Designing mathematics hybrid classrooms in high school: the cases of Nicoletta and Lorenza

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Online math videos are spreading in the world wide web, but little research in Mathematics Education focuses on the ways teachers use these videos in their teaching practices, if any. In particular, we claim that it is of crucial importance to investigate how teachers' beliefs and goals influence their choices about planning and delivering mathematics lessons that resort to this kind of resources. In the present chapter we present two cases that add complexity to the case of Valeria, described in the previous chapter.

Keywords*: centrality of online resources; frequency of use; in-the-moment teachers' decision making; teachers and technology;*

Introduction

Online math videos are spreading and teaching formats like the flipped classroom resort to these resources, especially at university. Our research project aims at understanding the possible uses of math videos in secondary mathematics classrooms and focuses on the ways teachers can introduce and use the videos. In particular, we resort to the lens of analysis provided by Schoenfeld (2011), who understands teacher's decision making as a selection of goals consistent with her orientations and resources, and we interpret the degree of centrality and the frequency of use of videos in relation to the promotion of student-directed instruction. In the previous chapter, the case of Valeria has been analysed and it allowed us to partly respond to our research question, that is: in what ways is enhancing self-directed learning a consequence of video use in terms of centrality and frequency? In one class, videos are peripheral but self-directed learning is enhanced. In the other class, videos are central but teacher-paced instruction takes place. A possible interpretation of this is that, in the case of Valeria, beliefs related to the students are stronger if compared to those related to the positive impact of the use of technology in promoting self-directed learning. Hence, Valeria's case allows us to provide a first answer to the question: how are different, contrasting goals and beliefs, related? In order to answer the general research question, which involves the ways of enhancing self-directed learning as a consequence of video use in terms of centrality and frequency, the cases of Nicoletta and Lorenza are added. The methodology described in the previous chapter applies also to these two cases.

The Case of Nicoletta

The Teaching Context

Nicoletta teaches in a poorly technologically-equipped school, and yet, Nicoletta's students were used to watch mathematical videos at home. Nicoletta said that she wants her students to enroll the whole Pre-calculus MOOC on the platform www.pok.polimi.it, and she explicitly added that she expected

that class C students' major difficulties would be with logging in and with understanding the organisation of the courseware, since they were used to other websites.

In the interview, Nicoletta told us that her major *goal* is to enhance her students' ability to operate and learn from MOOC videos:

"*I want my students to do not panic if some steps in a mathematical procedure are not made explicit, if one cannot grasp some mathematical concepts, or terms, at first or if different parameters are used*".

Nicoletta mentions a potential difficulty that can arise when a MOOC lecturer makes use of terminology and symbols that are different from the ones used in a classroom. Nicoletta is aware that videos are not interactive: *"there exists a chance that the students will not engage with videos at home"*. Namely, she is worried that, in case a student did not grasp a concept or did not see a connection between the content of the video and the in-class activity previously done, she would give up. This is particularly important for a teacher like Nicoletta, who believes that her students tend to do the least possible to succeed in mathematics, and hence tend to not complete the tasks assigned if they have a feeling that an effort is required. Thus, two long-term goals emerge, namely: that class C students become both fluent with mathematics discussed in the videos, and resilient.

Lesson Image

Nicoletta planned to assign a 5-minutes long video to be watched at home. The video recalls some math concepts that had been already learnt by class C students. In it, the graph and some properties of the exponential function $f(x)=a^x$ are explained. The particular cases of e^x and e^{-x} are discussed and the logarithmic function is also recalled. (The reader may notice that this video was planned to be watched by Valeria's class A in classroom at the end of the lesson). Nicoletta planned to assign some questions to be answered at home. In her words,

"*I will assign the theoretical video, which recalls definitions and properties, and other two practicum videos which show the solution of exercises. The questions I will assign to my students will enable them to reflect on how a video can be watched, which questions can one pose to oneself, how exercises can be solved*".

We can notice that Nicoletta's concern about her students' becoming able to deal with the (new, different) mathematical language of the video emerges in the choice of questions she wants to assign: no further exercises, but questions about 'how' to approach the content. Examples of questions posed by Nicoletta are: Did you find some mathematical words that are new to you? If yes, how did you deal with them? In case you did not understand a statement in the video, what did you do? Did you notice some difference between the way an exercise is solved in the video, and the way we solve it in class? If yes, which ones?

Nicoletta continued

"*I want to see my students' answers in advance, hence I will collect their work through emails*".

Class C homework were expected to be checked by the teacher, even if for them it was not the first time watching a video. We can say that Nicoletta wants to have control on her class C, but we can also say that this practice would allow Nicoletta to prepare the lesson in advance, in accordance to

her students' answers. In her lesson plan, it emerges the centrality of the video: Nicoletta declared that in class she would "*recall the parts in the video where the graphs of exp(x) and of exp(-x) are shown simultaneously*". She, indeed, plans to make explicit reference to the video.

"*I will ask my students to draw the graph of f(x) and f(-x) for the following functions: a parabola of the form* ax^2+bx+c*, sin(x) and cos(x). This will prompt the students to notice symmetries in some cases and I will introduce the definition of an even function focusing on the features on examples drawn by the students. The students will work in groups*".

Nicoletta also mentions the good quality of graphs in the MOOC videos and in fact she wants to exploit one of the graphs in the video to introduce the definition of even functions, instead of drawing her own at the lightboard. The MOOC video is central throughout Nicoletta's lesson image: even a new concept is planned to be introduced using the hints offered by the video.

Nicoletta's lesson plan for class C is as follows: she will divide the class in groups of four students each and she will assign a paper to each student (5 minutes). Then, the teacher will recall the graphs of y=exp(x) and y=exp(-x) watched in the video at home by the students (5 minutes). She will navigate the class while the students will be doing groupwork activity on the basis of the assigned paper (25 minutes). Once the students, in small groups, would have drawn the graphs of the parabola, of the sine and of the cosine, she would recall the part in the video were f(x) and f(-x) are shown and in a frontal lesson she will introduce the notion of even function (15 minutes). In the remaining 10 minutes, she plans to answer to the students' questions that will arise.

In the implemented lesson in class C, something unexpected happen that is worth of attention. We now report and analyse it.

In-the-moment Decision Making

We met Nicoletta 15 minutes before the start of her lesson. She said that very few students did their homework and she suspected that the majority of them had not watched the video. Her suspicion was grounded on her belief that the class is not motivated and tend to do the least possible in mathematics. A first decision was requested on Nicoletta's side: to either show the video at the beginning of the lesson, or to go on with the planned lesson and not to show the video. Eventually, she decided to start the lesson with the video, saying: "*it will last just for a few minutes, and to show the video won't compromise the lesson*". We interpret her use of the verb 'to compromise' in terms of time: since the video lasts a few minutes, her concern about the fact that she would not have enough time to do all that was planned is relieved. This unplanned decision further speaks to the centrality that the MOOC video has for Nicoletta's lesson: it is not possible to carry it out, without having watched the video.

To our view, however, to show the video at the beginning of the lesson changes the planned lesson in a substantial, even subtle way. First of all, the students were requested to do the homework, and they did not. The nature of the homework was reflective, and the students lost the opportunity to think about their ways of approaching a mathematical video. Nicoletta's goal to exploit the video in order to introduce a graphical approach to functions and to define the concept of evenness is prioritized with respect to her goal of having class C working (more) at home, individually, on a reflective task. Mathematical knowledge is prioritised, in class, with respect to student's individual reflection. The prioritization of this goal recruits resources throughout the lesson, as we now show, and discards other resources such as individual worksheets.

The students were divided in groups of four, but each individual student received a worksheet with some questions on it (as planned). Then, the class watched the video played by the teacher at the smartboard and on the given, individual worksheets the students sketched the graphs of exp(x) and exp(-x), of sin(x) and sin(-x), of cos(x) and cos(-x), while Nicoletta navigated the class and engaged in conversations with the students individually. Whilst being divided into groups, the majority of the class worked individually without collaborating with their group peers. It seems possible that the interaction of the teacher with individual students rather that with their groups promoted this behavior. There were two interesting exceptions: a group of four students (two girls and two boys) was really motivated and willing to do their best, so they interacted all the time proposing ideas and checking them within the group. They seemed seriously engaged with the task. Another group of four boys interacted a lot, almost having fun: they chatted rather than doing mathematics and pretended to be working while the teacher was passing by. After 15 minutes of this kind of 'work' in the class, the teacher decided to stop the groupwork, to play again the video from the point in which the lecturer was introducing exp(x) and exp(-x), and to comment the video. Some students, who had previously worked individually, intervened in the discussion. Nicoletta recapped the main features of the graphs of the two functions and then invited the students to go on with the worksheet. The dynamic of the class remained the same for another 10 minutes, with the students working individually and the group of two girls and two boys working together. The teacher stopped the work another time and invited the students to find out the general features of the drawn functions. The students were mostly silent, so she drew the functions at the smartboard and in a frontal lesson she introduced the definition of an even function.

We comment that there was congruity between the lesson image and the implemented lesson, even when unplanned decision making was necessary. For class C, self-directed learning was (expected to be) high during homework, but (in the actual lesson) it was low for the in-class part. We also notice a mismatch between the lesson image and the actual lesson in terms of the extent to which the students had the opportunity to engage with the activity. This is partly due to these particular students' unwillingness to work on the assigned tasks.

The Case of Lorenza

The Teaching Context

Lorenza teaches in a medium technologically-equipped school. For example, each class has a smartboard, but the students do not have tablets or laptops. Lorenza wanted to recap exponentials and logarithms and she wanted to exploit a feature of video-integrated lesson that in her words is

"*The advantages of using MOOC are: saving time, better understanding since the students can stop the videos, and favouring the students' self-confidence with technology*".

We recall that class D is not used to watch math videos. She adds

"*I also have non-math goals: to favour autonomy, to stimulate curiosity and to provoke critical thinking towards multimedia resources*".

Lorenza's goals can be classified into long-term goals within Schoenfled's view, since she also wants to develop critical thinking. Lorenza also shares with Valeria and Nicoletta the awareness that videos are not interactive:

"*Video-lessons are attended to at home, where students are comfortable, but at the same time there's a risk they won't work, compromising the efficacy of this pedagogical choice. A drawback is the impossibility to make synchronous questions and to receive answers from the teacher in the video. This flaw can be dealt with the day after, at school, with their teacher*".

Lorenza adds that the teacher is there, in class, to respond to any question arisen during home work.

Lorenza's reflections on her students make an impression that she sees her students as collaborative:

"*It's a class of only girls and they are really cooperative and collaborative with me. Some of them are good in math, but many of them have troubles with the subject*".

Lesson Image

Lorenza chose to assign the same videos assigned by Nicoletta at home: i.e., a recap of exponential functions and two practicum videos, where exponential and logarithmic equations and inequalities are solved. Lorenza planned to show class D students how to access the MOOC, in a previous lesson, and to assign them the exercises in the MOOC, both the ones that have a solution provided in the practicum videos and those which required to be solved in solitude. In class, she planned to discuss with the students their solutions, which she will collect via email in advance. In Lorenza's lesson image, we notice that she plans to spend a lesson commenting the videos ("*I will discuss with the students their solutions*"), watched at home.

"*In class, we will do more exercises*".

In Lorenza's lesson image, the video is peripheral since it is planned to be watched at home, then commented, but the very focus of the lesson would be on new exercises. The frequency of use of videos in class D is low, since they had been introduced for the first time in a previous lesson, but the students are not used to them. The students are offered an opportunity to engage in a non-instructional way with math at home, but the in-class lesson image is rather teacher-guided.

In-the-moment Decision Making

Having asked to send class D homework via email in advance, Lorenza was able to notice that *some* of her students (not *the huge majority*) did not send her their homework. Once in class, Lorenza firstly asked them why. She also asked how the students coped with the assignments: "*How did you feel with watching all these videos?*". To start with a question is a choice that is different both from Nicoletta and Valeria, but it was not unplanned, if we consider that Lorenza wants to start the lesson with a classroom discussion about students' difficulties. The students replied that the videos were clear, but they experienced difficulties with the assigned exercises and asked for teacher's assistance. Hence, the teacher engages the classroom in a rich discussion about "how to do" each exercise. They re-did the assigned exercises at the blackboard, stopping and commenting several times about general properties that emerged, but also on fine-grain details. We can notice that in class D the content of the videos was recalled and 'replayed' by the teacher who wrote the exercises on the blackboard.

Class D students proposed the activity they wanted to do, by asking the teacher to redo the mathematical procedures watched at home (this was unplanned by Lorenza), even if the teacher was the one who responded to the questions. The students actively engaged in the discussion, which aligns with Lorenza's opinion about her students' cooperative mood. We also notice that her way of conducting the lesson stimulates the students' critical thinking, since many times during the lesson they were not satisfied with the procedure recapped by Lorenza and wanted also to recap "why to do so". The students took a lot of notes, and in their notes we see many remarks concerning 'why' and 'how' to proceed, instead of just copy-pasting the exercise that was written (by the teacher) on the blackboard.

Discussion

Our research addresses the general imperative to understand how our students are 'ready' for new generation learning formats by focusing on teachers' orientations, goals and resources that shape different ways of using online material in secondary mathematics classrooms. To recall, Lorenza, Nicoletta and Valeria teach in four classes and they decided to use the MOOC videos to introduce the same mathematical topic, namely: exponentials and logarithms. They share the opinion that MOOC videos allow a teacher to optimise time and promote the students' self-confidence with technology. They appreciate a feature of MOOC videos, that is the possibility for a student to stop and rewatch any segment of the video-lesson, but at the same time they know that videos are not interactive, since no one can answer a student's question that may arise. Asynchronous interactions are the sole possibility in the chosen learning environment. In class, both Nicoletta and Valeria aim at recapping some mathematical concepts and at introducing new ones. Both Nicoletta and Valeria intend to split the class in small groups. Differently from Valeria and Nicoletta, Lorenza plans not to introduce new theory, nor to do problem solving, but to do exercises on the basis of class D homework.

Nicoletta conceives her lesson in a way that we can place somehow in between Valeria's classes A and B. Like class A, indeed, class C students are assigned a video as homework. Differently from class A, where the video was not planned to be recalled for the in-class activity, class C is planned to recall the content of the video several times: at the beginning of the lesson, recalling the graphs of exp(x) and exp(-x) and focusing on the symmetries; during the groupwork, drawing other symmetric/non-symmetric functions; at the end of the lesson, introducing the notion of evenness. Also in Valeria's class B the video is present throughout the lesson, but the kind of work the students are expected to do in class is similar to the one that class C did at home, namely: to reflect on how to watch a video of this sort. To comment on these differences, Anthony (2012)'s notions of frequency and centrality come to be useful to us: we would say that the frequency of video-integration is low in class B, where it is used only in that particular lesson because the students refuse innovative teaching formats, while it is high in classes A and C, where the students are used to work frequently with online material. However, in the specific lessons under study, the video-integration for class A was peripheral (since the classroom activity pivoted around a challenging problem), while it was central for classes B and C. The lesson in class B was designed to analyse the content of the video, and the lesson in class C was designed to start from the video watched at home and bring to the definition of a new property of real functions.

In the implemented lesson of Nicoletta, we can notice that the group work activity turned out to be very different from Valeria's class A and to some extent it was much closer to class B's activity, with

the teacher having control on what was going on. We further comment that self-directed learning was high in class A, both at home and in class, and it was low in classes B and C.

Lorenza's choice can be seen as an intermediate position between Valeria's class A and class B, too. Namely, in class A, the students are left alone in watching the video and Valeria does not care about possible difficulties that may arise, as if she sees her students able to deal with them; in class B, she wants to control everything and in the class she plays the video and checks how the students deal with it. Lorenza leaves her class D students alone at home (like class A), but she takes into account the possibility that some intervention would be needed in class (like class B). This scenario is similar, to some extent, to Nicoletta's lesson image, in which she planned to assign some work to be done alone at home, but to be sent to the teacher via email so that she knows the possible difficulties arising from the students.

Interestingly, like Nicoletta's class C, in the case of Lorenza's class D self-directed learning was high at home and low in class. However, the students in the two classes reacted very differently to their respective teachers' proposals: class C discarded homework and in class followed the teacher, while class D engaged (at least partly) in the assigned homework and co-participated to the teacher-directed lesson by proposing new directions to be taken, and the teacher followed them. Nicoletta's decision seemed not to be affected by her students' actions and behaviour during the in-class activity, while Lorenza was open to change her plans upon her students' requests. Another difference emerges from a comparison between classes A and D, which both watched the videos at home: class A had been left alone in dealing with the math content, while class D asked help to their teacher. Videos are played in classes B (and this was planned) and C (but this was unplanned, since the students had to watch them at home). The (content of the) videos had also been 'replayed' by the teacher at the blackboard in class D. The reasons why MOOC videos were 'replayed' in classes C and D are, however, different: in the former, it was because the students did not watch the assigned videos at home; in the latter, it was because the students *did* watched the videos at home, but asked for the teacher's assistance.

Valeria's case tells us also that in one of her classes self-directed learning is promoted even with a peripheral use of videos. And in the other class, despite central use of video, the lesson is teacher-guided. MOOC videos are central to both classes B and C. In the former, videos are central because the students have to learn how to 'use' them, while in the latter it is the mathematical content that necessarily needs to be introduced through the graphs and the terminology employed by the lecturer in the video. We have already commented that in classes A and D the videos are an add-on and different mathematical activities are central: problem-solving and exercises, respectively. In classes B and C the videos are central, yet unavoidable, in two different, contrasting ways. The four cases and the nuances of centrality they bring to the fore show how students deal with the videos and gradually become proficient users of online resources. Namely, the four classes under examination in this chapter can be further characterised by different stages of technology integration (see also Ruthven, 2002): Class B represents a case in which the teacher is mostly teaching how to use the tool, i.e. the video, hence she is concerned with the way the students learn how to manage it and she proposes an activity that is (inevitably?) teacher-guided and pivoting around the sense-making of the video. For this reason, the video is inevitably central to the math lesson. Class D represents a situation where the video as a tool is still a matter of concern for the teacher, and in fact she enacts a teacher-guided lesson where she is involved in troubleshooting, namely in answering to the questions arisen by her students during their previous, individual activity with the video(s). The video can become peripheral

at this stage of co-evolution of technical and conceptual understanding. Going further along the spectrum of stages, we find class C, where the teacher makes use of the video not for the sake of teaching how to approach it, but for teaching mathematics with it. At this stage, the video becomes again central and deeply related to the evolution of conceptual understanding, since the technical one is rather advanced. Finally, in class A the students are so confident with online teaching formats that they do not need assistance from the teacher, and the mathematical activity can go on with a peripheral role assigned to videos. This interpretation of the introduction of MOOC videos as going along subsequent and interconnected stages, allows us to give sense to the ups and downs in the centrality of video use, and making sense of its different uses. Moreover, it leads us to conclude that not necessarily MOOC videos should be central to the mathematical activity, nor should they be frequently used, for a self-directed learning formats to take place.

All in all, if we look at the four scenarios, and we try to find out similarities and differences among them, we can see that the relationship between video use, teachers' views, teachers' practices and teachers' being in their specific/individual classes is very complex. It is necessary to take this complexity into consideration to authentically understand the phenomenon and to contribute to a technologically-rich and deeper-thinking school in the future. Within such a complex scenario, in our four cases we took into consideration the dimensions of frequency and centrality, and the extent to which self-directed learning is promoted by each teacher in each class. Our conclusions, which follow this section and end the chapter, try to understand which scenarios can be meant as successful with respect to the promotion of self-directed learning and sketch possible future investigations.

Conclusions

Our findings can be briefly summarised in Table 1, from which it emerges that, for example in Valeria's case, frequent use of technology goes along with students' self-directed learning in class A, while infrequent use of technology is paired with rather traditional, teacher-guided lessons in class B. Classes B and C, as opposites of the spectrum, confirm a general research finding that having equipped the school with technology is not necessarily related to the use of technology in classes: Valeria teaches in a well-equipped school but she does not use technology in class B, while Nicoletta teaches in a poor-equipped school but her use of technology is central and high. Furthermore, Classes B and D relate to cases of teachers who, even if engaged in a specific research project aimed at introducing the use of MOOC videos in their class on a voluntary basis, turn out to use the videos infrequently.

Table 1. A summary of our findings with respect to the categories outlined.

	Centrality of MOOC videos	**Frequency of use of MOOC videos**	**Self-directed learning**
Class A	peripheral (only watched at home)	high (general use of technology)	promoted both at home and in class
Class B	central (shape the entire lesson)	low (very traditional)	not promoted

Class C	central (home-watched + re-played in class)	high (used to math videos)	promoted (expected) at home, not in class (implemented)
Class D	peripheral (meant as exercises)	low (general poorly equipped)	promoted at home, not in class

By looking at Table 1, which cases can be deemed as 'successful' with respect to enhancing self-directed learning? In class A, it is enhanced both at home and in class and, borrowing a metaphor from clinical research, we can conclude that class A represents a 'gold standard'. In class D, self-directed learning is encouraged at home and this can be seen as 'successful'. Classes B and C represent a failure in promoting self-directed learning, but grounding a possible explanation on the fact that both classes are 'difficult' does not satisfy us, because also class D is difficult but the teacher reached a (partial) success with respect to the aim. Our interpretation goes beyond a focus on the resources (namely, difficult students) and, given that all teachers in the study have positive orientations towards technology, it exploits the notions of frequency and centrality of video use. In both successful classes A and D, videos are *peripheral*. Being either high- or low- frequently used seems not to be very relevant with respect to promoting self-directed learning. In both unsuccessful classes B and C, videos are *central* and the fact that the students were already used to them (i.e., high frequency), or not (i.e., low frequency) seems not to play a crucial role. To draw the conclusion that central use of a resource does *not* enhance *what that it is designed to enhance* seems confusing, but we need to take into account the fact that *these particular students* have difficulties with mathematics, they are poorly motivated in doing mathematics and/or they refuse any innovation coming from the teacher. For this kind of students, a smooth approach with respect to the introduction of a new, somehow disruptive, online learning format seems to work better. A smooth approach to the introduction of MOOC videos assigns to them a peripheral place. In other words, in the cases of failure, the students may have perceived that MOOC videos had invaded their classes in the first time they met them, and as a consequence they have refused to work with MOOC videos. A general conclusion that we can draw is that it is advisable to introduce online mathematics resources in a peripheral way, especially with 'difficult' students.

The picture that emerges from the four cases is a picture of teachers struggling to find a way to integrate MOOC videos in their classes, instead of a systematic and well ordered picture of cases where different choices perfectly work. We recall that they are all expert teachers, and all they are technologically enthusiasts. As a consequence, the picture that emerges does not question a teacher's knowledge and it does not focus on her lack of either mathematical or technological knowledge. It does so *on purpose*. It is as if what makes sense for these teachers is no longer what is clear and precise, but we can use Nathalie Sinclair's words to investigate if they value an aesthetic of ambivalence, namely an aesthetic of simultaneous conflicting feelings, which brings to the fore confusion and loss in mathematics, while putting aside purity, coherence and connectedness as the very hallmarks of mathematics (Sinclair, 2018). Such an investigation is worth to be carried out as a follow-up of our research, since we agree with Sinclair (2018), who argues that this approach to learning phenomena may broaden our understanding of mathematics, and she warns us about the tremendous consequences of narrowing our view to consider only the beauty, certainty or the systematic nature

of the subject. Along this line of thinking, we aim at going on in analysing ‘bad cases’ that will allow us to unfold relevant issues concerning the uses of online resources in secondary math classrooms, and the meanings that underpin them.

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