Origin of oscillatory convection in a porous medium heated from below

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In order to investigate the significance of cyclic interaction in the evolution of fluctuating convective flow in a rectangular cell of porous material, the behavior of confined and unconfined flows are compared. In the latter case the constant pressure boundary condition at the top surface does not force the fluid to recirculate around the cell. However, regular oscillatory flow similar to that which occurs in an enclosed cell is still observed. This indicates that the oscillatory flow arises from the instability of the thermal boundary layer on the heated bottom surface. In both cases the frequency of the oscillatory flow is proportional to the Rayleigh number to the power of 3/2.

I. INTRODUCTION

In a previous paper,[1] we reported experimental and numerical observations of an unsteady convective flow regime in an enclosed square region of saturated porous material heated from below. The experiments were performed using a Hele–Shaw apparatus and the numerical methods used a direct solution method for solution of the stream function equation and an explicit energy semi-conservative method for the solution of the time dependent heat transport equation. Similar flows had previously been observed in the experiments of Caltagirone *et al.*[2] and have also been reported by Caltagirone[3,4] who used numerical techniques. Thus, the existence of these unsteady flows is well established and these various studies have uncovered some intriguing properties of them; however, the precise mechanism of these effects remains unclear. Drawing a parallel with convective flow in a homogeneous fluid layer, in which fluctuating flows of various kinds are also observed, Moore and Weiss[5] suggest that disturbances are formed by a cyclic "triggering" by their predecessors that have circulated around the convection cell. A similar concept was used by Welander[6] and Keller[7] to predict periodic convective effects in fluid loops in which thermal disturbances circulate around the loop in time to receive a "boost" on re-arrival over the heated portion. These instabilities were later observed in the experiments of Creveling *et al.*[8] On the other hand, Sparrow *et al.*[9] observed the evolution of thermal disturbances in experiments without major circulating motion of the fluid, and demonstrated that such disturbances were a result of instability of the thermal boundary layer. Foster[10] used a numerical method of solution for the circulating problem and also indicated the thermal boundary layer as the origin of the effects. However, Krishnamurti[11] observed an oscillatory convective behavior in fluid layer experiments, for which the Rayleigh and Prandtl numbers are very much less than that necessary for thermal instability of the boundary layer. Thus, it seems that in the homogeneous fluid layer problem either or both of the candidate mechanisms, cyclic triggering or thermal boundary layer instability, may give rise to oscillatory behavior.

To examine the origins of oscillatory flow further, we investigate the convective flow through a porous region in which the fluid is unconfined and can therefore flow into and out of the cell through the upper boundary, thus diminishing triggering of thermal disturbances by their circulating predecessors. Examination of the Rayleigh number dependence of the evolution time of the disturbances indicates that their regular occurrence results from a combination of triggering and thermal boundary layer instability.

II. DESCRIPTION OF THE PROBLEM

Assuming the validity of Darcy's law, the Boussinesq approximation, and that inertial effects are negligible, the governing equations for convective flow through a porous medium are

$$\nabla^2 \psi = -\frac{\partial \theta}{\partial x} , \qquad (1)$$

and

$$\frac{\partial \theta}{\partial t} = \nabla^2 \theta - R\left(\frac{\partial \psi}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi}{\partial x}\frac{\partial \theta}{\partial y}\right) , \qquad (2)$$

where x, y, and t are the nondimensional space and time coordinates (y vertically upward) and θ and ψ are the nondimensional temperature and stream function, respectively. The Rayleigh number R is defined as

$$R = (gak\,\Delta T\,\alpha/\kappa\nu)\,\lambda \ , \qquad (3)$$

where a is the depth of the porous layer, k is its permeability, κ is the thermal diffusivity of the fluid saturated medium, ν is the kinematic viscosity, α is the coefficient of thermal expansion of the fluid, ΔT is the temperature differential across the region, λ is the ratio of the volumetric heat capacity of the fluid to that of the saturated formation, and g is the acceleration due to gravity.

Previous studies of the case of a porous layer confined above and below show that the fluid remains stationary below a certain critical value of the Rayleigh number R_1, with heat transfer by conduction only. Above this value ($4\Pi^2$ for an infinitely wide layer)[12] the flow moves in steady convection cells[13] until the Rayleigh number reaches a second critical value R_2 at which unsteady disturbances first appear. This second value lies in the

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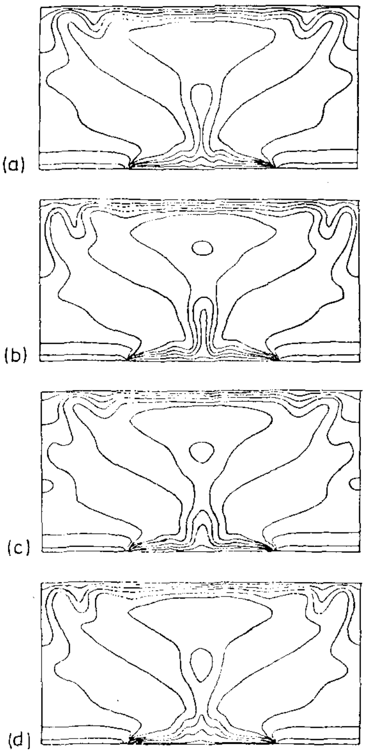

FIG. 1. A sequence of isotherm plots showing the oscillatory behavior of the flow at a Rayleigh number of 750. The region is heated along half of its base.

range 240–390 according to the various numerical, experimental, and analytical results.[1,2,4,14]

In Ref. 1 we discovered that the fluctuations in the flow became regular in the case of a square region heated along only half of its base, at Rayleigh numbers above 480. In such a region the flow exhibits ascending thermal disturbances over the heated portion of its lower boundary and descending disturbances at roughly half the frequency over the unheated portion (see Fig. 1). This behavior is strongly indicative of cyclic interaction or "triggering." In the present work the unconfined flow is investigated to determine the effect of preventing recirculation. Without recirculation of the convecting flow, the fluid will pass the heated portion of the boundary only once before leaving the system, thus allowing no opportunity for cyclic interaction.

To prevent recirculation of the fluid, the upper boundary of the region is taken to be an unconfined surface across which fluid may flow freely in or out. This situation occurs when the upper boundary is held at constant pressure (e.g., atmospheric) with sufficient recharge or discharge fluid supplied and removed above the surface. Such configurations exist naturally in single-pass hot springs and uncapped geothermal reservoirs. From the conservation of momentum and Darcy's law it follows that

$$U = -\partial p/\partial x \ ,$$

and the definition of the stream function gives

$$U = \frac{R}{\lambda}\frac{\partial\psi}{\partial y} \ ,$$

and

$$V = -\frac{R}{\lambda}\frac{\partial\psi}{\partial x} \ .$$

Thus, it is a consequence of the constant pressure condition that the derivative of the stream function normal to the boundary will be zero. Here, (U,V) is the mean flow velocity vector and p is the pressure.

With the three remaining boundaries impermeable, the boundary conditions for this region when heated nonuniformly from below may be obtained by modifying those given in Ref. 1; thus,

$$\begin{aligned}
&\psi = 0 \ , \quad x = 0,1, \quad y = 0 \ , \\
&\partial\psi/\partial y = 0 \ , \quad y = 1 \ , \\
&\theta = 1 \ , \quad x < \tfrac{1}{2}, \quad y = 0 \\
&\theta = 0 \ , \quad x > \tfrac{1}{2}, \quad y = 0 \ , \\
&\theta = 0 \ , \quad y = 1 \\
&\partial\theta/\partial x = 0 \ , \ x = 0,1 \ .
\end{aligned} \tag{4}$$

Retaining the thermal boundary condition $\theta = 0$ on the recharge/discharge surface deserves some comment. In a physical system, if such a boundary were cooled by convective heat transfer to the atmosphere, then the temperature at the surface would be described by

$$-k_c(\partial T/\partial Y) = h(T - T_\infty) \text{ at } Y = a \ , \tag{5}$$

where k_c is the thermal conductivity of the fluid filled medium, h is the convective heat transfer coefficient of the atmospheric interface, T and Y are the dimensional temperature and height, respectively, and T_∞ is the ambient atmosphere temperature. Now for a water saturated medium, k_c is typically 1.2 J/m/sec/°C, and for a horizontal interface in air h is of order 8×10^6 J/m^2/sec/°C. In a convective geothermal region, the temperature gradient is of order 0.5 °C/m close to the surface Thus, to preserve the heat flux balance given by Eq. (5), the temperature difference between T and T_∞ must be extremely small. For example, in the geothermal system described here, the difference in temperature between the surface and the atmosphere would be only 10^{-4}–10^{-5} °C. Therefore, it is satisfactory to employ the much simpler boundary condition $T = T_\infty$, and nondimensionally $\theta = 0$, as if the atmosphere were an infinite heat sink. Thus, the configuration selected is not only useful for examining the properties of the flow, it also represents a problem of practical interest.

In passing, it should be noted that an experimental investigation of flow regions with a constant pressure surface has already been performed by Bories and Thirrot[15]; however, they report a steady three-dimensional flow consisting of hexagonal cells where theory suggests two-dimensional cells.[14] Homsy[16] has pointed out that under the conditions of the experiment the thin layer of fluid above the medium used to visualize the flow is itself unstable to convective flow in hexagonal patterns driven by surface tension.

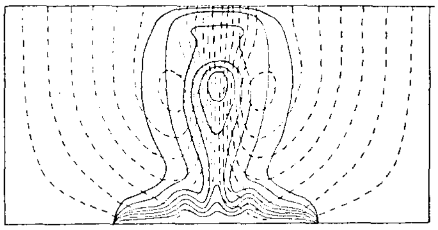

FIG. 2. Oscillatory flow at a Rayleigh number of 500 without major circulation of the flow. Solid lines are isotherms, broken lines streamlines.

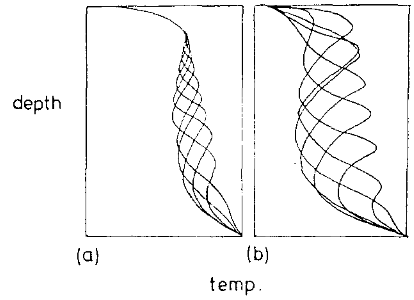

FIG. 4. A sequence of temperature profiles on a vertical section of the rising plume, at a Rayleigh number of 750. (a) For circulating flow, (b) noncirculating flow.

III. NUMERICAL SOLUTION

The numerical procedure used here is essentially the same as that of Ref. 1, namely, the noniterative odd-even reduction technique[17] for the solution of the stream function equation (1), and Arakawa differencing[18] for the representation of the advection terms in the heat transport equation (2). However, previously the fourth-order accurate, thirteen-point Arakawa template could only be used until the penultimate horizontal rows of the finite difference mesh where such a template required reference to points lying outside the mesh. For this reason, the smaller nine-point template, which is only accurate to second order, was used on the second and penultimate rows. However, Combarnous and Bories[19] have since pointed out that on horizontal, impermeable, isothermal boundaries the governing equations provide an additional set of conditions, and these may be used to alleviate the degradation in accuracy encountered here.

When the horizontal boundary is isothermal and impermeable, the temperature and stream function are both constant in x and t. Then, substitution in the governing equations provides the conditions

$$\partial^2\theta/\partial y^2 = 0 \ ,$$

and

$$\partial^2\psi/\partial y^2 = 0 \ . \qquad (6)$$

Since in this case the upper surface is not impermeable, these additional conditions may not be used on this boundary; however, at the lower boundary, which is of greater interest with regard to the evolution of thermal disturbances, the accuracy of the numerical representation may be increased to fourth order.

Calculations were performed for a range of Rayleigh numbers between 250 and 1000, using a mesh size of 33×33. Computations typically required about 0.25 sec per time step on an IBM 370/168 machine.

IV. RESULTS

Flows under the nonrecirculatory conditions exhibit the same regular evolution and flight of thermal disturbances as was seen in the enclosed flow of Ref. 1 (see Fig. 2 of a disturbance during flight). However, there are certain illuminating differences. With flow always normal to the upper boundary there is no natural convective transfer from it, and the descending flow consists entirely of cold fluid entering from the surface. Thus, there are no descending temperature disturbances. Second, the ascending disturbances evolve at similar frequencies but have greater amplitude and speed of flight than their enclosed system counterparts, compare Figs. 3(a) and 3(b). Third, the Rayleigh number at which the oscillatory flows first appear is somewhat lowered.

The smaller, secondary disturbance evident in the enclosed case [indicated by an arrow in Fig. 3(a)] becomes large enough in the recharge case so that it may almost be considered a disturbance within itself; however, closer examination of Fig. 3(b) reveals that the true cycle only repeats after the passage of the two temperature rises. Figure 2 may be compared to Fig. 5 of Ref. 1, which shows a disturbance during flight in the enclosed region at a Rayleigh number of 750. Even though the flow shown in Fig. 2 is at the lower Rayleigh number of 500, the thermal disturbance is much stronger (evident by the number of closed isothermal loops) and faster moving (as indicated by its elongation). Superimposing a set of temperature profiles taken through the rising plume at a sequence of times, Figs. 4(a) and (b) demonstrate similar differences between the flows.

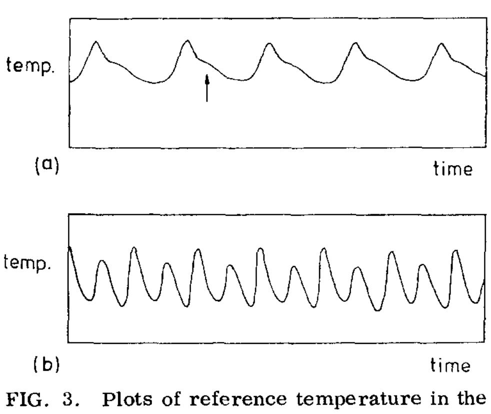

FIG. 3. Plots of reference temperature in the center of the rising plume of hot fluid at a Rayleigh number of 750. (a) For circulating flow, (b) noncirculating flow.

The results are summarized in Table I, with comparative values from the results of Ref. 1. Since the time periods given in Ref. 1 were determined for the descending disturbances rather than the ascending ones (which occur at roughly twice the frequency), the periods were re-evaluated for the ascending disturbance so that they

TABLE I. Summary of nonrecirculatory flows and comparative values from Ref. 1 (half-heated boundary).

	Nonrecirculatory			Recirculatory		
R	Nu_{min}	Nu_{max}	τ_p	Nu_{min}	Nu_{max}	τ_p
250	2.68	2.68	...	3.45	3.45	...
312	2.76	2.76	...	3.83	3.83	...
375	2.42	3.18	0.0124	4.20	4.20	...
500	2.19	3.88	0.0079	4.40	4.42	0.0091
750	2.34	4.78	0.0045	5.10	5.26	0.0053
1000	2.72	4.82	0.0034	7.81	8.29	0.0031

might be compared to the recharge case. For this reason the values given in Table I are different from those already quoted—Fig. 3(a) differs from Fig. 4 of Ref. 1 in the same regard.

V. THE THERMAL BOUNDARY LAYER

Since it now seems apparent that these oscillatory disturbances have their origin within the thermal boundary layer and are less likely to result from any triggering or resonance effects, it is advantageous to examine the growth characteristics of this layer. Suppose a horizontal plate $x > 0$ is heated impulsively to a temperature $\theta = 1$ in a cold ($\theta = 0$) half-space of saturated porous material in which the saturating fluid is moving horizontally across the plate with speed U. Then, the temperature of the quarter space above the plate is given approximately by

$$\theta = \mathrm{erfc}(y^2 U/4x)^{1/2}, \quad \text{for } x < Ut,$$

$$\theta = \mathrm{erfc}(y^2/4t)^{1/2}, \quad \text{for } x > Ut.$$

If we use Howard's[20] concept of a thermal boundary layer in which the temperature varies approximately linearly from that of the heated surface to some small percentage of that value, then, very approximately, the maximum "thickness" of such a layer would be proportional to $\sqrt{t}$. The maximum effective Rayleigh number across this layer would then also vary linearly with $\sqrt{t}$. Thus, the gestation period of the layer until the effective Rayleigh number reaches its lowest critical value, at which time the layer will begin to convect within itself, would be proportional to the overall Rayleigh number to the power of minus 2,

$$\tau_p \propto R^{-2}.$$

Clearly, this argument cannot be applied directly to cellular flow; however, a qualitative insight into the rate of evolution of thermal disturbances at various Rayleigh numbers may be obtained. In the cell situation the boundary layer is never completely dispelled by the flight of a thermal, but on the other hand it never reaches its maximum thickness for values of x within the cell with the velocities and times considered here. Wooding[21] in his Hele-Shaw experiments reported that the spacing of the thermals he observed (without major circulation in the flow) increased with the square root of time. Since the cell width for the first mode of convection is equal to its height, it follows that the depth of the layer in which the cells were forming was also dependent in some way on the square root of the time interval. This indicates the validity of this discussion although some departure from the $\tau_p \propto R^{-2}$ behavior should be expected to result from the circulating flow interrupting the gestation of the layer.

Figure 5 shows the variation of the oscillation period with Rayleigh number; for the results of Table I the period depends on Rayleigh number to the power of $(-3/2)$. Included in Fig. 5 are the oscillation periods for the half- and quarter-heated enclosed region and the minimum fluctuation periods for the uniformly heated enclosed regions of Ref. 1. In every case the Rayleigh number dependence is to the power $-3/2$ over the range of Rayleigh numbers considered.

To evaluate the importance of either triggering or thermal boundary layer instability, we also evaluated the effective Rayleigh number across the boundary layer, and the velocity variation upstream of the heater. In every case the effective Rayleigh number across the layer is greater than the critical value, the thermal boundary is clearly unstable. On the other hand, even though disturbances in the descending flow have been reduced almost to zero, there is a small but distinct variation in the velocity of the flow upstream of the heater, at exactly the frequency of the evolving disturbances in the thermal boundary layer.

VI. CONCLUSIONS

Having forced a loss of identity on a circulating thermal disturbance, the results indicate that the intermittency observed is not a resonant effect in which the disturbance receives a "boost" each time it passes the heater. However, it does appear that even though the thermal disturbances arise from instability of the thermal boundary layer over the heater, in fact, the regular flight of these disturbances is triggered by very small perturbations in the velocity field upstream, caused indirectly by previous disturbances.

The generation of the disturbances can be attributed to Howard's mechanism of thermal boundary layer instability for the following reasons:

(1) The effective Rayleigh number across the layer is unquestionably supercritical. It should be noted that

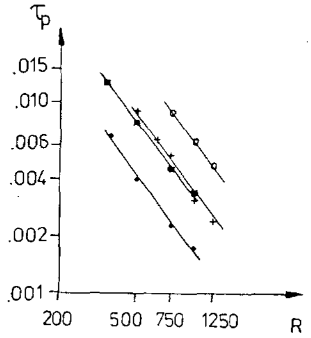

FIG. 5. Fluctuation period vs Rayleigh number. ● —uniformly heated regions, +—half-heated regions, o—quarter-heated regions, ■—half-heated regions without recirculation.

since in porous medium flow the Rayleigh number is dependent on a and not a^3 as in the fluid layer problem, the Rayleigh number across the boundary layer is not such a small fraction of the overall Rayleigh number as it would be in the fluid layer case.

(2) Remembering that the observed fluctuation time varies as $R^{-3/2}$, we note that the gestation time for the layer is somewhat shorter R^{-2}. If the mechanism is also governed by the circulation of a triggering perturbation, then we must consider the circuit time, which for a single disturbance varies like $R^{-1/2}$. To produce a variation like $R^{-3/2}$ requires that the number of disturbances circulating at one time be proportional to R.

(3) The second critical Rayleigh number R_2 at which the disturbances first appear is less in the unconfined region than in the confined region. When recirculation is prevented, cold fluid from outside the system is drawn into the proximity of the heated boundary, steepening the thermal gradient and increasing the effective Rayleigh number of the thermal boundary layer.

(4) In the recirculating case the descending disturbance occurs at roughly half the frequency of the ascending one, precluding complete circulation of all disturbances.

The appearance of the same fluctuation period/Rayleigh number dependence in all of the configurations considered so far demonstrates that these intermittent effects are not necessarily configuration dependent. The overall flow occurring outside the thermal boundary layer affects its behavior, as is evident from the difference in the regularity of the disturbances between the half-heated and the fully-heated cases (a factor which originally led us to look for a triggering mechanism), and provided that the location of their point of origin is not altered very much by this flow, the disturbances continue to evolve and are triggered regularly.

In concluding Ref. 1, we proposed that the convective regime in flow through porous media is influenced significantly by the choice of boundary and initial conditions. We have now also demonstrated, however, that the origination of the fluctuating flow is an inherent property of the boundary layer flow over the heated surface and only the regularity of its occurrence is dependent on the choice of boundary or initial conditions.

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