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ABSTRACT

Rutherford developed his revolutionary model of the atom based on experiments using helium nuclei scattered by a thin gold foil. In this paper, we review the measurements of the size of the helium nucleus, focusing on spectroscopic techniques. We touch on the “proton radius puzzle” that has incited a great deal of interest in high resolution spectroscopy of the most fundamental atoms. We find that for an absolute value of the size of the helium nucleus, the theoretical advance that will give a new determination is just around the corner.

KEYWORDS

Laser spectroscopy, Atomic Physics, Atomic structure, Quantum Electrodynamics

11 REVIEW ARTICLE

12 **The size of the helium nucleus: Then and Now**

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16 **ARTICLE HISTORY**

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18 **1. Introduction**

19 At the start of the 20th century, the prevailing model for the atom was the Thomson
20 “plum pudding” model. In this model, the charges are distributed over the size of the
21 atom, which was known to be in the order of Ångströms (10^{-10} m). To investigate
22 whether this model was correct, and in collaboration with Rutherford, Hans Geiger
23 and Ernest Marsden performed an experiment, in which they scattered highly energetic
24 α -particles (now known to be nuclei of the helium atom) off a gold foil. It was found
25 that the α -particles could be deflected over a large range of angles, including some
26 that were scattered back towards the source. On the other hand, most α -particles
27 went through without being deflected at all.

28 Both of these observations were at odds with the plum pudding model of the atom.
29 In particular, if the charges and mass are roughly evenly distributed across the atom,
30 most particles should show some deflection, as there would be no way to get through
31 without interaction. On the other hand, an interaction with a distributed charge would
32 result in a small deflection, as the α -particles had quite a high energy (~ 5 MeV). From
33 this, Rutherford deduced that most of the mass and the positive charge of the atom
34 must be concentrated in a small nucleus, thereby negating the “plum pudding” model
35 of the atom with a distributed charge. In contrast, if the mass and positive charge were
36 concentrated in the nucleus, α -particles missing this nucleus would zip right through,
37 and α -particles hitting the nucleus would be able to be scattered under large angles.
38 From this conclusion, the immediate question arises:

39 How big is this nucleus?

40 In his paper, Rutherford (1911) was able to derive that the charge on the gold
41 nucleus would have to be about 100 electron charges (we now know it is 79 electron
42 charges). It turned out much later, that the radius of a gold nucleus is ~ 7.0 fm. This
43 is to be compared to the radius of the electron cloud, which is about 150,000 fm. The
44 atom is indeed quite empty! Furthermore, Rutherford and Royds (1908) showed that
45 α particles were actually ionised helium nuclei. In this review, we will explore the size
46 of the α -particle, as it has been found then and now.

47 **2. Electron scattering**

48 The currently accepted value for the radius of the alpha particle from a range of
 49 electron scattering experiments is 1.681 ± 0.004 fm Sick (2008). The principle of the
 50 experiment has not changed since the first measurement from high-energy electron
 51 scattering, published by McAllister and Hofstadter (1956). The accuracy has however
 52 evolved considerably. The most accurate experiment to date has been published by
 53 Ottermann et al. (1985), by now already some time ago. They used a pressurised
 54 gas container in an evacuated detection chamber, and performed an angle-resolved
 55 scattering experiment using a 188 MeV incident electron beam. The differential cross-
 56 section was measured, and it was found that the best agreement was for a charge
 57 radius of 1.60 ± 0.10 fm. In figure 1, we show the basic principle of the differential
 58 cross-section experiment carried out in this paper.

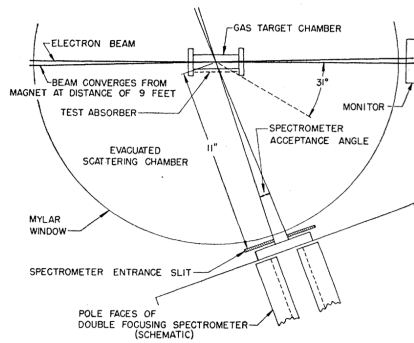


Figure 1. The electron scattering experiment, in which the angular deflection of electrons scattered from the gas target chamber is measured by the double focusing electron spectrometer. From reference McAllister and Hofstadter (1956)

59 **3. Spectroscopic measurements**

60 Over the past century, the framework of quantum mechanics has been found to be very
 61 powerful. Solving the spatial Schrödinger equation, one is able to find the rough struc-
 62 ture of the Hydrogen atom, and solve for the shape of the electron wave function. Then,
 63 the addition of spin to the electron and the nucleus add a number of features, giv-
 64 ing quite a good prediction of the level structure. Finally, Quantum-Electro-Dynamics
 65 (QED) corrections yield an extremely accurate picture of the energy levels.

66 With recent advances in spectroscopy, it is now possible to “count” optical cycles
 67 of a spectroscopy laser beam, as reviewed in Picqu and Hnsch (2019). In short, a beat
 68 signal is generated with a mode-locked laser, which has a frequency spectrum with a
 69 large number of equally spaced peaks at frequencies

$$f_n = f_0 + n f_r , \tag{1}$$

70 a “frequency comb”. One of these frequencies f_n is near enough to the spectroscopy
 71 laser frequency so that the beat frequency is in the radio-frequency (RF) region, and
 72 can be measured with a standard RF counter. The number n can be determined with
 73 a standard wavelength meter on the spectroscopy laser. The repetition frequency f_r is

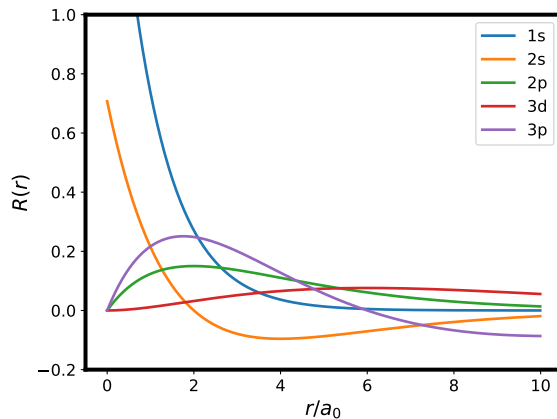


Figure 2. The radial wave functions in hydrogen. There is a significant fraction of the wave function inside the nucleus at $r = 0$, particularly for $l = 0$ states (1s and 2s in the figure). The horizontal axis is scaled by the Bohr radius a_0 .

74 also in the RF, and can easily be measured. Finally f_0 , which is less than f_r , can be
 75 determined by frequency doubling the frequency comb, and beating a low frequency
 76 component of this doubled beam to a high frequency component of the original laser
 77 beam. The beat frequency is f_0 , which can then be measured and locked.

78 **3.1. Background**

79 The radial part of the Schrödinger equation for an electron in a central potential with
 80 charge Z reads

$$-\frac{\hbar^2}{2m} \frac{d^2 u}{dr^2} + \left[-\frac{Ze^2}{4\pi\epsilon_0 r} + \frac{\hbar^2 l(l+1)}{2m r^2} \right] u = Eu \quad (2)$$

81 with radial wave functions as illustrated in Figure 2. From these distributions, it is
 82 apparent that there is a finite probability for the (2s) electron to be inside the nucleus.
 83 Here, the Bohr radius $a_0 = 4\pi\epsilon_0 \hbar^2 / (m_e e^2)$, with ϵ_0 the permittivity of the vacuum,
 84 the reduced Planck's constant \hbar , the electron mass m_e and the electron charge e .

85 A correction for the size of the nucleus can be found by considering the nucleus
 86 as a charged conducting sphere with radius R_n , and we set the potential to being
 87 constant inside this sphere. The correction can then be found by standard first order
 88 perturbation theory. It should be noted that for normal atoms the radius of the nucleus
 89 R_n is four orders of magnitude smaller than the Bohr radius a_0 , and that hence the
 90 energy shift is small.

91 The radial wave functions plotted in figure 2 are for Hydrogen, and are exact solu-
 92 tions. The available wave functions for helium are approximations, but Drake and Yan
 93 (1992) worked these out to an acceptable precision by to calculate the correction to the
 94 energy levels due to the electron wave function being inside the nucleus. Using these,
 95 Morton et al. (2006) determined the helium spectrum to a high level of accuracy, but
 96 without the influence of the size of the nucleus. Quantum Electro-Dynamics (QED)
 97 corrections, as calculated in this paper, are worked out in powers of the fine structure
 98 constant. By measuring the spectrum of helium with high resolution, and finding the

99 difference with the theoretical prediction, a value for the charge radius of the nucleus
100 can be found.

101 An interesting twist is that the QED corrections are identical for ^4He and its isotope
102 ^3He . Hence the uncertainties introduced by these imperfect corrections are cancelled
103 when the isotope shift is used to find the difference in radius between these two nuclei.
104 As the radius of the alpha particle is relatively well known, this could then serve as
105 a measurement of the ^3He nucleus. This charge radius is actually larger than that of
106 the alpha particle, even though it has fewer nucleons.

107 *3.2. Proton radius puzzle*

108 As the structure of the hydrogen atom can be calculated exactly, excluding the charge
109 radius of the nucleus (a proton), hydrogen was a prime candidate for a spectroscopic
110 experiment, where the energy difference between two states is measured with great
111 accuracy. A number of spectroscopic measurements before 2010 agreed with the results
112 from electron scattering as given by the CODATA value as can be found in Mohr et al.
113 (2016).

114 However, in 2010, everything changed with an experiment using muonic hydrogen
115 Pohl et al. (2010). In this experiment, the electron is replaced by a muon. As muons
116 and electrons, according to the standard model, differ only by their masses, this should
117 be a straightforward replacement. The advantage of using muons, which have a mass
118 about 200 times larger than the electron, is that, as the Bohr radius a_0 scales inversely
119 with the mass of the “electron”, the larger mass of the muon gives a 200 times smaller
120 radius of its orbit. Hence the size of the nucleus would yield a few million times larger
121 correction to the energy levels.

122 The main difficulty with these experiments is of course that one needs muons to be-
123 gin with, and these are produced in a nuclear reactor. In a landmark experiment, Pohl
124 et al. (2010) found a value for the charge radius of the proton of $r_p = 0.84184(67)$ fm,
125 which was as much as 5σ smaller than the accepted value of $0.8768(69)$ fm.

126 This led to a great deal of excitement, as any differences between hydrogen and
127 muonic hydrogen could point to physics beyond the standard model. For a treatise
128 on the standard model, refer to Oerter (2006). The precision of the muonic hydrogen
129 result is also helped by significant theoretical insight in its level structure, as worked
130 out by Antognini et al. (2013) and by Indelicato (2013).

131 An interesting development came in 2017, when new results in normal hydrogen by
132 Beyer et al. (2017) agreed with the muonic measurements and put the radius of the
133 proton at $r_p = 0.8335(95)$ fm. This was recently corroborated by a new result on the
134 $2s-2p$ splitting by Bezginov et al. (2019), which claims $r_p = 0.833(10)$ fm.

135 A recent review by Karr and et E. Voutier. (2020) beautifully summarises the efforts
136 to date to find the charge radius of the proton as well as that of the alpha particle. They
137 expect that the discrepancy between the CODATA value and the new spectroscopic
138 results will be resolved by better accuracy in both the scattering and spectroscopic
139 experiments contributing to this CODATA value, and that the muonic hydrogen result
140 will be confirmed by more normal hydrogen experiments in the future.

141 Very recently however, an experiment by Abi et al. (2021) shows a deviation of 4.2σ
142 of the muon magnetic moment from its theory prediction. This indicates that lepton
143 universality, meaning that a muon should behave identically to an electron apart from
144 the mass, is violated. Will this mean that there is new physics in the muonic atom
145 experiments, and the proton radius is closer to the CODATA value after all?

146 It is this author's view that this issue is far from resolved, as many measurements
 147 contribute to the CODATA value for the charge radius of the proton. We may see
 148 more evidence of physics beyond the standard model emerging from high accuracy
 149 experiments, and the proton radius puzzle may have only been the first of the road
 150 signs along the way.

151 4. Helium Experiments

152 A good candidate for the determination of the charge radius of the helium nucleus is
 153 afforded by the $2^3S \rightarrow 2^3P$ transition frequencies, which are around 1083 nm. As can
 154 be seen in Fig. 2, the $2s$ metastable lower state overlaps with the nucleus, whereas
 155 the $2p$ excited state has a significantly reduced overlap, as the amplitude of the wave
 156 function at $r = 0$ is zero for a p -state. However, these transitions are dipole allowed,
 157 and hence have a line width of ~ 1.6 MHz, much larger than the shift induced by
 158 the finite size of the nucleus. In an atomic beam experiment, careful spectroscopy by
 159 Pastor et al. (2004) of this transition has allowed for a new determination of the Lamb
 160 shift of the 2^3P states in helium.

161 A narrow transition in helium, which would allow for a more precise measurement
 162 of the transition frequency, is found in the $2^3S \rightarrow 2^1S$ transition, which is dipole and
 163 spin forbidden. The linewidth of this transition is $\Gamma = 2\pi \cdot 8$ Hz and the transition
 164 wavelength is around 1557 nm, in the telecom bracket. This line width is due to spon-
 165 taneous decay of the 2^1S state to the ground 1^1S state, but due to the twice forbidden
 166 nature of the transition, the $2^3S \rightarrow 2^1S$ transition has an Einstein A coefficient of
 167 $A \approx 9 \cdot 10^{-8} \text{s}^{-1}$, which should be compared to $A \approx 10^7 \text{s}^{-1}$ for the allowed transition
 168 discussed above. To excite the forbidden transition, a narrow-band laser is required as
 169 well as a long interaction time.

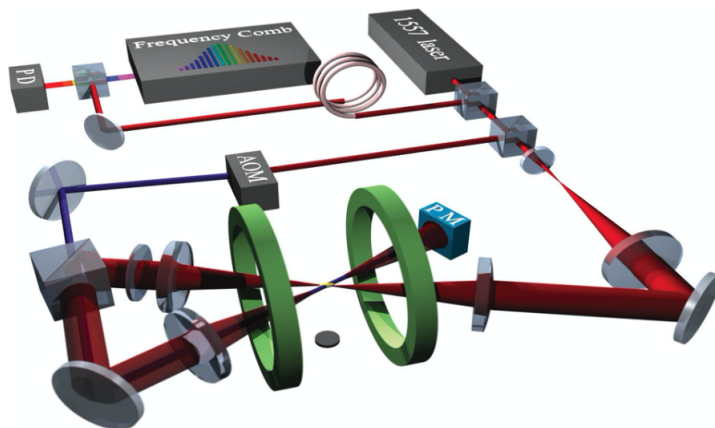


Figure 3. Schematic representation of the experiment by Van Rooij et al. (2011)

170 The first experiment exciting this transition and performing a high-resolution spec-
 171 troscopy experiment on both isotopes of helium was published by Van Rooij et al.
 172 (2011), who used the setup illustrated in Fig ???. A Bose-Einstein Condensate of
 173 metastable 2^3S atoms was captured in a dipole trap, formed by a laser close to res-
 174 onance with the forbidden transition. The ultracold atom sample and conservative

175 dipole trap allowed interaction times of up to six seconds. For the fermionic ^3He iso-
 176 tope, a degenerate Fermi gas was captured in the same trap. It should be noted that
 177 the hyperfine interaction adds an additional frequency shift, but that is known very
 178 accurately. The precision of this experiment for both isotopes was one part in 10^{11} ,
 179 limited by the laser linewidth and the large AC Stark shift of the atoms in the trap.
 180 The radius of the ^3He nucleus was determined to be 1.961(4) fm at the time. This
 181 was also limited by the accuracy of the QED corrections applied, which went up to
 182 α^4 . As mentioned, the QED corrections cancel when the difference in radius is investi-
 183 gated, and the difference in radii squared $\Delta r_c^2 = r_c^2(^3\text{He}) - r_c^2(^4\text{He})$ is proportional to
 184 the isotope shift. In this experiment, the difference $\Delta r_c^2 = 1.019(11) \text{ fm}^2$. This agreed
 185 reasonably well with a previous determination using the $2^3\text{S} \rightarrow 2^3\text{P}_0$ transition, which
 186 yielded $\Delta r_c^2 = 1.059(3) \text{ fm}^2$.

187 A little later, experiments by Cancio Pastor et al. (2012), using spectroscopy on the
 188 $2^3\text{S} \rightarrow 2^3\text{P}$ transitions, have also determined the difference between the charge radii
 189 of ^4He and ^3He , and found a significant shift of the transition frequency due to the
 190 size of the nucleus. More recent experiments on this same transition in ^4He by Zheng
 191 et al. (2017), using the ^3He data from Cancio Pastor et al. (2012), put the difference
 192 in charge radii to $\Delta r_c^2 = 1.028(2) \text{ fm}^2$, in good agreement with the ultracold atom
 193 experiment by Van Rooij et al. (2011), but with a higher precision.

194 A second generation of the ultracold atom experiment used a more narrow-band
 195 laser (linewidth 500 Hz) and a special magic wavelength trap, where the AC Stark
 196 shifts of the ^3S and ^1S states are identical. The 319 nm laser trap, described by
 197 Rengeling et al. (2016), was necessarily within a nanometre to an allowed transition,
 198 which led to a significant spontaneous emission rate. Therefore, interaction times were
 199 limited to one second. However, the reduced laser linewidth more than made up for the
 200 reduced interaction time, and the transition frequency could be determined to 1 part
 201 in 10^{12} for ^4He , a factor of 10 improvement, as reported by Rengeling et al. (2018). The
 202 experiment on ^3He is currently under way. Given the previous determination of the
 203 ^3He transition frequency, combined with the new determination of the ^4He frequency,
 204 yielded a difference in charge radius of the nuclei of $\Delta r_c^2 = 1.041(7) \text{ fm}^2$. If the new
 205 determination of the ^3He isotope yields a similar precision as the ^4He measurement,
 206 the precision of the difference in radii will reduce to less than 0.002 fm^2 .

207 In the singlet part of the spectrum, a new spectroscopic measurement by Huang et al.
 208 (2020) on the $2^1\text{S} \rightarrow 3^1\text{D}_2$ two-photon transition have also come to a determination of
 209 the difference in charge radius. This experiment uses a fluorescence method in a gas
 210 cell. The wavelength of the two-photon transition is 1009 nm. They determine the
 211 isotope shift to be 29.530246(18) GHz, leading to a derived difference in charge radius
 212 of $\delta r_c^2 = 1.059(25) \text{ fm}^2$.

213 Another experimental effort focuses on spectroscopy of the helium ion He^+ . As the
 214 helium ion only has one electron, like hydrogen, the level structure is exact. Again,
 215 the focus is on the two photon 1s–2s transition. In a new experimental effort, He^+
 216 ions are trapped and sympathetically cooled by beryllium ions. This eliminates the
 217 Doppler shift and increases the interaction time. The interrogation is then carried out
 218 with a 30 nm photon, generated by high-harmonic generation, and a 790 nm photon,
 219 as detailed in Krauth et al. (2019).

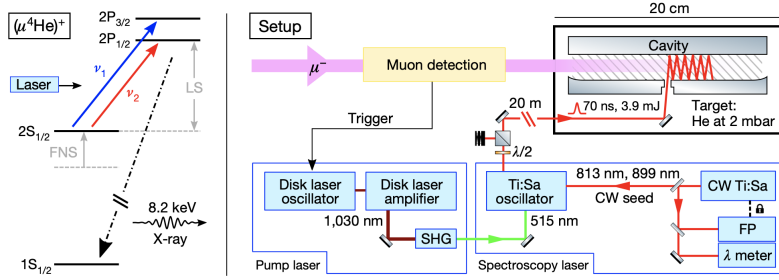


Figure 4. Schematic representation of the experiment by Krauth et al. (2021)

220 5. Exotic helium experiments

221 Very recently, the first result has been published on the charge radius of the helium
 222 nucleus using muonic helium by Krauth et al. (2021). This exciting new development
 223 benefits from the same increased sensitivity and a more convenient wavelength as the
 224 muonic hydrogen measurement discussed earlier. In fact the finite size of the nucleus
 225 accounts for 20% of the splitting between the 2s and 2p states. The linewidth of the
 226 observed 2s–2p transitions observed (319 GHz) is much larger than those used in
 227 normal atoms, leading to a 15 GHz precision on the frequency of the line centre, but
 228 that is more than offset by the gains in sensitivity. Their measurement of the radius
 229 of the α -particle $r_\alpha = 1.67824(13)_{\text{exp}}(82)_{\text{theo}}$ fm is the most accurate to date. As
 230 illustrated in Fig. 3, slow muons are decelerated by collisions with helium gas atoms.
 231 In the last collision, the muon takes the place of one of the electrons. The second
 232 electron is rapidly ejected by the internal Auger process, yielding a single muon bound
 233 to a helium nucleus, typically in a highly excited state. About 1% of the atoms decays
 234 into the 2s metastable state. A pulsed laser is triggered when a muon enters the
 235 detection chamber and performs an excitation to the 2p state, and the decay to the 1s
 236 ground state is observed through the emission of an X-ray photon subsequent to the
 237 laser pulse.

238 As with hydrogen, the muonic determination of the nuclear radius is by far more
 239 precise than previous spectroscopic determinations. It does however depend on lepton
 240 universality, which may be violated by physics beyond the standard model. This is
 241 an exciting development, and new spectroscopic determinations in normal helium are
 242 essential to understand this new physics.

243 A further recent development is the laser spectroscopy of pionic helium, as reported
 244 by Hori et al. (2020). A negatively charged pion π^- , formed by a down quark and an
 245 up antiquark, can also replace the electron in a helium atom. The pion is absorbed by
 246 the nucleus quickly, but there are some highly excited metastable states with lifetimes
 247 in the nanosecond region. As these highly excited states are relatively far removed
 248 from the nucleus, little information on the size of the nucleus can be expected from
 249 pion spectroscopy, but it may lead to insight into physics beyond the standard model.

250 6. Helium theory

251 Work is also continuing on the theoretical determination of the relativistic corrections
 252 to the helium spectrum. A recent advance by Patkóš et al. (2020) has now worked out

253 the corrections to the triplet states in helium to 7th order in the fine structure constant.
254 However, the numerical value of the correction, which would allow, in combination with
255 the experimental results, for an absolute determination of the charge radius of the α -
256 particle, has not been completed yet. This numerical value should not take long to
257 find, and we look forward to an exact determination.

258 A significant body of work working out the theory of the Lamb shift and fine struc-
259 ture in muonic helium has been published by Diepold et al. (2018) and also for ^3He
260 by Franke et al. (2017). The accuracy achieved in the muonic helium experiments is
261 clearly helped by these detailed investigations.

262 7. Conclusion

263 The size of the alpha particle, or rather its charge radius, remains an interesting
264 testbed for QED and Quantum Chromo Dynamics (QCD) calculations. Since the
265 days of Rutherford there have been extensive experiments on the charge radius of the
266 helium nucleus using electron scattering, yielding a value accurate to a few parts per
267 thousand. The latest spectroscopic determination using muonic helium yielded a result
268 for the charge radius with a precision of a few parts per million. The spectroscopic
269 measurement of the transition frequencies in normal helium are accurate to one part
270 in 10^{12} , and combined with state-of-the-art calculations of the QED corrections to
271 the level structure, should also yield an accurate determination of the helium nuclear
272 charge radius. It will be interesting to see how that compares to the muonic helium
273 measurement. More than a century after Rutherford's revolutionary model of the atom,
274 there is still plenty to discover in the structure of the atom.

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