Essays on Household Finance

Nguyen Quynh Chi

A thesis submitted in fulfilment of the requirements for the degree of

Doctor of Philosophy in Finance

The University of Auckland, 2021

ABSTRACT

The thesis comprises three studies on household finance and retirement income planning. The first study with a specific focus on South Korea examines whether individuals should defer claiming old-age pension benefits, and if so, whether it is beneficial to take out a reverse mortgage loan (RML) to fund the income gap during deferral years. I use the life-cycle portfolio choice setting to test these retirement income strategies. The results show that incentivising RMLs as a tool to delay pension benefits is welfare-enhancing for the average consumer. In addition, cash-poor investors are found to experience a welfare loss if using the distribution from individual retirement accounts as a sole source to fund the pension delay. Meanwhile, individuals who get the most out of RMLs are those who have a low level of required living standards and an initial money balance that is small but enough to survive the first five years of retirement.

The second study with a Singapore context sets out to explore the influence of unemployment-induced career breaks on retirement income adequacy under fully-funded defined-contribution pension plans, taking into consideration (i) the timing of the breaks, (ii) the duration of the breaks, (iii) and the severity of the impact that job losses might have on post-interruption wages. I find that due to the scarring effect of job loss on post-interruption wage profiles, the sooner the career break, the larger its impact on pension adequacy. Moreover, the results point out that median-income workers, who suffer job displacement during the career path, can still support themselves beyond subsistence in retirement. However, job losses do significantly decrease the retirement nest egg of displaced workers. I also discover that the duration of career breaks has a significant impact on the pension adequacy of displaced workers.

The third study with a U.S focus investigates how self-perceived health status interacts with the portfolio choice of homeowners and renters over their life cycle and how this relationship differs between homeowners and renters. I first develop a dynamic optimisation life-cycle model for the joint determination of health investment, housing, and portfolio allocation; and then perform empirical analysis, using data from the Panel Study of Income Dynamics (PSID), to examine whether the empirical findings support the life-cycle model predictions. The results indicate that the life-cycle model can explain key facts about stockholding profiles of homeowners and renters across different levels of health status in PSID. The proportion of total wealth in stocks is low overall and is positively related to health for both homeowners and renters. However, I find that renters' share of total wealth in stocks responds more strongly to health status deterioration.

ACKNOWLEDGEMENTS

First of all, I would like to express my heartfelt gratitude to Professor Dimitris Margaritis, Dr Joelle Fong, and Dr Helen Lu. I am very lucky to have them as my teachers, supervisors, and mentors on this special journey. Without their continuous support, I would not have come this far. I truly appreciate Professor Dimitris for his experienced guidance, constructive feedback, and helpful recommendations along the way. I have enjoyed our discussions over the past four years, and they do help me a lot in cultivating my reasoning. I am grateful to Dr Joelle Fong for showing me how to work smart and stay organised. My research has also been greatly benefited from her expertise in the field of pension and insurance economics. I also want to thank Dr Helen Lu for spending time and effort to discuss my papers, review drafts, and ask smart questions.

I have been extraordinarily fortunate to be welcome by the Department of Accounting and Finance, the University of Auckland, and Lee Kuan Yew School of Public Policy, the National University of Singapore, during my PhD study. Having opportunities to conduct research in both New Zealand and Singapore has tremendously broadened my horizon. I would like to thank the University of Auckland for granting me a Doctoral Scholarship as well as generous funding for conference travel. I would also like to extend my gratitude to Professor Henk Berkman, Associate Professor Alastair Marsden, Associate Professor Alfred Wu, and Dr Paul Geertsema for sharing their research experience and providing me with helpful comments. I appreciate all feedback and opinions from seminar participants at the 2019-2021 NZFC doctoral symposiums and at the University of Auckland. I thank all the friends that I made in New Zealand and Singapore while doing my PhD. My journey has become more meaningful with their company.

I should add that I have always been blessed to be supported unconditionally by my parents, my parents-in-law, my brother, and most importantly, my husband – Nicolas Vu Duc Thien. Without his love and understanding, we could not have reached where we are today. Thank you for spending time discussing a wide variety of topics with me over the years.

Last but not least, I would like to thank readers for taking the time to read this thesis.

Auckland, 28th February 2021

Chi Nguyen

LIST OF TABLES

LIST OF FIGURES

INTRODUCTION

In recent decades, rapid population ageing and stretched fiscal resources have led to significant pension reforms. In many retirement-income systems around the world, there has been a major shift from defined-benefit pension plans - where pension benefits are pre-determined on the basis of earnings history, tenure of service, and age - to defined-contribution pension plans - where pension benefits are not known in advance, but rather depend on individual portfolio choice and investment risks. On the one hand, individuals have more investment choices. On the other hand, they have to take responsibility for their investment decisions and their retirement income security. The field of household finance – the positive and normative study of how households allocate financial resources to achieve their goals (Guiso and Sodini, 2013) – has, therefore, attracted a lot of attention not only from practitioners but also from researchers and policymakers over the past two decades.

This thesis consists of three studies that investigate household financial decisions over the life cycle. Set within both Asian and U.S. backdrops, these three independent papers relate to the literature on life-cycle portfolio choice and retirement income planning. The first study, as shown in Chapter 1, focuses on household financial decisions during the retirement period. To be more specific, it seeks to find (i) whether reverse mortgage loans (RMLs) – a type of home-equity released products - can serve as a tool to delay old-age pension benefits, and (ii) which financial asset allocations should be adopted during the retirement period. Overall, this study contributes to the current literature in two areas. To begin with, it is among the first to investigate the strategy of taking out RMLs to support the pension delay in a life-cycle portfolio choice setting, the first to introduce the modified-term payment option of RMLs into the life-cycle model, and the first to test the effect of this strategy on individuals' portfolio choice over the retirement period. Second, the present work provides a more realistic solution for the household optimisation problem by adding the progressive retirement income tax and progressive property tax into the model.

The country of interest in this chapter is South Korea. The main reason for choosing South Korea is that it is perhaps the only Asian country which has been relatively successful at promoting RMLs. What the South Korean government has been doing to tackle rapid population ageing and old-age income poverty, therefore, can be an excellent example to learn from for many other countries in Asia where these issues are currently more profound than anywhere else in the world. Regarding the research method, I develop a life-cycle model in which a Korean senior at the age of 61 considers whether to postpone claiming old-age pension benefits until she/he reaches age 66. The individual lives with a probability of 100% at the age of 61, and their survival probability declines over time. During retirement, he/she faces stochastic stock returns and has to decide the optimal portfolio allocation and consumption choice in each period to maximise the discounted expected utility. Funding sources for the pension delay can come from the distribution of individual retirement accounts (IRAs), RMLs, and proceeds obtained from downsizing the house.

The results show that incentivising RMLs as a tool to delay pension benefits is welfare-enhancing for an average consumer. In addition, cash-poor investors are found to experience an average loss of 53.87% (with a standard deviation of 11.87%) in a certainty equivalent consumption level if using the money from IRAs as a sole source to fund the pension delay. At the same time, individuals who have a low level of required living standard and an initial financial wealth that is small but enough to survive the first five years of retirement are shown to benefit the most from RMLs. In terms of the portfolio allocation over the retirement period, I find that the optimal risky asset share is negatively correlated with financial wealth and fairly stable at 40-50%, with the total financial wealth ranging from 50 mil to250 mil KRW. Another finding is that progressive income taxation might have an effect on the optimal portfolio choice. However, this effect is only captured within certain starting points of the income ranges, specifically those subject to the highest and second-highest tax brackets. I also conduct a policy experiment involving a change in the modified-term payment method of the Korean-style reverse mortgages. The experiment suggests that if the money balance from the IRA is large enough to partly aid the delay of pension benefits, it would be better for investors to constrain the proportion of the RML principal limit converted into the fixed-term annuity at 50%.

The second study moves away from financial decisions in retirement and towards financial decisions in the working period. Specifically, this chapter examines the influence of unemployment-induced career breaks on retirement income adequacy under the current design of Singapore's national pension system - best known as one of the world's oldest and largest defined-contribution pension systems (McCarthy et al., 2002; Fong et al., 2011), taking into consideration (i) the timing of the breaks, (ii) the duration of the breaks, (iii) and the severity of the impact that job losses might have on post-interruption wages. In general, this study makes two contributions to the literature. First, in contrast to earlier studies that focus on EU and OECD countries where public earnings-related pension systems are generally defined-benefit or point systems, this chapter examines the impact of unemployment-related career breaks on pension adequacy under fully-funded defined-contribution plans. This is important as significant pension reforms around the world over the past few decades have brought about the increasing popularity of defined-contribution pension plans (Munnel, 2006; Dedry et al., 2017). Hence, looking specifically at how unemployment-related career breaks can impede retirement income adequacy under fully-funded defined-contribution pension plans can offer a valuable lesson. Second, as the middle class is becoming less secure in terms of opportunities, income, and jobs, and as economic inequality is worsening in many societies, this study seeks to determine whether new policies are necessary to mitigate the risks from wage depression and unemployment for middle-income families, who have not been the target of social support policies in many countries around the world.

To test the effect of job displacement on pension adequacy of a median-income earner in Singapore, who joins the labour market in 2018, I devise a simulation model of earnings, social security contributions, asset accumulations and decumulations, as well as housing patterns over the life cycle. First, I find that due to the scarring effect of the unemployment spell on post-interruption wage profiles, the sooner the break occurs, the larger its impact on pension adequacy is. Second, the results point out that median-income workers can support themselves well beyond subsistence in retirement. However, job losses do significantly decrease the retirement nest egg of displaced workers. Finally, the duration of the career break is found to considerably reduce the retirement nest egg at age 55.

The third study extends the previous ones by investigating household financial decisions both in the working and retirement periods. Specifically, I explore how self-perceived health status interacts with the portfolio choice of homeowners and renters over their life-cycle and how this relationship differs across homeowners and renters. On the whole, this study makes two contributions to the literature on the household portfolio choice over the life cycle. First, it is one of the first studies to model the relationship between health and portfolio choice separately for homeowners and renters. This approach enables us to examine whether the correlation between health and stock allocation for homeowners is stronger or weaker than that for renters. Previous works on portfolio choice often focus either on the relationship between homeownership and risky asset allocation, ignoring the health-related considerations (e.g., Grossman & Laroque, 1990; Cocco, 2004; Yao & Zhang, 2005; Chetty et al., 2017; Vestman, 2018); or on the correlation between health status and households' risky asset allocation, overlooking the influence of housing tenure choice on the household portfolio problem (e.g., Edward, 2008; Hugonnier et al., 2013). The second contribution of this chapter is that it introduces health risk as a critical source of background risk, besides labour income risk, to the portfolio choice problem during the working period. Prior research that examines the life-cycle portfolio choice problem in the presence of health risk primarily targets the retirement period (e.g., Edward, 2008; Yogo, 2016).

With regard to the research method, I first develop a dynamic optimisation life-cycle model for the joint determination of health investment, housing, and portfolio allocation, in which individuals face labour income risk while working, and house price risk, stochastic stock returns as well as health risk over the life cycle. Second, using data from the Panel Study of Income Dynamics, I examine whether the empirical findings support the life-cycle model predictions. The results indicate that the life-cycle model can explain key facts about stockholding profiles of homeowners and renters across different levels of health status in PSID. In addition, the proportion of total wealth in stocks is low overall and is positively related to health for both homeowners and renters. However, I find that renters' share of total wealth in stocks responds more strongly to the deterioration of health status. These investigations are crucial to the determination of households' optimal portfolio allocation over the life cycle. First, over recent decades, population ageing has presented challenges to the sustainability and the adequacy of many retirement income systems. As such, pension system reforms across many countries have called for mandatory saving accounts, individual retirement accounts, or individual investment accounts where the retirement wealth of participants depends largely on investment risks and their portfolio decision. Second, if health expenditure is considered an investment in health capital, just as bonds and stocks are investments in financial wealth, and housing expenditure is an investment in housing wealth, investors' health-related considerations and homeownership status should impose additional discipline to a more comprehensive asset allocation problem between health, housing, and financial assets.

CHAPTER 1: REVERSE MORTGAGE AS A TOOL TO DELAY OLD-AGE PENSION BENEFITS

Abstract

This study develops a dynamic stochastic optimisation life-cycle model that incorporates the stochastic process of risky asset returns, progressive tax rules on property value and retirement income to solve for the optimal consumption and optimal portfolio choice of Korean retirees during the retirement years. The representative agent is a homeowner, currently at the beginning of full-retirement age, who is considering (i) whether to postpone old-age pension benefits for five years and (ii) which financial strategies are appropriate to follow. Funding sources for the pension delay can come from the distribution of individual retirement accounts (IRAs), reverse mortgage loans (RMLs), and proceeds obtained from downsizing the house. The results show that incentivising RMLs as a tool to delay pension benefits helps an average consumer's liquid wealth survive almost 25 years in retirement, while under the case with no delaying, the money balance of the IRA runs down to zero after 17 years. In addition, cash-poor investors are found to experience an average loss of 53.87% (with a standard deviation of 11.87%) in certainty equivalent consumption if using the distribution of IRA as a sole source to fund the pension delay. At the same time, individuals who make the best use of RMLs are those who have a low level of required living standard and an initial money balance that is small but enough to survive the first five years of retirement.

1.1. Introduction

In most countries, while the elderly are typically at the bottom end of the income distribution, the majority are homeowners who have already paid off all or most of their mortgages. Moreover, housing wealth often accounts for the largest proportion of their total wealth. People, therefore, are starting to think about unlocking housing wealth to mitigate the problem of income inadequacy in retirement. In this context, reverse mortgage loans are designed for older homeowners to borrow against their home equity without periodic repayments and without moving out of the house. Despite having long suffered from a negative public perception, reverse mortgage has become a more popular financial product over the past decade in many countries such as the United States, United Kingdom, Canada, Australia, Japan, Korea, and Taiwan (Heo et al., 2016; Merton & Lai, 2016).

At the same time, population ageing is believed to be one of the most significant social transformations for many countries in the twenty-first century. As people nowadays are living longer, they have to plan for a longer period without labour income. Consequently, delaying public pension benefits has been increasingly considered an effective way to enhance retirement income security in three ways. First, Coile (2002), Shoven (2014), and Maurer (2016) indicate that a 7-8% increase in pension benefits per year of delay is likely to be far more than most people can earn on their own, especially in a low-interest financial environment. Second, even though the increase in pension benefits per year of delay is actuarially fair, risk-averse individuals value higher deferred benefits as this income source often makes up a significant portion of their total income in retirement. Additionally, in most OECD countries, public earnings-related pension systems are defined-benefit schemes, providing retirees with a lifetime income source that is hedged against longevity risk, inflation risk, as well as investment risk (Gustman & Steinmeir, 2015). Third, delaying public pension benefits is particularly relevant to those in better-than-average health as it gives them access to higher payments for an anticipated longer-than-average lifetime (Huebner et al.,2016; Maurer & Mitchell, 2020).

In view of the arguments outlined above, many financial planners have been promoting the use of reverse mortgage as a tool to postpone claiming old-age pension benefits. This approach allows homeowners to use reverse mortgage loans to replace the income they would otherwise receive in public pension benefits during the deferral years. However, in August 2017, the Consumer Financial Protection Bureau (CFPB) issued a report warning that the costs and risks of taking out a reverse mortgage are found to exceed the additional lifetime amount gained by postponing old-age pension benefits. In response to this report, Pfau (2017) argues that the CFPB has ignored several aspects of the retirement income planning practice, including (i) spending goals and (ii) the presence of an investment portfolio or individual retirement account (IRA). As such, CFPB (2017) does not provide any meaningful conclusion on what individuals should do to build an efficient retirement-income plan, which takes into account public pension claims, home equity, and investment portfolio. In contrast to CFPB (2017), Pfau (2017) incorporates individuals' investment portfolios in the retirement-income strategy. He finds that delaying public pension is beneficial. In addition, funding the delay through a reverse mortgage has a higher probability of meeting retirement spending goals than funding via distributions from investment portfolios. The author also demonstrates that the reduced distribution needs from the individual retirement portfolio and the reduced sequence risk[1] help offset reverse mortgage costs and protect individuals' overall net worth in a down market. Nevertheless, Pfau (2017) fails to address the importance of portfolio choice in the retirement income strategy by simply assuming a fifty-fifty asset allocation for stocks and bonds in investment portfolios.

Hence, in this study, I try to bridge the gap by studying several retirement income strategies in a life-cycle portfolio choice setting. To be more specific, this paper seeks to answer three research questions, including (i) whether individuals should defer claiming old-age pension benefits, and if so, (ii) whether they should take out reverse mortgage to fund the income gap during deferral years, and (iii) which financial asset allocations investors should follow during the retirement.

In this paper, I choose South Korea as the country of interest. First, South Korea is currently one of the fastest ageing countries in the world[2]. Second, according to the OECD, among developed nations, Korea has the highest rate of elderly poverty, with nearly half of Korean seniors at the age of 65 or more living in relative poverty (Jones & Urasawa, 2014). Another notable characteristic of the elderly population in South Korea is that their homeownership ratio at retirement is quite high, estimated at approximately 65% in 2016[3]. With the majority of total wealth locked up in home equity, Korean seniors now need to consider a financial plan that can free up their housing wealth so that they can expect a more stable lifestyle for a more extended period. Finally, South Korea is perhaps the Asian country that has a relatively more successful RM program compared to the few other Asian countries where RM has become available such as China (Hongkong), Singapore, Japan, and Taiwan. Its success mainly comes from flexible payment methods, low interest rates, tax incentives, and strong promotion from the government. According to the 2013 report of the Korean Housing Finance Corporation, with the encouragement fromthe government, 73% of Korean elderly prefer not to live with their children, 30% do not plan to leave a bequest to their children, and 87% do not want to increase the financial burden to their children. The case of South Korea, therefore, can be an excellent example to learn from for many other countries in Asia where population ageing has become one of the most significant social issues in the twenty-first century.

---

[1] Sequence risk is the risk that the timing of withdrawals from an individual retirement account may harm the investor's overall return.

[2] According to the National Statistics Office, Korea is ageing faster than any other developed countries. Specifically, Korea became an 'ageing society' just 17 years ago, and it is now on the verge of becoming an 'aged society' with the percentage of elderly in the total population projected to hit the level of 14% in 2020. In the past, it took Japan, Germany and France 24 years, 40 years and 115 years respectively to make the same transition

[3] 2016 Statistics of homeownership, Statistics Korea

I develop a life-cycle model in which a Korean senior at the age of 61 considers whether to postpone claiming old-age pension benefits until she/he gets 66. The individual lives with a probability of 100% at the age of 61, and their survival probability declines over time. During retirement, he/she faces stochastic stock returns and has to decide the optimal portfolio allocation and consumption choice in each period to maximise the discounted expected utility. This research considers four plans for senior homeowners who enter retirement at age 61 and no longer work. The first one, which is the benchmark for the welfare analysis, allows the senior to claim pension benefits right after retirement (Case 1). In the second plan, the senior decides to delay old-age pension for five years and use the distribution from their investment portfolio as a replacement for the deferred pension benefit (Case 2). Similarly, in the third plan, the agent decides to postpone the old-age pension for five years but take out a reverse mortgage to bridge the income gap due to the pension delay (Case 3). Lastly, to fund the old-age pension benefit postponement, the agent chooses to downsize his/her housing stock (Case 4). The optimal share of liquid wealth invested in risky assets and the optimal consumption level are then estimated using numerical dynamic programming. After the individual optimisation problem is solved for all above-mentioned financial plans, the simulation is done for investors with the old-age pension benefit at age 61 ranging from 3 million to 11 million KRW per year, the initial money balance of the IRA varying from 25 million to 115 million KRW, and current house value is ranging from 50 million to 400 million KRW.

The examination of an average individual's financial wealth path across all cases indicates that taking out a reverse mortgage loan to replace the old-age pension income during deferral years is beneficial. This financial strategy helps the individual’s liquid wealth survive almost 25 years in retirement while under the case with no delaying, the money balance of the IRA runs down to zero after 17 years. In addition, the welfare analysis shows that under Case 2, investors whose liquid wealth cannot last for the first five years of the retirement period will experience an average loss of 53.87% (with a standard deviation of 11.87%) in the certainty equivalent consumption level. Also, individuals who get the most out of the financial strategies under case 3 are those who have a low level of old-age pension benefit and an initial money balance that is small but enough to survive the first five years of retirement.

In terms of the portfolio allocation over the retirement period, I find that the optimal risky asset share is negatively correlated to financial wealth and quite stable at 40-50%, with the total financial wealth ranging from 50 mil-250 mil KRW. Since old-age pension benefits work as a guaranteed income stream during retirement, individuals who possess the same level of old-age pension but a lower level of financial wealth generally have greater exposure to risk-free assets, tilting them towards a more aggressive portfolio allocation. Besides, given a fixed level of financial wealth, the higher the old-age pension benefit, the higher the optimal portfolio share invested in stocks. Therefore, under Case 1, in which investors decide not to delay old-age pensions, the optimal risky asset share will be lower than that under the three remaining cases. Another finding is that progressive income tax might have an effect on the optimal portfolio choice. However, this effect is only captured within certain starting points of the income ranges subject to the highest and second-highest income tax brackets. Also, a rise of 9 percentage points in the marginal income tax rate from the second-highest to the highest tax bracket is found to associate with an increase of 1-3 percentage points in the optimal stock share.

I also conduct a policy experiment. First, some individuals need to convert a large portion of their home equity into a five-year fixed annuity if their house value is lower than 100 million KRW under case 3. Second, it is not recommended to use up home equity quickly during the first few years of retirement. Therefore, in this section, I experiment with one possible change in the modified-term payment method of the Korean-style reverse mortgage. Correspondingly, the proportion of total loan limit that can be converted into a five-year annuity is now fixed at 50%. This experiment shows that in case the money balance from the IRA is large enough to partly aid the delay of pension benefits, it would be better for investors to limit the proportion of loan limit converted into the fixed-term annuity at 50%.

Overall, the contributions of this study to the current literature fall within two areas. First, this study examines various financial strategies that help individuals find the optimal consumption and portfolio choice during the retirement period in a life-cycle model setting. Although there have been several studies on the saving behaviour and portfolio choice of retirees (Campbell et al., 2001, Cocco et al., 2005, Blau, 2008, Hurd & Rohwedder, 2008, Andreasson et al., 2017) as well as on the utilisation of reverse mortgage as a way to unlock home equity during retirement (Michelangeli, 2010, Huang et al., 2013, Nakajima et al., 2017), this thesis is among the first to investigate the strategy of taking out RMLs as a tool to delay claiming public pension benefits in a life-cycle model setting. It is also the first to introduce the modified-term payment option of RMLs into the life-cycle model and the first to test the effect of this strategy on individuals' portfolio choice over the retirement period.

Second, this study provides a more realistic solution for the household optimisation problem by adding the progressive retirement income tax and progressive property tax into the model.

Therefore, the effect of progressive taxation on portfolio choice can be analysed in the study. Several studies capture the income tax effect in the model, but the tax rate is generally set at the level of the average income tax rate. Only very few studies have been able to introduce the progressive tax system into the model (Horneff et al., 2015). However, the impact is tested only at the federal level and is ignored at state and local levels due to the complicated income tax rules in the US. Moreover, the property tax is omitted in these studies.

The rest of the paper is structured as follows. I first discuss the literature on the retirement-income system, the retirement-income planning with the focus on old-age pension benefit claiming strategy, and the use of reverse mortgage loan as a way to support the postponement of pension benefits and the life-cycle portfolio choice. Next, an actuarial model of the Korean-style reverse mortgage is described in Section 1.3. Section 1.4 presents the life-cycle model with the Korean-style reverse mortgage. Section 1.5 summarises the numerical methods used to solve the individual optimisation problem, followed by the calibration of parameters in Section 1.6. Section 1.7 discusses the solutions to the life-cycle model as well as simulation results based on these policy functions. After that, the scenario analysis, welfare analysis, and policy analysis are illustrated in Section 1.8, Section 1.9, and Section 1.10, respectively. Section 1.11 concludes with a discussion of the results and extensions for future work.

1.2. Literature Review

1.2.1. Design of pension systems across countries

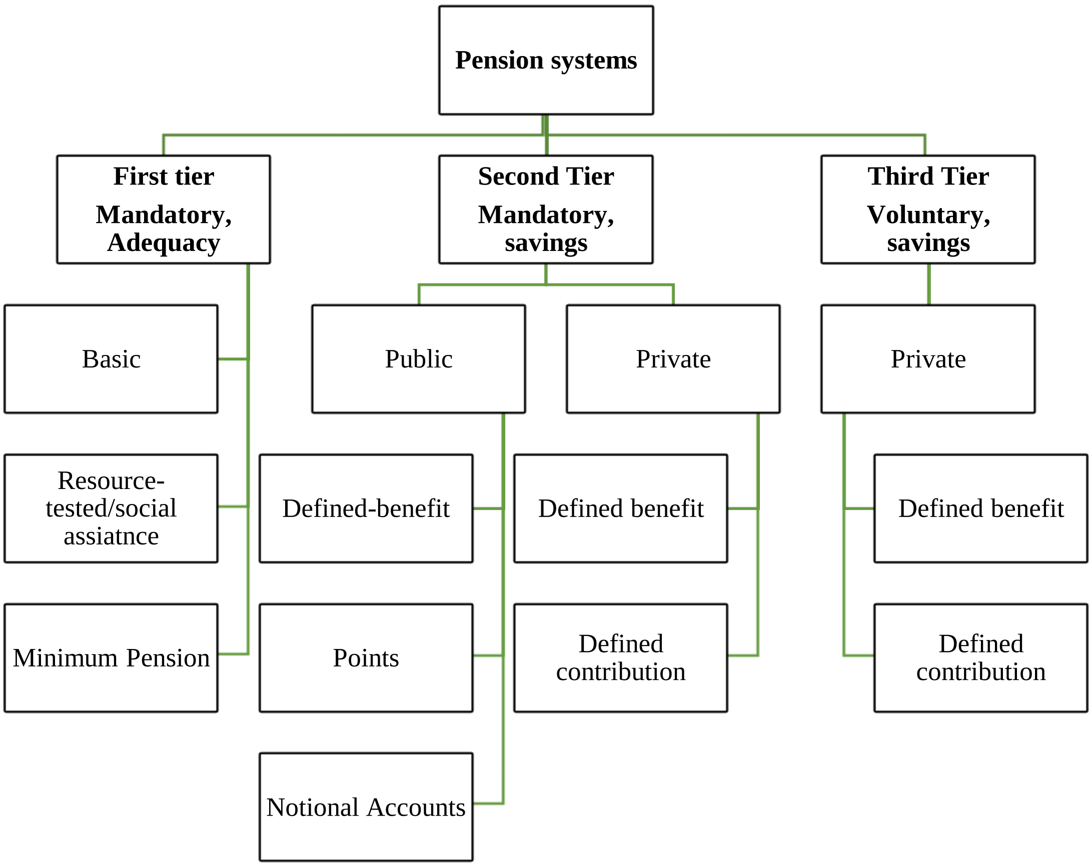
Retirement-income systems are diverse across countries. Setting up a unified classification of pension systems is, therefore, not an easy task. In reviewing the architecture of national pension systems in all OECD and G20 members, the report **Pensions at a Glance 2017** introduced a taxonomy of retirement-income provision based on the role and objective of each part of the pension system. This framework consists of two mandatory tiers and a voluntary provision (Figure 1.1). The first tier is a redistributive part that aims to prevent poverty in the old ages. In other words, it helps achieve the minimum living standard. The second tier is an earnings-related part designed to pursue a certain standard of living in retirement. Voluntary provision comprising individual savings plans or employer-provided sources makes up the third tier. This categorisation is entirely in line with the three-pillar pension format outlined by the World Bank in 1994. Pillar 1- a standardised, state-run pension scheme - primarily focuses on reducing poverty. Pillar 2 - a funded system that employees and employers both contribute into

– is targeting a higher-than-minimum living standard. And Pillar 3- voluntary private funded accounts including private savings and insurance – is encouraging self-fulfilment in retirement.

The first tier is also known as a non-contributory pension provision since the basic benefits under this provision are not funded by pensioners' contributions but by regional or national tax revenues. Non-contributory schemes consist of three main types: resource-tested pensions, basic pensions, and minimum pensions. Resource-tested plans pay a higher benefit to poorer pensioners and vice versa. When it comes to the basic pension scheme, the benefit is either the same for every retiree or is set based on the number of working years. Under the minimum pension scheme, the benefit is determined as the minimum pension of either a specific contributory plan or all combined plans. Although pension plans linked with the contribution history during the working period have become a dominant element of the retirement income system in many countries, the non-contributory scheme is necessary to bring out an adequate and sustainable old-age pension system. According to the social pension database worldwide, there were 112 countries establishing the social-safety net of this type in 2017.

The second tier, the earnings-related component, comprises mandatory contributory plans with contributions linked to earnings during the working years. In this type of pension provision, both employers and employees contribute a certain percentage of workers' salaries. This amount of money is then put into a pension fund and used to pay their lifetime pension. Earnings-related pension plans can be managed by either public or private sectors to provide pension benefits in retirement for the population at large. This mandatory scheme is often the largest income-maintenance program in many countries. Publicly-managed pension schemes are largely financed on a pay-as-you-go basis, which means that the pensions paid out today to retirees are funded by the contributions collected from today's workers. Conversely, in a funded system, contributions are poured into a scheme to create a pool of assets (investments) and blocked until retirement. These assets are used in part (partially-funded) or in full (fully-funded) to pay future benefits.

Figure 1.1: Different types of retirement-income system



Note: Reprinted from *Pensions at a Glance 2017: OECD and G20 Indicators* by OECD, 2017, retrieved from *http://dx.doi.org/10.1787/pension_glance-2017-en*.

Under the publiclymanaged contributory provision, there are three types of pension schemes, including defined-benefits, points, and notional accounts. In regard to point schemes, the pension benefit is determined by multiplying the sum of pension points accumulated during the working period by a pension-point value. Of all the OECD members, there are four countries (France, Estonia, Germany, and Slovakia) following this type of national occupational pension plans. Notional-account schemes are found in five OECD members (Italy, Latvia, Norway, Poland, and Sweden). The notional account records contributions in an individual account and a rate of return is applied to the account balance each period. When individuals reach their retirement age, the accumulated notional capital is converted into a stream of the lifetime pension.

The privately-managed occupational pension schemes are divided into defined-benefit and defined-contribution plans. In a defined-benefit pension scheme, employers or plan sponsors promise to pay employees or plan members a specified periodic payment when they retire. The pension entitlement is linked to employees' earnings history and years of service. Besides, it does not depend on portfolio choice and investment returns. By contrast, in a defined-contribution plan, the contribution formula is pre-determined. However, as benefits paid to plan members depend on investment risk and returns, they cannot be known in advance. In 2017, defined-benefit pension plans were provided by the public sector in 18 OECD members. Privately-managed defined-benefit plans existed in only three countries, including the Netherlands, Switzerland, and Iceland. At the same time, defined-contribution plans were compulsory in 10 countries.

The third tier is the voluntary pension schemes, categorised into defined-benefit and defined-contribution plans, which are similar to the privately-managed occupational pension schemes. However, it differs from the second tier in the sense that contributions to this tier are not mandatory and not earnings-related.

1.2.2. The pension system in South Korea and the National Pension Scheme

The pension system in South Korea is made up of four pillars (Table 1.1). First, the non-contributory "zero-pillar" with basic old-age pension benefits aims to provide elderly people with a minimum level of protection. The basic old-age pension was introduced in 2008 to enhance the elderly welfare by offering a monthly pension payment to the people in need. This pension covered individuals aged 65 and over who earned below a specific amount. By 2012, the basic old-age pension was providing only 16% of the minimum cost of living and benefited 67% of the retirees over the age of 65. It was extended in 2014 to cover all the people aged 65 and over at the bottom 70th percentile of the income distribution. The maximum benefit under this scheme is 204,010 KRW per month. Seniors who earn less than 306,015 KRW per month from the National Pension Scheme get an additional 204,010 KRW per month. Another scheme under the zero-pillar is the basic livelihood security program, providing social assistance for seniors who have an income below the national poverty line and have no family support.

The next pillar under the management of the public sector is the first pillar or the National Pension Scheme. This public pension scheme is a defined-benefit pension plan with contributions linked to workers' earnings history. Each month, employees have to contribute 4.5% of their gross earnings, and the employers give the remaining 4.5%. The objective of the first pillar is to provide a partial replacement of income during the working period. Created in 1988 and established via the National Pension Act, 1986, the National Pension Scheme covers employees between 18 and 59 years old, which makes up approximately 36-40% of the Korean population. In 2016, there was 44.6% of the elderly aged 65 and over being the beneficiary of this scheme, and the maximum level of pensionable income was 4.34 million KRW per month.

Table 1.1: The four-pillar-pension system in South Korea

Pillar	Income support system
Zero pillar	Basic Old-Age Pension
	Basic Livelihood Security Programme
First pillar	National Pension Scheme (Old-age pension)
Second pillar	Company pension (Defined benefit plans, defined contribution plans)
Third pillar	Individual savings for retirement (Individual Retirement Accounts)

The latest update from Statistics Korea shows that in 2017, the monthly average old-age pension was reported at 520,000 KRW per month for the beneficiaries aged 55-79. The insured person with a contribution history of at least ten years can start receiving old-age pension benefits from the age of 61. As the old-age pension benefit – which only stops when the pensioner dies - is adjusted for inflation, this guaranteed source of retirement income can hedge retirees against the inflation risk and longevity risk.

The second pillar of the Korean Pension system is the privately-managed company pension provision. The Korean Corporate Pension scheme was first introduced in 2005 to replace the previous severance pay system. This retirement income provision is still a voluntary occupational pension plan. As of June 2017, only 15.6% of 1.68 million local companies have adopted the corporate pension scheme. According to the Ministry of Strategy and Finance (South Korea), however, all local companies are required to introduce a retirement pension plan by 2022. There are two types of pension plans under the second pillar, including defined-benefit (DB) and defined-contribution (DC) plans. In regard to defined-benefit plans, pension entitlements are pre-determined and known in advance. Under defined-contribution plans, retirees have a range of investment options (portfolio allocation) to choose and pension benefits depend on the investment returns. The exposure to risky assets in a defined-contribution account was relaxed to reach the limit of 70% in 2015. It follows that DC participants will have to bear the investment risk. Under the DB pension plan, the investment risk is transferred to employers. In addition to employers’ contributions to pension plans, DB and DC participants can make an additional contribution of up to 12 million KRW per year.

Finally, the third pillar is a voluntary pension provision with contributions being made by individuals. The voluntary individual pension scheme has been available in the form of Individual Retirement Accounts (IRA) since 1994. Seniors will have to make investment decisions (portfolio allocation) and have to bear the investment risk. In other words, IRAs work in the same way as DC plans do. The only difference between these two plans lies in who contributes to the account. The lump-sum retirement benefits from corporate pension schemes can be poured into IRAs, and the accumulated balance will follow the investment rule (portfolio allocation) set by the individual.

1.2.3. The retirement-income planning

1.2.3.1. Old-age pension benefit claiming strategy: the case of South Korea

Under a publicly-managed and defined-benefit pension plan, the earnings-related pension benefits paid out in the future are pre-determined based on the earnings history, years of working, and the time when retirees claim pension benefits. In many OECD countries with DB pension schemes, individuals can claim pension benefits between early retirement age and late retirement age, which often takes place five years after the full-benefit retirement age.

In South Korea, the National Pension Scheme is a publicly-managed and earnings-related retirement-income provision established through the National Pension Act. According to Article 61 under this Act, when a person whose contribution was made for at least ten years reaches the age of 61, he or she shall receive the old-age pension for the duration of his/her lifetime. It is also demonstrated in the article that when a person aged more than 55 who has contributed to the National Pension Scheme for ten years and over is not involved in income-earning activities, he or she may receive the early old-age pension from the time of claiming onwards. Next, the amount of old-age pension and the amount of early old-age pension are defined in Article 63. For an insured period of 20 years and over, the amount of old-age pension is equal to the basic pension amount[4]. Meanwhile, for an insured period of over ten years but less than 20 years, the old-age pension amount is determined by adding 5% of the basic pension amount for every year over and above ten years to the amount equivalent to 50% of the basic pension amount. The early old-age pension is obtained by multiplying the old-age pension amount by the ratio of 70%, 76%, 82%, 88%, and 94% for the case of claiming benefits from the age of 56, 57, 58, 59, and 60 respectively.

In addition to the early claim of pension benefits, delaying pension benefits is accepted under the National Pension Act. The postponement may be granted before the age of 66 on no more

[4] "The basic pension amount is calculated as half of the targeted replacement rate (46% in 2016) times the individual's lifetime average earnings valorised in line with nominal wage growth and half of the average earnings of the insured measured over the previous three years and valorised in line with prices"- Retrieved from https://www.oecd.org/els/public-pensions/PAG2017-country-profile-Korea.pdf

than one occasion for a beneficiary aged 61 to 65 who wishes to defer the pension benefit. In other words, the pension benefit can be deferred for a maximum of five years. For every year of delay, the pensioner can earn an additional 7.2% of the old-age pension amount. So, if a person decides not to claim old-age pension benefits at the age of 61 but wait for five years until he or she gets 66, he or she will get an additional benefit of 36% of an original amount at age 61 plus the adjustment for inflation every year in the deferral period.

1.2.3.2.Funding the old-age pension delay

If an individual decides not to claim old-age pension benefits when he/she reaches the age of 61 and plans to postpone the claim for five years, he/she needs to carefully consider how to bridge the income gap during the deferral.

The first and least costly way an individual should consider if they want to delay claiming old-age pension benefits is to keep working. Not only does that helps Korean seniors bridge the old-age pension delay, but it may also put the need to draw down the retirement savings forward, allowing the money balance in the individual retirement account to continue growing.

However, working longer may not be the answer for most Korean seniors aged 61 or over. In this case, the other option is to withdraw an additional amount from the investment portfolio to replace the deferred pension benefits. Goda et al. (2017) show that approximately 31-34% of pensioners who claim prior to the full retirement age have an individual retirement account (IRA) that could support at least two additional years of pension benefit delay, and around 24-26% could fund the four-year-pension-deferral with the distribution from the IRA alone. These results suggest that distributions of IRAs can be used as an alternative income source for the deferred pension benefits in case individuals choose to delay. Also, Meyer & Reichenstein (2010) indicate that exploiting retirement savings first and cutting back the drawdowns later, when pension benefits kick in, can extend the investment portfolio's survival by up to seven years.

Third, seniors aged 61 can also purchase an immediate annuity[5] for a fixed number of years until they start receiving the pension benefits. This option is similar to the second one. Pfau (2017) claims that by combining an annuity with delaying pension benefits, the retirees' investment portfolio would likely last longer than if they had claimed benefits early. He argues that locking in an annuity helps investors avoid decumulating the IRA early in a down market,

[5] An immediate annuity is the type of annuity, in which individuals make a lump-sum contribution and receive an ongoing, guaranteed income stream for a specified period of time.

which can drain the portfolio more quickly. However, it should be noted that once seniors buy a fixed-term annuity, they cannot get the money back.

Finally, rather than relying on financial wealth as a funding source for the pension benefit delay, individuals can consider unlocking the housing wealth as a way to finance the deferral. People can choose to downsize the housing stock to free up a part of the wealth locked in their home equity or tap into their home equity through RMLs. For Korean seniors, the bonus of taking out RMLs is that borrowers can enjoy (i) the tax advantage on retirement income, (ii) the 25% reduction of property tax, and (iii) the exemption from registration tax, local education tax, the special tax for rural development, the obligatory purchase of National Housing Bonds. Pfau (2017), on the study of the funding sources for retirement efficiency improvements, finds that using RMLs to fund pension benefit delay can increase the probability of meeting consumption goals in retirement. The author also demonstrates that the reduced distribution needs from IRAs and the reduced sequence risk help offset reverse mortgage costs and protect the overall net worth in a down market.

In this study, I focus on the use of an RML as a financial tool to support the pension benefit delay. An overview of RMLs and the Korean-style reverse mortgage will be discussed in the next section.

1.2.3.3. Reverse mortgage as an income source to replace deferred pension benefits

1.2.3.3.1. Reverse mortgage loans: An overview

Although specific rules for reverse mortgage transactions vary with the governing laws, there are some common characteristics of RMLs across countries. In general, RMLs allow the elderly to borrow money against the value of their primary residence to enhance their current consumption. No repayments of principal or interests are required until the homeowners pass away or leave the house -the point at which the house must be sold and the proceeds from the sale of the house are used to repay the outstanding loan balance. Reverse mortgage is a non-recourse loan. Specifically, when the property is sold, and the sale cannot cover the outstanding loan balance, the borrowers or the heirs to the house have no responsibility for the shortfall. Lenders, therefore, will bear the risk when the value of the loan exceeds the amount getting from the sale of the property. There are two ways that can make a non-recourse loan possible. First, lenders will set the maximum proportion of home equity (Loan to Value ratio – LTV) that can be borrowed. This ratio is determined based on several aspects, including (i) the life expectancy of the individual and their spouse, (ii) the expected value of the actual loan interest rate, and (iii) the house price appreciation rate. Second, initial and annual mortgage insurance premiums are charged to borrowers and deposited into a mutual mortgage insurance fund. In short, lenders are more willing to make riskier loans as they are backed by the mutual mortgage insurance fund. While the initial mortgage insurance premium is a one-time payment paid at the loan closing, the annual mortgage insurance premium is an ongoing expense – defined as a certain percentage of the loan balance – throughout the life of the loan.

Although borrowers can delay the repayment schedule, the loan interest and the annual insurance premium will keep compounded annually, resulting in the rising debt balance over time. Moreover, there is also an up-front cost, which usually is set at 2-4% of the current house value. In addition to the up-front cost (counselling fee, origination fee, third-party fees, and initial mortgage insurance premium), periodic insurance premium and loan interest are the costs charged on the outstanding loan balance. For these reasons, reverse mortgages can be an expensive financial product. On top of that, borrowers also face a moving risk since a homeowner who moves out after a short period of time since taking out RMLs must repay the lesser of the house value and the outstanding debt balance, losing the up-front costs so quickly (Michelangeli, 2010).

Nakajima & Telyukova (2017) indicate that there are six main differences between a traditional mortgage and a reverse mortgage. First, a reverse mortgage borrower does not have to make periodic payments of interest and principal, while the opposite is true for a conventional mortgage. Second, to be qualified for an RML, individuals need to reach a certain age depending on the governing laws and have to be homeowners living in their house. One more requirement for a reverse mortgage borrower is that the other mortgages must have been repaid at the time of taking out the RML. Compared to a conventional mortgage, reverse mortgage lenders do not require income history or credit history since repayments are not made on the basis of borrowers' income. Third, reverse mortgage borrowers are required to participate in a counselling session with an approved counsellor. Fourth, the due date is not pre-determined, and repayment is required when the borrowers move out or pass away. Fifth, unlike traditional mortgage loans, RMLs are non-recourse. The last difference is that borrowers have various options to receive payments from the RMLs. Though each country has its own design for the payment methods from RMLs, some payment options are popular across countries: tenure-option (a fixed-monthly annuity for the lifetime), term-option (a fixed-monthly annuity for a fixed period), modified tenure, and modified term options (the combination of a certain percentage of the loan limit set aside and tenure/term features).

Over the last few decades, research on reverse mortgages has attracted growing attention from academics and policy-makers. Back in the 1990s, studies on RMLs (Venti & Wise, 1991; Merrill et al., 1994; and Miceli & Sirmans, 1994) often focus on developing actuarial models for various reverse mortgage plans. In general, an RML is a financial product that can be viewed as a put option in which the house is the underlying asset and the outstanding loan balance at the time the homeowner moves out or passes away is the exercise price (Merrill et al., 1994; Merton & Lai, 2016; Choi et al., 2020). A reverse mortgage, therefore, links to some fundamental risks due to the uncertainty of house price dynamics and homeowners' longevity. On the lenders' side, typical risks might include longevity risk, interest rate risk, house price risk, and maintenance risk (Merton & Lai, 2016). By contrast, on the borrowers' side, there is little risk, such as moving risk – the probability of moving out so quickly after the loan closing (Michelangeli, 2010), explaining the high cost of RMLs.

More recently, the literature on reverse mortgage loans has drawn attention to the design of RM products and the characteristics of RM borrowers and non-borrowers. Red-foot, Scholen and Brown (2007) help explain how to better design RMLs by interviewing actual borrowers and those who considered RMLs but eventually decided not to use them. The results from the survey show that the main reasons why borrowers take out reverse mortgages include (i) funding for everyday expenses (50% of 946 borrowers), (ii) improving the quality of life (73% of 946 borrowers), (iii) funding for emergencies or unexpected events (78% of 946 borrowers), (iv) funding for medical expenses (28% of 946 borrowers), and (v) funding for property taxes/insurance (29% of 946 borrowers). They also asked non-borrowers what could be the reasons to look into reverse mortgages. 68% of non-borrowers mentioned improving quality of life as the main reason to ponder RMLs, while 66% and 43% considered funding the unexpected events and funding the home repairs, respectively, their primary motives. Fisher et al. (2007) point out that some potential reasons why people feel reluctant to take out an RML involve the aversion to debt, the desire to preserve their house for future generations, high costs, and the strategy of keeping home equity as a last resort for major economic shocks or health crises. In terms of the effect that house price might have on the uptake of RMLs, Shan (2011) addresses that higher house prices lead to more reverse mortgage originations, explaining the significant growth in the U.S. reverse mortgage market in the mid-2000s. Rodda et al. (2004) indicate that because the total loan limit is determined based on the house value at the time of taking out RMLs, seniors can only benefit from the appreciation in house value via debt restructuring, which is worth doing if the costs associated with refinancing can be adjusted downward through a reduction in the insurance premium. It is also vital to note that the high up-front costs and periodic insurance premiums of reverse mortgages are attributed to the non-recourse feature. Indeed, Nakajima & Telyukova (2017) demonstrate that if this feature were removed, the demand for RMLs would increase by 73%.

Another strand of literature on reverse mortgage sets out to answer whether it is beneficial for retirees to take out RMLs. Davidoff (2015) examines the conditions under which reverse mortgages may be beneficial to homeowners but in a circumstance where many of the idiosyncratic risks are absent. Michelangeli (2010), Huang et al. (2013), and Nakajima & Telyukova (2017) are closest to this paper in approach. They all use the life-cycle model to investigate the welfare gain/loss from using RMLs. While Michelangeli (2010) and Huang et al. (2013) assume that borrowers have to borrow the maximum amount at the time of loan closing, Nakajima & Telyukova (2017) model the line-of-credit RML. Michelangeli (2010) and Huang et al. (2013) both agree that RMLs are welfare-enhancing for many retirees but not necessary for all. More specifically, Michelangeli (2010) shows that in the presence of compulsory moving shocks, RMLs are a very bad option for "house-rich, but cash-poor" households as this plan reduces expected utility to the same degree as 14 percent loss in financial wealth. Huang et al. (2013) indicate that for house-rich and cash-poor homeowners, the optimality of the plan depends on the extent of retirees’ cash shortage, and the best time of entering into RMLs is about five years after retirement. Nakajima & Telyukova (2017) develop a model of housing and saving/borrowing decisions in retirement. In the model, households are allowed to choose between owning and renting, and homeowners can choose to sell their house at any time or to borrow against their home equity. They find that the average welfare gain from RMLs is $ 252 (0.84% of annual after-tax income at age 65) per homeowner who values the option of being able to use their home equity sometime during retirement via the line of credit RML and $ 1770 (5.13% of household income at age 65) per borrower who eventually uses RML. Also, low-income households are found to value the option more than high-income households. Finally, they indicate that the higher the house value, the higher the welfare gain.

1.2.3.3.2. The Korean-style reverse mortgage

Reverse mortgage has a relatively short history. It first appeared in a bank in Maine, the United States, in 1961. However, the systematisation of RMLs was only brought about in 1987 by the U.S. federal government. Since then, it has become available in several countries such as Australia, Canada, China (including Hongkong), some European Union countries, Korea, New Zealand, Singapore. Although this financial product has gained considerable attention recently, the take-up rate is still modest even in countries such as the U.S., U.K., and South Korea, where the home-equity conversion program has been running for a considerable time. Merton & Lai (2016) indicate that South Korea is perhaps one of the few Asian countries that have achieved relative success on the reverse mortgage program. Even though the high homeownership rate at retirement and demographic ageing play a role in this success, flexible payment options are considered the main reason. In addition, three other factors are shown to create a positive effect on the reverse mortgage take-up rate in South Korea. First, the program is guaranteed by the government. Second, the interest rate charged on the outstanding loan balance is capped low. Third, many tax advantages are granted for reverse mortgage users. In fact, what currently makes the reverse mortgage program in South Korea unique, compared to that in other countries, is that the Korean government is taking proactive steps to promote this financial product. According to the report "2013 Residential Annuity Demand Condition" of Korean Housing Finance Corporation (KHFC), under the encouragement of the government, of all Korean elderly, there were 73% favouring the option of not living with children, 30% do not plan to leave a bequest to their children, and 87% trying not to increase the financial burden to their children. The Korean reverse mortgage program has, therefore, grown rapidly since its introduction in 2007.

JooTaekYeonKeum is the name of the Korean-style reverse mortgage, often referred to as JTYK. This program was launched by the Korean government in July 2007. Before the implementation of JTYK, all reverse mortgages were provided by banks and insurance companies without any support from the government. Kim & Yoo (2006) illustrate that there are three main features that make these private reverse mortgages differ from a typical government-guaranteed reverse mortgage program. First, under the private reverse mortgage program, the tenure payment options (monthly annuity for a lifetime) were not offered. Second, these loans were not designed for the elderly only. Finally, borrowers had to bear all risks, including interest rate risk, longevity risk, and house price risk. In the early 2000s, South Korea experienced a significant change in its demographic structure, resulting in a high ratio of ageing cohorts. At the same time, severe income poverty was witnessed among the Korean elderly population, encouraging the government to introduce a new retirement-income maintenance program. That is when JTYK came into play. This newly-employed reverse mortgage program has attempted to fix the limitations of the previous reverse mortgage program. The government established the JTYK guarantee fund to protect borrowers against some fundamental risks such as house price variations and the risk of discontinued payments. In return, borrowers must meet the requirements on the insurance premium and other charges.

The Korean-style reverse mortgage has two key characteristics. First, the borrower and their spouse can reside in their house and receive a pre-determined monthly payment over the borrower's lifetime. Since 2012, a "non-borrowing spouse" is protected under the condition that if the borrower passes away, his or her spouse can remain in the house without making any repayment to lenders. However, the age of non-borrowing spouses will be taken into consideration when lenders define the total loan limit that borrowers can get from their own house. In case the non-borrowing spouse is much younger than the borrower, the protection for the spouse can significantly reduce the initial principal limit. Second, borrowers under the JTYK program can choose a lump-sum payment up to a certain limit at any time to cover some unexpected expenses such as medical bills.

Regarding the requirements that Korean seniors have to satisfy to be eligible for RMLs, the first condition indicates that the homeowner or spouse must be 55 years old or older at the time of collateralising the house. Second, the property value must be less or equal to 900 million KRW. If the borrower has more than one property, the sum of property value must be equal to or lower than 900 million KRW. The house value is appraised by a credible external institution at the time of application.

Before 2016, there were three payment plans available: tenure, modified-tenure, and term plan. Tenure plans offer monthly annuities until the homeowner and partner die or move out of the house. When borrowers choose the modified-tenure type, a certain percentage of the initial principal limit is set aside as an available line of credit, and the remaining balance is converted into a lifetime annuity. Lump-sum withdrawals from the line of credit are permitted within 50% of the total loan limit and can be made anytime during the loan period. Under the term plan, a maximum of 45% of the initial principal limit can be withdrawn on any occasion before the loan is terminated, and 5% of the loan limit is paid out when the payment term ends. Besides, the remaining amount of the loan limit is converted into a fixed-term annuity (5, 10, 15, 20, 25, and 30 years). It is also important to note that lum-sump withdrawals are for three purposes, including (i) repayment for the housing debt, ii) returning the deposit amount to the tenant, iii) some general costs occurred – house maintenance costs, health costs, or educational cost. In 2016, the KHFC launched three types of modified reverse mortgage (Naejibyeongeum). The first type is the "loan-repayment-type" reverse mortgage, which sets out to expand the range of sizeable initial withdrawal (50% to 70% of the total loan limit). This type of RMLs aims to reduce the burden of the mortgage loans that already existed in retirees' debt balance. The second type of Naejibyeongeum plan is not a direct reverse mortgage but a product associated with the Bogeumjari loan. People who are in their forties and fifties can enjoy a favourable Bogeumjari loan interest rate if they promise to register for a reverse mortgage plan in the future. The last one targets the elderly with only one house of lower than 150 million KRW. Under this special tenure-payment plan, the monthly payment can increase by 15% at maximum. By far, the tenure-payment method has been the most popular choice, followed by the modified tenure-payment. According to the Korean Housing Finance Corporation, in 2016, there were 6615 (1901) cases picking the tenure plan (the modified-tenure plan), accounting for 64.2% (18.4%) of all the cases.

Apart from the various choices of payment plans, the JTYK borrowers can enjoy the exemption from registration tax, education tax, the obligatory purchase of National Housing Bonds, and a special tax for farming and fishing villages. Distributions from a JTYK plan are not considered taxable income but rather loan advances. Also, 25% of property tax is reduced when taking out a JTYK. Last but not least, the maximum of 2 million KRW per year on loan interest is deducted from taxable pension income.

Contrary to the above-mentioned benefits, JTYK borrowers have to pay up-front costs and annual insurance premiums. The KHFC regulates loan conditions and imposes guarantee fees for JTYK borrowers. The loan interest rate is set equal to an adjustable three-month Certificate Deposit (CD) rate plus 1.1%, which is currently lower than the interest rate of any other regular mortgage loan. This rate is charged on the outstanding loan balance at the end of each year. In case a JTYK loan is funded by private banks, the KHFC will control the margin rate to keep the loan interest rate more favourable than the interest rate imposed on a traditional mortgage. As borrowers join the JTYK program, they have to pay an initial insurance premium and annual insurance premiums. Regarding this type of cost, 1.5 percent of the total home value is charged at the time of loan closing (up-front costs), and 0.75% of the loan balance is charged annually until an RML becomes due. When an RML is terminated, lenders usually collect the principal and interest by selling the home. If the proceeds from selling the property are higher than the total outstanding loan balance, heirs can claim the residual value. In case the outstanding loan balance exceeds the amount received from the sale of the house, borrowers, and heirs do not need to cover the shortfall.

In terms of house prices at the time of loan closing, as stated in the 2016 annual report of the KHFC, the average value was 283 million KRW. Heo et al. (2016) summarise the results of the 2014 JTYK Demand Survey conducted by the KHFC. In-depth interviews were carried out with 600 JTYK borrowers and 3000 non-JTYK borrowers. They show that the percentage of house prices that were lower than 100 million KRW at the time of origination was 15.7% among non-borrowers, while the rate was only 5.8% for the case of JTYK borrowers. By contrast, the percentage of house prices between 400 and 600 million KRW was found much higher among JTYK users (15%) than among non-JTYK borrowers (7.2%). There were 45% (27.2%) of the borrowers having house prices at the time of taking out JTYK loans ranging from 200-400 (100-200) million KRW. In addition, the proportion of housing wealth in the total wealth among JTYK borrowers was remarkably larger than that among non-JTYK borrowers, indicating that the primary users of the Korean-style reverse mortgage were relatively house-rich and cash-poor.

The number of Korean elderly has been climbing up so fast over the last two decades. To overcome the problems of demographic ageing and poverty among older people, the Korean government has proactively encouraged the development of the JTYK program. There was a noticeable increase in the number of JTYK borrowers from 6486 cases in 2015 to 10309 cases in 2016, an increase of 58.9%. Still, the uptake of RMLs in South Korea is modest compared to the potential demand estimated at half a million homeowners aged 60 and over (Cho et al., 2004).

1.2.4. The life-cycle portfolio choice framework

In financial economics, a large number of studies on portfolio choice have been developed from the seminal work of Markowitz (1952). One of the most important assumptions in the Markowitz mean-variance model is that every individual makes decisions in only one period. However, in the real world, people will rebalance their portfolios in accordance with changes in their income, accumulated wealth, and investment opportunities. In this manner, investment decisions in one period might be affected by the portfolio allocation in the previous periods. As the standard mean-variance framework cannot solve the multi-periods optimisation problem of individual investors, there has been a growing interest in the study of dynamic portfolio choice, which aims at finding the optimal solution for investors with a long-term horizon.

Samuelson (1969) and Merton (1969, 1971) are the first to study lifetime portfolio selection using dynamic stochastic programming. They show that in the perfect-markets setting without the presence of labour income risk, the optimal share of wealth invested in stocks (i) is constant and independent of wealth and age, (ii) and depends only on the level of risk aversion and the moments of assets' excess return. However, in the realistic life-cycle setting, labour income is non-tradable and cannot be capitalised. Bodie et al. (1992) incorporate non-tradable labour income into the standard life-cycle model of portfolio and consumption decisions. Their findings indicate that with certain future labour income, young individuals in the working period are recommended to invest proportionally more stocks in their portfolios compared to the retired individuals. Koo (1998) and Heaton & Lucas (1997) take into account the uncertainty of the labour income process in their studies. However, they both assume that individuals continue working over their lifetime. Viceira (2001) fixes the issue mentioned above by introducing retirement into the dynamic model of the optimal portfolio and consumption choices in the presence of labour income risk. In line with the results from previous papers, Viceira (2001) points out that the optimal risky asset share is explicitly larger for employed individuals than for the retirees with idiosyncratic labour income risk. Besides, the author shows that portfolio rules in the retirement period match the standard solution described in Samuelson (1969) and Merton (1969). Because there is no labour income risk in retirement and the investment opportunity is constant, the optimal fraction of financial wealth put into stocks is found to be proportional to the equity risk premium and inversely proportional to the constant relative risk aversion.

In the study of the effects that different mandatory retirement plans might have on savings behaviour and portfolio choice during life-cycle, Campbell et al. (2001) reason that though future labour income is uncertain, it can be considered as an implicit holding of a risk-free asset. That way, early in life, when the implicit exposure to risk-free assets under the form of labour income is substantial, investors tend to allocate a more significant portion of his/her liquid wealth (which is often small at the early stage of life) into stocks. As the investor reaches the age of forty, the liquid wealth increases relative to labour income, making the implicit holdings of the risk-free asset less significant. Accordingly, less exposure to risk-free assets encourages a smaller proportion of liquid wealth invested in stocks. After retirement, liquid wealth generally decreases over time as individuals earn no labour income when they stop working. As such, there will be greater exposure to risk-free assets (in the form of defined-benefit pensions) during retirement, leading to a more aggressive portfolio allocation. In a study regarding life-cycle portfolio choice of individuals with non-tradable labour income and borrowing constraints, Cocco et al. (2005) compare the optimal portfolio rules from their model with the investment rule of thumb proposed by Malkiel (1999) and the optimal portfolio choice presented in Merton (1969). They find that although the utility cost of following the conventional advice, which suggests that the allocation of risky assets should be reduced with age (Malkiel, 1999), is relatively small, it increases as investors are less risk-averse. In addition, the welfare loss is found in the case of following the investment strategy suggested by Merton (1969).

Gomes et al. (2008) investigate the behaviour of investors under the life-cycle framework with flexible labour supply. They also calculate the utility cost of constraining portfolio composition to mimic the common default investment choices in the defined-contribution pension plans. The results support prior findings in that during the working period, the exposure to risky assets in the portfolio decreases as people age. However, variable labour supply significantly increases the optimal risky asset share. Post-retirement, it is found that the optimal share of financial wealth invested in stocks goes up as the decumulation of the individual investment portfolio over time leads to the increasing dominance of bond-like pension benefits over other sources of wealth. Some works that track this reasoning closely include Chai et al. (2011) and Maurer et al. (2013).

More recently, Fagereng et al. (2017) re-examine the life-cycle portfolio choice of investors in the presence of labour income risk, asset market participation costs, and a small probability of stock market crash using the large panel data of Norwegian households from 1995 to 2009. First, the optimal risky asset share is found to decrease with financial wealth, consistent with findings in previous literature. With a given level of labour income, investors with lower levels of financial wealth or total wealth have a higher exposure to risk-free assets, thus tend to invest more aggressively in stocks. Second, they find that adding participation costs to the model of Cocco et al. (2005) has little impact on the conditional risky asset share or its age profile. However, introducing a small probability of a market crash decreases the optimal risky asset share at all ages except for very young ages. Finally, estimation results illustrate that the risky asset share is high and roughly constant during young ages at a level of nearly 50%, and this rate slightly decreases with the pace of less than 1% per year from the age 45 until retiring, before remaining fairly stable around 30% during retirement.

1.3. Actuarial Model of Reverse Mortgage Loans in South Korea

One of the ways that seniors can get hurt from RMLs is that they are persuaded to take more money up-front than they actually need. In this case, while the money borrowed may be sitting in a bank account earning practically no interest, the outstanding loan balance on RMLs is accruing interest each month, eating up borrowers' home equity quickly. Hence, RMLs should be used wisely to achieve financial goals in the retirement period. In this study, I consider RMLs only for replacing the old-age pension benefits in five deferral years starting from full-benefit retirement age. Therefore, in this model, Korean seniors are recommended to choose the modified-term payment option, which is the combination of line of credit option and fixed-term payment option (5, 10, 15, 20, 25, or 30-year term). Under this plan, there will be a certain percentage of the total loan limit set aside as an available line of credit. The remaining amount will be converted into a fixed-term annuity.

In this study, the agent chooses the 5-year-term payment option for the RML to replace the old-age pension benefits that he or she might receive at age 61 during the five deferral years. The pricing formula for a Korean-style reverse mortgage under the modified-term plan is described below.

1.3.1. The loan-to-value ratio and loan limit

The loan limit is the maximum amount of money that an individual can borrow from an RML. This amount is determined by the house value at the time applying for an RM loan – $\mathbf{H_{i,1}}$ and the Loan-to-Value ratio **($\mathbf{LTV_i}$)**

$$LTV_i = (\frac{1+g}{1+k})^T$$

where **T** is the life expectancy of agent **i** at age 61**; g** is the house price appreciation rate, and **k** is the expected value of the actual loan interest rate.

The actual loan interest rate is defined as the sum of the 3-month CD (Certificate of Deposit) interest rate and 1.1%. The up-front cost of taking out an RM loan in Korea equals 1.5% of the house value at the loan closing- $\mathbf{H_{i,1}}$. The loan limit, thus, is computed as $\mathbf{H_{i,1}*LTV_i-1.5\%*H_{i,1}}$

Following Ma & Kim (2017), the house value appreciation rate and the expected value of actual loan interest rate in the current Korean RM model are set at 2.2% per year and 5.24% per year correspondingly. In this thesis, I set T equal to 25 years. The loan-to-value ratio is, therefore, estimated to be 48.06%[6].

1.3.2. Modified-term payment option

The minimum duration of the loan term under the fixed-term option for a Korean-style reverse mortgage is five years. I assume that in order to fund an old-age pension delay, Korean retirees choose the modified-term payment option, which is the combination of the line of credit option and the 5-year-term payment option. A lump-sum withdrawal from the available line of credit can be made anytime during the loan period. However, one withdrawal is only allowed within 50% of the total loan limit.

Under this payment option, I first need to calculate the present value (PV) of a 5-year annuity with an annual cash inflow set equal to the old-age benefit at age 61. The PV has to be smaller than the loan limit or the initial principal limit. Therefore, in case the calculated PV exceeds the loan limit, it will be set equal to the loan limit. The annual payment (monthly payment= annual payment/12) $\mathbf{pmt_{i,t}}$ is computed as:

$$pmt_{i,t} = \frac{PV}{\sum_{t=0}^{T-1} \frac{1}{((1+m)(1+k))^t}}$$

In which:

k: the expected value of the actual loan interest rate

m: annual guarantee fee (0.75%) charged annually on the loan balance

T: number of periods receiving payments

[6] According to Ma, S., & Deng, Y. (2013), the maximum loan-to-value ratio at age 60 is estimated at the level of 41.82% and 44.19% respectively for constant monthly payment and graduate monthly payment options. Kim, J. H.T, & Li, J.S.H (2017), on the other hand, derives the maximum loan-to-value ratio of around 54-56% for the case of fixed-term payment at age 60.

1.3.3. The annual insurance premium and loan interest rate

Annual insurance premium $\mathbf{aip_{i,t}}$ is calculated as below:

$$\mathbf{aip_{i,t}=(OLB_{i,t}+pmt_{i,t}-repay_{i,t})*m}$$

where $\mathbf{OLB_{i,t}}$ is the outstanding loan balance at the beginning of period t (at the end of period t-1), $\mathbf{repay_{i,t}}$ is the annual repayment, if any, and **m** is the annual guarantee fee (0.75%) charged annually on the outstanding loan balance.

The accrued interest on outstanding loan balance- $\mathbf{Int_{it}}$ is defined as:

$$\mathbf{Int_{i,t}=(OLB_{i,t}+pmt_{i,t}-repay_{i,t}+aip_{i,t})*r_t}$$

$\mathbf{r_t}$ is the actual loan interest rate at year t

1.3.4. The dynamic of the outstanding loan balance

Outstanding loan balance at the beginning of year t+1 or at the end of year t: $\mathbf{OLB_{i,t}}$

$$\mathbf{OLB_{i,t+1}= OLB_{i,t}+pmt_{i,t}-repay_{i,t}+aip_{i,t}+Int_{i,t}}$$

$$\mathbf{OLB_{i,t+1}= (OLB_{i,t}-repay_{i,t}+pmt_{i,t})*(1+m)*(1+ r_t)}$$

1.4. The life-cycle model with Korean-style reverse mortgage

The microeconomic optimisation problem can be summarised as follows. A consumer enters retirement with an initial money balance from the individual retirement account, housing wealth, and public pension benefit (under a defined-benefit pension plan) if claiming right after leaving the workforce. In each period, the retiree receives his/her pension benefit from the National Pension Scheme and has to decide the optimal consumption, the withdrawal amount from the individual retirement account, and the allocation of financial wealth between risk-free and risky assets in order to maximise the discounted lifetime utility. The risky asset return is allowed to follow a stochastic process. Retirement income in each period is subject to retirement income tax, and the retiree’s house is subject to property tax. When the individual enters retirement, he/she lives with a probability of 100%, the survival probability declines over time, and the agent is assumed to live up to age 85.

In this study, I examine the four following financial strategies for senior homeowners who enter retirement at age 61 and no longer work:

- Case 1: Agent i does not delay claiming old-age pension benefit (starting claiming Old-age pension benefit from age 61). In each period, she/he needs to find the optimal risky asset share (% of financial wealth should be put into risky assets), optimal consumption level, and withdrawal amount from the IRA.
- Case 2: Agent i decides to delay claiming old-age pension benefit until the age of 66 (five years after the full retirement age). He/she needs to withdraw an extra amount from his/her IRAs to replace the old-age pension benefit that she would receive if claiming right after stop working.
- Case 3: Agent i decides to delay claiming old-age pension benefit until the age of 66. Instead of withdrawing an extra amount from IRAs, the agent decides to convert a portion of his/her home equity into cash via an RML.
- Case 4: Agent i decides to delay claiming old-age pension benefit until the age of 66. However, instead of using the extra money from IRAs or using an RML as an alternative to old-age pension income, he/she chooses to downsize housing stock. At the beginning of age 61, agent i decides to downsize his/her housing stock by x% (if possible – depends on minimum available housing stock). The amount obtained from downsizing the house is set equal to the present value of all payments from in Case 3.

1.4.1. Time parameters

The investor's age is denoted by **t** (t=1 is equivalent to the age of 61). According to the latest statistics about the health status of populations in OECD countries (OECD, 2017), the life expectancy of Korean males at the age of 65 in 2017 is 18.4 years, while the figure is 22.6 for females. In this model, senior investors are assumed to live for a maximum of T periods, up until 85 years old. T, therefore, equals 25 years.

1.4.2. Consumption preferences

The within-period preference over consumption is represented by the time-separable CRRA utility function:

$$u(C_{i,t})=\frac{C_{i,t}^{1-\gamma}}{1-\gamma}$$

Where $C_{i,t}$ is the level of consumption in period t, and $\gamma>0$ is the coefficient of relative risk aversion (CRRA).

1.4.3. Bequest preferences

Following De Nardi (2004) and Ameriks et al. (2011), the bequest utility function is constructed as:

$$B(Q_{i,t}) = \frac{\varpi}{1-\gamma}(\phi+\frac{Q_{i,t}}{\varpi})^{1-\gamma}$$

The variable $Q_{i,t}$ denotes the level of wealth that the agent bequeaths to the inherited at death. $B(Q_{i,t})$ measures the direct utility gained from leaving bequests. The strength of the bequest motive is controlled by the parameter ϖ , and the degree to which bequests are luxury goods is denoted as ϕ (the threshold lifetime consumption up to where the retiree leaves no bequest).

As recorded in the OECD Income Distribution Database (OECD, 2017), there are 45.7% of Korean seniors (aged over 65) living in poverty (defined as earning 50% or less of median household disposable income) in 2015. Furthermore, according to the results of wave 11 -17 of the Korean Labour and Income Study Panel (2008-2014), older people aged 61 and over declared that their two main purposes for saving were the provision against accidents and diseases (precautionary saving) and the provision for old-age (life-cycle saving). In addition, Lee et al. (2009) and Park (2014) show that family transfers (especially supports from children) played a significant role in bringing income security for the elderly in South Korea. In view of all reasons mentioned above, I do not incorporate bequest motive in this analysis, as for those who lack cash and need to find alternative income sources such as unlocking home equity or downsizing the house to enhance retirement living standards; strategic bequests are too expensive. In this case, retirees leave bequests because of lifespan uncertainty, which is often referred to as accidental bequests. In general, retirees do not derive utility from leaving unintentional bequests (Hendricks, 2002).

1.4.4. State variables and control variables

The state-space in period t consists of variable $\mathbf{TW_{i,t}}$ where $\mathbf{TW_{i,t}}$ is agent ***i***'s total non-housing financial wealth at the beginning of period t.

Let assume that all financial wealth $\mathbf{W_{i,t}}$ of agent ***i*** will be transferred to an IRA when he or she turns 61. Under the corporate pension scheme, employees in Korea are either Defined-benefit (DB) or Defined-contribution (DC) plan participants. DB and DC plan participants can open an IRA for individual savings and for investing their lump-sum retirement benefits with tax deferral benefits. Besides, DB and DC participants can make an additional contribution of up to 12 million KRW annually to their IRAs.

$\mathbf{TW_{i,t}}$ comprises two components: $\mathbf{W_{i,t}}$ (money balances from IRAs) and $\mathbf{Op_{i,t}}$ (old-age pension benefits). In each period, the agent has to decide how much he or she will consume (non-housing consumption $\mathbf{C_{i,t}}$) each year and how much he or she needs to withdraw from the IRA. Let the annual withdrawal rate from IRAs be $\mathbf{wdr_{i,t}}$**.** In addition, the retiree will need to allocate his/her financial wealth into two types of financial assets: a risk-free asset and a risky one. In other words, they have to choose the risky asset share **($\boldsymbol{\alpha_{i,t}}$)** in each period. To sum up, control variables include $\mathbf{C_{i,t}}$**,** $\mathbf{wdr_{i,t}}$, and $\boldsymbol{\alpha_{i,t}}$.

1.4.5. Income flows

Let $\mathbf{X_{i,t}}$ be retirement income year **t.**

<u>Case 1</u>: Agent i does not delay claiming old-age pension benefits. Instead, he/she starts claiming the old-age pension benefits at age 61.

Income inflows include withdrawal amount from the IRA and the old-age pension benefit

$$\mathbf{X_{i,t}=W_{i,t}* wdr_{i,t}+ Op_{i,t}}$$

<u>Case 2</u>: Agent i decides to delay claiming old-age pension benefits until the age of 66. He/she needs to withdraw an extra amount from his/her IRA to replace the old-age pension benefit in the five deferral years.

Age 61-65: t=1➔5

The one and only source of cash inflow at this stage is the withdrawal amount from the IRA.

At the beginning of period t, $\mathbf{X_{i,t}=W_{i,t}* wdr_{i,t}}$

Age 66-85: t=6➔25 (t>5)

Income inflows at this stage include the withdrawal amount from IRA and the old-age pension benefit at the age of 66 ($Op_{i,t}$)

$$X_{i,t}=W_{i,t} * wdr_{i,t} + Op_{i,t}$$

$Op_{i,t}=Op_{i1}*(1+\text{inflation rate})^{(t-1)}*1.36$ with Op_{i1} be the old-age pension benefit at age 61 if agent i claims pension benefits at age 61. As the old-age pension benefit increase by 7.2% per year of delay, after five years, the pension benefit at age 66 will be 36% higher than the benefit that the individual would receive if claiming right after retirement.

In short, retirement income $X_{i,t}$ can be computed as:

$$X_{i,t}= W_{i,t} * wdr_{i,t} + Op_{i,t} * Defer_{i,t}$$

In which:

$$Op_{i,t}= Op_{i1}*(1+\text{inflation rate})^{(t-1)}*1.36$$

$$Defer_{i,t}=0 \text{ if } t \leq 5 \text{ and } Defer_{i,t}=1 \text{ if } t>5$$

<u>Case 3</u>: Agent i decides to delay claiming old-age pension benefits until the age of 66. Instead of withdrawing an extra amount from IRAs, the agent decides to convert a portion of their home equity into cash via an RML.

$X_{i,t}= W_{i,t-1}*wdr_{i,t}+ Defer_{i,t}* Op_{i,t}+ pmt_{i,t}$ with pmt_{it} is the periodic repayment from an RML during the five years of pension delay **($pmt_{i,t}$ is not taxable income in Case 3)**

$Op_{i,t}=Op_{i1}*(1+\text{inflation rate})^{t-1}*1.36$ with Op_{i1} be the old-age pension benefit at age 61 if agent i claims benefits when turning 61.

$$Defer_{i,t}=0 \text{ if } t \leq 5 \text{ and } Defer_{i,t}=1 \text{ if } t>5$$

<u>Case 4</u>: Agent i decides to delay claiming old-age pension benefits until the age of 66. He/she chooses to downsize the house. At the beginning of age 61, agent i decides to downsize his/her housing stock by x% (if possible – depends on minimum available housing stock). Money obtained from downsizing the house is set equal to the present value of all periodic payments pmt_{it} in Case 3.

$X_{i,t}= W_{i,t-1}*wdr_{i,t}+ Defer_{i,t}* Op_{i,t}+ pmt_{i,t}$ **($pmt_{i,t}$ is taxable income in Case 4)**

$\mathbf{Op_{i,t}=Op_{i1}*(1+inflation\ rate)^{(t-1)}*1.36}$ with Op_{i1} be the old-age pension benefit at age 61 if agent i claims benefits when turning 61.

$\mathbf{Defer_{i,t}=0}$ **if** $\mathbf{t \le 5}$ **and** $\mathbf{Defer_{i,t}=1}$ **if** $\mathbf{t>5}$

1.4.6. Retirement income tax and property tax

Under South Korean taxation, there are two main types of taxes that individual ***i*** will have to pay, including annuity income tax (pension income tax) and property tax. Each year, agent ***i*** pays $\mathbf{atax_{i,t}}$- the amount of annuity income tax, and $\mathbf{ptax_{i,t}}$- the amount of property tax.

1.4.6.1.Tax rules for Case 1, Case 2, and Case 4

Let $\mathbf{Ded_{it}}$ be the annuity income deduction for taxable income[7]

$\mathbf{Ded_{i,t}}$=	$X_{i,t}$ if $X_{i,t}$<=3.5 million KRW
	3.5 mil+ 0.4*($X_{i,t}$-3.5) if 3.5<$X_{i,t}$<=7 mil
	4.9 mil+ 0.2*($X_{i,t}$ -7) if 7<$X_{i,t}$<=14 mil
	6.3 mil+ 0.1*($X_{i,t}$ -14) if $X_{i,t}$>14 mil

The tax base of annuity income, in this case, is $\mathbf{Tbase_{i,t}=X_{i,t}-Ded_{i,t}}$

$\mathbf{atax_{i,t}}$=	$\mathbf{Tbase_{i,t}}$ *0.06 if $\mathbf{Tbase_{i,t}}$<=12 (million KRW)
	0.72 mil+ 0.15*$\mathbf{(Tbase_{i,t}-12)}$ if 12< $\mathbf{Tbase_{i,t}}$ <=46 mil
	5.82 mil+ 0.24*$\mathbf{(Tbase_{i,t}-46)}$ if 46< $\mathbf{Tbase_{i,t}}$ <=88 mil
	15.9 mil+ 0.33*$\mathbf{(Tbase_{i,t}-88)}$ if $\mathbf{Tbase_{i,t}}$ >88 mil

$\mathbf{H_{i,t}=H_{i1}*(1+g)^t}$, $\mathbf{H_{i1}}$ is the house price at the beginning of age 61**, g** is the house price appreciation rate (%/year)

$\mathbf{ptax_{i,t}}$=	**0.1%***$\mathbf{H_{i,t}}$ if $\mathbf{H_{i,t}}$<=60 (million KRW)
	0.06 mil+ 0.15%*$\mathbf{(H_{i,t}-60)}$ if 60<$\mathbf{H_{i,t}}$<=150 mil
	0.195 mil+ 0.25%*$\mathbf{(H_{i,t}-150)}$ if 150<$\mathbf{H_{i,t}}$ <=300 mil
	0.57 mil+ 0.4%*$\mathbf{(H_{i,t}-300)}$ if $\mathbf{H_{i,t}}$ >300 mil

1.4.6.2.Tax rules for Case 3

Tax benefits for Korean-style reverse mortgage users (JTYK users)

Agent i is fully exempted from registration tax, education tax, the obligation purchase of national housing bonds, and the special tax for farming and fishing villages. Distributions from RMLs are not considered taxable income but rather loan advances. Therefore, $\mathbf{pmt_{i,t}}$ will **not**

[7] According to the Income Tax Act in South Korea. Retrieved from https://elaw.klri.re.kr/eng_service/lawView.do?lang=ENG&hseq=28557

be considered **taxable income**. Besides, a **maximum of 2 million KRW of accrued interest** (**max($Int_{i,t}$, 2mil KRW)**) on the outstanding loan balance each year will be deducted from the taxable income of JTYK users. Finally, 25% of property tax is reduced when taking out a JTYK. **Taxable income in Case 3-** $Tincome_{i,t}=X_{i,t}-pmt_{i,t}-Int_{i,t}$ with $pmt_{i,t}=0$ **when** $t>5$

Let $Ded_{i,t}$ be the annuity income deduction for taxable income

$Ded_{i,t}=$	$Tincome_{i,t}$ if $Tincome_{i,t}<=3.5$ million KRW
	3.5 mil+ 0.4*($Tincome_{i,t}$-3.5) if 3.5<$Tincome_{i,t}$<=7 mil
	4.9 mil+ 0.2*($Tincome_{i,t}$-7) if 7<$Tincome_{i,t}$<=14 mil
	6.3 mil+ 0.1*($Tincome_{i,t}$-14) if $Tincome_{i,t}$>14 mil

The tax base of annuity income, in this case, is $Tbase_{it}=Tincome_{it}-Ded_{it}$

$atax_{i,t}=$	$Tbase_{i,t}$ *0.06 if $Tbase_{i,t}$<=12 million KRW
	0.72 mil+ 0.15*($Tbase_{i,t}$-12) if 12< $Tbase_{i,t}$ <=46 mil
	5.82 mil+ 0.24*($Tbase_{i,t}$-46) if 46< $Tbase_{i,t}$ <=88 mil
	15.9 mil+ 0.33*($Tbase_{i,t}$-88) if $Tbase_{i,t}$ >88 mil

$H_{i,t}=H_{i1}*(1+g)^t$, H_{i1} is the house price at the beginning of age 61, **g** is the house price appreciation rate (%/year).

$\overline{ptax}_{i,t}$ =	0.1%*$H_{i,t}$ if $H_{i,t}$<=60 million KRW
	0.06 mil+ 0.15%*($H_{i,t}$-60) if 60< $H_{i,t}$ <=150 mil
	0.195 mil+ 0.25%*($H_{i,t}$-150) if 150< $H_{i,t}$ <=300 mil
	0.57 mil+ 0.4%*($H_{i,t}$-300) if $H_{i,t}$ >300 mil

The property tax under Case 3 is reduced by 25%, so it should be equal to $ptax_{i,t}=0.75*\overline{ptax}_{i,t}$

1.4.7. Financial assets

There are two financial instruments in this model –a risk-free asset and a risky one. A senior individual can invest non-negative amounts in the risky asset, which yields a stochastic rate of return $R_{s,t}$, and in the risk-free asset that has a certain return R_f. The current holding of the risky asset in relation to liquid wealth is expressed by $\alpha_{i,t}$.

Short-sales in both these two assets are ruled out: $0\leq \alpha_{i,t} \leq 1$

The overall return on the investor's portfolio in period t (between the beginning of year t and the beginning of year t+1) will be:

$$R_{p,t}= R_f*(1- \alpha_{i,t})+ R_{s,t}* \alpha_{i,t}= R_f+ (R_{s,t}-R_f)* \alpha_{i,t}$$

In order to specify the stochastic process for $\mathbf{R_{s,t}}$, following the common practice, returns on risky assets are assumed to be **log-normally distributed**. $\log R_{s,t} \sim \mathrm{N}(\phi + R_f - \sigma_\phi^2 / 2, \sigma_\phi^2)$ where ϕ is the equity premium over the risk-free rate R_f.

This lognormal distribution rule guarantees that

$$\mathbf{Log\ E_t[R_{s,t}] = E_t[log(R_{s,t})] +}\ \sigma_\phi^2 / 2 = \phi + R_f - \sigma_\phi^2 / 2 + \sigma_\phi^2 / 2 = \phi + R_f$$

The lognormal distribution of returns on risky assets is then discretised into seven grid points by Gauss-Hermite quadrature.

1.4.8. The consumption floor and the required living standard

In 2014, the minimum cost of living in South Korea for a single person was reported at the level of 603 thousand KRW per month or approximately 7.2 million KRW per year (Jones & Urasawa, 2014). This figure is computed based on 11 categories of consumer goods. As the historical inflation rates are 0.71%, 0.97%, and 1.94% per year, respectively, for the years 2015, 2016, and 2017, the consumption floor for an individual in 2018 is assumed to be 7.5 million KRW. (C_{floor}=7.5 million KRW)

In South Korea, the earnings-related replacement rate after 40 years of contributions was 46% in 2016 and is being reduced every year from 2008 until reaching 40% in 2028. The required living standard can be defined as:

C_{min}= Old-age pension benefit/replacement rate.

C_{min} is then compared with C_{floor}. If $C_{min} \leq C_{floor}$, the minimum acceptable living standard is set to be C_{floor}. Otherwise, C_{min} is set as the required living standard of senior individuals.

1.4.9. Budget constraint

Each period, agent i chooses the optimal consumption to maximise the expected discounted lifetime utility. This level of consumption **($C_{i,t}$)** can be used to determine the withdrawal amount from the IRA. As the sources of income in each period can come from the old-age pension benefit, the withdrawal money from the IRA, and the payment from RML, agent i's monthly expenses, which include **C_{it}, $atax_{it}$, and $ptax_{it}$**, are limited within this budget.

Case 1: $X_{i,t}=W_{i,t}*wdr_{i,t}+Op_{i,t}=C_{i,t}+atax_{i,t}+ptax_{i,t}$

Case 2: $X_{i,t}=W_{i,t}*wdr_{i,t}+Op_{i,t}*Defer_{i,t}=C_{i,t}+atax_{i,t}+ptax_{i,t}$

Case 3: $X_{i,t}=W_{i,t}*wdr_{i,t}+Op_{i,t}*Defer_{i,t}+pmt_{i,t}=C_{i,t}+atax_{i,t}+ptax_{i,t}$

Case 4: $X_{i,t}=W_{i,t}*wdr_{i,t}+Op_{i,t}*Defer_{i,t}+pmt_{i,t}=C_{i,t}+atax_{i,t}+ptax_{i,t}$

$Op_{i,t}=Op_{i1}*(1+\text{inflation rate})^{t-1}*1.36$ with Op_{i1} be the old-age pension benefit at age 61 if agent i claims benefits when turning 61.

$Defer_{i,t}=0$ **if** $t \le 5$ **and** $Defer_{i,t}=1$ **if** $t>5$

1.4.10. The dynamics of IRA balance and total non-housing wealth

All financial wealth W_{i1} of agent ***i*** will be transferred to an IRA when he or she turns 61. $W_{i,t+1}$ is the money balance of investor ***i***'s IRA at the beginning of period t+1 and is determined as:

$$W_{i,t+1}= (W_{i,t}*(1-wdr_{i,t}))*R_{p,t}$$

$$TW_{i,t+1}=W_{i,t+1}+Op_{i,t+1}$$

$$TW_{i,t+1}= (TW_{i,t}-C_{i,t}-atax_{i,t}-ptax_{i,t})*R_{p,t}+Op_{i,t+1}$$

$$A_{i,t}= TW_{i,t}-C_{i,t}-atax_{i,t}-ptax_{i,t}$$

where $A_{i,t}$ is remaining wealth after all expenditures have been set aside for period t.

1.4.11. The individual optimisation problem

During the retirement period, individuals have to choose the optimal consumption level and the optimal portfolio choice to maximise the expected discounted lifetime utility over consumption.

The expected discounted lifetime utility derived from consumption can be presented as:

$$E_1 \sum_{t=1}^{T} \beta^{t-1} (\prod_{j=0}^{t-2} p_j)(p_{t-1} \frac{C_{i,t}^{1-\gamma}}{1-\gamma})$$

In which:

β <1: a discount factor or time preference rate

p_j: the probability that the investor is alive at t+1, provided that he/she is alive at period t

C_{it}: level of consumption in time t

γ >0: coefficient of relative risk aversion (CRRA)

Then the recursive definition of the corresponding value function is given by:

$$\mathbf{V_{i,t} = u(C_{i,t}) + \beta\, E_t(p_{t+1} * V_{i,t+1})}$$

And the investor's optimisation problem can be rewritten in the form of Bellman equation as follows:

$$[1]\ \mathbf{V_{i,t}\left(TW_{i,t}\right) = \max_{C_{i,t},\alpha_{i,t}} \{u\left(C_{i,t}\right) + E_t(\beta * p_{t+1} * V_{i,t+1}\left(TW_{i,t+1}\right))\}}$$

s.t.

(1) Laws of motion for the state variables

Case 1

$X_{i,t}=W_{i,t}*wdr_{i,t}+ Op_{i,t}$

$W_{i,t}= (W_{i,t-1}*(1-wdr_{i,t}))* R_{p,t}$

$TW_{i,t+1}= W_{i,t+1}+Op_{i,t+1}$ with $Op_{i,t+1}= Op_{i,t}*(1+ \text{inflation rate})$

$TW_{i,t+1}= (TW_{i,t}-C_{i,t}-atax_{i,t}-ptax_{i,t})*R_{p,t} + Op_{i,t+1}$

Case 2

$X_{i,t}=W_{i,t}*wdr_{i,t}+Op_{i,t} *Defer_{i,t}$

$Op_{i,t}=Op_{i1}*(1+\text{inflation rate})^{t-1}*1.36$

$Defer_{i,t}=0$ if $t\leq 5$ and $Defer_{i,t}=1$ if $t>5$

$W_{i,t}= (W_{i,t-1}*(1-wdr_{i,t}))*R_{p,t}$

$TW_{i,t+1}= W_{i,t+1}+Op_{i,t+1}$

$TW_{i,t+1}= (TW_{i,t}-C_{i,t}-atax_{i,t}-ptax_{i,t})*R_{p,t} + Op_{i,t+1}$

Case 3,4 $X_{i,t}=W_{i,t}*wdr_{i,t}+Op_{i,t}*Defer_{i,t}+pmt_{i,t}$

$Op_{i,t}=Op_{i1}*(1+\text{inflation rate})^{t-1}*1.36$

$Defer_{i,t}=0$ if $t\leq$ **5** and $pmt_{i,t}=0$ & $Defer_{i,t}=1$ if $t>5$

$W_{i,t}= (W_{i,t-1}*(1-wdr_{i,t}))*R_{p,t}$

$TW_{i,t+1}= W_{i,t+1}+Op_{i,t+1}$

$TW_{i,t+1}= (TW_{i,t}-C_{i,t}-atax_{i,t}-ptax_{i,t})*R_{p,t} + Op_{i,t+1}$

(2) The stock return process

A stochastic process of risky asset's return is assumed to follow a geometric Brownian motion $\log R_{s,t} \sim N(\phi + R_f - \sigma_\phi^2/2, \sigma_\phi^2)$.

$R_{p,t}= R_f*(1- \alpha_{i,t})+ R_{s,t}* \alpha_{i,t}= R_f+ (R_{s,t}-R_f)* \alpha_{i,t}$

(3) Retirement income and Property tax rules

Refer to 4.6

(4) Budget constraint

$X_{i,t}= C_{i,t}+atax_{i,t}+ptax_{i,t}$

$0\leq C_{i,t} \leq X_{i,t}$

(5) No short sales for financial assets

$0\leq \alpha_{i,t} \leq 1$

(6) Required living standard

$C_{i,t}\geq C_{min}$

1.5. Numerical Methods

1.5.1. The analysis of the optimisation problem

1.5.1.1. The first-order condition of the objective function with respect to C_{it}

The first-order condition for **<1>** with respect to C_{it} is:

$$[2]\ u'(C_{i,t}) = -E_t(\beta * p_{t+1} * V'_{i,t+1}(TW_{i,t+1}) * \frac{\partial TW_{i,t+1}}{\partial C_{i,t}})$$
$$= -E_t(\beta * p_{t+1} * V'_{i,t+1}(TW_{i,t+1}) * (-R_{p,t}))$$
$$= E_t(\beta * p_{t+1} * V'_{i,t+1}(TW_{i,t+1}) * (R_f + \alpha_{i,t} * (R_{s,t} - R_f)))$$

And the Envelope theorem holds that

$$[3]\ V'_{i,t}(TW_{i,t}) = E_t(\beta * p_{t+1} * V'_{i,t+1}(TW_{i,t+1}) * \frac{\partial TW_{i,t+1}}{\partial TW_{i,t}})$$
$$= E_t(\beta * p_{t+1} * V'_{i,t+1}(TW_{i,t+1}) * R_{p,t})$$
$$= E_t(\beta * p_{t+1} * V'_{i,t+1}(TW_{i,t+1}) * (R_f + \alpha_{i,t} * (R_{s,t} - R_f)))$$

The left-hand side (LHS) of <3> is substituted for the right-hand side (RHS) of <2> to get:

$$[4]\ u'(C_{i,t}) = V'_{i,t}(TW_{i,t})$$

Next, rolling this equation one period forward yields:

$$[5]\ u'(C_{i,t+1}) = V'_{i,t+1}(TW_{i,t+1})$$

From <2> and <5>, I get the Euler equation for consumption:

$$[6]\ u'(C_{i,t}) = E_t(\beta * p_{t+1} * u'(C_{i,t+1}) * (R_f + \alpha_{i,t} * (R_{s,t} - R_f)))$$

As $TW_{i,t+1} = (TW_{i,t} - C_{i,t} - atax_{i,t} - ptax_{i,t}) * R_{p,t} + Op_{i,t+1} = A_{i,t} * R_{p,t} + Op_{i,t+1}$, I define the expected value function in the period t+1 as follows:

$$[7]\ \upsilon_{i,t}(A_{i,t}) = E_t(\beta * p_{t+1} * V_{i,t+1}(A_{i,t} * (R_f + \alpha_{i,t} * (R_{s,t} - R_f)) + Op_{i,t+1}))$$

The above definition implies that

$$[8]\ \upsilon'_{i,t}(A_{i,t}) = E_t(\beta * p_{t+1} * V'_{i,t+1}(A_{i,t} * (R_f + \alpha_{i,t} * (R_{s,t} - R_f)) + Op_{i,t+1}) * (R_f + \alpha_{i,t} * (R_{s,t} - R_f)))$$

From <5> and <8> and policy function $C_{i,t}(TW_{i,t})$, it follows that:

$$[9]\ \upsilon'_{i,t}(A_{i,t}) = E_t(\beta * p_{t+1} * u'(C_{i,t+1}(A_{i,t} * (R_f + \alpha_{i,t} * (R_{s,t} - R_f)) + Op_{i,t+1}) * (R_f + \alpha_{i,t} * (R_{s,t} - R_f)))$$

Finally, the first-order condition of <1> with respect to C_{it} can now be rewritten as:

$$[10]\ u'\left(C_{i,t}\right) = \upsilon'_{i,t}\left(TW_{i,t} - C_{i,t} - atax_{i,t} - ptax_{i,t}\right)$$

1.5.1.2. The first-order condition for the objective function with respect to $\alpha_{i,t}$

The first-order condition for **<1>** with respect to $\boldsymbol{\alpha_{i,t}}$ is:

$$[11]\ \frac{\partial u(C_{i,t})}{\partial \alpha_{i,t}} = -E_t(\beta * p_{t+1} * V'_{i,t+1}\left(TW_{i,t+1}\right) * \frac{\partial TW_{i,t+1}}{\partial \alpha_{i,t}})$$

$$0 = -E_t(\beta * p_{t+1} * V'_{i,t+1}\left(TW_{i,t+1}\right) * A_{i,t} * \frac{\partial R_{p,t}}{\partial \alpha_{i,t}})$$

$$0 = E_t(\beta * p_{t+1} * V'_{i,t+1}\left(TW_{i,t+1}\right) * A_{i,t} * (R_{s,t} - R_f))$$

1.5.2. The behaviour in the last period

The solution to the individual optimisation problem can be found using the backward induction principle. An appropriate condition that can be used to initiate a backward induction algorithm is that investor i consumes all remaining resources in the last period.

$$[12]\ TW_{i,T} = C_{i,T} + atax_{i,T} + ptax_{i,T}$$

$$[13]\ TW_{i,T} = W_{i,T} + Op_{i,T}$$

$$[14]\ X_{i,T} = TW_{i,T}\ \&\ wdr_{i,T} = 1\ \&\ \alpha_{i,T} = 0$$

$$\begin{aligned}[15]\ V_{i,T}(TW_{i,T}) &= u(C_{i,T}) \\ &= u(TW_{i,T} - atax_{i,T} - ptax_{i,T}) \\ &= \frac{(TW_{i,T} - atax_{i,T} - ptax_{i,T})^{1-\gamma}}{1-\gamma} \\ &= \frac{((A_{i,T-1} * (R_f + \alpha_{i,T-1} * (R_{s,T-1} - R_f)) + Op_{i,T}) - atax_{i,T} - ptax_{i,T})^{1-\gamma}}{1-\gamma}\end{aligned}$$

1.5.3. Solving the next-to-last period

Next, the optimisation problem in the next-to-last period of life is defined as:

$$[16]\ V_{i,T-1}\left(TW_{i,T-1}\right) = \max_{C_{i,T-1}, \alpha_{i,T-1}} \{u\left(C_{i,T-1}\right) + E_{T-1}(\beta * p_T * V_{i,T}\left(TW_{i,T}\right))\}$$

From the equation $V_{i,T}(TW_{i,T}) = u(C_{i,T})$, the definition of the expectation operator $\mathbf{E_{T-1}(\beta^*p_T^*V_{i,T}\left(TW_{i,T}\right))}$, and the fact that $\mathbf{p_T}$ is non-stochastic, the objective function becomes:

$$\textbf{[17]}\ \mathbf{V_{i,T-1}\left(TW_{i,T-1}\right) = \max_{C_{i,T-1},\alpha_{i,T-1}} \{\frac{{C_{i,T-1}}^{1-\gamma}}{1-\gamma} + E_{T-1}(\beta^*p_T^*u\left(C_{i,T}\right))\}}$$

$$\mathbf{= \max_{C_{i,T-1},\alpha_{i,T-1}} \{\frac{{C_{i,T-1}}^{1-\gamma}}{1-\gamma} + \beta p_T \,* }$$

$$\mathbf{* \int_{-\infty}^{+\infty} \frac{((A_{i,T-1}*(R_f+\alpha_{i,T-1}*(R_{s,T-1}-R_f))+Op_{i,T})-atax_{i,T}-ptax_{i,T})^{1-\gamma}}{1-\gamma} dF(R_s)\}}$$

where **F** is the cumulative distribution function of $\mathbf{R_s}$**.**

In general, solving the objective function <17> is equivalent to finding the policy function $C_{i,T-1}(TW_{i,T-1})$ that returns optimal consumption in period T-1 for any given level of $TW_{i,T-1}$. However, there is no typical analytical solution to this optimisation problem. For any given $TW_{i,T-1}$, I need to use the numerical analysis to find $\mathbf{C_{i,T-1}}$ and $\boldsymbol{\alpha_{i,T-1}}$ that maximises **<17>.**

1.5.4. Discretisation of exogenous state space

For every $A_{i,T-1}$, the integral must be computed numerically, taking a great deal of time. It will save time if a discrete approximation to the lognormal distribution of risky asset returns' process is constructed.

First, I define a set of n points on the [0,1] interval as the elements of the set $\psi=\{0,1/n,2/n,\ldots,1\}$. The inverse of the R_s, distribution F^{-1}, is called, and these points are then determined by $\psi_a^{-1} = F^{-1}(\psi_a)$. Then the conditional mean of R_s in each of the intervals numbered 1 to n is:

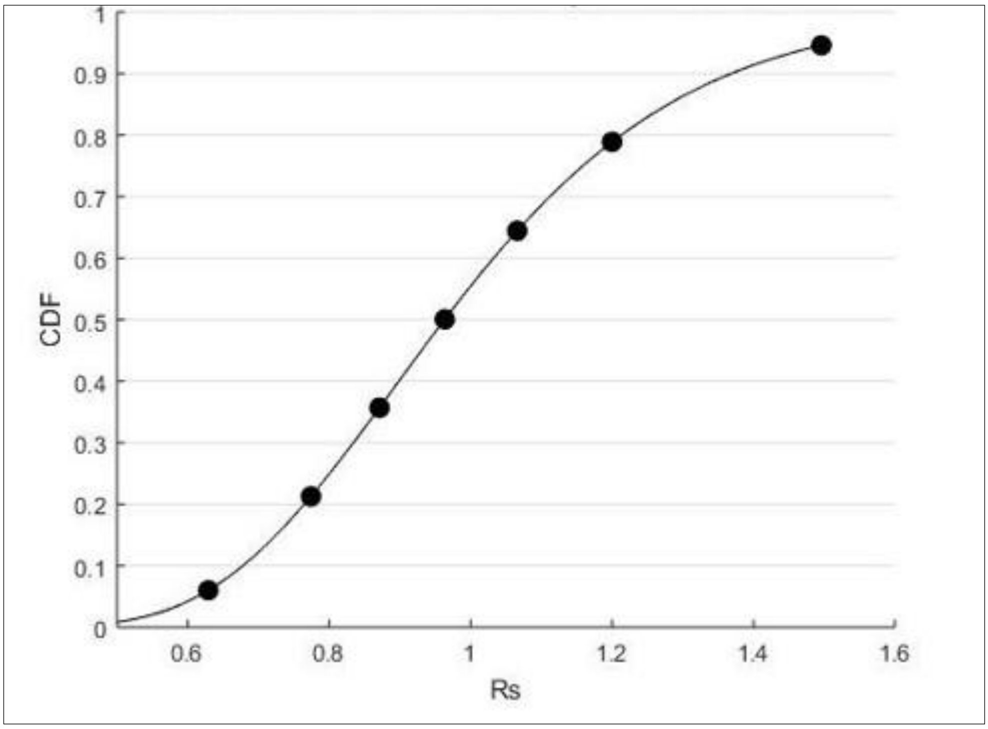
$$\textbf{[18]}\ \mathbf{R_a \equiv E\left[R_s \middle| \psi_{a-1}^{-1} \leq R \leq \psi_a^{-1}\right] = \int_{\psi_{a-1}^{-1}}^{\psi_a^{-1}} \vartheta dF(\vartheta)}$$

In Figure 1.2 below, the solid continuous curve represents the "true" cumulative distribution function $\mathbf{F(R_s)}$ for a lognormal distribution with $\boldsymbol{E\left[R_s\right] = e^{\phi+R_f}}$ ($\boldsymbol{\phi}$ is the equity premium over the risk-free rate R_f) and $\boldsymbol{\sigma_\phi}$.

Recalling the definition of $\upsilon_{i,t}(A_{i,t})$, for t=T-1, I get:

$$[19]\ \upsilon_{i,T-1}(A_{i,T-1})=\beta*p_T*\frac{1}{n}*\sum_{a=1}^{n}\frac{\left((A_{i,T-1}*(R_f+\alpha_{i,T-1}*(R_a-R_f))+Op_{i,T})-atax_{i,T}-ptax_{i,T}\right)^{1-\gamma}}{1-\gamma}$$

Figure 1.2: Discrete approximation to the lognormal distribution F(Rs)



Source: Author's calculation

So the optimisation problem can be rewritten as:

$$[20]\ V_{i,T-1}\left(TW_{i,T-1}\right)=\max_{C_{i,T-1},\alpha_{i,T-1}}\{\frac{C_{i,T-1}^{1-\gamma}}{1-\gamma}$$

$$+\beta*p_T*\frac{1}{n}*\sum_{a=1}^{n}\frac{\left((A_{i,T-1}*(R_f+\alpha_{i,T-1}*(R_a-R_f))+Op_{i,T})-atax_{i,T}-ptax_{i,T}\right)^{1-\gamma}}{1-\gamma}\}$$

Finally, the first-order conditions of the optimisation problem with respect to $\mathbf{C_{i,T-1}}$ and $\boldsymbol{\alpha_{i,T-1}}$ are computed as follows.

$$[21]\ u'\left(C_{i,T-1}\right) = E_t(\beta * p_T * V'_{i,T}\left(TW_{i,T}\right) * (R_f + \alpha_{i,T-1} * (R_{s,T-1} - R_f)))$$

$$= E_{T-1}(\beta * p_T * (TW_{i,T} - atax_{i,T} - ptax_{i,T})^{-\gamma} * (R_f + \alpha_{i,T-1} * (R_{s,T-1} - R_f)))$$

$$= E_{T-1}(\beta * p_T * ((A_{i,T-1} * (R_f + \alpha_{i,T-1} * (R_{s,T-1} - R_f)) + Op_{i,T}) - atax_{i,T} - ptax_{i,T})^{-\gamma} * (R_f + \alpha_{i,T-1} * (R_{s,T-1} - R_f)))$$

$$= \beta * p_T * \frac{1}{n} * \sum_{a=1}^{n} \left\{ \begin{array}{l} \left(A_{i,T-1} * (R_f + \alpha_{i,T-1} * (R_a - R_f)) + Op_{i,T} - atax_{i,T} - ptax_{i,T}\right)^{-\gamma} \\ * (R_f + \alpha_{i,T-1} * (R_a - R_f)) \end{array} \right\}$$

$$[22]\ \frac{\partial u(C_{i,T-1})}{\partial \alpha_{i,T-1}} = -E_{T-1}(\beta * p_T * V'_{i,T}\left(TW_{i,T}\right) * \frac{\partial TW_{i,T}}{\partial \alpha_{i,T-1}})$$

$$0 = E_{T-1}(\beta * p_T * V'_{i,T}\left(TW_{i,T}\right) * A_{i,T-1} * (R_{s,T-1} - R_f))$$

$$= \beta * p_T * \frac{1}{n} * \sum_{a=1}^{n} \left\{ \begin{array}{l} \left(A_{i,T-1} * (R_f + \alpha_{i,T-1} * (R_a - R_f)) + Op_{i,T} - atax_{i,T} - ptax_{i,T}\right)^{-\gamma} \\ * A_{i,T-1} * (R_a - R_f) \end{array} \right\}$$

1.5.5. Discretisation of endogenous state space

In order to solve the optimisation problem numerically, a set of grid points for Ai,t is needed.

Zainhofer, F. (2008) summarises several aspects to be considered when setting up grids for endogenous state variables. First, the number of grid points needs to be carefully chosen since the larger the number of points, the longer it takes to approximate the solution, and the smaller number of points may imply a less precise approximation. Next, the starting and ending points have to be specified in accordance with the nature of the underlying economic problem. Finally, another critical issue is the spacing between grid points. The grid can be equally spaced or densely spaced at lower values and coarsely spaced at higher values.

Following Cocco et al. (2005), I use an equally-spaced grid for variable Ai,t. The starting and ending points and the number of grid points will be determined in the model calibration section.

1.5.6. Method of endogenous grid-points

Now, given a particular value of $A_{i,T-1}$, a percentage of financial wealth put into risky assets (**$\alpha_{i,T-1}$**) can be found following the numerical maximisation routine **<22>.** Following Carroll (2006), I use Matlab programs to solve the optimisation problem. The next step is to apply the computed $\alpha_{i,T-1}$ into **<20>** and find the corresponding **$C_{i,T-1}$**, which is the consumption level that maximises investor ***i***'s utility.

According to retirement income tax rules, there are seven tax brackets to be considered **<23>:**

$0 \le X_{i,t} \le 3.5$	$atax_{i,t} = 0$
$3.5 \le X_{i,t} \le 7$	$atax_{i,t} = 0.036 X_{i,t} - 0.126$
$7 \le X_{i,t} \le 14$	$atax_{i,t} = 0.048 X_{i,t} - 0.21$
$14 \le X_{i,t} \le 18.78$	$atax_{i,t} = 0.054 X_{i,t} - 0.294$
$18.78 \le X_{i,t} \le 56.56$	$atax_{i,t} = 0.135 X_{i,t} - 1.815$
$56.56 \le X_{i,t} \le 103.22$	$atax_{i,t} = 0.216 X_{i,t} - 6.396$
$103.22 \le X_{i,t}$	$atax_{i,t} = 0.297 X_{i,t} - 14.757$

The budget constraint requires that

[24] $X_{i,T-1} = C_{i,T-1} + atax_{i,T-1} + ptax_{i,T-1}$

With the above-computed $C_{i,T-1}$, $ptax_{i,T-1}$, retirement income tax rules <23>, it is possible to find the pension income $X_{i,T-1}$ in period T-1 and the corresponding $atax_{i,T-1}$.

Consequently, the total financial wealth $TW_{i,T-1}$, money balance $W_{i,T-1}$ and withdrawal rate associated with IRA at the beginning of period T-1 can be determined by the following equations:

$TW_{i,T-1} = A_{i,T-1} + C_{i,T-1} + atax_{i,T-1} + ptax_{i,T-1}$

$W_{i,T-1} = TW_{i,T-1} - Op_{i,T-1}$

$wdr_{i,T-1} = (X_{i,T-1} - Op_{i,T-1}) / W_{i,T-1}$

$C_{i,T-1}$, $A_{i,T-1}$, $atax_{i,T-1}$, $X_{i,T-1}$ are unique optimal values that correspond to the solution of the optimisation problem in a single state. These $TW_{i,T-1}$ grid points are endogenously derived in contrast to the common solution method of setting up grid points of **$TW_{i,T-1}$** and then using a root-finding process to achieve the corresponding optimal **$\alpha_{i,T-1}$ and $C_{i,T-1}$**.

1.5.7. Interpolated consumption function

After solving for $C_{i,T-1}$, a set of ($TW_{i,T-1}$, $C_{i,T-1}$) pairs can be generated in order to yield an interpolated consumption function $\bar{C}_{i,T-1}(TW_{i,T-1})$. I can, therefore, define an approximation to the consumption function $\bar{C}_{i,T-1}(TW_{i,T-1})$ using piecewise linear interpolation.

Since in order to solve for the previous period by backward induction, I need to incorporate an interpolated consumption function $\bar{C}_{i,T-1}(TW_{i,T-1})$ into the first-order condition of the objective function in period T-2 with respect to $C_{i,T-2}$ and $\boldsymbol{\alpha}_{i,T-2}$.

$$[25]\quad \frac{\partial u(C_{i,T-2})}{\partial \alpha_{i,T-2}} = E_{T-2}(\beta * p_{T-1} * V'_{i,T-1}(TW_{i,T-1}) * A_{i,T-2} * (R_{s,T-2} - R_f))$$

$$0 = E_{T-2}(\beta * p_{T-1} * u'(C_{i,T-1}(TW_{i,T-1})) * A_{i,T-2} * (R_{s,T-2} - R_f))$$

$$0 = E_{T-2}(\beta * p_{T-1} * (\bar{C}_{i,T-1}(TW_{i,T-1}))^{-\gamma} * \frac{\partial \bar{C}_{i,T-1}}{\partial TW_{i,T-1}} * A_{i,T-2} * (R_{s,T-2} - R_f))$$

From <25>, optimal risky asset share $\boldsymbol{\alpha}_{i,T-2}$ can be found, and a similar process can be applied to find the optimal consumption level $\mathbf{C}_{i,T-2}$ and withdrawal rate from IRA ($\mathbf{wdr}_{i,T-2}$) in period T-2.

1.5.8. Recursion

When the optimisation problem has been solved up to the period T-1 and an approximated $\bar{C}_{i,T-1}(TW_{i,T-1})$ has been generated, the solution method is to use <25>, <22>, <21> and work back progressively from period T-1 to the beginning of the examined period. The Matlab algorithm to solve the investor ***i***'s optimisation problem is presented below:

(a) For any points in the grid of $A_{i,T-1}$, numerically compute the values $\alpha_{i,T-1}$ and $C_{i,T-1}$ from

$$u'(C_{i,T-1}) = \beta * p_T * \frac{1}{n} * \sum_{a=1}^{n} \left\{ \begin{array}{l} \left(A_{i,T-1} * (R_f + \alpha_{i,T-1} * (R_a - R_f)) + Op_{i,T} - atax_{i,T} - ptax_{i,T}\right)^{-\gamma} \\ * (R_f + \alpha_{i,T-1} * (R_a - R_f)) \end{array} \right\}$$

(b) Construct a corresponding list of values of $C_{i,T-1}$ and $TW_{i,T-1}$ from $TW_{i,T-1} = A_{i,T-1} + C_{i,T-1} + atax_{i,T-1} + ptax_{i,T-1}$, the retirement income tax rules, budget constraint equation and property tax related to the housing value in period T-1

(c) Construct a piecewise linear interpolation $\bar{C}_{i,T-1}(TW_{i,T-1})$

(d) The next period to solve is T-2. With each given value of $A_{i,T-2}$, the corresponding upper bound and lower bound of the $TW_{i,T-2}$ will be set, and these two bounds will be used to match the examined $A_{i,T-2}$ with the appropriate piece of linear interpolation $\bar{C}_{i,T-1}(TW_{i,T-1})$. Accordingly, the optimal risky asset share in period T-2 can be calculated from the following equation:

$$E_{T-2}(\beta * p_{T-1} * (\bar{C}_{i,T-1}\left(A_{i,T-2} * (R_f + \alpha_{i,T-2} * (R_{s,T-2} - R_f)) + Op_{i,T-1}\right))^{-\gamma} * \frac{\partial \bar{C}_{i,T-1}}{\partial TW_{i,T-1}} * A_{i,T-2} * (R_{s,T-2} - R_f)) = 0$$

After that, the optimal consumption level in period T-2 is found based on the following equation**:**

$$u'\left(C_{i,T-2}\right) = E_{T-2}(\beta * p_{T-1} * u'\left(C_{i,T-1}(TW_{i,T-1})\right) * (R_f + \alpha_{i,T-2} * (R_{s,T-2} - R_f)))$$

$$= \beta * p_{T-1} * \frac{1}{n} * \sum_{a=1}^{n} \left\{ \begin{array}{l} \left(A_{i,T-2} * (R_f + \alpha_{i,T-2} * (R_a - R_f)) + Op_{i,T-1} - atax_{i,T-1} - ptax_{i,T-1}\right)^{-\gamma} \\ * (R_f + \alpha_{i,T-2} * (R_a - R_f)) \end{array} \right\}$$

(e) Repeat steps (b) to (d) for every period from T-2 backward.

1.6. Model Calibration

1.6.1. Time

As stated in the previous sections, individual **i** enters the model at age 61 and leaves the model at age 85. Each period in the model is calibrated to correspond to one year. The number of periods to solve in this optimisation problem is T=25.

1.6.2. Preferences

According to Attanasio et al. (1999), the relative risk aversion coefficient range, which is commonly used in the economics literature is from 1 to 3. In the baseline model, the CRRA is set at the level of 3. The inter-temporal discount factor β is calibrated to 0.96.

1.6.3. Survival probabilities

The survival probabilities between age x and x+n are taken from the 2016 Korean life table provided by the Korean Statistical Information Service (KOSIS). The survival probabilities at age x conditional on being alive at age x-1 and the survival probabilities at age x conditional on being alive at age 61 are, then, reported in Table A.1 and Table A.2 (Appendix A).

1.6.4. Risky asset returns

The real return on the Korean Composite Stock Price index[8] (returns that have been adjusted for inflation rate during the examined period)[9] has a mean of 8% and a standard deviation of 18% from June 2003 to June 2018. Based on these estimates, real stock returns are calibrated with $\overline{R_s}$ =8% per year and σ_s =18% annually. An equity premium of 5.65% is estimated for the case of South Korea, according to Damodaran (2016). The proxy for South-Korean risk-free rate is the 10-year Government Bond yield. Historical data from November 2003 to June 2018 shows that the average 10-year Government bond yield[10] is approximately 3.5% per year. However, the monthly rate has stabilised at around 2.5% per year since June 2017. In this study,

---

[8] The Korean Composite Stock Price index monthly data from June 2003 to June 2018 is loaded from the Korea Stock Exchange official website http://global.krx.co.kr/main/main.jsp

[9] Monthly CPI data from June 2003 to June 2018 is collected from Economic Statistic System- Bank of Korea, historical annual inflation rates stay at the level of around 1-1.5% per year.

[10] Data from https://www.investing.com/rates-bonds/south-korea-government-bonds

therefore, the risk-free rate is calibrated to R_f=2.5% a year, and the equity risk premium is set at 5.5%, which is slightly lower than Damodaran's estimate.

1.6.5. Defining the grid for endogenous state space

The individual's optimisation problem is solved by the method of endogenous grid points. Regarding this method, in each period, I need to start with a set of $\mathbf{A_{i,t}}$ grid points.

I first choose the lower bound of $\mathbf{A_{i,T-1}}$ by using the fzero function (MatLab) to find the value of total financial savings at which the risky asset share is set at the maximum level of 1 (100%). After that, the upper bound is defined by adding 100 million KRW to the lower bound. The number of points in $\mathbf{A_{i,T-1}}$ grid is set equal to 100 points. From the period T-2 backward, the lower bound in the current period is determined by adding 0.5 million KRW to the previous lower bound, and the upper bound in the current period is then computed as the sum of the lower bound in the same period and 100 (million KRW).

1.6.6. Parameters in the actuarial model of Korean-style reverse mortgage

As being analysed in the section "**Actuarial Model of Reverse Mortgage Loans in South Korea,"** the house appreciation rate and the expected value of actual loan interest rate in the current Korean RM model are set at 2.2% per year and 5.24% per year, respectively. The loan-to-value ratio is estimated to be 48.06%.

The payment plan from RML is the modified-term payment option that combines the 5-year fixed term annuity and an available line of credit.

Further, repayment each period ($repay_{i,t}$) is set equal to 0 as in this study, individual **i** is finding an alternative income source to replace the deferred old-age pension benefits in the first five years of the retirement period. The actual loan interest rate is used to calculate the interest amount and the outstanding loan balance. In the life-cycle model, this actual loan interest rate is calibrated at 5.24% per year.

The parameters used in the model are summarised in Table A.3 (Appendix A).

1.7. Results for the baseline case

The annual old-age pension benefit at age 61 and the current house value are set at 6 million KRW and 100 million KRW, respectively. The initial financial wealth or the money balance of the IRA ranges from 25 million KRW to 115 million KRW. Using these inputs and all parameters in Table 4, I solve the baseline models for all cases (Case 1, 2, 3, and 4).

This section describes the investor's optimal financial asset allocation, optimal consumption level, the withdrawal amount from the IRA. The impact that the progressive income tax might have on the portfolio choice is also investigated.

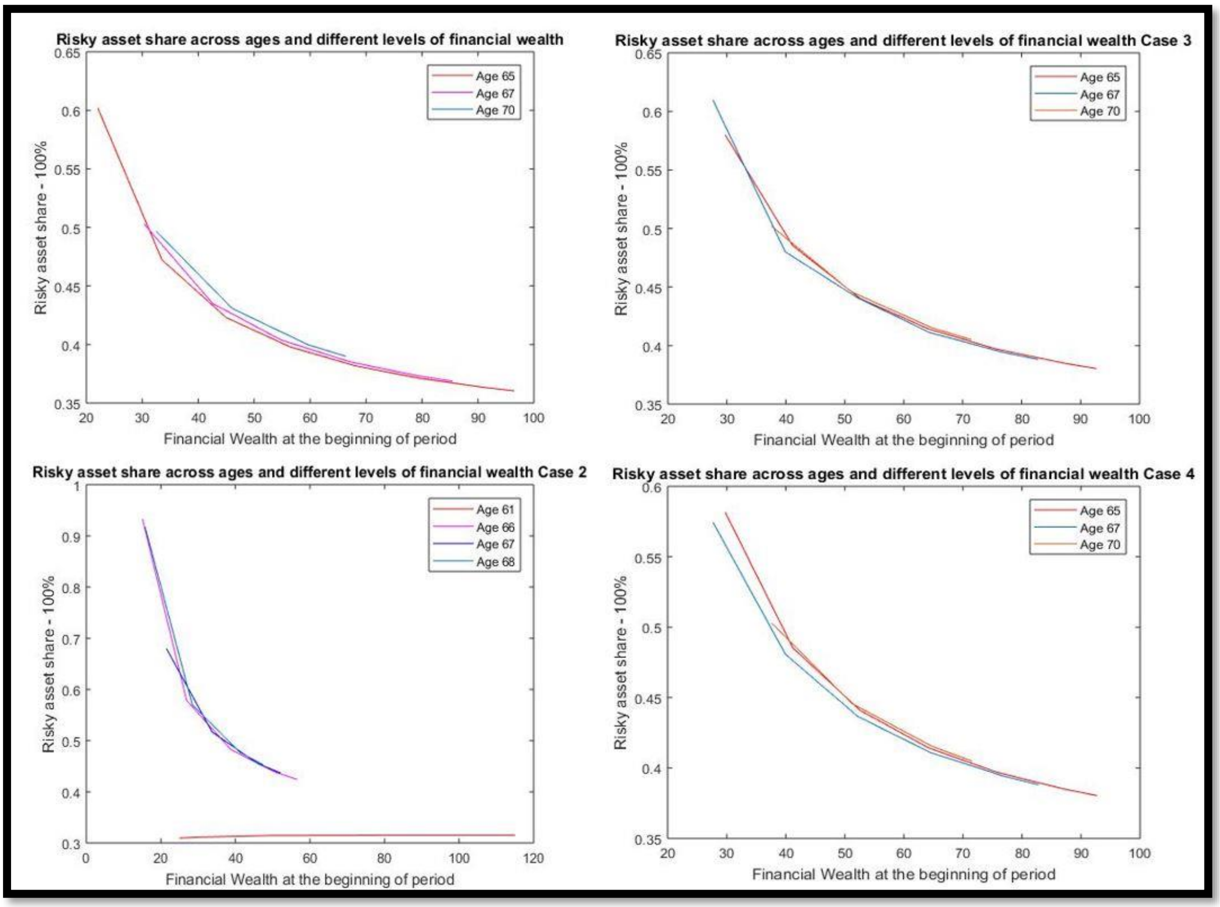
Portfolio choice across ages and different levels of financial wealth

As can be noticed from Figure 1.3, each curve in Case 1, 2, 3, and 4 represents the optimal portfolio rule at a particular age. The exposure to risky assets in the portfolio is negatively related to wealth. In particular, the proportion of wealth invested in stocks tends to decrease for wealthier retirees. Investors who have an IRA balance ranging from 20 to 30 million KRW are found to invest 50%-60% of their financial wealth in risky assets, whereas those with an IRA balance of 80-100 million KRW have 35-40% of their financial wealth invested in risky assets. It is clear that during the retirement period, old-age pension benefits act as an alternative for risk-free asset holdings. Compared to investors with a higher level of financial wealth, individuals with less wealth tend to be more aggressive in choosing the optimal risky asset share. One possible explanation is that for poorer investors, old-age pension benefits often make up a more significant portion of their financial wealth, or in other words, poorer investors have greater exposure to risk-free assets, giving them more incentive to increase their exposure to risky assets. Another observation is that, given the specific level of financial wealth, the optimal risky asset shares are fairly stable during retirement. The horizon effect in portfolio choice appears to be negligible during the retirement period. This finding does not support the traditional investment advice suggesting that the proportion of wealth that investors put into risky assets should decline with age. However, this result agrees with Ameriks et al. (2004) and Cocco et al. (2005), who find no empirical evidence supporting the gradual reduction in risky asset share as investors age during retirement.

For Case 2, in order to sustain the required living standard in the first five years (13-15 million KRW per year) of the retirement period without receiving any benefits from the old-age pension scheme, investor i has to depend entirely on the income source from IRA. The money balance of IRA in Case 2 will thus be exhausted much sooner compared to the three remaining cases. With the initial financial wealth ranging from 25 to 115 million KRW, the money balance can hardly survive beyond the age of 75. That said, in Case 2, there are not enough points to reflect the optimal portfolio rule across various levels of wealth at age 70. Moreover, as mentioned above, in the first five years of Case 2, there is no old-age pension benefit serving as a guaranteed income source, resulting in lower exposure to risk-free assets in the portfolio and hence tilting investors towards a more conservative portfolio. To be more specific, under

Case 2, the optimal risky asset shares at age 61 are lower than that at older ages and stay just over 30% across different levels of financial wealth.

Figure 1.3: Risky asset share across ages and different levels of financial wealth

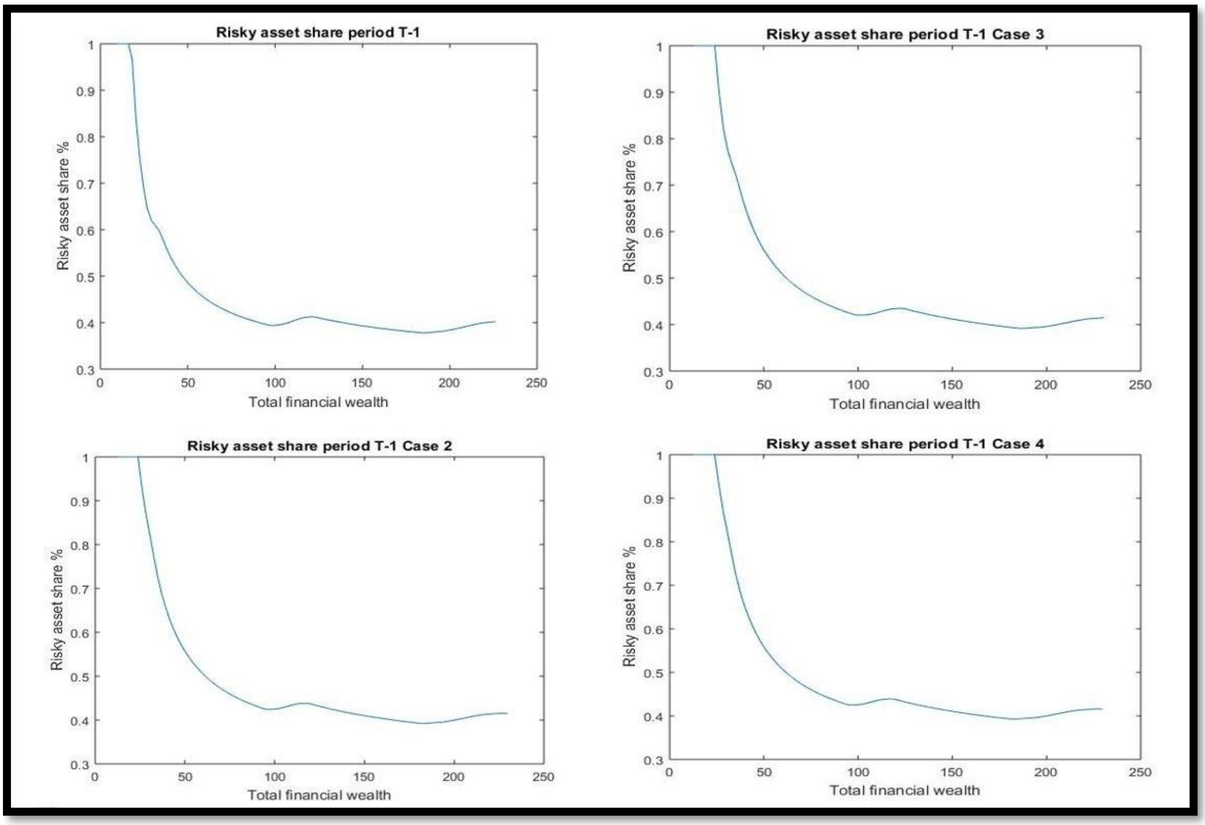


Note: Financial wealth is measured in millions of KRW. *Source: Author's calculation*

Progressive income tax and portfolio choice

In analyzing the optimal risky asset share $\boldsymbol{\alpha_{i,T-1}}$, it can be obviously seen from Figure 1.4 that $\boldsymbol{\alpha_{i,T-1}}$ is not a monotonic function of total financial wealth- $TW_{i,T-1}$. Although, in general, the optimal risky asset share is negatively correlated with total financial wealth, there are some $TW_{i,T-1}$ points around the range of 100 to 120 million KRW and 180 to 230 million KRW experiencing an upward trend in the risky asset share. Interestingly, these observations are captured right after the jumps from the lower income tax bracket to the higher one, in particular, from the fifth to the sixth and from the sixth to the seventh income tax bracket. Figure 1.4, therefore, sheds some light on the effect that the progressive income tax might have on the optimal portfolio choice.

Figure 1.4: Risky asset share across different levels of total financial wealth in T-1



Note: Financial wealth is measured in millions of KRW. *Source: Author's calculation*

Another thing that should be noticed from Figure 1.4 is that when shifting from the third to the fourth[11], or from the fourth to the fifth income tax bracket[12], the fraction of wealth put into stocks smoothly follows a downward trend without any turning point. This observation implies that the effect of the progressive tax rule on portfolio choice is only captured at the second-highest and highest income tax bracket. Table 1.2 conveys how the optimal portfolio choice varies when there is a jump from a lower to a higher income tax bracket, especially from the fifth to the sixth and from the sixth to the seventh tax bracket.

Within the sixth and seventh income tax brackets, the first and the last points affected by the progressive income tax rule are marked in bold. The point which starts the new income tax bracket is also the first point influenced by the progressive tax rule. For each of these starting points, the absolute increase in income tax amount due to the change in the tax bracket is defined as the difference between the tax amount computed based on the actual tax bracket and

[11] The third income tax bracket corresponds to the cases in which total financial wealth ranges from 9 million KRW to 16 million KRW. The fourth income tax bracket is associated with the cases in which total financial wealth ranges from 18 million KRW to 25 million KRW.

[12] The fifth income tax bracket corresponds to the cases in which total financial wealth ranges from 27 million KRW to 96 million KRW

the amount computed based on the previous tax bracket. As can be seen from Table 1.2, this absolute increase for the highest and second to highest tax brackets is significantly higher than that of the other remaining brackets in all cases. If the absolute increase in tax amount caused by a jump in the marginal income tax rate is used as a proxy for the effect of the progressive income tax rule, the above inspection might explain why an upward trend in risky asset share is discovered only within the top highest tax brackets. In this case, the progressive tax rule's effect might surpass the wealth effect in determining the optimal portfolio allocation. Similarly, if the percentage point change in the marginal income tax rate is used to measure the impact that the progressive income tax rule might have on the risky asset share, I find that an increase of 9 percentage points in the marginal income tax rate from the second-highest to the highest tax bracket corresponds with an increase of 1-3 percentage points in the optimal stock share.

The optimal consumption and withdrawal amount from the IRA

Figure 1.5 displays the optimal consumption, the withdrawal amount from the IRA and the money balance in IRA, averaging across different states of risky stock returns, for the agent who has an initial financial wealth of 25 million KRW and an old-age pension benefit of 6 million KRW at age 61. The IRA's balance declines with age in all cases and is fully depleted around 65 years old or sooner. As the old-age pension benefit is assumed to account for 45% of spending needs in retirement, the required living standard for the investor with the benefit of 6 million KRW in the first year of the retirement period is equal to13.5 million KRW.

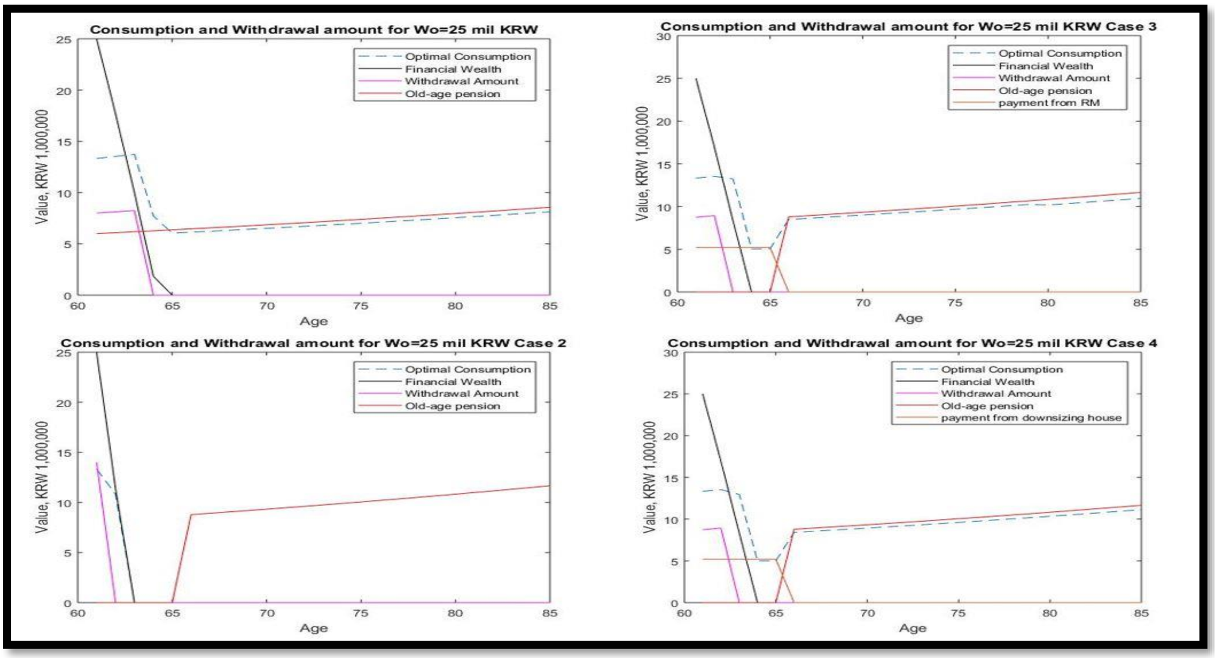


Figure 1.5: Consumption and withdrawal amount (Initial FW=25 mil KRW)

Note: Financial wealth is measured in millions of KRW. *Source: Author's calculation*

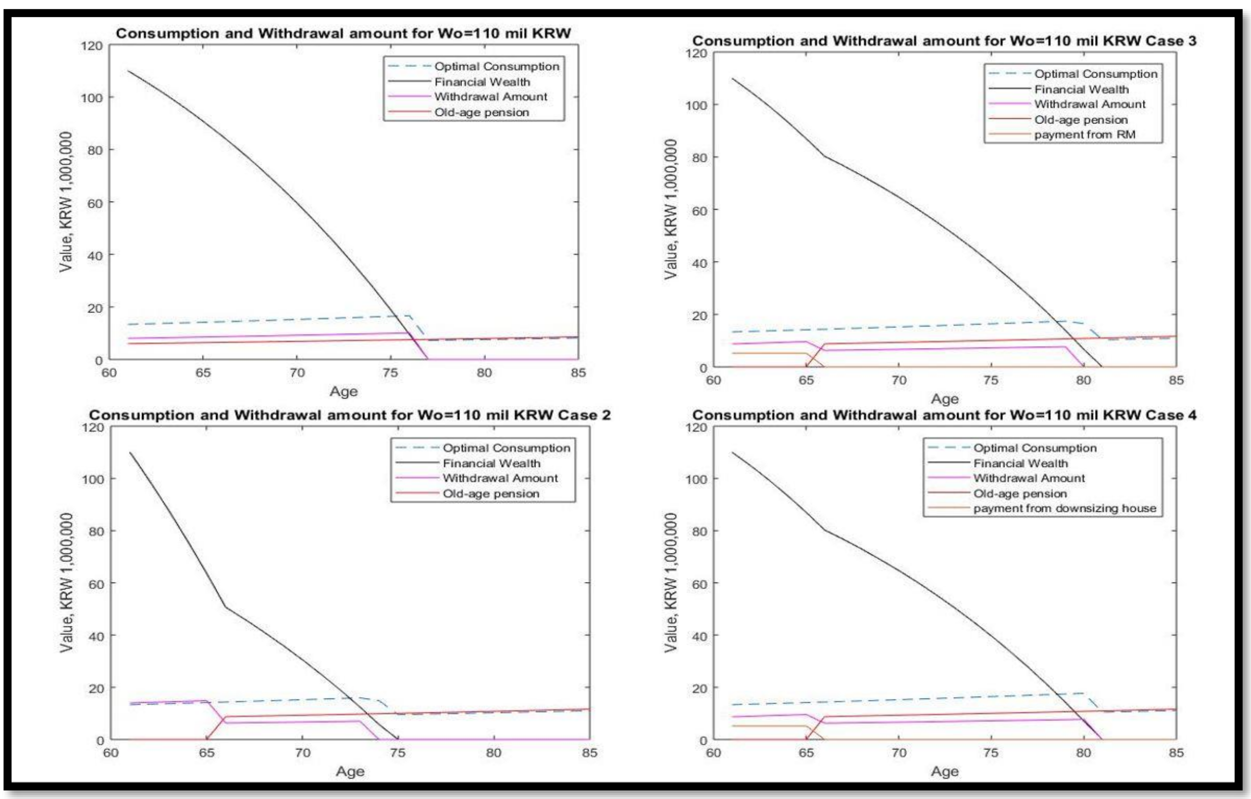
Table 1.2: The effect of progressive income tax on portfolio choice in period T-1

Case 1				Case 2				Case 3				Case 4			
Total wealth in T-1 (million KRW)	Tax-bracket	Risky asset share	Absolute change in tax amount (KRW)	Total wealth in T-1 (million KRW)	Tax-bracket	Risky asset share	Absolute change in tax amount (KRW)	Total wealth in T-1 (million KRW)	Tax-bracket	Risky asset share	Absolute change in tax amount (KRW)	Total wealth in T-1 (million KRW)	Tax-bracket	Risky asset share	Absolute change in tax amount (KRW)
16.03	3	1		12.84	3	1		17.33	3	1		12.84	3	1	
18.21	4	0.9658	1076.8	15.05	4	1	185.6	19.54	4	1	3061.6	15.06	4	1	282.4
24.72	4	0.6909		23.83	4	1		26.09	4	0.9047		21.65	4	1	
26.88	5	0.644	1,935.2	26.09	5	0.9282	101.2	28.48	5	0.8235	50,859.8	23.84	5	1	526.2
96.34	5	0.3947		93.29	5	0.4265		97.93	5	0.4213		93.30	5	0.4272	
98.55	**6**	**0.3935**	47,386	**95.50**	**6**	**0.4244**	45,497	**100.41**	**6**	**0.4202**	117,601	**95.51**	**6**	**0.4251**	46,472
100.82	6	0.3945		97.77	6	0.4247		102.68	6	0.4206		97.78	6	0.4253	
103.09	6	0.3955		100.04	6	0.425		104.95	6	0.421		100.05	6	0.4256	
105.35	6	0.3973		102.30	6	0.4262		107.21	6	0.4226		102.31	6	0.4268	
107.60	6	0.3999		104.55	6	0.4282		109.46	6	0.4247		104.56	6	0.4288	
109.84	6	0.4030		106.79	6	0.4309		111.69	6	0.4273		106.80	6	0.4315	
114.27	**6**	**0.4087**		**117.78**	**6**	**0.4383**		**122.68**	**6**	**0.4348**		**117.79**	**6**	**0.4389**	
183.60	6	0.3781		180.55	6	0.3927		183.29	6	0.3928		180.56	6	0.3931	
185.85	**7**	**0.3777**	65,211	**182.79**	**7**	**0.3921**	63,371	**185.74**	**7**	**0.3918**	58,019	**182.81**	**7**	**0.3925**	64,350
188.13	7	0.3786		185.07	7	0.3929		188.03	7	0.3918		185.09	7	0.3932	
190.41	7	0.3794		187.35	7	0.3936		190.31	7	0.3925		187.37	7	0.3939	
192.69	7	0.3803		189.64	7	0.3942		192.59	7	0.3932		189.65	7	0.3946	
194.97	7	0.3811		191.92	7	0.3949		194.87	7	0.3938		191.93	7	0.3953	
197.25	7	0.3820		194.19	7	0.3957		197.15	7	0.3945		194.21	7	0.3961	
199.51	7	0.3837		196.46	7	0.3973		199.42	7	0.3954		196.47	7	0.3976	
226.15	**7**	**0.4022**		**227.47**	**7**	**0.4159**		**230.51**	**7**	**0.4148**		**227.48**	**7**	**0.4162**	

Note: The marginal tax rates applied to the fourth, fifth, sixth, and seventh tax brackets are 6%, 15%, 24%, and 33%, respectively.

When the minimum living condition binds, the initial money balance of 25 million KRW can only survive the first five years, three years, and four years of the retirement period under Case 1, 2, and the two remaining cases, respectively. Once the IRA is exhausted, old-age pension benefits will act as a sole source of income, and the optimal consumption level is determined just by subtracting the income tax and property tax from the pension benefit. It follows that beyond the age of 65, the consumption path in all cases closely tracks the old-age pension benefit. The optimal drawdown each year is figured out on the basis of the optimal consumption level and slightly increases in the first few years since the required living standard is indexed to inflation. As soon as there is nothing left in the IRA, the withdrawal amount runs down to zero. One big issue in Case 2 is that in the first five years, the investor has no pension benefits and depends entirely on the drawdown from the IRA. Hence, for poor-cash investors, it seems impossible to delay old-age pension benefits for five years without unlocking their home equity or downsizing their housing stock. Case 3 and 4 yield a better consumption path as these financial strategies provide an alternative income source in the five deferral years.

Figure 1.6: Consumption and withdrawal amount (Initial Financial Wealth=110 mil KRW)

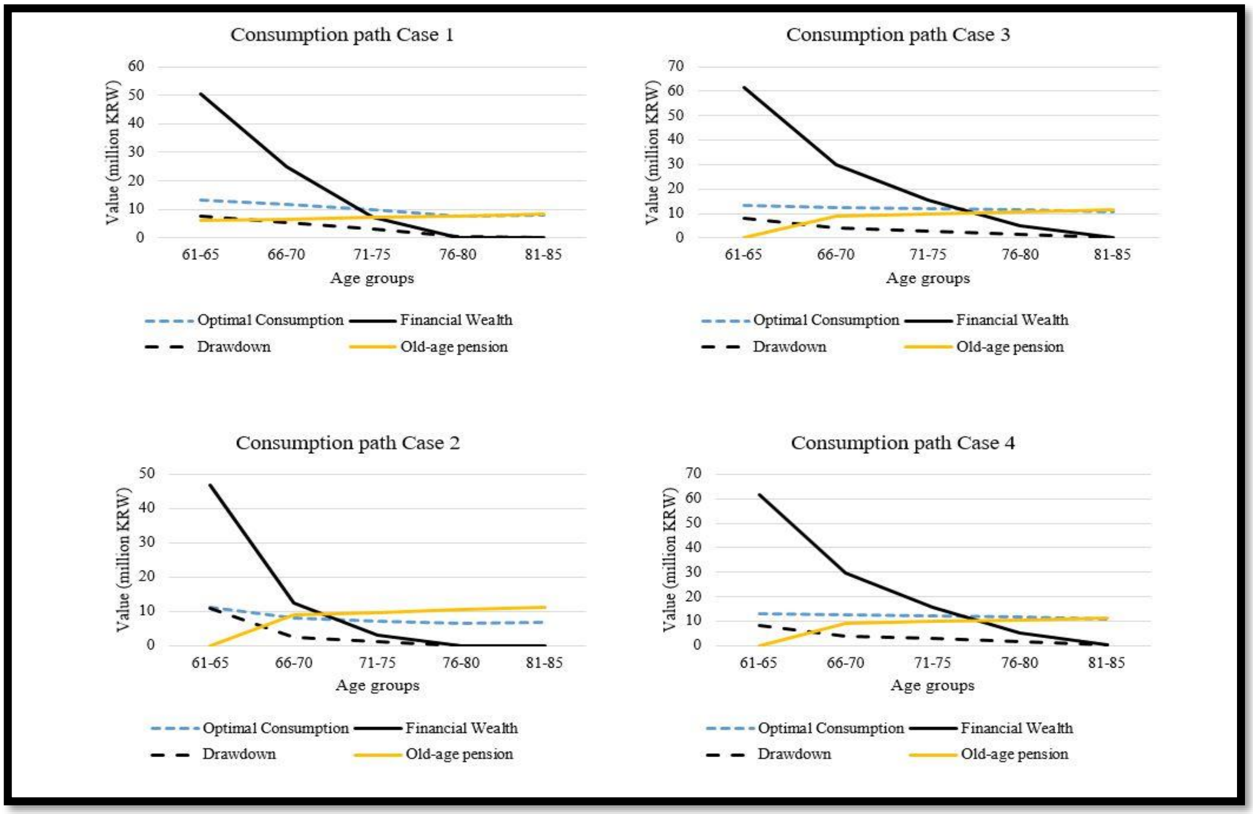


Source: Author's calculation

When the initial financial wealth at age 61 is 110 million KRW, the average money balance can survive at max 21 years in retirement under the financial strategies in case 3 and case 4. At the same time, under case 2, the balance of the IRA can serve as an essential income source before getting exhausted entirely at age 75. As can be seen from Figure 1.6, if the agent does not want to tap into the home equity or downsize the house, with the initial money balance of 110 million KRW, he or she can consider the financial strategy under Case 2. Nevertheless, welfare analysis is necessary to find out which strategy is optimal.

Figure 1.7 depicts the consumption path of an average consumer. The blue dotted lines in all panels indicate that the average consumption is quite smooth but slopes down over the retirement years. This outcome is consistent with Campbell et al. (2001) and Cocco et al. (2005). During retirement, the individual consumes more than his/her old-age pension benefit until the IRA's balance is exhausted. In case 1, the liquid wealth runs down to zero after approximately 17 years, while in case 3 and case 4, the average liquid wealth can survive almost 25 years of the retirement period. The drawdown from the IRA is illustrated by black dotted lines. This amount gradually declines with age before decreasing to zero. In case 2, without the income source from old-age pension benefits in the first five years, the average consumption of the age group 61-65 tracks the withdrawal amount from the IRA very closely. When it comes to the next age groups, the average consumption level stays well below the old-age pension benefit as many poor-cash investors cannot survive five deferral years.

Figure 1.7: Consumption path of the average consumer during the retirement period



Source: Author's calculation

1.8. Scenario Analysis

The optimal portfolio choice and optimal consumption path under the baseline scenario have been studied in the previous section. However, investors may differ along other dimensions. For instance, some individuals may be more impatient or risk-averse than others. These differences may have important effects on the optimal consumption rule and financial strategies during retirement years. In this section, I consider how the heterogeneity of preferences affects the optimal behaviour of investors during retirement years. In the baseline model, the coefficient of relative risk aversion, γ, is 3, and the time discount factor, β, is 0.96. In this section, the optimal behaviour is calculated for highly risk-averse investors with a CRRA of 7 and impatient investors with a β of 0.8.

Table 1.3 reports three scenarios in which the investor's preferences shift away from the benchmark parameters. It also compares the optimal behaviour in these three scenarios with the results under the baseline model. First, in the scenario with higher CRRA (γ=7), the cross-sectional means of the simulated optimal consumption levels in each age group for all cases are accordingly lower than that in the baseline model. One possible explanation is that highly risk-averse individuals tend to consume less and save more. When people enter retirement and stop working, there is generally no labour income risk, and at the same time, old-age pension benefits guarantee a lifetime income stream. As such, the precautionary savings motive is not as strong during this period as during the working phase. The increase in the CRRA from $\gamma=3$ to $\gamma=7$ has a negligible effect on the consumption behaviour of the average consumer. Specifically, in the first two age groups, a slight reduction of around 0.8% in the optimal consumption amount is witnessed in all cases. The rate is a bit larger in the remaining age groups- 1.2%-2.8% decrease in the level of consumption as moving from a lower to a higher coefficient of relative risk aversion.

Obviously, the highly risk-averse individual invests less of their financial wealth in risky assets. In the scenario with the CRRA of 7, the proportion of financial wealth invested in stocks is significantly lower than that in the benchmark scenario. Moreover, within the first three age groups, the impact that an increase in the level of risk-aversion has on the optimal portfolio choice is notable. For example, under case 1, as the CRRA increases from $\gamma=3$ to $\gamma=7$, a reduction of approximately 25 percentage points in the average optimal risky asset share is observed for the age groups 61-65, 66-70, and 71-75. When it comes to the older groups, due to the downward trend in the money balance of the IRA during retirement, the portion of old-age pension benefit in total wealth generally increases with age, implying greater exposure to risk-free assets compared to the younger investors. Subsequently, the last two age groups are found to allocate an extremely high proportion of financial wealth into risky assets, from 93% to 100%. Also, because short sales are not allowed, there will be little or no impact on the optimal risky asset share from the increase in CRRA within the oldest and second oldest groups.

Table 1.3: Effect of heterogeneity of preferences on consumption, financial wealth, and portfolio choice

Consumption (million KRW)												
	Baseline model				CRRA γ=7				β=0.8			
Age	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
61-65	13.32	11.18	13.12	13.10	13.30	11.16	13.10	13.08	13.77	11.26	13.63	13.60
66-70	11.64	8.00	12.59	12.56	11.53	7.91	12.54	12.51	11.64	8.00	12.59	12.56
71-75	9.79	7.16	12.18	12.14	9.54	6.90	11.96	11.92	9.53	7.03	12.14	12.11
76-80	7.82	6.35	11.64	11.66	7.53	6.35	11.28	11.29	7.33	6.35	10.91	10.92
81-85	7.89	6.84	10.79	10.98	7.89	6.84	10.64	10.82	7.89	6.84	10.64	10.82
Financial Wealth (million KRW)												
	Baseline model				CRRA γ=7				β=0.8			
Age	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
61-65	64.1	46.7	61.5	61.6	63.4	46.2	60.8	60.8	62.5	46.4	59.8	59.8
66-70	36.8	21.6	38.9	38.9	34.8	20.5	36.9	36.9	33.8	21.1	35.5	35.6
71-75	17.4	12.9	25.3	25.3	15.3	12.0	22.7	22.8	14.0	12.4	21.4	21.5
76-80	8.4	10.5	15.6	15.7	7.9	10.5	13.5	13.6	7.7	10.5	12.0	12.1
81-85	8.3	11.3	11.5	11.5	8.3	11.3	11.3	11.3	8.3	11.3	11.3	11.3
Liquid Portfolio Share in Risky asset												
	Baseline model				CRRA γ=7				β=0.8			
Age	C1	C2	C3	C4	C1	C2	C3	C4	C1	C2	C3	C4
61-65	0.49	0.51	0.50	0.50	0.27	0.37	0.28	0.28	0.49	0.51	0.50	0.50
66-70	0.70	0.85	0.72	0.71	0.55	0.75	0.56	0.56	0.70	0.86	0.72	0.72
71-75	0.88	0.97	0.84	0.84	0.82	0.94	0.74	0.74	0.92	0.98	0.87	0.86
76-80	1.00	1.00	0.95	0.95	1.00	1.00	0.93	0.93	1.00	1.00	0.99	0.99
81-85	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Regarding the change in the total financial wealth in accordance with the increase in the level of risk-aversion, a similar pattern is observed, as $TW_{i,t}$ appears to be lower for the scenario with higher CRRA. A slight reduction in the optimal consumption amount and a downward trend in optimal risky asset share might explain the smaller level of total wealth.

The last column of Table 1.3 shows the optimal consumption, total financial wealth, and portfolio allocation decisions for impatient investors (β = 0.8). In the neoclassical theory of interest, Fisher (1930) defines time preference or human impatience as individuals' feelings about current and future consumption. The time preference is captured mathematically via the discount interest rate. As one might expect, the higher the time preference, the higher the required discount rate on returns receivable in the future, leading to the lower time discount factor β. Basically, these impatient investors consume more earlier in the retirement period and less later. On that account, when the time discount factor decreases to 0.8, the first age group 61-65 experiences a climb in the optimal consumption in all cases, while the opposite is true for the other remaining age groups. The liquid wealth will be decumulated faster as the investor will consume more in the first few years. With the old-age pension benefits being indexed to inflation and the IRA's balance decreasing over time, the exposure to risk-free assets increases as the investor ages. It follows that the representative investor will keep a higher percentage of financial wealth in risky assets in the scenario of higher time preference. A slight increase in the optimal risky asset share for all cases is found as the time discount factor declines from 0.96 to 0.8.

1.9. Welfare Analysis

The welfare analysis is conducted on the basis of standard consumption-equivalent variation. To be more specific, for each case from 1 to 4, I calculate the constant consumption stream that makes the investor ***i*** as well-off in expected utility terms as the simulated consumption path. Relative utility gain/loss is obtained by measuring the difference in the equivalent consumption among the four cases. The difference is then divided by the minimum living standard (expenses) so that I can examine the welfare gain/loss as a percentage of the required living costs.

To begin with, the individual optimisation problem is solved for a particular agent with the given level of old-age pension benefit at age 61, initial financial wealth, and current house value. The simulation is then done for all investors with the old-age pension benefit at age 61 ranging from 3 to 11 million KRW per year, the initial money balance of the IRA varying from 25 to 115 million KRW and current house value ranging from 50 to 400 million KRW. The optimal consumption paths are denoted by $\mathbf{C^1_{i,t}}$, $\mathbf{C^2_{i,t}}$, $\mathbf{C^3_{i,t}}$ and $\mathbf{C^4_{i,t}}$, respectively, for Case 1, Case 2, Case 3, and Case 4. Next, the expected lifetime utility is determined as follows:

$$\textbf{[26]}\ V^k = E_1\sum_{t=1}^{T}\beta^{t-1}\left(\prod_{j=0}^{t-1}p_j\right)\frac{(C^k_{i,t})^{1-\gamma}}{1-\gamma}\quad \textbf{(k=1,2,3,4)}$$

Therefore, $\mathbf{V^k}$ **(k=1,2,3,4)** represents the maximum lifetime utility of the agent who follows the optimal consumption and optimal portfolio allocation rules during the retirement period. Then, this discounted lifetime utility is converted into consumption units by calculating the equivalent constant consumption stream $\overline{C}^k_{i,t}(k=1,2,3,4)=\overline{C}^k(k=1,2,3,4)$. This conversion is given by the following equation:

$$[27]\ V^k = E_1 \sum_{t=1}^{T} \beta^{t-1} \left(\prod_{j=0}^{t-1} p_j\right) \frac{(\overline{C}^k)^{1-\gamma}}{1-\gamma} \quad (k=1,2,3,4)$$

Consequently, I have:

$$[28]\ \overline{C}^k = \left[\frac{(1-\gamma)V^k}{\sum_{t=1}^{T} \beta^{t-1} \left(\prod_{j=0}^{t-1} p_j\right)} \right]^{\frac{1}{1-\gamma}}$$

The welfare gain or loss between two particular cases is next determined by:

$$[29]\ \mathrm{Wf}_{i,j} = \frac{\overline{C}^i - \overline{C}^j}{\mathrm{MinLivSt}} * 100\%$$

where MinLivSt= Minimum living standard = max(Old-age pension benefit/ replacement rate, 7.5 million KRW)

The comparison of case 2 and case 1

Regarding the question of whether it is beneficial for senior individuals to delay old-age pension without any additional income source other than the drawdown from the IRA during the pension delay, I first compare the equivalent constant consumption stream of Case 2 with that of Case 1 and compute the percentage gain/loss relative to the minimum living standard determined at the beginning of the retirement period. The results are then categorised by three dimensions, including (i) the house value at the beginning of age 61, (ii) the initial financial wealth, and (iii) the old-age pension benefit at age 61, as shown in Table 1.4. The negative number - highlighted in red- represents the welfare loss if the investor follows financial strategies under Case 2. By contrast, the positive number indicates the welfare gain.

The old-age pension benefit at age 61 is assumed to account for only 45% of the required living standard. Hence, when this required living condition binds, investors have to withdraw a much higher amount of money from the IRA if there is no old-age pension benefit in the first five years of the retirement period. The money balance of IRA is, thus, decumulated at a faster rate in case 2 than in case 1. It can be noticed from Table 1.4 that with higher levels of the required living standard, the initial money balance needs to get bigger in order to experience a welfare gain from delaying old-age pension benefits under case 2. For instance, if the old-age pension benefit at age 61 is 6 million KRW per year, investors should have at least 50 million KRW in their individual retirement accounts to gain from financial strategies under case 2.

However, when the old-age pension benefit goes up to 11 million KRW, even the initial financial wealth of 110 million KRW cannot yield a welfare gain under case 2. Distinguishably, the red numbers in the upper right corner of the table show that in case investors initially have a large amount of money available in the IRA (over 90 million KRW) and at the same time, their old-age pension benefits at age 61 stay low at the level of under 5 million KRW per year, it is not recommended to exercise the financial strategies in case 2. On average, these investors lose around 3.3-4.5% in the equivalent constant consumption level. This finding can be explained by the fact that rich-cash investors can always satisfy their required living standard even without a 36% increase in old-age pension given by the pension delay. Accordingly, instead of following strategies under case 2, they can claim the old-age pension benefit at age 61 and thus, avoid depleting the money balance of IRAs so quickly. It can also be seen from Table 1.4 that given a particular level of initial money balance and old-age pension benefit at age 61, as moving from a low to a high current house value, the welfare loss if following the plan in case 2 tends to decrease. One potential reason is that a high property tax amount corresponding to a high house value might make the plan to increase their old-age pension benefits by delaying the pension claim for five years more favourable.

Table 1.4: Welfare Gain Case 2 vs. Case 1 (% of minimum living standard)

House Value (million KRW)	Initial Financial Wealth (million KRW)											Old-age pension (million KRW/yr)	Minimum Living standard (million KRW/yr)
	25	30	40	50	60	70	80	90	100	110	115		
50	-59	-63	3	4	3	2	1	-3	-6	-5	-4	3	8
50	-63	-67	0	2	2	2	1	1	-4	-7	-6	4	9
50	-59	-62	-68	0	2	2	2	2	1	0	-1	5	11
50	-58	-59	-64	-68	0	2	2	2	1	2	2	6	13
50	-56	-58	-61	-65	-69	0	2	2	2	2	2	7	16
50	-55	-56	-59	-62	-65	-69	0	2	2	2	2	8	18
50	-54	-55	-58	-60	-63	-66	-69	0	3	3	2	9	20
50	-53	-54	-56	-58	-61	-64	-66	-68	-2	3	3	10	22
50	-52	-53	-55	-58	-59	-61	-64	-67	-68	-3	2	11	24
100	-56	-60	5	4	4	3	1	-2	-5	-5	-4	3	8
100	-60	-64	0	2	3	2	2	1	-3	-7	-6	4	9
100	-57	-60	-66	0	2	2	2	2	2	1	-1	5	11
100	-56	-58	-63	-67	0	2	3	2	2	2	2	6	13
100	-55	-57	-60	-64	-67	0	2	2	3	2	3	7	16
100	-54	-55	-58	-61	-64	-67	0	2	2	2	3	8	18
100	-53	-54	-57	-59	-62	-64	-67	-2	3	3	3	9	20
100	-52	-53	-55	-58	-60	-63	-65	-68	-3	3	3	10	22
100	-52	-52	-54	-57	-58	-61	-63	-66	-68	-5	3	11	24
200	-46	-49	8	7	7	7	4	2	-3	-4	-4	3	8
200	-53	-57	-21	4	4	4	3	2	0	-4	-5	4	9
200	-52	-55	-59	-12	4	4	4	3	3	2	2	5	11
200	-51	-53	-57	-62	-8	4	3	4	3	3	3	6	13
200	-51	-52	-56	-59	-63	-6	4	4	3	3	3	7	16
200	-51	-52	-54	-57	-61	-64	-7	3	3	4	3	8	18
200	-49	-51	-54	-55	-58	-61	-64	-9	3	3	4	9	20
200	-49	-51	-52	-55	-57	-59	-62	-65	-12	4	4	10	22
200	-49	-50	-52	-54	-56	-58	-60	-62	-65	-14	1	11	24
400	-18	-20	-23	17	18	19	20	21	23	18	16	3	8
400	-31	-32	-36	11	13	12	12	15	12	9	6	4	9
400	-34	-36	-39	-43	9	11	10	9	11	9	8	5	11
400	-38	-38	-42	-45	-49	7	9	9	9	8	9	6	13
400	-39	-40	-43	-46	-49	-52	6	7	8	8	8	7	16
400	-40	-42	-44	-45	-48	-51	-54	4	6	6	7	8	18
400	-41	-42	-44	-46	-48	-50	-53	-55	3	7	7	9	20
400	-42	-42	-44	-46	-48	-50	-52	-54	-57	1	6	10	22
400	-43	-43	-45	-46	-48	-50	-51	-54	-55	-58	-35	11	24

Focusing on numbers in black (Table 1.4), which refer to the welfare gain, in cases with current house value from 50 to 200 million KRW, where the property tax effect is still weak, senior individuals experience only a small gain in the equivalent constant consumption level - around 1-8% of the required living standard at the beginning of the retirement period. Obviously, the higher the old-age pension benefit, the lower the percentage of welfare gain since the welfare gain is defined by dividing the difference between the equivalent constant consumption levels in two cases by the required living cost. As the current house value reaches 400 million KRW, the amount of property tax is significantly higher due to the progressive tax system. Subsequently, exercising financial plans under case 2 will be more advantageous. The average welfare gain for individuals with a current house value of 400 million KRW and old-age pension benefit of 3 million KRW per year is roughly 19% of their minimum living standard or about 1.4 million KRW equivalently. The rate gradually decreases as the pension benefit goes up; however, the absolute gain is quite stable around the level of 1-1.5 million KRW per year.

The comparison of case 3 and case 1

As discussed above, if the initial money balance of the IRA stays low at 20-30 million KRW, investors are not recommended to follow financial strategies under case 2 even if the minimum living standard at age 61 is only 8 million KRW per year. Moreover, as the required living standard increases, the initial financial wealth needs to get bigger so as to make plans under case 2 beneficial. Hence, cash-poor investors may find it difficult to delay old-age pension benefits until they reach 66 years old without any alternative income source other than the drawdown from their IRA. The investor now decides to delay old-age pension for five years and take out an RML to replace the pension benefits in five deferral years. In this section, the welfare gain/loss from undertaking plans in case 3 is considered. In the case of the Korea-style RML, the loan-to-value ratio applied to individual investors with the current age of 61 and the life expectancy of 85 is approximately 48.06%. It follows that the maximum borrowable amount from RMLs is defined by subtracting the up-front cost from 48.06% of the current house value.

Table 1.5: Welfare Gain Case 3 vs. Case 1 (% of minimum living standard)- Convert k% of loan limit to 5-year term option

House Value (million KRW)	Initial Financial Wealth (million KRW)											Old-age pension -million KRW/yr	Per cent LOC
	25	30	40	50	60	70	80	90	100	110	115		
50	16	17	18	17	13	8	4	6	7	8	8	3	0.58
50	17	18	18	17	14	10	5	4	7	8	9	4	0.77
50	14	17	17	18	18	16	13	9	5	5	6	5	0.96
50	6	10	15	16	16	16	15	14	11	8	7	6	1
50	1	1	5	13	14	14	14	14	13	11	10	7	1
50	-7	-3	-1	2	13	13	13	12	13	12	12	8	1
50	-9	-11	-7	-2	-2	11	12	12	12	13	11	9	1
50	-12	-13	-10	-11	-4	-6	11	12	12	12	11	10	1
50	-16	-15	-17	-14	-14	-7	-9	9	10	10	10	11	1
100	17	18	20	19	16	10	5	6	8	9	9	3	0.29
100	18	19	20	18	16	11	6	4	7	9	9	4	0.38
100	14	18	18	20	19	17	14	10	6	6	6	5	0.48
100	13	14	17	18	19	19	18	16	13	10	8	6	0.58
100	11	13	17	17	19	19	18	18	17	14	14	7	0.67
100	11	11	14	17	18	19	20	19	18	17	17	8	0.77
100	10	11	13	16	18	19	19	19	19	19	18	9	0.86
100	9	11	12	14	17	18	19	19	19	19	20	10	0.96
100	8	8	10	12	13	17	17	18	19	18	19	11	1
200	20	21	24	23	22	18	11	9	9	9	10	3	0.14
200	20	20	22	22	21	17	12	7	8	9	9	4	0.19
200	15	19	21	21	22	21	18	15	10	8	8	5	0.24
200	13	15	19	20	21	22	20	19	16	14	12	6	0.29
200	12	13	17	19	19	20	21	21	19	17	16	7	0.34
200	11	12	15	18	19	20	20	20	20	19	19	8	0.38
200	11	12	14	17	18	19	20	20	20	20	20	9	0.43
200	10	11	12	14	18	19	20	20	20	20	20	10	0.48
200	10	10	12	14	16	18	19	20	20	20	20	11	0.53
400	22	28	31	37	41	44	47	46	43	35	33	3	0.07
400	20	27	30	32	36	37	37	36	29	23	21	4	0.1
400	18	19	25	28	29	32	32	32	31	26	23	5	0.12
400	15	18	23	24	26	28	29	29	30	28	28	6	0.14
400	14	14	18	23	23	25	26	27	27	27	27	7	0.17
400	13	14	17	21	23	24	24	25	25	26	26	8	0.19
400	12	13	14	17	21	23	24	24	24	25	25	9	0.22
400	11	12	13	16	19	21	22	23	24	24	24	10	0.24
400	11	11	13	15	16	21	21	21	23	23	24	11	0.26

The representative investor chooses the modified term payment option. Under this plan, there will be a portion of the total principal limit set aside as an available line of credit. The remaining amount is converted into a fixed-term annuity with the periodic payment equal to the pension benefit at age 61 if claiming right after retirement.

Currently, Korean Housing-Finance Corporation accepts the modified term payment method, which is the combination of the line of credit option and the term option. Investors can flexibly choose the portion of the loan limit that they want to convert into the fixed-term annuity, and withdrawals from the available line of credit can be made anytime during the loan term. However, these amounts are limited to 50% of the initial principal limit. It appears that in order to match the modified term payment option with the goal of replacing the pension benefits in the five deferral years, the current house value must be large enough so that a part of the loan limit can be transformed into the 5-year annuity with the annual payment at least equal to the old-age pension benefit at age 61. In this study, I define a cash-poor investor as the person whose initial financial wealth cannot sustain for the first five years of the retirement period provided that the required living standard is met. At the same time, a house-poor investor is identified as the one whose current house value at age 61 cannot satisfy the plan of replacing the pension benefits in the five deferral years.

Table 1.5 shows the result of the welfare comparison between case 3 and case 1. The red negative figure indicates the welfare loss while the black positive rate represents the welfare gain. The percentage of total loan limit being transformed into the fixed-term annuity is denoted by percentLOC. As one might expect, house-poor and cash-poor individuals need over 50% of the loan limit to be converted into a 5-year annuity in order to meet the requirement in living conditions. The opposite is true for house-rich investors under this payment method. Overall, with current house value ranging from 100 to 400 million KRW, investors experience a welfare gain from plans under case 3 regardless of the level of initial financial wealth. When the amount of initial money balance and the level of the required living standard are kept unchanged, the welfare gain increases with the current house value. For instance, investors who have the initial old-age pension benefit of 5 million KRW per year and possess a 100 million KRW house, on average, gain 13.53% from financial plans under case 3. With all parameters except for the current house value kept constant, the average welfare gain will reach 15.75% (31.26%) if investors now have a 200-million (400-million) KRW house.

I now consider investors with the current house value of 50 million KRW. Under the modified-term payment method from the RML, with the old-age pension benefit ranging from 3 to 7 million KRW per year, these investors averagely gain 11.48%. As can also be seen from table 1.5, there are some individuals who are considered house-poor and cash-poor investors, experiencing a welfare loss when taking out RMLs to replace old-age pension benefits in the five deferral years. Individuals- whose house value stays at 50 million KRW and initial pension benefit stays at 8 million KRW a year- need at least 50 million KRW in the IRA to gain under case 3. For those who have lower than 50 million KRW in the IRA, the average welfare loss is 3.15%. Things get worse when it comes to the investors with the house value of 50 million KRW but with a high level of the old-age pension benefit. For example, retirees who have the initial pension benefit of 9 million KRW per year and the current house value of 50 million KRW will experience, on average, a loss of 6% if their initial finance wealth is lower than 70 million KRW. By contrast, individuals who benefit the most from the financial strategies under case 3 are those who have a low level of old-age pension benefit and have an initial money balance that is small but enough to survive the first five years of retirement under case 3.

The comparison of case 4 and case 3

Another way that can support the delay of old-age pension benefits for the first five years of the retirement period in case investors do not want to take RMLs is to downsize their house. In this section, I conduct the welfare analysis between Case 4 and Case 3. In case 4, investors decide to downsize their house, and the amount attained from the downsizing is then converted into a 5-year annuity. In order to make Case 4 and Case 3 comparable, I set the proceeds obtained after selling the old house and buying a smaller one equal to the proportion of the reverse mortgage loan limit converted to a 5-year annuity. Based on these proceeds and a round-trip transaction cost of 23%[13], the portion of the current housing stock that needs to be reduced can be determined.

---

[13] According to Ministry of Land, Infrastructure and Transport in South Korea, the transaction costs that you have to pay when buying a house include (i) Registration cost: Registration tax (2% of the house price), Education tax (0.2% of the house price), stamp duty (0.01-0.2% of the house price), (ii) Real estate agent's fee: 0.8-1.2% of the house price, (iii) Legal fees: 0.25-1% of the house price, (iv) Sales and Transfer taxes: Acquisition tax (2% of the house price), special tax for rural development (0.2% of the house price), VAT (10% of the house price), and (v) National Housing Bonds: 5% of the house price.

Next, I conduct the welfare comparison between Case 3 and Case 4 in Table 1.6. There are three crucial differences between these two cases. First, it is the tax advantage that investors will forego if they follow plans under case 4 instead of plans under case 3. The payment from RMLs is not considered taxable income, while the opposite is true for the income from the 5-year annuity in case 4. Also, RM borrowers can benefit from the reduction of 25% in property tax. In case people choose to downsize the housing stock, on the one hand, the property tax will reduce; on the other hand, the retirement income tax will go up. Second, it is home equity at the end of life that distinguishes these two cases. Under case 3, although borrowers are not required to make any periodic repayment, loan interest, and annual insurance premium accrue on the OLB over time. When borrowers pass away or move out of the house, they need to pay back the OLB. By contrast, under case 4, there will be no debt balance increasing over time, and individuals can leave their house as a bequest to their children when they pass away. Without a doubt, under case 3, reverse mortgage borrowers can still leave a bequest to their heirs if their house value at the end of life exceeds the OLB. The probability of leaving bequest under case 3, hence, depends on the house price dynamics. Third, RMLs can protect senior homeowners from significant drops in house prices, while downsizing cannot do so. Since RMLs are non-recourse loans, borrowers or the heirs do not have to take responsibility for any shortfall if the house value at the time of loan termination is lower than the OLB. For this reason, financial strategies under case 3 can insure homeowners against the downside risk to house prices.

The results show that financial strategies under case 4 track plans under case 3 quite closely except for the house value and home equity at the end of life. So, the percentage gain in the equivalent constant consumption level when moving from case 3 to case 4 is relatively small at around 0-0.5% for the house value ranging from 50 to 100 million KRW. The rate appears to be higher for the house value of 200 million KRW. This finding can be explained by the reason that investors might experience a welfare gain from case 4 when property tax reduction exceeds the increase in annual income tax in the first five years. When the house is downsized, the property tax amount imposed on house-rich individuals (200 million KRW) will reduce more significantly than in the case of lower house value (50-100 million KRW).

Table 1.6: Welfare Gain Case 4 vs. Case 3 (% of minimum living standard)

House Value (million KRW)	Initial Financial Wealth (million KRW)											Old-age pension (million KRW/yr)	Downsize
	25	30	40	50	60	70	80	90	100	110	115		
50	0.1	0.2	0.3	0.4	0.2	0.1	0.1	0.3	0.3	0.4	0.4	3	0.4
50	-0.1	0.0	0.1	0.3	0.2	0.1	0.0	0.0	0.1	0.1	0.2	4	0.48
50	-0.3	-0.3	-0.3	0.0	0.1	0.2	0.1	0.1	0.0	0.1	0.2	5	0.55
50	-0.3	-0.4	-0.3	-0.1	0.0	0.1	0.3	0.2	0.1	0.1	0.1	6	0.57
50	-0.4	-0.4	-0.4	-0.3	-0.1	-0.1	0.0	0.1	0.1	0.1	0.1	7	0.57
50	-0.3	-0.4	-0.5	-0.3	-0.3	-0.2	-0.3	-0.2	-0.2	0.0	-0.1	8	0.57
50	-0.3	-0.3	-0.4	-1.0	-0.4	-0.7	-0.5	-0.4	-0.6	-0.5	-0.4	9	0.57
50	-0.4	-0.3	-0.8	-0.4	-0.9	-0.4	-1.2	-1.2	-1.0	-0.7	-0.9	10	0.57
50	-0.3	-0.3	-0.3	-0.3	-0.3	-0.4	-0.4	-0.3	-0.2	-0.2	-0.2	11	0.57
100	-0.4	-0.3	-0.3	0.0	0.2	0.1	0.0	0.0	0.0	0.1	0.2	3	0.3
100	-0.5	-0.5	-0.3	-0.1	0.1	0.1	0.0	-0.2	-0.1	-0.1	0.0	4	0.33
100	-0.5	-0.6	-0.6	-0.4	-0.3	-0.1	0.1	0.1	0.0	0.0	0.0	5	0.37
100	-0.5	-0.5	-0.4	-0.3	-0.2	-0.1	0.0	0.1	0.2	0.1	0.1	6	0.4
100	-0.2	0.0	0.0	0.0	0.1	0.3	0.5	0.2	0.1	0.2	0.1	7	0.44
100	-0.2	-0.2	-0.1	0.1	0.3	0.2	0.3	0.1	0.2	0.1	0.1	8	0.48
100	-0.6	-0.4	-0.4	-0.6	-0.3	-0.1	0.0	0.0	0.0	0.1	0.1	9	0.51
100	-0.5	-0.8	-1.3	-0.4	-0.5	-0.4	-0.3	-0.2	-0.2	-0.5	-0.2	10	0.55
100	-0.6	-0.5	-0.6	-1.2	-0.8	-0.6	-0.7	-0.6	-0.6	-0.6	-0.4	11	0.57
200	-0.3	-0.6	-0.6	0.2	0.6	1.2	0.5	0.0	-0.1	0.1	0.2	3	0.24
200	-0.2	-0.1	0.2	0.5	0.7	0.8	0.5	0.2	0.1	0.4	0.5	4	0.26
200	-0.2	0.2	0.2	0.3	0.6	0.7	0.8	0.8	0.4	0.5	0.6	5	0.28
200	2.1	0.9	0.9	0.6	0.6	0.8	0.7	1.1	1.1	0.8	0.6	6	0.3
200	1.0	1.5	1.5	1.0	1.6	0.8	1.0	1.4	1.3	0.9	0.8	7	0.31
200	1.0	0.9	1.0	1.1	1.0	1.3	1.0	1.5	0.7	1.4	1.2	8	0.33
200	0.7	0.7	0.7	0.7	1.0	1.0	1.0	0.9	0.8	1.3	0.9	9	0.35
200	0.4	0.2	0.6	0.5	0.4	0.5	0.6	0.7	0.7	0.7	0.8	10	0.37
200	0.2	0.2	0.3	0.0	-0.8	0.4	0.4	0.4	0.5	0.5	0.6	11	0.39
400	-4.6	-0.4	-0.1	-0.7	-0.9	-1.2	0.2	0.1	-0.1	0.3	1.2	3	0.21
400	-1.4	-0.5	-0.2	-0.6	-1.3	-0.7	-0.1	0.2	-0.1	-0.2	1.1	4	0.22
400	-1.0	-2.0	-0.5	-0.4	-0.9	-0.5	-0.9	0.2	0.0	0.3	0.9	5	0.23
400	-2.3	-1.8	-0.5	-0.1	-0.8	-0.4	-0.6	-0.2	0.1	-0.1	0.4	6	0.24
400	-2.2	-1.7	-2.7	-0.1	0.0	-0.3	0.0	-0.5	0.1	0.0	0.3	7	0.25
400	-0.4	-1.3	-1.5	0.0	-0.3	-0.1	-0.1	0.6	0.6	0.0	-0.1	8	0.26
400	-1.8	-0.9	-1.2	-1.5	-0.1	-0.3	-0.1	-0.5	0.9	0.2	0.0	9	0.27
400	0.1	-1.0	-1.0	-1.8	-0.1	0.0	-0.1	-0.2	0.2	1.4	0.5	10	0.28
400	-0.3	-0.6	-0.8	-1.6	-1.0	-0.1	0.0	-0.2	-0.4	1.4	0.4	11	0.29

I do not incorporate the utility from leaving a bequest, as for those who lack cash and need to find alternative income sources such as unlocking home equity or downsizing the house to enhance retirement living standards, strategic bequests are expensive. Without bequest motives, the certainty equivalent consumption level in case 4 is approximately identical to that in case 3. However, for those who are considering to postpone claiming pension benefits for the first five years of their retirement and are not short on cash, motives to bequest can be a major obstacle when it comes to the decision of whether to take out RMLs. In this case, bequest motives lead individuals to remain homeowners until a later age (Cocco et al., 2020), meaning the downsizing option (Case 4) overweighs the RML option (Case3). Nevertheless, one important feature of RMLs, namely, the benefit of ageing in place, should also be incorporated in the model. The utility derived from remaining in the same house (Case 3) might offset the utility from leaving a bequest via the downsizing option (Case 4). Cocco and Lopes (2020) find that with the inclusion of both the ageing in place benefits and utility of bequests, RMLs tend to benefit the individuals who have a weak bequest motive, those who have high levels of other pre-existing debt, and those who have low levels of financial wealth and pension income relative to housing wealth, which is consistent with our model predictions.

1.10. Policy experiment

Bringing our attention back to Table 1.5, which summarises the welfare comparison between case 3 and case 1, it can be clearly seen that some individuals need to convert a large portion of their home equity into a five-year fixed annuity if their house value is lower than 100 million KRW. Generally, it is not recommended to use up home equity quickly during the first few years of retirement. Therefore, investors whose IRAs can partly support the old-age pension delay should not think about converting a significant part (over 50%) of the reverse mortgage loan limit into a five-year fixed annuity. In this section, I experiment with one possible change in the modified-term payment method of the Korean-style reverse mortgage. Correspondingly, the proportion of total loan limit that can be converted into a five-year annuity is now fixed at 50%. The optimisation problem is solved again for case 3, and the simulation is re-conducted. Table 1.7 shows the result of the welfare comparison between case 3 and case 1 under the new policy in the payment method.

On the condition that the current house value is 50 million KRW, the new payment option from the RM loan can provide investors with an annual payment of approximately 2.6 million KRW per year for the first five years of the retirement period and the 11.64 million line of credit. The amount of 2.6 million KRW per year is indeed not enough to cover the old-age pension benefit at age 61 if investors choose to delay claiming. However, when the required living standard is not substantial, and the money balance from the IRA can partly support the postponement of the old-age pension benefits, 50% of the principal limit might be a safer choice for investors. In particular, for those who have the initial old-age pension benefit of 3-4 million KRW per year, the welfare gain is still witnessed in all levels of initial financial wealth, varying from 25 million to 115 million KRW. With a higher level of the old-age pension benefit, investors need a larger amount of money in the IRA to experience welfare gain from the financial strategy under case 3.

The same logic is applied to the case with the current house value of 100 million KRW. Under the new modified term option, each year during the first five years of retirement, investors receive 5.21 million KRW from the RML. Now, if the old-age pension benefit at age 61 is lower than 8 million KRW per year, senior individuals will gain if following strategies under case 3 regardless of the level of initial financial wealth. Investors who have the initial money balance of 25-60 million KRW and the old-age pension benefit of 3-5 million KRW a year are the ones who get the most out of financial plans under case 3. On average, they gain 21.25% in terms of the equivalent constant consumption. In case the pension benefit at age 61 is over 8 million KRW, if the initial money balance is not exhausted before the old-age pension benefit is claimed at age 66, investors can consider using only 50% of the RM loan limit to bridge the income gap in the five deferral years.

Table 1.7: Welfare Gain Case 3 vs. Case 1 (% of minimum living standard)- Convert 50% of loan limit into 5-year term annuity

House Value (million KRW)	Initial Financial Wealth (million KRW)											Old-age pension -million KRW/yr	Min Living standard (million KRW/yr)
	25	30	40	50	60	70	80	90	100	110	115		
50	14	15	15	16	12	7	3	4	6	6	7	3	8
50	1	12	12	12	11	8	4	2	2	3	4	4	9
50	-11	-5	10	10	10	10	9	7	4	2	1	5	11
50	-17	-17	-12	9	9	9	9	9	7	6	5	6	13
50	-25	-22	-24	-17	8	8	8	8	8	7	6	7	16
50	-27	-28	-26	-20	-21	7	7	7	7	7	7	8	18
50	-29	-30	-29	-30	-24	-26	7	8	8	8	7	9	20
50	-31	-32	-33	-31	-34	-27	-29	6	7	7	7	10	22
50	-33	-32	-35	-33	-34	-34	-30	-32	4	7	6	11	24
100	28	29	30	26	20	12	13	16	17	18	18	3	8
100	24	25	25	23	19	13	7	9	12	13	14	4	9
100	15	18	19	20	20	18	14	10	6	6	7	5	11
100	6	11	15	16	17	17	16	14	12	9	8	6	13
100	2	1	6	14	14	15	15	15	14	12	12	7	16
100	-6	-3	-2	2	13	13	14	14	13	13	13	8	18
100	-9	-11	-7	-2	-2	12	13	13	13	13	12	9	20
100	-12	-13	-10	-11	-5	-5	11	12	12	12	12	10	22
100	-16	-15	-17	-14	-14	-7	-9	9	11	10	11	11	24

1.11. Conclusion

This study develops a realistic life-cycle model which incorporates (i) the stochastic process of risky asset returns and (ii) the progressive tax rules on property's value and retirement income to solve the dynamic stochastic optimisation problem for the optimal consumption and portfolio choice of Korean retirees during the retirement period. The representative agent in the study is a homeowner and is currently at the beginning of age 61. He/she is considering whether to postpone old-age pension benefits for five years and which financial strategies are appropriate to follow. As mentioned previously, there are four strategies being examined in this paper: (i) claiming old-age pension benefits at age 61; (ii) delay old-age pension benefits for five years and fund the delay by the distribution from IRAs; (iii) delay old-age pension benefits for five years and fund the delay by taking out RMLs; and (iv) delay old-age pension benefits for five years and fund the delay by downsizing the house.

Regarding the portfolio allocation in retirement, the results show that the optimal risky asset share is negatively correlated to total financial wealth. Since old-age pension benefits work as a guaranteed income stream during retirement, indicating the exposure to risk-free assets in the financial portfolio, individuals who possess a lower level of financial wealth appear to have a more aggressive portfolio allocation. Similarly, given a fixed level of financial wealth, the higher the old-age pension benefit, the higher the optimal share invested in stocks. So, under case 1, in which investors decide not to delay old-age pensions, the optimal risky asset share tends to be lower than that under the three remaining cases. Moreover, I find that the optimal risky asset share is not a monotonic function of total non-housing wealth, suggesting that the progressive income tax rule might have an influence on the optimal portfolio allocation. Interestingly, the effect of the progressive tax rule on portfolio choice is only captured at the second-highest and highest income tax bracket. Last but not least, given a specific level of financial wealth, the optimal share invested in equities is fairly stable during the retirement period. This result does not support the traditional investment advice recommending that the allocation of risky assets in the portfolio should decline with age. However, it agrees with Ameriks et al. (2004) and Cocco et al. (2005), who find no empirical evidence supporting the gradual reduction in risky asset share as investors age during retirement.

The welfare comparison among the four cases is also analysed using benchmark parameters. I find that if undertaking strategies under case 2, investors whose liquid wealth cannot last for the first five years of the retirement period (cash-poor investors) will experience an average loss of 53.87% (with a standard deviation of 11.87%) in the certainty equivalent consumption level. Financial strategies under case 3 and case 4 support the goal of delaying old-age pension for five years by providing an alternative income source to replace the pension benefits in deferral years. In terms of RMLs, the payment method is selected based on the aim of finding an alternative income for the deferred pension benefits. Thus, the modified term payment option, in which a portion of the total loan limit is converted into a five-year fixed annuity, and the remaining amount is set aside as an available line of credit, is chosen. I define a house-poor investor as the one whose current house value cannot satisfy the plan of replacing the pension benefits in the five deferral years under case 3. It is found that under the current design of the modified term payment option, house-poor and cash-poor investors are not recommended to take out reverse mortgage loans. On average, these investors lose 9% in the equivalent constant consumption if implementing plans under case 3 instead of case 1. Individuals who get the most out of the financial strategies under case 3 are those who have a low level of old-age pension benefit and have an initial money balance that is small but enough to survive the first five years of retirement under case 3. In addition to the use of RMLs, downsizing the house can be another option aiding the delay of old-age pension. The welfare comparison between case 4 and case 3 shows that the difference in the equivalent constant consumption levels under these two cases is relatively small.

I also conduct a policy experiment. First, some individuals need to convert a large portion of their home equity into a five-year fixed annuity if their house value is lower than 100 million KRW under case 3. Second, it is not recommended to use up home equity quickly during the first few years of retirement. Therefore, in this section, I experiment with one possible change in the modified-term payment method of the Korean-style reverse mortgage. Correspondingly, the proportion of total loan limit that can be converted into a five-year annuity is now fixed at 50%. This experiment shows that in case the money balance from the IRA is large enough to partly aid the delay of pension benefits, it would be better for investors to limit the proportion of loan limit converted into the fixed-term annuity at 50%.

This research is of particular interest for several reasons. First, the population in many countries, especially those in Asia, is ageing more rapidly than ever before, making it hard to sustain the traditional pension system where the mandatory earnings-related pension scheme is mostly defined-benefit plans. Policymakers, therefore, can find this work useful as it might provide insights on how to reduce the financial burden on fiscal resources and how to design a new payment method from the RML that can enhance retirement security for senior people. Second, over the past few decades, many pension systems have undergone a radical shift from defined-benefit plans to defined-contribution plans, requiring individuals to be more responsible for their investment decisions and financial security over the life cycle. Hence, from this study, individuals can find a suitable financial strategy that makes the most of their housing wealth, financial wealth, and old-age pension benefits during retirement. It is important to note that this paper focuses on investors' behaviour in the retirement period. One potential direction for future research is to extend the study to the working phase to see whether the availability of equity-released products such as RMLs has an influence on the portfolio decision before retirement.

CHAPTER 2: DISRUPTED CAREERS AND RETIREMENT ADEQUACY UNDER FULLY-FUNDED DEFINED-CONTRIBUTION PENSION PLANS

Abstract

Previous literature has extensively investigated the impact of job displacement on wages and earnings. Nonetheless, research on the consequences of job displacement for retirement income security is limited. Furthermore, among several papers examining the effect of career breaks on pension benefits, no single study has adequately covered the implication of post-interruption wage profiles on the adequacy of retirement income. I address this gap by assessing the influence of unemployment-induced career breaks on retirement income adequacy under the current design of Singapore's national pension system - best known as one of the world's oldest and largest defined-contribution pension systems (McCarthy et al., 2002; Fong et al., 2011), taking into consideration (i) the timing of the breaks, (ii) the duration of the breaks, (iii) and the severity of the impact that job losses might have on post-interruption wages. I find that due to the scarring effect of job loss on post-interruption wage profiles, the sooner the break, the larger its impact on pension adequacy. Moreover, the results point out that median-income workers, who suffer job displacement during their career path, can still support themselves well beyond subsistence in retirement. However, job losses do significantly decrease their retirement nest egg, with the level of impact ranging from SG$ 12,800 to SG$ 153,267 (in 2018 dollars). I also discover that further extending the length of unemployment by six months can incur a reduction of at least SG$ 8,550- SG$ 11,970 (in 2018 dollars) in the retirement nest egg at age 55.

2.1. Introduction

Globally, advances in robotics, artificial intelligence (AI), and machine learning are embracing the new age of automation, significantly affecting the future of work. To stay competitive in such an ever-changing technological world, companies need to update their business models more frequently, so do employees. Whenever an innovation is happening faster than human labour can adapt, workers are at risk of being displaced, and their careers are at risk of being interrupted.

The most immediate impact of having a career path punctuated by periods of unemployment, for most people, is losing their critical source of income. However, in the long run, a more fragmented career might also have a persistent effect on post-interruption earnings profiles. In particular, a large body of empirical research on job loss effect shows that displaced workers often suffer a significant reduction in earnings in the first year following job displacement; and even six years after the layoff event, this loss still amounts to 10% to 20% of the pre-interruption earnings (Jacobson et al.,1993; Couch & Placzek, 2011; Jarosch, 2015; Krolikowski, 2017; Michaud, 2018; Jung & Kuhn, 2018). Although the adverse impact of job loss on wages and earnings has been extensively investigated in previous literature, research on the consequences of unemployment-induced career interruptions for retirement income security is limited.

Recently, there have been several papers examining the effect of career breaks on pension benefits (De Freitas et al.,2011; Potrafke, 2012; OECD, 2015; Bravo et al., 2017; European Commission/Social Protection Committee, 2018). However, no single study has adequately covered the implication of post-interruption earnings profiles on the adequacy of retirement income. For instance, De Freitas et al. (2011), OECD (2015), and European Commission/Social Protection Committee (2018) assume that the re-entry wages of displaced workers are the same as in their old position. Doing so might lead to the underestimation of the actual impact that career breaks impose on pension benefits. Among the few exceptions that address the impact of post-displacement wages on pension entitlements, Bravo et al. (2017) indicate that the impact increases substantially under more fragmented careers with a drop in the gross pension benefit of a displaced worker relative to that of non-displaced workers ranging from 9.98% to 40.81%. Nevertheless, their study fails to recognise the changing pattern of post-displacement wage growth rate by simply assuming a constant salary raise of 2% per year for both non-displaced and displaced workers.

As such, in this study, I aim to address this gap by using administrative data from the Singapore Ministry of Manpower, which was previously unavailable to researchers, to construct the age-wage profiles of workers with uninterrupted career paths across different income percentiles, and allow the movement from a higher to a lower income group when the job displacement occurs. Correspondingly, I assess the influence of unemployment-induced career breaks on retirement income adequacy under the current design of Singapore's national pension system, taking into consideration (i) the timing of the breaks, (ii) the duration of the breaks, (iii) and the severity of the impact that job losses might have on post-interruption wages.

This research also relates to the literature on retirement income adequacy. Even though there has been a standard conception that pension adequacy is determined by the degree of poverty eradication and consumption smoothing that a system can provide to pensioners (European Commission, 2003; Barr & Diamond, 2006; Draxler et al., 2009, Grech, 2013; Knoef et al., 2016), the majority of analyses involve only one dimension of consumption smoothing (Hui, 2012; Chia & Tsui, 2019). These studies often use the income replacement rate (IRR), identified as the ratio of pension payouts to pre-retirement earnings, to evaluate the extent to which these retirement benefits can sustain individuals' pre-retirement living standards. The problem is that using solely IRR may overstate the success of the social safety net for the lower-income group. Thus, following McCarthy et al. (2002), I employ an additional measure which is the subsistence replacement rate (SRR) - defined as the retirement wealth accumulated at a certain age in relation to the present value of future expenditures necessary to meet basic needs in retirement - to assess how well a national retirement income system can protect its pensioners against poverty risk.

To examine the effect of job displacement on pension adequacy of a median-income earner in Singapore, who joins the labour market in 2018, I devise a simulation model of earnings, social security contributions, asset accumulations and decumulations under Singapore's national pension system, as well as housing patterns over the life cycle. The findings suggest that due to the scarring effect of the unemployment spell on post-interruption wage profiles, the sooner the break occurs, the larger its impact on pension adequacy is. To be more specific, for a median-income female (male) worker, an early-career break, which happens after the first ten years of working, can bring about a fall of 60-90 (30-60) percentage points in the SRR; whereas postponing the break by ten years can reduce the impact by 30-50 (20-40) percentage points. Moreover, the results point out that median-income workers can support themselves well beyond subsistence in retirement, with an SRR of over 100% in all case scenarios. However, job losses do significantly decrease the retirement nest egg of displaced workers, with the level of impact ranging from SG$ 12,800 to SG$ 153,267 (in 2018 dollars). I also find that further extending the length of unemployment by six months can incur a reduction of at least SG$ 8,550 to SG$ 11,970 (in 2018 dollars) in the retirement nest egg at age 55.

These findings are of particular interest for several reasons. First, in contrast to earlier studies that focus on EU and OECD countries where public earnings-related pension systems are generally defined-benefit or point systems, this paper examines the impact of unemployment-related career breaks on pension adequacy within the context of fully-funded defined-contribution plans. Furthermore, over the past few decades, significant pension reforms around the world have brought about the increasing popularity of defined-contribution (DC) plans (Munnel, 2006; Dedry et al., 2017). Hence, looking specifically at how unemployment-related career breaks can impede retirement income adequacy under Singapore's national pension system– best known as one of the world's oldest and largest DC pension systems (McCarthy et al., 2002; Fong et al., 2011) – can offer a valuable lesson. Second, as the middle class is becoming less secure, in terms of opportunities, income, and jobs, worsening economic inequality in many societies, I seek to determine whether new policies are necessary to mitigate the risks from wage depression and unemployment for middle-income families, who have not been the target of social support in m countries around the world.

The rest of this paper is organised as follows. I first summarise the theoretical background and related literature about (i) the effect of career interruptions on pension benefits and (ii) the measurement of retirement income adequacy. Next, I provide an overview of Singapore's national pension system in Section 2.3. Section 2.4 presents the data and model specification as well as it includes a discussion on (i) earnings profiles of Singaporean workers with uninterrupted career paths; (ii) earnings profiles of workers with career paths punctuated by one or several periods without a job; (iii) CPF policy parameters; (iv) housing consumption; and (v) pension payout benefits under the CPF. Section 2.5 reports results on (i) the pension adequacy of a median-income earner with an uninterrupted career path under the CPF and (ii) the impact of unemployment-induced career breaks on pension adequacy. After that, I conduct a sensitivity analysis for the case of single-income families in Section 2.6. Section 2.7 concludes with some policy recommendations.

2.2. Theoretical Background

2.2.1. The effect of career interruptions on pension benefits and post-interruption wages

I would expect in a pension system where contributions are closely linked with benefits, the higher an individual's earnings and the more uninterrupted the career, the higher the pension benefits will be. Correspondingly, there are two ways by which unemployment-related career breaks can adversely affect pension entitlements. First, career breaks attributable to retrenchment, termination of fixed-term contracts can entail **contribution gaps** during individuals' working life, leading to reduced pension payouts if measures to plug those gap years are not available. Second, job displacement can induce **lower levels of post-break earnings**, and thus lower **levels of contribution during the working phase**.

Previous research that investigates the effect of unemployment spells on pension benefits often assumes that re-entry wages of displaced workers are the same as in their old positions (OECD, 2015; De Freitas et al., 2011; European Commission/Social Protection Committee, 2018). As such, their findings can only point out the impact of contribution gaps on pension benefits, which appears to be negligible. For instance, OECD (2015) shows that for every year without a job, on average, old-age pension drops slightly more than 1%. In line with OECD (2015), European Commission/Social Protection Committee (2018) highlights that the highest loss in the income replacement rate of a person who suffers three years without a job after ten years of working, as reported in Germany, Luxembourg, Slovakia, barely exceeds five percentage points. Among few exceptions that address the impact of post-displacement wages on pension entitlements, Bravo et al. (2017) indicate that under the case of an employment break with a minor wage penalty, the impact is marginal; however, when the earnings scarring effect becomes relevant, the impact increases substantially with a drop in the gross pension benefit of a displaced-worker relative to that of non-displaced workers ranging from 9.98% to 40.81%. However, their study fails to recognise the changing pattern of post-displacement wage growth rates by simply assuming a constant salary raise of 2% per year for both non-displaced and displaced workers.

A large body of empirical research that investigates wages and earnings following job displacement[14] shows that displaced workers often experience large and persistent earnings losses. For example, in assessing the earnings losses of displaced workers from Pennsylvania during the period from 1980 to 1986, Jacobson et al. (1993) find that in the event of being laid off, prime-age workers with six or more years of tenure suffer a reduction of around 25% of their pre-displacement earnings, and this loss does not fall significantly even after three years of separation from the previous employment. More recent studies also indicate that in the first year following the job displacement, earnings fall by 35% to 40%, and six years after the layoff event, losses still amount to 10% to 20% of pre-displacement earnings (Couch & Placzek, 2011; Jarosch, 2015; Jung

[14] Job displacement is defined as leaving work due to a plant or business closing or due to being laid off or fired (Stevens, 1997). Mass-layoffs refer to the case in which firms experience a substantial reduction in employment, as in Jacobson et al. (1993), a decrease of more than 30 percent in a firm's employment is considered a "substantial" decrease.

& Kuhn, 2018)[15]. In terms of the relationship between the tenure profile and earnings losses after job displacement, Topel (1990) finds that the dip in earnings and long-term losses generally rise with tenure. More specifically, von Wachter et al. (2009) point out that earnings losses of displaced workers with at least six years of tenure are larger than that of workers with at least three years of tenure, and losses are still substantial and last up 20 years even in the case of lower pre-displacement tenure. To date, specific human capital theory[16], job-matching theory[17], and information revelation theory[18] are the three key underlying theories that explain the empirical evidence on reduced earnings of displaced workers (Carrington & Fallick, 2017).

Previous literature has not always made a clear distinction between earnings losses and wage losses following job displacement. One notable exception is Stevens (1997). By examining separately the long-term wage and earnings losses of U.S. displaced workers in the period from 1968 to 1988, the author discovers that in the year of displacement, earnings reduce by roughly 25% while wages decrease by more than 12%, and over the five years after the displacement event, both wage and earnings are highly variable, but earnings appear to recover faster than wage does. A more recent study by Krolikowski (2017) shows that lost employment and reduced wages explain respectively 70% and 30% of the initial loss in annual earnings; however, over time, while the employment effect becomes less relevant, the opposite is true for the wage effect with losses in wages being responsible for roughly 75% of the earnings reduction in the fourth year after the job displacement.

[15] The size of earnings losses due to job displacement varies depending on the source of data – largely driven by the availability of a comparison group in the analysis, the examined period, or the studied location. For instance, Couch & Placzek (2011) use longitudinal administrative data on employees and firms in Connecticut from 1993 through 2004. Jarosch (2015) use German administrative data on wages, employment status, gender, and age of a person who has been employed and subject to social security in any point between 1974 and 2010. Jung & Kuhn (2018) investigate U.S. data from the monthly CPS files and the Occupational Mobility and Job Tenure supplements for the period from 1980 to 2007.

[16] Human capital is perceived as "acquired capabilities which are developed through formal or informal education at school and at home, and through training, experience and mobility in the labour market" (Mincer, 1984). Models of specific human capital (Becker,1962; Fallick, 1993; Mincer, 1962; Shaw, 1984) are based on the notion that specific on-the-job training or learning-by-doing generates human capital that might only valuable at the current firm or at a particular set of jobs; and being laid off when firms fail or scale down might leave the worker with only the general human capital, thus inducing lower levels of earnings in the next employment

[17] Models of job matching (Jovannovic, 1979, 1984; Mortensen & Pissarides,1994; Davis & von Wachter, 2011; Jung & Kuhn, 2018; Ortego-Marti, 2016; Krolikowski, 2017) reason that earnings fall upon displacement since workers were, on average, in high-quality job matches before being laid off.

[18] Information revelation theory builds on the idea that job displacement in the event of partial layoffs may reveal to future employers something unfavorable about the displaced workers' performance at their previous firms, lowering a future employer's estimate of the workers' productivity and ultimately their respective earnings in the new job (Gibbons & Katz, 1991; Kahn, 2013; Michaud, 2018).

In the search and matching model of Jung & Kuhn (2018), sources of earnings losses in the first year experiencing the layoff event are broken down into a wage loss effect, an increased job instability effect, and a selection effect which relates to the construction of the control (non-displaced workers) and treatment (displaced workers) groups within the available data sets. Their findings highlight that in the first year following the job displacement, losses in annual earnings are approximately 37% of earnings in previous employment, and six years after the displacement, earnings reduction still amount to 11% of pre-displacement earnings. Notably, the wage loss effect, which captures the difference in wages between control and layoff groups, is found to represent from 35% to 48% of the changes in annual earnings within the first six years after the job displacement.

To sum up, unemployment itself, in the event of either a mass layoff or not, carries an average wage penalty of about 13%-16% in the first year (Krolikowski, 2017; Michaud, 2018; Jung & Kuhn, 2018), falling to approximately 6% in the sixth year after job displacement. Unemployment duration adds a further penalty as longer unemployment spells may lead to greater loss of human capital[19] during the period without a job, inducing a larger wage loss in the next employment. In particular, Addison & Portugal (1989) illustrate that an increase of 10% in the length of unemployment lowers the wage on the subsequent job by 0.6%-0.8%. In line with the aforementioned paper, Cooper (2014) also confirms that workers who are unemployed for 26 weeks or more experience a larger negative income effect and have lower earnings even after 10 to 15 years compared to those with a shorter duration of unemployment.

2.2.2. Measurement of retirement income adequacy

Regarding the discussion about the retirement adequacy of a national pension system, it is necessary to define its fundamental goals. It has been commonly agreed in the previous literature that a national pension system has two primary objectives, including (i) poverty alleviation and (ii) consumption smoothing (European Commission, 2003; Barr & Diamond, 2006; Draxler et al., 2009, Grech, 2013; Knoef et al., 2016). In line with these goals, the adequacy level of a pension system is generally determined based on two approaches. The first approach is to set a social

[19] The clarification on how unemployment duration affects post-displacement earnings dates back to the theory of human capital depreciation initiated by Mincer & Polachek (1974). According to this theory, human capital built up on the job is one of the main determinants of individuals' wage rates. Discontinuities in the career employment in consequence of unemployment spells often cause a reduction in labour income since they imply a disruption in the accumulation of human capital as well as the depreciation of human capital stock acquired in the past.

standard for adequacy, which helps assess the poverty eradication goal. According to this approach, I can use two different metrics to evaluate the retirement system adequacy: (i) a wealth-based measure, which compares the retirement wealth accumulated at a certain age with the present value of future expenditures (McCarthy et al., 2002; Bajtelsmith & Rappaport, 2018); and (ii) an income-based measure, which compares retirement income with the poverty levels of income[20] (Haveman et al., 2007; Caminada et al., 2012).

The second approach traces back to the well-known life-cycle hypothesis, suggesting that people intend to smooth their consumption/standards of living over their lifetimes by accumulating when they work and dis-saving when they retire. Under this hypothesis, retirement income is considered adequate when it can replace a portion of a paycheck earned in the working phase so as to maintain individuals' pre-retirement living standards (Moore & Mitchell, 1997; Munnell & Soto, 2005; Moore et al., 2010; MacDonald et al., 2011). The most common measure underlying this logic is the income replacement rate, identified as the ratio of pension payouts to pre-retirement earnings. Broadly, the replacement rate literature determines the target income replacement rate or calculates the historical or projected income replacement rate of the target population. The computation of income replacement rates varies as there are different perspectives on the measurement period of pre-retirement earnings, as well as on what should be included in the numerator – the post-retirement income, and in the denominator – the pre-retirement income.

When used as a tool to assess the adequacy of a national pension system, the replacement rate should include pre-retirement earnings in the denominator and pension benefits in the numerator. Recently, several papers have highlighted that conventional studies on income replacement rates failed to recognise the benefits of homeownership in retirement. Biggs & Schieber (2014) argue that ignoring the advantage of being a homeowner in old age can lead to an overestimation of the level of income to be replaced, and thus, underestimate the retirement adequacy of the social security system in the US. Correspondingly, recent studies have incorporated "imputed rent"- the amount that homeowners would have to pay to earn a right to live in their house if they did not own it - into the calibration of both pre-retirement and post-retirement income (Munnell & Soto,

[20] Caminada et al., 2012 pointed out three ways of setting the poverty line: an absolute standard, a relative standard, and a subjective standard. For example, the US poverty line is based on an absolute yardstick, which remains constant in real terms over time while the EU-poverty line is set at a certain percentage of the median income in each country. The subjective poverty line is based on the opinions of respondents regarding what they consider an adequate standard of living.

2005; Munnell et al., 2006; Moore et al., 2010; MacDonald et al., 2011; Chia & Tsui, 2019). In terms of the measurement period of pre-retirement labour income, there are diverse views. While according to several papers, the level of income prior to retirement that pension benefits should replace is earnings in the final year of working (McCarthy et al., 2002; Hui, 2012); in other studies, the denominator is the average lifetime income (Chia & Tsui, 2019).

When it comes to the target optimal income replacement rate, Boskin & Shoven (1987) demonstrated that a replacement rate of less than unity is in agreement with the life-cycle hypothesis. Also, the literature on pension economics has widely accepted that the level of income necessary to maintain pre-retirement consumption should at least equal 70% of previous earnings (Schieber, 2004; McGill et al., 2005; Haveman et al., 2007; Knoef et al., 2016). Taking the measurement period of pre-retirement earnings into account, the World Bank recommended that for a middle-income earner, the net pension benefit should replace at least 53% of net earnings[21] in the final year of working and 78% of net average lifetime earnings (World Bank, 1994, p.294).

2.3. The Singapore's Central Provident Fund

Singapore's Central Provident Fund (CPF) is a national mandatory retirement savings scheme run by the central government. This national pension scheme operates on a fully-funded basis and is a defined-contribution pension plan. When first introduced in 1955, the CPF's main objective was to ensure the retirement income security of Singaporeans. Over time, its mission has developed into achieving multi-faceted goals. The CPF now aims at helping Singaporeans accumulate enough savings to (i) support retirement consumption, (ii) finance a home purchase, (iii) pay for health care as well as provide insurance protection. To this end, under the CPF system, employees, during the working phase, together with their employers, will have to set aside a percentage of gross monthly salary into three separate accounts, namely, Special Account (SA), Medisave Account (MA), and Ordinary Account (OA). While savings in SA are for retirement security, and thus, locked up until age 55, savings in OA can be withdrawn for pre-retirement needs and, most importantly, can be used for funding a house purchase.

[21] Taxes and savings are subtracted from the pre-retirement income that needs to be replaced as retirees no longer have to contribute to the national pension system or pay taxes on their employment earnings.

Before the age of 55, except for the uses identified above, CPF assets must be preserved. However, upon reaching 55 years old, active CPF members can withdraw a portion of their CPF savings after setting aside a required retirement sum into a Retirement Account (RA), which is then managed by the CPF Board. From 2015 onwards, in order to meet the required retirement sum, active CPF members who turn 55 can either set aside a full retirement sum (FRS) from the OA and SA; or set aside at least their basic retirement sum (BRS), which equals to one half of the FRS if they are the owner of at least one residential property. The FRS was SG$ 155000 in 2014, SG$ 161000 in 2015 and 2016, and SG$ 166000 in 2017. From 2017 to 2020, the FRS is set to increase by 3% per annum.

To protect pensioners against longevity risk given the rapid population ageing, since 2009, the Singapore government has introduced the CPF LIFE scheme- an annuity scheme that provides members with a lifetime monthly benefit starting from their payout eligible age, which is currently 65 years old. At present, there are three CPF LIFE plans: the LIFE Standard plan, LIFE Basic plan, and LIFE Escalating plan. Under the CPF LIFE Standard Plan, all the savings from RA will be converted into the annuity premium at the point of policy issuance, and the deducted amount combined with all interest earned on the premium will be poured into the Lifelong Income Fund and factored into the monthly payout benefit. In the LIFE Basic Plan, a portion - from 10% to 20%, depending on age and gender- of the RA savings will be set aside as the annuity premium to form the Lifelong Income fund, the remaining amount stays in the RA and provides a monthly benefit until reaching 90 years old. From 90 years old, members under the LIFE Basic plan receive monthly income from the Lifelong Income fund for as long as they live. In January 2018, the CPF LIFE Escalating Plan was initiated to provide the lifetime annuity payouts, which are fully indexed to inflation. This new plan is similar to the LIFE standard plan; however, it has lower initial payouts, which increase by 2% per year.

2.4. Data and Model Specification

In this study, I assess the impact of unemployment-induced career interruptions on the adequacy of the pension payouts under the CPF system for median-income earners in Singapore who joined the labour market in 2018. Regarding the earnings profiles of workers at different income percentiles, I use data from the Singapore Comprehensive Labour Force Survey, which were previously unavailable to researchers to (i) construct the uninterrupted lifetime age-wage profile, and then, together with data on Singapore's labour market from the Ministry of Manpower, (ii) devise several layers of labour income profiles for those who experience job displacement during their working phase. From the age-wage profiles of displaced and non-displaced workers, I next move to the accumulation phase of the retirement nest egg under the CPF by discussing the CPF policy parameters and housing consumption during the working period. I also cover the CPF payout phase in the section of retirement income under the CPF. Notably, in this study, all the numbers in absolute value are presented in 2018 dollars (real terms).

2.4.1. An uninterrupted lifetime age-wage profile

I assume that female workers participate in the labour force from the age of 23, while male workers start working at age 25[22]. Regardless of gender, workers retire at the age of 65. Two critical factors that affect the earnings profile of an entrant worker over the life cycle include (i) the starting wage when they enter the labour market and (ii) the wage growth rates during their working years.

In Singapore, data on the labour force market are managed by the Manpower Research and Statistics Department, Ministry of Manpower. Upon researchers' request, they provided us with unpublished data on the number of employed residents by gross monthly income from work (excluding employer CPF), age-groups, and gender for the period of 2008-2018. I use these data to first find the gross monthly income of male/female workers at 30th, 50th and 70th income percentiles for eleven age groups (15-24, 25-29, 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, 60-64, 65-69 and 70+) from 2008 to 2018. The results from the previous step are then used to (i) calibrate the starting wages of male/female workers and (ii) compute the real wage growth rates of male and female workers at different income percentiles over time.

Regarding the starting wages of male and female workers in 2018, I derive the calibration from the labour income (excluding employer CPF) at 15^{th}- 50^{th} income percentiles for three age groups, namely, 15-24, 25-29, and 30-34[23]. I assume the midpoint of a particular age group to be a representative point. Consequently, the salaries at different income percentiles of workers aged 20, 27, and 32 are equal to those of the group aged 15-24, 25-29, and 30-34 correspondingly. I also suppose that wages increase with age at a constant rate from age 20 to 32, and then use that flat

[22] Due to mandatory military service, generally, male Singaporeans enter the labour market two years later than their female counterparts.
[23] I need the gross monthly income of three age-groups 15-24, 25-29, and 30-34 to avoid over-/under-estimating the percentage difference in wages of workers aged between 23 and 27.

rate to interpolate wages of males at 25 and females at 23 from wages of male/female workers at age 27. Table 2.1 presents the starting wages of entrant male and female workers in 2018.

Table 2.1: Starting Wages of Entrant Workers (in 2018 dollars)

Percentiles	Male (Age 25)	Female (Age 23)
15P	$1,588	$1,430
30P	$2,121	$2,005
50P	$2,784	$2,687

Source: Author's calculation

When it comes to the estimation of wage growth rates over the life cycle, the literature on earnings replacement rates has devised two different approaches. While some studies use a deterministic projection to construct age-earnings profiles, assuming either a constant wage growth rate over the life cycle (McCarthy et al., 2002; Hui, 2012)[24] or time-varying wage growth rates (Chia & Tsui, 2019)[25], other works rely on a stochastic approach to model the earnings path over the working life (Bosworth et al., 2000; MacDonald & Moore, 2011; Biggs et al., 2015). Pre-retirement income forecasts based on stochastic models require a proper panel dataset in which the earnings record of each employed resident is tracked over the years. Unfortunately, such micro-panel data sets are not available in Singapore. Hence, I choose the deterministic approach with time-varying real wage growth rates to model pre-retirement income profiles.

To determine the annualised real wage growth rate for employed residents, I first rescale the gross monthly income series in the period of 2008-2018 to the same reference year (2018) by adjusting the series for the 10-year annualised inflation rate using the MAS core inflation measure[26] from 2008 to 2018. I then find the gross monthly income at each income percentile for male and female

[24] McCarthy et al. (2002) assume a constant annual real earnings growth of 2%. Hui (2012) also assume constant annual wage growth rates of 4.1%, 5.2%, and 5.9% respectively for three different levels of education, namely, Secondary and Lower Secondary, Post-Secondary, and Tertiary.

[25] Chia & Tsui (2019) use historical data on the Singapore Labour Force from 2001 to 2011 provided by the Ministry of Manpower to compute the real-wage growth paths of workers.

[26] The CPI core inflation measure by the Monetary Authority of Singapore (MAS) excludes the components of "Accommodation" and "Private Road Transport." The MAS Staff Paper in August 2011, conducted by Ong et al. (2011), points out two reasons why "accommodation" and "private transport" should be excluded from the calculation of core inflation. First, they are volatile and significantly influenced by administrative policies. Second, changes in the cost of these two items do not necessarily equal changes in cash expenditures for most resident households in Singapore, taking into account the high homeownership rate and long intervals between new car purchases for a typical resident.

workers aged 20-24 as the time interval in the youngest age group is currently not matched with the other remaining age groups.

Next, the goal is to calibrate the annual wage growth rate from age 25 (23) to age 64. As an illustration, I describe the method to identify the real wage growth rate of a worker aged 25-29 for each income percentile level. The basic idea is that cohort A, which comprises all workers aged 20-24 in 2008, will become the group aged 25-29 in 2013 and then turn into the group aged 30-34 five years later. Following this logic, I set the average annual income growth rate from age 25 to age 29 in cohort A equal to the annualised income growth rate of this cohort over a 10-year interval so that I do not over-estimate or under-estimate the real wage growth. To come up with the wage growth rate for a worker from the age 25 to age 29, given that he/she was 25/23[27] in 2018, I even out the cohort effect by averaging the annualised income growth rate among three cohorts: Cohort A-5, Cohort A, and Cohort A+5[28]. The same process is repeated for the other five-year age bands, including 30-34, 35-39, 40-44, 45-49, 50-54, 55-59, and 60-64.

With (i) starting wages of entrant male and female workers at 15th -50th income percentiles in 2018, and (ii) the annual real wage growth rate for each income percentile from age 25 (23) to age 64, I map out the age-earnings profile of workers over their working phase. Figures 2.1a and 2.1b illustrate the resulting earnings paths of male and female workers at different income percentiles. These earnings paths generally follow a hump-shaped pattern where real wage growth is faster when workers are young, reaches a peak around the mid-career phase, and gradually decreases when they approach retirement. The justification for the hump-shaped age-wage profile traces back to the human capital model[29] (Becker, 1962; Mincer, 1974). It follows that at the beginning of the working phase, workers aim at accumulating their skills and knowledge so as to increase their productivity, and consequently, their earnings prospects. Whereas, as workers approach retirement, their employment preferences change, leading to reduced efforts in upgrading skills or even reduced work hours before fully retiring (Quinn et al., 2019). Due to this changing pattern in

---

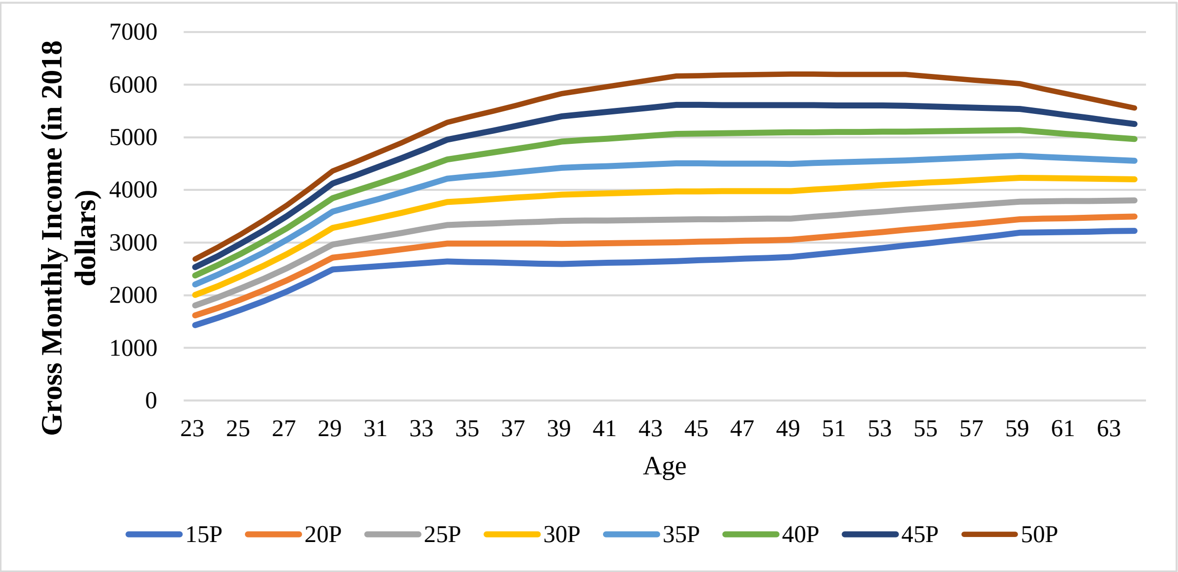
[27] One thing that should be noted from the case of female workers is that I set the real wage growth rate from age 23 to age 24 equal to the annualised income growth rate from age 25 to age 29.

[28] Cohort A-5 (A+5) contains all employed-residents who are 5 years younger (older) than Cohort A. The information on the gross monthly income of Cohort A-5 when they reach 30-34 years old is not available, and at the same time, I also lack the income data for Cohort A+5 when they were 20-24 years old. Taking the average of the annualised income growth rate among three cohorts may also help avoid over/under-estimating the real wage growth rate of a worker from age 25 to age 29.

[29] Human capital is perceived as “acquired capabilities which are developed through formal or informal education at school and at home, and through training, experience and mobility in the labour market” (Mincer, 1984)

mid- or late-career employment, the real income may grow at a decreasing rate during the mid-career phase or even fall for near-retirement individuals.

(a)



(b)

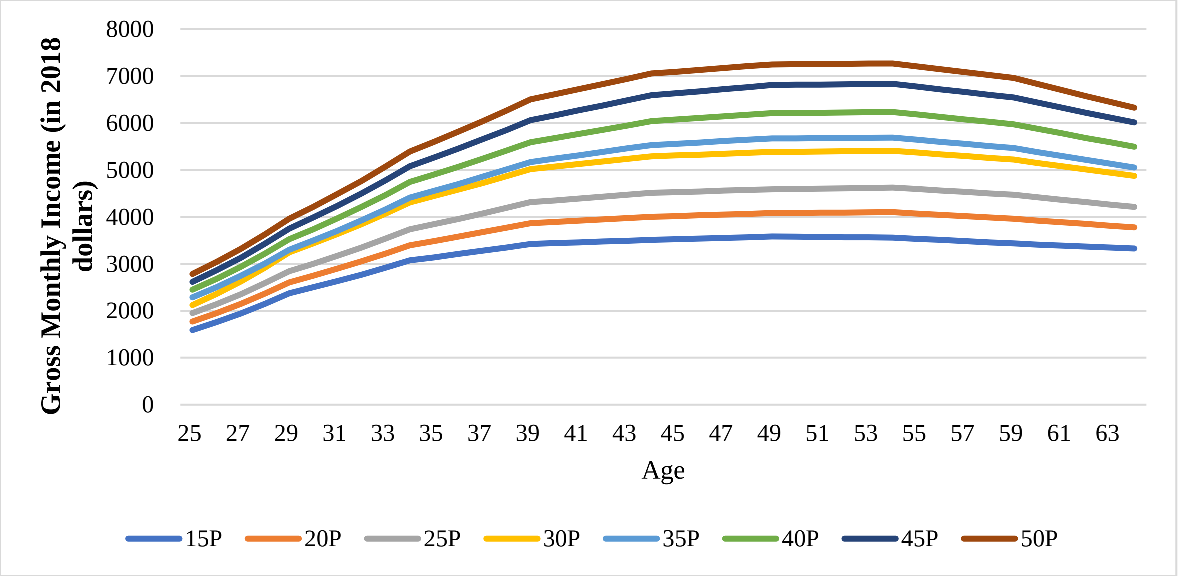


Figure 2.1. (a) Age-earnings profile of male workers at 15th-50th income percentile. (b) Age-earnings profile of female workers 15th- 50th income percentile.

Source: Author's calculations

For the reason that the age-earnings profile is hump-shaped, and people tend to smooth their consumption over the lifetime (Modigliani & Brumberg (1954), Ando & Modigliani (1963)), total savings are also hump-shaped, meaning that during the early stage of the working phase, more savings are accumulated, and will gradually decrease as people approach retirement. In the baseline case, I assume that workers stay in their respective income percentiles over the life cycle. In cases where workers experience job displacement during their working phase, I allow movements to lower-income percentiles.

2.4.2. Career paths punctuated by periods of job displacement

The rise of highly advanced automation, such as AI- and robotics-based automation, is expected to replace jobs of not only low-skilled labourers but also traditionally stable professions with predictable and repetitive tasks, for instance, lawyers, accountants, and even doctors. In Singapore, profiles of retrenched workers in recent years indicate a significant increase in the share of the retrenched who previously had professional, managerial, executive, and technician (PMET) jobs[30] (from 37% in 2008 to nearly 70% in 2018). Three possible clarifications for this trend are: (i) the increasing proportion of PMETs in the labour force[31] (from 50.9% in 2008or to 56.9% in 2018) as a result of the shift in Singapore's economic growth strategy towards a high-tech value-added manufacturing and rapid development of the services industry; (ii) the scaling down of businesses in sunset industries, such as traditional banking, where most PMETs are employed; and (iii) the lack of skills required by new jobs in sunrise industries among unemployed PMETs. Accordingly, those retrenched PMETs might find it increasingly difficult to obtain jobs in the same industry and may have to take up inferior jobs. While inherently, in this new age of automation, the worst effects are on PMETs, which belong to the middle-class, there has not been any social safety net designed to protect those middle-income earners from labour market risks[32].

To evaluate the effect of career interruptions due to unemployment spells on pension adequacy under the CPF, I consider several dimensions of unemployment-related career breaks, including (i) the timing of the break; (ii) the duration of the break; and (iii) the degree at which the real wage

[30] MOM, Labour Market Statistical Information, Table: Retrenchment, 2018, Retrenched Employees by occupational group, industry, and reasons for retrenchment https://stats.mom.gov.sg/Pages/Retrenchment-Tables2018.aspx
[31] MOM, Labour Market Statistical Information, Time Series Table: Employment, Employed Residents by Occupation and Sex https://stats.mom.gov.sg/Pages/EmploymentTimeSeries.aspx
[32] Their income is not low enough to qualify for Workfare Income Supplement Scheme (WIS) and the Silver Support Scheme (SS), whose targets are those at the bottom 20th percentile of income earners.

is affected after the break. First, in such a rapidly changing technological world, job displacement can happen at any point during the working phase. It has been evident in the 2018 Singapore labour market report that among those retrenched in the fourth quarter of 2018, 34% are from the group aged 40-49, 33.6% are from the group aged 50-59, and 23.6% are from those aged 30-39. This study, hence, sets three different timing scenarios of unemployment spells: (i) at the beginning of employment path – after the first ten years of working; (ii) in the middle of the employment path – after twenty years of working; and (iii) at the end of career path – after thirty years of working.

Second, regarding the **duration of unemployment-related career breaks**, previous studies, which focus on OECD and EU countries, often assume a long duration from one to even ten years out of work. Nevertheless, it is unrealistic to model a career break of multiple years in the case of Singapore. Unlike most OECD countries, Singapore does not implement any unemployment benefits schemes dedicated to supporting the unemployed since the government considers the best way to assist retrenched or unemployed individuals is to assist them to rejoin the labour market as soon as possible instead of directly handing out financial support. Statistics on unemployment in 2018 by the Singapore Ministry of Manpower (MOM)[33] highlight that the median duration of unemployment is 11, 10, and 13 weeks, respectively, for three age groups, namely 30-39, 40-49, and 50 & over. In the baseline analysis, I choose six months[34] as the duration of an unemployment-related break since (i) six months reflect better the risk that employees have to encounter in the next few decades, and (ii) in Singapore, the duration of six months being unemployed is a benchmark for long-term unemployment. I also assume that six months without a job do not bring about the depreciation in human capital acquired in the past.

Lastly, taking into account the severity of job displacement's effect on the post-interruption wage, I consider two alternative cases: (i) a reduction of 10% - and (ii) a reduction of 20% - in wages after displacement. The lowest level of wage penalty after unemployment-related career breaks – 10% - stems from the Singapore Government guidelines on Variable Wage Components. According to the guidelines, the Ministry of Manpower suggests that employers should implement

[33] For details, see Table 87 https://stats.mom.gov.sg/Pages/Unemployment-Tables2018.aspx

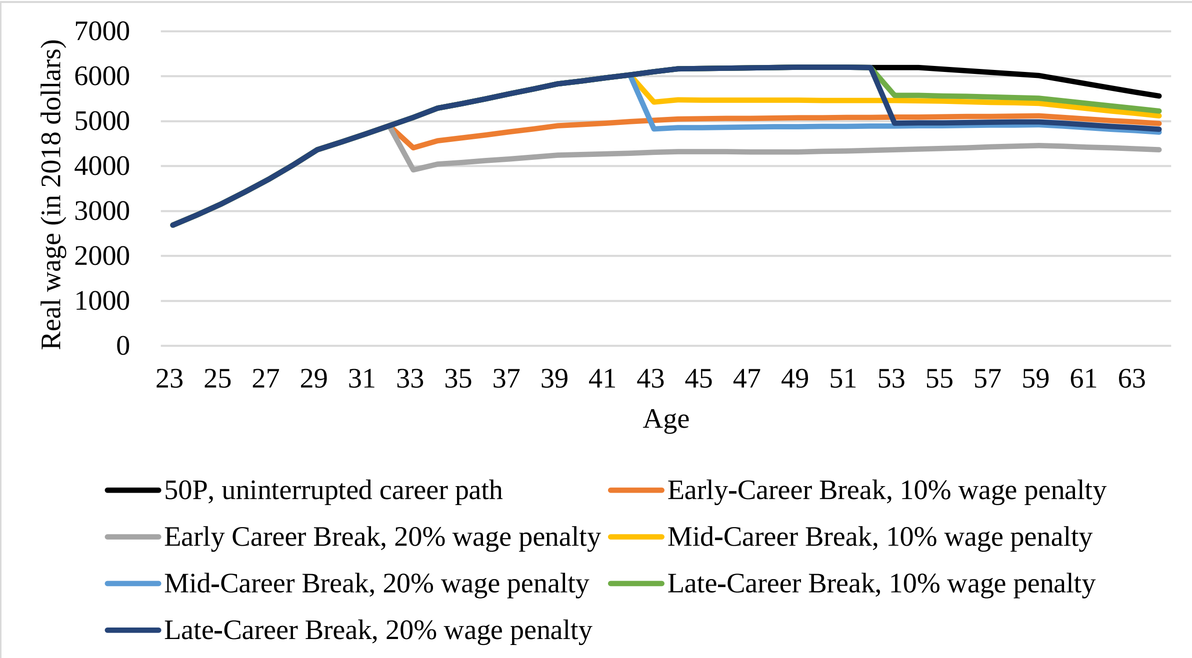
[34] The proportion of retrenched PMETs who re-enter the labour force within the six months after job displacement has decreased over the past few years from 67.1% in 2015 to 59.5% in 2018 (see https://stats.mom.gov.sg/Pages/ReEntryIntoEmploymentTimeSeries.aspx). According to a recent update of MOM on the number of retrenched employees by occupational group, there were almost seven thousand of PMETs losing their jobs in 2018. Hence, a large number of PMETs remained jobless even after six months.

a monthly variable component (MVC) in employees' monthly wages so that businesses can bring down wage costs immediately when encountering short-term economic shocks or changes in the business environment. The recommended MVC level is 10% of the monthly salary[35]. The fact that a company has to cut parts of its employment instead of adjusting the MVC in bad times can serve as a source of information for new employers when negotiating wages of displaced workers (Gibbons & Katz, 1991; Kahn, 2013; Michaud, 2014). For that reason, I use a 10% reduction in wages after job displacement as the lower limit of the unemployment impact on wages. The upper limit of 20% wage penalty comes from the previous empirical research on wages and earnings following a job displacement, which finds an average decline of 13%-16% in wage post-interruption.

Combining the timing and the duration of unemployment spells as well as different levels of wage penalty after job displacement, I come up with age-wage profiles of fragmented career paths. Figure 2.2a (b) compares age-real wage profiles of median-income female (male) workers who experience one unemployment episode at different points in time during the career path with that of non-displaced median-income employees. As the age-earnings profiles are hump-shaped, real wage grows at a faster rate during the early stage of the employment path than in the mid- or late-career phase. Correspondingly, the earlier the career break occurs, the larger the impact it has on the employees' earnings paths. For instance, while being displaced after the first ten years in the workforce, individuals move from 50th to 40th income percentile in case of 10% wage penalty, or from 50th to 30th percentile in case of 20% wage penalty; whereas breaks in mid-career and late-career stages only lead to a fall from 50th to 45th income percentile in case of 10% reduction in wages, or from 50th to 40th percentile in case of 20% reduction in wages.

[35] For details, see https://www.mom.gov.sg/~/media/mom/documents/statistics-publications/flexible-wages-for-smes-english.pdf

(a)



(b)

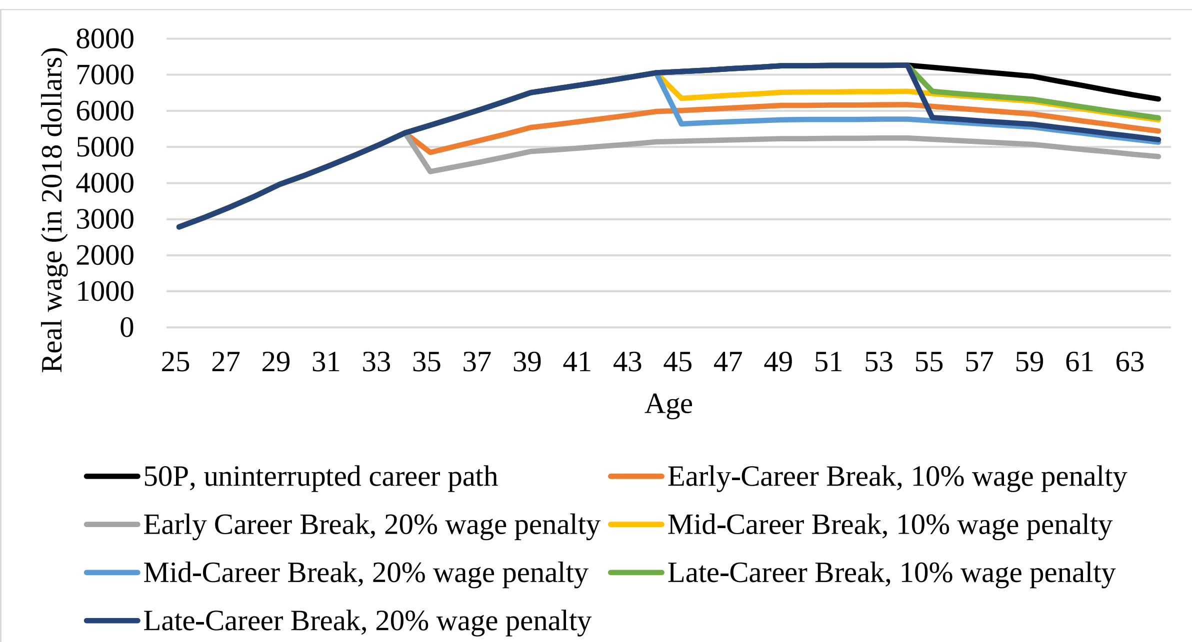


Figure 2.2: (a) Age-wage profiles of female workers - uninterrupted vs interrupted career paths with one unemployment episode. (b) Age-wage profiles of male workers - uninterrupted vs interrupted career paths with one unemployment episode.

Source: Author's Calculation

2.4.3. CPF policy parameters

Current pension policies as of December 2019 and the Government's plans to reform the pension system in the next ten years are taken into account to calibrate CPF parameters used in the analysis.

According to data on the currently effective CPF contribution and allocation rates[36], an employee aged 55 or younger must contribute 20% of their monthly salary, together with their employer, to form a total contribution of 37% of the monthly wage into three CPF accounts. For older groups, these rates decline: (i) for the group aged 55 to 60, the total contribution reduces to 26% with equal shares between employee and employers; (ii) for the group aged 60 to 65, the total contribution accounts for 16.5% with 9% coming from employers; (iii) for the group aged 65 and over, the overall contribution is 12.5% with 7.5% from employers. Recently, at the 2019 National Day Rally, Prime Minister Lee Hsien Loong announced plans to increase the contribution rates for workers aged between 55 and 70. As a result, by 2030, the total contribution rate will go up to 37% and 26%, respectively, for the groups aged 55-60 and 60-65. Next, this total contribution from employee and employer is allocated to OA, SA, and MA, as shown in **Table 2.2**.

Table 2.2: Rates of contribution and allocation by 2030

Employee Age (Years)	Contribution (% of wage)		Total contribution (% of wage)	% Total contribution credited to		
	Employer	Employee		OA	SA	MA
35 & below	17	20	37	23	6	8
Above 35-45	17	20	37	21	7	9
Above 45-50	17	20	37	19	8	10
Above 50-55	17	20	37	15	11.5	10.5
Above 55-60	17	20	37	15	11.5	10.5
Above 60-65	13	13	26	12	3.5	10.5

Source: CPF website at https://www.cpf.gov.sg. All the rates above are for monthly wages of SG$ 750 and above.

In general, while allocations into OA decrease with age, allocations into SA and MA become larger over time, requiring people to save more for retirement and medical expenses when they get older. All these rates apply to wages up to an ordinary salary ceiling of SG$ 6000[37] per month, and the

[36] For details, see https://www.cpf.gov.sg/Employers/EmployerGuides/employer-guides/paying-cpf-contributions/cpf-contribution-and-allocation-rates

[37] I assume the Ordinary Salary Ceiling to remain constant in real terms over time

additional wage ceiling is determined by subtracting the total ordinary wages subject to CPF from SG$ 102,000[38].

There are two ways by which CPF members can grow their accounts. First, they can choose to invest their OA and SA savings under the CPF Investment Scheme. Second, they can earn default interest on their OA, SA, and MA balances. In this study, I focus on the second mechanism. Currently, CPF members aged 55 and younger can earn default interest rates of up to 3.5% per annum on their OA fund and up to 5% per annum on their SA and MA balances, including an extra interest of 1% per annum on the first SG$ 60000 of the combined OA and SA balances (with up to SG$ 20000 from the OA). On turning age 55, members can enjoy an additional 1% extra interest on the first SG$ 30000 of the combined OA and SA balances (with up to SG$ 20000 from the OA). Since all the numbers in simulation models are presented in real terms, I need to adjust the nominal interest amount for inflation. Using data on the core inflation rates from 1998 to 2018 published by the MAS, I set the average long-term core inflation equal to 1.57% per year.

2.4.4. Housing consumption

In regards to Singapore's housing system, there are two distinctive characteristics. First, while in most countries, public housing targets the low-income class, in Singapore, it is an option for almost all socio-economic classes. The Singapore government ensures housing affordability for a substantial proportion of the society by providing a generous subsidy in the form of the reduced sale price of HDB (Housing and Development Board) flats – government-built apartments sold on a 99-year lease agreement - as well as additional housing grants after taking into account the household income level. Second, the purchase of an HDB flat can be financed through the mandatory savings system – the CPF.

Depending on the budget and housing needs, households can choose to buy an HDB flat from a wide range of sizes and locations. Normally, HDB flats are categorised by the number of rooms with more rooms indicating a larger flat in size[39]. In Singapore, households can buy an HDB flat from the resale market or via the Build-to-Order scheme (BTO). A flat under the BTO scheme is subsidised; hence, its price is often significantly lower than the price of an equivalent flat in the

[38] I assume the Additional Wage Ceiling to remain constant in real terms over time

[39] The number of rooms includes the number of living rooms and bedrooms. For instance, a three-room HDB flat would have two bedrooms and one living room; a four-room HDB flat can have three bedrooms and one living room, and a five-room flat has three bedrooms plus two additional rooms (living and dining rooms)

resale market. However, citizens can only get a BTO flat as a couple or a family. I assume that married couples purchase HDB flats under the BTO scheme. The representative male worker is set to get married at age 30 with a female partner of the same income percentile who is 28 years old. I consider the purchase of a four-room (five-room) HDB flat when the couple is married. I use the average prices of new HDB flats with different sizes in 2018[40] to proxy for HDB flat prices in the model. Thereby, the prices used are SG$ 305,000 for a four-room BTO flat and SG$ 401,000 for a five-room BTO flat. Since an entrant worker is assumed to buy a BTO flat after five years of working, I adjust the house prices in 2018 dollars for real house price appreciation over the five years. Based on the price range of new flats in HDB Annual Reports from 2016 to 2018, I obtain a real annualised growth rate of 2.36%[41]. In this analysis, the real housing appreciation rate is set at a lower level of 2% per year to reflect the Government's intention of keeping the house price appreciation moderate.

Starting from September 2019, households buying new HDB flats for the first time can enjoy additional government supports through the Enhanced Housing Grants (EHG). Different household income ranges will be eligible for different levels of grants (Table 2.3). With an average household income level of approximately SG$ 8240 per month, the representative household qualifies for a grant of SG$ 10,000. At the time of signing the lease agreement contract, they each use their accumulated OA balance to make the down payment. The net amount after subtracting the down payment and the housing grant from the house price is then financed using HDB loans, which are normally a 25-year fixed-rate mortgage. The mortgage-financing rate is 2.6% per annum. The monthly mortgage will be deducted from the Ordinary Account and shared equally between husband and wife. I adjust the nominal monthly mortgages for a long-term inflation rate of 1.57% per annum. The real monthly mortgages are then discounted to their respective value at the time of house purchasing before averaging over the loan period.

[40] The price range of BTO flats for fiscal year 2018/2019 is reported in the HDB Annual Reports 2018/2019. https://services2.hdb.gov.sg/ebook/AR2019-keystats/html5/index.html?&locale=ENG&pn=15
[41] The nominal growth rate of 3.93% per year is adjusted for the core inflation rate of 1.57% per year (2016-2018)

Table 2.3: Enhanced Housing Grant for flat applications received from September 2019

Average Monthly Household Income* Over 12 Months	EHG
Up to $1,500	80,000
$1,501 to 2,000	75,000
$2,001 to 2,500	70,000
$2,501 to 3,000	65,000
$3,001 to 3,500	60,000
$3,501 to 4,000	55,000
$4,001 to 4,500	50,000
$4,501 to 5,000	45,000
$5,001 to 5,500	40,000
$5,501 to 6,000	35,000
$6,001 to 6,500	30,000
$6,501 to 7,000	25,000
$7,001 to 7,500	20,000
$7,501 to 8,000	15,000
$8,001 to 8,500	10,000
$8,501 to 9000	5,000

Source: HDB website at https://www.hdb.gov.sg

Table 2.4 tabulates house prices, housing loans and the average real monthly housing mortgage in two different scenarios: buying a four-room BTO flat (4R) versus buying a five-room BTO flat (5R). The last column in Table 2.4 shows the average monthly mortgage payment as a percentage of household income at the time of purchase. For median-income couples, the real monthly mortgage accounts for 8.09% and 12.46% of their monthly income, respectively, for the purchase of a four-room BTO flat and a five-room BTO flat. This periodic housing cost burden in Singapore is much lower than that in most OECD countries. For example, the mortgage burden of middle-income families usually accounts for over 15% of their disposable income in countries such as the US (20%), the UK (17.5%), Germany (16%), Netherlands (15.5%), Australia (23%), and Canada (16%)[42]. On top of that, in Singapore, the housing mortgage burden as a share of income after tax

[42] Housing Costs over Income, OECD Affordable Housing Database- https://www.oecd.org/els/family/HC1-2-Housing-costs-over-income.pdf

and CPF contribution is lower than the mandatory portion of monthly salary deposited into the Ordinary Account, meaning that the CPF contributions in the OA can fully cover the monthly mortgage payments.

Table 2.4: Housing consumption and financing

Percentiles	Housing Types	Price of Housing (2023)	Housing Grants	Housing Loan	Combined Income at the time of purchase	Average real monthly housing mortgage	Monthly mortgage as % of combined income
50P	4R	336,745	10,000	201,948	8,238	666	8.09%
50P	5R	442,736	10,000	307,939	8,238	1,026	12.46%

Source: Author's calculation. All absolute numbers are in 2018 dollars.

2.4.5. Retirement income under the CPF

When it comes to the decumulation phase under CPF, although members can choose to withdraw part of their savings at age 55, they cannot cash out the entire amount accumulated in their OA and SA balances. Particularly, they have to set aside a required sum from the combined OA and SA balances to form their Retirement Account (RA), which will then be locked and managed by the CPF Board until their payout eligible age.

Upon reaching age 65, which is the current payout eligible age, Singapore citizens or permanent residents born in 1958 or after, who have at least SG$ 60,000 in their RA six months before age 65, will automatically be enrolled in the CPF LIFE scheme. Prior to 2018, there were two plans in effect: the CPF LIFE Standard plan and the CPF LIFE Basic plan. Despite hedging individuals against longevity risk, they still leave inflation risk untouched. In January 2018, the Government initiated the CPF LIFE Escalating Plan to provide lifetime annuity payouts, which are fully indexed to inflation. In other words, under the Escalating Plan, annuity payouts remain unchanged in real terms. As real annuity is more consistent with the goal of smoothing real consumption (Brady, 2010), I assume in this study that at the age of 65, individuals choose the CPF LIFE Escalating Plan.

In the simulation model, the representative male and female workers will not withdraw any part of their CPF savings except for housing financing purposes. Hence, all the CPF monies in OA and SA net of withdrawals for housing will be fully annuitised under the CPF LIFE Escalating Plan.

Premiums for this real annuity are paid in two tranches, with one starting from the age 55 via the set up of RA and one starting ten years later via the new contributions from age 55 to age 64. Since after the age of 55, monies in OA, SA, MA, and RA can earn the same nominal default rate of 4%[43], I leave the amount excess of Full Retirement Sum, which equals SG$ 171,000 in 2018, in the RA.

Monthly annuity payouts are determined from the combined OA, SA and RA balances at the beginning of age 65. The first payout begins at age 65. While the formulae to calculate the CPF LIFE payouts are not publicly available, the CPF Board does provide the CPF LIFE Estimator[44] tool to estimate the monthly payouts under three CPF LIFE plans. The Board also specifies some of the parameters used in their computation. For instance, the discount rates in the valuation model range from 3.75% to 4.25% per annum, and the life expectancy for males is shorter than that of females. Based on these clues, I calculate the real annuity payout under the CPF Escalating Plan using the following formula:

$$PV_{65} = \sum_{t=65}^{n} \frac{a*(1+g)^{t-65}}{(1+r)^{t-65}}$$

where PV_{65} is the total OA, SA and RA balances at the beginning of age 65, r is the discount rate, and **a** is real annuity payout during retirement. g indexes the annual growth rate that the CPF Board uses to adjust the real annuity payout to keep up with inflation each year. This rate is currently set at 2% per annum. n represents the maximum age that an individual could reach. I set the maximum age equal to 100 and 97 years old, respectively, for female and male retirees. Also, the discount rate used in this valuation model is assumed to be 4% per annum, which is the middle point in the range of discount rates used by the CPF Board's Estimator.

Table 2.5 presents a comparison[45] between the computed real annuity payouts and the results from the CPF LIFE Estimator. For the most part, the real annuity payouts from the calculations match quite well the upper limit of the initial monthly payouts at age 65 found by the CPF Board's

[43] the Special, MediSave and Retirement Account (***SMRA***) ***rate***

[44] For details, see https://www.cpf.gov.sg/eSvc/Web/Schemes/LifeEstimator/LifeEstimator

[45] As under the Escalating Plan, pension payouts will basically remain constant in real value during retirement, it is reasonable to compare the initial monthly payout at 65 (in 2018 dollars) estimated by the CPF Board with the real annuity payout calculated using the aforementioned valuation formula.

Estimator under the Escalating Plan. This is because the CPF LIFE Estimator does not account for the contributions to the CPF from age 55 to age 64.

Table 2.5: Real annuity payouts under the CPF LIFE Escalating Plan

		Accumulated OA and SA balance at 55	Total Premiums at 65	CPF LIFE Estimator-Escalating Plan Initial monthly payout at 65	Real annuity payout - Author's Calculation
Female	5 room	$459,504	$803,422	$2346-$2636	$2678
	4 room	$527,013	$889,279	$2745-$3081	$2964
Male	5 room	$432,923	$766,428	$2434-$2723	$2721
	4 room	$497,257	$850,674	$2781-$3112	$3021

Source: Author's Calculation. Note that all numbers in absolute value are presented in 2018 dollars.

2.5. Results and Analysis

When it comes to the measurement of pension adequacy under the CPF, previous studies generally use the income replacement rate (IRR), a ratio of retirement income to pre-retirement earnings (Hui, 2012; Chia & Tsui, 2019). However, using solely IRR may overstate the success of the social safety net. We, therefore, assess the retirement adequacy of Singapore's CPF employing two indicators: (i) IRR, (ii) Subsistence Replacement Rate[46]- a ratio which compares the combined OA and SA balances at the age of 55 with Full Retirement Sum (in 2018 dollars).

Regarding the computation of IRR, several authors reason that retirement income needs are not to replace the entire or gross pre-retirement income since median-income retirees generally reduce their spending in retirement (Hurd et al.,2013; Velarde et al., 2014), and do not have to pay taxes on pension income as well as contribute to the earnings-related pension systems (Brady, 2010; MacDonald and More, 2011; Purcell, 2012). In agreement with this rationale, I only focus on the net replacement rate, defined as a percentage of pre-retirement income net of taxes and social security contributions that retirement income can replace. The average retirement income in the numerator of the net replacement rate is the real monthly payout under the CPF LIFE Escalating Plan. For the choice of denominator, there are still diverse views on whether it should include earnings in the last working year, peak earnings, or average lifetime earnings (Biggs & Springstead, 2008; Chia & Tsui, 2019). To better evaluate retirement adequacy, instead of

[46] McCarthy et al. (2002) first termed this adequacy criterion as "Subsistence Replacement Rate".

replacing income in a particular age or a particular period, I use the average lifetime income as the denominator. On top of that, following Chia & Tsui (2019), taking into account the high homeownership rate[47] in Singapore and net benefits of owning a home in retirement, I incorporate net imputed rent pre-retirement and post-retirement in the calibration of pre-retirement and post-retirement earnings. In essence, net imputed rent is determined by subtracting the costs of homeownership from the subletting rental rate. Chia & Tsui (2019) demonstrate that homeownership comes at the costs of maintenance and mortgage financing, yet, they fail to recognise the forgone interest on the down payment. I address this gap in this analysis.

In addition to IRR, it is also important to look at how well a pension system can protect its residents from poverty. Full Retirement Sum has been set by the Government to approximately equal the amount needed to buy a subsistence-level single life annuity. It, thus, can serve as a benchmark to assess the retirement income adequacy under CPF. Official statistics in the CPF Annual Reports from 2015 to 2018 show that over 40% of active members at age 55 were unable to meet the Retirement Sum. Although housing over-consumption and housing financing via mandatory CPF savings have been identified as major culprits (Koh et al., 2008; Hui, 2012; Chia & Tsui, 2019), in this study, I look at how unemployment-induced career breaks during the working life can harm the possibility of reaching the Full Retirement Sum at age 55.

2.5.1. Retirement Adequacy of median-income earners with an uninterrupted career path

In Singapore, Full Retirement Sum is a threshold below which income in retirement would be inadequate for subsistence. In fact, the Full Retirement Sum of SG$ 171,000 in 2018 dollars can only provide a real monthly payout of about SG$ 928- SG$ 1038 for a single female elderly, and SG$ 1020- SG$ 1135 for a single male elderly. As representative agents keep all the CPF savings in OA, SA, and RA without withdrawing any amount except for housing financing purposes, in retirement, they can expect a real monthly payout of SG$ 2678 - SG$ 2964 for females and SG$ 2721- SG$ 3021 for males depending on the housing consumption (Table 2.5). These figures are significantly larger than the subsistence level of retirement income based on the Full Retirement Sum. Notably, as depicted in Table 6, even when a couple decides to buy a five-room BTO flat, resulting in an HDB loan of over SG$ 300,000 at the time of purchase, their real CPF LIFE payouts

[47] Department of Statistics (2018), Homeownership rate of resident households was 91% in 2018

during retirement are at least 2.5 times larger than the subsistence level (269% and 253% respectively for females and males). Choosing the smaller house – a four-room BTO flat- can further increase the subsistence replacement rate to approximately 300% for both husband and wife.

Table 2.6: Subsistence Replacement Rate of non-displaced median-income workers

	Flat types	Accumulated OA, SA balance at age 55	FRS at age 55	Subsistence Replacement Rate
Female	5 room	459,504	171,000	269%
	4 room	527,013	171,000	308%
Male	5 room	432,923	171,000	253%
	4 room	497,257	171,000	291%

Source: Author's Calculation

With the accumulated balances from OA and SA at age 55 far exceeding the Full Retirement Sum, median-income workers, regardless of gender, appear to support themselves well above the subsistence level. Nevertheless, to evaluate the success of a pension system, I should also assess another adequacy goal, which is smoothening real consumption. Hence, I compute the net income replacement rate of a median-income earner with an uninterrupted career path to gauge how well the retirement income under the CPF LIFE Escalating plan can replace the pre-retirement income net of taxes and CPF contributions. The results are then displayed in Table 2.6.

To calculate real annual earnings before retirement, I deduct (i) personal income taxes and (ii) the annual CPF contributions based on both ordinary wages and annual bonus which is set equal to one month salary, from pre-retirement gross earnings. I use the current personal income tax rules[48] to calibrate the income tax payable. It is important to note that CPF LIFE payouts are not taxable. **Table 2.7** summarises the average monthly lifetime income, the income in the last working year, the real monthly payout under the CPF LIFE Escalating plan, and the resulting net income replacement rates, respectively, for the two benchmarks mentioned above. The findings show that in the case of buying a five-room HDB flat, a median-income female (male) worker's pension benefits can replace 58% (51%) of the average lifetime income, yet can only replace 48% (43%) of the final year's income. In both cases, these estimates slightly exceed the results of McCarthy

[48] Data from the Inland Revenue Authority of Singapore (IRAS), retrieved from https://www.iras.gov.sg/irashome/Individuals/Locals/Working-Out-Your-Taxes/Income-Tax-Rates/

et al. (2002) and Hui (2013)[49] , who find that pension income of a median-income earner can replace 30%-40% of the final year's income; and Chia & Tsui (2019)[50] who predicts that annuity payouts under the Standard Plan, adjusted for inflation, can replace 52.6% (49.3%) of female (male) workers' average income over the working period.

Table 2.7: Net Income Replacement Rate of non-displaced median-income workers

	Flat Types	Real Annuity Payout- CPF LIFE	Average Lifetime income	Final Year's Income	Net IRR: Working lifetime	Net IRR: Final Year
Female	5 room	2,678.2	4,611.3	5,559.3	58.1%	48.2%
	4 room	2,964.4	4,611.3	5,559.3	64.3%	53.3%
Male	5 room	2,721.5	5,292.3	6,325.5	51.4%	43.0%
	4 room	3,020.6	5,292.3	6,325.5	57.1%	47.8%

Source: Author's Calculation

If the couple purchases a four-room BTO flat rather than a five-room one, the net income replacement rate can increase by six percentage points to reach a high of 57%-64%, which is comparable to the average net replacement rate of 63% under mandatory pension schemes across all OECD countries in 2017 (OECD, 2017). The net replacement rate for a median-income earner in Singapore is still smaller than that in many OECD countries, for example, Netherlands (100.6%), Austria (91.8%), Denmark (80.2%) but also significantly higher than that in countries such as Germany (50.5%), Japan (40%), Korea (45.1%), and New Zealand (43.2%).

Recently, several papers (Munnell & Soto, 2005; Munnell et al., 2006; Moore et al., 2010; MacDonald et al., 2011) highlight that conventional studies on income replacement rates failed to recognise the benefits of homeownership in retirement. As homeowners do not have to pay rent, which would have been one of the largest expenditure items for retired residents if they did not own a house, they can save less. This perspective is essential for all retirement systems where there are high homeownership rates among retirees. Taking into account the high homeownership rate in Singapore, Chia & Tsui (2019) incorporate imputed rent - the amount that homeowners would have to pay to earn a right to live in their house if they did not own it - in the calibration of both pre- and post-retirement income. They compute pre-retirement and post-retirement net imputed

[49] Since McCarthy et al. (2002) and Hui (2012) assume a constant income growth rate over the working life, income in the last working year is particularly high, leading to a low IRR.
[50] Chia & Tsui (2019) assess the age-wage profiles of a young entrant worker in 2012.

rent by adjusting the gross rent for the homeownership costs before and after retirement. However, in calculating the cost of owning a home, they fail to recognise the forgone interest on the down payment. In this analysis, I compute the real interest on the down payment that the representative couple could earn if they did not purchase their house. To come up with the pre-retirement real interest lost on the down payment, I average all real interest amounts incurred each year from the time of house buying until the end of age 64. On the contrary, post-retirement real interest lost on down payment covers the period from age 65 to age 100.

Following Chia & Tsui (2019), I derive the median subletting rental income for each flat type from the publicly available rental statistics[51] provided by the Housing Development Board (HDB). The amount of property tax is determined based on the current property tax rates[52]. The average housing maintenance costs for non-retiree households and retiree households are taken from the Report on the Household Expenditure Survey 2017/2018[53]. I calculate the average mortgage interest payments by separating the interest amount and the principal amount in the periodic mortgage payment. The nominal interest amounts are then adjusted for the long-term inflation rate of 1.57% per annum before being discounted to the time of house purchase and averaged over the loan period. By subtracting all the homeownership costs from the rental income, I find a pre-retirement net imputed rent of SG$ 724.3 (701.6) per month per person for the owner of a four-room (five-room) flat. Post-retirement net imputed rents are higher as retiree households incur less home maintenance: SG$ 803.4 (884.1) per month per person for the case of a 4-room (5-room) flat. (Table 2.8)

[51] Rental statistics in 2018 from the Housing & Development Board, Ministry of National Development, Singapore. Retrieved from https://www.hdb.gov.sg/cs/infoweb/residential/renting-out-a-flat-room/renting-out-your-flat/rental-statistics

[52] Data from the Inland Revenue Authority of Singapore (IRAS). Retrieved from https://www.iras.gov.sg/irashome/Property/Property-owners/Working-out-your-taxes/Property-Tax-Rates-and-Sample-Calculations/

[53] Retrieved from https://www.singstat.gov.sg/publications/households/household-expenditure-survey

Table 2.8: Net imputed rent pre and post-retirement by dwelling types (2018 $)

Flat types	Subletting rental rate	Property tax	Average mortgage interest payments	Home maintenance per household: non-retiree	Home maintenance per household: retiree	Real Interest Lost on Down payment (Pre- and post-retirement): Pre	Real Interest Lost on Down payment (Pre- and post-retirement): Post	Net imputed rent per person (Pre- and post-retirement): Pre	Net imputed rent per person (Pre- and post-retirement): Post
4R	2,000	53.3	200.8	183.6	181.9	113.7	157.9	724.3	803.4
5R	2,200	61.3	309.3	312.4	212.6	113.7	157.9	701.6	884.1

Source: Rental statistics from https://www.hdb.gov.sg. Home maintenance costs for non-retiree and retiree households are derived from DOS, 2019 (Table 11, 18A, and 45). All numbers presented above are determined on a monthly basis.

Adjusting both the numerator and denominator of the net replacement rate for the pre and post-retirement imputed rent leads to a higher percentage of average lifetime income that can be replaced by pension payouts under the CPF LIFE Escalating Plan. In detail, for the owner of a four-room (five-room) flat, including net imputed rent increases the net replacement rate by roughly 6.3 (8.8) percentage points, which is slightly lower than the impact found using the method of Chia and Tsui (2019) of 7 (9.5) percentage points. (Table 2.9)

On the whole, when compared with the commonly accepted target income replacement rate of 70% in the literature, or with standards proposed by World Bank (1994)[54], the net replacement rate with imputed rent of a median-income earner in Singapore falls slightly behind. Nonetheless, the success in achieving the poverty alleviation goal of the Singapore pension system is worth noting. In the absence of unemployment-induced career interruptions, if the representative couple limits their housing choice within BTO flats, their accumulated OA and SA balance can total almost three times subsistence at age 55.

[54] the net pension benefit should replace at least 53% of net earnings[54] in the final year of working and 78% of net average lifetime earnings (World Bank, 1994, p.294).

Table 2.9: Net IRR with pre and post-retirement net imputed rent

	Flat types	Real Annuity Payout- CPF LIFE	Average Lifetime income	Pre-retirement net imputed rent (per person)	Post-retirement net imputed rent (per person)	IRR with imputed rent	IRR without imputed rent
Female	5 room	2,678	4,611	701.6	884.1	67.0%	58.1%
	4 room	2,964	4,611	724.3	803.4	70.6%	64.3%
Male	5 room	2,721	5,292	701.6	884.1	60.2%	51.4%
	4 room	3,021	5,292	724.3	803.4	63.6%	57.1%

Source: Author's Calculation

2.5.2. The impacts of unemployment-induced career breaks on pension adequacy

In this section, I assess whether, under the current design of Singapore's social security system, median-income workers who suffer job displacement during their employment careers can secure pension income adequacy. There are two ways by which unemployment-related career breaks can adversely affect pension adequacy. First, career breaks attributable to retrenchment, termination of fixed-term contracts can entail **contribution gaps**, leading to reduced pension payouts if measures to plug those gap years are not available. Second, job displacement can induce **lower levels of post-break wages** and, thus, lower **levels of contribution during the working phase**. In fact, in determining the post-interruption age-wage profiles, the timing of the career break and the severity of the impact that job displacement might have on post-break wages go hand in hand. Hence, in what follows, I analyse the impacts of unemployment-induced career breaks on pension adequacy via two channels: (i) examining the implication of the timing and severity of the career break on pension adequacy of displaced workers, and (ii) the implication of the duration of the career break on pension adequacy of displaced workers.

2.5.2.1.The timing and severity of career breaks and pension adequacy of displaced workers

In the baseline analysis, I assume that the representative workers experience only one unemployment episode of six months. It is important to recall that (i) job displacement can happen in the early-career stage (after the first ten years of working), in the mid-career stage (after the first twenty years of working), or in the late-career stage (after the first thirty years of working); and

(ii) when it happens, it can induce a wage penalty of 10% or 20%. To evaluate the retirement income adequacy of median-income earners with one unemployment episode during the career path, I compute both the subsistence replacement rate and net income replacement rate with imputed rent.

Regarding the poverty eradication goal, Table 2.10 exhibits the subsistence replacement rate for median female and male workers across different unemployment scenarios and different levels of housing consumption. With one unemployment episode during the working life, there are six different scenarios, as depicted in the first column of Table 2.10. Obviously, as for female workers, job displacement is assumed to occur before age 55; experiencing unemployment during the career path will reduce the contribution to the CPF before the combined OA and SA balances are set aside at the end of age 54, resulting in a lower SRR. The situation is a bit different for male counterparts. In case the job displacement takes place during male workers' late-career phase, which is set at age 55, it does not affect the SRR because, at the end of age 54, the accumulated balance from OA and SA is already locked up and transferred to a new account – RA. Overall, in all case scenarios, median-income earners who suffer six months without a job can support themselves well beyond subsistence. In particular, workers hardest hit by job loss can accumulate at least 1.79 times the FRS upon reaching age 55.

Table 2.10: Subsistence replacement rate of workers with a single career break

	Subsistence Replacement Rate			
	Female, 5-room	Female, 4-room	Male, 5-room	Male, 4- room
50P	269%	308%	253%	291%
Early-Career Break, Wage penalty of 10%	214%	254%	232%	269%
Early Career Break, Wage penalty of 20%	179%	219%	198%	235%
Mid-Career Break, Wage penalty of 10%	247%	287%	245%	282%
Mid-Career Break, Wage penalty of 20%	232%	272%	239%	276%
Late-Career Break, Wage penalty of 10%	261%	301%	253%	291%
Late-Career Break, Wage penalty of 20%	259%	299%	253%	291%

Source: Author's Calculation. 50P refers to the case of median-income workers with an uninterrupted career path.

In assessing the magnitude of job loss on retirement nest egg at age 55, I compare the subsistence replacement rate of workers who experience one unemployment episode over the working life with that of workers with uninterrupted career paths, as shown in figure 2.3.

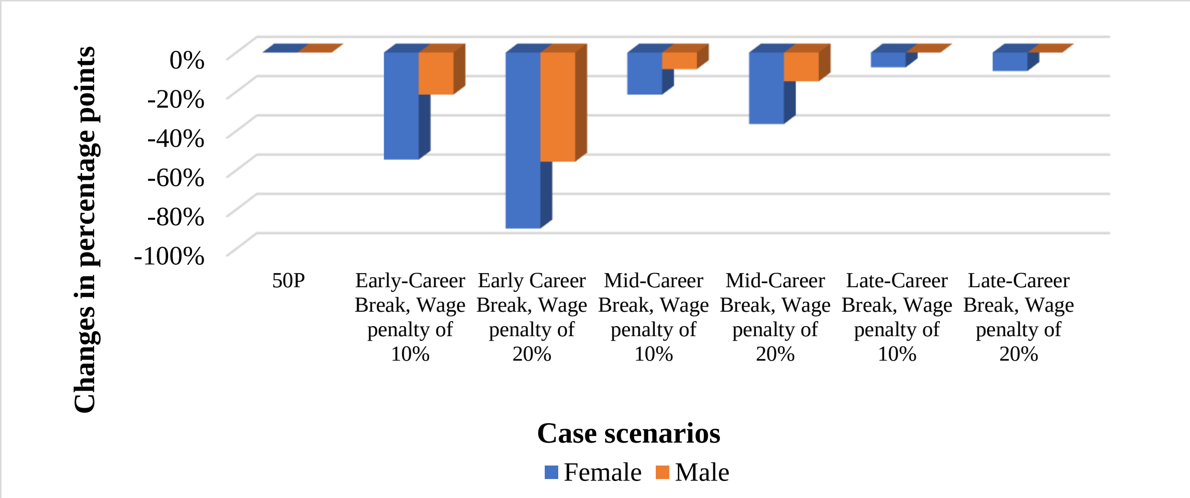


Figure 2.3: The impact of one unemployment episode on subsistence replacement rate.

Source: Author's Calculation

Notwithstanding that workers are still able to afford their retirement beyond the minimum standards of living after losing a job once in the working life, unemployment-related career breaks appear to have a sizeable impact on median-income workers' retirement nest egg. Notably, the sooner the career break occurs, the larger the impact is. This is because the longer the workers can secure their position in the 50th income percentile, the more contributions they deposit into their CPF accounts during their career. In particular, for a median-income female (male) worker, an early-career break, which happens after the first ten years of working, can bring about a fall of 60-90 (30-60) percentage points in SRR; whereas postponing the break by ten years can reduce the impact by 30-50 (20-40) percentage points.

For the real consumption smoothing goal, I calculate the net income replacement rate with imputed rent, as illustrated in Table 2.11. The examination of the net income replacement rate among people whose career is disrupted by a single unemployment episode indicates that their real annuity payouts under the CPF LIFE Escalating Plan can replace at least 60.8% and at most 70.6% of pre-retirement earnings net of income taxes and CPF contributions. Hence, compared with the net income replacement rate of non-displaced workers, the rate of displaced workers does not change considerably. However, this result should be interpreted cautiously because both the numerator and denominator change when job displacement occurs, unlike SRR with a fixed denominator. On the one hand, suffering a wage reduction will move workers from the 50th income percentile to lower percentiles, causing a decline in average lifetime income or a fall in the denominator of net replacement rate. On the other hand, lower-income percentiles mean lower contributions and thus lower pension benefits in retirement or a smaller numerator of net replacement rate.

Table 2.11: Net income replacement rate of workers with a single career break

	Income Replacement Rate with imputed rent			
	Female, 5-room	Female, 4-room	Male, 5-room	Male, 4- room
50P	67.0%	70.6%	60.2%	63.6%
Early-Career Break, Wage penalty of 10%	65.3%	69.3%	64.8%	68.7%
Early Career Break, Wage penalty of 20%	64.0%	68.4%	64.1%	68.4%
Mid-Career Break, Wage penalty of 10%	66.6%	70.4%	62.5%	66.1%
Mid-Career Break, Wage penalty of 20%	66.4%	70.4%	64.4%	68.2%
Late-Career Break, Wage penalty of 10%	66.6%	70.2%	60.8%	64.2%
Late-Career Break, Wage penalty of 20%	66.8%	70.6%	61.6%	65.2%

Source: Author's Calculation. 50P refers to the case of median-income workers with an uninterrupted career path.

When the average lifetime income decreases by a greater extent than the pension payout does, the net income replacement rate increases, as can be seen in the case of displaced male workers; whereas, when the opposite is true, the net income replacement rate decreases as observed in the case of displaced female workers. In either way, compared to the effect of job displacement on SRR, changes in net income replacement rates are considerably smaller, not exceeding five percentage points. Furthermore, an analysis of the correlation between the career state at which the job displacement occurs and the net income replacement rate of displaced workers reveals that a larger change in net IRR comes with an earlier career break, which is consistent with the findings related to SRR.

From the analysis above, even in the worst-case scenario, a median income earner still lives above the poverty line in retirement as their amount of CPF savings at age 55 can total almost 1.79 times the recommended full retirement sum; and at the same time can replace at least 64% of their pre-retirement net earnings. However, more fragmented career paths can lead to a massive loss in their aggregated retirement savings, with a larger impact witnessed in the case of an earlier career break.

2.5.2.2.The duration of career breaks and pension adequacy of displaced workers

In the previous section, I highlighted the impact of unemployment-related career breaks on pension adequacy, mostly via the direct implication of the timing and severity of the career break on the post-interruption earnings profile. Hence, the impact of contribution gaps incurred due to the discontinuity in the career path has not been covered. In what follows, I further examine the case in which job displacement induces a reduction in wages but does not entail any periods without a job and compare its effect with the impact that a six-month unemployment break has on pension adequacy. As explained in the previous section, when I use net income replacement rate as a measurement of pension adequacy, the impact of job displacement is reduced. Therefore, to fully capture the effect of unemployment duration on pension adequacy, I only examine changes in subsistence replacement rate of displaced workers relative to that of non-displaced workers across different case scenarios.

In the baseline analysis, I assume that an unemployment episode lasts for six months. A shorter period without a job would certainly cause a less significant loss in CPF contributions. To assess the implication of the duration of the unemployment period on pension adequacy, I keep all parameters constant except for the length of the unemployment spell. I re-estimate the CPF contribution record based on earnings profiles of median-income workers who suffer job displacement without any periods of unemployment yet experience a reduction of 10% (20%) in wages post-displacement. The accumulated OA and SA balance at age 55, the Subsistence Replace Rate, and the real annuity payout under the CPF LIFE Escalating under various cases are then computed.

Before discussing the correlation between the duration of the career break and pension adequacy, it is worth noting that the difference between the case of a six-month break and the case of a zero-

-month break lies solely in the earnings in the first year following the layoff event, but not in the whole earnings profiles from the job displacement event onwards. Hence, comparing the impact of job displacement on the retirement income adequacy under these two cases can help minimise the involvement of the timing of the displacement event as well as the severity of job loss effect on post-interruption wages. Table 2.12 compares the impact of job displacement on subsistence replacement rates under the case of a six-month break and a zero-month break. Except for the case of late-career break among male workers where job displacement occurs after the formation of RA, thus having no impact on the subsistence replacement rate, other cases see a reduction in the retirement nest egg at age 55.

Table 2.12: Impact of job loss on SRR - six-month break versus zero-month break

	Change in Subsistence Replacement Rate relative to that of non-displaced workers					
	Female, 0 month	Female, 6 month	Female, Difference in impact	Male, 0 month	Male, 6 month	Male, Difference in impact
50P	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Early-Career Break, Wage penalty of 10%	-48.2%	-54.5%	6.3%	-14.6%	-21.4%	6.8%
Mid-Career Break, Wage penalty of 10%	-14.7%	-21.4%	6.7%	-1.2%	-8.5%	7.2%
Late-Career Break, Wage penalty of 10%	-1.8%	-7.5%	5.7%	0.0%	0.0%	0.0%
Early Career Break, Wage penalty of 20%	-84.0%	-89.6%	5.6%	-49.5%	-55.6%	6.0%
Mid-Career Break, Wage penalty of 20%	-30.5%	-36.4%	5.9%	-7.9%	-14.7%	6.8%
Late-Career Break, Wage penalty of 20%	-4.3%	-9.4%	5.1%	0.0%	0.0%	0.0%

Source: Author's Calculation.

As shown in Table 2.12, the difference between the case of a zero-month break and a six-month break brings about a gap of approximately five to seven percentage points in subsistence replacement rate, meaning that extending the length of unemployment by another six months would reduce the retirement nest egg at age 55 by an amount equal to 5%-7% of the Full Retirement Sum (SG$ 8,550- SG$11,970) in 2018 dollars.

It can also be seen from Table 2.12 that the implication of the duration of the unemployment period on subsistence replacement rate does vary across different career stages at which the job displacement occurs and across the different degrees at which the re-entry wage is affected by the job loss event. First, the consequence of an additional six-month without a job for pension adequacy appears to be more profound when the job displacement event happens at the mid-career phase. This is because the age-real wage profiles of median-income earners follow a hump-shaped pattern, with peak earnings occurring at the mid-career stage, resulting in the highest amount of earnings loss in the case of mid-career breaks. Second, the implication of an additional six-month staying unemployed on subsistence replacement rate decreases when the severity of job loss effect on post-interruption wages increases. This is because the difference in earnings profiles between the case of a six-month break and a zero-month break equals the six-month salaries in the first year after the job displacement; thus, the higher the level of wage loss, the lower the level of salary in the first year after the event, and the lower the gap between two cases.

2.5.3. Sensitivity Analysis

In the previous section, I highlighted the impact of unemployment-related career breaks on pension adequacy via two channels: (i) the direct implication of job displacement on post-interruption earnings profile, and (ii) the implication of the duration of the unemployment period on pension adequacy. It is also essential to look at the case of single-income families where one person is solely responsible for the financing of the house purchase. In that case, retirement income adequacy after periods of job loss is much more worrisome when compared with the financial situation of dual-income couples.

In the baseline analysis, I assume households to be dual-income couples. The down payment of their BTO flat is financed by the accumulated OA balances of both husband and wife. Besides, periodic mortgage payments are also equally shared between the couple and are paid out from their OA funds. For sensitivity analysis, I consider the case when the husband (or the wife) is the sole income earner of the family. Since only the income earner's OA savings are used for the down payment and the monthly mortgage payment, more CPF monies are utilised for the housing financing purpose, eating up the savings for retirement. I run simulations assuming two different types of housing consumption: a 3-room BTO flat with an average price of SG$ 207,000 (in 2018) and a 4-room BTO flat with an average price of SG$ 305,000 (in 2018). First, I re-calculate subsistence replacement rate, net income replacement rate without imputed rent for the sole income earner keeping the same amount of housing consumption as in the baseline cases (4-room BTO flat). Second, I still take into account the sole-financing but allow a purchase of a smaller flat (a 3-room BTO flat), reducing a housing cost of almost SG$ 100,000.

Under the Enhanced Housing Grant (EHG) scheme, lower-income households will receive a higher amount of housing grants, which equals SG$ 50,000 in the sensitivity analysis. After subtracting the housing grants, the down payment from the house price, I come up with a housing loan of SG$ 222,800-SG$ 225,900 in the case of buying a 4-room BTO flat, or SG$ 114,560-SG$ 117,700 in the case of buying a 3-room BTO flat.

Table 2.13 summarises the subsistence replacement rate and net income replacement rate of a single-income family under two types of housing consumption and across various career paths. When a median-income earner, regardless of gender, is a sole earner of the family, buying a four-room flat type means that their family can, at best, accumulate a nest egg of approximately two times larger than the FRS at age 55. Opting for a smaller BTO flat would increase the subsistence replacement rate by roughly 80 percentage points. Overall, if the career path of the sole earner is smooth and not affected by periods of unemployment, the couple family can still live beyond the subsistence level during retirement if their housing choices are limited within 3-room or 4-room BTO flats. When the career path of the representative worker is likely to be punctuated by periods of unemployment, selecting a smaller size flat at the time of purchasing would be a safer choice. In the worst-case scenario where the wife as the sole earner of the family experiences an early-career break with a loss of 20% in their wage post-interruption, choosing a four-room flat from the inception might put the family at risk of living below the poverty line in retirement: their subsistence replacement rates at 55 are 125%, lower than the OECD's recommended rate of 141%[55]. These findings imply that single-income families should be more cautious when making their housing consumption decisions.

[55] In determining the ratio of the couple to the single elderly household budgets, recent OECD publications (OECD, 2011; OECD, 2008) use square root equivalence scale which is the square root of household size, or around 1.41.

In line with the analysis of subsistence replacement rate, the examination of net income replacement rate yields a similar conclusion. In the case of buying a 3-room BTO flat, a single-income family can secure a net replacement rate of at least 65% in the worst-case scenario. However, choosing a bigger size flat would reduce the net IRR to a level below 55% in the worst-case scenario.

Table 2.13: Pension adequacy of a single income family (husband or wife is a sole income earner) under various types of housing consumption

	Male, 4-room		Male, 3-room		Female, 4-room		Female, 3-room	
	SRR	IRR	SRR	IRR	SRR	IRR	SRR	IRR
50P	204%	53%	281%	61%	215%	59%	296%	68%
Early-Career Break, Wage penalty of 10%	183%	56%	260%	66%	161%	56%	241%	66%
Early Career Break, Wage penalty of 20%	149%	55%	226%	65%	125%	54%	206%	65%
Mid-Career Break, Wage penalty of 10%	196%	54%	273%	65%	194%	58%	274%	67%
Mid-Career Break, Wage penalty of 20%	190%	56%	266%	65%	179%	57%	259%	67%
Late-Career Break, Wage penalty of 10%	204%	53%	281%	64%	208%	58%	288%	67%
Late-Career Break, Wage penalty of 20%	204%	54%	281%	64%	206%	58%	286%	68%

Source: Author's Calculation. IRR stands for the net income replacement rate with imputed rent. SRR stands for the subsistence replacement rate.

2.6. Conclusion

I have employed a simulation model to assess (i) whether a median income earner in Singapore who enters the labour market in 2018 can adequately save for their retirement, and more importantly (ii) whether this median-income worker is adequately prepared for worse case scenarios where their career paths are interrupted by periods of unemployment. I use administrative data from the Singapore Ministry of Manpower, which has been previously unavailable to researchers, to construct the age-real wage profiles of workers with uninterrupted career paths across different income percentiles. Career paths punctuated by periods of job displacement are then categorised by the (i) timing of the career breaks; (ii) duration of the breaks; (iii) frequency of unemployment during the working life; and (iv) degree at which the post-break wage is affected by the unemployment spell. When the median-income worker in the simulation model is hit by job displacement, he or she will be forced to move from the 50th income percentile to the lower-income percentile group. Based on the uninterrupted earnings profiles across various income percentiles, I build up the respective earnings profiles for the fragmented career paths defined above.

I take into account pension policies as of December 2019 and the Government's plans to reform the pension system in the next ten years so as to better reflect the situation of future retirees who happened to enter the labour market in 2018. CPF contributions by workers and their employers, as well as the allocation of the total contribution to OA, SA, and MA, are computed based on these policy parameters and the age-real wage profiles. Of particular importance in the discussion about the pension payout adequacy under the CPF is the link between public housing policies and the retirement income system. Uniquely in Singapore, public housing is an option for most social classes made possible by generous subsidies and housing grants. Another notable feature is that the purchase of a public flat can be financed via the mandatory savings system. I consider two levels of housing consumption for a median-income couple: a four-room BTO flat and a five-room BTO flat. The down payment and periodic mortgage payments are made via the monies accumulated in the OA of both husband and wife.

From all the ingredients mentioned above, I calculate all CPF balances during the working life and the pension payout benefits under the CPF LIFE scheme. Since real annuity payout represents better the goal of smoothing real consumption over time, I choose the CPF LIFE Escalating Plan for the representative workers when they turn 65 years old. To assess the adequacy of pension income, I use two indicators, including the Subsistence Replacement Rate, which compares the combined OA and SA balances at age 55 with the Full Retirement Sum recommended by the Government, and the Net Income Replacement Rate, which compares the real annuity payout under the CPF LIFE scheme and pre-retirement earnings.

Overall, the findings suggest that a median-income worker with an uninterrupted career path, regardless of gender, appears to support themselves well beyond the subsistence level with the accumulated OA and SA balance at age 55, far exceeding the Full Retirement Sum (in 2018 dollars). When it comes to the consumption smoothing goal, in the case of buying a five-room HDB flat, a median-income earner's pension benefits can replace 51%-58% of the average lifetime income. If the couple opts for a four-room flat instead of a five-room one, the net replacement rate can increase by six percentage points to reach a high of 57%-64%, which is higher than the average net replacement rate of 63% under mandatory pension schemes across all OECD countries in 2017. Furthermore, following Chia and Tsui (2019), to account for the high homeownership rate in Singapore and the net benefits of owning a home in retirement, I incorporate pre- and post-net imputed rent into the calculation of net IRR and find that including net imputed rent increases net replacement rate by roughly 6.3 (8.8) percentage points for the owner of a four-room (five-room) HDB flat.

In assessing the pension adequacy of workers who suffer a single unemployment spell, I find that even in the worst-case scenario, they can still build a retirement nest egg of at least 1.79 times larger than the Full Retirement Sum at age 55. However, job losses do significantly decrease the retirement nest egg of displaced workers, with the level of impact ranging from 7.5% to 89.6% of the FRS (SG$12,800-SG$153,267) in 2018 dollars. Furthermore, the results point out that due to the scarring effect of job displacement on post-interruption wage profiles, the sooner the career break occurs, the larger the impact is. To be more specific, for a median-income female (male) worker, an early-career break, which happens after the first ten years of working, can bring about a fall of 60-90 (30-60) percentage points in the SRR; whereas postponing the break by ten years can reduce the impact by 30-50 (20-40) percentage points. Compared to the effect of job displacement on SRR, changes in net income replacement rates are considerably smaller, not exceeding five percentage points. However, a larger change in net IRR is found to associate with an earlier career break, which is consistent with the findings related to Subsistence Replacement Rate. I also find that extending the length of unemployment by six months can incur a reduction of at least SG$ 8,550- SG$ 11,970 (in 2018 dollars) in the retirement nest egg at age 55.

In addition, I examine the pension adequacy of single-income families as retirement income adequacy after periods of job loss is even more worrisome for this household type than for dual-income couples. Findings indicate that in such a case as a single career break in the early stage with a 20% reduction in wage, opting for a four-room BTO flat or bigger will put the household at risk of living below the poverty line in retirement. Hence, single-income families should be more cautious when making their housing consumption decisions.

In today’s constantly-evolving technological world, where workers are more likely than ever before to experience job displacement during the career path, it appears that the CPF is doing well in helping young entrant workers accumulate enough savings to sustain their retirement consumption beyond the subsistence level. However, for the middle-income group, the main objective is not to live beyond subsistence but to grow their nest egg as large as possible. It has been shown in this research that more fragmented career paths can lead to a substantial loss in their aggregated retirement savings. However, their loss might not be large enough to make them fall into the lowest 20% of Singaporean income earners to be eligible for any social assistance schemes. In helping the middle class ease the risks from wage depression and unemployment, various government-supported programs have been introduced in Singapore, for example, SkillFuture since 2014 and Adapt & Grow Initiative since 2016. Nevertheless, in a recent survey conducted by Skillsoft- a global corporate learning provider- about upskilling employees for the digital workplace in the Asia Pacific, Singapore firms are shown not to do enough to assist their internal employees to reskill or upskill. In the events that (i) internal employees have a high possibility of being replaced by external resources, and (ii) learning new skills to adapt to new requirements in the workplace takes time, the significant impact of unemployment duration on pension income adequacy calls for a measure to mitigate the gaps in the CPF contribution history. Moreover, as being displaced in the early-career stage might have a scarring effect on post-interruption earnings profile, career support programs for the unemployed should put more emphasis on the early- and mid-career break. Finally, in Singapore, a threshold for the level of housing consumption may be appropriate for single-income families because they are more likely to save inadequately in retirement.

CHAPTER 3: HEALTH STATUS AND PORTFOLIO CHOICE OF HOMEOWNERS AND RENTERS OVER THE LIFE CYCLE

Abstract

Although previous literature has studied the relationship between health and portfolio choice, little research has been done to incorporate both health and homeownership status in the household financial asset allocation problem. I address this gap by investigating how self-perceived health status interacts with the portfolio choice of homeowners and renters over their life-cycle and how this relationship differs between homeowners and renters. First, I develop a dynamic optimisation life-cycle model for the joint determination of health investment, housing, and liquid portfolio allocation, in which individuals face labour income risk while working, and house price risk, stochastic stock returns, as well as health risk over the life cycle. Second, using data from the Panel Study of Income Dynamics (PSID) with a U.S. focus, I perform empirical analysis to examine whether the empirical findings support the life-cycle model predictions. The results indicate that the life-cycle model can explain key facts about households' stock allocation across health status and different housing tenure choices in PSID. The proportion of total wealth in stocks is low overall and positively related to health for both homeowners and renters. However, I find that a change in the share of stocks in total wealth corresponding with the deterioration of health status is significantly larger for renters than for homeowners.

3.1. Introduction

Economic theory predicts that an increase in background risk should tilt investors towards safer assets. Increasingly perceived as a source of background risk, health-related issues should, therefore, act as an important input in the household financial asset allocation problem. Homeownership is another ingredient that plays a central role in forming family financial risk-taking as homeowners and renters have been shown to react very differently in determining portfolio allocation (Yao & Zhang, 2005; Vestman, 2018). Although previous literature has studied the relationship between health and portfolio choice, little research has been done to incorporate both health and homeownership status in the household financial asset allocation problem. In this paper, I address this gap by investigating how self-perceived health status interacts with the portfolio choice of homeowners and renters over their life-cycle. I also look at how this relationship differs between homeowners and renters. These investigations are crucial to the determination of households' optimal portfolio allocation over the life cycle. Indeed, if health expenditure is considered an investment in health capital, just as bonds and stocks are investments in financial capital, and housing expenditure is an investment in housing capital, investors' health-related considerations and homeownership status should be parts of a more comprehensive portfolio decision between health, housing and financial assets. Furthermore, over the last few decades, population ageing has posed challenges to the sustainability and adequacy of the retirement income system across many countries. Accordingly, pension system reforms worldwide have called for mandatory saving accounts or individual retirement accounts where the retirement wealth of participants depends largely on investment risks and their portfolio decision. As a result, retirement income adequacy is increasingly depending not only on the level of financial resources but also on how these amounts are allocated across different asset classes such as stocks, bonds, housing and health capital over the life cycle.

This research makes two contributions to the literature on the household portfolio choice over the life cycle. To begin with, it is among the first to model the relationship between health and portfolio choice separately for homeowners and renters. This approach enables us to examine whether the correlation between health and stock allocation for homeowners is stronger or weaker than that for renters. Previous works on portfolio choice often focus either on the relationship between homeownership and risky asset allocation, ignoring the health-related considerations (e.g., Grossman & Laroque, 1990; Cocco, 2004; Yao & Zhang, 2005; Chetty et al., 2017; Vestman, 2018); or on the correlation between health status and households' risky asset allocation, overlooking the influence of housing tenure choice on the household portfolio problem. (e.g., Edward, 2008; Hugonnier et al., 2013).

The second contribution of this paper is that it introduces health risk as a critical source of background risk, besides labour income risk, to the portfolio choice problem during the working period. Prior research that examines the life-cycle portfolio choice problem in the presence of health risk primarily targets the retirement period (e.g., Edward, 2008; Yogo, 2016). In this study, health enters the investor's decision problem through three channels- the preference channel, the background risk channel, and the life expectancy channel (e.g., Hugonnier et al., 2013; Yogo, 2016). First, the preference channel implies that health and non-health consumption can either be substitutes or complements (Edwards, 2008). For instance, improved health can increase the consumption of leisure activities, whereas worse health can increase the consumption of labour-saving activities, such as cleaning services. This channel explains how optimal non-health consumption reacts with changes in health status. Second, the background risk channel captures the fact that poor health often triggers large out-of-pocket medical expenditure, which drains financial wealth and increases the marginal utility of consumption. These unexpected medical costs activate individuals' precautionary saving motive, urging them to lower their exposure to risky assets. However, the agent in this study is allowed to endogenously respond to changes in their health status by investing in their health capital in each period to reduce their exposure to health shocks, which again induces financial risk-taking. Third, the life-expectancy channel signifies that better health reduces mortality risk, meaning a longer investment horizon. This channel implies that investors with a longer investment horizon tend to have a higher proportion of their liquid wealth held in stocks.

In terms of research methodology, first, I develop a dynamic optimisation life-cycle model for the joint determination of health investment, housing, and portfolio allocation. In this model, individuals face labour income risk in the working period while facing house price risk, stochastic stock returns, and health risk over the life cycle. Second, using data from the Panel Study of Income Dynamics (PSID), I perform empirical analysis to examine whether the empirical findings support the life-cycle model predictions based on the policy functions and simulation results. I incorporate four stochastic processes, including health accumulation process, labour income process, stock returns process, and housing returns process, in the life-cycle model. To calibrate the health accumulation process and the labour income process, I use data on self-reported health status, health expenditure, labour income, and asset holdings for a sample of single and couple families aged over 20 years old in the PSID. Other remaining inputs, involving the stock returns process and the housing returns process, are determined following Yogo (2016) and Yao & Zhang (2005).

The two most important ingredients in my model are identified as the health accumulation process and homeownership status. With regard to the health accumulation process, individuals enter each period with an initial health capital. Health stock then depreciates at a stochastic rate, affecting both the marginal utility of consumption and the probability of dying. Health expenditure in this model is an investment in the sense that its impact on health can last for more than one period. In each period, apart from making asset allocation decisions, individuals choose health expenditures based on changes in their health. Therefore, I build a health accumulation process in which health expenditure is an endogenous response to the stochastic rate of health depreciation. The PSID merged dataset covering every two years from 1999 to 2017 is used to estimate health transition probabilities and the stochastic health depreciation rate. Regarding homeownership status, I examine the portfolio choice problem separately for homeowners and renters. To become a homeowner, individuals have to meet the down payment requirement and finance the rest via a mortgage debt. Additionally, a substantial fraction of the market value of the house (the liquidation cost) will be borne by homeowners at the time of selling. Finally, in each period, homeowners have to pay maintenance costs and property taxes. Unlike homeowners, renters are not in charge of periodic maintenance costs and property taxes and can relocate without bearing any transaction costs. In terms of earnings uncertainty during the working phase, I follow the existing literature on life-cycle portfolio choice in the presence of labour income risk to model the labour income process (Viceira, 2001; Cocco, 2005; Yao & Zhang, 2005; Love, 2013).

The life-cycle model is solved by numerical dynamic programming and simulated at a two-year frequency from the age of 40 to the age of 84. First, the numerical solutions for homeowners and renters both show that optimal health investment decreases in health status. In other words, better health conditions are associated with a lower share of total wealth spent on health care. A possible clarification is that treatment in poor health often has a much larger impact on health than preventive care in excellent health.

Second, the optimal portfolio rules of the life-cycle model indicate that regardless of the homeownership status, health status is found to be negatively related to the proportion of stocks in liquid wealth. To understand this relationship, it is useful to recall that although labour income during the working period is uncertain, it is more bond-like than stock-like and often considered a risk-free asset in the household portfolio. As such, a lower share of labour income in financial wealth would tilt investors towards a more conservative portfolio (Campbell et al., 2001; Cocco et al., 2005)**.** Besides, as people tend to finance a large medical expenditure out of their liquid wealth instead of using illiquid assets (Berkowitz & Qiu, 2006), liquid wealth as a share of total wealth appears to increase in health, which can be referred to as the crowding-out effect of medical expenditure on liquid wealth. Correspondingly, with labour income kept constant, better health would mean a lower share of labour income in liquid wealth, resulting in a more conservative liquid portfolio.

Third, I find that the optimal share of stocks in total wealth – including both housing and liquid wealth – is positively related to health status. The stock share in total wealth is defined by multiplying the ratio of liquid wealth to total wealth and the ratio of stock share to liquid wealth. These two ingredients react differently to changes in health status. Hence, the mechanism with a stronger effect would determine the sign of the correlation between stock share in total wealth and health status. In the baseline case, as health status moves from poor to excellent, the crowding-out effect of medical expenditure on liquid wealth is stronger, resulting in a positive relationship between the proportion of stocks in total wealth and health status.

Based on health distribution at a given age and the range of state variables, I simulate a population of 100,000 individuals starting at the age of 40 using the optimal solution to the life-cycle model. From the simulation results, I report the mean of stock share in total wealth separately for homeowners and renters at various ages over the life cycle. I find that as health deteriorates, the decrease in the renters' stock share in total wealth is significantly larger than that for homeowners. Compared to homeowners, renters have a significantly larger percentage of total wealth held in liquid assets. Accordingly, if labour income is comparable, renters tend to have a more conservative portfolio choice. When the health condition moves from "good" to "poor", a change in the proportion of stocks in total wealth is, therefore, often more significant for renters than for homeowners.

I also conduct an empirical analysis using censored regression models (RE Tobit) to estimate the correlation between households' stock allocation and the health status of the household head. In this process, I control for (i) the age, education level, and labour income of the household head; (ii) the total asset and total debt of the household; (iii) family size; (iv) the presence of health insurance; and (v) the out of pocket medical costs of the household. The empirical findings confirm the optimal rules of the life-cycle portfolio choice model. In general, poorer health is associated with a lower share of total wealth held in stocks for both homeowners and renters. This relationship, however, is stronger for renters. To facilitate the comparison of PSID data and simulation results, I obtain the predicted value of households' portfolio share in stocks from the above regression models. These values support the results of the simulation.

The rest of this paper is organised as follows. I first summarise the related literature on (i) household portfolio choice in a life-cycle setting and (ii) the demand for health with costly health investment. Next, I describe the life-cycle model of consumption and portfolio choice in Section 3.3. This section presents the ingredients of the optimisation problem, which include (i) state variables and control variables, (ii) four stochastic processes used in the model, (iii) the budget constraints and the dynamics of state variables, and (iv) the objective function of the optimisation problem and the normalisation of the problem. Section 3.4 explains the calibration of model inputs using data from PSID. After that, I report the optimal policy functions and the simulation results of the life-cycle model in Section 3.5. Section 3.6 provides an analysis to test whether the empirical findings support the results of the life-cycle model. Section 3.7 concludes with a discussion of open issues and possible extensions for future research.

3.2. Literature Review

This study builds on two well-recognised but separated frameworks, namely, the life-cycle portfolio choice problem and the demand for health with costly health investment.

The asset allocation in a life-cycle setting dates back to the renowned work of Merton (1971), showing that in perfect markets without the presence of labour income risk, the optimal share of wealth invested in stocks is constant, independent of wealth and age, and depends only on the level of risk aversion together with the moments of assets' excess return. Over time, there have been a large number of papers attempting to incorporate fundamental sources of background risk - such as labour income risk (e.g., Heaton & Lucas, 1997; Viceira, 2001; Cocco, 2005; Fagereng et al., 2017), health risk (e.g., Edward, 2008; Hugonnier et al., 2013; Yogo, 2016) and house price risk (e.g., Cocco, 2004; Yao & Zhang, 2005; Yogo, 2016; Chetty et al., 2017)- in the life-cycle portfolio choice problem to reproduce patterns observed in the data. First, previous literature argues that people who face uninsured labour income risk tend to accumulate precautionary savings, increase the labour supply, and reduce exposure to risks that they can avoid, for example, lowering the share invested in stocks (e.g., Kimball, 1993; Heaton & Lucas, 1997; Heaton & Lucas, 2000). However, the results from the life-cycle portfolio choice model with uncertain earnings show that the effect of labour income risk on risky asset allocation, though carrying the same sign as theoretically predicted, is relatively small in size, leaving the "low allocations to equity" puzzle unresolved.

Second, many models have incorporated housing assets into household portfolios to better explain the level of stockholdings observed in data. Unlike other financial assets, a house – often the largest asset owned by households- serves a dual purpose of a durable consumption good from which owners derive utility and an investment vehicle that allows investors to accumulate home equity[56]. Prior research shows that the investment in housing may affect risky asset allocation via several mechanisms, including (i) the illiquidity channel - where liquidation costs reduce the incentive to move houses as well as investors' exposure to stock (e.g., Grossman & Laroque, 1990; Chetty & Szeidl, 2007; Chetty et al., 2017), (ii) the house price risk channel – where the positive correlation between house price risk and labour income shocks makes housing assets riskier, causing households to shift their financial wealth toward safer assets (e.g., Fratantoni, 2001; Cocco, 2004), (iii) the diversification effect- where the low correlation between stock returns and housing returns encourages investors to increase the equity proportion in their liquid wealth (e.g., Yao & Zhang, 2005), (iv) the hedging effect – where owning a house hedges against rent uncertainty, making houses less risky than perceived, hence increasing people's exposure to stock (e.g, Sinai & Souleles, 2005; Chetty et al., 2017), and (v) the committed expenditure channel- where mortgage commitments lead to reduced stockholdings (e.g., Fratantoni, 1998; Yao & Zhang, 2005; Chetty et al., 2017). Regarding the magnitudes of the impact that housing has on the liquid financial portfolio, Fratantoni (1998) finds that all things equal, a 10% decrease in mortgage debt would increase stock share in household liquid wealth by 1.5%, while Yamashita (2003) discovers an elasticity of risky asset share with respect to the property value of roughly -0.1. Chetty et al. (2017) point out that studying the effect of housing on risky asset allocations requires separating the effect of home equity and mortgage debt. They find (i) that with home equity kept constant, increases in house value reduce stockholdings, whereas, with steady house value, increases in home equity raise stockholdings, (ii) and that keeping home equity constant, if households spend 10% less on their housing, the stock share of liquid wealth would increase by one percentage point.

Third, another strand of life-cycle portfolio choice has been elaborated to link the health-related decisions with the other financial decisions in households. Nevertheless, so far, dynamic portfolio choice models with health risks have primarily focused on the retirement period (Rosen & Wu, 2004; Edward, 2008; Yogo, 2016). The results show that portfolio share in stocks is positively correlated with health status, or in other words, individuals with poorer health tend to hold safer portfolios, and risky health may explain 20% of the age-related decline in risk-taking behaviours after retirement. These findings are mainly driven by two economic mechanisms. The first

[56] Home equity is the net housing wealth after subtracting mortgage debt from house value.

mechanism is the precautionary saving motive. Future health shocks can trigger out-of-pocket medical spending, which erodes financial wealth, driving people to save more to offset this background risk and lower the demand for risky assets at the same time. The second mechanism is the horizon effect (Bodie et al., 1992). The horizon effect clarifies why healthier individuals, who have a longer life expectancy, tend to invest a greater share of financial wealth in stocks in response to the risk of outliving the retirement nest egg.

When it comes to the modelling of health capital, Grossman (1972) provides a major theoretical framework for analyzing the demand for health and health expenditure. One of the key assumptions of the Grossman model is that the rate of health depreciation is assumed known. Several papers have relaxed this constraint to make the health depreciation rate exogenously determined (Hren, 2012). Liljas (1998) demonstrates that health depreciation depends on stochastic health status, and optimal health capital increases with the initial level of health and the uncertain incidence of illness. In a model with exogenous and stochastic health expenditures, the inability to insure uncertainty over large out-of-pocket medical spending creates a strong motive to save liquid assets. In fact, individuals can endogenously adjust health expenditures in response to their health status and wealth. Yogo (2016) builds a health accumulation model in which health expenditure endogenously responds to the stochastic health depreciation rate with the distribution depending on present health status, age, and health transition probabilities.

3.3. The life-cycle model of consumption and portfolio choice

The intention of this research is to model the joint determination of homeownership, health investment, and risky asset allocation of an individual over the life cycle. The individual enters the model at the age of 40, works until the age of 64, when he/she can start claiming his/her social security benefits and dies at the age of 84. Each period in the model is calibrated to match two years in real life.

During the working period, the investor faces labour income risk, health risk, house price risk, and risky stock returns. Also, in each period of the working life, he/she can be either a homeowner or a renter and has to choose the amount of health investment in response to their realised health depreciation, the amount of non-health consumption, the proportion of liquid wealth invested in stocks to optimise the utility from consumption, housing, and health. Entering the retirement period, at the age of 65, the individual starts claiming the old-age pension benefit and have to make a decision on health expenditure, non-health consumption, housing adjustments, and risky asset allocation.

Individual features of this model have appeared in the previous literature. In particular, several portfolio choice models allow health expenditure to respond endogenously to health risk, but they either focus on the retirement period or do not incorporate housing (Hugonnier et al., 2013; Yogo, 2016). Also, a large number of studies incorporate labour income risk in the analysis of household portfolio choice but overlook the impact of health risk on other financial decisions (e.g., Heaton & Lucas, 1997; Viceira, 2001; Cocco, 2005; Fagereng et al., 2017). Several papers investigate the joint decisions of housing and portfolio choice but do not take into account health-related investment decisions (Cocco, 2004; Yao & Zhang, 2005; Chetty et al., 2017).

3.3.1. State variables and control variables

3.3.1.1.State variables

During the working period, the investor acquires housing services by either renting (R_t^0=0) or owning a house (R_t^0=1). In order to become a homeowner, the agent has to meet the down payment requirement, which is to pay a fraction (δ) of the house value, and the rest can be financed through a mortgage. Also, in case the agent is a homeowner, there will always be costs incurred as a result of moving. A substantial fraction (***b***) of the market value of the current residence will be borne by the homeowner when he/she decides to sell the house. Hence, if the agent considers buying a house a consumption decision, the frequency of housing adjustments is often low over the life cycle. As moving is costly, I allow the agent to buy a house only once during the working years and do not allow him/her to move after buying the property. Additionally, in each period, a homeowner needs to spend a proportion (***tx***) of the house value to pay the maintenance costs and property taxes. Unlike homeowners, renters are not in charge of periodic maintenance costs and property taxes and can relocate without bearing any transaction costs. However, they have to pay a portion (***rent***) of the market value of the rental house each period.

With regard to health-related conditions, individuals enter each period with initial health capital (H_{t-1}), and health stock depreciates at a stochastic rate $0 \leq w_t \leq 1$, affecting both the marginal utility of consumption and the probability of dying.

The state-space $X_t = \left\{ P_t^Y, R_{t-1}^0, D_{t-1}, P_t^D, P_0^D, H_{t-1}, TW_t, yr_t \right\}$ consists of $\mathbf{TW_t}$ where $\mathbf{TW_t}$ is the total wealth net of housing liquidation costs at the beginning of period t, P_t^Y **-** permanent labour income in period t if ***t<t*** $t<t_R$ (t_R -retirement age) **or** the pension benefit in period t if $t>=t_R$, $\mathbf{D_{t-1}}$ - the size of an existing house if the agent is the homeowner at the beginning of period t, $\mathbf{P_t^D}$ **-** the price per unit of housing stock at the beginning of period t ($\mathbf{P_t^D D_{t-1}}$ is the market value of the agent's own house or rental house), $\mathbf{R_{t-1}^0}$ - the homeownership status at the beginning of period t which takes the value of 1 if the agent owns a house and 0 otherwise, $\mathbf{P_0^D}$ - the price per unit of housing stock at the time of buying, and yr_t – the remaining term of the mortgage at the beginning of period t in case the individual is a homeowner.

3.3.1.2.Control variables

If the agent is a renter in the previous period ($\mathbf{R_{t-1}^0}$**=0),** he/she needs to decide whether to adjust the housing stock and which size is optimal **(**D_t**).** If the agent is a homeowner in the previous period ($\mathbf{R_{t-1}^0}$**=1),** he/she will stay in place and make no housing adjustments. Moreover, individuals have to decide on how much they will invest in health capital in response to the realised health depreciation (I_t), how much to consume (non-health consumption-C_t), and how to optimally allocate the saving amount (A_t) into two different types of financial assets: a risk-free bond with a real gross return of R_f, and a risky stock with a real gross return of R_t^S. While R_t^S is assumed to follow a stochastic process, R_f is kept constant over time. The proportion of liquid wealth invested in stock is denoted by α_t . To sum up, control variables include $K_t = \left\{ C_t, D_t, I_t, A_t, \alpha_t \right\}$.

3.3.2. Labour income process

During the working years, the agent receives labour income $\mathbf{Y_t}$ in period $\boldsymbol{t}$. Before the retirement age $\boldsymbol{t_R}$, the labour income process is given by:

$$Y_t = P_t^Y \varepsilon_t \text{ and } P_t^Y = exp\{f(t, Z_t)\} P_{t-1}^Y \nu_t \quad (t < t_R)$$

where $\boldsymbol{f(t, Z_t)}$ is a deterministic function of age and other characteristics $\boldsymbol{Z_t}$ such as marital status and family size. P_t^Y is the permanent component of the agent's labour income, $\boldsymbol{\nu_t}$ and $\boldsymbol{\varepsilon_t}$ are the persistent shock and temporary shock to the labour income accordingly. $\mathbf{ln}\boldsymbol{\nu_t}$ and $\mathbf{ln}\boldsymbol{\varepsilon_t}$ are assumed to be i.i.d normally distributed shocks with $\ln \nu_t \sim N(-\sigma_\nu^2/2, \sigma_\nu^2)$ **and** $\ln \varepsilon_t \sim N(-\sigma_\varepsilon^2/2, \sigma_\varepsilon^2)$**.** In this model, I allow the shock to permanent income to correlate with the shock to the housing returns as well as the shock to stock returns.

After retirement at age 65 ($t>=t_R$), the agent receives an income stream under the form of old-age pension benefits, which is modelled as a constant fraction ***(θ)***-replacement rate- of his/her labour income in the last working year.

3.3.3. Health accumulation process and health expenditure

The health capital model builds on the groundwork of Grossman (1972). The representative agent starts the period t with the initial health capital $\boldsymbol{H_{t-1}}$ and realises the rate of health depreciation $\boldsymbol{w_t}$ ($0 \leq \boldsymbol{w_t} \leq 1$). The distribution $\boldsymbol{w_t}$ depends on the state space, which includes the initial health capital $\boldsymbol{H_{t-1}}$. The individual dies if $\boldsymbol{w_t} = \mathbf{1}$ when their health capital depreciates completely.

Following Yogo (2016), I allow health expenditure $\boldsymbol{I_t}$ to endogenously respond to the realised health depreciation. The key economic property that differentiates health capital from other financial assets is that health investment is irreversible, meaning individuals cannot reduce their health stock via negative expenditure. Another feature of health investment is that it is subject to decreasing returns in the sense that treatment for people in poor health has a much larger impact on their health condition than preventative care for people in good health.

Hence, the health accumulation process is modelled as below:

$$H_t = (1 - w_t)H_{t-1} + \psi[(1 - w_t)H_{t-1}]^{1-\psi} I_t^{\psi}$$

in which the parameter $\psi \in (0,1]$ features the law of diminishing marginal returns (Ehrlich & Chuma, 1990).

3.3.4. Housing Services and Mortgage

The investor in this study derives the utility from housing services via either renting or owning. I assume in the model that the quality of housing services corresponds to the size of the house **(D_t)**. The gross rate of return on the price per unit of the housing stock $R_t^D = P_{t+1}^D / P_t^D$ is modelled as follows:

$$R_t^D = \bar{R}_D \varepsilon_t^D$$

where $ln\varepsilon_t^D \sim N(-\sigma_D^2 / 2, \sigma_D^2)$ is independently and identically distributed, and σ_D^2 is the variance of the housing returns. The shock to housing returns is allowed to correlate with the shock to permanent labour income.

In order to buy a house, the agent needs to pay a required fraction - δ - of the house value (down payment) and finance the rest via a mortgage. I assume that the individual, at the time of house buying, only pays δ (%) of the house value and funds the remaining via a 16-year fixed-rate mortgage. If $P_0^D D_0$ is the house value at the time of buying, the mortgage debt balance at that time would be $(1-\delta)P_0^D D_0$. The annual discount rate is denoted by R_m. Then, the periodic debt repayment is given by: $(1-\delta)P_0^D D_0 r_m$ in which $r_m = 2 * R_m / (1 - 1/(1+R_m)^\wedge 16)$. Note that one period in the life-cycle model equals two years in real life.

3.3.5. Financial assets

The individual can invest non-negative amounts in the risky asset, which yields a stochastic gross rate of return R_t^S and in the risk-free asset that has a constant gross return R_f. The risky asset share in liquid wealth in period **t** (savings after all expenses are set aside - A_t) is denoted by α_t. Short-sales in both these two assets are ruled out: $0 \leq \alpha_t \leq 1$

The overall return on the investor's portfolio in period t (between the beginning of year t and the beginning of year t+1) will be:

$$R_t^p = R_f(1-\alpha_t) + R_t^S\alpha_t = R_f + \alpha_t(R_t^S - R_f)$$

In order to specify the stochastic process of stock returns**,** I construct the following model:

$$R_t^S = \bar{R}_s\varepsilon_t^s$$

The stochastic component of risky asset returns (ε_t^s) is assumed to be i.i.d log-normally distributed $\ln\varepsilon_t^s \sim N(-\sigma_s^2/2, \sigma_s^2)$ where σ_s^2 is the variance of the risky asset returns. Also, ε_t^S is set to correlate with the shock to permanent labour income.

This log-normal distribution rule guarantees that

$$\ln E_t\left[\varepsilon_t^s\right] = E_t\left[\ln(\varepsilon_t^s)\right] + \sigma_s^2/2 = -\sigma_s^2/2 + \sigma_s^2/2 = 0$$

3.3.6. Budget constraint

Each period, the agent starts with the total wealth TW_t, which includes non-housing financial wealth, labour income in working years or pension benefits in retirement, and home equity net of liquidation cost. Cash outflows in the corresponding period are limited within the total net wealth TW_t. These expenses consist of (i) consumption C_t; (ii) health expenditure I_t at the relative price Q_t (ii) rental cost $\alpha_r P_t^D D_t$ in case the agent is a renter; or maintenance cost and property taxes $tx P_t^D D_t$ in case the agent is a homeowner, (iii) mortgage debt repayment $P_0^D D_0(1-\delta)r_m$ in case the agent is a homeowner with a positive outstanding mortgage balance, (iv) home equity net of liquidation costs in case the agent is a homeowner $P_t^D D_t(1-b)-P_0^D D_0(1-\delta)r_m(yr_t-1)$; and (vi) savings after all expenses are set aside A_t.

In case the agent is a homeowner, the budget constraint is expressed as follows:

For $yr_t \geq 1$

$$TW_t = C_t + I_t + A_t + P_t^D D_t tx + P_0^D D_0(1-\delta)r_m + P_t^D D_t(1-b) - P_0^D D_0(1-\delta)r_m(yr_t-1)$$

For $yr_t = 0$

$$TW_t = C_t + I_t + A_t + P_t^D D_t tx + P_t^D D_t(1-b)$$

In case the agent is a renter, the budget constraint is expressed as:

$$TW_t = C_t + I_t + A_t + \alpha_r P_t^D D_t$$

3.3.7. The dynamics of financial wealth and total wealth

The financial wealth or savings after all expenses have been set aside in each period $\mathbf{A_t}$ will be allocated into two different types of financial assets, including a risk-free bond (R_f) and a risky stock (R_t^S). At the beginning of the next period ***t+1,*** the agent's financial wealth is $A_t\left[R_f+\alpha_t(R_t^S-R_f)\right]$

Hence, the total wealth net of house liquidation costs complies with the following dynamics:

If the agent is a homeowner, and $yr_t \geq 1$

$$TW_{t+1} = A_t(R_f+\alpha_t(R_t^S-R_f)) + P_t^Y \exp\left\{f(t+1,Z_{t+1})\right\}\nu_{t+1}\varepsilon_{t+1} + P_t^D D_t(1-b) - P_0^D D_0(1-\delta)r_m(yr_t-1)$$

If the agent is a homeowner, and $yr_t < 1$

$$TW_{t+1} = A_t(R_f+\alpha_t(R_t^S-R_f)) + P_t^Y \exp\left\{f(t+1,Z_{t+1})\right\}\nu_{t+1}\varepsilon_{t+1} + P_t^D D_t(1-b)$$

If the agent is a renter

$$TW_{t+1} = A_t(R_f+\alpha_t(R_t^S-R_f)) + P_t^Y \exp\left\{f(t+1,Z_{t+1})\right\}\nu_{t+1}\varepsilon_{t+1}$$

3.3.8. Objective function and individuals' optimisation problem

The agent derives utility from housing, health, and consumption. Following Yogo (2016), I model utility over consumption and housing using Cobb-Douglas function, and model utility from health and non-health consumption using the constant elasticity of substitution function:

$$\mathrm{U}(C_t, \mathrm{D}_t, H_t) = [(1-\alpha)(C_t^{1-\phi} D_t^{\phi})^{1-1/\rho} + \alpha H_t^{1-1/\rho}]^{1/(1-1/\rho)}$$

The parameter $\phi \in (0,1)$ is the utility weight on housing, the parameter $\alpha \in (0,1)$ is the utility weight on health, and the parameter $\rho \in (0,1)$ is the elasticity of substitution between health and non-health consumption.

The investor's bequest motive is represented by a function of bequeathed wealth, which comprises the non-housing wealth and the proceeds from liquidating the house in case the investor is a homeowner at the time of death. The utility over the bequest is computed as:

$$B(TW_t, \mathrm{P}_t^D) = \mathrm{TW}_t \left(\frac{\phi}{(1-\phi)\mathrm{P}_t^D}\right)^{\phi}$$

This indirect utility function corresponds to a Cobb-Douglas function over consumption and housing (i.e., $C_t^{1-\phi} D_t^{\phi}$), reflecting the fact that financial wealth and housing are not perfectly substitutable forms of bequest (Yao & Zhang, 2005; Yogo, 2016)

Over the life cycle, individuals have to choose the amount of health investment, housing tenure choice (rent or buy), housing stock, the amount of consumption, and proportion of savings invested in risky assets ($K_t = \{C_t, R_t^o, D_t, I_t, \alpha_t\}$) to maximise the expected discounted lifetime utility. The recursive definition of the corresponding value function is given by:

$$\mathrm{V}_t = \left\{ (1-\beta) U(C_t, \mathrm{D}_t, H_t)^{1-1/\sigma} + \beta E_t [p_{t+1} bstr^{\gamma} B(TW_{t+1})^{1-\gamma} + (1-p_{t+1}) V_{t+1}^{1-\gamma}]^{\frac{1-1/\sigma}{1-\gamma}} \right\}^{\frac{1}{1-1/\sigma}}$$

where the parameter $\beta \in (0,1)$ is the subjective discount factor, the parameter $bstr \geq 0$ is the strength of bequest motive, and p_{t+1} -the probability of dying in period t+1- is estimated based on the distribution of health depreciation rate. I use Epstein-Zin preferences instead of the Constant Relative Risk Aversion (CRRA) class of preferences to specify the objective function as I want to separate two distinct features of preferences: (i) risk aversion ($\gamma > 1$-relative risk aversion) – which signifies the preference for less uncertainty over more uncertainty, and (ii) intertemporal substitution ($\sigma > 0$- elasticity of intertemporal substitution) – which indicates the preference for current consumption over future consumption or vice versa.

3.3.9. The normalisation of the optimisation problem

The optimisation problem can be simplified by using **TW_t** – investor's total wealth net of liquidation costs – as a normaliser to reduce the number of variables in the state space if homogeneity of the objective function is kept. I redefine state variables and control variables of the life-cycle model as their values relative to total wealth.

New state variables include $y_t = \frac{P_t^Y}{TW_t}$, $\bar{d}_t = \frac{D_{t-1}P_t^D}{TW_t}$, $d_{0,t} = \frac{D_0 P_0^D}{TW_t}$, $\bar{h}_t = (1-w_t)\frac{H_{t-1}Q_t}{TW_t}$ (H_{t-1} is the health capital at the beginning of period **t**, Q_t is the relative price of health care), R_{t-1}^0 - the homeownership status at the beginning of period ***t***, which takes the value of 0 if renting and 1 if owning a house, yr_t is the remaining term of the mortgage contract at the beginning of period ***t***.

New control variables include $c_t = \frac{C_t}{TW_t}$, $i_t = \frac{Q_t I_t}{TW_t}$, $d_t = \frac{D_t P_t^D}{TW_t}$, $a_t = \frac{A_t}{TW_t}$, α_t

I set $\bar{h}_t = (1-w_t)\frac{H_{t-1}Q_t}{TW_t}$ and $h_t = \frac{H_t Q_t}{TW_t}$, hence, the normalised health accumulation process is:

$$h_t = \bar{h}_t + \psi(\bar{h}_t)^{1-\psi} i_t^{\psi}$$

In fact, to preserve the homogeneity of the objective function, three additional assumptions are necessary. First, the distribution of health depreciation rate w_{t+1} depends on initial health only through $\bar{h}_t$. Second, the distribution of health depreciation rate w_{t+1} depends on the state space $\{t, \bar{h}_t\}$, but not the other way around. Third, health insurance coverage, which is incorporated in the relative price of health care Q_t depends on initial health only through $\bar{h}_t$. The estimation of health transition probabilities, and health insurance coverage using data from the Panel Study of Income Dynamics is presented in Section 4.

The objective function after normalisation becomes:

$$v_t = \left\{ (1-\beta)\mathbf{u}_t^{1-1/\sigma} + \beta E_t[\mathbf{G}_t^{1-\gamma}(\mathbf{p}_{t+1}\, bstr^{\gamma}\, \mathbf{b}_{t+1}{}^{1-\gamma} + (1-p_{t+1})\mathbf{v}_{t+1}^{1-\gamma})]^{\frac{1-1/\sigma}{1-\gamma}} \right\}^{\frac{1}{1-1/\sigma}}$$

where

$$u_t = \frac{U(C_t, D_t, H_t)}{TW_t} = [(1-\alpha)(c_t^{1-\phi}(\frac{d_t}{P_t^D})^{\phi})^{1-1/\rho} + \alpha(\frac{h_t}{Q_t})^{1-1/\rho}]^{\frac{1}{1-1/\rho}}$$

$b_{t+1} = (\frac{\phi}{(1-\phi)\mathrm{P}_t^D})^{\phi}$, $G_t = \frac{TW_{t+1}}{TW_t}$, and $b_{t+1}G_t = \frac{B(TW_{t+1})}{TW_t}$

The budget constraint after normalisation is:

For homeowners

$$yr_t \geq 1: \; 1 = c_t + i_t + a_t + d_t tx + d_{0,t}(1-\delta)r_m + d_t(1-b) - d_{0,t}(1-\delta)r_m(yr_t - 1)$$

$$yr_t = 0: \; 1 = c_t + i_t + a_t + d_t tx + d_t(1-b)$$

For renters

$$1 = c_t + i_t + a_t + \alpha_r d_t$$

The dynamics of total wealth after normalisation are:

For homeowners with $yr_t \geq 1$

$$G_t = a_t(R_f + \alpha_t(R_t^S - R_f)) + y_t \exp\left\{ f(t+1, Z_{t+1}) \right\} \nu_{t+1}\varepsilon_{t+1} + d_t R_t^D(1-b) - d_{0,t}(1-\delta)r_m(yr_t - 1)$$

For homeowners with $yr_t < 1$

$$G_t = a_t(R_f + \alpha_t(R_t^S - R_f)) + y_t \exp\left\{ f(t+1, Z_{t+1}) \right\} \nu_{t+1}\varepsilon_{t+1} + d_t R_t^D(1-b)$$

If the agent is a renter

$$G_t = a_t(R_f + \alpha_t(R_t^S - R_f)) + y_t \exp\left\{ f(t+1, Z_{t+1}) \right\} \nu_{t+1}\varepsilon_{t+1}$$

The law of motion of the state variables after normalisation follows:

$$y_{t+1} = \frac{y_t}{G_t} exp\left\{ f(t+1, Z_{t+1}) \right\} \nu_{t+1} \varepsilon_{t+1}$$

$\bar{d}_{t+1} = \frac{R_t^D}{G_t} \bar{d}_t$ if the agent is a homeowner, $\bar{d}_{t+1} = \frac{R_t^D}{G_t} d_t$ if the agent is a renter, $d_{0,t+1} = \frac{1}{G_t} d_{0,t}$

$$\bar{h}_{t+1} = \frac{(1 - w_{t+1})Q_{t+1}\bar{h}_t}{Q_t G_t} (1 + \psi(\frac{i_t}{\bar{h}_t})^{\psi})$$

3.4. Calibrating the life-cycle model with the Panel Study of Income Dynamics

The Panel Study of Income Dynamics is designed to collect information on employment, income, wealth, education, and health of individuals and families in the United States every year between 1968 and 1999 and every two years starting from 2001. I merge family data with individual data using the unique family ID number in each interview year. The merged dataset, covering every two years from 1979 to 2017, is then used to measure the key inputs and outputs of the life-cycle model. Since data on health-related conditions of the head and wife (husband) of the household is not available for the period before 1999, I estimate health transition probabilities, the rate of health depreciation, the relative price of health care using the merged dataset spanning from 1999 to 2017. Whereas, to calibrate the labour income process, I use the merged dataset covering the period from 1979 to 2017.

3.4.1. Labour income process

I use the PSID merged dataset covering every two years between 1979 and 2017 to estimate the labour income process. In this section, I illustrate the description of the sample and estimation method. The labour income in this study is defined broadly as the total reported labour income (which includes wages/salaries, bonus, overtime, and commissions from main jobs and extra jobs) plus (i) workers compensation, (ii) unemployment compensation in case the individual is unemployed, (iii) supplemental security income for the aged, disabled people who have little or no income, (iv) social security, annuities and other pensions for retirees, (v) other welfare, (vi) child support, and (vii) total transfers (which includes helps from relatives and helps from others), all this for the head and wife (husband) of the household separately. Defining labour income this way helps avoid overstating labour income risk since there might be multiple ways of self-insuring against pure labour income shocks, for example, various welfare programs or helps from relatives and friends. Next, I deflate the labour income using the Consumer Price Index from 1979 to 2017 with 1979 as the base year.

The sample spans from 1979 to 2017, so individuals appear at most 20 times. From that point, I only keep those with at least six years of information available and estimate an unbalanced panel. Also, as there are few observations for households with female heads, in this study, I target households with male heads only to estimate the labour income process. From the new sample, I eliminate students, retirees, housewives, and permanently disabled people to estimate the structure of the labour income shocks.

Following Cocco (2005), to control for education, I split the sample into three groups: individuals who have not finished high-school education, individuals who have finished high-school education but have not finished college education, and individuals who are college graduates. In a fixed-effects context, it is not allowed to have education endogenously changing over the life cycle. Since few individuals are found to change their education group, I drop observations who experienced changes in their education level during the examined period.

$$Y_{it} = P_{it}^{Y} \varepsilon_{it} \text{ and } P_{it}^{Y} = exp\left\{ f(t, Z_{it}) \right\} P_{i,t-1}^{Y} \nu_{it}$$

For each education group, I assume that t (age) and Z_{it} (personal characteristics other than age and fixed family effects) are additively separable. Vector Z_{it} includes marital status and family size.

I then regress the logarithm of deflated labour income on a full set of year dummies, age dummies, family size, and marital status for three educational groups (Table 3.1). The coefficients of the age dummies are significant, and the results are consistent with the findings in existing literature (Cocco, 2005; Love, 2013).

Table 3.1: Labour Income process: fixed-effects regression

Independent variable	No high school		High school		College	
log real income	coefficient	t-statistic	Coefficient	t-statistic	Coefficient	t-statistic
Family size	-0.0505	-2.12	-0.0161	-2.42	-0.0561	-4.51
Marital Status	0.3961	3.49	0.1136	2.54	0.2610	4.51
Constant	8.3738	18.64	8.7845	69.87	8.0748	18.63
Age dummies	Yes		Yes		Yes	
Year dummies	Yes		Yes		Yes	
n	896		9113		4159	
T-bar	13		13.3		13.2	
σ_{ε}^{2}	0.4601		0.2486		0.3007	
R2-within	9.32		9.07		21.24	
F-statistic	1.19		12.63		18.84	

Source: Author's calculation

The age dummies are then fitted by a third-order polynomial and a fifth-order polynomial to obtain the age-earnings profile to solve the optimisation model. (Table 3.2)

Table 3.2: Labour Income process: coefficients in the age polynomial

	No high school		High School		College	
	3rd order	5th order	3rd order	5th order	3rd order	5th order
Constant	-2.1827	-25.3467	-0.6728	-0.9191	-0.6103	-14.0891
Age	0.1867	3.1918	0.0519	0.0004	0.1030	1.7573
Age^2/10	-0.0396	-1.5379	-0.0015	0.0686	-0.0059	-0.7932
Age^3/100	0.0026	0.3627	-0.0005	-0.0286	-0.0005	0.1815
Age^4/1000		-0.0419		0.0046		-0.0205
Age^5/10000		0.0019		-0.0003		0.0009
Replac. rate	0.7651	0.7651	0.5974	0.5974	0.5558	0.5558

Source: Author's calculation

The endogenous variable in the fitted polynomials is the sum of all age dummies estimated in the first stage fixed-effects regression shown in Table 3.1. The exogenous variables are the third-order (fifth-order) age polynomials for the working life period. The replacement ratio, which measures retirement income as a fraction of pre-retirement income, is calculated as the ratio of the average of the above-defined labour income for retirees in a specific education group to the average of labour income in the last working year.

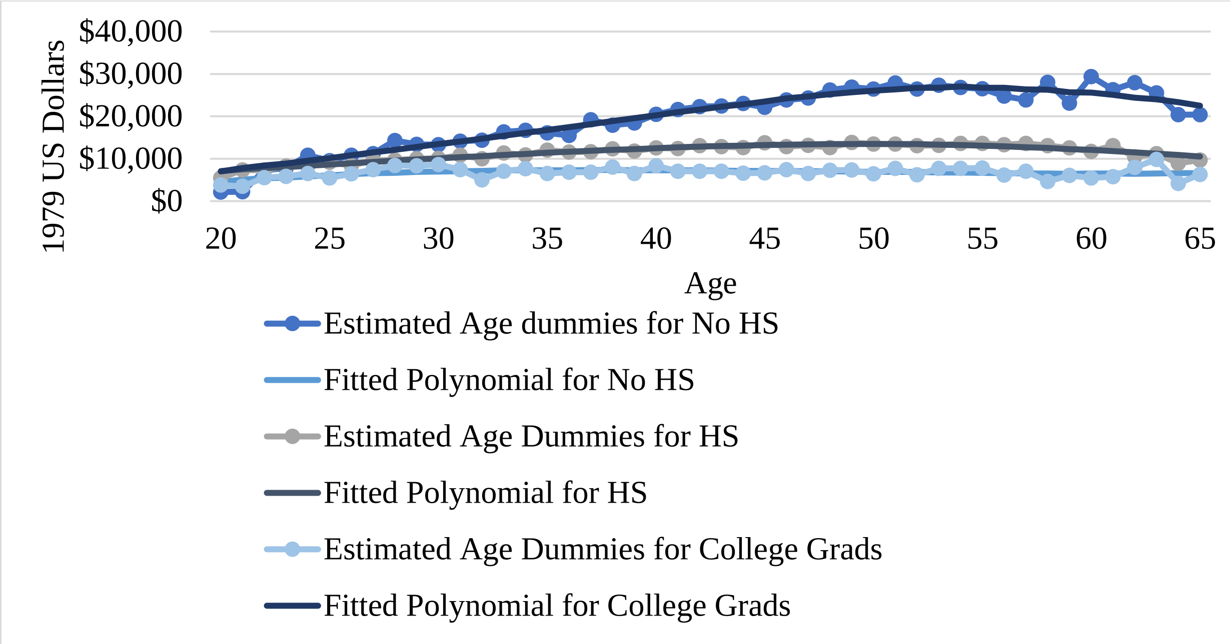


Figure 3.1: Labour income processes estimated from the PSID for the three different education groups: individuals without high school education, individuals with high school education but without a college degree, and college graduates. For each education group, the solid line plots the average earnings profile with respect to the estimated age dummies from the first stage fixed-effects regression (Table 3.1) and the connected scatterplot displays the average age-income profile with respect to the fitted third-order polynomial from Table 3.2. *Note:* No HS= without high school diploma, HS= high-school graduates, College Grads= College graduates.

Figure 3.1 depicts the resulting average age-earnings profile for each of the three education groups, which generally follows the hump-shaped pattern. Incomes for the group with no high school education and the group with college dropouts closely track one another despite a level difference, whereas college graduates have a much steeper earnings profile earlier in life.

Next, I examine the error structure of the labour income process. The estimation routine follows the variance decomposition method described in Carroll & Samwick (1997) and Cocco et al. (2005). Table 3.3 reports the estimated transitory and permanent shocks to labour income for each of the three education groups. I define r_{id} as: $r_{id} \equiv log(Y^*_{i,t+d}) - log(Y^*_{it})$ with $d \in \{1,2,...,19\}$ and $log(Y^*_{it}) \equiv log(Y_{it}) - \hat{f}(t, Z_{it})$. From there, I get $Var(r_{id}) = d * \sigma^2_\varepsilon + 2 * \sigma^2_v$. As the PSID sample spans over 20 waves every two years between 1979 and 2017, I include all possible series of r_d to get the estimates of σ^2_ε and σ^2_v by running the OLS regression of $Var(r_{id})$ on d and a constant term.

In this study, I restrict the estimates of σ_{ε}^{2} and σ_{v}^{2} to be the same across individuals in a particular education group.

Table 3.3: Variance decomposition and correlation with stock returns

	No high school	t-statistic	High School	t-statistic	College	t-statistic
Transitory (σ_{ε}^{2})	0.1093	1.67	0.0833	1.87	0.0814	1.63
Permanent (σ_{v}^{2})	0.0463	1.94	0.0361	2.47	0.0331	2.37

Source: Author's Calculation

3.4.2. Health transition probabilities

I use the PSID merged dataset covering every two years between 1999 and 2017 to estimate the health transition probabilities. I only keep observations that are heads and spouses in families since health-related data is solely available for heads and spouses. The primary measure of health in this study is the self-assessed general health status. At each interview, individuals' health can be either poor, fair, good, very good, or excellent. To augment the health status with an additional sixth state – dead, I use the sequence number in the family data file each year, which provides a means of identifying (i) an individual's status with regard to their family unit, and (ii) whether the individual died by the time of the current interview. From the merged dataset, I also create the dummies for doctor-diagnosed health issues, smoking, and difficulty with several daily activities.

In each period t, individuals report their health status h_t^*. Health status is determined based on a latent variable h_t, which captures the unobservable health, through a response function:

$$h_t^* = \begin{cases} 0 \quad \text{Dead} & \text{if } \bar{h}_t < \bar{h}_1 \\ 1 \quad \text{Poor} & \text{if } \bar{h}_1 \le \bar{h}_t < \bar{h}_2 \\ 2 \quad \text{Fair} & \text{if } \bar{h}_2 \le \bar{h}_t < \bar{h}_3 \\ 3 \quad \text{Good} & \text{if } \bar{h}_3 \le \bar{h}_t < \bar{h}_4 \\ 4 \quad \text{Very Good} & \text{if } \bar{h}_4 \le \bar{h}_t < \bar{h}_5 \\ 5 \quad \text{Excellent} & \text{if } \bar{h}_5 \le \bar{h}_t \end{cases}$$

Table 3.4: Future health status in relation to present health status

Future health	(1)				(2)			
Explanatory Variable	Coefficient for females		Interaction effect for males		Coefficient for females		Interaction effect for males	
	Coeff.	z	Coeff.	z	Coeff.	z	Coeff.	z
Health status								
1. Poor	-1.2464***	-3.71	0.3426	0.68	-0.7617*	-2.24	-0.7411	-1.3
2. Fair	-0.7920***	-4.64	0.3898	1.59	-0.7304***	-4.26	-0.4023	-1.06
4. Very Good	0.3111**	2.62	0.3192*	2.37	0.3301**	2.78	-0.4112	-1.30
5. Excellent	0.6972***	4.69	0.1569	0.87	0.7544***	5.06	-0.5736	-1.71
Age	-0.0154***	-11.91	-0.0060***	-3.10	-0.0108***	-7.88	-0.0045	-2.19
Age*Healthstatus								
1. Poor	0.0067	1.53	-0.0159*	-2.21	0.0065	1.48	-0.0109	-0.51
2. Fair	-0.0003	-0.13	-0.0023	-0.59	0.0012	0.48	-0.0027	-1.31
4. Very Good	0.0053***	3.00	-0.0007	-0.25	0.0040*	2.23	-0.0013	-1.27
5. Excellent	0.0106***	4.53	-0.0026	-0.78	0.0076***	3.21	-0.0030	-1.82
LnWealth	0.0306***	3.90	0.0177	1.50	0.0252***	3.19	0.0211	1.78
LnWealth*Healthstatus								
1. Poor	-0.0555*	-2.40	0.0522	1.41	-0.0763***	-3.28	0.0600	1.61
2. Fair	-0.0088	-0.60	-0.0069	-0.29	-0.0116	-0.8	-0.0027	-0.12
4. Very Good	0.0245*	2.25	-0.0253	-1.55	0.0245*	2.24	-0.0252	-1.54
5. Excellent	0.0506***	3.62	-0.0028	-0.14	0.0513***	3.67	-0.0043	-0.21
famsize	-0.0174***	-3.26			-0.0220***	-4.09		
Smoking	-0.1180*	-2.51	0.0547	0.83	-0.1189*	-2.52	0.0822	1.25
Smoking*Healthstatus								
1. Poor	0.2461	1.54	0.0004	0.00	0.2104	1.31	0.0201	0.09
2. Fair	0.1180	1.34	-0.0473	-0.37	0.1422	1.6	-0.0548	-0.43
4. Very Good	-0.1465*	-2.24	0.1633	1.80	-0.1547*	-2.36	0.1796	1.97
5. Excellent	-0.2018*	-2.15	-0.0964	-0.78	-0.2013*	-2.14	-0.0697	-0.56
Doctor-diagnosed issues								
1.WthStroke					-0.0137	-0.21	0.1127	1.13
1.WthHeartAttack					-0.0931	-1.12	0.0551	0.55
1.WthHeartDisease					0.0349	0.57	0.1240	1.52
1.WthHighBloodPressure					-0.1337***	-5.27	0.0171	0.48
1.WthAsthma					-0.1376***	-4.11	-0.0557	-1.07
1.WthLungDisease					-0.1663***	-3.28	-0.0819	-1.05
1.WthDiabetes					-0.2531***	-6.68	-0.0287	-0.53
1.WthArthritis					-0.1208***	-4.03	0.0663	1.46
1.WthCancer					-0.0297	-0.7	0.0838	1.3
Difficulty with daily activities								
1.WthProbBathing					-0.1013	-1.21	0.1058	0.70
1.WthProbDressing					-0.1390	-1.52	-0.0624	-0.41
1.WthProbEating					0.0711	0.51	0.3275	1.49
1.WthProbWalk					-0.1515***	-3.12	-0.0169	-0.22
Observations	36,993				36,993			
Pseudo R2	0.1864				0.1923			

Source: Author's Calculation

*Statistical significance is identified as follows: *p<0.05, **p<0.01, ***p<0.001.*

I use an ordered probit model to estimate the future health status two years from the current interview from a set of explanatory variables, which include the current self-reported health status, age, family size, the logarithm of total net wealth, smoking and their interaction with current health status. All of these independent variables are then interacted with a male dummy. I also introduce additional explanatory variables such as dummies for doctor-diagnosed health issues and difficulty with several daily activities in another regression to see whether the present health state can fully explain heterogeneity in the future health state.

Columns (1) and (2) of Table 3.4 respectively report the estimated coefficients and z-statistics for the ordered probit model without and with the presence of dummies for doctor-diagnosed issues and difficulty with daily activities. I choose the health state "Good"- $h_t^* = 3$ as the reference state in both models, explaining why there are only four states of present health status ($h_t^* = 1,2,4,5$) presenting in the regression results. A negative coefficient on the predictor signifies a higher probability of the "Dead" health state ($h_t^* = 0$), while a positive coefficient on the predictor implies a higher probability of the "Excellent" health state ($h_t^* = 5$). For instance, age is a statistically significant predictor of future health status, and the negative coefficient on age indicates that older individuals are more likely to die.

I control for total net wealth ***(TW$_t$)*** since the relevant measure of health ***(i.e. w$_t$)*** in the life-cycle model depends on the state space ***(x$_t$)***, which includes total net wealth. As the total net wealth belongs to the whole family of the examined individual, I also incorporate family size as an explanatory variable in the regression. The results show that coefficients on total net wealth and family size are statistically significant. The positive sign of coefficients on the total net wealth points out that people with a higher level of net wealth are more likely to stay in the best health state ($h_t^* = 5$) in the next period. Whereas, the negative sign of the coefficient on family size implies that all things equal, an individual living in a family with a bigger size is more likely to stay in the worst health state ($h_t^* = 0$) in the next period.

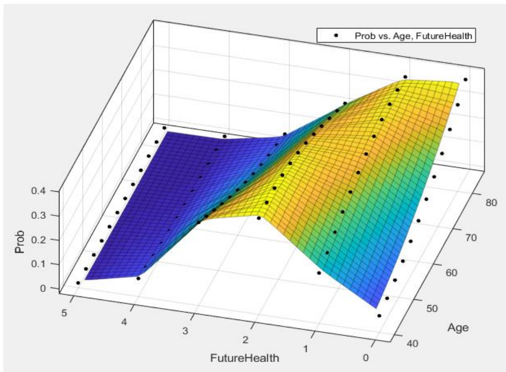
The coefficient on the present health status has a slightly different interpretation. The negative coefficients for "Poor" and "Fair" health states ($h_t^* = 1,2$) demonstrate that compared to individuals in the reference health state ($h_t^* = 3$), people currently in poor and fair health have a higher probability of dying in the next period. The positive coefficients for "Very Good" and "Excellent" health states ($h_t^* = 4,5$) suggest that compared to people in the reference health state, individuals who are currently in very good and excellent health are more likely to be in the best health state in the next period.

Furthermore, smoking is a negative and statistically significant predictor of future health, indicating that compared to non-smokers, people who smoke are more likely to die in the next period. I let age, total net wealth, labour income, and smoking interact with present health status to test whether the marginal effects of these predictors on future health vary with the current health status. For example, the age effect on future health is weaker for people currently in the "Very Good" and "Excellent" health status than for those in the "Good" health status. Additionally, the interaction with a male dummy allows for the possibility that the marginal effects of all the explanatory variables on the future health change across genders.

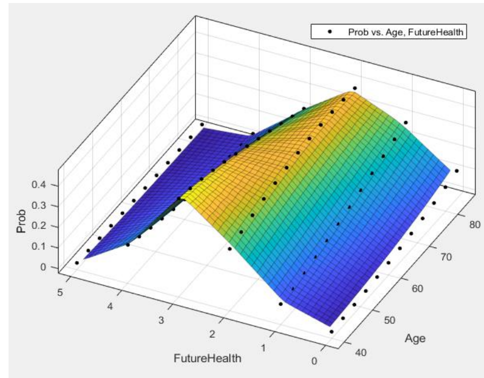
I introduce doctor-diagnosed issues and difficulty with daily activities as extra measures of present health status. As a result, dummies for high blood pressure, asthma, lung disease, diabetes, arthritis, and walking difficulties are statistically significant predictors of the future health state, suggesting that the self-reported health status does not fully grasp heterogeneity in health. However, after controlling for the doctor-diagnosed issues and difficulty with daily activities, coefficients on current self-reported health status remain statistically significant.

The estimates from column (1) of Table 3.4 are then used to predict the dynamics of health for an individual, who (i) is between 40 to 84 years old, (ii) has a family size of one person, and the average total net wealth and labour income for his/her age, (iii) is a non-smoker at the time of the interview, and (iv) has not used health care services two years before the interview, hence has no information on doctor-diagnosed issues. Figure 3.2 depicts the health transition probabilities for individuals by present health status and age. It shows that health status is persistent, and current health status is an important explanatory variable of future mortality. At any given age, compared to people currently in a better health state, individuals who are in poor health are more likely to die in the next period. By contrast, people in excellent health are less likely to die in the next period.

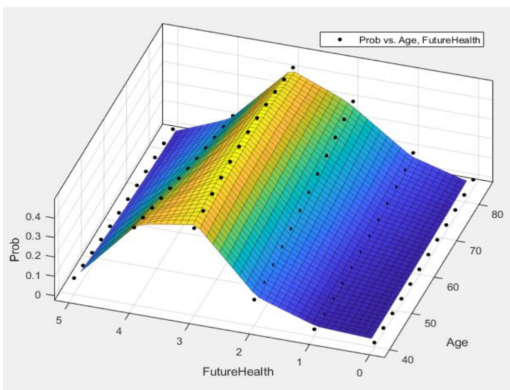
Current health status H_t=1 (Poor)



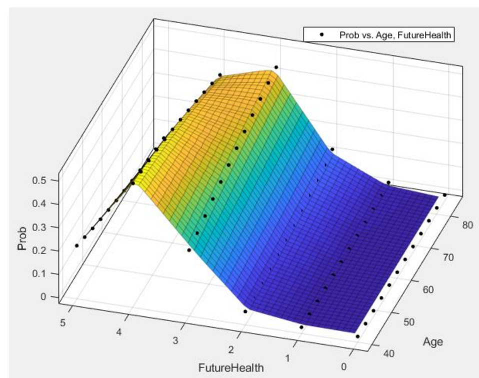
Current health status H_t=2 (Fair)



Current health status H_t=3 (Good)



Current health status H_t=4 (Very Good)



Current health status H_t=5 (Excellent)

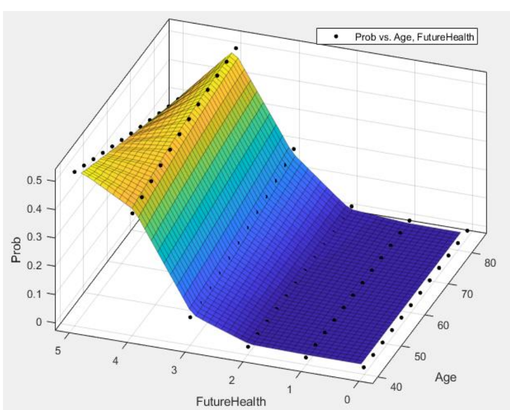


Figure 3.2: Health transition probabilities for individuals by current health status and age. The probabilities of future health status are estimated from the ordered probit model presented in column (1) of Table 3.4. The predicted probabilities are for individuals, who (i) are between 40 to 84 years old, (ii) have a family size of one person, the average total net wealth and labour income for his/her age, (iii) are non-smokers at the time of the interview, and (iv) have not used health care in the two years before the interview, hence have no information on doctor-diagnosed issues.

The predicted transition probability from the health state ***i*** in period **t** to the health state ***j*** in period **t+1** in the absence of health investment is denoted as $Pr(h^*_{t+1} = j \mid h^*_t = i)$. After estimating the dynamics of health, I calibrate the health depreciation rate at period t+1 before making any health investment:

$$w_{t+1} = \begin{cases} 0 & \text{With } Pr(h^*_{t+1} = 0 \mid h^*_t = i) \\ (\bar{h}_i - \bar{h}_1)/((\bar{h}_5 - \bar{h}_1)*5/4) & \text{With } Pr(h^*_{t+1} = 1 \mid h^*_t = i) \\ (\bar{h}_i - \bar{h}_2)/((\bar{h}_5 - \bar{h}_1)*5/4) & \text{With } Pr(h^*_{t+1} = 2 \mid h^*_t = i) \\ (\bar{h}_i - \bar{h}_3)/((\bar{h}_5 - \bar{h}_1)*5/4) & \text{With } Pr(h^*_{t+1} = 3 \mid h^*_t = i) \\ (\bar{h}_i - \bar{h}_4)/((\bar{h}_5 - \bar{h}_1)*5/4) & \text{With } Pr(h^*_{t+1} = 4 \mid h^*_t = i) \\ (\bar{h}_i - \bar{h}_5)/((\bar{h}_5 - \bar{h}_1)*5/4) & \text{With } Pr(h^*_{t+1} = 5 \mid h^*_t = i) \end{cases}$$

Conditional on the health status $\bar{h}_i \leq h_t < \bar{h}_{i+1}$ in period t

3.4.3. The relative price of health care

In the PSID merged dataset from 1999 to 2017, respondents also provide information about (i) the total amount of health insurance premiums, (ii) the out-of-pocket expenditure for nursing home and hospital bills, (iii) the out-of-pocket expenditure for doctors, outpatient surgery and dental bills, (iv) the out-of-pocket expenditure for prescriptions, in-home medical care, special facilities, and other services as well as (v) the total medical expenditure including both the total out-of-pocket expenditure and the costs covered by Medicare, Medicaid or other health insurance. These health care expenditures are reported at the family level for two combined years prior to the time of the interview. Hence, to capture the correlation between health status and medical spending, I examine single persons only and eliminate observations with a family size of two persons and over. I define the relative price of health care as the ratio of the out-of-pocket medical expenditure[57] to the total medical expenditure[58]. I then use a Tobit model to estimate the out-of-pocket health expenditure share in relation to current health status, age, and total net wealth (Table 3.5).

[57] Out of pocket health expenditures include health insurance premium, expenditures for nursing home, hospital bills, doctor, outpatient surgery and dental bills, for prescriptions, in-home medical care, special facilities, and other services – that are not covered by health insurance.

[58] Total medical expenditures include out of pocket health expenditures and health costs covered by health insurance (both private and public health insurance)

Table 3.5: Estimation of the out-of-pocket medical expenditure share

Explanatory variables		Out of pocket health expenditure/Total medical expenditure: Coefficient	t-statistic
Health status			
	1. Poor	-0.1457	-3.47
	2. Fair	-0.0604	-2.13
	4. Very Good	0.0608	2.54
	5. Excellent	0.0827	2.66
Age		0.0035	5.29
LnWealth		0.0051	3.80
Constant		0.2900	7.82

This table reports the estimation results of a Tobit model for the proportion of total medical expenditure that is paid out-of-pocket. Independent variables include the current self-reported health status, age, the logarithm of net wealth. The sample consists of individuals with a family size of one person.

The out-of-pocket medical expenditure share is then predicted for individuals, who (i) are between 40 to 84 years old, and (ii) have the average total net wealth for their age. Let $q_t(h_t^*)$ be the predicted out-of-pocket medical expenditure share for health status h_t^* in period **t**. I model the relative price of health care as: $Q_t = e^{q(1-t)} q_t(h_t^*)$ with $e^{q(1-t)}$ referring to as the growth in the relative price in health care. q is calibrated as the average log growth rate of the consumer price index for medical care relative to the consumer price index for all items less medical care.

Figure 3.3 illustrates the relative price of health care by current health status and age. The results show that the relative price of health care rises with health status. For instance, the relative price of health care is roughly 0.3 for individuals in poor health at age 40, which is significantly lower than 0.53 for those in excellent health at the same age. Indeed, the health insurance coverage in the poor health status is better than that in the excellent health status. This result is consistent with the fact that health insurance plans which also cover preventive services are more expensive than those with treatment services only. The relative price of health also increases in age. For example, the relative price of health care for individuals in the excellent health status at age 40 is 0.53, while that for individuals in the excellent health status at age 56 is 0.8.

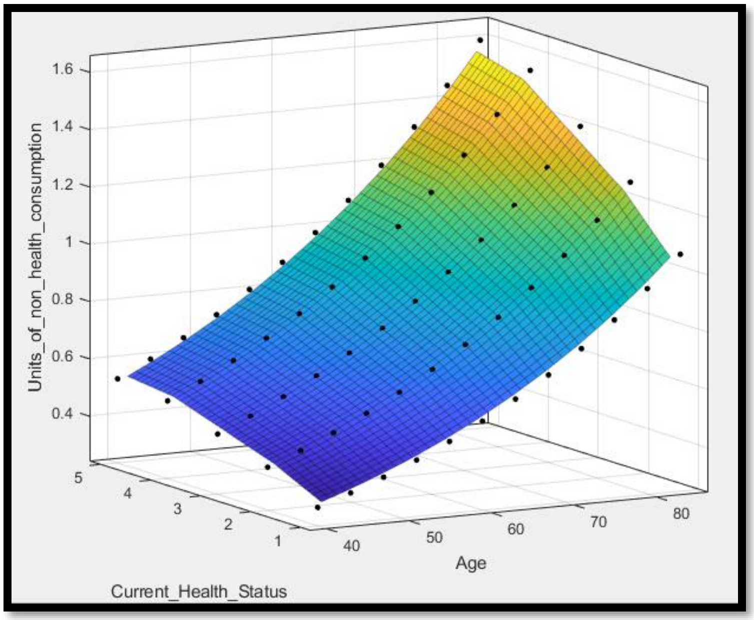


Figure 3.3: Relative price of health care based on q=1.9% per year

3.5. Asset allocation in the life-cycle model

3.5.1. The policy function of optimal consumption and portfolio choice

Table 3.6 summarises parameters used to calibrate the life-cycle model for homeowners and renters. Following the common practice in the life-cycle portfolio choice literature (Cocco et al., 2005; Love, 2013), I set the subjective discount factor equal to β=0.96 annually. Similarly, the relative risk aversion is fixed at γ=5 in the benchmark case to reflect the low portfolio share in stocks. Other preference parameters, which include the elasticity of intertemporal substitution, the utility weight on housing, the utility weight on health, and the elasticity of substitution between health and non-health consumption, are set to match with those in Yogo (2016). However, the utility weight on housing in this study is set at 0.6 for households with male heads, which is lower than the level of 0.9 in Yogo (2016). The reason is that while Yogo (2016) solves the portfolio problem in retirement, this research covers both the working and retirement period, resulting in a lower portfolio share in housing and a lower utility weight on housing.

Following Lockwood (2018), the strength of bequest motive *bstr* is determined as $bstr = \frac{m}{1-m}$ in which m ∈ [0, 1) is the marginal propensity to bequeath out of wealth. In the benchmark model, m is set at 0.5, the strength of bequest motive is 1 correspondingly.

Next, the age-earnings profiles and the structure of labour income shocks estimated in the previous section are used to calibrate the labour income profiles of high-school and college graduates in the life-cycle model. I allow the permanent shock to labour income to correlate with shock to housing return and shock to stock return.

I calibrate stock returns with $\overline{R_s}$ =8% and σ_s =0.18 annually. I assume an equity premium of 5.5%, which is higher than the historical estimate of 4.5% from 1958 to 2008 in Yogo (2016). Housing returns in the model are calibrated with $\overline{R_D}$ =3% and σ_D =0.031 annually. To become a homeowner, the agent has to satisfy the down payment requirement, which equals 40% of the house value at the time of buying. The remaining is financed via a 16-year fixed-rate mortgage with an annual mortgage rate of 3%. Moreover, each year, homeowners have to pay for the maintenance cost and property tax, which altogether account for 2.5% of the current house value. The liquidation cost associated with selling the house is set at 6% of the current house value. For renters, I assume an annual rental cost equals 6% of the current house value.

Finally, health parameters are drawn from a log-normal distribution $\mathbf{\log h_t \sim (\mu_h, \sigma_h)}$ to match the observed health distribution of household heads at age 40, and following Yogo (2016), I set the rate of return to health investment equal to 0.2.

Table 3.6: Parameters in Life-cycle model

Parameter	Symbol	Value
Preferences		
Subjective discount factor	β	0.96
Elasticity of intertemporal substitution	σ	0.5
Relative risk aversion	γ	5
Utility weight on housing	ϕ	0.6
Utility weight on health	α	0.1
Elasticity of substitution between non-health consumption and health	ρ	0.7
Strength of the bequest motive	bstr	1
Labour income profile of college graduates		
Standard deviation of permanent shock to labour income	σ_v	0.18
Standard deviation of temporary shock to labour income	σ_ε	0.28
Correlation of housing return and permanent shock to labour income	ρ_{Dv}	0.2
Correlation of stock return and permanent shock to labour income	ρ_{Sv}	0.15
Correlation of housing return and stock return	ρ_{Ds}	0
ReplacementRate		0.5558
Labour income profile of high-school graduates		
Standard deviation of permanent shock to labour income	σ_v	0.19
Standard deviation of temporary shock to labour income	σ_ε	0.29
Correlation of housing return and permanent shock to labour income	ρ_{Dv}	0.2
Correlation of stock return and permanent shock to labour income	ρ_{Sv}	0.15
Correlation of housing return and stock return	ρ_{Ds}	0
ReplacementRate		0.5974
Financial assets		
Inflation rate	inf	0.02
Average risk-free return	R_f	0.025
Average stock return	$\overline{R}_s$	0.08
Standard deviation of stock returns	σ_s	0.18
Housing		
Down payment requirement		0.4
Average housing return	$\overline{R}_D$	0.03
Standard deviation of housing returns	σ_D	0.031
Annual maintenance cost and property tax	tx	0.025
Liquidation cost		0.06
Annual rental cost	Rrent	0.06
Annual mortgage rate	Rm	0.03
Mortgage term (years)		16
Health		
Average of log health	μ_H	-10
Standard deviation of log health	σ_H	1.9
Returns to health investment	ψ	0.2

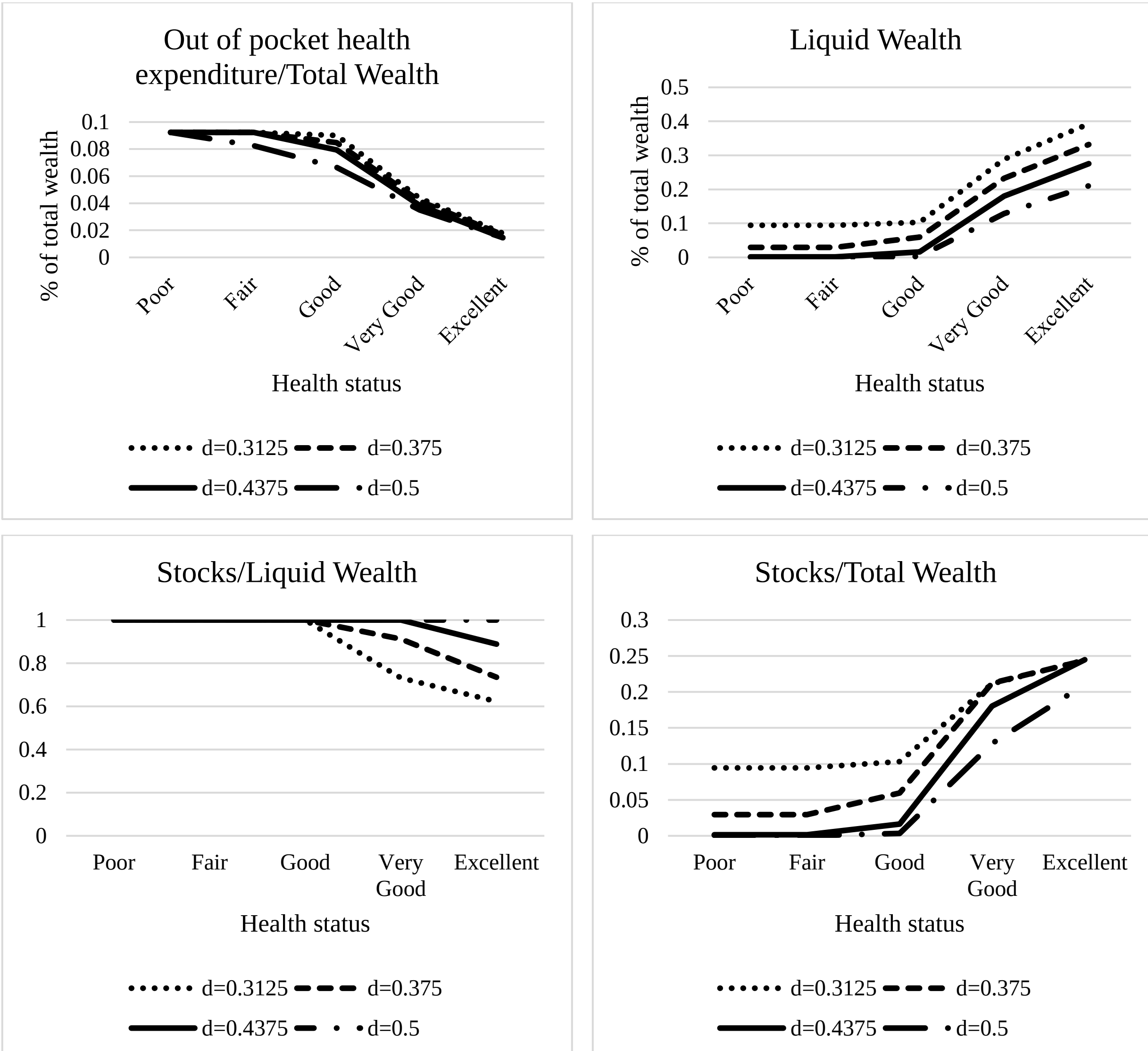


Figure 3.4: Optimal out of pocket health expenditure and portfolio rules for homeowners. The optimal health consumption and portfolio rules for homeowners at age 60 are reported as functions of self-perceived health status. The baseline case corresponds to d_t=0.375, and higher (lower) share of housing wealth corresponds to d_t=0.4375, d_t=0.5 (d_t=0.3125). The labour income-to-total wealth ratio and the remaining years of mortgage debt are fixed at y_t=0.15 and yr_t=0, respectively.

I solve the optimisation portfolio choice problem separately for homeowners and renters using numerical dynamic programming, as explained in Appendix C. Figure 3.4 illustrates the optimal rules of health consumption and the optimal stock allocation for homeowners as functions of the self-perceived health status. The optimal out-of-pocket health expenditure decreases in health status. In other words, better health conditions are associated with a lower share of total wealth spent on health care. A possible clarification is given by the fact that treatment in poor health often has a much larger impact on health than preventive care in excellent health. As also captured in

Figure 3.4, a higher share of housing wealth in total wealth is associated with a lower share of total wealth invested in health care, reflecting the crowding-out effect of housing investment on health investment. However, this crowding-out effect is mild. Increasing the housing wealth share from dt=0.3125 to dt=0.5 decreases the optimal out of pocket health expenditure share in total wealth of a homeowner in fair health at age 60 by approximately 1.5 percentage points. In this model, health expenditure is an endogenous response to a change in health status. Besides, people tend to finance a large medical expenditure out of their liquid wealth instead of using illiquid assets (Berkowitz & Qiu, 2006). Hence, a negative relationship between the optimal out of pocket health expenditure (as a share of total wealth) and the self-reported health condition can translate into a positive relationship between the liquid wealth (as a share of total wealth) and the health status, which can be referred to as the crowding-out effect of medical expenditure on liquid wealth.

The optimal share of stocks in financial wealth decreases in health status. To understand this relationship, it is useful to recall that although labour income during the working period is uncertain, it is more bond-like than stock-like and often considered a risk-free asset in the household portfolio. As such, a greater share of labour income in financial wealth would tilt investors towards a higher proportion of financial wealth held in stocks. Since liquid wealth as a share of total wealth increases in health status, the same labour income as a share of total wealth would yield a lower share of labour income in financial wealth as health status moves from poor to excellent. Better health is, therefore, associated with a more conservative liquid portfolio, as shown in Figure 3.4.

By contrast, the optimal share of stocks in total wealth increases in health. First, the stock share in total wealth is defined by multiplying the liquid wealth share in the total wealth with the stock share in the liquid wealth. Second, these two ingredients react differently to changes in health. Hence, the mechanism with a stronger effect would determine the sign of the relationship between the optimal share of stocks in total wealth and health status. The solution to the life-cycle model shows that as health status moves from poor to excellent, the crowding-out effect of medical expenditure on liquid wealth is much stronger, resulting in the positive relationship between the proportion of total wealth in stocks and health status.

Figure 3.5 depicts the optimal rules of health consumption and portfolio choices for renters as functions of the self-reported health status. The following discussion will focus on the baseline case evaluated at age 40, dt=0.5 (the value of rental house-to-total wealth ratio), and yt=0.5 (the labour income-to-total wealth ratio). In general, the direction of the relationship between each control variable (including out-of-pocket health expenditure, liquid wealth and stocks) and health status is the same for both renters and homeowners. However, for renters, their total wealth is not tied up in housing assets. Hence, I would expect a larger crowding-out effect of health expenditure on liquid wealth as well as a larger diversification effect of labour income on stocks in the liquid portfolio in the case of renters than in the case of homeowners, as shown in Figure 3.5.

Figure 3.5: Optimal out of pocket health expenditure and portfolio rules for renters. The optimal health consumption and portfolio rules for renters at age 40 are reported as functions of self-perceived health status. The baseline case corresponds to d_t=0.5 (the value of rental house-to-total wealth ratio), and the labour income-to-total wealth ratio is fixed at y_t=0.5.

3.5.2. Simulation results

On the basis of the health distribution at a given age and the value range of state variables in the 1999-2017 PSID data, the policy functions of the life-cycle model are used to simulate a population of 100,000 individuals starting at the age of 40. Table 3.7 reports the mean of stock share in total wealth separately for homeowners and renters at various ages over the life cycle. The simulation results show that the stock share in total wealth is low overall for both homeowners and renters. However, the optimal share of stocks in total wealth is significantly lower for renters. For instance, I find that while the average share of total wealth held in stocks for homeowners in excellent health at age 60 is 32%, that figure for renters in excellent health at the same age is only 12%. Besides, consistent with the life-cycle portfolio choice literature, the results confirm a hump-shaped pattern in age of the stockholding profiles with the optimal risky asset share increasing in age during the working phase while decreasing in age during the retirement period for both homeowners and renters. Most importantly, the portfolio share in stocks is positively related to health status at each age for both homeowners and renters. Nevertheless, renters' stock share in total wealth responds more strongly as health status deteriorates from "Good" to "Poor". Compared to homeowners, renters have a significantly larger percentage of total wealth held in liquid assets. Accordingly, if labour income is comparable between homeowners and renters, renters tend to have a more conservative portfolio choice. When health condition moves from "good" to "poor", a change in the proportion of stocks in total wealth is, therefore, often more significant for renters than for homeowners.

Table 3.7: Stock allocation for homeowners and renters in the simulated model

	Age				
	40	52	60	72	80
Panel A: Stock allocation for homeowners (% of financial and housing wealth)					
Poor	7	8	14	2	5
Fair	6	13	17	14	11
Good	13	20	20	18	15
Very Good	14	26	31	19	17
Excellent	19	28	32	22	21
Panel B: Stock allocation for renters (% of financial and housing wealth)					
Poor	1	1	1	1	5
Fair	3	4	1	3	5
Good	6	8	7	11	9
Very Good	11	10	11	12	11
Excellent	11	12	12	14	12

3.6. Empirical Analysis

3.6.1. Self-perceived health status and stock allocation of homeowners and renters

I used Tobit models to estimate how households' stock allocation correlates with the health status of the household head after controlling for (i) the age, education level, and labour income of the household head; (ii) the total asset and total debt of the household; (iii) family size; (iv) the presence of health insurance; and (v) the out of pocket medical costs of the household.

Table 3.8 summarises the results from the full specifications of household portfolio share in stocks. Column (1) and column (2) show the estimation results for homeowners and renters, respectively. I choose the health state "Good"- $h_t^* = 3$ as the reference state in both models. In these specifications, I find a statistically significant and positive correlation between health status and the proportion of total wealth held in stock for homeowners. For instance, the marginal effects indicate that compared to the stockholding level of homeowners in good health, the stock share is 2.03 percentage points lower for homeowners in poor health and 1.35 percentage points higher for those in excellent health. Similarly, for renters, poorer health is associated with a lower stock share. The share of total wealth in stocks of renters decreases by 1.97 percentage points when health status deteriorates from "Good" to "Poor". Moreover, better health might not necessarily induce renters' risk-taking behaviour as "very good" ("excellent") health exhibit an insignificant effect of about 0.09 (0.17) percentage-point increase compared to the reference health status. A potential explanation is that decreasing returns on health investment suggest that the increase in health expenditure as health decreases from "Excellent" to "Good" is often much smaller than that in case health deteriorates from "Good" to "Poor", leading to a reduced crowding-effect of medical expenditure on liquid wealth and ultimately a much smaller increase in the share of stocks in total wealth.

Another measure of health that I use in these regression models is the total out of pocket medical expenditures of the family. I also find a statistically significant and positive correlation between the out of pocket medical expenses and the proportion of wealth held in stocks for homeowners. One possible mechanism behind this relationship is that households might be reluctant to finance their out of pocket medical costs by selling stocks and choose instead to pay for these expenses out of more liquid assets, resulting in a higher share of stocks in net wealth. In addition, this result might be associated with the discretionary or ‘luxury’ nature of some medical expenditures (Love & Smith, 2009).

In these models, I only examine single and couple households to reduce the impact of other unobserved household characteristics, such as the health status of other family members other than heads and spouse, on the health-related and investment decisions of the household. The estimated coefficient on family size indicates that for homeowners, couple households tend to have a higher share of total wealth in stocks compared to single households, whereas for renters, the opposite is true.

Column (3) reports the estimation results for the whole sample of homeowners and renters. In this censored regression model, I use a dummy variable, “Renter”, which equals 1 if the agent is a renter, and its interaction with other independent variables to test whether renters’ stock allocation reacts differently to changes in health status compared to that of homeowners. The marginal effects point out that as health status moves from “Good” to “Poor”, the decrease in the stock share of renters is 5.79 percentage points larger compared to that of homeowners, and this difference is statistically significant at 1 per cent level. To understand this finding, it is useful to recall from the discussion of optimal rules for the life-cycle portfolio choice problem that the crowding-out effect of medical expenditure on liquid wealth is much stronger for renters than for homeowners since renters’ wealth is not tied up in housing assets. As such, the decrease in the stock allocation corresponding with the decrease in health status appears to be significantly larger for renters than for homeowners.

Table 3.8: Proportion of total wealth held in stocks in relation to the health status of the household head

	Portfolio share in stocks (Stocks/NW)			
	(1) Homeowners	(2) Renters	(3) Elasticity for homeowners	(3) Interaction effect for renters
Healthstatus				
(1) Poor	-0.0479**	-0.2091**	-0.0522**	-0.1224**
	(-2.94)	(-2.57)	(-2.55)	(-2.8)
(2) Fair	-0.0453***	-0.0584	-0.0549***	0.0050
	(-4.78)	(-1.23)	(-4.62)	(0.2)
(4) Very Good	0.0201**	0.0079	0.0244**	-0.0197
	(3.16)	(0.26)	(3.07)	(-1.17)
(5) Excellent	0.0293***	0.0158	0.0427***	-0.0412*
	(3.48)	(0.43)	(4.12)	(-2.07)
Education				
(0) No HS	-0.0914***	-0.1367**	-0.1006***	
	(-6.64)	(-2.51)	(-6.74)	
(2) College	0.1124***	0.111***	0.1104***	
	(13.65)	(3.74)	(12.59)	
AgeHead	0.0061***	0.0134***	0.0073***	
	(4.78)	(3.22)	(5.66)	
AgeHead^2	-0.00004**	-0.0001***	-0.00005***	
	(-3.17)	(-3.45)	(-4.49)	
LnAsset	0.1272***	0.2586***	0.1459***	0.0068
	(55.46)	(37.02)	(52.39)	(1.81)
LnDebt	-0.0046***	0.0051	-0.0064***	0.0107***
	(-6.35)	(1.71)	(-7.39)	(6.59)
LnLaborIncome	0.0029***	-0.0022	0.0029**	-0.0038
	(3.53)	(-0.55)	(2.86)	(-1.79)
famsize	0.0231**	-0.0908***	0.0301**	-0.0743***
	(2.77)	(-3.39)	(2.98)	(-4.64)
WthHealthIns	-0.0003	-0.0984*	0.0174	-0.1076***
	(-0.02)	(-2.29)	(0.92)	(-3.81)
LnOOPmedcost	0.0183***	-0.0008	0.0221***	-0.0244***
	(11.35)	(-0.17)	(11.11)	(-7.96)
Renter			0.7005***	
			(12.68)	
_cons	-1.916***	-2.9399***	-2.2708***	
	(-42.22)	(-22.09)	(-44.26)	
Marginal effects				
Healthstatus				
(1) Poor	-0.0203**	-0.0197**	-0.0202***	-0.0579**
(2) Fair	-0.0193***	-0.0061	-0.0144***	-0.0089
(4) Very Good	0.0092**	0.0009	0.0057**	-0.0039
(5) Excellent	0.0135***	0.0017	0.0098***	-0.0102
AgeHead	0.0028***	0.0014***	0.0021***	
Age2	-0.00001**	-0.00002***	-0.00002***	
EDucation_HD				
0	-0.0347***	-0.0128**	-0.0242***	
2	0.0532***	0.0122***	0.0327***	
LnAsset	0.0574***	0.0277***	0.0419***	0.0308***
LnDebt	-0.0021***	0.0005	-0.0011***	0.0035***
LnLaborIncome	0.0013***	-0.0002	0.0006*	-0.0011
famsize	0.0104**	-0.0097***	0.0039	-0.0268***
WthHealthIns	-0.0002	-0.0105**	-0.0023	-0.0454***
lnOOPmedcost	0.0082***	-0.0001	0.0047***	-0.0065***
Year dummies	Yes	Yes	Yes	
Observations	22452	19897	42349	
rho	0.5633	0.3776	0.5235	
Log-likelihood value	-7918	-4259	-13718	
Wald $\chi2$	4552	1542	6447	
Health vars $\chi2$ (4)	58.44	8.49	60.51	

The dependent variable is the total wealth share in stocks held both inside and outside of individual retirement accounts of a household. T-statistics are reported in parentheses below the coefficient estimates. Three RE Tobit models are run respectively for the sample of homeowners, the sample of renters, and the whole sample in the Panel Study of Income Dynamic from 1999-2017. rho is the variance share of unobserved heterogeneity. Statistical significance is identified as follows: *p<0.05, **p<0.01, ***p<0.001. *The unconditional marginal effects of all variables and interaction terms are also reported. Wald joint-significance tests are included for the model as a whole, the set of four health indicators.*

3.6.2. The comparison between empirical findings and simulation results

On the basis of the censored regression models (1) and (2) in Table 3.8, I estimate the proportion of household wealth held in stocks by health status and age (Table 3.9) to facilitate the comparison of the PSID data and the simulated results from the life-cycle model.

Since the education level enters the life-cycle model only via the labour income process, it is not incorporated in the life-cycle model when individuals approach retirement and stop working. Taking this into account, I do not control for the education level during the retirement period when estimating the average share of stocks in total wealth for individuals in the PSID sample. Panel A shows the mean of stock share in total wealth for homeowners who are single, college-graduated, and have the average asset, debt and labour income for their cohort and age. These values match well with the simulation results from Panel A in Table 3.7. In general, the portfolio share in stocks of homeowners is positively related to health status and exhibits a hump-shaped pattern in age.

Table 3.9: Stock allocation for homeowners and renters in PSID

	Age				
	40	52	60	72	80
Panel A: Stock allocation for homeowners (% of financial and housing wealth)					
Poor	8	10	12	3	6
Fair	6	15	16	14	11
Good	13	21	22	20	15
Very Good	14	26	31	19	17
Excellent	18	28	34	24	20
Panel B: Stock allocation for renters (% of financial and housing wealth)					
Poor	1	3	3	2	8
Fair	3	4	4	4	2
Good	5	6	8	12	17
Very Good	6	8	9	8	6
Excellent	7	8	13	14	5

Note: Panel A and B report the predicted value of households' portfolio share in stocks from the censored regression models (1) and (2) in Table 3.8. The predicted stock share in panel A is for homeowners who are single, college-graduated and have the average asset, debt, and labour income for their cohort and age. The predicted stock share in panel B is for renters who are single, high-school graduated, and have the average asset, debt, and labour income for their cohort and age.

Panel B illustrates the predicted share of total wealth held in stocks for renters who are high-school graduates, have a family size of one person, and have the average asset, debt and labour income for their cohort and age. The results indicate that during the working period, renters' stock share is positively related to health status, which matches quite well with the simulation results in Table 3.7. However, in the retirement period, this relationship is not preserved, which might be explained by the limited number of retirees who do not own a house yet have a "good"-to-"excellent" health status in the PSID sample.

3.7. Conclusion

The results show that a dynamic optimisation life-cycle model for the joint determination of health investment, housing, and portfolio allocation, in which individuals face labour income risk while working, and house price risk, stochastic stock returns as well as health risk over the life cycle, can explain key facts about households' stock allocation across health status and different housing tenure choices. Overall, the proportion of total wealth invested in stocks is low and positively related to health for both homeowners and renters. However, the findings indicate that a change in the share of stocks in total wealth corresponding with the deterioration of health status is significantly larger for renters than for homeowners.

One implication of the results is that health is useful for explaining cross-sectional differences in stock allocation across households. Another implication is that portfolio strategy should be designed separately for renters and homeowners since their stock allocations appear to react differently when health status moves from "Good" to "Poor". Both of these two points are critical in two ways. First, many workers nowadays have defined-contribution retirement savings plans as well as individual investment accounts where they have to make a set of decisions, which may include how much to contribute, how to allocate money across various investment vehicles, and how to distribute plan assets during the retirement period. These financial decisions do have a great impact on the retirement income adequacy of individuals. It is, hence, important to devise sensible asset allocations in accordance with specific risk profiles. Second, investor risk profiling is always at the centre of retirement income planning. This paper suggests that without proper knowledge of the investor's health condition and homeownership status, it might be impossible to come up with an appropriate investment strategy. In other words, facts about health-related considerations and homeownership impose additional disciplines on models of households' portfolio choice over the life cycle.

In this study, I allow the health investment decision to respond endogenously to changes in health status. The question is, what if individuals' labour supply also responses endogenously to health shocks, and if so, how these adjustments might affect the portfolio choice over the life cycle. Another possible extension to the current work is to allow married couples to make decisions on health investment and portfolio allocation based on the health status and survival of both partners. Finally, it is also interesting to look at the effect that some health care policies, such as ObamaCare, might have on households' financial asset allocation.

CONCLUSION

This thesis comprises three studies examining household financial decisions over the working and retirement period. The first study sets out to find (i) whether using RML[59] as a funding source for delaying old-age pension is beneficial and (ii) which financial asset allocations investors should follow during the retirement period. The second study explores the impact of unemployment-induced career breaks on retirement income adequacy under fully-funded defined-contribution pension plans, taking into consideration (i) the timing of the breaks, (ii) the duration of the breaks, (iii) and the severity of the impact that job losses might have on post-interruption wages. The final study investigates (i) how self-perceived health status interacts with the portfolio choice of homeowners and renters over their life-cycle and (ii) how this relationship differs between homeowners and renters.

Summary of main findings and contributions

This section summarises the main results of the research and its fundamental contributions to the existing literature. The first study shows that it is beneficial for an average consumer to take out RMLs as a tool to delay old-age pension benefits. Moreover, cash-poor investors are found to suffer an average loss of 53.87% (with a standard deviation of 11.87%) in the certainty equivalent consumption level if money from the IRA is the sole source to finance the pension delay. These results support the findings of Michelangeli (2010) and Huang et al. (2013). In particular, they all agree that RMLs are welfare-enhancing for many retirees but not necessary for all, and the optimality of the plan depends on the level of cash shortage. The findings also point out that individuals who have a low level of required living standards and an initial financial wealth that is small but enough to sustain for the first five years of retirement are most beneficial from taking out RMLs. Regarding portfolio allocation, the optimal risky asset share is found to have a negative correlation with financial wealth and be fairly stable over the retirement period. Hence, the horizon effect on portfolio choice appears to be minimal during retirement. This finding does not support the traditional investment advice suggesting that the proportion of wealth invested in risky assets should decline with age, yet agrees with Ameriks et al. (2004) and Cocco et al. (2005), who find no empirical evidence supporting the gradual reduction in risky asset share as investors age during retirement. Last but not least, the results indicate that

[59] RML refers to reverse mortgage loan

progressive income tax might have an impact on portfolio choice. However, this impact is only detected within several starting points of the highest and second-highest income tax bracket.

Although the idea of using RMLs to fund the old-age pension delay has been around for quite some time (CFPB, 2017; Pfau, 2017), little research has been done to address the presence of IRAs[60] and the importance of financial asset allocation in this retirement income plan. The first study, therefore, bridges this gap by investigating several retirement income strategies in a life-cycle portfolio choice setting. In addition, this study contributes to the literature on RMLs (Michelangeli, 2010, Huang et al., 2013, Nakajima & Telyukova, 2017) by introducing the modified-term payment option into the life-cycle portfolio choice model. Finally, this study provides a more realistic solution for the household optimisation problem by adding the progressive retirement income tax and progressive property tax into the model. Several studies do capture the income tax in the model, but the tax rate is generally set equal to the average income tax rate. Property tax is also omitted in these studies.

The second study shows that the sooner the unemployment-induced career break occurs, the larger its impact on pension adequacy is. To be more specific, for a median-income female (male) worker, an early-career break, which happens after the first ten years of working, can bring about a fall of 60-90 (30-60) percentage points in the SRR; whereas postponing the break by ten years can reduce the impact by 30-50 (20-40) percentage points. Another main finding is that median-income workers can support themselves well beyond subsistence in retirement; however, job losses do significantly decrease the retirement nest egg of displaced workers, with the level of impact ranging from SG$ 12,800 to SG$ 153,267 (in 2018 dollars). Finally, the duration of the career break is found to have a significant impact on the retirement nest egg at age 55. Specifically, further extending the length of unemployment by six months can lead to a reduction of at least SG$ 8,550 to SG$ 11,970 (in 2018 dollars) in the retirement nest egg at age 55.

This study contributes to the existing literature by examining the effect of job displacement on retirement income adequacy (De Freitas et al.,2011; Potrafke, 2012; OECD, 2015; Bravo et al., 2017; European Commission/Social Protection Committee, 2018). Unlike earlier papers that focus on EU and OECD countries where public earnings-related pension systems are generally defined-benefit or point systems, this study targets fully-funded defined-contribution pension plans.

[60] IRA refers to Individual retirement accounts

In recent decades, the traditional defined-benefit pension scheme has been pointed out to be vulnerable to rapid population ageing. Accordingly, there has been a major shift from the defined-benefit pension system to the defined-contribution pension system. Investigating how unemployment-related career breaks can impede retirement income adequacy under fully-funded defined-contribution pension plans can, thus, offer a valuable lesson. For instance, this study recommends that career support programs for the unemployed should put more emphasis on the early- and mid-career break as being displaced in the early-career stage might have a scarring effect on post-interruption earnings profile, leaving a remarkable effect on the retirement income adequacy under defined-contribution plans. Furthermore, the results suggest that new policies are needed to mitigate the risk of wage depression and unemployment for middle-income families. Although they can support themselves well beyond subsistence in retirement, job losses significantly reduce their retirement nest egg. Without strategies to tackle this issue, economic inequality would become worse.

The third study provides evidence supporting the positive correlation between health status and portfolio choice for both homeowners and renters, using both the life-cycle portfolio choice model and empirical analysis. This result is consistent with the findings of Rosen & Wu (2004), Edward (2008), Yogo (2016). In general, they all find that individuals with poorer health tend to hold safer portfolios. The findings also show that a change in the share of stocks in total wealth corresponding with the deterioration of health status is significantly larger for renters than for homeowners. One implication of this investigation is that health is useful for explaining cross-sectional differences in stock allocation across households. Another implication is that homeownership and health status should be taken into account when it comes to the design of default investment options in defined-contribution pension plans since homeowners’ and renters’ stock allocations are shown to react differently when health status declines from “Good” to “Poor.”

This study contributes to the literature on the household portfolio choice over the life cycle by modelling the relationship between health and portfolio choice separately for homeowners and renters. Previous works on portfolio choice often focus solely on either the relationship between homeownership and risky asset allocation (Grossman & Laroque, 1990; Cocco, 2004; Yao & Zhang, 2005; Chetty et al., 2017; Vestman, 2018) or the correlation between health status and households' risky asset allocation (Edward, 2008; Hugonnier et al., 2013). The second contribution of this study is that it introduces health risk as an additional source of background risk, besides labour income risk, to the household optimisation problem during the working period. Prior research that studies the life-cycle portfolio choice problem in the presence of health risk primarily targets the retirement period (e.g., Edward, 2008; Yogo, 2016).

Limitations of research

This section briefly discusses the limitations of the research. In the first study, there are two issues that have not been addressed. First, the utility derived from housing consumption is not taken into account in the optimisation problem. Although doing so might not affect the answer of whether using RMLs as a funding source for the pension delay is more favourable than claiming pension benefits at age 61, adding the utility from housing will facilitate the comparison between using RMLs and downsizing the house to finance the pension delay. The second issue is that house prices are assumed to grow constantly at 2% per annum. It is true that people are recommended to take out RMLs if house price decreases over time since RMLs hedge against the downside risk to house value. Nevertheless, in reality, house price follows a stochastic process. In this study, house prices are set to follow a deterministic trend because I want to study the impact of progressive income tax rules other than the impact of house price dynamics on portfolio choice.

The second study uses administrative data from the Singapore Ministry of Manpower to construct the age-real wage profiles of workers with uninterrupted career paths across different income percentiles. This data source only provides aggregate data, but not microdata. Hence, the impact of educational background, occupation, and industry on the earnings profile over the working period has not been considered.

In the third study, I use data from the Panel Study of Income Dynamics to calibrate health transition probabilities, the stochastic health depreciation rate, and the relative price of health care. As the information on health expenditure, household wealth, and its various components (including stocks) is not broken down to the individual level, it is assumed that financial decisions in one household unit depend largely on the characteristics of the household head (including health status, age, and labour income). Although only households with a family size of one or two people are included in the final sample to eliminate the effect of the health condition of other family members on the portfolio choice, in households with a family size of two persons, there would still be a chance that the health status of the remaining partner other than the household head plays a role in identifying the household portfolio allocation.

Recommendations for future research

The findings of this thesis put forward the following insights for further research. The first study finds that taking out RMLs to bridge the income gap during the pension delay is welfare-enhancing for the average consumer. It also points out the case in which individuals might experience welfare loss if using RMLs and the case in which individuals most benefit from using RMLs. The objective of tapping into home equity via RMLs is to finance the old-age pension delay. Accordingly, the modified-term payment method is chosen. Under the modified-term payment option, a certain portion of the loan limit is converted into a fixed-term annuity. Overall, the exposure to risk-free assets under the case of using RMLs to delay pension benefits is greater than that under the case of an immediate claim. Hence, individuals with the support of RMLs tend to have a more aggressive portfolio choice. One possible direction for future research is to extend the model to the working phase to see whether the availability of home equity release products such as RMLs in retirement has an influence on the demand for risky assets in the working period.

The second study indicates that due to the scarring effect of the unemployment-induced career breaks on the post interruption wage profiles, the sooner the break occurs, the larger its impact on pension adequacy is. However, the impact of the educational background, occupation, and industry on the labour income profile has not been addressed in this study. Therefore, we can further investigate how technological disruption might affect retirement income adequacy under specific occupations and industries.

The third study assumes that health investment decision responds endogenously to changes in health status. However, I do not allow a change in the health condition to result in a change in the labour supply. The question that comes to mind is, what if individuals' labour supply also responds endogenously to health shocks, and if so, how these adjustments might affect the portfolio choice over the life cycle. Another possible extension to the current work is to allow married couples to make decisions on health investment and portfolio allocation based on the health status and survival of both partners.

APPENDICES

Appendix A: Chapter 1

Table A.1: Survival probability at age x conditional on being alive at age x-1

Age	Survival probability at age x conditional on being alive at age x-1	Age	Survival probability at age x conditional on being alive at age x-1
61	1	74	0.92153
62	0.97029	75	0.92153
63	0.97029	76	0.8534
64	0.97029	77	0.8534
65	0.97029	78	0.8534
66	0.95454	79	0.8534
67	0.95454	80	0.8534
68	0.95454	81	0.74029
69	0.95454	82	0.74029
70	0.95454	83	0.74029
71	0.92153	84	0.74029
72	0.92153	85	0.74029
73	0.92153	86	0

Table A.2: Survival probability at age x conditional on being alive at age 61

Age	Survival probability at age x conditional on being alive at age 61	Age	Survival probability at age x conditional on being alive at age 61
61	1	74	0.5065
62	0.97029	75	0.4668
63	0.9415	76	0.3984
64	0.9135	77	0.3399
65	0.8864	78	0.2901
66	0.8461	79	0.2476
67	0.8076	80	0.2113
68	0.7709	81	0.1564
69	0.7358	82	0.1158
70	0.7024	83	0.0857
71	0.6473	84	0.0634
72	0.5965	85	0.0469
73	0.5497	86	0

Table A.3: Parameters for the baseline model

Time parameters	
Entry age	61
Life-expectancy (Maximum age)	85
Maximum number of years delay claiming Old-age pension benefits (if delay)	5
Consumption preferences	
Coefficient of relative risk aversion γ	3
Discount factor β	0.96
Consumption floor	7.5 million KRW/year
Replacement rate of Old-age pension benefits	45%
Minimum acceptable living standard	Old-age pension benefit/0.45
Portfolio returns	
Risk-free rate	2.5%/year
Equity risk premium	5.5%/year
Standard deviation of real risky asset return	18%/year
Survival Probabilities	Table 3
RM parameters	
House value at age 61	100 million KRW
Loan to Value ratio	48.06%
House value appreciation rate	2.2%/ year
Expected value of actual loan interest rate	5.24%/year
Inflation rate	1.5%/year (Section 6.4. footnote)
Retirement Income Tax	Section 4.6 and 5.3.3
Property Tax	Section 4.6
Old-age pension benefits at age 61	6 million KRW/year
Round-trip transaction cost for a house trading deal	23% buying cost

Appendix B: Chapter 3

Numerical solution of the life-cycle model

B.1. Investor's behaviour in the period T-1

Life-cycle models of portfolio choice have specific characteristics that make them unsolvable by the analytical and standard numerical methods. The solution to the individual optimisation problem can be found using the backward induction principle. At the beginning of period T, the probability of dying is set equal to 0 ($p_T = 1$), the investor's optimisation problem in the period T-1 is restated as:

$$v_{T-1} = \max_{k_T} \left\{ (1-\beta)\mathrm{u}_{T-1}^{1-1/\sigma} + \beta E_{T-1}[\mathrm{G}_{T-1}^{1-\gamma}(\mathrm{p}_T\, \upsilon^{\gamma}\, \mathrm{b}_T^{\;1-\gamma})]^{\frac{1-1/\sigma}{1-\gamma}} \right\}^{\frac{1}{1-1/\sigma}}$$

s.t.

$c_{T-1} > 0$, $a_{T-1} > 0$, $i_{T-1} > 0$, $0 \le \alpha_{T-1} \le 1$, $d_{T-1} > 0$ and budget constraint equations specified in Section 3.3.9

The expected value function depends not only on the initial state and the set of choices within the current period but also on the stochastic process of exogenous state space. Specifically, in the last period ***T-1***, the agent faces the stochastic risky asset returns ($R_{T-1}^{S} = \bar{R}_S \varepsilon_{T-1}^{s}$), the stochastic house price appreciation rate ($R_{T-1}^{D} = \bar{R}_D \varepsilon_{T-1}^{D}$), and the stochastic health depreciation rate w_{T-1}

B.2. Discretisation of exogenous state space

For every combination of state space and set of choices, the integral must be computed numerically, taking a great deal of time. To reduce the computational burden, I discretise the distribution of risky asset returns and housing returns.

As $ln\,\varepsilon_t^i \overset{i.i.d}{\sim} N(-\sigma_i^2/2, \sigma_i^2)$, I have $f(\ln \varepsilon_t^i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp(\frac{-(\ln \varepsilon_t^i - (-\sigma_i^2/2))^2}{2\sigma_i^2})$ $i = D, S$

It follows that

$$E_{T-1}[\mathrm{G}_{T-1}^{1-\gamma}(\mathrm{p}_T\, \upsilon^{\gamma}\, \mathrm{b}_T^{\;1-\gamma})] = \upsilon^{\gamma} E_{T-1}\left[h(\ln \varepsilon_{T-1}^{D}, \ln \varepsilon_{T-1}^{s}, x_{T-1}, \mathrm{k}_{T-1})\right]$$

If X is normally distributed $N(\mu,\sigma^2)$, the expectation of h(X) corresponds to the following integral:

$$E(h(X)) = \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} exp(\frac{-(X-\mu)^2}{2\sigma^2}) h(X) dX$$

I use the Gauss-Hermite quadrature to approximate the expectation of value function in the next period. In numerical analysis, the Gauss-Hermite quadrature rule is presented as follows:

$$\int_{-\infty}^{+\infty} exp(-x^2) f(x) dx \approx \sum_{i=1}^{n} w_i f(x_i)$$

Hence, I need to replace $\frac{-(X-\mu)^2}{2\sigma^2}$ in the expectation of $h(X)$ with a new variable Y so that $E(h(X))$ can match with the integral in the previous form. Now, I obtain:

$$\begin{aligned} E(h(X)) &= \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}} exp(-Y^2) h(\sqrt{2}\sigma Y + \mu) d(\sqrt{2}\sigma Y + \mu) \\ &= \int_{-\infty}^{+\infty} \frac{\sqrt{2}\sigma}{\sigma\sqrt{2\pi}} exp(-Y^2) h(\sqrt{2}\sigma Y + \mu) d(Y) \\ &= \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} exp(-Y^2) h(\sqrt{2}\sigma Y + \mu) d(Y) \\ &= \frac{1}{\sqrt{\pi}} \sum_{i=1}^{n} w_i h(\sqrt{2}\sigma Y_i + \mu) \end{aligned}$$

where w_i is the Gauss-Hermite quadrature weight, Y_i is the quadrature nodes, and n is the number of points used in the discretisation of the stochastic process. Y_i are the roots of the physicists' version of the Hermite polynomial $H_n(y) = (2y - \frac{d}{dy})^n$ and the associated weights

$$w_i = \frac{2^{n-1} n! \sqrt{\pi}}{n^2 \left(H_{n-1}(y_i)\right)^2}$$

I set $\dfrac{(\ln\varepsilon_t^D - (-\sigma_D^2/2))}{\sqrt{2}\sigma_D} = Y^D$ and $\dfrac{(\ln\varepsilon_t^S - (-\sigma_s^2/2))}{\sqrt{2}\sigma_s} = Y^s$ where Y^D and Y^s are the roots of Hermite polynomial $H_n(Y^D)$ and $H_n(Y^s)$ respectively, the expectation of value function in the next period becomes: (correlation between the risky asset returns and house price appreciation rate $\rho_{D,S} = 0$)

$$E_{T-1}\left[h(\ln\varepsilon_{T-1}^D, \ln\varepsilon_{T-1}^s, x_{T-1}, \mathbf{k}_{T-1})\right] =$$

$$\frac{1}{\sqrt{\pi}}\sum_{i=1}^{n}\sum_{j=1}^{m} w_i w_j h(\sqrt{2}\sigma_h Y_i^D - \frac{\sigma_D^2}{2}, \sqrt{2}\sigma_s Y_j^s - \frac{\sigma_s^2}{2}, x_{T-1}, k_{T-1})$$

where w_i, w_j are the corresponding weights of quadrature nodes Y^D and Y^s, n and m are the number of points used in the discretisation of risky asset returns and housing returns.

During the working period, the agent faces labour income shocks, including a persistent ($\ln v_t$) and a transitory component ($\ln\varepsilon_t$). I also allow the shock to permanent income to correlate with the shock to housing returns $\rho_{D,v} \neq 0$, and with the shock to stock returns $\rho_{s,v} \neq 0$

For the bivariate normal distribution, the conditional distribution for one of the variables, given the value for the other variable, is normally distributed. So,

$ln\varepsilon_t^D \mid ln v_t \sim N(\mu_{\varepsilon_t^D|v_t}, \sigma^2_{\varepsilon_t^D|v_t})$ where $\mu_{\varepsilon_t^h|v_t} = \mu_{\varepsilon_t^D} + \dfrac{\rho_{D,v}\sigma_D}{\sigma_v}(ln v_t - \mu_{v_t}) = \dfrac{-\sigma_D^2}{2} + \dfrac{\rho_{D,v}\sigma_D}{\sigma_v}(ln v_t + \dfrac{\sigma_v^2}{2})$,

and $\sigma^2_{\varepsilon_t^D|v_t} = \sigma_D^2(1-\rho_{D,v}^2)$.

$ln\varepsilon_t^S \mid ln v_t \sim N(\mu_{\varepsilon_t^S|v_t}, \sigma^2_{\varepsilon_t^S|v_t})$ where $\mu_{\varepsilon_t^s|v_t} = \mu_{\varepsilon_t^s} + \dfrac{\rho_{s,v}\sigma_s}{\sigma_v}(ln v_t - \mu_{v_t}) = \dfrac{-\sigma_s^2}{2} + \dfrac{\rho_{s,v}\sigma_s}{\sigma_v}(ln v_t + \dfrac{\sigma_v^2}{2})$,

and $\sigma^2_{\varepsilon_t^s|v_t} = \sigma_s^2(1-\rho_{s,v}^2)$.

The expectation of the value function in the next period during the working period is expressed as follows:

$$E_t\left[h(\ln\varepsilon_t,\ln\nu_t,\ln\varepsilon_t^D,\ln\varepsilon_t^s,\mathbf{x}_t,\mathbf{k}_t)\right]$$

$$=\int_{-\infty}^{+\infty}\iiint h(\ln\varepsilon_t,\ln\nu_t,\ln\varepsilon_t^D,\ln\varepsilon_t^s,\mathbf{x}_t,\mathbf{k}_t)f(\ln\varepsilon^D\,|\ln\nu)d(\ln\varepsilon^D)f(\ln\varepsilon^S\,|\ln\nu)d(\ln\varepsilon^s)f(\ln\nu)d(\ln\nu)f(\ln\varepsilon)d(\ln\varepsilon)$$

Let $\frac{(\ln\varepsilon_t^D-(-\sigma^2_{\varepsilon_t^D|\nu_t}/2))}{\sqrt{2}\sigma_{\varepsilon_t^D|\nu_t}}=Y^D$, I obtain $\ln\varepsilon_t^D=\sqrt{2}\sigma_D\sqrt{1-\rho^2_{\mathrm{D},\nu}}Y^D-\frac{\sigma_D^2}{2}+\frac{\rho_{\mathrm{D},\nu}\sigma_D}{\sigma_\nu}(\ln\nu_t+\frac{\sigma_\nu^2}{2})$

where Y^D are the roots of Hermite polynomial $H_n(Y^D)$

And set $\frac{(\ln\nu_t-(-\sigma_\nu^2/2))}{\sqrt{2}\sigma_\nu}=Y^\nu$, I get $ln\nu_t=\sqrt{2}\sigma_\nu Y^\nu-\frac{\sigma_\nu^2}{2}$. Substituting the equation of $ln\nu_t$ into the equation of $ln\varepsilon_t^D$: $\ln\varepsilon_t^D=\sqrt{2}\sigma_D\sqrt{1-\rho^2_{\mathrm{D},\nu}}Y^D-\frac{\sigma_D^2}{2}+\rho_{\mathrm{D},\nu}\sigma_D\sqrt{2}Y^\nu$

Similarly, I discretise the distribution of the shock to stock returns and get $\ln\varepsilon_t^S=\sqrt{2}\sigma_s\sqrt{1-\rho^2_{\mathrm{s},\nu}}Y^s-\frac{\sigma_s^2}{2}+\rho_{\mathrm{s},\nu}\sigma_s\sqrt{2}Y^\nu$

Finally, let $\frac{(\ln\varepsilon_t-(-\sigma_\varepsilon^2/2))}{\sqrt{2}\sigma_\varepsilon}=Y$ and $\frac{(\ln\mathrm{v}_t-(-\sigma_\nu^2/2))}{\sqrt{2}\sigma_\nu}=Y^\nu$ with Y and Y^ν are the roots of Hermite polynomials $H_n(Y)$ and $H_n(Y^\nu)$accordingly.

The approximation of the expected value function in the next period is shown below:

$$E_t\left[h(\ln\varepsilon_t,\ln\nu_t,\ln\varepsilon_t^D,\ln\varepsilon_t^s,\mathbf{x}_t,\mathbf{k}_t)\right]=\frac{1}{\sqrt{\pi}}\sum_l^s\sum_k^\varepsilon\sum_j^\nu\sum_i^h w_l w_k w_j w_i h(\ln\varepsilon_t,\ln\nu_t,\ln\varepsilon_t^D,\ln\varepsilon_t^s,\mathbf{x}_t,\mathbf{k}_t)$$

where w_l,w_k,w_j,w_i are the corresponding weights of the quadrature nodes Y^s, Y, Y^ν, Y^D.

B.3. Discretisation of endogenous state space

In order to solve the optimisation problem numerically, I need to build up a set of grid points for state space $x_t=\left\{y_t,R^0_{t-1},\bar{d}_t,d_{0,t},\bar{h}_t,yr_t\right\}$, and control variables $k_t=\left\{c_t,i_t,d_t,a_t,\alpha_t\right\}$

Zainhofer, F. (2008) summarises several aspects to be considered when setting up grids for endogenous state variables. First, the number of grid points needs to be carefully chosen since the larger the number of points, the longer it takes to approximate the solution, and a smaller number of points may imply a less precise approximation. Next, the starting and ending points have to be specified in accordance with the nature of the underlying economic problem. Last, another critical issue is the spacing between grid points. The grid can be equally spaced or densely spaced at lower values and coarsely spaced at higher values.

Following Cocco et al. (2005), I use an equally-spaced grid for all variables. The starting and ending points and the number of grid points are specified in the section Model Calibration.

B.4. Interpolated Value Function and recursion

The value function $\hat{v}_{T-1}(x_{T-1})$ is constructed using 5D- linear interpolation in case the agent starts the period as a homeowner and 3D- linear interpolation in case the agent starts the period as a renter. When the optimisation problem in the period ***T-1*** has been solved and an approximated $\hat{v}_{T-1}(x_{T-1})$ has been generated, the solution method is to follow the same procedure applied in period ***T-1*** and work back progressively to the beginning of the examined period.

B.5. Partial derivatives of the objective function

In this section, I compute the partial derivatives of the objective function to make the optimisation routine faster and more accurately.

The partial derivative of the objection function v_t with respect to consumption c_t is:

$$\frac{\partial v_t}{\partial c_t} = v_t^{1/\sigma} \left\{ \begin{array}{l} (1-\beta)\mathrm{u}_t^{-1/\sigma} \dfrac{\partial u_t}{\partial c_t} - \beta E_t[\mathrm{G}_t^{1-\gamma}(\mathrm{p}_{t+1}\upsilon^{\gamma}\mathrm{b}_{t+1}{}^{1-\gamma} + (1-p_{t+1})\mathrm{v}_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\sigma}{1-\gamma}} * \\ *E_t[\mathrm{G}_t^{-\gamma}(\mathrm{R}_f + \alpha_t(\mathrm{R}_t^s - \mathrm{R}_f))(\mathrm{p}_{t+1}\upsilon^{\gamma}\mathrm{b}_{t+1}{}^{1-\gamma} + (1-p_{t+1})\mathrm{v}_{t+1}^{1-\gamma})] \end{array} \right\}$$

where

$$\frac{\partial u_t}{\partial c_t} = (1-\alpha)(1-\phi)(\frac{u_t}{c_t})^{1/\rho}(\frac{d_t}{c_t P_t^D})^{\alpha(1-1/\rho)}$$

The partial derivative of the objection function v_t with respect to health investment i_t is:

$$\frac{\partial v_t}{\partial i_t}=v_t^{1/\sigma}\left\{\begin{array}{l}(1-\beta)\mathrm{u}_t^{-1/\sigma}\frac{\partial u_t}{\partial i_t}-\beta E_t[\mathrm{G}_t^{1-\gamma}(\mathrm{p}_{t+1}\upsilon^{\gamma}\,\mathrm{b}_{t+1}{}^{1-\gamma}+(1-p_{t+1})\,\mathrm{v}_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\sigma}{1-\gamma}}*\\ *E_t[\mathrm{G}_t^{-\gamma}(\mathrm{R}_f+\alpha_t(\mathrm{R}_t^s-\mathrm{R}_f))(\mathrm{p}_{t+1}\upsilon^{\gamma}\,\mathrm{b}_{t+1}{}^{1-\gamma}+(1-p_{t+1})\,\mathrm{v}_{t+1}^{1-\gamma})]\end{array}\right\}$$

where

$$\frac{\partial u_t}{\partial i_t}=\alpha\psi^2\frac{u_t^{1/\rho}[\bar{h}_{t-1}(1+\psi(\frac{i_t}{\bar{h}_{t-1}})^{\psi}]^{1-1/\rho}}{Q_t^{1-1/\rho}(\bar{h}_{t-1}^{\psi}i_t^{1-\psi}+\psi i_t)}$$

The partial derivative of the objection function v_t with respect to housing d_t is:

$$\frac{\partial v_t}{\partial d_t}=v_t^{1/\sigma}\left\{\begin{array}{l}(1-\beta)\mathrm{u}_t^{-1/\sigma}\frac{\partial u_t}{\partial d_t}-\beta E_t[\mathrm{G}_t^{1-\gamma}(\mathrm{p}_{t+1}\upsilon^{\gamma}\,\mathrm{b}_{t+1}{}^{1-\gamma}+(1-p_{t+1})\,\mathrm{v}_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\sigma}{1-\gamma}}*\\ *E_t[\mathrm{G}_t^{-\gamma}((\mathrm{tx}+1-\mathrm{b})(\mathrm{R}_f+\alpha_t(\mathrm{R}_t^s-\mathrm{R}_f))-R_t^D(1-\mathrm{b}))(\mathrm{p}_{t+1}\upsilon^{\gamma}\,\mathrm{b}_{t+1}{}^{1-\gamma}+(1-p_{t+1})\,\mathrm{v}_{t+1}^{1-\gamma})]\end{array}\right\}$$

if the agent is a homeowner

$$\frac{\partial v_t}{\partial d_t}=v_t^{1/\sigma}\left\{\begin{array}{l}(1-\beta)\mathrm{u}_t^{-1/\sigma}\frac{\partial u_t}{\partial d_t}-\beta E_t[\mathrm{G}_t^{1-\gamma}(\mathrm{p}_{t+1}\upsilon^{\gamma}\,\mathrm{b}_{t+1}{}^{1-\gamma}+(1-p_{t+1})\,\mathrm{v}_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\sigma}{1-\gamma}}*\\ *E_t[\mathrm{G}_t^{-\gamma}(\alpha_r(\mathrm{R}_f+\alpha_t(\mathrm{R}_t^s-\mathrm{R}_f)))(\mathrm{p}_{t+1}\upsilon^{\gamma}\,\mathrm{b}_{t+1}{}^{1-\gamma}+(1-p_{t+1})\,\mathrm{v}_{t+1}^{1-\gamma})]\end{array}\right\}$$

if the agent is a renter

where

$$\frac{\partial u_t}{\partial d_t}=(1-\alpha)\phi(\frac{u_t}{c_t})^{1/\rho}\frac{c_t}{d_t}(\frac{d_t}{c_tP_t^D})^{\alpha(1-1/\rho)}$$

The partial derivative of the objection function v_t with respect to risky asset share α_t is:

$$\frac{\partial v_t}{\partial \alpha_t}=v_t^{1/\sigma}\left\{\begin{array}{l}\beta E_t[\mathrm{G}_t^{1-\gamma}(\mathrm{p}_{t+1}\upsilon^{\gamma}\,\mathrm{b}_{t+1}{}^{1-\gamma}+(1-p_{t+1})\,\mathrm{v}_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\sigma}{1-\gamma}}*\\ *E_t[\mathrm{G}_t^{-\gamma}(\mathrm{a}_t(\mathrm{R}_t^s-\mathrm{R}_f))(\mathrm{p}_{t+1}\upsilon^{\gamma}\,\mathrm{b}_{t+1}{}^{1-\gamma})+\frac{\partial(\mathrm{G}_t^{1-\gamma}(1-p_{t+1})\,\mathrm{v}_{t+1}^{1-\gamma}))}{\partial\alpha_t}]\end{array}\right\}$$

It is important to note that $\frac{\partial v_t}{\partial c_t} = \frac{\partial v_t}{\partial i_t} = 0 \Leftrightarrow \frac{\partial u_t}{\partial c_t} = \frac{\partial u_t}{\partial i_t}$ and

$(1-\beta)\mathbf{u}_t^{-1/\sigma} \frac{\partial u_t}{\partial c_t} \neq 0 \Rightarrow \beta E_t[\mathbf{G}_t^{1-\gamma}(\mathbf{p}_{t+1}\upsilon^{\gamma}\mathbf{b}_{t+1}{}^{1-\gamma} + (1-p_{t+1})\mathbf{v}_{t+1}^{1-\gamma})]^{\frac{\gamma-1/\sigma}{1-\gamma}} \neq 0$. Hence, in order for the partial derivative of v_t with respect to risky asset share α_t to equal to 0, the following condition should be met:

$$E_t[\mathbf{G}_t^{-\gamma}(\mathbf{a}_t(\mathbf{R}_t^s - \mathbf{R}_f))(\mathbf{p}_{t+1}\upsilon^{\gamma}\mathbf{b}_{t+1}{}^{1-\gamma}) + \frac{\partial(\mathbf{G}_t^{1-\gamma}(1-p_{t+1})\mathbf{v}_{t+1}^{1-\gamma}))}{\partial \alpha_t}] = 0$$

I also have:

$$\mathbf{v}_{t+1}(y_{t+1}, \bar{\mathbf{d}}_{t+1}, \mathbf{d}_{0,t+1}, \bar{\mathbf{h}}_{t+1}, yr_{t+1}) = \mathbf{v}_{t+1}(y_{t+1}, \bar{\mathbf{d}}_{t+1}, \mathbf{d}_{0,t+1}, \bar{\mathbf{h}}_{t+1}, \mathbf{de}_{t+1} = \mathbf{d}_{0,t+1}(1-\delta)r_m(yr_{t+1}-1))$$

With

$$y_{t+1} = \frac{y_t}{G_t} exp\left\{ f(t+1, Z_{t+1}) \right\} \nu_{t+1}\varepsilon_{t+1}$$

$\bar{d}_{t+1} = \frac{R_t^D}{G_t}\bar{d}_t$ if the agent is a homeowner, $\bar{d}_{t+1} = \frac{R_t^D}{G_t}d_t$ if the agent is a renter, $d_{0,t+1} = \frac{1}{G_t}d_{0,t}$

$$\bar{h}_{t+1} = \frac{(1-w_{t+1})Q_{t+1}\bar{h}_t}{Q_t G_t}(1+\psi(\frac{i_t}{\bar{h}_t})^{\psi})$$

$\mathbf{de}_{t+1} = \frac{de_t}{G_t} * \frac{yr_t - 1}{yr_t}$ with $\mathbf{de}_{t+1}$ is the outstanding mortgage balance at the beginning of period t+1

➔ $\mathbf{v}_{t+1}(x_{t+1}) = \vartheta_{t+1}(y_t, \bar{\mathbf{d}}_t, \mathbf{d}_{0,t}, \bar{\mathbf{h}}_t, \mathbf{de}_t, \nu_{t+1}, \varepsilon_{t+1}, R_t^D, R_t^S, w_{t+1}) * \frac{1}{G_t}$

➔ $\frac{\partial(\mathbf{G}_t^{1-\gamma}(1-p_{t+1})\mathbf{v}_{t+1}^{1-\gamma}))}{\partial \alpha_t} = 0$

REFERENCES

Addison, J. T., & Portugal, P. (1989). Job displacement, relative wage changes, and duration of unemployment. *Journal of Labor Economics*, 7(3), 281-302.

Agarwal, S., Pan, J., & Qian, W. (2019). Age of decision: Pension savings withdrawal and consumption and debt response. *Management Science* 66(1), 43-69

Ameriks, J., & Zeldes, S. P. (2004). How do household portfolio shares vary with age? TIAA-CREF Working Paper, Columbia University, New York.

Ameriks, J., Caplin, A., Laufer, S., & Van Nieuwerburgh, S. (2011). The joy of giving or assisted living? Using strategic surveys to separate public care aversion from bequest motives. *The Journal of Finance*, 66(2), 519-561.

Ando, A., & Modigliani, F. (1963). The life-cycle hypothesis of saving: Aggregate implications and tests. *American Economic Review*, 53(1), 55-84.

Andreasson, J. G., Shevchenko, P. V., & Novikov, A. (2017). Optimal consumption, investment, and housing with a means-tested public pension in retirement. *Insurance: Mathematics and Economics*, 75, 32-47.

Attanasio, O. P., Banks, J., Meghir, C., & Weber, G. (1999). Humps and bumps in lifetime consumption. *Journal of Business & Economic Statistics*, 17(1), 22-35.

Bajtelsmit, V. L., & Rappaport, A. (2018). Retirement adequacy in the United States. *Journal of Financial Service Professionals*, 72(6), 71-86.

Barr, N., & Diamond, P. (2006). The economics of pensions. *Oxford Review of Economic Policy*, 22(1), 15-39.

Becker, G. S. (1962). Investment in human capital: A theoretical analysis. *Journal of Political Economy*, 70(5, Part 2), 9-49.

Berkowitz, M. K., & Qiu, J. (2006). A further look at household portfolio choice and health status. *Journal of Banking & Finance*, 30(4), 1201-1217.

Biggs, A. G., & Schieber, S. J. (2014). Is there a retirement crisis? *National Affairs*, 20, 55-75.

Biggs, A. G., Pang, G., & Schieber, S. J. (2015). Measuring and communicating social security earnings replacement rates. *The Journal of Retirement*, 2(4), 69-85.

Biggs, A. G., & Springstead, G. R. (2008). Alternate measures of replacement rates for social security benefits and retirement income. *Social Security Bulletin*, 68(2), 1-19.

Blau, D. M. (2008). Retirement and consumption in a life cycle model. *Journal of Labor Economics*, 26(1), 35-71.

Bodie, Z., Merton, R. C., & Samuelson, W. F. (1992). Labor supply flexibility and portfolio choice in a life cycle model. *Journal of Economic Dynamics and Control*, 16(3-4), 427-449.

Boskin, M. J., & Shoven, J. B. (1987). Concepts and measures of earnings replacement during retirement. In Bodie, Z., Shoven, J. B., & Wise, D. A. (Eds.), *Issues in Pension Economics* (pp. 113-146). University of Chicago Press.

Bosworth, B., Burtless, G., & Steuerle, E. (2000). Lifetime earnings patterns, the distribution of future social security benefits, and the impact of pension reform. *Social Security Bulletin*, 63(4), 74-98.

Brady, P. (2010) Measuring retirement resource adequacy. *Journal of Pension Economics and Finance*, 9(2), 235–262

Bravo, J., & Herce, J. A. (2017). On the influence of employment breaks on pension benefits. *Papers Mi Jubilación,* 19, 1-30.

Broadbent, J., Palumbo, M., & Woodman, E. (2006). *The shift from defined benefit to defined contribution pension plans-implications for asset allocation and risk management*. Retrieved from BIS database: https://www.bis.org/publ/wgpapers/cgfs27broadbent3.pdf

Caminada, K., Goudswaard, K., & Koster, F. (2012). Social income transfers and poverty: A cross-country analysis for OECD countries. *International Journal of Social Welfare,* 21(2), 115-126.

Campbell, J. Y. (2006). Household finance. *The Journal of Finance*, 61(4), 1553-1604.

Campbell, J. Y., & Viceira, L. M. (1999). Consumption and portfolio decisions when expected returns are time-varying. *The Quarterly Journal of Economics*, 114(2), 433-495.

Campbell, J. Y., Cocco, J. F., Gomes, F. J., & Maenhout, P. J. (2001). Investing retirement wealth: A life-cycle model. In Campbell, J. Y., & Feldstein, M. (Eds.), *Risk Aspects of Investment-Based Social Security Reform* (pp. 439–482). University of Chicago Press.

Carrington, W. J., & Fallick, B. (2017). Why do earnings fall with job displacement? *Industrial Relations: A Journal of Economy and Society*, 56(4), 688-722.

Carroll, C. D. (2006). The method of endogenous gridpoints for solving dynamic stochastic optimization problems. *Economics Letters*, 91(3), 312-320.

Carroll, C. D., & Samwick, A. A. (1997). The nature of precautionary wealth. *Journal of Monetary Economics*, 40(1), 41-71.

Chai, J., Horneff, W., Maurer, R., & Mitchell, O. S. (2011). Optimal portfolio choice over the life cycle with flexible work, endogenous retirement, and lifetime payouts. *Review of Finance*, 15(4), 875-907.

Chetty, R. and A. Szeidl (2007). Consumption commitments and risk preferences. *The Quarterly Journal of Economics*, 122(2), 831-877.

Chetty, R., Sándor, L., & Szeidl, A. (2017). The effect of housing on portfolio choice. *The Journal of Finance*, 72(3), 1171-1212.

Chia, N. C., & Tsui, A. K. (2019). Nexus between housing and pension policies in Singapore: measuring retirement adequacy of the Central Provident Fund. *Journal of Pension Economics & Finance*, 18(2), 304-330.

Cho, D. H., Park, C. K., & Ma, S. R. (2004). A study on the adoption of reverse mortgage system. Korea Financial Supervisory Service.

Choi, K. J., Lim, B., & Park, J. (2020). Evaluation of the Reverse Mortgage Option in Korea: A Long Straddle Perspective. *International Journal of Financial Studies*, 8(3), 55.

Cocco, J. F. (2004). Portfolio choice in the presence of housing. *The Review of Financial Studies*, 18(2), 535-567.

Cocco, J. F., Gomes, F. J., & Maenhout, P. J. (2005). Consumption and portfolio choice over the life cycle. *The Review of Financial Studies*, 18(2), 491-533.

Cocco, J. F., & Lopes, P. (2020). Aging in place, housing maintenance, and reverse mortgages. *The Review of Economic Studies*, 87(4), 1799-1836.

Coile, C., Diamond, P., Gruber, J., & Jousten, A. (2002). Delays in claiming social security benefits. *Journal of Public Economics*, 84(3), 357-385.

Consumer Financial Protection Bureau (2017). The costs and risks of using a reverse mortgage to delay collecting social security. Office for Older Americans.

Cooper, D. (2014). The effect of unemployment duration on future earnings and other outcomes. Working Papers No. 13-8. Boston, MA: Federal Reserve Bank of Boston.

Couch, K. A., Jolly, N. A., & Placzek, D. W. (2011). Earnings losses of displaced workers and the business cycle: an analysis with administrative data. *Economics Letters*, 111(1), 16-19.

Damodaran, A. (2016). Equity Risk Premiums (ERP): Determinants, Estimation and Implications – The 2016 Edition. Available at SSRN: http://dx.doi.org/10.2139/ssrn.2742186.

Davidoff, T. (2015). Can high costs justify weak demand for the Home Equity Conversion Mortgage? *The Review of Financial Studies*, 28(8), 2364-2398.

Davis, S. J., & Von Wachter, T. M. (2011). Recessions and the cost of job loss. NBER Working paper No. 17638. National Bureau of Economic Research.

De Freitas, N. E. M., Duc, C., Briard, K., Mage, S., & Legendre, B. (2011). Career Interruptions: how do they impact pension rights? *The Geneva Papers on Risk and Insurance-Issues and Practice*, 36(3), 440-457.

De Nardi, M., French, E., & Jones, J. B. (2009). Life expectancy and old age savings. *American Economic Review*, 99(2), 110-15.

Dedry, A., Onder, H., & Pestieau, P. (2017). Ageing, social security design, and capital accumulation. *The Journal of the Economics of Ageing*, 9, 145-155.

Draxler, Juraj and Jorgen Mortensen (2009) Towards sustainable but still adequate pensions in the EU: Theory, trends and simulations. ENEPRI Research Report No.67. Brussels: Centre for European Policy Studies.

Edwards, R. D. (2008). Health risk and portfolio choice. *Journal of Business & Economic Statistics*, 26(4), 472-485.

Ehrlich, I., & Chuma, H. (1990). A model of the demand for longevity and the value of life extension. *Journal of Political Economy*, 98(4), 761-782.

European Commission (2003). *Adequate and sustainable pensions*, Luxembourg: Publications Office. Retrieved from EU database: https://op.europa.eu/s/oMOE

European Commission/Social Protection Committee (2018). *The 2018 pension adequacy report: Current and future income adequacy in old age in the EU*. Retrieved from EU database: https://op.europa.eu/s/oMOF.

Fagereng, A., Gottlieb, C., & Guiso, L. (2017). Asset market participation and portfolio choice over the life-cycle. *The Journal of Finance*, 72(2), 705-750.

Fallick, B. C. (1993). The industrial mobility of displaced workers. *Journal of Labor Economics*, 11(2), 302-323.

Fisher, J. D., Johnson, D. S., Marchand, J. T., Smeeding, T. M., & Torrey, B. B. (2007). No place like home: Older adults and their housing. *The Journals of Gerontology Series B: Psychological Sciences and Social Sciences*, 62(2), S120-S128.

Fisher, I. (1930). *Theory of interest: As determined by impatience to spend income and opportunity to invest it*. New York, NY: The MacMillan Company.

Fong, J. H., Mitchell, O. S., & Koh, B. S. (2011). Longevity risk management in Singapore's national pension system. *Journal of Risk and Insurance*, 78(4), 961-982.

Fratantoni, M. C. (1998). Homeownership and investment in risky assets. *Journal of Urban Economics* 44(1): 27-42.

Fratantoni, M. C. (2001). Homeownership, committed expenditure risk, and the stockholding puzzle. *Oxford Economic Papers*, 53(2), 241-259.

Gibbons, Robert, and Lawrence Katz. 1991. Layoffs and Lemons. *Journal of Labor Economics*, 9(4), 351– 80.

Goda, G. S., Ramnath, S., Shoven, J. B., & Slavov, S. N. (2017). The financial feasibility of delaying Social Security: evidence from administrative tax data. *Journal of Pension Economics & Finance*, 17(4), 419-436.

Gomes, F. J., Kotlikoff, L. J., & Viceira, L. M. (2008). Optimal life-cycle investing with flexible labour supply: A welfare analysis of life-cycle funds. *American Economic Review*, 98(2), 297-303.

Grossman, M. (1972). On the concept of health capital and the demand for health. *Journal of Political Economy*, 80(2), 223-255.

Grossman, S. J. and G. Laroque (1990). Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods, *Econometrica*, 58(1), 25– 51.

Grech, A. G. (2013). How best to measure pension adequacy. CASE Papers. Centre for Analysis of Social Exclusion, London School of Economics.

Guiso, L., & Sodini, P. (2013). Household finance: An emerging field. In Constantinides, G. M., Harris, M., & Stulz, R. M. (Eds.), *Handbook of the Economics of Finance* (Vol. 2, pp. 1397-1532). Elsevier.

Gustman, A. L., & Steinmeier, T. L. (2015). Effects of social security policies on benefit claiming, retirement and saving. *Journal of Public Economics*, 129(C), 51-62.

Haveman, R., Holden, K., Wolfe, B. L., & Romanov, A. (2007). The sufficiency of retirement savings: Comparing cohorts at the time of retirement. In Mitchell, O. S., Madrian, B., & Soldo, B. J. (Eds.), *Redefining Retirement: How will Boomers Fair* (pp. 36-69). Oxford University Press.

Heaton, J., & Lucas, D. (1997). Market frictions, savings behavior, and portfolio choice. *Macroeconomic Dynamics*, 1(1), 76-101.

Heaton, J., & Lucas, D. (2000). Portfolio choice and asset prices: The importance of entrepreneurial risk. *The Journal of Finance*, 55(3), 1163-1198.

Hendricks, L. (2002). Intended and accidental bequests in a life-cycle economy. Working Papers No. 2133407, Department of Economics, W. P. Carey School of Business, Arizona State University.

Heo, Y. C., An, S., & Hong, B. E. (2016). Reverse Mortgage as an income stabilizer for the elderly in Korea. *Asian Social Work and Policy Review,* 10(1), 103-112.

Horneff, V., Maurer, R., Mitchell, O. S., & Rogalla, R. (2015). Optimal life-cycle portfolio choice with variable annuities offering liquidity and investment downside protection. *Insurance: Mathematics and Economics*, 63(C), 91-107.

Hren, R. (2012). Theoretical shortcomings of the Grossman model. *Bulletin: Economics, Organisation and Informatics in Healthcare*, 28(1), 63-75.

Huang, M., Chen, B., & Deng, Y. (2013). Welfare effects of developing the reverse mortgage market in China: An individual and social perspective. *Asia-Pacific Journal of Risk and Insurance*, 8(1), 27-55.

Hubener, A., Maurer, R., & Mitchell, O. S. (2016). How family status and social security claiming options shape optimal life cycle portfolios. *The Review of Financial Studies*, 29(4), 937-978.

Hugonnier, J., Pelgrin, F., & St-Amour, P. (2013). Health and (other) asset holdings. *The Review of Economic Studies*, 80(2), 663-710.

Hui, W.T. (2012). Macroeconomic trends and labour welfare: A focus on retirement adequacy. In *Singapore Perspectives 2012 - Singapore Inclusive: Bridging Divides* (pp. 37-58)

Hurd, M. D., & Rohwedder, S. (2008). The retirement consumption puzzle: Actual spending change in panel data. NBER Working paper No. 13929. National Bureau of Economic Research.

Hurd, M. D., & Rohwedder, S. (2013). Heterogeneity in spending change at retirement. *The Journal of the Economics of Ageing,* 1, 60-71.

Jacobson, L. S., LaLonde, R. J., & Sullivan, D. G. (1993). Earnings losses of displaced workers. *American Economic Review,* 83 (4), 685-709.

Jarosch, G. (2015). Searching for job security and the consequences of job loss. CEPR Discussion Paper DP15814. Centre for Economic Policy Research, London.

Jones, R. and S. Urasawa (2014). Reducing the High Rate of Poverty Among the Elderly in Korea. OECD Economics Department Working Papers, No. 1163, OECD Publishing, Paris, https://doi.org/10.1787/5jxx054fv20v-en

Jovanovic, B. (1979). Job matching and the theory of turnover. *Journal of Political Economy*, 87(5, Part 1), 972-990.

Jovanovic, B. (1984). Matching, turnover, and unemployment. *Journal of Political Economy*, 92(1), 108-122.

Jung, P., & Kuhn, M. (2018). Earnings losses and labour mobility over the life cycle. *Journal of the European Economic Association*, 17(3), 678-724.

Kahn, L. B. (2013). Asymmetric information between employers. *American Economic Journal: Applied Economics*, 5(4), 165-205.

Kim, J. H., & Li, J. S. (2017). Risk-neutral valuation of the non-recourse protection in reverse mortgages: A case study for Korea. *Emerging Markets Review*, 30(C), 133-154.

Kim, S. J., & Yoo, S. J. (2006). A study on the deciding cause of choosing reverse mortgage: Analysis of the feature of reverse mortgage user. *The Korea Spatial Planning Review*, 50, 125-146.

Kimball, M. S. (1990). Precautionary saving in the small and in the large. *Econometrica*, 58(1), 53-73.

Kimball, M. S. (1993). Standard risk aversion. *Econometrica*, 61(3), 589-611.

Klos, A., & Weber, M. (2006). Portfolio Choice in the presence of non-tradable income: An experimental analysis. *German Economic Review*, 7(4), 427-448.

Knaack, P., Miller, M., & Stewart, F. (2020). Reverse mortgages, financial inclusion, and economic development: Potential benefit and risks. Policy research working paper series No. 9134, The World Bank.

Knoef, M., Been, J., Alessie, R., Caminada, K., Goudswaard, K., & Kalwij, A. (2016). Measuring retirement savings adequacy: developing a multi-pillar approach in the Netherlands. *Journal of Pension Economics & Finance*, 15(1), 55-89.

Koh, B. S., Mitchell, O. S., Tanuwidjaja, T., & Fong, J. (2008). Investment patterns in Singapore's central provident fund system. *Journal of Pension Economics & Finance,* 7(1), 37-65.

Koo, H. K. (1998). Consumption and portfolio selection with labor income: Evaluation of human capital. *Mathematical Finance,* 8(1), 49–65.

Krolikowski, P. (2017). Job ladders and earnings of displaced workers. *American Economic Journal: Macroeconomics,* 9(2), 1-31.

Kunze, A. (2002). The timing of careers and human capital depreciation. IZA Discussion Papers No. 509, Institute of Labor Economics (IZA)

Laitner, J., & Juster, F. T. (1996). New evidence on altruism: A study of TIAA-CREF retirees. *American Economic Review,* 86(4), 893-908.

Light, A., & Ureta, M. (1995). Early-career work experience and gender wage differentials. *Journal of Labour Economics*, 13(1), 121-154.

Liljas, B. (1998). The demand for health with uncertainty and insurance. *Journal of Health Economics*, 17(2), 153-170.

Lockwood, L. M. (2018). Incidental bequests and the choice to self-insure late-life risks. *American Economic Review*, 108(9), 2513-50.

Love, D. A. (2013). Optimal rules of thumb for consumption and portfolio choice. *The Economic Journal*, 123(571), 932-961.

Love, D. A., Palumbo, M. G., & Smith, P. A. (2009). The trajectory of wealth in retirement. *Journal of Public Economics*, 93(1-2), 191-208.

Ma, S., & Deng, Y. (2006). Evaluation of reverse mortgage programs in Korea. Working Paper No. 8568. USC Lusk Center for Real Estate.

Ma, S., Kim, C. (2017) Building a new reverse mortgage model for elderly people with low price house. The 21st Annual Conference of Asia-Pacific Risk and Insurance Association, Poznan, Poland.

MacDonald, B. J., & Moore, K. D. (2011). Moving beyond the limitations of traditional replacement rates. Available at SSRN: http://dx.doi.org/10.2139/ssrn.2003895

Malkiel, B. G. (1999). *A Random Walk down Wall Street: Including a Life-cycle Guide to Personal Investing*. W.W Norton & Company, Inc.

Manyika, J. (2017). A future that works: AI automation, employment and productivity. McKinsey Global Institute Research, New York.

Manyika, J., Lund, S., Chui, M., Bughin, J., Woetzel, J., Batra, P., Ko, R. and Sanghvi, S. (2017). Jobs lost, jobs gained: Workforce transitions in a time of automation. New York, NY: McKinsey Global Institute.

Markowitz, H. (1952). Portfolio selection. *The Journal of Finance*, 7(1), 77-91.

Maurer, R., Mitchell, O. S., Rogalla, R., & Kartashov, V. (2013). Life-cycle portfolio choice with systematic longevity risk and variable investment-linked deferred annuities. *Journal of Risk and Insurance*, 80(3), 649-676.

Maurer, R., & Mitchell, O. S. (2020). Older peoples' willingness to delay social security claiming. *Journal of Pension Economics & Finance*, 1-16. doi:10.1017/S1474747219000404

McCarthy, D., Mitchell, O. S., & Piggott, J. (2002). Asset-rich and cash-poor: retirement provision and housing policy in Singapore. *Journal of Pension Economics & Finance*, 1(3), 197-222.

McGill, D. M., Brown, K. N., Haley, J. J., & Schieber, S. J. (2005). *Fundamentals of Private Pensions* . Oxford University Press.

McKenzie, C. R., Liersch, M. J., & Finkelstein, S. R. (2006). Recommendations implicit in policy defaults. *Psychological Science*, 17(5), 414-420.

Mercer (2014). Melbourne Mercer Global Pension Index 2014. Melbourne, Australia: Australian Centre for Financial Studies.

Mercer (2018). Melbourne Mercer Global Pension Index 2018. Melbourne, Australia: Australian Centre for Financial Studies.

Merrill, Sally, Meryl Finkel, and Nandinee Kutty (1994). Potential beneficiaries from reverse mortgage products for elderly homeowners: An analysis of American Housing Survey. *Real Estate Economics*, 22(2), 257–99.

Merton, R. C. (1969). Lifetime portfolio selection under uncertainty: The continuous-time case. *The Review of Economics and Statistics*, 51(3), 247-257.

Merton, R. C. (1971) Optimum consumption and portfolio rules in a continuous-time model. *Journal of Economic Theory*, 3(4), 373-413.

Merton, R. C., & Lai, R. N. (2016). On an efficient design of reverse mortgages: A possible solution for aging Asian populations. Available at SSRN: http://dx.doi.org/10.2139/ssrn.3075087

Meyer, W., & Reichenstein, W. (2010). Social Security: When to start benefits and how to minimize longevity risk. *Journal of Financial Planning*, 23(3), 49-59.

Miceli, T. J., & Sirmans, C. F. (1994). Reverse mortgages and borrower maintenance risk. *Real Estate Economics*, 22(2), 433-450.

Michaud, A. M. (2018). A quantitative information theory of worker flows and wage dispersion. *American Economic Journal: Macroeconomics*, 10(2), 154-183.

Michelangeli, V. (2010). Does it pay to get a reverse mortgage? Working paper, Bank of Italy.

Mincer, J. & S. Polachek. (1974). Family investment in human capital: Earnings of women, *Journal of Political Economy,* 82(2), S76-S108.

Mincer, J. (1962). On-the-job training: Costs, returns, and some implications. *Journal of Political Economy*, 70(5, Part 2), 50-79.

Mincer, J., & Ofek, H. (1982). Interrupted work careers: Depreciation and restoration of human capital. *Journal of Human Resources*, 17(1), 3-24.

Mitchell, O. S., & Piggott, J. (2004). Unlocking housing equity in Japan. *Journal of the Japanese and International Economies*, 18(4), 466-505.

Modigliani, F., & Brumberg, R. (1954). Utility analysis and the consumption function: An interpretation of cross-section data. In Kurihara, K. (Ed.) *Post-Keynesian Economics* (pp. 388-436). Rutgers University Press.

Moon, H. (2011). The Korean pension system: Current state and tasks ahead. Seoul: Korea Development Institute.

Moore, J. F., & Mitchell, O. S. (1997). Projected retirement wealth and savings adequacy in the health and retirement study. NBER Working Paper No. 6240. National Bureau of Economic Research.

Moore, K. D., Robson, W. B., & Laurin, A. (2010). Canada's looming retirement challenge: Will future retirees be able to maintain their living standards upon retirement? Commentary Pension paper No. 317. C.D. Howe Institute.

Mortensen, D. T., & Pissarides, C. A. (1994). Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, 61(3), 397-415.

Munnell, A. (2006). Employer-Sponsored Plans: The Shift from Defined Benefit to Defined Contribution. In Clark, G. L., Munnell, A. H., Williams, K., & Orszag, J. M. (Eds.), *Oxford Handbook of Pensions and Retirement Income* (Vol. 13). Oxford University Press.

Munnell, A. H., & Soto, M. (2005). What replacement rates do households actually experience in retirement? Available at SSRN: http://dx.doi.org/10.2139/ssrn.1150172

Munnell, A. H., Webb, A., & Delorme, L. (2006). A new national retirement risk index. *Issues in Brief*, (48). Center for Retirement Research at Boston University.

Nakajima, M., & Telyukova, I. A. (2017). Reverse mortgage loans: A quantitative analysis. *The Journal of Finance*, 72(2), 911-950.

OECD (2008). *Growing Unequal? Income Distribution and Poverty in OECD Countries*. OECD Publishing, Paris, https://doi.org/10.1787/9789264044197-en.

OECD (2011). *Divided We Stand: Why Inequality Keeps Rising*. OECD Publishing, Paris, https://doi.org/10.1787/9789264119536-en.

OECD (2015). *Pensions at a Glance 2015: OECD and G20 indicators*, OECD Publishing, Paris, https://doi.org/10.1787/pension_glance-2015-en.

OECD (2017). *Pensions at a Glance 2017: OECD and G20 Indicators*, OECD Publishing, Paris, https://doi.org/10.1787/pension_glance-2017-en.

Ong, D., Soo, G., Choy, M., & Ng, E. (2011). A review of the core inflation measure for Singapore. MAS Staff Paper No. 51, Monetary Authority of Singapore.

Ortego-Marti, V. (2016). Unemployment history and frictional wage dispersion. *Journal of Monetary Economics*, 78, 5-22.

Orton, M., & Rowlingson, K. (2007). *Public Attitudes to Economic Inequality* (Vol. 4). York: Joseph Rowntree Foundation.

Pfau, W. D. (2017). *Reverse Mortgages: How to Use Reverse Mortgages to Secure Your Retirement*. McLean, Virginia: McLean Asset Management Corporation.

Potrafke, N. (2007). The timing of employment breaks: how does it affect pension benefits? Empirical evidence from Germany. DIW Discussion Papers No. 710, German Institute of Economic Research.

Potrafke, N. (2012). Unemployment, human capital depreciation and pension benefits: an empirical evaluation of German data. *Journal of Pension Economics & Finance*, 11(2), 223-241.

Purcell, P. J. (2012). Income replacement ratios in the health and retirement study. *Social Security Bulletin*, 72(3), 37-58.

Quinn, J. F., Cahill, K. E., & Giandrea, M. D. (2019). Transitions from career employment among public-and private-sector workers. *Journal of Pension Economics & Finance*, 18(4), 529-548.

Redfoot, D. L., Scholen, K., & Brown, S. K. (2007). *Reverse Mortgages: Niche Product or Mainstream Solution?* Washington, DC: AARP's Public Policy Institute.

Rodda, D. T., Lam, K., & Youn, A. (2004). Stochastic modeling of federal housing administration home equity conversion mortgages with low-cost refinancing. *Real Estate Economics*, 32(4), 589-617.

Rosen, H. S., & Wu, S. (2004). Portfolio choice and health status. *Journal of Financial Economics*, 72(3), 457-484.

Samuelson, P. A. (1969). Lifetime portfolio selection by dynamic stochastic programming. *The Review of Economics and Statistics*, 51(3), 239-46.

Schieber, S. J. (2004). Retirement income adequacy: Good news or bad? *Benefits Quarterly*, 20(4), 27-39.

Shan, H. (2011). Reversing the trend: The recent expansion of the reverse mortgage market. *Real Estate Economics*, 39(4), 743-768.

Shaw, K. L. (1984). A formulation of the earnings function using the concept of occupational investment. *Journal of Human Resources*, 19(3), 319-340.

Shoven, J. B., & Slavov, S. N. (2014). Does it pay to delay social security? *Journal of Pension Economics & Finance*, 13(2), 121-144.

Sinai, T., & Souleles, N. S. (2005). Owner-occupied housing as a hedge against rent risk. *The Quarterly Journal of Economics*, 120(2), 763-789.

Spivey, C. (2005). Time off at what price? The effects of career interruptions on earnings. *Industrial and Labor Relations Review*, 59(1), 119-140.

Stevens, A. H. (1997). Persistent effects of job displacement: The importance of multiple job losses. *Journal of Labor Economics*, 15(1, Part 1), 165-188.

The World Bank (1994). *Averting the Old Age Crisis: Policies to Protect the Old and to Promote Growth*. Washington, DC: World Bank Group.

Theunissen, G., Verbruggen, M., Forrier, A., & Sels, L. (2011). Career sidestep, wage setback? The impact of different types of employment interruptions on wages. *Gender, Work & Organization*, 18(1), e110-e131.

Topel, R. (1990). Specific capital and unemployment: Measuring the costs and consequences of job loss. *Carnegie-Rochester Conference Series on Public Policy*, 33(1), 181-214.

The Economist (2018). The automation readiness index: who is ready for the coming wave of automation? London: The Economist Intelligence Unit Limited.

Van Ooijen, R., Alessie, R., & Kalwij, A. (2015). Saving behaviour and portfolio choice after retirement. *De Economist*, 163(3), 353-404.

Velarde, M., & Herrmann, R. (2014). How retirement changes consumption and household production of food: lessons from German time-use data. *The Journal of the Economics of Ageing*, 3, 1-10.

Venti, Steven F. and David A. Wise (1991). Ageing and the income value of housing wealth, *Journal of Public Economics*, 44(3), 371-397.

Vestman, R. (2018). Limited stock market participation among renters and homeowners. *The Review of Financial Studies*, 32(4), 1494-1535.

Viceira, L. M. (2001). Optimal portfolio choice for long-horizon investors with non-tradable labour income. *The Journal of Finance*, 56(2), 433-470.

Von Wachter, T., Song, J., & Manchester, J. (2009). Long-term earnings losses due to mass-layoffs during the 1982 recession: an analysis using longitudinal administrative data from 1974 to 2004. Discussion Papers 0708-16. Columbia University, Department of Economics.

Wachter, J. A., and M. Yogo (2010). Why do household portfolio shares rise in wealth? *The Review of Financial Studies*, 23(11), 3929-3965.

Yamashita, T. (2003). Owner-occupied housing and investment in stocks: an empirical test. *Journal of Urban Economics*, 53(2), 220-237.

Yao, R., and H. H. Zhang (2005). Optimal life-cycle asset allocation with housing as collateral. *The Review of Financial Studies*, 18(1), 197-239.

Yogo, M. (2016). Portfolio choice in retirement: Health risk and the demand for annuities, housing, and risky assets. *Journal of Monetary Economics*, 80(C), 17-34.

Zainhofer, F. (2008). *Life-cycle Investing and Occupational Old-Age Provision in Switzerland*. Gabler Verlag.