45 000 years for the USA, one in 500 000 years for France, and one in 10 000 years for South Africa. Given that these values are based on the system performance at seasonal peak, the annual average return period for failures would be substantially more conservative.

The tank reliabilities above are substantially higher than what can be considered reasonable for a water supply system. The most likely reason is that design guidelines have to cater for all types of systems and thus are inherently conservative in their approach. While design guidelines might determine a realistic tank sizing for a system with a very long supply pipeline combined with high user and fire demand requirements, the example shows that this will not necessary be the case for a 'typical' system. The power of stochastic analysis lies in the fact that each system can be analyzed based on local conditions and requirements.

Designing for risk

Stochastic analysis gives the designer the ability to determine the required capacity for a specific tank based on an acceptable risk of failure. The question is now what an acceptable risk of failure is for municipal storage tanks?

The consequential damage associated with non-supply in a water distribution system for short periods is small compared to many other engineering fields. Loss of life is unlikely since little water is needed for sustaining life and critical industries such as hospitals and factories have the ability to maintain their own storages in preparation for a failure event.

Kwietniewski and Roman (1997) formulated reliability requirements for water supply systems. They analyzed data on people's feelings on sanitary threats, comfort and good living conditions to identify three reliability criteria: fail frequency, mean repair time and proportion of time for which water is available. Their recommendations on supply interruption frequency vary between 1 and 12 events per year for a maximum mean interruption time of 24 h. In a survey of 100 families they found that 74 % of consumers considered a 24 h interruption to supply once per year ‘tolerable’. Only 55 % of consumers found two 12 h failures ‘tolerable’, indicating that users are more sensitive to the number of failures than the failure duration.

Given the sensitivity of tank reliability to capacity and the uncertainties inherent in the input data, it seems risky to design the system for one or more failures per year. On the other hand the relatively small consequences of a tank failure do not warrant a failure return period of one in 100 years. In our opinion a failure rate of one failure in 10 years during the seasonal peak is a reasonable design value for municipal storage tanks. For the example system, the required tank capacity for a failure return period of 10 years is 18 hours of seasonal peak demand or 28 hours of annual average demand. This represents reductions of 49 %, 55 % and 44 % in the required tank capacities of the USA, France and South Africa for the example system respectively.

CONCLUSIONS

A stochastic method is proposed for analyzing the reliability of municipal storage tanks. The method is based on a Monte Carlo analysis and incorporates deterministic and probabilistic components of consumer demands, fire demands and pipe failures.

The proposed method is applied to a water supply system with a common layout and typical parameters values. It is shown that the average number of failures per year is an exponential function of the tank size. The number of failures is very sensitive to the tank size and an order of magnitude reduction in the number of failures can be obtained by increasing the tank capacity by roughly one third.

The results show that the average failure duration is not greatly affected by the tank size. An analysis of a large number of failures generated for specific tank capacity showed that the failure durations follow a Weibull distribution. This distribution can be used to determine the tank capacity if the maximum failure duration is a critical parameter. It is also shown how the Weibull parameters can be used to estimate the number of tank failures that need to be generated to ensure reliable results.

Finally, a criterion of one failure in ten years under seasonal peak conditions is proposed as an acceptable level of reliability for municipal storage tanks. This measure reduced the required tank capacity for the example system by roughly half compared to the USA, French and South African design guidelines. The power of the proposed method lies in its ability to analyze site-specific conditions to determine an appropriate tank capacity, rather than rely on design guidelines that have to cater for all types of systems and thus result in overly conservative tank sizes for most tanks.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

C_{dow} = day-of-week demand factor

C_h = hourly demand factor

D_{ave} = average demand for the period studied

D_d = average demand for day d

D_h = average demand for hour h

d = day counter

E = expected value

e = base of the natural logarithm

f = function

h = hour counter

i = lag counter

k = number of occurrences

m = number of daily autocorrelation lags

n = number of points

$N(t)$ = cumulative number of events at time t

P = probability of occurrence,

S = survival function,

T = duration of an event / failure duration data

t = time / duration of an event

t_α = duration related to α

α = exceedance probability

β_0 = position parameter of a Weibull distribution

Δt = time between successive events

ε_d = daily random demand component

λ = rate parameter of a Poisson process

μ = mean

σ = standard deviation / scale parameter of a Weibull distribution

σ_D = standard deviation of daily random demand component

τ = time interval,

υ_d = daily demand residual function

υ_h = hourly demand residual function

φ_i = daily auto regression coefficient for lag i

Table 1. Some tank design criteria for low-density residential areas

Country	Storage requirements		
	Balancing (B)	**Fire (F)**	**Emergency (E)**
USA	35 % of peak day	680 m^3	1 peak day
France	30 % peak day	120 m^3	2 average days
South Africa	Included in E	108 m^3	2 average days

Note: The requirements represent our interpretation of the guidelines for a typical low-density residential area. Where more than one option is given, the most stringent was usually selected. Where no fire duration was specified, a value of 2 hours was used.

Table 2. Typical tank sizing for low-density residential areas

Country	Storage elements (m^3)			Total storage required		
	Balancing	Fire	Emergency	(m^3)	(h ave demand)	(h seasonal peak demand)
USA	2 419	680	6 912	10 011	52	35
France	2 074	120	9 216	11 410	59	40
South Africa	0	108	9 216	9 324	49	32

Figure 1 Example network layout

Figure 2 Annual average number of tank failures as a function of the tank capacity

Figure 3 Average tank failure duration as a function of the tank capacity

Figure 4 Failure duration distribution for a tank with a capacity of 12 h of seasonal peak demand.

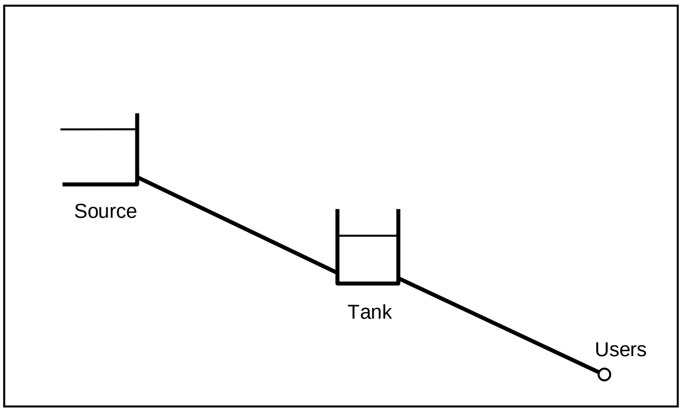

Figure 1 Example network layout

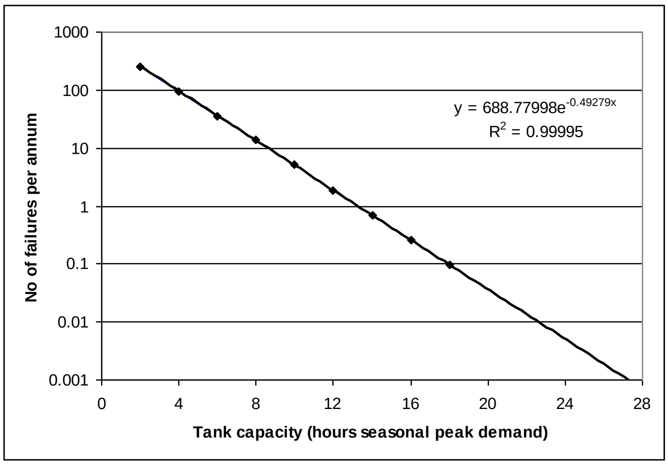

Figure 2 Annual average number of tank failures as a function of the tank capacity

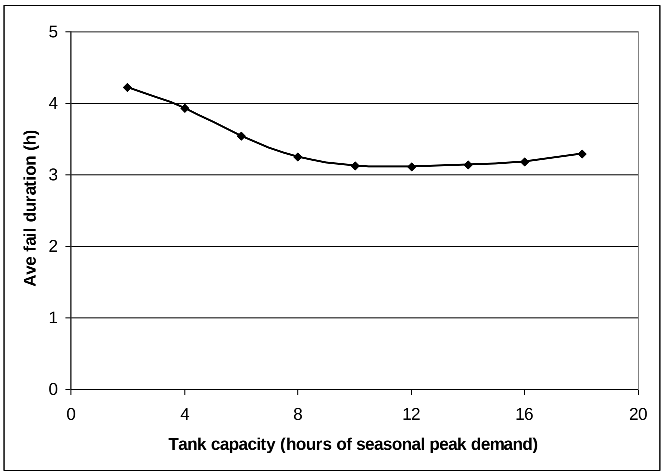

Figure 3 Average tank failure duration as a function of the tank capacity

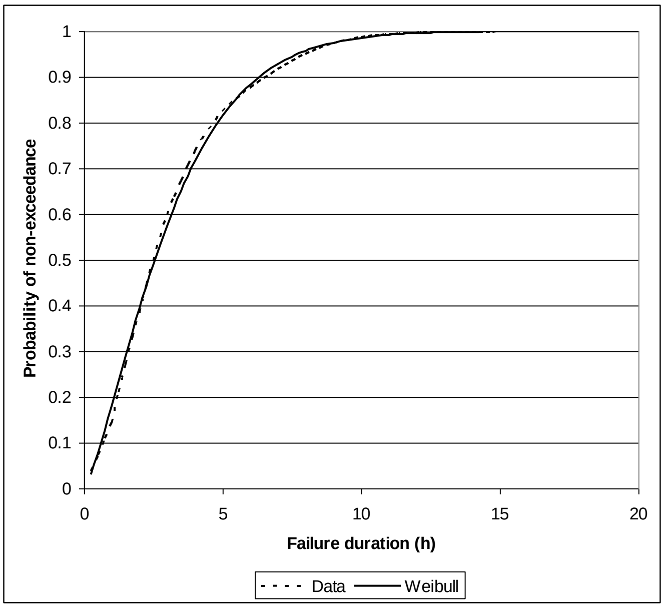

Figure 4 Failure duration distribution for a tank with a capacity of 12 h of seasonal peak demand.