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Non-routine problem solving through the lens of self-efficacy

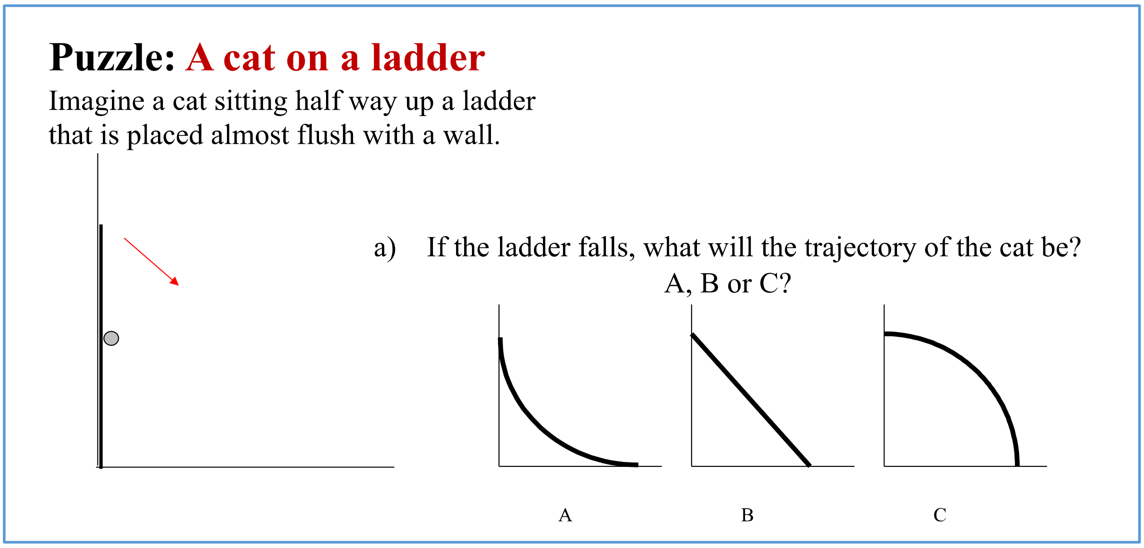
This study involved an intervention designed to examine whether employability prospects for STEM students studying mathematics could be improved. It incorporated use of non-routine problem solving in second year mathematics courses at two New Zealand universities. From a theoretical standpoint, we conceptualised a novel construct called *lateral thinking self-efficacy,* which is defined as a learner's confidence in their ability to solve non-routine problems. It relates to the creative thinking ability needed for solving innovative real-life problems in the work place, and hence is pertinent in transfer of mathematical learning to novel domains. The findings suggest that the attitude profiles of students with high and low *lateral thinking self-efficacy* differ significantly with respect to three dimensions spanning their affective domain. Further, a significant difference between genders with regard to *lateral thinking self-efficacy* was observed, with a significantly greater proportion of confident males than females, although there was no significant association between gender and non-routine problem solving performance. These results raise questions about equity with regard to employability prospects for females in STEM companies and have implications for the underrepresentation of women in STEM fields.

Keywords: employability; self-efficacy; undergraduate mathematics; STEM education; gender

Introduction

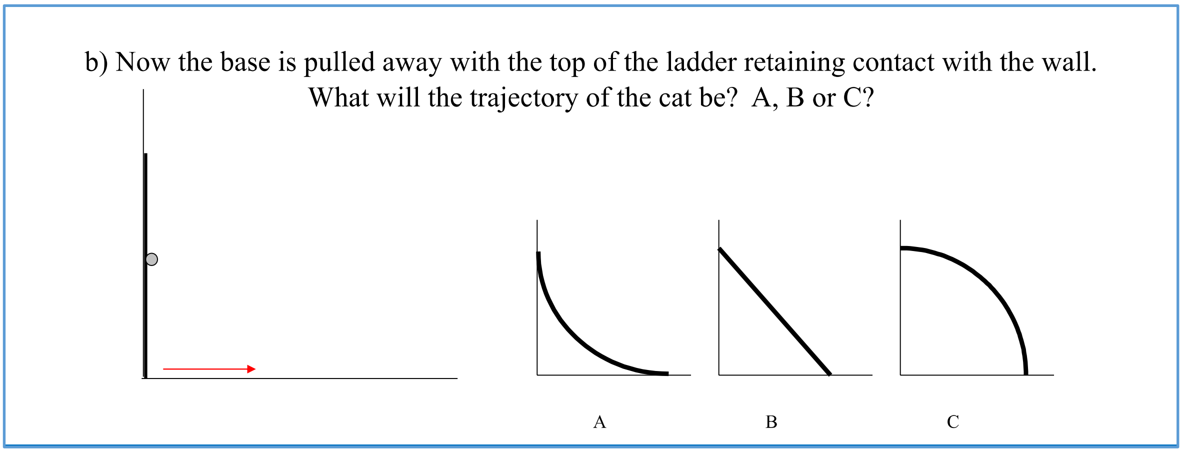
Imagine yourself as a graduate, with an affinity for mathematics, going to your first job interview. As part of the interview process, you are asked to solve the following puzzle:

Figure 1. Example of a puzzle – part (a)



The correct answer is C. You got it right! Next, you are presented with the same scenario but with a change in assumptions:

Figure 2. Example of a puzzle – part (b)



According to numerous observations, it is likely that your answer is A or B. If so, sorry, it is wrong and your chance of getting the job might have slipped away! The solution is rather simple if using a particular approach and basic mathematics (see Appendix). As pointed out by Klymchuk (2017), many companies use puzzles as part of their job interview process to evaluate candidates' problem-solving skills and select the best of the best. They believe that the ability to solve puzzles relates to the creative thinking needed for solving innovative real-life problems.

In today's fast-changing labour market, the most in-demand occupations, such as data scientists, app developers, or cloud computing specialists, did not exist ten, or even five years ago. The most recent report by the World Economic Forum predicts that technological breakthroughs, rapidly shifting the frontier between work tasks performed by humans and those performed by machines and algorithms, are likely to transform global labour markets: "technological change and shifts in job roles and occupational structures are transforming the demand for skills at a faster pace than ever before." (The Future of Jobs 2018, World Economic Forum, p. 22). To succeed in such a rapidly evolving employment landscape, recent graduates need to demonstrate not only their content mastery but also intellectual flexibility and ability to adopt to novel settings. As pointed out by Barnett and Ceci (2002, p. 613) "Much of the financial and human investment in education has been justified on the grounds that formal schooling helps inculcate general skills that transfer beyond the world of academia and thus help students become more productive members of society."

In the last decade there has been an increased focus globally on identifying graduate attributes and employability skills (Bridgstock, 2009; Clarke, 2017; Kensington-Miller, Knewstubb, Longley, & Gilbert, 2018). Attributes highly valued by employers include the so-called 'C' skills: Creativity (which is often associated with lateral thinking), Curiosity, and Critical thinking. Some involved in university education see the need to produce well-rounded individuals with higher thinking skills as of paramount importance, leading to less emphasis on the employability of students. However, while the issue of training students for future employment is by no means the only consideration in gaining a university education, often the value of their qualification for employability is a major concern for the students themselves.

In light of the above, the importance of transfer of learning has been highlighted in recent considerations of graduate attributes in many higher educational settings (Litchfield, Frawley, & Nettleton, 2010). In their comprehensive literature review on transfer of learning Pugh and Bergin (2006, p. 147) state that “Without transfer, the relevance of formal schooling is limited. Therefore, anything that significantly affects the probability of transfer should be studied carefully, and techniques that foster transfer should be implemented in schools and other instructional settings”. In their research synthesis, focused on the importance of motivational influences on transfer, Pugh and Bergin (2006) provided a thorough analysis of such factors by reviewing studies that examined the influences of motivation on cognitive processes related to transfer. In doing so they identified four motivational constructs, namely: achievement goals, interest, self-efficacy, and intentional transfer, and recommended that future research utilise them.

In this research, we recognised and conceptualised the main phenomenon of our study guided by Pugh and Bergin’s (2006) synthesis of motivational influences on transfer of learning. Thus, the tool of our analysis aligned with self-efficacy, one of the four motivational constructs they identified. We investigated the mechanisms involved in the transfer of learning in the context of non-routine problem solving through this lens of perceived self-efficacy, which is defined by Bandura (1997, p. 3), as “beliefs in one’s capabilities to organize and execute the courses of action required to produce given attainments”. In the context of transfer, “self-efficacy usually refers to confidence in the ability to do or learn a skill that can transfer to another domain” (Pugh & Bergin, 2006, p. 153). A body of knowledge has been accumulated across many educational domains pertaining to the mechanisms by which it influences transfer of learning.

In summary, research suggests that self-efficacy is positively associated with transfer of learning through mechanisms that include its influence on cognitive engagement (metacognition, use of effective strategies) and persistence (Ford, Weissbein, Smith, Gully, & Salas, 1998; Pugh & Bergin, 2006).

This study was situated within a long-term project with an overarching goal of improving employability attributes of graduates by enhancing students' generic thinking skills and enabling transfer of learning through the regular use of non-routine problems. It was conducted alongside an intervention studying the influence of incorporating regular use of non-routine problem solving in STEM courses on employability prospects, and had three aims:

- To conceptualise the specificity of students' confidence with regard to applying creativity to solving non-routine problems by defining a construct of *lateral thinking self-efficacy* and exploring its discriminant validity;
- To investigate performance in solving non-routine problems and the role of *lateral thinking self-efficacy* with respect to gender; and
- To analyse the interplay between students' *lateral thinking self-efficacy* and other aspects of students' perspectives on non-routine problem solving: their feelings, beliefs and attitudes toward it.

We note that the testing of the efficacy of the intervention was not an aim of the study and will be addressed in future research.

Self-efficacy and mathematics

Mathematics-specific self-efficacy has been shown to be an important construct for mathematics education, and is generally viewed as a belief about one's capacity for doing mathematics (e.g. Pajares & Miller, 1994). Methods for measuring it have been developed and used to demonstrate that students who develop higher mathematical self-efficacy tend to show great interest, effort, persistence, help-seeking behaviour, and, ultimately, greater mathematics achievement than those who feel that their efforts in mathematics have less efficacy (Pajares & Miller, 1994; Schukajlow, Achmetli, & Rakoczy, 2019; Skaalvik, Federici, & Klassen, 2015).

The relationship between mathematics self-efficacy and the process of career choice emphasises the importance of self-evaluation of competence as much as the learners' actual performance. As Hackett and Nancy (1989) note: "In fact, cognitions concerning competence may be a much more critical factor than measured abilities in both educational and career choice processes, particularly for women pursuing non-traditional options" (p. 271). Further, Hannula pointed out that "[p]erhaps the most robust research finding in mathematics-related affect is that female students have on average lower self-efficacy in mathematics than male students" (Hannula, 2016, p. 2; see also Else-Quest, Hyde, & Linn, 2010). According to the findings of Hackett and Nancy (1989), the gender differences in the realisation of mathematics self-efficacy expectations are not statistically significant, hence suggesting that "to a great extent sex differences in mathematics self-efficacy expectations are correlated with sex differences in mathematics performance." (p. 270).

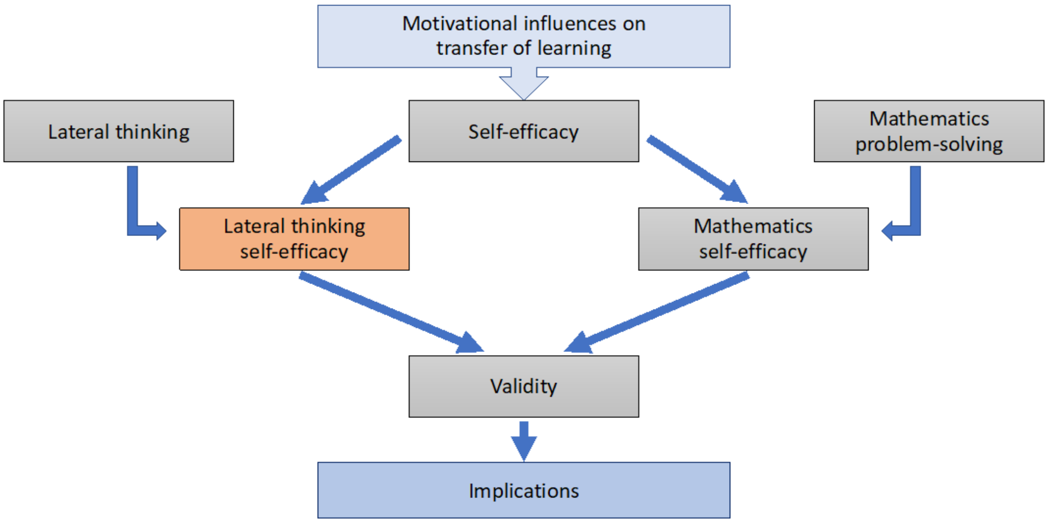
Self-efficacy and lateral thinking

Creativity, a central focus of our study, has been defined in a number of ways in the literature. One is the idea that analytical thinking is convergent and vertical, while creative thinking is divergent and lateral. This concept of lateral thinking is due to de Bono (1990) who distinguishes it from analytical, saying that "Lateral thinking is concerned with changing patterns. Instead of taking a pattern and then developing it as is done in vertical thinking, lateral thinking tries to restructure the pattern by putting things together in a different way."

(1990, p. 38). It also involves changing the focus of attention in order to apply analytical thinking in another direction. Thus, he maintains (*ibid.*) that rather than being an attack on vertical thinking lateral thinking is a method of making it more effective by adding creativity to it.

As mentioned above, a key theoretical concept here is self-efficacy. Due to its behavioural and situational specificity, Bandura (1997) described self-efficacy as a construct that includes a subset of task contingent skills that vary in importance based upon task demands. Oriented in this way, we define a novel construct of *lateral thinking self-efficacy* in order to capture the specificity of students' confidence in regard to applying creativity to solving non-routine problems. Central to this is the distinction between this construct and one capturing a learner's perceived confidence in mathematical problem-solving – mathematics self-efficacy – which is their "belief in advance about their likelihood of successfully performing a particular mathematical task" (Foster, 2016, p. 273). Guided by our overarching aim to improve learners' employability prospects, we saw value in developing a more nuanced perspective by conceptualising the main phenomenon of our study as *lateral thinking self-efficacy*, which is a learner's confidence in their ability to solve non-routine problems. This relates to the creative thinking ability needed for solving innovative real-life problems in the work place, and hence is pertinent in transfer of mathematical learning to novel domains. The connections between these different constructs used in this study are illutrated in Figure 3.

Figure 3. Conceptual diagram: *lateral thinking self-efficacy* construct



One assumption in this explorative study is that the construct of *lateral thinking self-efficacy* in the context of mathematical problem-solving is instrumental in understanding the mechanisms involved in transfer of mathematical competence to novel domains. Through the analysis of empirical evidence, we anticipated seeing that the construct of *lateral thinking self-efficacy* in the context of mathematical problem-solving is principally distinct from the well-known construct of mathematics self-efficacy. In that way, we set out to explore its discriminant validity. Moreover, we assumed that *lateral thinking self-efficacy* can serve as a useful tool in analysing students' performance and attitudes towards non-routine problem solving, thus providing means for delineation of implications related to learners' employability prospects.

Method

Intervention: site and design principles

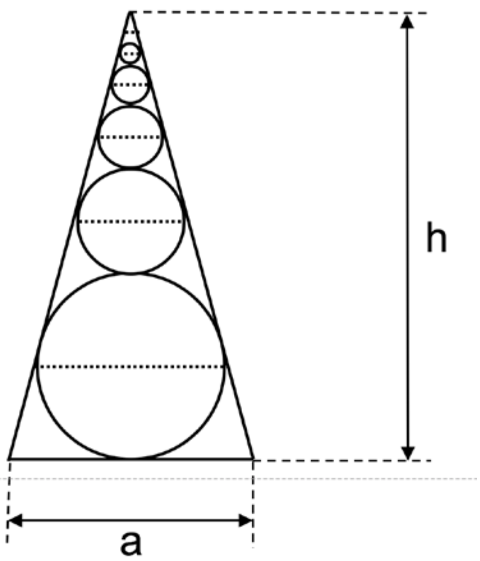
The research, conducted at two New Zealand universities: the University of Auckland and Auckland University of Technology, employed a case study methodology. The team of mathematics education researchers (the authors), in collaboration with lecturers, designed, developed and implemented an intervention in two second-year mathematics courses that formed a single case, with individual student responses as the unit of analysis. The project was guided by an aim to design an intervention that was practical and could be easily integrated into existing university courses by slightly tuning the instructional practice, and hence require only a small developmental investment. Accordingly, slight changes were made to the course lectures alone. None of the other components of the instructional delivery (tutorials, assignments, tests and examinations) were altered. In the middle of every lecture, instructors involved in the trial inserted an unfamiliar non-routine problem and asked students to spend a few minutes trying to solve it. The students were free to choose either to work in a small group or individually. Afterward, possible solutions to the problem were briefly discussed. The ways a solution would emerge varied. Sometimes students would offer a possible answer and, if correct, they would be prompted by the lecturer to elaborate on their solution. At other times, students did not volunteer solutions and so were presented with an explanation by the lecturer. The duration of this activity was in the range of 2-5 minutes.

The courses in the trial were delivered over a standard New Zealand semester with 12 teaching weeks. There were three 50-minute lectures per week and, with the exception of the intervention, all were conducted in lecturing style, using direct instruction mode. In total, the scale of our intervention was small, amounting to solving and discussing up to 30 non-routine problems, but comprising less than 5% of lecture time.

Non-routine problems

In the trial, we defined non-routine problems to be of the following types: puzzles, paradoxes or sophisms. By a puzzle, we mean a non-standard, unstructured question presented in an entertaining way. Some authors treat a puzzle as an antithesis to a mechanical and boring routine problem that can only be solved through the drudgery of long, complex calculations (Gnadig, Honyek & Riley, 2001). Often authors distinguish between a puzzle and a procedural problem noting an important characteristic of puzzles is that they cannot be solved by rote and hence are invaluable in making students think (Thomas, Badger, Ventura-Medina, & Sangwin, 2013). Along the same line, in our project, non-routine problems are characterised by the addition of cognitive incongruity triggering surprise, curiosity and/or confusion. Here are two examples: Consider Fig. 4. What is the sum of the diameters of the infinitely many circles inscribed into this isosceles triangle? The solution becomes almost obvious if you use lateral thinking and change your perspective in considering the question. Second example: Fifty five players start a (singles) tennis tournament. How many matches will be played if a player who loses a match leaves the tournament? (See the Appendix for solutions)

Figure 4. Puzzle: infinite sum of diameters.



By a paradox, we mean a surprising, unexpected, counter-intuitive statement that looks invalid but in fact is true. A sophism is characterised by intentionally invalid reasoning that looks formally correct, but in fact contains a subtle mistake or flaw. The existing literature confirms that the use of non-routine problems in teaching and learning engages students’ emotions, creativity and curiosity and enhances their critical thinking skills and ability to engage in lateral thinking, or thinking “outside the box”. The theoretical pedagogical perspective of the project was based on the Puzzle-Based Learning approach developed by Michalewicz and Michalewicz (2008) that has been adopted in many educational settings worldwide. A number of publications have reported positive feedback from students, lecturers and researchers on the regular use of this pedagogical strategy (see Falkner, Sherwood, & Michalewicz, 2012; Klymchuk, 2017; Thomas et al., 2013).

Data collection and analysis

A student survey was conducted in class at the end of the semester using a paper-based questionnaire comprising ten questions, including three on demographics. The students were prompted to provide answers in an open-ended way (e.g. a ‘Yes or No’ question was followed by ‘Why?’ or “Please give the reasons’) with space for any unsolicited comments that they might have. The design of the questionnaire was guided by the specific aims of the study, including operationalisation of the construct of *lateral thinking self-efficacy* and investigation of the interplay between students’ *lateral thinking self-efficacy,* performance, gender, prior-achievement, and their feelings, beliefs and attitudes towards non-routine problem solving (the questionnaire can be found here http://doi.org/[10.17608/k6.auckland.8009921]). In total, we recorded and analysed data from 137 students (81 males, 53 females, 3 unidentified) from two second-year mathematics courses, with a response rate of 97% of those present in the lecture.

The analysis was conducted in two sequential phases. The first phase involved qualitative analysis of students’ responses eliciting their perspectives on aspects of the intervention. In the second phase, we employed a quantitative technique used to investigate frequency counts in each of the themes identified in phase one. These were used to determine the relative frequency of differential responses to the phase one themes from students with high versus low *lateral thinking self-efficacy*. The sequential two-phase data analysis approach was based on recent methodological advances in mathematics education research (Moyer, Robison, & Cai, 2018). It has been an effective approach to uncover statistically significant differences in the empirical manifestation of theoretical constructs, and was aligned with a recommendation to reduce reliance on Likert-style instruments in attitude research and move toward greater use of narratives (see Di Martino & Zan, 2015).

Results

Consistently with research pertaining to validation of instruments assessing self-efficacy, we operationalised *lateral thinking self-efficacy* of individual learners by evaluating their perception of confidence in solving non-routine problems at the end of the trial. The analysis was carried out using a Chi-square test of independence. This comprised two variables: X, the number of students within three categories representing students' grades in the pre-requisite mathematics courses (the grades were grouped into the following grade bands: A-grade band: A+, A, A-; B-grade band: B+, B, B-; and C-grade band: C+, C, C- to ensure categorical frequency counts were large enough) and Y, dichotomous representing the number of students' 'Yes' or 'No' responses to the question 'Do you feel confident solving the puzzles?' The null hypothesis for this Chi-square independence test is that the students' grades in the pre-requisite mathematics courses and whether they feel confident in solving the puzzles are independent (see Fig. 5). In this case no association between the categorical variables was observed and so they were deemed to be independent ($\chi^2(2)=1.14$, $p=.566$, n.s.), meaning that prior performance in mathematics does not appear to say anything about confidence in solving non-routine problems.

Figure 5. Confidence in solving non-routine problems by grade in prerequisite mathematics course.

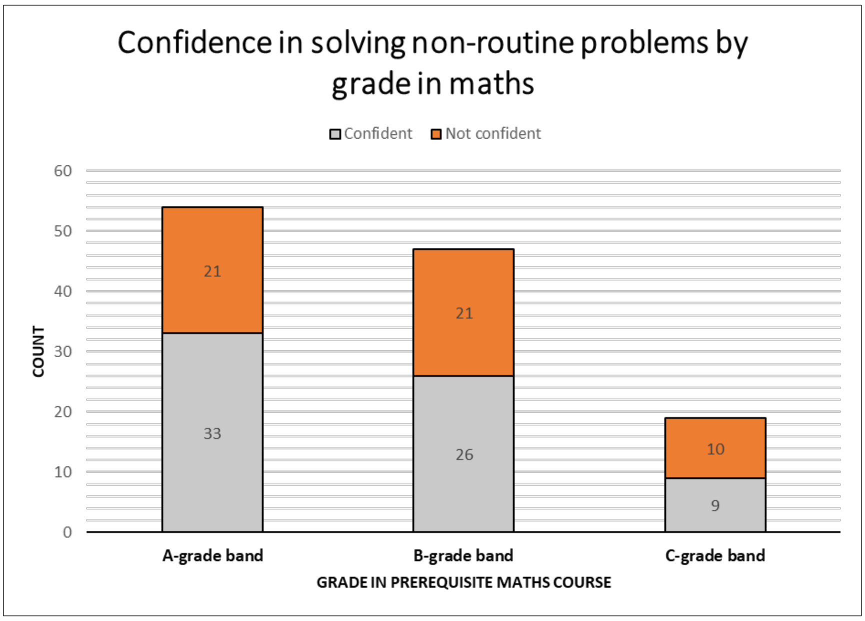
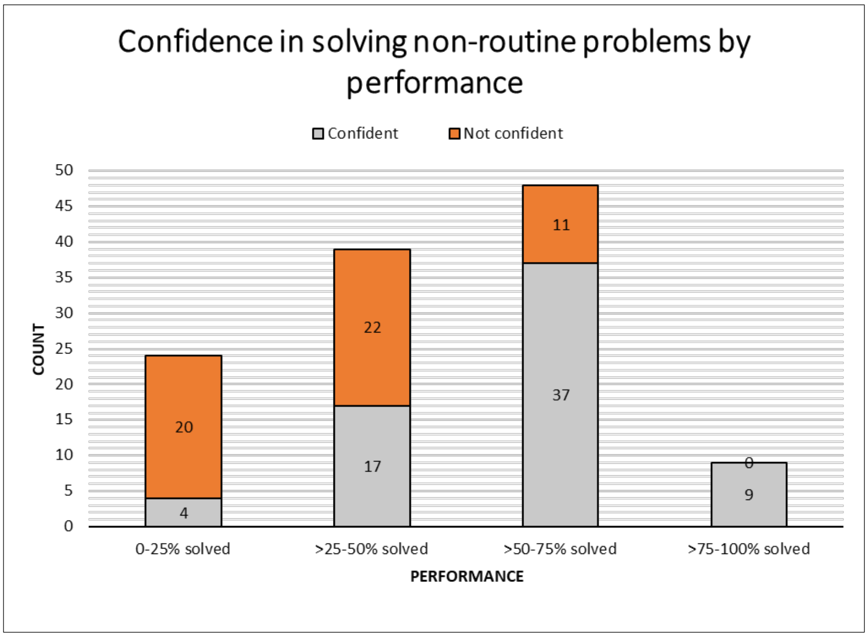


Figure 6. Confidence in solving non-routine problems by performance on those problems.



This lack of a significant relationship underscores the difference between our construct of *lateral thinking self-efficacy* and mathematics self-efficacy, since there is an extensively researched body of knowledge demonstrating a statistically significant reciprocal relationship between mathematics self-efficacy and mathematics performance (Hackett & Betz, 1989; Hannula, 2016, Pajares & Miller, 1994; Usher & Pajares, 2008).

In sharp contrast with the finding above, we were able to make the following observation when keeping Y the same but changing the variable X to four categories representing students' self-reported performance in solving non-routine problems during the course of the trial (see Fig. 6). In this case there was a highly significant difference ($\chi^2(3)=33.21$, $p<.000001$), identifying an association between students' confidence in solving non-routine problems and their performance in solving them. The effect size of the association, measured by Cramér's *V*, is equal to 0.53, which is considered substantial, indicating a moderate association between the categorical variables.

Gender

The impact of gender differences on many expressions of confidence has been reported across many domains, and so, recognising the importance of this field of study, we investigated the existence of an association between gender and confidence in solving non-routine problems (see Fig. 7).

Figure 7. Confidence in solving non-routine problems by gender.

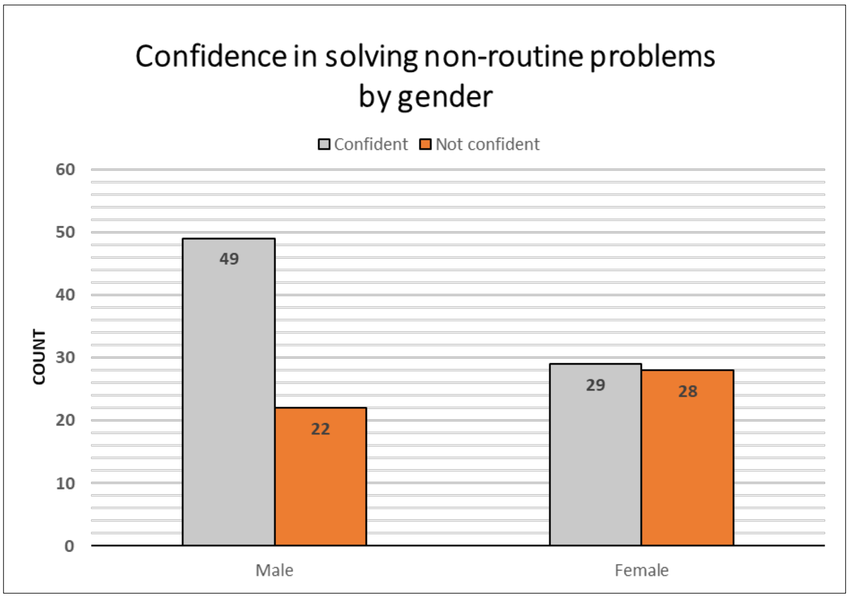
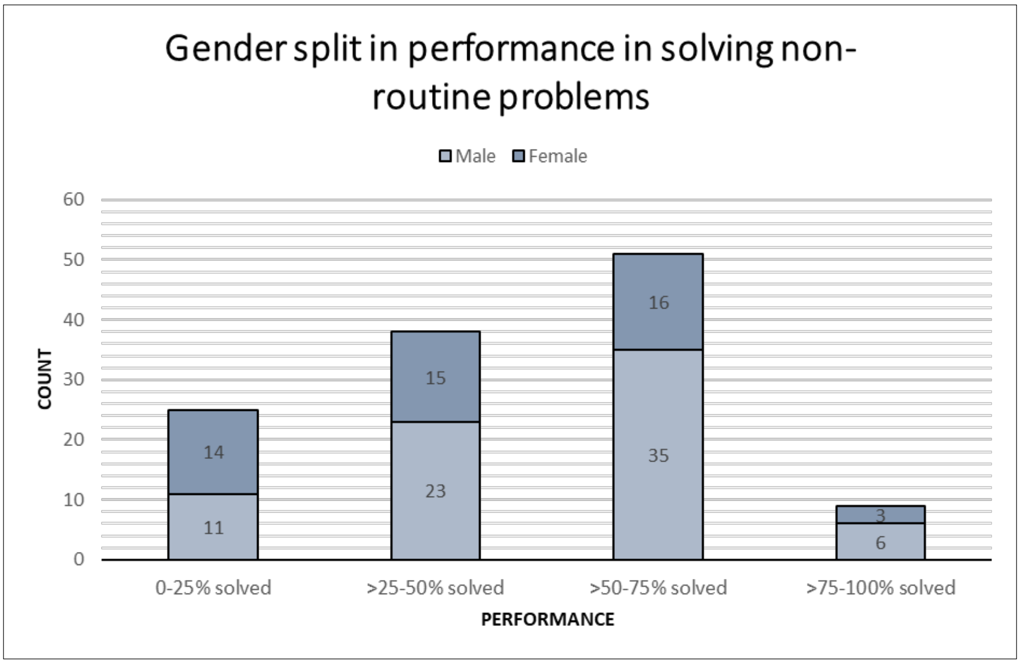


Figure 8. Gender split in performance categories in solving non-routine problems.



In this case the Chi-square test for independence was significant ($\chi^2(1)=4.37$, $p<.05$) implying an association between the variables gender and confidence in solving non-routine problems (Cramér's $V=.18$). A *z*-test for proportions confirmed that the proportion of confident males was significantly greater than the proportion of confident females ($z=2.09$, $p=.037$). This was despite there being no significant association between gender and non-routine problem solving performance, as shown in Figure 8, $\chi^2(3)=4.41$, $p= .221$, n.s.

The combination of the two sets of data in regard to gender underscores an important distinction between our construct of *lateral thinking self-efficacy* in mathematics-related contexts and mathematics self-efficacy. As noted in the introduction, the gender differences in the realisation of mathematics self-efficacy expectations are not statistically significant, hence suggesting that gender differences in mathematics self-efficacy expectations are correlated with gender differences in mathematics performance. However, our findings suggest a statistically significant difference between genders in regard to *lateral thinking self-*

efficacy in mathematically-related contexts, while showing no association with self-reported performance utilising lateral thinking.

Students' attitudes profiles towards non-routine problem solving

In order to discern possible relationships between students' *lateral thinking self-efficacy* and other aspects of their perspectives pertaining to non-routine problems, we analysed each response from the questionnaire for explicit or implied evidence of student feelings, beliefs, and attitudes towards non-routine problem solving. During the trial we purposely did not use the term 'non-routine' problems to minimise conforming reactivity of our subjects. Instead, we referred to the intervention as solving puzzles. In the questionnaire the students were asked about the main differences between puzzles and routine problems, whether or not they think that solving the puzzles can enhance their generic thinking skills and problem-solving skills, and whether or not they see any other benefits for them in solving the puzzles. We used a similar approach to that of Moyer, Robison and Cai, (2018), in which the students' responses are read and annotated with the aim to identify all of the major attitudinal themes directly articulated or indirectly implied by the students. We grouped students' responses into thematic chunks and assigned codes to indicate potential themes that each chunk suggested. After a few rounds of revision each researcher independently further refined the codes and revised decisions about the grouping into thematic chunks. As a result we agreed on eight major themes that could be used to represent the students' expressed and implied attitudes towards non-routine problem solving (see first column in Table 1).

In line with Moyer et al. (2018), while controlling for *lateral thinking self-efficacy* (high versus low), we performed a categorical content (or thematic) analysis where categories are defined, with separate examples of the text extracted, classified and gathered into the categories, or themes. We then recorded frequency counts of responses that appeared in each theme in order to determine the relative frequencies with which the students from the two groups responded similarly or differently to the themes that emerged (Table 1).

The eight themes that were identified using thematic analysis were grouped into three dimensions (categories): Vision of non-routine problems (differentiating disposition), Enhancement utility disposition, and Emotional disposition based on Di Martino and Zan's analytical model (2015). We followed Moyer et al.'s (2018) methodology in employing a three-valued analytic rubric to evaluate students' attitudes on each of the themes consisting of positive, neutral or negative ratings (values: +1, 0 or -1, respectively). This methodology enables one to test whether there are any differences in the proportions of students assigned positive ratings for each dimension and theme. Moyer et al. (2018) argue that adding a third (neutral) value is necessary to capture a value of "students' attitudes on a particular theme if they did not have a passage that fell within that theme, if they had both positive and negative passages within that theme, or if a passage that fell within the theme could not be coded as either positive or negative" (p. 122). Their approach has been shown to have satisfactory reliability.

Using this approach, all three authors independently assigned ratings for 137 students using the rubric developed after the initial coding round. After comparison, the researchers discussed and reached consensus on all students' ratings. For each student, we produced a profile capturing their attitudes towards non-routine problem solving by assigning a value for each theme and then summing the values up within each dimension to come to a conclusion about the overall positive, neutral or negative score for each of the three dimensions of attitude.

The coding protocol that we followed was based on the intent to capture students' attitude profiles in a holistic way. The passages grouped within a given theme were not restricted by the questions that were targeting different themes. Because of the narrativeprompting questions, students often responded with a multi-passage answer to a single question. The multi-passage answer frequently contained passages addressing more than one theme or dimension. Consistently with Moyer et al. (2018)'s coding protocol, each passage in a response was assigned to no more than one theme. For example, in response to Question 1, in which the student was asked about the main differences between puzzles and routine problems, the response "Have to think out of the box more for puzzles, be suspicious of any simple sounding answer. Figure out the trick, before attempting any mathematical solution or explanation. Just need the lost bit for regular questions" was coded as two passages contributing (+1) to the Lateral thinking/creativity theme and (+1) to the Trick theme. Both themes belong to the Vision dimension (differentiating disposition). An answer to the same question from another student "Allows the use of out-of-the-blue thinking in the course which is hardly used - it is much more interesting" was coded as (+1) for the Lateral thinking/creativity theme and as (+1) for the Engagement theme (expression of interest), which is a theme from the Emotional disposition dimension of attitude. The Employability theme contains passages in which students discuss the usefulness of puzzles for employment prospects, job interviews, and real-life practical applications. The following juxtaposition of two passages illustrates the contrasting opinions of two students: "Yes; Problem-solving. At the end of the day, that's why people pay other people: to solve problems they can't. It underpins the fabric of the working world" was coded as (+1), whereas the passage "Chews class time when you could actually be learning something" was assigned a (–1) rating. A more detailed description of the eight themes with coding examples is available here http://doi.org/[10.17608/k6.auckland.8009921].

Table 1. Frequency counts by lateral thinking self-efficacy of positive ratings on the eight themes of the three dimensions of students' attitudes towards non-routine problem solving

Dimensions of students' attitudes and their constituent themes	High LT self-efficacy *n*=72	Low LT self-efficacy *n*=58	*z* value	Cohen's *d*
Vision of non-routine problem solving (differentiating from routine problem solving)	47	37	0.18	-0.010
Lateral thinking/creativity theme	30	24	0.03	0.225
Trick theme	6	4	0.31	-0.549
Challenge theme	5	13	-2.83*	-1.310
Enhancement utility dimension	62	44	1.50	0.219
Problem solving theme	66	47	1.79	0.310
General thinking theme	63	49	0.50	0.220
Employability theme	13	7	0.94	0.365
Emotional disposition dimension	28	8	3.18**	0.474
Enjoyment theme	22	7	2.52*	-0.083
Engagement theme	13	3	2.22*	1.222

*p<.05. **p<.01. LT=lateral thinking

Students' attitudes profiles vs lateral thinking self-efficacy

To analyse the interplay between a student's *lateral thinking self-efficacy* and other aspects of the students' perspectives pertaining to non-routine problems, we analysed the distribution of students' positive ratings for the eight themes (Table 1). Seven students were excluded from our reporting since their *lateral thinking self-efficacy* was not identified. With one expected cell count less than five, Fisher's exact test (2 × c) was conducted between the two categories of *lateral thinking self-efficacy* and the frequency counts of positive ratings on the eight themes. A statistically significant difference in proportions was observed, p=.036 (Monte Carlo (2-sided), 99% CI [0.031, 0.041]). Thus, the evidence suggests that the profiles of students with high *lateral thinking self-efficacy* differ significantly from students with low *lateral thinking self-efficacy* with respect to their affective domain spanning the three dimensions: vision; enhancement utility; and emotional disposition towards non-routine problem solving.

Vision dimension (differentiating disposition): results

For each student, we assigned a value of +1, 0, or -1 on each dimension if the sum of the ratings on the corresponding themes was positive, zero, or negative, respectively. Overall for this dimension, the majority of students, both those with high and low *lateral-thinking self-efficacy*, believed that non-routine problems were different to routine problems: 65% and 64%, for high and low categories respectively. Only six students expressed a view that there are no characteristic distinctions associated with non-routine problem solving (5.56% and 3.45%, respectively for the high and low *lateral thinking self-efficacy* categories). The rest had neutral views. None of these differences were significant.

We report results for the three differentiating themes. Almost the same proportions of students viewed non-routine problem solving as requiring lateral/creative thinking from the high and the low *lateral thinking self-efficacy* categories (41.66% and 41.36%, respectively). There was no significant difference between the positive ratings in the Trick theme, which were low—8.33% and 4%, respectively for the high and low *lateral thinking self-efficacy* categories. However, a significant difference was observed in the proportion of positive ratings in the Challenge theme (z=-2.54, p=.011) implying that the proportion of students with high *lateral thinking self-efficacy* (6.94%) who viewed non-routine problem solving as a challenge was significantly less than the proportion of students with low *lateral thinking self-efficacy* (22.41%).

Enhancement utility dimension: results

Overall, a large majority of students believed that non-routine problem solving is a useful activity that would enhance their future prospects, regardless of their *lateral thinking self-efficacy* profiles: 86.1% and 75.9%, for high and low categories respectively. Only ten students had overall negative ratings for this dimension, expressing a view that there is nothing useful to gain from spending time in class on solving non-routine problems (5.6% and 10.3%, for the high and low *lateral thinking self-efficacy* categories, respectively). The rest had neutral views. None of these differences were significant (see Table 1 for *z*-test values).

The results for the three themes in this dimension were that very high proportions of students viewed non-routine problem solving as enhancing their problem solving skills—91.7% and 81.0% for the high and the low categories, respectively. Furthermore, similarly high proportions of students in both groups viewed non-routine problem solving as enhancing their generic thinking skills—87.5% and 84.5% in the high and the low categories, respectively. The proportions of students with positive ratings on the Employability theme were 18.1% for the high and 12.1% for the low *lateral thinking self-efficacy*. None of these differences were significant.

Emotional disposition dimension: results

A *z*-test for proportions confirmed a significant difference in positive ratings on this dimension for students with different levels of *lateral thinking self-efficacy* (z=3.18, p=.002), implying that the proportion of students with high *lateral thinking self-efficacy* having positive emotional dispositions towards non-routine problem solving was significantly greater than the proportion of students with low *lateral thinking self-efficacy* (38.9% and 13.8%, respectively).

The results for the two themes comprising this dimension showed that there is a significant difference in positive ratings on both of these themes. A *z*-test for proportions confirmed there were significantly more students with high *lateral thinking self-efficacy* with positive ratings on the Enjoyment theme than with low *lateral thinking self-efficacy* (z=2.52, p=.012). Furthermore, a *z*-test for proportions showed significantly more students with high *lateral thinking self-efficacy* with positive ratings on the Engagement theme than with low *lateral thinking self-efficacy* (z=2.223, p=.027).

Discussion

The starting point for our project was to contribute to the field of transfer of learning that is of vital importance to education. Without transfer, the relevance of formal mathematical schooling is limited. Therefore, many researchers have called for studies to identify significant influences on transfer and have emphasised the need to develop techniques that foster transfer to be implemented in instructional settings (Pugh & Bergin, 2006; Schoenfeld, 1999).

In this study, in the context of examining STEM employability prospects of students studying mathematics, we conceptualised the specificity of students' confidence in regard to applying creativity to solving non-routine problems by defining the novel construct of *lateral thinking self-efficacy* and used it as an investigation tool. When fully validated, the utility of this construct as a tool offers a way forward to unpack the complexity of mechanisms involved in the transfer of mathematical learning to novel domains, since it relates to the creative thinking needed for solving innovative real-life work place problems.

As noted in the introduction, an extensive body of research pertaining to mathematics self-efficacy and its relation to the career choices of learners emphasises the importance of self-evaluation of competence as much as the learners' actual performance. Our findings extend gender-dependent differences to the construct of *lateral thinking self-efficacy* by providing evidence that the proportion of males reporting confidence in solving non-routine problems was significantly greater than the proportion of females. This was despite no significant association between gender and self-reported non-routine problem solving performance. This important distinction raises key questions about equity with regard to employability attributes and prospects for females in particular. The especially unfortunate part of these inferences is how heavily they factor into crucial decisions such as the types of jobs females choose to apply for, in lieu of actual facts. Could an implication be that female STEM graduates are less likely to apply for jobs at companies, such as Microsoft, Google and many other technology companies (Klymchuk, 2017) that use puzzles as part of their job interviews? It is plausible that the manifestation of gender difference identified in *lateral thinking self-efficacy* is a contributing factor to the existence of the glass ceiling – the unseen, yet unbreachable barrier that keeps women from rising to the upper rungs of the corporate ladder, regardless of their qualification or achievement – a phenomenon particularly affecting STEM fields. In this sense, our findings bring to the fore the importance of monitoring and developing *lateral thinking self-efficacy* in mathematics-related contexts in order to enhance employability attributes of all learners and to reduce the underrepresentation of women in STEM companies.

Moreover, overall, we observed a statistically significant association between students' performance in solving non-routine problems and their *lateral thinking self-efficacy*. The causal relationship is not clear. A mutually reinforcing mechanism of self-efficacy and performance could be at play. Nevertheless, future research could focus both on ways to enable improvement in students' performance in solving non-routine problems and on enhancing their mastery experiences – a major source of self-efficacy (Usher & Pajares, 2008), thus enabling improvement in *lateral thinking self-efficacy*.

Further unpacking the role of the construct, we report that the attitude profiles of students with high and low *lateral thinking self-efficacy* are significantly different with respect to their affective domain spanning the three dimensions of vision, enhancement utility and emotional disposition towards non-routine problem solving. More fine-grained analysis revealed significant positive association between high *lateral thinking self-efficacy* and positive emotional dispositions towards non-routine problem solving.

This explicit finding signposts future research directions in order to improve *lateral thinking self-efficacy*. The role of emotions and beliefs in problem solving has been recently identified as one of the important areas of research on mathematics-related affect (e.g. Goldin, 2000; Hannula, 2016). From a broader perspective, an extensive body of research has demonstrated that students' emotions have profound effects on their academic engagement and performance, particularly the impact of positive and negative moods on problem-solving. Experimental evidence suggests that a positive mood promotes flexible, creative, and holistic ways of solving problems and reliance on generalised, heuristic knowledge structures (e.g. Pekrun & Linnenbrink-Garcia, 2012). Given these experimentally grounded considerations, together with our findings, we can hypothesise possible ways to improve *lateral thinking self-efficacy* by targeting an activation of positive emotions during the stages of non-routine problem solving. In particular, future research efforts could be focussed on supporting students' transition through their emotional states while solving such problems. Hence, it may be beneficial to consider moderating students' epistemic emotions during similar interventions by ensuring that they experience the enjoyment of understanding a solution, as opposed to feeling frustrated when a solution is not explained well. This could be a clearly defined intent for the future stages of the project.

Study limitations

With any study there are limitations to be considered when judging the validity and generalisability of its results. Firstly, the surveying process here was based on a non-probability sample since it was conducted in class, hence selecting students who are more likely to attend lectures than in a random sample. Secondly, we need to consider that there is: the potential for students to say what they think the researcher wants; a need to ensure that the questions genuinely address student attitudes toward non-routine problems; and a possibility of misinterpreting responses. We hope we have taken sufficient measures to mitigate these potential problems. Further, from a methodological perspective, our approach based on a sequential two-phase data analysis, which combines qualitative and quantitative methods to perform statistical analysis of the scores of thematic responses, is still relatively novel. It has only recently been employed as an effective way to uncover statistically significant differences in empirical manifestation of theoretical constructs (see Moyer et al., 2018). Therefore, given the use of this recent advance in our analysis, we note that a degree of caution should be exercised when results are extrapolated into tentative general conclusions. We are also conscious that, due to the short-term and exploratory nature of the intervention, we have not been able to examine the longer-term influences of *lateral thinking self-efficacy*. Finally, although there are positive signs that *lateral thinking self-efficacy* is a valid construct, its operationalisation has not been fully validated and this requires further investigation.

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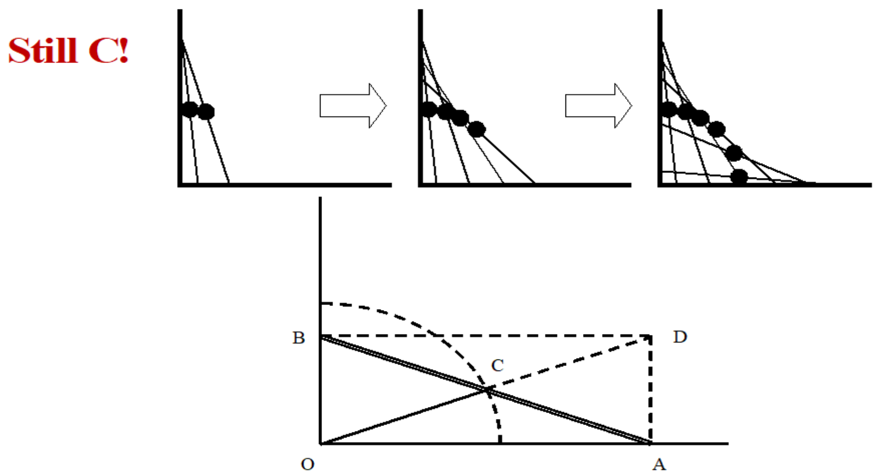
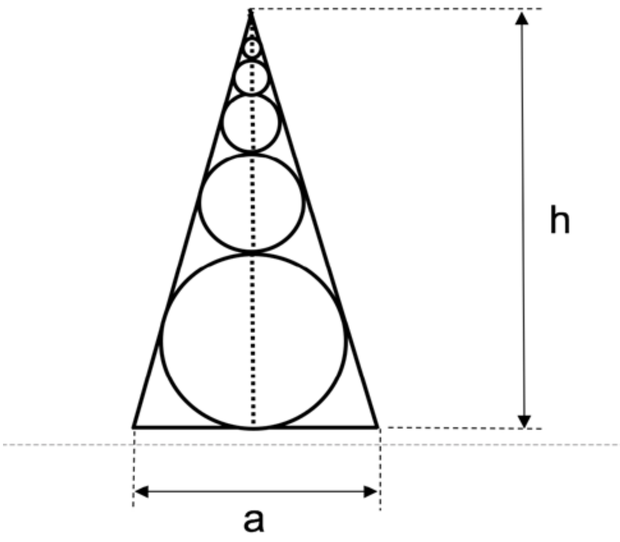
Appendix

Answer to the puzzle about a cat sitting half way on the ladder with the base being pulled away:



Solution to the inscribed circles problem: the sum of diameters is h



Tennis tournament answer: Fifty four (54 players to be eliminated need 54 matches.

Illustrating ‘start at the end’ problem-solving strategy.)