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# Broadband Teleportation and Entanglement in Cascaded Open Quantum Systems

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# Abstract

Quantum optics provides powerful means to probe quantum mechanics. In this thesis, we study various aspects of quantum phenomena arising in quantum optical systems. Part I studies broadband quantum teleportation. After presenting three different methods of analyzing the standard teleportation protocol, we study the interplay between various bandwidths in determining the fidelity of a broadband quantum field teleportation. Explicit formulae for the degrees of first- and second-order coherence for the teleportation of resonance fluorescence are derived for this purpose. Part II studies entanglement arising in cascaded open quantum (optical) systems. First, a detailed laser model is produced within quantum trajectory theory to study the total decoherence rate of a laser-driven qubit. Second, using this model, we address the issue of laser quantum state, viewed in connection with separability of the laser-driven-qubit system. Third, a measure of entanglement within quantum trajectory theory called ‘Contextual Entanglement’ is calculated for a few simple systems and compared with the ‘Entanglement of Formation’. Lastly, we introduce a method to quantify entanglement (based on the contextual entanglement) between a source and the field it emits, which we call the ‘Entanglement Spectrum’. It is applied to study the entanglement between a laser-driven qubit and the field the qubit scatters.



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*Changsuk Noh*  
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# List of abbreviations

Stochastic electrodynamics (SED)  
Einstein, Podolsky, and Rosen (EPR)  
Quantum trajectory theory (QTT)  
Gea-Banacloche (GB)  
van Enk and Kimble (vEK)  
Entropy of Entanglement (EE)  
Local operations and classical communications (LOCC)  
Entanglement of formation (EF)  
Distillable entanglement (DE)  
Relative entropy of entanglement (REE)  
Contextual entanglement (CE)  
Entanglement spectrum (ES)





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