

HYDRODYNAMIC OPTIMISATION OF A POINT WAVE-ENERGY CONVERTER USING LABORATORY EXPERIMENTS

by

Scott Kelly

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Summary

Investment in renewable energy technology, such as wave power, is increasingly seen as a beneficial and economically-viable alternative to existing fossil-based power plants. New Zealand has particularly energetic ocean waves and is well positioned to harness wave energy as a source of power. This thesis presents an in-depth analysis of scale-model experiments conducted on a novel WEC (Wave Energy Converter) device called the 'spar fork'. The model was built using Froude scaling, with a dimensional scale ratio of 1:25 between model and prototype.

Tests were performed on the device in a 20m long wave flume, with waves being created and measured in the flume using a specially designed control system created in LabVIEW 8.0. Wave flume operation was validated by showing strong correlation between experimental results and dispersion theory. Sub-surface velocity profiles were shown to compare well with Stokes' Second Order theory. Wave reflection analysis showed that reflected wave heights for the newly-installed flume beach were approximately 5% of the total wave height, which is well inside the acceptable range for wave flume modelling - that is, less than 10%.

Friction-torque for the spar fork model (representing the generator resistance of the prototype) was evaluated using pendulum decay methods. With software developed in MATLAB, angular velocities of the moving parts of the spar fork were measured using image processing and particle tracking methods. From the measurements taken, the power generated could be calculated for a number of wave states, also providing the opportunity to test a number of spar fork configurations.

It was found that the optimum spar fork design will include a significantly buoyant spar section tightly moored to the sea floor by no less than two mooring lines. The arm of the spar fork was optimum when its length was equal to the wave height. It is recommended that active float size is also maximised, as both these variables increase the amount of torque that can be generated. Designing the spar to be of natural heave frequency less than, but approaching the frequency of the design wave climate is particularly advantageous for power generation (although challenging physically). Further device recommendations are given in the conclusions.

For the expected wave climate on the west coast of New Zealand (using significant wave properties), the total power output of the tested device is shown to increase from 2.2kW for the original design to 5.4kW after optimisation.

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Nomenclature

Variables used to define waves

k	angular wave number ($2\pi / \lambda$)
x	horizontal displacement from frame of reference (m)
a_i	incident wave amplitude (m)
F_0	magnitude of the force created from a wave (N)
H_{\max}	maximum wave height (m)
p	pressure (Pa)
a_r	reflected wave amplitude (m)
H_s	significant wave height (m)
T_s	wave period (s)
\bar{T}	significant wave period (s)
a_x	subsurface horizontal wave particle acceleration (ms^{-2})
u	subsurface horizontal wave particle velocity (ms^{-1})
u_0	subsurface horizontal wave amplitude particle velocity (ms^{-1})
a_y	subsurface vertical wave particle acceleration (ms^{-2})
w	subsurface vertical wave particle velocity (ms^{-1})
y	vertical displacement from the still water level (m)
h	water depth (m)
A	wave amplitude
ω	wave angular frequency (rads^{-1})
f	wave frequency (s^{-1})
V_g	wave group speed (ms^{-1})
H	wave height (m)
λ	wave length (m)
M, M_0	wave moments (N.m)
T	wave period (s)
v	wave phase speed or wave celerity (ms^{-1})
η	wave surface profile from still water level (m) (also used to represent efficiency)

Common variables

m_a	added mass (kg)
\hat{I}_{yy}	area moment of inertia (m ⁴)
u	breach height above the still water level of object (m)
$L_{m,p}$	characteristic length of model/prototype (m)
ω_d	damped natural frequency (rads ⁻¹)
T_d	damped period (s)
ρ	density of water (998 kgm ⁻³)
D_d	diameter of disc (m)
ξ	dimensionless damping factor
ζ	distance between wave capacitance probes (m)
F_d	drag force (N)
τ_f	friction torque (N.m)
L, r, d	geometric properties of object (length, radius, distance) (m)
g	gravitational constant (ms ⁻²)
F_g	gravitational force (N)
$\nu_{m,p}$	kinematic viscosity of model/prototype (m ² s ⁻¹)
E_K	kinetic energy (J)
l	length of pendulum (m)
δ	logarithmic decrement for an exponentially decaying pendulum (also used by Isaacson (1991) to represent the phase difference between probes)
m	mass (kg)
I	mass moment of inertia (N.s ² .m)
P_{\max}	maximum possible power achievable (W)
ω_n	natural frequency (rads ⁻¹)
$\omega_{n,p}$	natural frequency in pitch (rads ⁻¹)
σ	phase difference between the wave and the wave force (rad) (also used to represent the statistical quantity of standard deviation)
τ	phase difference between an object and a wave (rad)
ε	phase difference between an object relative to a wave (rad)
E_P	potential energy (of a wave) (J)
U_T	potential energy (of a pendulum) (J)
P	power (W)
θ_R	relative angular displacement of between arms on the spar fork (rad)
Δ	release height of pendulum (deg)
α	scale factor ratio (usually with subscript 'L' designating length)
c	spring stiffness (modulus) (N.m ⁻¹)

$c_{\theta\theta}$	spring stiffness in pitch ($\text{N}\cdot\text{m}^{-1}$)
U_0	subsurface velocity amplitude (ms^{-1})
t	time (s)
E_T	total energy contained within a wave (kinetic plus potential)
a	virtual mass ($m_a + m$) (kg)
b	viscous damping coefficient ($\text{N}\cdot\text{sm}^{-1}$)
F_v	viscous force (N)
∇	volume (m^3)
A_w	water plane area (m^2)
U	wave flume current (ms^{-1})
\mathbf{W}_T	work (J)

