Designing mathematics hybrid classrooms in high school: the case of Valeria

Chiara Andrà[1,2], Domenico Brunetto[2], and Igor' Kontorovich[3]

[1]Università del Piemonte Orientale, Italy; [2]Politecnico di Milano, Italy; [3]The University of Auckland, New Zealand

Online mathematics learning is spreading among tertiary education. One of the challenges for secondary mathematics teachers is to prepare their students to learn in this new format. MOOC videos delivered online for free are one of these formats and it is necessary to investigate how teachers' beliefs and goals influence their choices about planning and delivering mathematics lessons that resort to this kind of resources. In this chapter, the case of Valeria illuminates important relations between a teacher's beliefs about her students, and her attitudes towards technology.

Keywords*: centrality of online resources; frequency of use; in-the-moment teachers' decision making; teachers and technology;*

Introduction

In the recent decades, online teaching and learning are spreading among tertiary education: from online resources that students can obtain in a few clicks almost on every topic, to Massive Open Online Courses (MOOCs) that have become a standard in many universities worldwide. As a matter of fact, the majority of tertiary students around the world seems to engage in online learning of some sort and, as a consequence, some online mathematics forums have thousands of members and receive hundreds of posts every day (van de Sande, 2011). These new learning formats, such as MOOCs, promote self-directed learning, since the quality of students' engagement with and their understanding of the mathematical content in the MOOC videos depends on the extent to which they search for sources of knowledge in order to better understand and deal with the content of the video, when this is not immediately clear to them (Fredriksen, Hadjerrouit, Monaghan & Rensaa, 2017). Furthermore, to engage in learning formats like MOOC videos has been proven to be a crucial ability for the success of students in tertiary-level mathematics (Niegemann, Domagk, Hessel, Hein, Hupfer & Zobel, 2008).

Despite of positive learning outcomes from MOOC videos and online resources in general, and despite our current and future students are part of an online generation by definition, students should not be assumed to be online learners from birth, at least not when mathematics and statistics are under discussion. Indeed, contemporary research recurrently shows that online environments are not as helpful as it could be assumed in overcoming classical challenges of higher mathematics education (e.g., see Tall, 1991 for abstractness, formality and complexity; Moore, 1994 for proving; Bosch, Fonseca & Gascon, 2004 for new didactical contracts). Moreover, these environments entail new challenges on their own (Fredriksen et al., 2017), and tertiary students tend to resist to online teaching formats. Our claim is that introducing online learning formats at secondary level would help both to smooth the transition from high school to university, lowering difficulties with mathematics, and to prepare the students to welcome these new formats once enrolled at university, getting used to them and appreciating the advantages. This claim is supported by numerous research findings, as we briefly recall in what follows. In their fundamental study, Clark and Lovric (2008) contend that at the basis of the leap between secondary school and university there is a shock: the students have to move from the procedural mathematics the students are used to at school, to the conceptual understanding that university mathematics entails. According to Hibert and Lefevre (1986), conceptual knowledge describes knowledge of the principles and relations between pieces of information in a certain domain, while procedural is the knowledge of the ways in which to solve problems quickly and efficiently. Some researchers (e.g., Gamer and Gamer, 2001) found that teacher-paced instruction favours the development of procedural knowledge, while student-directed instruction favours the development of conceptual knowledge. A teacher-paced lesson provides the students with a linear and organised exposition of knowledge, while a student-directed one engages students in groupwork activities, classroom discussion and in the production of meanings that are inevitably other than final or authorised: they are personal and provisional, not universal and absolute. The use of video clips from university lectures in secondary mathematics classrooms has the potential of both acclimatising the students to the language and the teaching style of university math professors and of promoting student-paced learning formats, since it may happen that the students watch the videos and try to get sense of the content without any guidance from the teacher, or in case parts of the videos are not clear for the students, search for other sources in order to make sense of the content, or propose new activities. All this entails a production of meanings, from the students, that is personal and that emerges from the mathematical activity in which each student is engaged. With Niegemann et al. (2008), we maintain that video-based mathematics lessons combine self-directed and externally regulated learning types of instructional formats: in fact, the math teacher plays a major role in introducing the videos to the class, and the focus of this chapter is on the role that teachers' beliefs, goals and resources play in shaping the way(s) video clips are introduced in different mathematics classes.

The value of instructional videos for enhancing learning is recognised by many scholars, inside and outside Mathematics Education. Berk (2009), for example, notes that the variety of video formats, the ease with which the technology can facilitate their use in the classroom, the number of video techniques an instructor can use, and the research on multimedia learning that provides the theoretical and empirical support for their use as an effective teaching tool, prompt teachers to use video clips in their teaching. According to Berk (2009), findings from Cognitive Neuroscience support positive implications for educational practice, and the use of video clips in teaching is more appropriate for introductory courses, for introducing complex topics in any course, for lower achieving students, and for visual/spatial learners. Furthermore, research findings on the effectiveness of videos embedded in multimedia classes are very encouraging: a large number of studies in teacher education, for example, have produced significant results favouring videos (e.g., Sherin, 2003). However, and despite the promising findings in education, a few researches are dedicated to the use of math videos in classrooms, at secondary level. In order to contribute to this *problematique*, the present chapter focuses on the use of math videos in secondary school.

More specifically, the research project *Flip Math* at the Politecnico di Milano is aimed at preparing high school students to tertiary studies through turning their traditional face-to-face classrooms into hybrid learning environments. In the last six years, dozens of teachers in Italy engaged in the *Flip Math* project. The teachers engaged in the project driven by their interests in introducing online components into their mathematics lessons and in enhancing self-directed learning. The online components consisted of instructional videos that were taken from the first Italian Mathematics MOOC initially developed by the Politecnico di Milano. The MOOC was developed for first year Engineering students who needed a recap of a variety of topics often captured under the term of "pre-calculus". The MOOC emphasised mathematical terminology, formal definitions, and problem solving skills. The MOOC is a research-informed initiative that has been developed to be useful as a whole but also in a modular mode, where videos and activities can be used in actual classroom to provoke exploratory discussions. Taken together, these characteristics turn the MOOC into a rich educational resource that is capable to support teachers in achieving the desired change[1].

A particular sample of experienced and technologically enthusiast secondary mathematics teachers engaged in the *Flip Math* project: until now, all teachers have an average of teaching experience of twenty years (five years is the minimum). Furthermore, all of them come with evidence of impressive technological knowledge and skills. The research project turns out to be a project investigating how mathematics teachers with good technological knowledge and with long experience in teaching develop new learning formats entailing MOOC videos. Two issues related to the sample emerge at this point: one regards the generalisability of the results of our research, conducted on a purposive sample of experienced and technology-enthusiast teachers, and the other one regards the shift of focus that our research undertook. As regards the former, we maintain that the issues that emerge have a "general" nature, as we will argue in the discussion. As regards the latter, if it is true that instructional videos have learning potentialities (as revealed by promising research findings on teacher education, for example), and if it is true that learning formats such as the flipped classroom are taking the lead in many countries because they are effective, it is also true that it is important to investigate how the teachers introduce and use math videos in classroom. Up to date, and up to our knowledge, this phenomenon has received little attention in research. The research focus is not on examining the effects of introducing online learning formats at secondary level on students, but it is on understanding the role of teachers' beliefs, goals and resources in their decision making during the use of instructional videos in mathematics classrooms. An outcome of the research is the identification of different ways in which experienced and technologically literate math teachers appropriate MOOC videos in their teaching practice.

Technological literacy is not the only factor that influences a teacher's introduction of online resources such as MOOC videos in her classroom, as Anthony (2012) noted. Other individual characteristics such as a teacher's beliefs, or the perceived usefulness of technology, are essential factors to determine both the frequency and the centrality of MOOC videos use in mathematics classrooms. We, thus, adopt Schoenfled's (2011) lens and focus on a teacher's orientations, goals and resources to examine how they influence a teacher's choice of using MOOC videos in her class. It is well known, in fact, that orientations, goals and resources play a key role, but how do they interact, especially when a conflict between two different goals, or between two contrasting beliefs, takes place, is less researched. The aim of the present chapter is to showcase examples from our extensive data and discuss the role of teachers' beliefs in shaping her choices when she introduces her classroom to MOOC videos. To this aim, we summarise the conceptual framework that informs our lens of analysis in the next section.

[1] The interested reader can freely log in the course at www.pok.polimi.it

Conceptual Framework

Schoenfeld (2011) argues that, when people make decisions in well-practiced domains, as it is for teachers the domain of teaching, it is possible to model decision making in a quite precise way as a function of knowledge and other intellectual, social, and material resources, goals, and orientations. ‘’Well-practiced’’ activities are those for which a person has had enough time to develop expertise, knowledge and routine that shape the majority of her actions. According to Schoenfeld (2011), any person enters into a particular context, such as a classroom, with a specific body of resources, goals, and orientations. For example, a teacher starts her lesson with a specific body of resources, goals and orientations, but a teacher engages in a new research project with the same specific body, as well. Whatever the situation, a teacher takes in and orients to it. Certain resources become essential and are activated. Goals are either established or recalled (if pre-existing). Consistently with the goals, and within the constraints and the potentialities offered by the resources, decisions are made. An interesting point of Schoenfeld’s model is the distinction between familiar and not familiar situations: in the former case, Schoenfeld (2011) notes that actions can be automatic and they can consist mostly of implementation of scripts, frames or routines. In the latter case, there is not an established routine, hence decision making is made by a mechanism that can be modeled as a function of the subjective expected values of available options, given the orientations of the teacher. When a teacher joins a research project like *Flip Math*, she finds herself engaged in both familiar (i.e., routine classroom practices) and unfamiliar situations, which can subvert the routine.

The basic assumption of Schoenfeld’s (2011) framework is that beliefs and orientations are an essential factor shaping teachers’ decision-making, behaviours and professional development. Schoenfeld proposes to understand beliefs as “perceptions on the part of individuals that shape the ways in which they frame or orient themselves to any particular context, and thus shape they ways they act in that context” (Schoenfeld, 2011, p.460). This assumption needs to be complemented with additional considerations.

First of all, it is necessary to define what is meant by “beliefs” and “orientations”. To this end, Skott (2015) maintains that there are four characteristics of beliefs that seem to be shared by almost all researchers: *conviction* - beliefs can be thought of as knowledge that is true at least in the eyes of the beholder; *commitment* - beliefs are value-laden and relate to motivation; *stability* - beliefs are stable and change only after substantial, new experiences; *impact* - beliefs influence one’s individual perception as well as her practice. In Schoenfeld’s (2011) view, orientations include beliefs, but also values, preferences and tastes. Values are the deep affective qualities which education aims to foster through the school subject of mathematics (Bishop, Seah & Chin, 2003). Values and beliefs are related to each other since there is a presence of underlying values in beliefs: values are conceptualised as beliefs held by individuals to which they attach special priority or worth, and by which they tend to order their lives. As such, values can be thought of as guides to behaviour. A category of values is related to an individual’s preferences and tastes: i.e., personal values (e.g., when choosing a new car, one may consider the price, or the color, or the design, or functionality). Economical reasons, design, and functional importance are referred to as personal values (see Shimada & Baba, 2015).

The second consideration is that orientations are systemic and contextual. Schoenfeld (2011) posits that in many situations, an activation of certain beliefs triggers the activation of other, related beliefs, and in such a way, a belief system contributes to the choices that teachers make. Skott (2015) suggests to see the *context* as a constraint on the opportunities for "belief enactment", acknowledging a central role to social interactions that takes place in the context.

According to Schoenfeld's framework, teachers' decisions and behaviours also depend on goals and available resources (including: knowledge, materials, personal and interpersonal skills):

> Every sequence of actions can be seen as consistent with a series of goal prioritizations that are grounded in the teacher's beliefs and orientations, and the selection, once a goal has been given highest priority, of resources intended to help achieve that goal (Schoenfeld, 2011, p. 460).

A goal, whether explicit or tacit and unarticulated, is something that a teacher wants to accomplish. Goals can be classified in terms of grain sizes. For instance, a short-term goal can be associated to a single lesson, while a long-term goal can refer to what a teacher wants her students to know as a result of a school experience. In many research projects with teachers, their genuine goals become difficult to observe, since the project's agenda dominate teachers' prioritizations. This one, even if it is the most widespread, should not be the only possibility, as different forms of teacher-researcher interactions may take place (see for example Wagner, 1997). Among the forms of teacher-research interaction identified by Wagner, ours is a case of *co-learning agreements*. In co-learning partnerships, the goals, methods and principles of inquiry are negotiated openly to maximize the learning and the fruits of learning for both researchers and teachers. Therefore, co-learning agreements essentially reduce asymmetry in the roles of the researchers and teachers. According to Penuel, Fishman, Cheng and Sabelli (2011), co-learning agreements focus on problems of practice from multiple stakeholders' perspectives, concerns with developing capacity for sustaining change in educational systems and calls for breaking down barriers that isolate those who design and study educational innovations and those who implement them. Rather than accepting a readymade research agenda, teachers work with researchers on translating both sides' goals into a mutually beneficial agenda. Teachers are invited to work side by side with the researchers in developing, validating and implementing data-collection tools, and practice collective reflection and data analysis. A particularly important stage in co-learning inquiry is formulating conclusions and implications. Here teachers can bring forth an established sense of what works in education and what makes education work, "a feel for the breadth, depth, and complexity of education as an institution that cannot be picked up by reading about it or observing it" (Labaree, 2003, p.16). Our claim is that specifically in this form of teacher-researcher interaction it is possible to 'see' a teachers' goals.

Finally, Schoenfeld's (2011) notion of resources include all kinds of 'goods' that are available for a teacher. For example, the tools in the classroom, students' knowledge, teachers' knowledge, interpersonal skills and relations with students. To this respect, focusing on technology integration in general, Anthony (2012) argues that recent research offers insights into why technology integration efforts have not had a greater impact on teaching practice and student achievement: in Anthony's view, a strand of research has emphasized that teachers' individual characteristics, such as technological literacy (a resource in Schoenfeld's view), and constructivist beliefs or the perceived usefulness of technology (orientations with Schoenfeld's words), are essential factors that influence technology use; while another strand of research has focused on institutional conditions, such as school and district settings, as factors supporting or constraining technology use (resources in Schoenfeld's model). Indeed, Anthony (2012) stressed the role of resources (understood in terms of Schoenfeld's views) in shaping technology adoption by a teacher. Moreover, important for our study is Anthony's focus on both technology planning at school- or district-level, and technology integration that teachers enact in their classrooms. Anthony argues that district-level system of planning and teacher-level system of integration connect in ways that mediate teachers' ability to implement technology with her students. Along this line of reasoning, we pay specific attention to the teacher's ability to utilize the potential of video-integrated lessons for the benefits of students' learning and, adapting Anthony (2012)'s dimensions of technology integration, we focus on "centrality" and "frequency" as dimensions that are useful to understand the ways and the extent of MOOC videos introduction in mathematics classrooms. Anthony, in fact, understands the centrality of technology use as either seamlessly embedded or peripheral to classroom routines. The frequency of technology use is accounted to as ranging from daily to less than once a month. 'Infrequent' technology use occurs when it is used a few times per month, or less. 'Frequent' technology use refers to weekly or daily use. Technology use is 'peripheral' to a majority of classroom routines when technology is used as an add-on resource for remediation or enrichment. It is 'central' when technology use is integral to a majority of classroom routines and is an essential teaching and learning tool. It is so much embedded in the classroom activity that learning would have assumed completely different forms without it.

The focus on frequency and centrality as indicators of 'quality' of technology use is widespread in literature, since researchers seem to agree that technology promotes student-centered learning (see Clark-Wilson, Robutti and Sinclair, 2014), and as such it becomes central to know how much technology is employed by teachers, especially in those countries where huge economical investments had been done in order to equip the school with a variety of ICT (Information Communication Technology) tools. A general research finding is that technology integration mainly depends on a teacher's orientations. The general finding on technology use, and in particular on ICT, seems to hold also for MOOC videos. Research findings on teachers' use of technology, and more broadly of innovative pedagogical settings, reveal that there is considerable gap between the *frequency* of use of these innovative tools and the economic investments that had been done. From one's hand, researchers report an increase in the students' academic performance in technologically-rich environments (Cuban, 2001; Hannafin & Foshay, 2008; Wenglinsky, 2005), on the other's hand, despite access to funding and equipment, a recent study found that less than 35% of teachers in U.S. districts that received specific funding have integrated online learning in their instruction on at least a weekly basis (U.S. Department of Education, 2008). Outside U.S., international surveys indicate low ICT integration in mathematics classrooms, though such indicators mask considerable variation between and within countries and provide little detail into how and why teachers use ICT. For example, in a survey of 42 countries within the Organisation for Economic Cooperation and Development (OECD), on average 32% of students reported that they, or their classmates, performed at least one of a range of seven mathematical tasks on a computer in the last month. A further 14% reported that only teachers demonstrated the use of computers – consistent with a finding of infrequent computer use in mathematics instruction (OECD, 2015). The 2011 Trends in International Mathematics and Science Study (Mullis et al., 2012) reported that only a quarter of students on average used computers at least monthly during mathematics lessons. Like technology in general, also online learning formats in particular are little exploited by teachers and their use is infrequent. As it emerges from this literature review, and as it is cogently argued in Clark-Wilson, Robutti and Sinclair (2014), there exists a gap between the corpus of research findings that show how technology of different sort promotes self-directed learning, and the actual practices that are observed in math classrooms. Videos can contribute to promote student-directed instruction, since for their nature they encapsulate both a rather trasmissive modality, which might attune to the modalities of use that are more spread among teachers, and they may require the students to activate resources in order to understand them and to give them meaning. However, to achieve this purpose a priority is to understand the role that teachers' beliefs, goals and resources play when videos are used in math classrooms. The metric of success for the use of math videos is not the centrality or the frequency *per se*, but the promotion of self-directed instruction in the different scenarios that emerge from our observations.

Methodology

In what ways is enhancing self-directed learning a consequence of video use in terms of centrality and frequency? In order to answer to this question, through the lens of Schoenfeld's (2011) model, we explore the roles that orientations and goals can play in the lessons teachers plan and deliver, and in particular whether MOOC videos are central to classroom practices, how frequently are they used and to what extent non-instructional engagement with the mathematical content takes place. So, in this chapter we aim at answering the sub-question: how are different, maybe contrasting, beliefs and goals, related when teachers use videos in their classes? In this chapter, we analyse two lessons, and in the next chapter we analyse other two lessons, which represent a variety of possible scenarios.

The context of the research

Flip Math is an exploratory, longitudinal research project where teachers and researchers collaborate closely towards developing pedagogies that are powerful in hybrid learning environments. Teachers join the project on a voluntary basis, hence the sample for data analysis is purposive. Being based upon co-learning agreements, at the heart of the project are three activities: (i) intensive meetings where teachers develop innovative instructional methods, (ii) experimental lessons where the methods are implemented, and (iii) the theory-driven fine-grained analysis which is carried on and discussed with the teachers. Specifically, we use Schoenfeld's (2011) model of resources, goals and orientations for making sense of teachers' lesson designs and in-the-moment decision-making. Overall, in the last six years we have collected data from a variety of classrooms with students across skill levels. In particular, three teachers, fictionally named as Valeria, Nicoletta and Lorenza, joined the Flip Math project with their classes. In this chapter we consider the case of Valeria, while in the next chapter we consider also the cases of Nicoletta and Lorenza. We present the methodological details concerning the three teachers in this chapter, anticipating some information about the teachers whose data will be analysed subsequently, because we would like to provide the reader with the full picture regarding our sample.

Participants: the teachers and their students

Valeria, Nicoletta and Lorenza were three highly-experienced secondary mathematics teachers: Valeria has 30 years of teaching experience, she attends professional development courses for in-service teachers regularly and she is a tutor for beginner teachers; Nicoletta has 6 years of teaching experience, she collaborates in research projects at the Politecnico di Milano, and she has an outstanding mathematical knowledge; Lorenza has 5 years of teaching experience, she has a phd in Mathematics Education, and she regularly participates in International research conferences for researchers in Mathematics Education. Nicoletta and Lorenza are also lecturers at university and are used to online teaching formats like online lecturing, forum interactions and blended learning. The purposive selection of a group of teachers that have a strong technological knowledge, positive attitudes towards online learning and good teaching expertise allows us to ground our findings on the basis of a rich teaching context, rather than focusing on a teacher's weaknesses.

During the school year 2014/15, when our research was carried out, Valeria was teaching two grade-11 classes (the second-to-last year in high school in Italy), which we refer to as class A and B. During the school year 2016/17 Nicoletta was teaching a grade-12 class (first-to-last year in Italy) and Lorenza was teaching a grade-10 class. We refer to them as classes C and D, respectively: they will be analysed and discussed in the next chapter. Valeria's class B, and Nicoletta and Lorenza's classes C and D are described by the teachers as "difficult" to some extent. For Valeria, her class B students are not open to innovation and complain every time she proposes something that is not routine. She, thus, wants to include them in the project to see if some change can happen. Nicoletta talks about class C as a class where the students are not motivated to do mathematics, they rather hate the subject and tend to do the minimum requested by the teacher in order to get good marks. Nicoletta joined the project because her class was already using math videos and she wanted to coordinate and confront with other colleagues. Lorenza says that her students are very nice, hard working and collaborative, but have serious difficulties with mathematics. She aimed at conducting a research on her students' difficulties and she wanted to be part of a team of both researchers and teachers. To sum up, we can say that classes B, C and D are described as 'difficult' by the teachers in three different ways:

- class B does not accept anything new and is not used to video-integrated lessons;
- class C is not motivated in doing mathematics and is used to math videos;
- class D has difficulties with mathematics and is not used to videos.

Interestingly, three out of four classes were chosen by the teachers not because the students were 'the best possible', but for rather the opposite reasons, namely because the teachers wanted to improve the learning environment for these students and we believe this is a feature of co-learning agreements that has positive outcomes for both the teachers involved and the researchers, who have the possibility to meet true, actual students.

Valeria, Nicoletta and Lorenza were invited to use videos from the Pre-Calculus MOOC course developed at the Politecnico di Milano for students enrolling in the first-year university courses. The mathematical content reflects (and recaps) the mathematical curriculum of the last years in Italian high school, namely logic and set theory, pre-calculus, algebra, analytical geometry and trigonometry, probability and statistics. The course is structured in six weeks, each one corresponding to one of the above listed topics. Each week is divided into modules, and each module opens with a theoretical video, which serves the purpose of recapitulating the main concepts, then a series of practicum videos follow and they serve the purpose of showing typical exercises; at the end of each module there is a quiz. Each week is made of 3 or 4 modules. These MOOC videos represented a novelty for all the teachers (also for Nicoletta, who was used to other math videos available on line).

Data Collection

After watching the MOOC videos at home, all the teachers were invited to imagine a scenario for integration of some of the videos in their respective classes. Our data come from the meetings with the teachers after they watched the videos, and from video-recorded lessons where the chosen videos had some role. We met Valeria for four times before entering her class, where we observed, took field notes and videotaped two lessons of two hours each for each class. Valeria shared with us all the material produced online before and after the lessons. The videotaped lessons took place in two subsequent days, and on the same day we were present in class A for the first two hours, and in class B for the second two hours. We met Nicoletta in person for two times before the lesson, and we also met her on Skype once. She wrote for herself and shared with us many considerations about her class and her goals. Since the three lessons took place in three subsequent weeks, we had time to meet in between the lessons, and she made comments to what had happened. We met Lorenza five times before her lesson and we shared with her also the theoretical framework we were going to use in order to analyse the data. We observed and videotaped two lessons, in two subsequent weeks, but had no chance to meet her in between them. We met Lorenza once after the lesson. After their lessons, we presented and discussed with Valeria and Nicoletta our theoretical lens of analysis, and they shared various comments.

Our data analyses was concerned with: (i) the teaching context, which refers to how a teacher describes her class, the school and the pedagogical setting she is used to implement in the class; (ii) the lesson image, namely the lesson plan paired with the teacher's expectations about the lesson; and (iii) in-the-moment decisions that the teachers made during the lessons.

A *teaching context* emerges from the analysis of teachers' resources, orientations and long-terms goals. The aim of this analysis is to understand different scenarios for the introduction of MOOC videos. A teacher can be concerned with the technological infrastructure of the school, teacher's perceptions of students and their abilities, and ideas regarding how videos can be used in a lesson. At this stage, it can emerge how much frequently MOOC videos are (intended to be) used by a teacher, as well as which technology that may support their use is available.

A *lesson image* comes into being from the interactions between researchers and teacher after the teachers have watched the MOOC videos and before enacting the imagined lesson. Following Schoenfeld's view, at this phase decisions should be made regarding the mathematical topics of the lessons that will be carried out, the kind of activity and the (short-term) goals of these lessons. Accordingly, the discussions between researchers and teachers are focused on designing students' engagement with the videos and on the activities that precede and proceed the engagement. For instance, mathematical problems and tasks can be created for reaching the lessons' goals. We analyse how MOOC videos are related with the tasks assigned to the students. At this stage, it can emerge whether MOOC videos are central or peripheral to the mathematics classroom activity, namely whether the use of this resource is unavoidable to reach the learning outcome, or whether it is used as a sort of add-on. It is also at this stage that it emerges the extent to which teachers want to promote self-directed learning.

After the meetings during which the teaching contexts and the lesson images emerged, few days later Valeria, Nicoletta and Lorenza went to teach in their respective classrooms. We videotaped the lessons carried out in each class. The videos provided us with data on teachers' *in-the-moment decision making*. The data contain video-recordings, notes of the teachers, students's worksheets, and field notes taken by the observing researchers. We look at matches/mismatches between the lesson image and the actual lesson. We interpret a match as a further evidence for the importance of declared resources, orientations and goals. If there's a mismatch, we will analyse the gap between *declared orientations and goals* and the actual *beliefs in use* and *goals carried over*. As a side note on the reliability of our results, we specify that all the data had been analysed side-by-side by three of us followed by extensive discussions of our interpretations with the teachers, as co-learning agreements entail.

The Case of Valeria

The Teaching Context

Valeria teaches in a technologically highly-equipped school and has experience in supervising technology-related projects. Valeria told us that in her teaching she often uses various technological devices, such as laptops and an interactive whiteboard, as well as software, such as GeoGebra. She also adopts educational environments like Wikispaces. When we asked her to explain how she uses them, she specified that she devises a learning trajectory for each one of her classes, from grade-9 (first year at high school in Italy) to grade-13 (the last year). She does not assume that students are able to work with technology "on their own", and hence, she gradually introduces software in grades 9 and 10. At grade-11 she starts assigning homework that involves software, and designs consequent lessons on the basis of students' submission. In this way, she believes that the students gradually learn to appreciate both mathematics and technology.

Valeria also shared that technology helps her in representing mathematical ideas, spare time and ease on her communication with the students. When we asked Valeria to explain her interest in integrating MOOC videos in her lessons, her response addressed her personal long-term goals and orientations:

"*It is innovative in terms of didactic methods, and it is an opportunity to use videos produced by experts to develop the teaching around the students. I am very willing to take part in this because it allows me to sharpen my teaching style, to improve myself, and to be involved in new challenges, which is useful for finding out new ways for reducing the gap between students and math*".

Valeria watched a number of MOOC videos from the Pre-calculus course. Her reaction indicated some of her orientations:

"*It looks like a good way to foster the collaborative learning, which is also a keyword in the last ministerial directive*".

She concluded that she would need to design new practices for integrating the videos. Valeria, in fact, is used to work with software like Geogebra, where the students have to manipulate mathematical representations and solve problems. In the case of MOOC videos, the students are confronted with mathematical content that is somehow ready-made and not necessarily related to problem-solving activities.

Regarding her long-term goals related to students' learning, Valeria explained:

"*I want my students to approach technologies and multimedia with critical thinking, they should be able to use software and applets properly, they should view this kind of videos, which are very dense, getting the main idea and being able to discern the details*".

Paired with her belief that MOOC videos provide new learning opportunities for her students, it also emerges another belief from Valeria's words, related to differences in her students: she anticipates

"*Not all my classes fit for MOOC videos*".

Lesson Image

To recall, Valeria chooses two classes for this project, referred to as class A and B. Valeria decided to develop two lessons (one for each class), with a common (short-term) goal: to recall the properties of monomial functions (e.g., x^n) and their inverses – i.e., root functions. Valeria's students learned these topics in lower grades. She chose to use the same six-minute video clip for both classes A and B. In the clip, a lecturer addressed the graphs and definitions of monomial and root functions for natural, integer, rational and real exponents. The root functions were presented as inverses of monomial functions, and the notions of oddness, evenness, and symmetry were briefly explained[1].

Class A

Valeria planned to ask the students of Class A to watch the MOOC video at home before the lesson and to answer a list of questions that addressed the definitions, properties, graphs, and relations among monomial and root functions. At the lesson, she planned to divide the students in small groups and to engage them in solving a challenging problem. More precisely, her lesson plan was to spend the first five minutes to divide the students in groups of four, give each group a sheet of paper with the problem on it, and then leave the students alone doing problem-solving for about 25 minutes. The subsequent 15 minutes were planned to be dedicated to a classroom discussion of the problem, then she planned to show a second video clip on exponential functions for introducing students to a new topic[2]. The video lasts 4 minutes, so another 10 minutes are left for Valeria's short frontal lesson.

Valeria considered different challenging problems (some of them have been proposed by us), and eventually chose a 'paper-folding' problem (see Figure 1). The mathematical heart of the problem lies at a geometrical progression that emerges when a piece of paper is consequently folded in halves. The connection between the video and the problem should be found in the function $y=x^n$. Valeria wants her students to recall the properties of this function (and of its inverse, the root function), and then to work on a problem that exploits the notion of power (where the basis is fixed and the exponential varies, to introduce the class to exponential functions), and reaches an unexpected outcome: after relatively few foldings, the paper becomes really thick.

Valeria explained that she chose this problem because "*it comes from the real life and is very mathematical. Moreover, it shows a non-trivial connection between functions and geometric sequences*". To discuss the connection with real life and the mathematical nature of the problem is outside the aim of this chapter, as well as it is not the aim of this work to present and discuss the students' answers to the problem. What is central to our chapter is that, in Valeria's view, the MOOC video is a tool that can help the students to recall some math content, and it is expected to be handled by the students alone, at home, without any guidance from the teacher. Valeria's usage of the video clip is quite similar to her usage of the challenging problem: her beliefs and orientations towards class A shape her expectation that the students will be able to learn from the MOOC video by themselves. In fact, the students are planned to be left alone watching the video (at home) and left alone in solving the problem (in class), before a (short) classroom discussion. We can say that this activity, at least in Valeria's intentions, promotes non-instructional engagement with mathematical content, which is one of the key features of online learning at tertiary level. By non-instructional engagement with a learning activity we mean that students are left free to choose what to do, which resources use and which sources of information resort to, in order to attain an assigned learning task. Instead of being the teacher who decides (and have a full control) on where the classroom activity should go, the students are put in a situation where they have genuine possibilities to choose. This a case of self-directed learning. Are MOOC videos central to the planned activity in class A? To answer to this question, we should consider that the first video serves the purpose of recalling a math content already 'seen' by the students, but they are able to participate to the in class activity even without having watched it. The second video, planned to be watched at the end of the lesson, serves the purpose of recapitulating concepts that would have emerged during small group activity. Hence, group work and problem solving are central to the planned lesson in class A, while MOOC videos are peripheral. Moreover, we are prone to conclude that the frequency of video use is high, both because it is planned for both home work and in class activity, and because in general this class makes a frequent use of online resources.

Consider a thin piece of paper. At the beginning fold it in half, then fold the folded paper in half, then again, and so on.

1. How can we describe this situation mathematically? 2. What will be the thickness of the folded paper after 100 folds, if the thickness of the original piece of paper is 0.1 mm?

Figure 1: The folding-paper problem designed by Valeria

Class B

The students of Class B were planned to watch the first video twice during the lesson. After the first time (6 minutes), the students would create a table with concepts from the video that were familiar and unfamiliar for them (10 minutes). Then, Valeria wanted to replay particular parts of the video clip to help the students with identifying definitions of the key concepts (another 20 minutes). Afterwards, the students were planned to be divided into small groups and discuss the concepts of oddness, evenness and inverse functions. In group discussions the students would have been requested to sketch examples of such functions with GeoGebra and summarize their group work (the remaining 20 minutes). In the case of class B, the teacher *did not plan to assign any homework*: the video would be watched in classroom, and the teacher would have had full control of how many times they watch it, and on what it is relevant to dwell on. In class B the students were not planned to be left free to explore the video.

In class B the students are expected to be divided in small groups, but in Valeria's words there's no indication that she is searching for a challenging problem for her class B students (even if, indeed, the assigned task can be perceived as challenging by the students of class B). We have commented that Valeria's usage of the video in class A is quite similar to her usage of a challenging problem. It seems that the same pairing can be made for class B: teacher-guided watching of the clip and teacher-guided filling of the table. However, the MOOC video is central to this lesson plan, since the in class activity pivots around its first watching and a subsequent replay of some of its parts. The students of class B are expected to work for the entire lesson on the content of the video. But, use of MOOC video is infrequent in this class, since the students are not used to online learning resources.

In-the-moment Decision Making

Valeria started the lesson in Class A with asking if the students watched the video and if they had any question about the concepts that were discussed there. There were no questions, and the class turned to the group work on the paper-folding problem. All the groups answered question 1 correctly.

In one of the groups, the students were working on question 2 and asked Valeria how she came up with the number of 0.1 mm. Valeria redirected the question to the class and asked them to consider possible ways for determining the thickness of a piece of paper. After the lesson, Valeria told us that this question was unplanned. The groups approached Valeria's question differently. One of the groups tried to measure the thickness with standard rulers. When a student in another group noted these unsuccessful attempts, she recalled that "*there exists an instrument for measuring thin things, but I don't remember how it's called*". Then her group engaged in looking for thickness gauge in the Internet and exploring how it works. A student in another group connected between Valeria's question and the given problem, and suggested to fold the paper several times until the thickness becomes measurable with a standard ruler. His group liked the idea and engaged in developing a formula for the thickness of a piece of paper as a function of the measured thickness of the folded paper and the number of folds. Another student noted that this solution is not always practical because the number of times that a piece of paper can be folded is quite limited[3].

If we focus on in-the-moment decisions of Valeria in class A, we can comment that the actual lesson resembles her original plans quite well: the students reported that they watched the video at home, then the students engage in the problem solving activity and the unplanned work on question 2 allowed Valeria to invite the entire class to think about a real-life problem from multiple perspectives. We can see a match between declared beliefs and in-the-moment decision in Valeria's involvement of the entire class in searching for an answer to the second question: the students are expected to solve the problem by themselves, working in groups.

A difference with respect to the planned lesson is that the students did not watch the second video clip on exponential functions. We can infer that another Valeria's belief emerges, namely her appreciation of students' work more than a teacher's 'giving knowledge'. Instead of stopping the students' activity in order to watch the second video (transmissive pedagogy) and to stick to the plan, she decided to let the classroom activity go on and focus on the mathematical relations between physical quantities (connective pedagogy). Her valuing problem-solving activity prioritises certain actions, so the MOOC videos in the implemented lesson become even more peripheral than planned.

In Class B, after showing the video clip for the first time, Valeria invited the students to create a table of familiar and new concepts, as it was planned. However, the students asked to watch the video again because they did not pay attention at the first time. After the second time, they still did not engage in filling in the table and Valeria decided to change the plan: she created a table for the whole class on the whiteboard and replayed particular segments of the video, around ten seconds each, with the concepts and properties that she considered important (e.g., image and domain, graphs). After each video-segment, the students pointed at the central concepts in the segment (practically, repeated the concepts' names) and Valeria explained them by extending the explanations of the lecturer in the video clip. Such interactions continued until the end of the lesson and it did not leave time for the planned group work. The way the lesson in class B turned out to be managed confirms our inference that Valeria has different goals for her two classes, and in particular that in class B she is rather concerned with students' engagement with videos. Also for class B, thus, unplanned decisions were driven by the beliefs and orientations that emerged in the interviews before the lesson start (i.e., the teaching context and lesson image).

Discussion

If we compare the lesson images and in-the-moment decisions regarding classes A and B, we notice that there are not relevant mis-matches. Valeria's lesson plan allows us to have a 'good enough' idea of what actually happened in the two classes, a few days later. Even her decisions for unplanned facts do not take us by surprise: in-the-moment decisions align with her declared goals and beliefs. This situation is interesting since it is not very common: teachers activate different beliefs and thus act differently in different contexts (as Skott, 2015, observes), and different, sometimes contrasting goals emerge. However, this is not the case for Valeria: it seems that Valeria is dealing with a familiar situation.

Valeria's case becomes more interesting, if we consider the striking differences between the two classes: in class A, a high frequency of MOOC video use, despite of being peripheral, is connected to self-directed learning that (at least, apparently) could have been taken place even without the introduction of the MOOC video. In class B, we notice very low frequency of online resources, but the MOOC video becomes central to the lesson, where we see a teacher-guided activity where the students have a few possibilities to explore and manipulate the mathematical content that is offered to them. However, it is exactly this striking difference between A and B that can help us finding a reason for such a strong match between the planned and the implemented lesson, the beliefs enacted and the goals emerged in either context: to our interpretation, class A is 'too good' and class B is 'too poor' in Valeria's expectations, that (with her long and deep teaching experience) she can plan a lesson that resembles more or less exactly what that is going to take place in the actual class. In other words, self-directed learning takes place in class A even without the introduction of a new learning format, which lays at the background. In class B, the MOOC video is brought at the foreground by the teacher, exactly because the class is not used to self-directed learning formats, but the MOOC video is used in a way that is all but promoting non-instructional engagement.

The striking difference between classes A and B allows us to further notice that a hierarchy emerges in Valeria's belief system, namely: her expectations about her students take the priority when it is time to decide the mathematical activity, while the way the video can be used in order to promote self-directed learning comes secondly. This tells us that the classroom is not passive in the eyes of the teacher: she knows that the students respond to her prompts, and she knows how do they respond. However, her (negative) expectations about class B somehow freeze Valeria from proposing something 'really new' to them. In this way, the students' different engagement with the task in classes A and B results to be less influenced by the use of technology and more a reflection of the different mathematical attitudes, or motivation to learn. This was anticipated by Valeria in her lesson image, and things went the way they were expected: as a conclusion, Valeria's beliefs about her classes shaped the way the two lessons resulted, and the students' reaction. In order to understand the role of a teacher's beliefs and resources in shaping the use of videos, more nuances need to be added before we can make some conclusions. The cases of Nicoletta and Lorenza are, thus, presented in the next chapter.

Endnotes

[1] The video can be watched in Italian at https://youtu.be/15wQiw8fOLs

[2] The video can be watched in Italian at https://youtu.be/MT9dSVmq2Ek

[3] We refer the reader to the work of Gallivan for an exploration of the idea of folding paper *n* times, and an empirical proof for 12 folds. The reader can also see http://pomonahistorical.org/12times.htm

References

Anthony, A.B. (2012). Activity theory as a framework for investigating district-classroom system interactions and their influences on technology integration. *Journal of Research on Technology in Education, 44*(4), 335–356.

Berk, R. A. (2009). Multimedia teaching with video clips: TV, movies, YouTube, and mtvU in the college classroom. *International Journal of Technology in Teaching and Learning, 5*(1), 1–21.

Bishop, A., Seah, W.T., & Chin, C.(2003).Values in mathematics Teaching－The Hidden Persuaders?, In: *Second International Handbook of Mathematics Education*, Dordrecht, NL: Kluwer Academic Publishers. Printed in Great Britain (pp.717-765).

Bosch, M., Fonseca, C., & Gascon, J. (2004). Incompletud de las organizaciones matematicas locales en las instituciones escolares. *Recherches en Didactique des Mathématiques, 24*(2–3), 205–250.

Clark, M. and Lovric, M. (2008). Suggestion for a theoretical model for secondary-tertiary transition in mathematics. *Mathematics Education Research Journal, 20*(2), 25-37.

Clark-Wilson, A., Robutti, O., and Sinclair, N. (2014). *The mathematics teacher in the digital era: an international perspective on technology focused professional development*. Dordrecht, NL: Springer.

Cuban, L. (2001). *Oversold and underused: computers in the classroom.* Cambridge, MA: Harvard University Press.

H. Fredriksen, S. Hadjerrouit, J. Monaghan & R.J. Rensaa (2017). Exploring tensions in a mathematical course for engineers utilizing a flipped classroom approach. In: *Proceedings of the 10th Conference of the European Society for Research in Mathematics Education*. Dublin, IR: CERME.

Gamer, B. E. and Gamer, L. E. (2001). Retention of concepts and skills in traditional and reformed applied calculus. *Mathematics Education Research Journal, 13*(3), 165-184.

Hannafin, R. D., & Foshay, W. R. (2008). Computer-based instruction’s (CBI) rediscovered role in K–12: An evaluation case study of one high school’s use of CBI to improve pass rates on high-stakes tests. *Educational Technology Research and Development, 56*(2), 147–160.

Hibert, J. and Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analyis. In: *Conceptual and procedural knowledge; The case of mathematics,* pp, 1-23.

Labaree, D. F. (2003). The peculiar problems of preparing educational researchers. *Educational researcher*, *32*(4), 13-22.

Moore, R.C. (1994). Making the transition to formal proof. *Educational Studies in Mathematics, 27*, 249-266.

Niegemann, H.M., Domagk, S., Hessel, S., Hein, A., Hupfer, M., & Zobel, A. (2008). *Kompendium multimediales Lernen.* Springer-Verlag.

Penuel, W. R., Fishman, B. J., Haugan Cheng, B., & Sabelli, N. (2011). Organizing research and development at the intersection of learning, implementation, and design. *Educational Researcher, 40*(7), 331–337.

Schoenfeld, A. H. (2011). Toward professional development for teachers grounded in a theory of decision making. *ZDM, 43*(4), 457-469.

Sherin, M. G. (2003). New perspectives on the role of video in teacher education. *Advances in Research on Teaching, 10*, 1–27.

Shimada, I. & Baba, T. (2015). Transformation of Students' Values in the Process of Solving Socially Open-Ended Problems. In: *Proceedings of the 39th Conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 161–168). Hobarth, AU: PME.

Skott, J. (2015). Towards a participatory approach to 'beliefs' in mathematics education. In B. Pepin & B. Roesken-Winter (Eds.), *From beliefs to dynamic affect systems in mathematics education* (pp. 3-23), Heidelberg, DE: Springer.

Tall. D. H. (1991). *Advanced Mathematical Thinking*. Kluwer, Dordrecht.

U.S. Department of Education. (2010). Transforming American education: Learning powered by technology: National Educational Technology Plan 2010. Washington, DC: Office of Educational Technology.

van de Sande, C. (2011). A description and characterisation of student activity in an open, online, mathematics help forum. *Educational Studies in Mathematics, 77*, 53-78.

Wagner, J. (1997). The unavoidable intervention of educational research: A framework for reconsidering researcher-practitioner cooperation. *Educational Researcher, 26*(7), 13-22.

Wenglinsky, H. (2005). *Using technology wisely: The keys to success in schools*. New York: Teachers College Press.