On the probability of ventricular fibrillation due to electric shock

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Abstract

The probability that a randomly chosen individual will, when exposed to a specified electrical shock, undergo fatal ventricular fibrillation can be regarded as a function of random variation in the human population along two dimensions. The first is the individual's body impedance characteristic: for this we introduce a new two-parameter model that improves on the simpler one-parameter model used in previous work. The second is the individual's current tolerance: for this we codify some curves used in previous practice. We also consider methods of solving the resulting shock circuit, and show in particular that it is possible for the fixed-point iteration method to give incorrect results.

Contents

1 Introduction **2**

2 The rate of shocks **3**

3 Modelling the human body impedance characteristic **4**

4 Modelling the human current tolerance **8**

5 Solving the shock circuit **10**

6 The probability of ventricular fibrillation **11**

7 Conclusion **13**

1 Introduction

The statistical modelling of electrical shock hazards is both decades old ([16]) and still of current interest ([9, 11]). While there are several possible approaches, there is general agreement that a key concept is that of the *ventricular fibrillation probability*. This is the probability that an individual randomly chosen from the population will, when subjected to electrical shock of a specified kind, undergo ventricular fibrillation leading to death. It is generally accepted that ventricular fibrillation is the principal cause of fatalities due to electrical shock.

The paradigm of the present paper is well-established ([11, 2]). A *shock* consists of a given voltage applied for a given length of time, possibly via a given series impedance, between two given points on a human body. An item of electrical equipment creates such shocks from time to time at a rate λ_s per unit time; these may be modelled as a Poisson or other random process. The probability that a shock results in a fatality is identified with P_{fib}, the probability that a randomly selected person subjected to the shock will undergo ventricular fibrillation. The long-run average rate of fatalities is thus $\lambda_s P_{fib}$ per unit time. We devise models that allow this rate to be calculated in particular examples. Alternatively, the desired fatality rate may be specified, and the models used to determine a value for some parameter of the shock (*e.g.* the maximum allowable voltage).

A shock circuit consists of a human body in series with another given impedance, powered by a source of given voltage (and frequency). To determine the current that flows in the circuit, it is necessary to model the impedance of the human body; such a model must be statistical in nature as there is substantial variation between individuals. (The body impedance is principally resistive. Although the skin exhibits some capacitive behaviour, this is relatively small ([1]) and will be neglected: for the purposes of the present paper, references to "impedance" are to resistance only.) Once the current is determined, another statistical model is required to estimate the probability that it results in ventricular fibrillation.

The necessary calculations are captured in the Australian industry-standard (but closed-source) software known as Argon ([3]). The purpose of the present paper is to examine the standard underpinning the Argon software suite, and to replicate, evaluate, and improve on the approach adopted therein.

2 The rate of shocks

The occurrence of shocks can be modelled in the following simple way. A given piece of equipment is assumed to experience faults from time to time at random. Simultaneously, contact events between humans and objects that will become live during a fault also occur from time to time at random. The faults and the contacts are modelled as independent Poisson processes with rates λ_f and λ_c (per unit time) respectively; every fault is assumed to have duration τ_f and every contact to have duration τ_c. A shock occurs when a fault event overlaps in time with a contact event (a "coincidence").

For this model, it is straightforward to calculate the rate at which fault-contact coincidences occur. Without loss of generality, each event of the Poisson process representing faults can be taken to be the start of a period of duration τ_f during which the fault is in effect; if a contact is initiated during this period, a coincidence occurs. The start of the fault period also marks the end of another period of duration τ_c; a coincidence will also occur if a contact is initiated during this period. In total, then, each fault event is associated with a period of duration $\tau_f + \tau_c$ during which the initiation of any contact event will give rise to a coincidence. Since the fault and contact processes are independent, the expected number of coincidences arising from a single fault event is $\lambda_c(\tau_f + \tau_c)$. Therefore, the rate at which coincidences (or shocks) occur is $\lambda_s = \lambda_f \lambda_c(\tau_f + \tau_c)$ per unit time.

Note that the foregoing analysis takes no account of the possibility of faults coinciding with each other; similarly contacts. Should two faults overlap with each other in time, they are counted as separate events, and any coincidences they give rise to are counted separately. Similarly, the exceptionally unlucky individual whose contact period overlaps with two distinct faults (which may or may not overlap with each other) is considered to have experienced two coincidences, and to have been exposed to two shock hazards. While the realism of this neglect may be problematic, the effect on the results should be small, provided that faults and contacts are both rare events.

We also note that this analysis differs somewhat from a slightly different approach published elsewhere ([15]), which involves a detailed calculation of the *probability of at least one* coincidence occurring during a given time interval. The analysis in the present paper would instead tell us the *expected number* of coincidences during the time interval. The two values will be very similar if coincidences are rare enough to make it very unlikely that two or more occur during the interval. However, the latter approach is better suited to our purposes in this paper, as the expected number of coincidences can be multiplied by a fibrillation probability to obtain the expected number of

Table 1: Body impedance statistics at various voltages (hand-to-hand, dry, large contact area, AC 50-60 Hz)

Voltage	Impedance (Ω)		One-parameter model		Two-parameter model
(V)	5th percentile	50th percentile	mean(log)	sd(log)	β
25	1750	3250	8.086	0.376	1.000
50	1375	2500	7.824	0.363	0.735
75	1125	2000	7.601	0.350	0.523
100	990	1725	7.453	0.338	0.413
125	900	1550	7.346	0.330	0.339
150	850	1400	7.244	0.303	0.280
175	825	1325	7.189	0.288	0.250
200	800	1275	7.151	0.283	0.229
225	775	1225	7.111	0.278	0.209
400	700	950	6.856	0.186	0.106
500	625	850	6.745	0.187	0.067
700	575	775	6.653	0.181	0.040
1000	575	775	6.653	0.181	0.040

fatalities.

3 Modelling the human body impedance characteristic

The available data on human body impedance is illustrated in Figure 1 (left; see also [1]). Estimates are tabulated of the 5th, 50th, and 95th percentiles of the impedance at a finite set of voltages ranging from 25V to 1000V; the first two of these quantiles are reproduced in Table 1. Similar tables are available for other current paths (e.g. left-hand-to-right-foot, or foot-to-foot) and for contact made with wet or saltwater-wet skin.

A simple modelling approach ([11]) is to fit a lognormal distribution to the given quantiles for each given voltage. This choice of distribution is suggested by data ([6]); it also has the desirable feature that it guarantees non-negativity. The lognormal distribution, which has two parameters, is fit exactly to the given values for the 5th and 50th percentiles. The 95th percentile value is ignored as being of lesser interest for present purposes – it is the individuals with low body impedance who will carry the highest currents and so are most likely to perish from electric shock – although in fact it is reproduced quite well by the fitted lognormal distributions. Fitted parameters

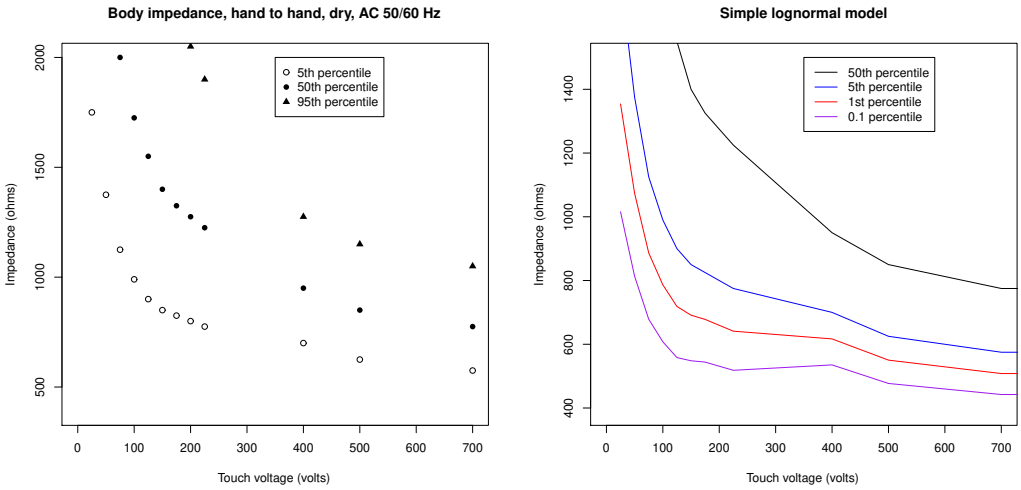

Figure 1: Body impedance data (left) and a simple lognormal fit (right).

(mean and standard deviation of the log(impedance)) are recorded alongside the data in Table 1.

To model the body impedance characteristics of individuals, we need a further assumption regarding the correlation between impedances at different voltages. The simplest thing is to posit a perfect correlation, i.e. that each individual's body impedance coincides with the same population quantile at all voltages. So, for example, an individual whose body impedance at 25V happens to match the population median will have a body impedance at 1000V matching the population median for that statistic also. The body impedance functions of the human population are thus modelled as a one-parameter family: any individual's body impedance is completely specified, as a function of voltage, by the population quantile with which the individual coincides.

This simple lognormal model can be used to extrapolate to lower quantiles of impedance, as shown in Figure 1 (right). Doing so reveals a main flaw in this approach: for the most extreme quantiles, the impedance is no longer a monotone function of voltage. This is unappealing because we expect on physical grounds that the impedance characteristic of any individual will be monotone decreasing in voltage; therefore, any quantile of the population should be monotone decreasing also. It is readily apparent why the simple lognormal model fails in this way: the impedance (or rather, its logarithm) at low voltages has greater variance than at high voltages, and so there must be some quantile at which the low-voltage impedance falls below the high-

voltage impedance.

The physical basis for the voltage-dependence of the body impedance lies in the behaviour of the skin, which presents a relatively large impedance to low voltages, but breaks down for voltages on the order of a few hundred volts. This points the way towards an improved model: the skin and the subdermal tissue can be treated separately:

$$Z_b(V) = Z_{skin}\ \beta(V)\ +\ Z_{subd}. \tag{1}$$

That is, the body impedance function of any individual is the sum of two terms, representing skin and subdermal impedances in series. Only the skin impedance is voltage-dependent. The body impedance functions of the human population are thus modelled as a two-parameter family: each individual has their own values for the two parameters Z_{skin} and Z_{subd} while $\beta(V)$ is a fixed function common to all individuals. We assume that Z_{skin} and Z_{subd}, which we must now regard as random variables, are independent and both lognormally distributed.

Note that the impedance curves of two different individuals may cross, something not possible with the simple lognormal model. Note also that the impedance function of any individual is guaranteed to be decreasing with voltage (provided the function β is decreasing).

The model (1) is fitted to the given 5th and 50th percentile impedance values at the tabulated voltages $V_1, \ldots, V_n$. In so doing, we actually have fewer fitted parameters than the simpler model. Given tabulated quantiles at the n voltages, the simple lognormal model fits $2n$ parameters: two for each specified voltage. But (1) fits only $n + 3$ parameters: two each for the distributions of Z_{skin} and Z_{subd}, and $n - 1$ for the values of $\beta(V_2), \ldots, \beta(V_n)$. (Without loss of generality, we may take $\beta(V_1) = 1$ for the smallest voltage V_1.) Given $2n$ tabulated percentiles (the 5th and 50th for each voltage) we thus have an overdetermined system to solve, and must seek the best fit.

We take a likelihood approach to the fitting. Let the vector of parameters be θ. Suppose that θ determines a univariate cumulative distribution function $F(\cdot;\theta)$, and we have data indicating that the 5th percentile of this distribution should be q_5 and the 50th percentile q_{50}. That is, $F(q_5;\theta) = 0.05$ and $F(q_{50};\theta) = 0.50$, or as nearly so as can be managed. The log-likelihood function corresponding to this condition is

$$\theta \mapsto 0.05 \log F(q_5;\theta) + 0.45 \log(F(q_{50};\theta) - F(q_5;\theta)) + 0.5 \log(1 - F(q_{50};\theta)).$$

Given a collection of conditions of this form, we determine the value of θ that best fits them all by maximizing the sum of the corresponding log-likelihood functions.

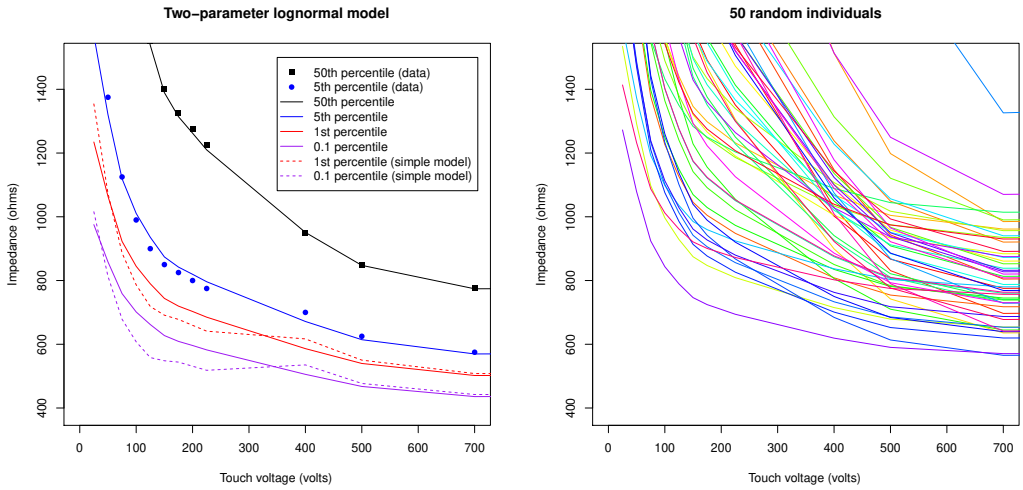

Figure 2: The two-parameter model of body impedance: quantiles (left) and 50 randomly sampled individuals (right).

In the particular case of interest here, we have $n = 13$ different distributions F, corresponding to the different voltages. The parameter vector θ is of size 16:

$$\theta = (\mu_{skin}, \sigma_{skin}, \mu_{subd}, \sigma_{subd}, \beta_2, \ldots, \beta_n)$$

where mu_{skin}, σ_{skin} are the mean and standard deviation of $\log Z_{skin}$; similarly μ_{subd}, σ_{subd}; and $\beta_2, \ldots, \beta_n$ are the values of $\beta(V_2), \ldots, \beta(V_n)$.

In carrying out this nonlinear optimization, we have the difficulty that for given θ, the distribution $F(\cdot;\theta)$ is that of the sum of two independent lognormally-distributed random variables, and so there is no closed-form analytical expression for $F(q;\theta)$. We overcome this problem in the following way. First, draw large independent random samples $(W_i^{skin})_{i=1}^N$ and $(W_i^{subd})_{i=1}^N$ according to the standard normal distribution; we used $N = 10^5$. These samples, once drawn, remain fixed throughout the fitting. Then for any θ, $(Z_{ik})_{i=1}^N$ defined by

$$Z_{ik} = \exp\left(\mu_{skin} + \sigma_{skin} W_i^{skin}\right) \beta_k + \exp\left(\mu_{subd} + \sigma_{skin} W_i^{subd}\right)$$

is an independent random sample drawn according to the distribution of body impedances at voltage V_k $(k = 1, \ldots, n)$ corresponding to θ. We use the proportion of values in this sample that are less than q as an approximation to $F(q;\theta)$.

Fitted values of the $\beta(V_i)$ parameters are recorded alongside the data in Table 1. The corresponding fitted distributional parameters for $\log(Z_{skin})$

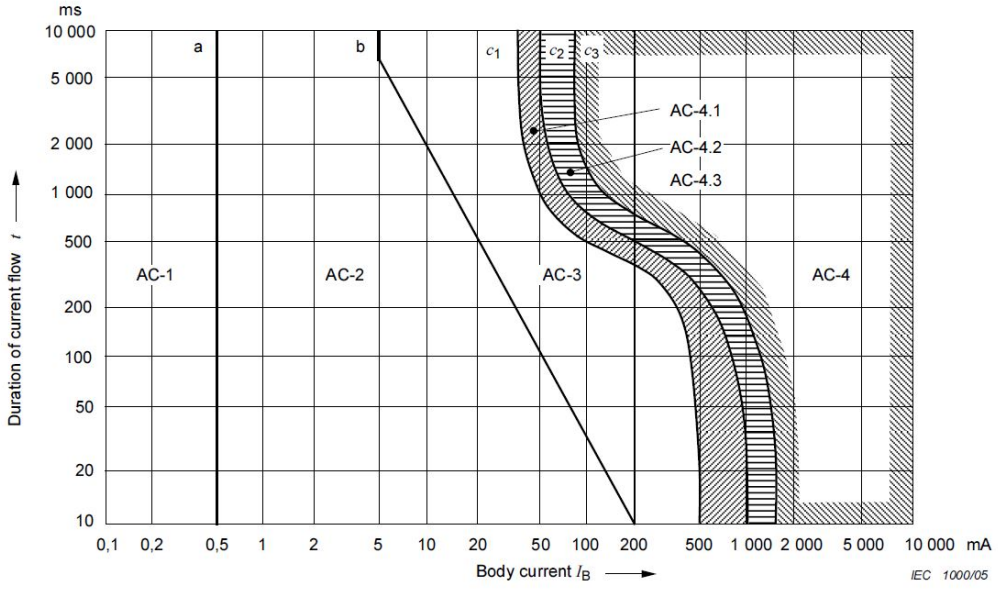

Figure 3: Human current tolerance.

are $\mu_{skin} = 7.84$ and $\sigma_{skin} = 0.62$; those for $\log(Z_{subd})$ are $\mu_{subd} = 6.49$ and $\sigma_{subd} = 0.19$. Figure 2 illustrates the model obtained. A good fit has been achieved to the required 5th and 50th percentile values. The extrapolated 1st percentile and 0.1 percentile curves are monotone decreasing as expected, and for high voltages are in good agreement with those extrapolated by the simple lognormal model. (This, too, is as expected, since the skin term becomes negligible at high voltages). The models differ most at around 200V. Figure 2 (right) shows that it is not only possible, but quite common for the impedance characteristics of different individuals to cross: one individual may have relatively high-impedance skin and a low-impedance internal body, while another has low-impedance skin and a high-impedance internal body.

4 Modelling the human current tolerance

In this section we turn our attention to modelling the ability of individuals to tolerate current through the body without triggering ventricular fibrillation and death. As with most published work in this area, we begin with the diagram in Figure 3; see also [1] and (in earlier form) [5, 4]. Note that the current tolerance depends on the duration of the exposure: short electric shocks are less harmful than prolonged ones. Of interest are the three S-

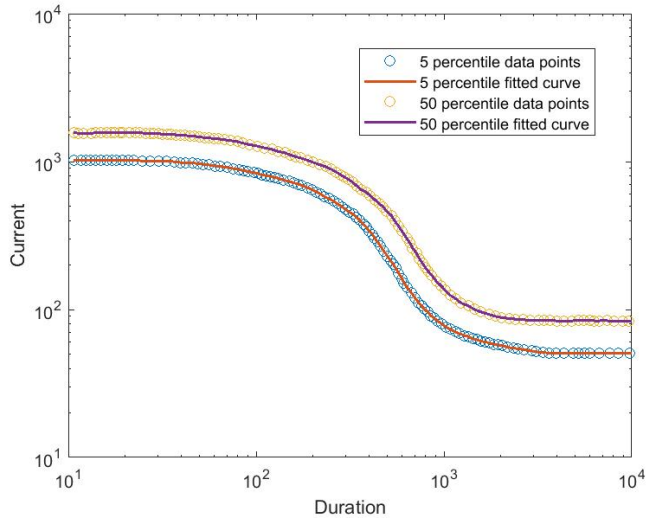

Figure 4: Spline curve fitted to the current-tolerance thresholds.

curves in Figure 3, which represent thresholds at which ventricular fibrillation will occur in 5%, 50%, and 95% of exposed individuals.

We first attempt to standardize a numerical representation of these curves. Two approaches were trialled: one a generalized logistic function fitted with ceiling, floor and growth rate estimated by minimizing the root mean squared error, the other a polynomial spline. Comparison of the residuals suggested that the spline approach was favoured over the logistic estimate (and results did not seem to vary much on a substantive basis when the number of steps chosen for fitting the spline were increased from 13 to 113).

A univariate piecewise polynomial (pp) is specified by its break sequence `breaks` and the coefficient array `coefs` of the local power form of its polynomial pieces. The ppform of a polynomial spline of order k provides a description in the terms of the break points $\psi_1, \psi_2, \psi_3, ..., \psi_l + 1$ and the local polynomial coefficients C_{ji} of its l pieces.

$$P_j(x) = \sum_{i=1}^{k} (x - \psi_j)^{k-i} C_{ji}, \quad j = 1 : l, \tag{2}$$

We use a cubic spline: the order is $k = 4$. The break sequence is assumed to be strictly increasing with l polynomial pieces that make up the ppform. The resulting interpolation is depicted in Figure 4.

We define an individual's current tolerance (also known as the threshold of ventricular fibrillation) as the minimum current sufficient to cause ventricular fibrillation. Since the current tolerance varies between individuals,

it is necessary to fit a probability distribution for it. There is reasonable evidence from animal experiments (see [7]) that a lognormal distribution has the right shape, although it is not the only possibility. (See, *e.g.* [14], where a three-parameter variant of the lognormal distribution is fitted, or [12], where normal and log-triangular distributions are also considered.) We take the straightforward approach of fitting a standard two-parameter lognormal distribution exactly to the required 5th and 50th percentiles of the distribution, these values being obtained from the spline interpolation above. It is then possible to directly calculate the probability of fibrillation for a given current and exposure time.

5 Solving the shock circuit

In this section we consider the shock circuit comprising a voltage source connected across two impedances in series, one of which is a human body. The other impedance – which may represent footwear, gloves, soil, paving material, etc. – is assumed (as with the body impedance) to be purely resistive, and to have a fixed value Z_s. The source voltage (also known as the *prospective touch voltage*) will be denoted V_s.

As discussed in Section 3, the impedance of the human body is voltage-dependent; we denote it $Z_b(V)$ at voltage V, and define this quantity for all voltages V by linear interpolation between the tabulated voltages $V_1, \ldots, V_n$. For completeness, we define $Z_b(V) = Z_b(V_1)$ for $0 \leq V < V_1$ and $Z_b(V) = Z_b(V_n)$ for $V > V_n$. (The first of these approximations has poor accuracy, but this is not a concern as the very lowest voltages, and highest impedances, are not hazardous in any case.)

The voltage across the body (known as the *touch voltage*) V_t then satisfies

$$V_t = \frac{Z_b(V_t)}{Z_s + Z_b(V_t)} V_s \tag{3}$$

or equivalently

$$Z_b(V_t) \; - \; \frac{Z_s V_t}{V_s - V_t} \; = \; 0. \tag{4}$$

Provided Z_b is a decreasing function, it is clear that there exists a unique solution to (4) with $0 < V_t < V_s$, since the left-hand-side of (4) is a strictly decreasing continuous function of V_t on that interval which takes positive values as $V_t \to 0$ and negative values as $V_t \to V_s$.

It was noted in Section 3 that the simple lognormal model may have some quantiles which are not decreasing functions of the voltage; this raises the mathematical possibility of multiple solutions to (4). Such cases, however,

are found to exist only for very extreme quantiles corresponding to a fraction circa 10^{-20} (or less) of the population, and we will not consider them further.

Equation (3) could be solved by fixed-point iteration on the function $f(v) = V_s Z_b(v)/(Z_s + Z_b(v))$ (see [11]). That is, a sequence $(v_k)_{k=0}^{\infty}$ is defined by letting $v_0 = V_s$ and $v_k = f(v_{k-1})$ for $k = 1, 2, \ldots$; it is then observed experimentally that in cases of practical interest, this sequence always seems to converge to the solution of the equation. However, fixed-point iteration is well-known to be a method that works in some situations but not in others, and in some (relatively rare) cases this approach to solving (3) may fail.

Example. Let Z_s =10kΩ and V_s =3200V, and let the exposed individual have a body impedance coinciding with the 98th percentile of the population according to the simple lognormal model given in Section 3. According to this model, the body impedances are 2170Ω, 1391Ω, and 1248Ω at 225V, 400V, and 500V respectively. Solving (3) reveals the touch voltage to be V_t =395.59V, with the corresponding body impedance being 1410.6Ω and body current 280mA. However, if the above fixed-point iteration is initiated from 395V (or any close approximation of the solution) it will not converge to the correct value, but will eventually alternate between values 390.26V and 401.4V, which constitute a 2-cycle for f.

In general, a necessary condition for the successful convergence of the fixed-point iteration is $|f'(V_t)| \leq 1$ (see, for example, [8] or [13]). We have

$$f'(v) = \frac{V_s Z_s Z_b'(v)}{(Z_s + Z_b(v))^2}$$

which for the above numerical example gives $f'(V_t) \approx -1.09$.

An alternative and more reliable method of solving (3) is a simple bisection search for the touch voltage V_t satisfying (4); this method is used for calculations for the present paper.

6 The probability of ventricular fibrillation

A key modelling assumption is that the body impedance curve and the current tolerance are statistically independent. This assumption seems to be made everywhere (*e.g.* [2]) where both variables are considered, although it is far from clear that it is justified. For example, some association is found between an animal's body weight and its current tolerance ([10]); it is easily imaginable that an individual's body weight or size will also affect the body impedance.

Suppose, however, that the body impedance curve and the current tolerance are independent random quantities. On this assumption, it is in prin-

ciple straightforward to calculate the probability of ventricular fibrillation resulting from a given shock. Denote by I the corresponding (random) body current: this depends only on the body impedance curve. Adopt a lognormal model for the current tolerance as suggested in Section 4, with log(current tolerance) having mean μ_f and standard deviation σ_f. Then the fibrillation probability for given I is

$$P(\text{fibrillation} \mid I) \;=\; \Phi\left(\frac{\log I - \mu_f}{\sigma_f}\right),$$

where Φ denotes the standard normal cumulative distribution function. The unconditional fibrillation probability is

$$P(\text{fibrillation}) \;=\; E\left[P(\text{fibrillation} \mid I)\right] \;=\; E\left[\Phi\left(\frac{\log I - \mu_f}{\sigma_f}\right)\right]. \qquad (5)$$

Since I is a function of the body impedance curve only, we have an expectation of a function of this random curve.

For the simple lognormal model of body impedance, a random individual's body impedance curve is drawn from a one-parameter family: the sole parameter p (with $0 < p < 1$) specifies that the body impedance curve of the individual in question matches the population's p-quantile curve. The resulting current in the shock circuit is then a function of p; if we denote it $g_1(p)$ then 5 becomes

$$P(\text{fibrillation}) \;=\; \int_0^1 \Phi\left(\frac{\log g_1(p) - \mu_f}{\sigma_f}\right)\, dp. \qquad (6)$$

That is, the required probability is obtained as a univariate integral. Even though the integrand is somewhat complicated – to evaluate it for a single value of p requires solving the shock circuit as described in Section 5 – it is a simple matter to numerically evaluate such an integral ([11]).

For our two-parameter model of body impedance, a random individual's body impedance curve is given by (1); the parameters Z_{skin} and Z_{subd} appearing there are independent lognormally-distributed random variables. The resulting current is a function of these two parameters only: $I = g_2(Z_{skin}, Z_{subd})$. Hence

$$P(\text{fibrillation}) \;=\; E\left[\Phi\left(\frac{\log g_2(Z_{skin}, Z_{subd}) - \mu_f}{\sigma_f}\right)\right]$$

or equivalently

$$P(\text{fibrillation}) \;=\; E\left[\Phi\left(\frac{\log g_2(e^{\mu_{skin}+\sigma_{skin}W_1}, e^{\mu_{subd}+\sigma_{subd}W_2}) - \mu_f}{\sigma_f}\right)\right],$$

where W_1, W_2 are independent standard normal random variables. It would be possible to express this expectation as a double integral, analogously to (6), but we do not pursue that approach here. Instead, we content ourselves with simple Monte Carlo simulations to evaluate the fibrillation probabilities.

Example. A given item of equipment produces shocks by fault-contact coincidences at rate $\lambda_s = 0.01$ per annum – that is, the shocks are 1-in-100 year events. All the shocks are assumed to have duration 500ms and protective series impedance $Z_s = 1\text{k}\Omega$. What is the maximum prospective touch voltage consistent with a fatality rate of 10^{-6} per annum?

The given rates imply that the fibrillation probability must be $P_{fib} = 10^{-4}$. Figure 3 indicates that for shocks of this duration, the current tolerance is 200mA at the population median and half of this value at the 5th percentile; a lognormal model of current tolerance thus has mean (log current) equal to $\mu_f = \log(200)$ and standard deviation (log current) equal to $\sigma_f = \log(0.5)/\Phi^{-1}(0.05) = 0.421$. Using Monte Carlo samples of $n = 10^5$ body impedance curves suggests that the required prospective touch voltage may be taken to be 110V; for this value of V_s, the estimated P_{fib} is $(93.1 \pm 1.3) \times 10^{-6}$ for the simple lognormal model of body impedance, and $(96.3 \pm 1.2) \times 10^{-6}$ for the two-parameter model.

7 Conclusion

As in earlier work, this paper has decomposed the fibrillation-probability problem into two parts: individuals are assumed to vary both in their current tolerance and their body impedance characteristic. This decomposition reduces the problem to one of calculating a bivariate expectation. Body impedance is the more complicated variable, and for this we have devised a new two-parameter model that may be more realistic than the simpler model previously used.

We have also followed the approach of explicitly allowing for a series impedance in the circuit ([11]). We have discovered that the resulting circuit is not always solved correctly by fixed-point iteration, and proposed an alternative.

One aspect of the problem not addressed in the present paper is the nature of the series impedance discussed in Section 5. The Argon software ([3]) models this impedance explicitly in terms of types of footwear and paving surfaces. (For shocks other than via the feet, no series impedance appears to be allowed for.) The reduction of the geometry and materials of a given situation to a single series impedance value requires separate models beyond those we have considered.

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