BARELY BAIRE SPACES AND HYPERSPACES

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Abstract. We prove that if the Vietoris hyperspace $\mathcal{F}(X)$ of all non-empty closed subsets of a space X is Baire, then all finite powers of X must be Baire spaces. In particular, there exists a metrizable Baire space X whose Vietoris hyperspace $\mathcal{F}(X)$ is not Baire. This settles a problem of McCoy stated in [9].

1. Introduction

In this paper, all topological spaces are assumed to be infinite and at least Hausdorff. Also, all product spaces are endowed with the Tychonoff product topology. A space X is called *Baire* [8] if the intersection of any sequence of dense open subsets of X is dense in X. Alternatively, this notion can be formulated in terms of second category sets. The Baire category theory has numerous applications in Analysis and Topology. Among these applications are, for instance, the open mapping and closed graph theorems, the Banach-Steinhaus theorem in Functional Analysis, the Ellis' theorem on joint continuity of separately continuous group actions of locally compact groups on locally compact spaces [5], etc. Other applications of the Baire category theory are related to the existence of particular sets of functions, say the existence of a "big" subset of continuous functions on $[0, 1]$ that are nowhere differentiable, or the existence of a "big" subset of continuous functions on $[-\pi, \pi]$ whose Fourier series diverge on a dense subset of $[-\pi, \pi]$. On the other hand, still there are several interesting open questions about Baire spaces themselves, see [1].

Several function spaces can be identified as spaces of subsets (i.e., graphs of functions) endowed with a particular hyperspace topology. The Baire category theory of hyperspaces has proven to be also very usefull, see, for instance, [12] and [13]. In fact, what is usually happening here is to determine that the hyperspace of certain type is a Baire space. The present paper deals with this particular question in the case of Vietoris hyperspaces. In

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what follows, for simplicity, by a *hyperspace* of a space X, we will mean the family $\mathcal{F}(X)$ of all non-empty closed subsets of X endowed with the *Vietoris topology* τ_V. Let us recall that a canonical base for τ_V is given by all subsets of $\mathcal{F}(X)$ of the form

$$\langle \mathcal{V} \rangle = \left\{ F \in \mathcal{F}(X) : F \subset \bigcup \mathcal{V},\ F \cap V \neq \varnothing \text{ for any } V \in \mathcal{V} \right\},$$

where $\mathcal{V}$ runs over the finite families of non-empty open subsets of X. In the sequel, any subset $\mathcal{D} \subset \mathcal{F}(X)$ will carry the relative Vietoris topology τ_V as a subspace of $(\mathcal{F}(X), \tau_V)$.

The question whether $\mathcal{F}(X)$ of a Baire space X is still Baire is related to the notion of barely Baire spaces. A Baire space X is *barely Baire* [6] if there is a Baire space Y such that $X \times Y$ is not Baire. It has been an open problem whether such spaces do exist. The first Baire space X whose square X^2 is not Baire, constructed under the Continuum Hypothesis, is due to Oxtoby [11]. Then the example was improved to an absolute one by Cohen [4] relying on forcing. Finally, Fleissner and Kunen [6] constructed a metrizable Baire space X whose square X^2 is not Baire in *ZFC* by direct combinatorial arguments.

In [9], McCoy has studied hyperspaces of Baire spaces, and indicated that the following problem would be of interest, see [9, page 140].

Problem 1.1. Let X be a metrizable Baire space such that X^2 is not Baire. Must $\mathcal{F}(X)$ be Baire?

There are several results concerning hyperspaces of Baire spaces (see, for instance, [2, 3, 14]), but we were unable to find any reference about a possible progress in the solution of Problem 1.1. The main purpose of the present paper is to provide the negative solution of this problem by proving the following theorem.

Theorem 1.2. *Let X be a space. If $\mathcal{F}(X)$ is a Baire space, then all finite powers of X must be Baire spaces.*

Corollary 1.3. *There exists a metrizable Baire space X such that $\mathcal{F}(X)$ is not Baire.*

Other possible consequences are demonstrated in Section 4. The next section contains a preparation for the proof of Theorem 1.2. We would like to draw the reader's attention on Theorem 2.1, which may have some independent interest. Finally, the proof of Theorem 1.2 will be accomplished in Section 3.

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2. Finite powers and hyperspaces of finite sets

For a space X and $n < \omega$, we let $\mathcal{F}_n(X) = \{S \in \mathcal{F}(X) : |S| \leq n\}$. Note that $\mathcal{F}_n(X)$ is always a closed subset of $\mathcal{F}(X)$, and, in fact, $\mathcal{F}_1(X)$ is naturally homeomorphic to X. The latter fact means that the Vietoris topology is *admissible*, see [10]. It is clear that we may look at $\mathcal{F}_n(X)$ as an "unordered" version of X^n, and, in the light of this, our first step in the preparation for the proof of Theorem 1.2 is not surprising. Here, we prove the following natural result which may have some independent interest.

Theorem 2.1. *Let X be a space, and let $n \geq 1$. Then, $\mathcal{F}_n(X)$ is a Baire space if and only if X^n is a Baire space.*

The proof of Theorem 2.1 will rely on two other special sets as an interface between $\mathcal{F}_n(X)$ and X^n, namely,

$$[X]^n = \{S \in \mathcal{F}_n(X) : |S| = n\},$$

and

$$\mathcal{D}(X^n) = \{(x_1, \dots, x_n) \in X^n : |\{x_1, \dots, x_n\}| = n\},$$

where $n \geq 1$. Here, we identify X^1 with X, hence it makes sense to consider $\mathcal{D}(X^1)$, and clearly $\mathcal{D}(X^1) = X^1$. Also, $\mathcal{D}(X^n)$ will always carry the relative topology from X^n.

Now, we have the following two observations reducing the proof of Theorem 2.1 only to the sets $[X]^n$ and $\mathcal{D}(X^n)$.

Proposition 2.2. *Let X be a space, and $n \geq 1$. Then, $\mathcal{F}_n(X)$ is a Baire space if and only if $[X]^n$ is a Baire space.*

Proof. Note that $[X]^n$ is a τ_V-open subset of $\mathcal{F}_n(X)$, because X is Hausdorff. Hence, $[X]^n$ is a Baire space if so is $\mathcal{F}_n(X)$. Suppose now that $[X]^n$ is a Baire space, and take τ_V-open dense subsets $\mathcal{V}_k \subset \mathcal{F}_n(X)$, $k < \omega$. Next, set $\mathcal{G} = \bigcap\{\mathcal{V}_k : k < \omega\}$. Finally, take a finite family $\mathcal{W}$ of non-empty open subsets of X, with $\langle \mathcal{W} \rangle \cap \mathcal{F}_n(X) \neq \varnothing$, and let us show that $\langle \mathcal{W} \rangle \cap \mathcal{G} \neq \varnothing$. To this end, we distinguish the following two cases. If each $W \in \mathcal{W}$ consists only of isolated points of X, then $\langle \mathcal{W} \rangle$ consists only of isolated points of $\mathcal{F}(X)$, so $\langle \mathcal{W} \rangle \cap \mathcal{F}_n(X) \subset \mathcal{G}$. In the another case, some $W \in \mathcal{W}$ should contain a non-isolated point of X, so $\mathcal{H} = \langle \mathcal{W} \rangle \cap [X]^n \neq \varnothing$. However, each $\mathcal{U}_k = \mathcal{V}_k \cap [X]^n$, $k < \omega$, is open and dense in the Baire space $[X]^n$. Therefore, $\mathcal{D} = \bigcap\{\mathcal{U}_k : k < \omega\}$ is τ_V-dense in $[X]^n$. This finally implies that $\varnothing \neq \mathcal{H} \cap \mathcal{D} \subset \langle \mathcal{W} \rangle \cap \mathcal{G}$, which completes the proof. □

Proposition 2.3. *Let X be a space, and $n \geq 1$. Then, X^n is a Baire space if and only if $\mathcal{D}(X^n)$ is a Baire space.*

Proof. Just like before, $\mathcal{D}(X^n)$ is an open subsets of X^n, hence $\mathcal{D}(X^n)$ is a Baire space if X^n is Baire. To show the converse, we proceed by induction. Namely, $\mathcal{D}(X^1) = X^1$, so X^1 is Baire if $\mathcal{D}(X^1)$ is Baire. Then, suppose that both X^n and $\mathcal{D}(X^{n+1})$ are Baire spaces for some $n \geq 1$, and let us show that

X^{n+1} is also a Baire space. So, let $G = \bigcap\{V_k : k < \omega\}$ for some open dense subsets $V_k \subset X^{n+1}$, $k < \omega$, and let $W \subset X^{n+1}$ be a non-empty open subset. Also, let $\pi_j : X^{n+1} \to X$ be the projection onto the jth-factor of the product X^{n+1}, $1 \leq j \leq n+1$. We distinguish the following cases. If $W \cap \mathcal{D}(X^{n+1}) = \varnothing$, then there should be some $1 \leq m \leq n+1$ such that $\pi_m(W)$ contains an isolated point $x \in X$. Hence, there exists an open subset $H \subset W$, with $\pi_m(H) = \{x\}$. In this case, we let $Y = \{w \in X^{n+1} : \pi_m(w) = x\}$. Note that Y is an open subset of X^{n+1} because x is an isolated point of X, and it is also naturally homeomorphic to X^n. On the other hand, each $U_k = V_k \cap Y$, $k < \omega$, is open and dense in Y. Hence, by assumption, $D = \bigcap\{U_k : k < \omega\}$ is dense in Y. So, $\varnothing \neq H \cap D \subset W \cap G$. Consider finally the case when $T = W \cap \mathcal{D}(X^{n+1}) \neq \varnothing$. Observe that each $L_k = V_k \cap \mathcal{D}(X^{n+1})$, $k < \omega$, is open and dense in $\mathcal{D}(X^{n+1})$ because $\mathcal{D}(X^{n+1})$ is open in X^{n+1}. Hence, by assumption, $R = \bigcap\{L_k : k < \omega\}$ is dense in $\mathcal{D}(X^{n+1})$. Thus we get again that $\varnothing \neq T \cap R \subset W \cap G$, which completes the proof. □

To accomplish the proof of Theorem 2.1, we may now concentrate only on the sets $[X]^n$ and $\mathcal{D}(X^n)$. This final equivalence is based on the following probably known observation.

Proposition 2.4. *Let X and Y be spaces, and let $f : X \to Y$ be an open continuous surjection which is finite-to-one. Then, X is a Baire space if and only if Y is a Baire space.*

Proof. In fact, the one implication is trivial. Namely, if X is a Baire space, then Y is also a Baire space, because f is open and continuous. Suppose now that Y is a Baire space, and let $G = \bigcap\{V_k : k < \omega\}$ for some decreasing sequence of open dense subsets $V_k \subset X$, $k < \omega$. Also, let $W \subset X$ be a non-empty open set. Then, each $U_k = f(V_k \cap W)$, $k < \omega$, is open and dense in $H = f(W)$ because $g = f \upharpoonright W : W \to H$ is open and continuous. Hence, by assumption, $D = \bigcap\{U_k : k < \omega\}$ is dense in H because H is open in Y, and, in particular, there exists $y \in D$. Let $F_k = g^{-1}(y) \cap V_k$, $k < \omega$. Then, $\{F_k : k < \omega\}$ is a decreasing sequence of non-empty finite subsets because f is finite-to-one (hence, g as well). Therefore, $F = \bigcap\{F_k : k < \omega\} \neq \varnothing$ which implies that $W \cap G \neq \varnothing$. □

Proof of Theorem 2.1. Let X be a space, and let $n \geq 1$. According to Propositions 2.2 and 2.3, it suffices to show that $\mathcal{D}(X^n)$ is a Baire space if and only if $[X]^n$ is a Baire space. Towards this end, consider the map $f : \mathcal{D}(X^n) \to [X]^n$ defined by $f((x_1, \ldots, x_n)) = \{x_1, ..., x_n\}$, whenever $(x_1, \ldots, x_n) \in \mathcal{D}(X^n)$. Then, f is a continuous, open and finite-to-one surjection, hence Proposition 2.4 completes the proof. □

3. Proof of Theorem 1.2

In what follows, we need the following terminology. Let σ and γ be families of subsets of a space X. As usual, we say that σ is a *refinement* of γ if any element of σ is a subset of some element of γ. Now, we shall say that

σ is a *strong refinement* of γ if σ is a refinement of γ, and any non-empty element of γ contains some non-empty element of σ. In fact, we will need strong refinements to generate special Vietoris neighbourhoods. Let us draw the reader's attention that if σ and γ are finite families of non-empty open subsets, then $\langle\sigma\rangle \subset \langle\gamma\rangle$ if σ is a strong refinement of γ. Motivated by this, to any finite family γ of open subsets of X we will associate the set $\mathcal{SR}(\gamma)$ of all strong refinements σ of γ such that σ consists of open sets, and $|\sigma| = |\gamma|$. Note that if γ is disjoint, then any $\sigma \in \mathcal{SR}(\gamma)$ is also disjoint, while $\langle\gamma\rangle \neq \varnothing$ implies $\varnothing \neq \langle\sigma\rangle \subset \langle\gamma\rangle$ for every $\sigma \in \mathcal{SR}(\gamma)$.

The following simple observation will be our basic tool to work with strong refinements.

Proposition 3.1. *Let X be a space, $n \geq 1$, γ be a family of non-empty pairwise disjoint open subsets of X, with $|\gamma| = n$, and let $\mathcal{V} \subset \mathcal{F}(X)$ be a τ_V-open set which is dense in $[X]^n$. Then, there exists a $\sigma \in \mathcal{SR}(\gamma)$ such that $\langle\sigma\rangle \subset \mathcal{V}$.*

Proof. Follows from the fact that there exists a set $S \in [X]^n$ such that $S \in \langle\gamma\rangle \cap \mathcal{V}$. □

Lemma 3.2. *Let X be a space, $n \geq 1$, γ be a family of non-empty pairwise disjoint open subsets of X, with $|\gamma| \geq n$, and let $\mathcal{V} \subset \mathcal{F}(X)$ be a τ_V-open set which is dense in $[X]^n$. Then, there exists a $\sigma \in \mathcal{SR}(\gamma)$ such that $\langle\tau\rangle \subset \mathcal{V}$ for every $\tau \in [\sigma]^n$.*

Proof. We proceed by induction. Namely, suppose that $|\gamma| = n$. Then, by Proposition 3.1, there is a $\sigma \in \mathcal{SR}(\gamma)$ such that $\langle\sigma\rangle \subset \mathcal{V}$. Since $\tau = \sigma$ for every $\tau \in [\sigma]^n$, this σ is as required.

Suppose now that our statement is true for all families which consist of at most k non-empty pairwise disjoint open subsets of X, for some $k \geq n$, and take a family γ of non-empty pairwise disjoint open subsets of X, with $|\gamma| = k+1$. Next, take a $T_0 \in \gamma$, and consider the family $\gamma_0 = \gamma \setminus \{T_0\}$. Since $|\gamma_0| = k \geq n$, by our assumption, there exists a $\sigma_0 \in \mathcal{SR}(\gamma_0)$ such that $\langle\tau\rangle \subset \mathcal{V}$, $\tau \in [\sigma_0]^n$. Now we distinguish the following two cases. If $n = 1$, then, by Proposition 3.1, there is $\sigma_1 \in \mathcal{SR}(\{T_0\})$ such that $\langle\sigma_1\rangle \subset \mathcal{V}$. Hence, in this case, we may take $\sigma = \sigma_0 \cup \sigma_1$.

Suppose finally that $n > 1$, and let $[\sigma_0]^{n-1} = \{\tau_1, \dots, \tau_m\}$ where $\tau_i \neq \tau_j$ if $i \neq j$. By induction, for every $k \leq m$ we will construct families $\lambda_k \in \mathcal{SR}(\tau_k)$ and $\mu_k \in \mathcal{SR}(\sigma_0 \setminus \tau_k)$, and non-empty open subsets $T_k \subset T_0$ such that $\langle\{T_k\} \cup \lambda_k\rangle \subset \mathcal{V}$, $T_{k+1} \subset T_k$, and $\lambda_{k+1} \cup \mu_{k+1} \in \mathcal{SR}(\lambda_k \cup \mu_k)$. Namely, $|\{T_0\} \cup \tau_1| = n$, and, by Proposition 3.1, there exists a non-empty open subset $T_1 \subset T_0$ and $\lambda_1 \in \mathcal{SR}(\tau_1)$ such that $\langle\{T_1\} \cup \lambda_1\rangle \subset \mathcal{V}$, while we can take $\mu_1 = \sigma_0 \setminus \tau_1$. So, suppose that λ_k, μ_k, and T_k have been already constructed for some $k < m$. For convenience, let $\sigma_k = \lambda_k \cup \mu_k$, and

$$\tau^*_{k+1} = \{S \in \sigma_k : S \subset T \text{ for some } T \in \tau_{k+1}\}.$$

Note that $\tau^*_{k+1} \in \mathcal{SR}(\tau_{k+1})$ because $\sigma_k \in \mathcal{SR}(\sigma_0)$. Then, just like before we can construct the required $T_{k+1} \subset T_k$ and $\lambda_{k+1} \in \mathcal{SR}(\tau_{k+1})$, but now using the family $\{T_k\} \cup \tau^*_{k+1}$ instead of $\{T_0\} \cup \tau_1$. As for μ_{k+1}, we just take $\mu_{k+1} = \sigma_k \setminus \tau^*_{k+1}$. This completes the induction.

We finally complete the proof by taking $\sigma = \{T_m\} \cup \lambda_m \cup \mu_m$. Obviously, $\sigma \in \mathcal{SR}(\{T_0\} \cup \sigma_0) \subset \mathcal{SR}(\gamma)$. Take a $\tau \in [\sigma]^n$, with $T_m \in \tau$. Then, by construction, $\tau \in \mathcal{SR}(\{T_\ell\} \cup \lambda_\ell)$ for some $\ell \leq m$ because $T_m \subset T_k$ and $\lambda_m \cup \mu_m \in \mathcal{SR}(\lambda_k \cup \mu_k)$, for every $k \leq m$. So, $\langle \tau \rangle \subset \langle \{T_\ell\} \cup \lambda_\ell \rangle \subset \mathcal{V}$. $\square$

We now proceed to the proof of Theorem 1.2. So, let X be as in that theorem, and let $n \geq 1$. According to Theorem 2.1, it suffices to show that $\mathcal{F}_n(X)$ is a Baire space. Take a countable family $\{\mathcal{V}_k : k < \omega\}$ of τ_V-open subsets of $\mathcal{F}(X)$ which are dense in $\mathcal{F}_n(X)$, and let us show that $\mathcal{G} = \bigcap\{\mathcal{V}_k : k < \omega\}$ is dense in $\mathcal{F}_n(X)$. To this end, let λ be a finite family of non-empty open subsets such that $\langle \lambda \rangle \cap \mathcal{F}_n(X) \neq \varnothing$. In case $\bigcup \lambda$ is finite, then $\langle \lambda \rangle$ consists only of isolated points of $\mathcal{F}(X)$, hence there exists an $S \in \langle \lambda \rangle \cap \mathcal{F}_n(X)$ which is an isolated point of $\mathcal{F}(X)$. Clearly, in this case $S \in \mathcal{G}$. If $\bigcup \lambda$ is infinite, then $\langle \lambda \rangle \cap [X]^n \neq \varnothing$ because $\langle \lambda \rangle \cap \mathcal{F}_n(X) \neq \varnothing$. Hence, there should exist a finite family μ consisting of non-empty pairwise disjoint open sets such that $|\mu| = n$ and $\langle \mu \rangle \subset \langle \lambda \rangle$. In this case, for every $k < \omega$, we consider the collection Σ_k of all finite families σ consisting of non-empty pairwise disjoint open subsets of X such that

$$\sigma \text{ is a strong refiniment of } \mu, \text{ and } \langle \tau \rangle \subset \mathcal{V}_k \text{ for every } \tau \in [\sigma]^n. \tag{3.1}$$

Next, we consider the τ_V-open sets

$$\mathcal{U}_k = \bigcup\{\langle \sigma \rangle : \sigma \in \Sigma_k\},\ k < \omega,$$

and we are going to show that they are dense in $\langle \mu \rangle$. Take $k < \omega$ and a finite family ν of open subsets of X such that $\langle \nu \rangle \cap \langle \mu \rangle \neq \varnothing$. Since ν is finite and $|\mu| = n$, there now exists a finite family γ consisting of non-empty pairwise disjoint open subsets of X such that $|\gamma| \geq n$, and $\langle \gamma \rangle \subset \langle \nu \rangle \cap \langle \mu \rangle$. Since $\mathcal{V}_k$ is dense in $[X]^n$, by Lemma 3.2, this now implies the existence of a $\sigma \in \mathcal{SR}(\gamma)$ such that $\langle \tau \rangle \subset \mathcal{V}_k$ for every $\tau \in [\sigma]^n$. Thus, $\sigma \in \Sigma_k$, and therefore $\varnothing \neq \langle \sigma \rangle \subset \langle \gamma \rangle \cap \mathcal{U}_k$, so $\mathcal{U}_k$ is dense in $\langle \mu \rangle$. As a result, we get that $\mathcal{D} = \bigcap\{\mathcal{U}_k : k < \omega\}$ is a τ_V-dense subset of $\langle \mu \rangle$ because $\langle \mu \rangle$ is itself a Baire space being a τ_V-open subset of $\mathcal{F}(X)$. Therefore there exists an $F \in \langle \mu \rangle \cap \mathcal{D}$. Next, for every $W \in \mu$ pick a fixed point $x_W \in F \cap W$, and then set $T = \{x_W : W \in \mu\}$. Note that $|T| = |\mu| = n$, and, in particular, $T \in \mathcal{F}_n(X)$. Now, on one hand, for every $k < \omega$, we can find a $\sigma_k \in \Sigma_k$, with $F \in \langle \sigma_k \rangle$. On the other hand, we can define a special subfamily of σ_k by letting $\tau_k = \{S \in \sigma_k : S \cap T \neq \varnothing\}$. Then $|\tau_k| = |T| = n$ because σ_k is a pairwise disjoint strong refinement of μ, while $|T \cap W| = 1$ for every $W \in \mu$. Hence, according to (3.1), this implies that $T \in \langle \tau_k \rangle \subset \mathcal{V}_k$ for every $k < \omega$, so $T \in \mathcal{G}$. That is, $T \in \langle \mu \rangle \cap \mathcal{G} \subset \langle \lambda \rangle \cap \mathcal{G}$, which completes the proof. $\square$

4. Volterra spaces

A space X is *Volterra* if the intersection of any two dense G_δ-subsets of X is dense in X. Clearly, any Baire space is Volterra. In fact, a space X which contains a dense metrizable subspace is Baire if and only if it is Volterra [7].

According to Theorem 1.2, we have the following consequence.

Corollary 4.1. *If X is a metrizable space such that $\mathcal{F}(X)$ is Volterra, then all finite powers of X must be Baire.*

Proof. Note that $\mathcal{F}(X)$ contains a dense metrizable subspace. For instance, $\bigcup\{\mathcal{F}_n(X) : n = 1, 2, \dots\}$ is a metrizable subspace of $\mathcal{F}(X)$. Hence, this follows immediately by the mention result in [7] and Theorem 1.2. $\square$

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