Children's Everyday Mathematical Experiences (EMEs)

in Hong Kong Kindergarten Settings

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Dedication

This thesis is dedicated to the loving memory of my NaiNai, *谢美蕊/Xie Meirui*, you will always be remembered, and to my supportive and encouraging MaMa, *李小青/Li Xiaoqing*, and BaBa, *张自欢/Zhang Zihuan*. You have successfully made me the person I am becoming through your endless love, support, and encouragement. I am honoured to be your pride.

Abstract

This photo was taken in a Hong Kong (HK) kindergarten classroom. Two five-year-old children were using their knowledge of comparison to playfully and spontaneously explore the different length of pencils while others were doing a worksheet. When I was a kindergarten teacher in HK, I often saw children exploring mathematics in an informal and playful manner. I term these experiences as *everyday mathematical experiences* (EMEs), and these are what I focused in my study. The settings for my study were HK kindergartens, where formal mathematics learning and teaching play a dominant role. Within such a serious atmosphere, how might there be opportunities for children's informal and playful EMEs?

Previous studies have suggested that EMEs may be beneficial to children's activities and play in the moment as well as preparing children for formal education, and that the role of teacher is essential for encouraging and promoting EMEs in kindergartens. Although research on children's learning has expanded over the past three to four decades, empirical studies in relation to early childhood mathematics are limited, especially in the context of HK. Therefore, my study aimed to investigate the nature and content of children's EMEs in HK kindergarten settings, affordances and constraints that influence the availability of children's EMEs, and teachers' perceptions and practices regarding children's EMEs.

A qualitative approach of case study drawing on ethnographic techniques, framed by sociocultural theory, offered an in-depth understanding of children's EMEs. Children and teachers from two kindergartens in HK were the main participants of my study. Three main data collection methods were used: classroom observations, individual semi-structured interviews with teachers, and documentation. A range of strategies, including data triangulation, enhanced the credibility and dependability of the study. Key themes from the findings were identified through data analysis procedures.

The findings revealed that children's EMEs in HK kindergartens showed four features in that they were exploratory, playful, informal, and spontaneous. Findings also showed the content of children's EMEs. Children were capable of exploring and using a range of mathematical concepts and knowledge to construct EMEs. Additionally, the study found that diverse aspects of the physical environment, social environment, and classroom atmosphere in kindergartens both afforded and constrained children's construction of EMEs in HK kindergarten settings. An unexpected finding was that teachers' perceptions and practices, influenced by Chinese cultural beliefs, drove children to turn their EMEs into "underground" and hide from teachers in some situations. All these findings led to my argument that a deeper understanding of curriculum and the teacher's role are needed to improve teachers' responses to children's EMEs in HK kindergarten settings.

My study adds value to existing sociocultural research via the exploration of sociocultural factors that influenced children's EMEs and relevant teachers' perceptions and practices. It provides teachers with insights regarding how to enhance their values and practices in relation to EMEs so that they might align more closely with children's exploratory, playful, informal, and spontaneous nature in mathematics learning. For policy makers, my study provides intellectual support to the establishment of curriculum documents and policies so that it is possible to empower children to construct EMEs with surrounding objects, peers, and teachers or adults in order to facilitate informal mathematics learning in kindergartens.

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Chapter 1: Introduction

In this study, I investigate children's everyday mathematical experiences (EMEs) in Hong Kong (HK) kindergarten settings. EMEs involve young children's ways of learning and applying early mathematical concepts in their everyday experiences. The process of establishing the operational definition of EMEs for my study is addressed in Chapter 2. Drawing on this key concept, I explore the nature and content of children's EMEs in kindergartens, factors that influence the availability of EMEs, and teachers' perceptions and practices regarding EMEs.

This introductory chapter introduces the thesis; what it is about, why the research topic is important, how the study was conducted, and how the thesis is laid out to present the study. I first discuss the context of the study and thesis problem statement, then state the rationale and research purposes for my study, before providing an overview of the thesis.

Context of Study and Statement of the Problem

Because my study focused on exploring children's EMEs in HK kindergartens, it is important to begin by providing background information about kindergarten education and early childhood mathematics education in the context of HK, then state the problem.

Overview of Kindergarten Education in HK

In HK, kindergarten education is not compulsory. It refers to the education and care services provided by kindergartens and kindergarten-cum-child care centres, which are both registered with the Education Bureau (EDB). The main distinction between the two organisations is that kindergartens are just for 3- to 6-year-old children, but Kindergarten-cum-child care centres provide services for children from birth to 6 years. I use the term "kindergarten" to describe both organisations, as both provide kindergarten services for 3- to 6-year-old children. In HK kindergartens, children are enrolled in classes according to age-group, of which there are three: Nursery classes (K1) for 3- to 4-year-old children, lower classes (K2) for 4- to 5-year-old children, and upper classes (K3) for 5- to 6-year-old children (EDB, 2015a).

All kindergartens, including half-day and full-day kindergartens, are privately run and categorised either as “non-profit-making kindergartens” or “private independent kindergartens”, depending on whether they are sponsored by charitable organisations or private enterprises (EDB, 2015a). Most non-profit-making kindergartens adopt a “local curriculum” that aligns with the curriculum documents published by the HK Curriculum Development Council (CDC). Although private independent kindergartens operate within guidelines set by the EDB, the curriculum they provide may or may not align with the CDC’s curriculum documents. Some private independent kindergartens incorporate ideas from a range of curriculum philosophies, such as the Reggio Emilia, Montessori, and Waldorf approaches (EDB, 2015b). To understand children’s EMEs influenced by the HK curriculum documents, I focused on investigating children and teachers from non-profit-making kindergartens in my study.

Official curriculum documents structure and shape the curriculum of HK kindergartens. Since 1984, the CDC has formally launched five official curriculum documents[1]. The *Guide to the Pre-primary Curriculum (2006)* and *Kindergarten Education Curriculum Guide (2017)* are the two main documents that currently guide curriculum planning in kindergartens, particularly in non-profit-making kindergartens. In the 2017/2018 school year, the CDC (2017) launched its latest curriculum document, the *Kindergarten Education Curriculum Guide*. Compared to the previous curriculum document (CDC, 2006), the major changes noted in the current curriculum document (CDC, 2017) include (i) the names of three out of six learning areas[2], including mathematics learning, (ii) the

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[1] The five curriculum documents include *Guide to the Kindergarten Curriculum (1984)*, *Guide to the Kindergarten Curriculum (1993)*, *Guide to the Pre-primary Curriculum (1996)*, *Guide to the Pre-primary Curriculum (2006)*, and *Kindergarten Education Curriculum Guide (2017).*

[2] In the CDC (2006), the six learning areas referred to “early mathematics”, “science and technology”, “arts” “physical fitness and health”, “language”, and “self and society”. In the CDC (2017), the six learning areas referred to “*early childhood mathematics*”, “*nature and living*”, “*arts and creativity*”, “physical fitness and health”, “language”, and “self and society”.

promotion of "free play"[3], and (ii) the transition between kindergartens and primary schools. The earlier curriculum document (CDC, 2006) still has a profound influence on the curriculum in HK kindergartens, as it guided the kindergarten curriculum for 11 years. Hence, although I refer to information cited from the current curriculum document CDC (2017) in this thesis, the earlier curriculum document CDC (2006) has helped me to understand and explain teachers' perspectives and practices regarding EMEs and HK kindergarten education.

Official curriculum documents also structure and shape children's learning through pre-determining the main purpose, objectives, and areas of learning and development. Preparing for formal education is the main purpose of HK kindergarten education. According to the CDC (2017), "quality kindergarten education should be aligned with primary and secondary education in laying a firm foundation for the sustainable learning and growth of children" (p. 7). Moreover, to foster children's holistic development across the five "domains of ethics, intellect, physique, social skills and aesthetics" (p. 19), the CDC (2017) specifies the six learning areas noted earlier.

Although the CDC (2017) claims that "the classification of learning content into different learning areas […] is not intended to promote learning by subjects" (p. 28), the fact that children's learning is broken into distinct areas can be used to drive content. Content-based learning mainly focuses on "the acquisition of knowledge and mastery of concepts" (Wood et al., 2010, p. 3). As a result, many kindergartens in HK that followed official curriculum documents explicitly taught numeracy and promoted children's academic learning (Ho, 2006).

Official curriculum documents also structure and shape teachers' pedagogy and practice. Influenced by the growing focus on early childhood education (ECE) in the late twentieth century, HK kindergarten curriculum documents promote child-centred pedagogy and play-based approaches.

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[3] I put the word "free play" in inverted commas in this thesis to represent the "free play" described in the CDC (2017) and carried out in HK kindergartens, aiming to distinguish it from the free play noted by previous research.

For instance, in the 1980s, the Curriculum Development Committee[4] (1984) encouraged kindergartens to adopt “learning through play” as a pedagogical approach, even though it only described play as a supplement to teacher-directed learning activities and a strategy to facilitate children’s academic learning and physical development. Since 1996, the CDC (1996, 2006, 2017) has revised curriculum documents and encouraged teachers to adopt “real-life themes”, an “integrated approach”, and “learning through play” as three main pedagogical approaches for facilitating teaching and learning across the six learning areas. These pedagogical approaches currently influence education in HK kindergartens (Ho, 2015; Rao et al., 2010). However, teachers’ understandings of the pedagogical approaches proposed by curriculum documents may be limited by contextual factors, which will be discussed later in this chapter and Chapter 2.

After unpacking key ideas from the CDC (2006, 2017), I interpret constructivism, social constructivism, and sociocultural theory as three theories that underpin these documents. The CDC (2006) explicitly noted that it used “the perspective of constructivist learning theory” to understand children’s learning process (p. 7). Drawing on this perspective, among diverse pedagogical approaches, the CDC (2006, 2017) has highlighted the value of a child-centred approach, which aligns with a constructivist approach to education (Tandon, 2017), in HK kindergarten education. The two documents have described “child-centredness” as the core value of the kindergarten curriculum, with children being viewed as “unique individuals”. The CDC (2017) specifically notes the value of “children’s development and learning needs, experiences and interests” (p. 140). Thus, constructivism is the first theoretical underpinning of the documents.

The CDC (2017) also emphasises the importance of social interaction in kindergarten education. This notion aligns with the concept of social relations in children’s learning and development proposed by social constructivism and sociocultural theory. Additionally, the CDC (2017) suggests that “Children’s prior knowledge and experiences should be taken into account [by

[4] In Hong Kong the Curriculum Development Committee preceded the CDC.

teachers] to help them gradually develop levels of deeper understanding of new things" (p. 26). In the process, the CDC (2006, 2017) highlights the use of language by promoting daily conversations between children and teachers. The CDC (2006) has noted, "Most knowledge originates with language. […] 'Language is a key to full participation in life' (Lybolt & Gottfred, 2003, p. 25) and also a key to knowledge acquisition" (p. 22). These ideas align with Vygotsky's (1978) sociocultural theory, which views language as an important cultural tool and argues that cultural and historical elements should be considered alongside social interaction.

Underpinned by constructivism, social constructivism, and sociocultural theory, the CDC (2017) states that children's individual needs and interests, play-based practices, social interactions with peers and teachers, and teachers' roles and responsibilities are important considerations in helping children construct "appropriate and effective learning experiences" (p. 17) (as will be discussed again shortly in this chapter) in different learning areas, including mathematics. It suggests that teachers could (i) "design learning activities with abundant sensory stimulation" (p. 26) and appropriate content related to children's everyday life in particular (ii) create an environment that encourages children's "self-directed learning" (p. 26) in kindergartens every day; and (iii) participate in children's play by solving "problems for children during play at an appropriate time" (p. 67) and "invite children to share their experiences and feelings" (p. 67) after play. These statements indicate that the CDC (2017) values children's learning experiences through play and interacting with other people in kindergartens.

In summary, the endeavours of the HK government to support kindergarten education and facilitate children's learning and development demonstrate that the government recognises the importance of kindergarten education and the value of children's everyday learning experiences. Curriculum documents provide HK kindergarten education with a pedagogical framework that leads to learning and teaching within six learning areas, including mathematics. I introduce early childhood mathematics education in HK kindergartens in the next section.

Overview of Early Childhood Mathematics Education in HK Kindergartens

As noted earlier, the CDC (2017) has replaced the term “early mathematics” noted in the CDC (2006) with “early childhood mathematics”. In doing so, the CDC (2017) aims to “highlight that […] over-teaching should be avoided” (p. 28) and “to show explicitly that learning of mathematics at this stage should take into account children’s growth and their developmental needs” (p. 29). The Chinese version of the CDC (2017) adopts the term “幼兒數學/ jau3 ji4 sou2 hok6[5]”, which literally means children’s mathematics. However, in the English version of the CDC, this word is translated as early childhood mathematics, highlighting early childhood. Early childhood is defined as birth to six years (CDC, 2017). Therefore, early childhood mathematics education in HK refers to mathematics education for children under six years of age. Neverthless, the terminology used in the Chinese version of the CDC (2017) could limit teachers’ understandings of early childhood mathematics, as it does not show the nature of early childhood mathematics addressed in the English version, and most teachers and kindergartens refer to the Chinese version rather than the English one. This may lead to teachers’ over-teaching mathematics as they endeavour to prepare children for primary school.

There are two purposes of implementing early childhood mathematics education in HK, which reflect the expectations of the community and culture for children’s mathematics learning. First, preparing children for formal mathematics education. This purpose aligns with the primary goal of HK kindergarten education as noted earlier. In this case, the CDC (2017) views early childhood mathematics as a subject area of children’s learning. Second, facilitating children’s competence in “using mathematical concepts to solve practical problems in real-life situations” (CDC, 2017, p. 43) or through “everyday life experiences” (CDC, 2006, p. 31). This involves encouraging children to “be aware of the relationship between mathematics and life” (CDC, 2017, p.

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[5] This study used the Linguistic Society of Hong Kong Cantonese Romanization Scheme.

45). Therefore, early childhood mathematics is also taught as a daily living skill (e.g., solving mathematical problems that exist in everyday life situations) in HK kindergartens.

To prepare children for formal education, teachers play an essential role in facilitating children’s learning. The CDC (2017) suggests that teachers could adopt three main approaches to facilitate children’s learning across the six learning areas. Because the focus of my study is mathematics, I discuss the three main approaches adopted by teachers to facilitate children’s mathematics learning below.

The first approach is constructing the physical environment and preparing learning materials for children’s mathematics learning. The CDC (2017) indicates that “experimenting with objects can help reinforce the [mathematical] concepts in children” (p. 39), so “[teachers] can create real-life learning contexts and select suitable objects and teaching aids to facilitate children’s learning” (p. 41) in mathematics. “Interest corners”[6] play an important role in the classroom and are designed as spaces to encourage free choice activities. According to my working experience in an HK kindergarten, interest corners in classrooms usually include a book corner, a dramatic play corner, a language (i.e., English and Mandarin) corner, a mathematics and science corner, an art corner, a thematic learning corner, a toy corner, and/or a computer corner. More information about interest corners is presented in Chapter 3.

To prepare children for formal mathematics education and aid children’s mathematics learning, the CDC (2017) supports the approach of manipulating physical objects in kindergartens. Drawing on the belief that the manipulation of physical objects is beneficial to children’s mathematical competence, the CDC (2017) notes that “experimenting with objects can help reinforce the [mathematical] concepts in children” (p. 40). It also advises teachers to prepare “diversified materials for children to manipulate according to their developmental and learning

[6] In HK kindergartens, “in the interest corner, children can choose to take part in an activity they are interested in either on their own or in groups. Teachers should prepare diversified materials for children to manipulate according to their developmental and learning needs, thereby facilitating children’s learning in different areas and enhancing their learning interests” (CDC, 2017, p. 120).

needs, thereby facilitating children's learning in different areas [including mathematics]" (p. 120). However, the CDC (2017) has not provided a detailed description regarding the context for how these physical objects are used. As a result, teachers may prepare resources focused more on academic learning (e.g., number matching cards) or children's play (e.g., board games and Lego) for children.

The idea of learning mathematics by manipulating physical objects is associated with a research project called *Operational Mathematics Learning in Preschoo*l. This project was funded by the Quality Education Fund and promoted in 80 kindergartens in HK from 2006 to 2008. The project was conducted by Cheng (2002, 2008), who was also a member of the CDC committee on ECE that contributed to the launch of the CDC (2017), and her fellow researchers. Cheng (2002, 2008) believed that children can learn the basic concepts and principles in mathematics and develop logical thinking through operating physical objects. This belief aligns with the CDC (2017). As a consequence, many HK kindergartens facilitate children's mathematics learning using the teaching kit designed by Cheng (2002, 2008) and her colleagues. Their teaching kit mainly includes resources that explicitly highlight a mathematics learning focus (e.g., 10 x 10 pegboard).

Influenced by the operational mathematics project, the operational mathematics curriculum is an early childhood curriculum used in many kindergartens in HK aiming to introduce mathematics to children in an accessible manner (Cheng, 2007, 2008). According to Ni et al. (2010), the operational learning curriculum concretises mathematics (e.g. by using a 10 x 10 pegboard), but also connects children to their respective mathematics concepts (e.g. cardinal and ordinal numbers and place values), and encourages children to highlight their systematic structure (e.g. addition and subtraction). As a result, Ni et al. (2010) claimed that children can create an image of the logico-mathematical system of concepts and operations, thereby aiding their learning of abstract logico-mathematics in primary schools (Cheng, 2007, 2008).

Second, the CDC (2006, 2017) places value on the benefits of children's everyday experiences to meet the two purposes of implementing early childhood mathematics education mentioned earlier. The term "everyday" noted by the two curriculum documents indicates three meanings, including (i) informal; (ii) relating to children's daily life (the English version of CDC (2017) adopts the term "real-life"), such as "family, festivals and community" (CDC, 2017, p. 28); and (iii) regular happenings every day (e.g., the time allocation of child- and teacher-directed activities and play prescribed in the daily kindergarten timetables).

The CDC (2006) has asserted that everyday activities and play whereby children may develop mathematical concepts are the foundation of mathematics learning, claiming, "These activities and experiences are essential for children to develop their interest, positive attitude, necessary communication skills and thinking abilities for learning mathematics" (p. 31). That is, children's everyday experiences are beneficial to children's mathematics learning. The CDC (2017) also notes that being "closely related to children's life" is a feature of early mathematical concepts (p. 43). It encourages children to apply mathematical concepts in their life every day. Without further unpacking these statements, the two curriculum documents suggest that teachers use "daily activities and play" to "introduce and consolidate mathematical concepts" (CDC, 2006, p.31; CDC, 2017, p. 46). This idea aligns with Vygotsky's (1987) idea of "scientific concepts". According to Vygotsky (1987), scientific concepts are introduced to children through some form of adults' instruction.

Based on the descriptions of early childhood mathematics in the CDC (2017) and Vygotsky's (1987) notions of spontaneous and scientific concepts (as will be explained in Chapter 2), I use the term "everyday mathematical experiences (EMEs)" to refer to children's ways of learning and applying mathematical concepts in their everyday lives. These experiences can be found in their meaningful day-to-day activities and play with others in kindergartens. For example, two children are stacking blocks in the classroom. One says, "My building is tall", and the other responds, "No, my building is very tall". In this example, children initiate peer interaction and apply the concept of height in block play. They discover the difference in height and use verbal language to express their understandings (i.e., tall and very tall). I view such experiences as EMEs that children commonly construct in kindergartens. More details of children's EMEs will be presented in Chapter 2.

Daily communication between children and their peers and teachers is the third approach to facilitate children's mathematics learning. The CDC (2017) highlights the relationship between "children's language development and their formation of mathematical concepts" (p. 41). It suggests that teachers "use mathematical concepts more often to discuss with children the things they encounter in their everyday life and encourage children to communicate among themselves" (p. 40). However, verbal communications in the classroom may be challenged by factors such as the wording of curriculum documents, Confucian images of children, parents' expectations, teacher-child ratios, and class size. Thus, the statement of the problem is discussed as follows.

Statement of the Problem

As discussed above, HK curriculum documents place value on children's mathematical experiences constructed through everyday activities and play in kindergartens. However, in a study investigating mathematics teaching in HK kindergartens, Ng (2014) found "a gap between the recommendations made in [the 2006 curriculum document] and practices adopted in the classroom" (p. 5). Although the CDC (2006) did not recommend formal mathematics teaching, kindergartens and teachers still employed teacher-directed ways to teach mathematics. Some research results indicated that most HK kindergartens taught numeracy, used worksheets, set homework, and adopted examinations to promote children's academic and pre-academic learning (e.g., Opper, 1992; Ho, 2006).

The CDC (2017) restates the importance of child-centredness and play noted in the 2006 curriculum document, and since the 2017/2018 academic year has promoted "free play" in kindergartens (as will be described in Chapter 2). However, Hsieh's (2004) research has pointed out that teachers' resistance towards the child-centred approach makes it challenging to incorporate child-centred teaching and learning practices. Cheng's (2001) study found that many HK kindergarten teachers showed difficulty in understanding "learning through play". She further argued that these teachers' classroom practices indicated "over-simplified and distorted interpretation of 'play'" (p. 868), which led to teachers neglecting children's learning experiences in play. Thus, the inconsistency between curriculum documents and practices that align with the CDC (2017) may constitute tensions for children's construction of their learning experiences through everyday activities and play in kindergartens.

This inconsistency is due to a range of reasons including, the pedagogical objectives of kindergarten education, the lack of detailed explanations of key terminology used in curriculum documents, cultural factors, parents' expectations, and other contextual factors. These are discussed below.

The first possible reason for the inconsistency between curriculum documents and teachers' practices is that the main pedagogical objectives at the kindergarten level are to ensure children achieve formal literacy and numeracy skills through direct instruction (Pearson & Rao, 2006). Although the CDC (2017) "emphasises the process of inquiry instead of content knowledge transmission" (Mak et al., 2018, p. 144), the curriculum document still refers to developmental characteristics of children at different ages, including their mathematical learning, that are associated with content knowledge (e.g., 3- to 4-year-old children should be able to recite from 1-10). Following the developmental characteristics suggested by the CDC (2017), kindergartens pre-determine content knowledge-based learning objectives, including those related to mathematics, for children of different ages to achieve.

It is common for HK kindergartens to use formally scheduled mathematics time to help children achieve pre-determined mathematics learning objectives (Ng, 2014). Furthermore, previous research, such as Ng (2014), found that many HK kindergartens and teachers used commercially published resources to guide their planning of learning and teaching activities, including mathematics learning, as the learning objectives in the commercially published resources aligned with curriculum documents. In Ng's (2014) study conducted in HK kindergartens, the teachers admitted that the existing reference materials served them well in sequencing mathematics concepts for children in different age groups.

Second, vague, contradictory, or insufficient explanation of key terminology provided by the CDC (2017) can contribute to inconsistencies between curriculum documents and practices. For example, as noted earlier on p. 5, the CDC (2017) highlights “appropriate” and “effective” as two features of learning experiences that children should be able to construct in kindergartens. However, it provides vague descriptions of what these words mean within the context of kindergarten education. This may lead to teachers’ misunderstanding of how to facilitate children’s learning and apply the pedagogical approaches noted in the CDC (2017).

Another example is that although the CDC (2017) identifies play as an appropriate approach in the kindergarten curriculum, it also claims that “an appropriate amount of homework” is one way to “consolidate, extend or assess children’s learning performance” (p. 72). Salili et al. (2003) have argued that using homework to facilitate children’s learning is influenced by the achievement-oriented aspect of Chinese beliefs about education that emphasise intensive drilling and practice. However, the CDC (2017) only defines homework “as task(s) accomplished at home” (p. 82). Without a further and clear explanation of the appropriateness of such terms, most teachers see their role as helping children prepare for future formal education through homework and repetitive drilling to satisfy parents’ expectations (Chan, 2016). Of note, some of the teachers who participated in Cheuk and Hatch’s (2007) HK study, complained that homework had become a necessary task for children to complete in kindergartens.

Some HK-based studies have found that homework influenced children’s learning and play in kindergartens. According to Chan (2016), Cheuk and Hatch (2007), and Ho (2015), homework is undertaken in kindergartens so that teachers can ensure children know how to do their homework. Children are expected to complete one or two examples of the required homework and to finish the rest at home by the next school day. When doing homework at kindergarten, children have to sit still and straight, be quiet and pay attention to what and how to write correctly. Additionally, children are only permitted to join free choice activities once they have finished assigned tasks, such as their homework. Therefore, access to free choice activities depends on how quickly the children finish their homework. Children who are slower at finishing their tasks have less or no time to play.

Third, the role of Confucianism in HK education has been studied widely (e.g., Hong & Howes, 2014; Huang & Gove, 2015; Yim et al. 2011). Confucianism, developed by the Chinese philosopher Confucius, has had a profound influence on Chinese views of education, educational practices, and children's learning experiences (Yim et al. 2011). According to Confucianism, it is crucial to build up children's capabilities from a young age (Hong & Howes, 2014), which are believed to contribute towards their future success in different areas (Cheon, 2006). Confucian educational beliefs may not align with the constructivist, social constructivist, and sociocultural pedagogies that underpin the curriculum documents. Additional influences of Confucianism on ECE are discussed in Chapter 2.

Fourth, HK parents' expectations are another factor. The CDC (2017) states that, "parents care about their children with respect to their growth and needs in various domains of development" (p. 17). It also suggests that kindergartens should "take the initiative to understand parents' expectations and make appropriate adjustments to provide quality and special learning experiences for children" (p. 59). Nevertheless, this statement does not define "quality and special learning experiences". Pong and Chow's (2002) study showed that many HK parents were found to pay more attention to their children's achievement-oriented activities and schoolwork than social and cultural development. Fung (2009) indicated that most HK parents expected teachers to teach more academic concepts, including mathematical concepts, rather than allowing children to play. Accordingly, teachers may focus more on providing achievement-oriented activities in kindergartens.

Fifth, other contextual factors in HK, such as large class size, high teacher-child ratios, and a lack of funding may also influence classroom practices. Pan et al. (2008) observed that kindergarten routines required teachers to organise activities for 30 or even 40 children during the whole morning session, resulting in more teacher-initiated activities and interactions. Moreover, even though teachers created learning and playing areas for children in kindergarten classrooms, they provided limited choices and ways for children to interact with toys or materials because of lack of funding (Hu et al., 2017). Due to lack of funding, many kindergartens in HK are small in size and cannot afford an outdoor playground (Ho, 2015). Losing opportunities to freely play outside means “less opportunity for physical exercise, for learning about their environment, and for social interaction” (Robson & Smedley, 1996, p. 168).

After analysing the various reasons that may cause inconsistencies between ideas addressed in curriculum documents and practices adopted in kindergarten classrooms to align with the CDC (2017), a rationale for this study is addressed as follows.

Rationale

This study was undertaken by me for two main reasons:

Personal Interest and Experience

A reason for this study is my own background as a Chinese kindergarten teacher, who was trained and worked with children in a HK kindergarten for almost four years and is familiar with HK kindergarten education. This study stems from my interest in exploring children’s EMEs in HK kindergarten settings. When I was a kindergarten teacher in HK, during my interactions with children in kindergartens, their interest in using their own ways to learn and engage with mathematics inspired me to explore what was happening underlying children’s mathematical actions. Moreover, having a common language with children and teachers in HK, the knowledge and experiences of HK kindergarten education, and personal interest in exploring children’s EMEs makes me an appropriate person to conduct this study.

The Importance of Investigating Children’s EMEs in HK Kindergartens

The importance of early childhood mathematics in current HK kindergarten education triggered me to investigate children’s EMEs in HK kindergartens. According to the background information addressed above, HK kindergarten education focuses mainly on children’s academics, and promoting mathematics is a key concern in kindergarten education. Furthermore, the CDC (2006, 2017) has prescribed mathematics as one of the six learning areas in kindergarten education.

The importance of mathematics learning can also be seen in the emphasis on STEM (science, technology, engineering, and mathematics) education in kindergartens, which the HK government has promoted as a key emphasis in the primary and secondary school curriculum since 2015 (EDB, 2016). Although STEM education does not officially cover the kindergarten sector, several newspaper reports and kindergarten websites indicate that some kindergartens are paying attention to implementing STEM education (e.g., Bastille Post, 2016; Hong Kong Economic Journal, 2016). The possible reason for this phenomenon could be that many HK parents have set high academic-oriented goals for their children to prepare for their future schooling, as discussed earlier.

Additionally, as previously mentioned, the CDC (2006, 2017) has suggested that teachers adopt diverse approaches to extend children's everyday experiences across diverse learning areas, including mathematics. However, tensions and inconsistencies between curriculum documents and kindergarten practices may create challenges and barriers for teachers to understand children's everyday experiences, including EMEs. Thus, conducting a relevant study may provide insights for teachers to better understand the potential of children's everyday experiences in mathematics learning.

Research Purposes and Aims

The primary purpose of this qualitative case study is to investigate children's construction of EMEs in HK kindergartens. I adopted the lenses of sociocultural theory, affordances theory, and cultural beliefs to investigate children's EMEs.

In particular, my study aimed to explore the nature and content of children's EMEs, affordances and constraints that influence the availability of children's EMEs, and teachers' perceptions and practices regarding children's EMEs in HK kindergartens.

Thesis Overview

In the first chapter of this thesis, I have introduced my study by discussing the background information of kindergarten education and early childhood mathematics education in HK. Moreover, the statement of the problem, the rationale and significance of the study, and the purpose and aims of the study were detailed.

In Chapter 2, I provide a literature review on children's EMEs. Through the chapter, I first illustrate the process of establishing the operational definition of EMEs for this study by reviewing previous literature and HK curriculum documents. Second, I explore methods of investigating children's EMEs noted in previous empirical studies. Third, I investigate the role of EMEs in children's learning. Fourth, I address the theoretical underpinnings of my study to assist in efforts to understand children's EMEs in HK kindergartens at a theoretical level. The chapter concludes with an illustration of the research questions and a chapter summary.

Chapter 3 first presents the choice of methodology and methods for this study. The qualitative approach and interpretive paradigm are detailed before introducing case study methodology and the ethnographic techniques utilised in my study. Ethical considerations are provided before shifting the focus to the practicalities of data collection. To conclude this chapter, I address four steps of data analysis.

Chapters 4 and 5 present the study's findings and discussion with respect to (i) four main types of children's EMEs and the indicators or features of each of these, (ii) the mathematical content demonstrated by children while engaging in EMEs, (iii) affordances and constraints related to the physical environment, social environment, and classroom atmosphere, and (iv) teachers perceptions and practices regarding children's EMEs.

Chapter 6 concludes with a summary and highlights key findings and arguments uncovered in the study that answer the three research questions addressed in Chapter 2. I also describe the study's contributions and implications for teacher professional learning, teacher practices, and policy making. This chapter also outlines limitations of the study and suggestions for future studies. I end the thesis with personal reflections brought about by conducting this study.

Through presenting the whole thesis, I argue that a deeper understanding of curriculum and the teacher's role are needed to improve teachers' responses to children's EMEs in HK kindergartens.

Chapter 2: Literature Review on Children's Everyday Mathematical Experiences (EMEs)

The problem statement addressed in Chapter 1 navigates the review of literature related to my study. This literature review chapter serves five functions. First, it establishes the academic context of the study, which explored children's everyday mathematical experiences (EMEs) in kindergartens. Second, it is a path to establishing the operational definition of EMEs, the key concept of my study. Third, it discusses the debates arounds EMEs. Fourth, it establishes gaps that provide the academic rationale for my study. Fifth, it informs the research questions, methodology, and data analysis and interpretation of my study.

There are six sections in this chapter. Firstly, I provide a rationale for selecting sociocultural perspectives as the theoretical ground of my study. Secondly, I conceptualise key ideas associated with children's EMEs noted in previous literature and Hong Kong (HK) curriculum documents. I address the process of establishing the operational definition of EMEs in this section. Thirdly, I outline methods of investigating children's EMEs used in previous empirical studies. Fourthly, I explore the role of EMEs in children's learning. I also identify diverse factors that may affect children's EMEs in kindergartens from sociocultural perspectives. Fifth, I address the theoretical underpinnings of my study. I then provide the chapter summary and research questions at the end of this chapter.

Sociocultural Theoretical Framing

Although I adopted sociocultural approaches to frame my study, I noticed that many scholars have frequently used nativism and constructivism to investigate and examine young children's mathematical experiences. Thus, before proceeding to establishing the definition of EMEs for my study, it is important and useful to clarify why I adopted sociocultural perspectives, rather than nativism and constructivism, to guide my study.

Nativists believe that important elements for human beings to understand the world are innate; we do not have to learn these elements from experience, as they are part of our initial conditions (Samet & Zaitchik, 2017). International research from nativist perspectives has investigated young children's mathematical competence. Results revealed that children showed mathematical skills and competence from a very young age. Cognition research claims that babies, from birth, can detect numerical correspondences and abstract properties of objects and events (Starkey et al., 1990). However, researchers such as Cowie (1999) and Fodor (2001) critique nativist perspectives, arguing that nativism is a denial of empiricism, as human beings lack a way to explain what is innate compared to what human beings have not yet learned.

Without focusing on the critique of nativism, some researchers have argued that children are "biologically primed to learn [mathematical concepts]" (Ginsburg, 2006, p. 147). Gelman (2000) has indicated that "we are born with number-relevant mental structures that promote the development of principles for counting" (p. 36). It is noticeable that the statements made by Ginsburg (2006) and Gelman (2000) are inspired by constructivism.

Piaget's theoretical concepts play an important role in constructivism, which highlights that cognitive development or "intelligence" leads to the growth of language and precedes learning (Piaget, 1962). According to Piaget (1952), human beings, including children, create knowledge through the interaction between their experiences and ideas. Regarding mathematics, he claimed that children's logical-mathematical knowledge, constructing from acting on physical objects, is not present until they enter the concrete operation stage at around six to seven years of age (Piaget, 1952). However, this idea could be misleading in that children do not learn mathematical knowledge until a specific age. To Piaget (1962), learning is bound by development and age. However, Driscoll (1994) has argued that sometimes children can demonstrate unexpected cognitive strengths at lower ages than those indicated in Piaget's stages of knowledge development.

Although I did not use Piaget's theoretical concepts to shape my study, his ideas regarding children's exploration with physical objects during play inspired me to think about how physical objects influence children's ways of learning and applying mathematical concepts. He emphasised the importance of exploration during play for children's cognitive development (Piaget, 1952, 1962). According to Piaget (1952), exploration refers to the "discovery of new means through active experimentation" (p. 263). He believed that children would engage in "exploration by chance", which resulted in "the discovery of unknown phenomenon" if they were provided with "new objects" (Piaget, 1952, p. 255). More details about how Piaget's ideas of physical objects have been used in previous empirical studies and relevant critiques are discussed in the next section.

Sociocultural theory grew from Vygotsky's work, who believed that learning precedes development (Vygotsky, 1978) and, thus, holds the opposite position to Piaget (1952). Vygotsky (1987) explained that children's development is neither age-bound nor a set of "mechanical processes where each new phase begins only with the completion of the previous one" (p. 160). Chan (2014) has indicated that Vygotsky's (1987) statement highlights "the fluidity of children's learning and development" (p. 37) rather than a linear process noted by Piaget (1952, 1962). Vygotsky (1978, 1994) believed that social, cultural, and historical factors influence children's learning and development. Additionally, Vygotsky (1986) noted the importance of language and viewed language as developing thought. He further maintained that thought develops from society to the individual, rather than the other way around other way.

The sociocultural theoretical framing provides a basis for my study for three reasons: (i) I believe that children's ways of learning and exploring mathematics occurs in a sociocultural context, which aligns with Vygotsky's ideas. According to Vygotsky (1978), long before they enter school, "children have their own preschool arithmetic" (p. 84) learned in the course of their interactions with others, particularly others who are more competent in culturally valued skills and concepts; (ii) as discussed in Chapter 1, sociocultural theory is a theory that underpins HK kindergarten curriculum documents, such as those published by the Curriculum Development Council (CDC) (2006, 2017); and (iii) previous researchers have involved sociocultural perspectives in investigating children's ways of learning and exploring mathematics in early childhood settings (e.g., Benigno, 2012; Papandreou & Tsiouli, 2020). Research based on sociocultural perspectives focuses on the sociocultural basis of children's mathematical experiences. However, these kinds of international studies are limited. Thus, my study contributes to the field.

In this chapter, I use sociocultural perspectives to interpret previous literature in relation to children's ways of learning and applying mathematics. More details of the theoretical underpinnings of my study are addressed later in this chapter. Next, drawing on previous literature, I establish a working definition of EMEs for my study by explaining what children's EMEs are.

What are Children's EMEs?

My study adopted the term everyday mathematical experiences (EMEs) because I was interested in children's ways of learning and applying mathematics in their everyday life (as noted in Chapter 1). My intention was to use the word "everyday" to distinguish these kinds of mathematical experiences from mathematical experiences taught by teachers in a formal and didactic manner. However, few scholars have used this term. After ongoing searches using Google Scholar (which were undertaken right up until one month before this thesis was submitted), only one empirical doctoral study (i.e., Benigno, 2012) and one theoretical study (i.e., Tudge et al., 2008) have used this term as the research topic. In Benigno's (2012) study, she interpreted children's EMEs as the experiences that naturally unfold in children's daily activities at home, or in other everyday settings, prior to formal schooling. In Tudge et al.'s (2008) article, they described EMEs as children play with, talk about, and use mathematical concepts, such as shapes, time, and distance during their "typically occurring everyday activities" (p. 188). I only used their interpretations as a part of my definition of EMEs, as they focused on diverse settings that give meaning to "EMEs" (e.g., home, early childhood settings, and other everyday settings) rather than the sole kindergarten settings in my study.

Some researchers in the early childhood mathematics field have devoted themselves to investigating these kinds of mathematical experiences. Depending on what children do with mathematics during play and everyday activities, different terms to describe these experiences. For example, "everyday mathematics" (Ginsburg, 2006), "spontaneous focusing on numerosity" (Hannula & Lehtinen, 2005), "informal numeracy" (Purpura & Lonigan, 2013), "informal mathematical ideas" (Resnick, 1989), "informal mathematics" (Resnick, 1991), "emergent mathematical skills" (Schwarz & Shaul, 2018), and "everyday, naturally occurring mathematical activities" (Tudge & Doucet, 2004). Although this chapter uses the terms noted in original studies to engage in the discussion, I also generally term them as "informal and spontaneous mathematical experiences" for short. In doing so, I can distinguish these terms from the way I define EMEs. It is also important to note that the above researchers used the terms of "informal", "everyday", "spontaneous", "emergent", or "naturally occurring" without carefully defining these terms. Thus, in this chapter, I discuss their usage and how they align (or not) with my thinking. This will lead to establishing the definition of EMEs for my study.

As can be seen from the above terms, previous researchers focused on either mathematics or numeracy in their research. In my study, because curriculum documents, such as the CDC (2017), mainly adopt the term mathematics, I also use mathematics in EMEs to align with the HK context. However, I am aware that numeracy is both different from, and not an alternative to, mathematics (Steen, 2001). Numeracy is "the ability to process numerical and probabilistic information" (Park & Cho, 2019, p. 530). Some researchers view numeracy as the most important mathematical capacity to "make effective use of mathematics in contexts related to personal life" (Geiger et al., 2015, p. 611) and other situations. In the discussion below, I use both mathematics and numeracy according to the terms noted in original studies.

Additionally, previous scholars sometimes used the term mathematics or numeracy in noun form to describe informal and spontaneous mathematical experiences (e.g., Ginsburg, 2006; Resnick, 1991; Purpura & Lonigan, 2013). Without a clear explanation, the meaning of mathematics or numeracy in noun form is vague. This inspired me to clarify that I mainly use the term mathematics in adjective form to describe children's mathematics-related concepts, knowledge, and experiences in this thesis. More details about children's mathematical concepts, knowledge, and experiences will be unpacked later in this chapter.

In addition to the terminology, the defining of children's informal and spontaneous mathematical experiences by previous researchers was also inconsistent and vague. Without providing a clear and operational definition, some previous literature generally described the findings of examples of children's informal and spontaneous mathematical experiences (e.g., children deciding how some cookies should be fairly divided among siblings) (e.g., Ginsburg, 2006). Because these kinds of examples are specific, they did not enable me to have a clear understanding of the broader phenomenon. Therefore, it was important to construct a working definition of EMEs for my study, to show my understanding of children's EMEs and guide my data collection, data analysis, and discussion of findings.

The review of previous empirical studies showed that even though there have been variations across definitions of children's informal and spontaneous mathematical experiences, three aspects have been noted consistently across those definitions, including (i) contexts, (ii) purposes of applying or exploring mathematical concepts and knowledge by children, and (iii) children’s mathematical concepts and knowledge. To align with these three aspects and develop a definition of EMEs for my study, this section discusses: (i) children’s play and everyday activities in HK kindergartens; (ii) children’s informal and spontaneous mathematical application and exploration; (iii) early mathematical concepts and knowledge; and (iv) notions of children’s mathematical experiences in HK curriculum documents.

Children’s Play and Everyday Activities in HK Kindergartens

Although previous research has mainly noted children’s everyday practices as out-of-school practices (e.g., home), I argue that routine activities and play that take place at kindergartens are also children’s everyday practices in HK. As noted above, previous researchers sometimes used everyday mathematics interchangeably with informal mathematics or out-of-school mathematics without justification (e.g., Ginsburg, 2006). Arcavi (2002) has critiqued that “Everyday mathematics may consist of disparate tasks, depending on the question ‘Everyday for whom?’” (p. 13). Moschkovich (2002) has stated that it can be misleading to use different labels (e.g., everyday, academic, and school) to differentiate mathematical practices. In HK, children go to kindergarten every day from Monday to Friday. The CDC (2017) also highlights the role of play in kindergartens, which is different from the school and out-of-school contexts for informal and everyday mathematics noted in previous studies. Thus, I explored children's everyday practices in terms of mathematics that took place during play and routine activities in the participating kindergartens in my study. I involve children's *play and everyday activities* as a part of the operational definition of EMEs and discuss as follows.

Before shifting to the description of children's play in HK kindergartens, I unpack some ideas of play noted in the literature, particularly from sociocultural perspectives. Vygotsky (1978) viewed play as the leading activity for young children, proposing that "play contains all developmental tendencies in a condensed form and is itself a major source of development" (p. 102). Vygotsky's (1978, 2004) theorisation of play recognises that children creatively and agentively reproduce their sociocultural experiences through re-creating their past experiences in play. It is noticeable that when talking about play, Vygotsky (1978) meant only sociodramatic play for preschool children and primary school children. This means that play defined by Vygotsky does not involve processes and activities (e.g., games, manipulating objects, explorations, and physical activities) that most people, including teachers, also call "play" (Bodrova & Leong, 2015). According to Bodrova and Leong (2015), Vygotsky's definition of play shows features of (i) creating imaginary situations, (ii) taking on and acting out roles, and (iii) following rules determined by specific roles. Through play, social pretence and imagination offer potentially rich contexts that "situate" learning and allow children to explore their existing cultural knowledge, including mathematics (Vygotsky, 1978).

Previous studies regarding children's informal and spontaneous mathematical experiences emphasise the value of free play. Free play is free, spontaneous, fun, without adults' intervention, and carried out for its own sake (Salomonsen, 2019). From birth, children are naturally exposed to and participate in mathematical activities through play, songs, games, counting, and exploring their surroundings (Worthington & Van Oers, 2016). When elaborating on the notion of free play, previous researchers often associate emotions with children's play. Reikerås (2020) has indicated that free play is regarded as having intrinsic value in itself, in the sense that it represents fun and enjoyment to children. Vygotsky (1926/2003) also mentioned emotions when addressing human's thoughts. He stated that “anyone who thinks that emotion is a purely passive experience of the body and it does not cause any activity [including play] is designing the question in a wrong way” (Vygotsky, 1926/2003, p. 118, as cited in Mesquita, 2012). More ideas about sociocultural perspective of play and emotions are outlined later in the section of theoretical underpinnings.

Free play in early childhood settings creates many situations that are rich in opportunities for learning and stimulating children’s development, including aspects related to mathematics (Ginsburg et al., 2008). Kindergartens are an important arena that give children opportunities for valuable peer interaction (as will be discussed later) during free play and for the development of learning skills (Ray & Smith, 2010), including mathematical skills. For example, Reikerås’s (2020) study examined relations between play skills and mathematical skills of 1088 toddlers in Norwegian early childhood settings. She assessed their mathematical and play skills through structured observation during exploration and construction play, and rule-based play. She found that toddlers showed different play skills during exploration and construction play and that their exploration and construction play was strongly related to their level of mathematical skills. Her key findings suggest that play and mathematics learning are related skills, and that children’s play, including free play, is beneficial to children’s mathematics learning.

Previous research has also shown conflicting results when mathematics in children’s free play is studied. For instance, Seo and Ginsburg (2004) and Worthington and Van Oers (2016) found that children’s mathematics-related actions naturally and frequently occurred in their play. However, Gifford (2005) indicated that children only applied a limited range and degree of mathematical concepts and knowledge during their free play. By investigating children’s ways of learning and exploring mathematics during play, my study has implications for solving these addressing results.

In HK kindergartens, timetables prescribed by the curriculum document and planned activities that teachers teach didactically take up most of the children’s time. Play is, however, scheduled as an activity, namely “free play” and free choice activities. As noted in Chapter 1, the

CDC (2017) deliberately highlights “free play” as a new focus of the kindergarten curriculum. “Free play” has been promoted in all kindergartens since the 2017/2018 academic year, aiming to align with the pedagogical approach of “learning through play”. According to the CDC (2017), “free play” refers to “a behavioural activity evoked by the intrinsic motivation of children. It emphasises children’s autonomy and free participation and children are not limited by adults/teachers established or pre-set rules and goals. During free play, children can choose their own tools, ways to play, playmates and activity area” (p. 119).

“Free play” and free choice activities, however, occur at fixed times in the timetables of HK kindergartens. According to the CDC (2017), “Free play can be carried out during free choice activities and physical activities” (p. 58). It also suggests that half-day kindergartens “should arrange no less than 30 […] minutes every day […] for children to participate in free play” (p. 58). This may lead to boundaries (e.g., regarding time and types of activities) for free play and exclude some aspects of free play noted in the literature. For example, Guedes et al. (2020) have defined free play as all “moments during which children play freely around the classroom corners or on the playground” (p. 3). To some extent, “free play” and free choice activities may not be free and accessible for children in HK kindergartens because, although children are able to take part in activities and play with toys in the interests corners, these are prescribed by teachers and take place within a limited amount of time (Ho, 2015). These tensions in relation to “free play” may be caused by the CDC’s (2017) uncritical way of adopting the concept of “free play”. Also, Confucian views of education may also lead to these tensions. These factors will be discussed later in this chapter.

Some HK researchers have found that contemporary notions of learning through play and conventional teaching beliefs co-exist in HK kindergarten classrooms (Rao et al., 2010), and that these influence children’s play. Some HK scholars have found that kindergarten teachers frequently implement teacher-directed group activities alongside more child-centred and play-based practices (Hu et al., 2015) or “eduplay” (Rao & Li, 2009). Eduplay as a mixture of teacher-directed and play-based learning, incorporates play and learning in HK kindergartens (Rao & Li, 2009). Through eduplay, children play to learn, and teachers considers children's needs when designing eduplay activities. This means that "children play with learning-related purposes" (Ho, 2015, p. 282). This approach is different from those ideas of play that children guide their own play activities, with little interference from teachers (Biber, 1984). Accordingly, even though some changes have occurred in the HK kindergarten curriculum (e.g., promoting a more child-centred approach and learning through play), traditional Confucian pedagogy may be still found with teacher-directed, discipline-oriented, and other features of classroom practice (as will discussed later in this chapter).

In addition to "free play" and free choice activities, children also carry out a range of routine activities in HK kindergartens every day (e.g., circle time, snack time, and toilet time). These routines enable opportunities for playful and spontaneous interactions between children that may include mathematical concepts, such as counting the number of children waiting to wash their hands or talking about the shape of cookies during snack time. Previous research has investigated the ways in which learning opportunities for very young children, including opportunities for learning mathematics, occur during everyday caregiving routines (Lee & Lomas, 2015). Caregiving routines, such as mealtimes, provide rich opportunities for social referencing, joint attention, and use of rich mathematical language (Chen et al., 2017) and promote the building of shared meaning through communication (Johnston & Degotardi, 2020).

Björklund (2014) has explained that during routine activities, it is the interactions instigated and maintained by teachers that are pedagogically significant. Thus, she has argued that routine activities provide an important everyday context to build knowledge, including mathematical knowledge. However, adults or teachers did not often recognise and utilise these times as opportunities to promote children's learning (Degotardi, 2010; Klette et al., 2018). There is a dearth of research on this issue. Through exploring children's mathematical experiences in routine activities in HK kindergarten settings, this study fills this gap.

In summary, drawing on the above discussions about children's play and everyday activities in HK kindergartens, I involved *play and everyday activities* in the operational definition of EMEs. I also used play and everyday activities as a criterion for selecting relevant literature about children's construction of mathematical experiences. This literature is reviewed next.

Children's Informal and Spontaneous Mathematical Application and Exploration

Previous studies have investigated children's mathematical experiences constructed during play and everyday activities, which I termed as informal and spontaneous mathematical experiences as noted at the beginning of this section. Children's construction of mathematical experiences by applying and exploring mathematical concepts and knowledge during play and everyday activities is a complex process. In this subsection, I unpack this complex process through discussing literature related to (i) children's mathematical application and exploration with physical objects, peers, and teachers and (ii) everyday, informal, and spontaneous mathematical experiences.

Children's Mathematical Application and Exploration. Guided by sociocultural approaches and the purpose of investigating children's EMEs in kindergarten settings in my study, I mainly discuss children's mathematical experiences that are related to the kindergarten context in this subsection. I view children's mathematical application and exploration with physical objects, peers, and teachers as their mathematical experiences.

Physical Objects. While investigating children's mathematical application and exploration in everyday settings, a group of studies has paid attention to features of the physical environment where children are situated. Ginsburg and Seo (1999) have indicated that children in all cultures develop in an environment containing a multitude of objects and events that can support mathematics learning in everyday life. Ginsburg (2006) has described the mathematical attributes of the physical world in which children are situated in. He stated:

> A large number of parallel bars is on the side of babies' cribs; stalks of corn in a field are similarly arranged in rows; there is a larger number of candies or stones in one collection than other; the toy is under the chair, not on top of it; blocks can be cubes and balls are spheres; in the field, one cow is front of the tree and another behind it. Although varying in many ways, including the availability of books, schools and

> "educational" toys, all environments surely contain objects to count, shapes to discriminate, and locations to identify (p. 2).

According to Ginsburg (2006), mathematical attributes of the environment can be found everywhere. Drawing on the mathematical potential embedded in the physical environment, Ginsburg (2006) has further argued that objects and events in the physical environment are not themselves mathematics, but they afford mathematical thinking. In brief, children are universally provided with common "supporting environments" (Gelman et al., 1991) for at least some aspects of mathematical development. These ideas resonate with concepts of affordances theory (as will be discussed later in this chapter).

Different from those objects that feature in the natural environment of children's lives, in HK kindergartens, as discussed in Chapter 1, the CDC (2017) supports the use of objects specially designed for mathematics learning (e.g., a 10 x 10 pegboard) to aid children's mathematics learning during "free play" and free choice activities. It also notes that "experimenting with objects can help reinforce the [mathematical] concepts in children" (p. 40). Some support for the idea of operations promoted by the CDC (2017) comes from developmental psychology and the belief that children's thinking is inherently concrete (e.g., Montessori, 1964; Piaget, 1970). For example, Montessori (1964) promoted a hands-on approach with concrete objects that satisfy children's curiosity to act on and explore the world and act as a "self-correcting" mechanism, by which she believed that children could see and feel mistakes in a concrete form. Additionally, the emphasis on contextualised knowledge and active learning (Martin & Schwartz, 2005) also provides support for the idea of operations. For example, embodied cognition theories indicate that action and perception lead to symbolic thought (Pouw et al., 2014). Aligned with this idea, some studies have theorised operations as providing children with rich opportunities for embodied and active learning, including mathematics learning (e.g., Donovan & Alibali, 2021; Martin & Schwartz, 2005).

Many of the above studies, however, are based on Piaget's theory, as briefly introduced earlier in this chapter. Piaget (2013) indicated that mathematical understanding in the early years comes from children's active involvement with ordinary objects (e.g., tables and trees) rather than formal mathematics teaching. Building on this idea, prior research adopted tasks involving physical objects to investigate children's informal and everyday mathematics in pre-set situations (Starkey & Klein, 2008). Children were found to count, compare heights and volumes, and transform, compose, and decompose geometric shapes in pre-set situations, such as block play (Sarama & Clements, 2009; Verdine et al., 2014). However, Schuler (2011) has pointed out that although physical objects may provide opportunities for children to apply mathematical concepts or knowledge, the mathematical potential of the physical objects is not predefined and can be shaped by how the objects are engaged with during everyday activities. More studies in terms of children's informal and spontaneous mathematical experiences constructed through interaction with physical objects in natural settings, rather than pre-set situations, are needed to support Schuler's (2011) argument. This study contributes to Schuler's argument by exploring how children use mathematical concepts and knowledge to engage with physical objects and settings.

According to Ginsburg (2006), the existence of mathematical food for thought does not guarantee that children will digest these by themselves; teacher interaction is a key part of the process. That is, although physical objects enable children's mathematical thinking, social interactions with peers and teachers may also play an important role in constructing children's EMEs. Influences of social interactions are discussed next.

Social Interactions. Guided by sociocultural perspectives, it is important to investigate children's mathematical application and exploration with social partners in child-directed activities and play. In kindergartens, peers and teachers are the main social partners for children. Children from a very young age show an active role in learning and "appropriating" (Rogoff, 1995) mathematical concepts and knowledge through social interactions with peers and teachers in early childhood settings (Björklund, 2008, 2010). The concept of appropriation noted here means that "when an individual takes an artifact (psychological-cultural tool) and makes it his own" (Esteban-Guitart, 2014, p. 129). According to Rogoff (1995), appropriation is "a process of becoming, rather than acquisition" (p. 142), by which children could "change and handle a later situation in ways prepared by their own participation in the previous situation" (p. 142). That is, through participating in social interactions, children may use the mathematical concepts and knowledge appropriated from others to construct mathematical experiences in later situations. The terms appropriation and related ideas will be addressed and used throughout the thesis. I explore children's mathematics-related social interactions with peers and teachers during play and everyday activities below.

Peer Interaction. In kindergartens, peers are important social partners for children to interact with. Sociocultural theory suggests that children learn best from more knowledgeable others, including more advanced peers, who can scaffold or guide their exploration and learning (Vygotsky, 1978). Peers are paramount in facilitating children's learning through play, which makes it critical to understand their role in children's mathematical exploration (Williamson et al., 2020). However, the potential for peer interactions to promote mathematics learning through exploration in the form of play in children has yet to be fully explored, providing a rationale for this study to explore the impact of peer interaction on children's EMEs.

Researchers have investigated different types of children's social interaction with peers during play, ranging from playing individually, to parallel play (i.e., engaging in similar behaviour alongside, but not with, another child) and cooperative play (i.e., coordinating their behaviour during play) (Rubin et al., 1983). Zippert et al. (2019) have stated that cooperative play is essential for mathematics-related play, as communication between peers during play can elicit meaningful discussions of mathematical concepts. For example, playing shops with a peer could involve talk related to numbers (e.g., "How much is the apple?" and "The apple costs one dollar."). Drawing on Vygotsky's idea of Zone of Proximal Development (ZPD) (as will be discussed later), Zippert et al. have argued that children's cooperation with peers during play leads children to problem solve more effectively and in more advanced ways than they would have done independently.

Previous research suggests that children explore mathematics jointly with their peers. In Hendershot et al.'s (2016) study of children's mathematics conversation in home-based family childcare centres in the United States, naturalistic observations of 3- to 5-year-old children's mathematics-related peer conversations showed that they produced an average of four words about magnitude, eight words about enumeration, and almost 12 words about spatial concepts in 90 minutes. In contrast, children produced less than one mathematical word when playing on their own, and overall tended to produce fewer mathematical words with their teacher (one word about magnitude and four words about enumeration). Their study suggests that peers may encourage each other to talk about mathematics. However, whether this extends to nonverbal mathematical explorations is unclear. My study fills this gap by observing children's verbal and nonverbal mathematical explorations when constructing EMEs.

Additionally, previous research has found that peers contribute to children's mathematical exploration during play. In Williamson et al.'s (2020) New Zealand study, although their focus was not children's mathematical experiences, the findings generated from their naturalistic observations of 3- to 4-year-old children's peer play (i.e., constructing a domino track) showed that children "played around with [mathematical concepts, alongside] intellectual concepts about social rules, roles, responsibilities, power, and control" (p. 207). In Zippert et al.'s (2019) study conducted in the United States, mathematics-related materials were prepared for children in classroom-like settings. They used assessment tools in relation to children's mathematics ability and play to examine 3- to 5-year-old children's verbal and nonverbal mathematical exploration during play with peers. They found that an individual child's mathematical knowledge contributed to their peers' nonverbal mathematical exploration. Thus, I argue that peer interaction can help facilitate children's mathematics learning through exploring objects during peer play. However, peer play in Zippert et al.'s (2019) study took place in pre-set situations. Thus, more naturalistic observations of children's play in early childhood settings are needed to support my argument, providing a rationale for my study.

Drawing on the crucial role of peers in children's mathematical application and exploration, Zippert et al. (2019) have suggested that the extent to which children explore mathematical concepts during play with a peer and without adult assistance has been largely unexamined. Thus, my study contributes to the field by examining these subjects. In addition to peers, much research has included children's interactions with adults or teachers because previous scholars view the interaction with adults or teachers as an efficient way of facilitating children's mathematics learning. Next, children's interaction with teachers is addressed.

Interaction with Teachers. Teachers are important adult social partners for children to interact in kindergartens. In the early years, children apply mathematical ideas and intellectual tools that they appropriate from their culture and with teachers' guidance and assistance to develop mathematical knowledge and problem-solving skills (Carraher et al., 1985; Radziszewska & Rogoff, 1991). The CDC (2017) and a substantial body of research indicate that with teachers' support, children can develop a wealth of informal mathematical knowledge in early childhood (e.g., Funahashi & Hino, 2014; Rigelman, 2007; Swaminathan et al., 2011; Trawick-Smith et al., 2016).

Among these studies, on the one hand, some have focused on teacher interaction and its effectiveness in developing children's mathematical concepts during play and everyday activities (e.g., Swaminathan et al., 2011; Trawick-Smith et al., 2016). On the other hand, many other researchers have emphasised teachers' roles and responsibilities in developing children's mathematical knowledge or skills through formal teaching (e.g., Funahashi & Hino, 2014; Rigelman, 2007). I do not discuss teacher interaction in relation to formal mathematics teaching here, as my study does not align with these kinds of studies. Previous studies about mathematics-related teacher interaction in relation to play and everyday activities are discussed as follows.

In a qualitative study of four early childhood teachers conducted in the United States, Swaminathan et al. (2011) found that mathematics-related teacher interaction naturally occurred in play settings. They most frequently observed teacher interaction related to number concepts (e.g., as teachers encouraged children to count or compare amounts of objects and to identify and write numerals during play). They found that teachers engaged children in two processes of mathematical interactions: (i) in communication interactions when teachers initiated conversations with children about their mathematical thinking as they played; and (ii) in problem-solving interactions—as they posed challenging mathematics-related problems for children to solve. However, Swaminathan et al.'s study did not investigate how teacher interactions influence children's mathematical exploration during play.

Another study, by Trawick-Smith et al. (2016), was conducted in four classrooms of an early childhood setting in the United States. To examine the relationship between "teacher-child play interactions" and children's growth in mathematics ability, they used diverse measuring tools to examine children's mathematical knowledge. This included coding data collected through video-recordings of teacher-child interactions that occurred naturally during a one-hour free play period in each classroom. Their findings revealed that mathematics-related teacher interactions with children in play settings were important for children's mathematics learning during play. They also found that "good-fit interactions, in which teachers provide just the right amount of guidance to children in their play" (p. 728), had a great influence on children's mathematics learning. Trawick-Smith et al.'s study provides evidence for the relationship between diverse types of teacher interaction and children's mathematics learning during play. However, social interaction is a reciprocal process, and their study did not investigate the process of children's mathematical application and exploration during play, rather it focused on assessing children's mathematical performance. As a result, children's responses to teacher interaction are unknown. My study contributes to the field of mathematics research by focusing on children's responses to teacher interaction.

In addition to play, researchers have also suggested that teachers can facilitate mathematics learning by engaging in joint talk about mathematical concepts (e.g., Boonen et al., 2011; Klibanoff et al., 2006). Below I discuss two studies investigating the relationship between teachers' "math talk" (Klibanoff et al., 2006), the amount of mathematical words and phrases in teachers' speech, and children's mathematical abilities.

Klibanoff et al. (2006) conducted a study as part of a larger project that took place in 26 classrooms at 13 early childhood settings in the United States. They examined the relationship between teachers' math talk and the growth of 4- to 5-year-old children's mathematical knowledge over the school year. They used tools to measure children's mathematical knowledge at the beginning and end of the school year. They also used the techniques of audio-recording, questionnaires for observers, and a 2.5-3 hours classroom observation to collect data related to teachers' math talk. To compare math talk across the different classrooms, they only transcribed teachers' verbal speech during one-hour "circle time" (i.e., time for the entire class to get together and participate in discussions) and the time immediately following circle time. Their findings showed that the amount of math talk varied among different teachers. Moreover, the amount of teachers' math talk was significantly related to the growth of children's mathematical knowledge over the school year.

Another study conducted by Boonen et al. (2011) investigated the relationship between teachers' math talk and 5-year-old children's number sense within Dutch kindergarten classrooms. They video-recorded 35 teachers' verbal and non-verbal classroom practices during circle time. After analysing diverse categories of teachers' math talk, the research findings showed that there were significant positive relations between teachers' math talk categories, such as cardinality and conventional nominatives, and children's score on specific number sense tasks. However, children's number sense acquisition was negatively impacted by the diversity in teachers' math talk. The researchers suggest that teachers should be careful and selective with the amount of math talk that they offer to children.

These two studies provide evidence that engaging in math talk with teachers contributes to children's mathematics learning. However, research about teachers' interactions with children in terms of children's mathematical application and exploration in child-directed activities and play is still needed. This study fills this gap by observing teachers' practices in relation to children's EMEs.

Although teacher interactions are beneficial for children's mathematics learning during play and everyday activities, previous research has found that adults, including teachers, rarely joined or even constrained children's play. For example, Seo and Ginsburg (2004) observed 4- to 5-year-old children's mathematical competence during free play in early childhood settings in the United States. They found that most children participated in a significant number of varied mathematical activities through free play. However, the teachers in their research spent little time with children as they played. Although several studies support the finding that adults spend little time with children during free play (e.g., Ginsburg et al., 2008; Lee, 2006), relevant research is still needed to investigate the affecting factors underlying the phenomenon. My study fills this gap by exploring the nature of EMEs and teachers' perceptions regarding children's EMEs during play and everyday activities. Additionally, in the context of HK, due to some cultural beliefs, previous research has found that teachers also set boundaries for or constrained children's play and, therefore, EMEs during play (as discussed earlier on pp. 26-27). This will be discussed again later in this chapter.

In summary, the literature reviewed in this subsection specifically focused on discussing how children interact with physical objects, peers, and teachers to construct mathematical experiences during play and everyday activities in kindergartens. Accordingly, I add children's *interaction with surrounding objects, peers, and teachers or adults* to the definition of EMEs. Next, I discuss children's everyday, informal, and spontaneous mathematical experiences.

Everyday, Informal, and Spontaneous Mathematical Experiences. According to the previous literature regarding children's informal and spontaneous mathematical experiences noted earlier, the idea of everyday EMEs is a complex concept associated with two notions (i.e., informal and spontaneous). To establish the operational definition of EMEs, I unpack the notion of everyday by discussing the relationship between the concepts of everyday, informal, and spontaneous features in this subsection. I argue that informal and spontaneous are two aspects of the notion everyday.

Previous studies (e.g., Ginsburg, 2006) have used the term "everyday" interchangeably with "informal" to represent the meaning of out-of-school settings (e.g., home). In doing so, these studies tend to contrast children's everyday or informal mathematical experiences with formal and academic mathematical experiences. However, without a clear justification, the interchangeable use of everyday and informal may cause misunderstanding. Thus, it is necessary to clearly discuss the informal aspect of “everyday” addressed in these studies.

Regarding mathematical experiences, previous studies have interpreted the meaning of informal from two aspects. Firstly, researchers generally used informal mathematical experiences to describe children’s mathematical experiences that were not constructed in a formal school environment. For example, Resnick (1989, 1991) indicated that “informal mathematical knowledge” refers to the knowledge that children learn outside the school setting or from informal settings. Magnusson and Pramling (2018) have conceptualised children’s experience as the appropriation of knowledge about cultural tools (as will be unpacked later in this chapter) and practices. Thus, children’s informal mathematical experiences could consist of the informal mathematical knowledge and relevant practices noted by Resnick (1989, 1991). Resnick (1989, 1991) argued that children build up informal mathematical knowledge through everyday experiences and informal teaching by parents and social partners (e.g., siblings), rather than being taught in a formal schooling environment.

The idea of informal mentioned by Resnick (1989, 1991) may not be appropriate for HK kindergarten education for two reasons: (i) Resnick (1989, 1991) did not clearly define school and formal education in the research. Therefore, the definition may lead to confusion as to whether ECE is part of formal education. In HK, kindergarten education is not considered to be formal education, as it promotes learning through informal practices, such as play. Also, the CDC (2006, 2017) has indicated that kindergarten education aims to prepare children for formal schooling (i.e., primary education). And (ii) although kindergarten education in HK is not considered formal education, a formal schooling environment may include kindergartens, as many kindergartens in HK carry out formal mathematics guided by teachers (e.g., teaching written mathematical symbols or algorithms) (Ng, 2014). That is, kindergartens in HK can be a place where formal teaching takes place (Ng & Rao, 2008). In my study, I do not set a boundary between formal contexts and informal, or out-of-kindergarten contexts, to contrast informal with formal. However, inspired by Resnick’s (1989, 1991) ideas, I argue that children may bring their informal mathematical experiences constructed in out-of-kindergarten contexts to the kindergarten context, which is a type of EME explored by my study.

Children's mathematics learning begins at a very young age, before they enter ECE settings. Out-of-kindergarten contexts provide opportunities for meaningful exploration of mathematics by children (Guberman, 2004). Engagement in cultural practices outside of early childhood settings may have a profound impact on the knowledge that children bring to the classroom (Guberman, 2004). This idea aligns with the notion of "funds of knowledge", as each child brings their own funds of knowledge into the classroom that has been culturally and historically developed for a family's "household or individual functioning and well-being" (Moll et al., 2005, p. 73). Investigating children's EMEs in kindergarten settings does not mean separating children's out-of-kindergarten mathematical experiences or funds of mathematical knowledge, as children bring these to kindergartens, as I noted above. In HK, children's out-of-kindergarten contexts may include home, church, playground, tutoring centres, and so forth. It is noticeable that tutoring services in HK are available for children as young as 18 months to prepare children for interviews when applying for kindergarten (Bray, 2015; Eng, 2019). Although these out-of-kindergarten contexts were not my study focus, I also explored how these contexts influence children's EMEs in kindergartens.

Secondly, instead of contrasting informal with formal, some researchers view informal as a feature of children's informal mathematical experiences and note the difference between children's informal mathematical experiences and formal, didactic mathematical experiences. For example, Ginsburg (2006) described informal mathematical experiences (he adopted the term "everyday mathematics" in his study) as children's use of "informal skills and ideas relating to number, shape, and pattern as they play with blocks or read storybooks" (p. 145). In this description, he associates children's mathematical knowledge and skills with their everyday activities, rather than regarding them as a fixed knowledge or topic existing in textbooks.

The idea of informal learning theorised by Rogoff et al. (2016) enabled me to further unpack the meaning of informal and to understand the value of children's informal learning of mathematics and, therefore, EMEs in my study. To some extent, Ginsburg's (2006) idea of "everyday mathematics" aligns with the concept of informal learning proposed by Rogoff et al. (2016), as both studies pay attention to the informal feature of children's learning, rather than focusing on out-of-school settings as suggested by Resnick (1989, 1991). Rogoff et al. (2016) have indicated that informal learning takes place in diverse settings, including "everyday family and community settings that are not designed around instruction", "innovative schools", "children's 'underground' informal learning in schools", "after-school programs", "institutional settings that have an instructional as well as voluntary leisure focus, such as science centres and museums" (p. 358). In this sense, I argue that diverse activities in kindergartens, even teacher-directed mathematics learning activities in HK kindergartens, are contexts for children's informal learning of mathematics and, therefore, EMEs.

Rogoff et al. (2016) have proposed that the value of informal learning is often overlooked "because of its assumed inferior opposition to formal learning" (Hedges, 2021, p. 4). They have indicated shared features across the settings noted above to show the value of informal learning, which resonates with Vygotskian perspectives (Hedges, 2021).

> Informal learning is interactive and embedded in meaningful activity; Guidance is available to learners and their partners through social interaction and the structure of activities; Talk is conversational, not didactic; Involvement builds on individual initiative, interest and choice; Assessment occurs in support of contributing to the activity, not for external purposes; Participants hone their existing knowledge and skills and also innovate, developing new ideas and skills (pp. 359-360).

I view these features of the concept of informal learning proposed by Rogoff et al. (2016) as features of children's informal ways of learning and applying mathematics and, therefore, EMEs. These features guided the processes of data collection and analysis in my study. Both Hedges (2021) and Rogoff et al. (2016) have suggested the importance and value of practices and studies about children's informal learning, guiding my study to investigate children's informal learning of mathematics and, therefore, EMEs. I also used the idea of informal in my definition of EMEs, which will be addressed later.

The notion of everyday or informal is sometimes associated with being spontaneous. Vygotsky's (1987) idea of spontaneous concepts, as mentioned in Chapter 1, enabled me to unpack the notion of spontaneous. The CDC (2006) acknowledges that children can form mathematical concepts or construct mathematical experiences through everyday activities and play in kindergartens by highlighting, "Through a variety of activities such as hands-on experiment and play, [children] are also able to grasp basic mathematical concepts" (p. 31). This idea aligns with Vygotsky's (1987) idea of "everyday concepts", which refer to spontaneous concepts. I use the term "spontaneous concepts" to distinguish the meaning of everyday as spontaneous—as per Vygotsky's (1987) notion of everyday concepts—from the CDC's (2017) references to children's "informal" everyday experiences.

Drawing on Vygotsky's (1987) idea of spontaneous concepts, I argue that children's spontaneity shows situational and empirical features. Vygotsky (1987) identified spontaneous concepts as an intuitive understanding of everyday contexts, which are formed during children's hands-on activities. However, different from the notion of learning mathematical concepts "through a variety of activities such as hands-on experiment and play" (CDC, 2006, p. 31), Vygotsky (1987) believed that spontaneous concepts are related to what children encounter in everyday life. Children form spontaneous concepts during everyday activities through direct interaction with other people or the world. Thus, children's formation of spontaneous concepts shows situational and empirical features. Relevant empirical studies that align with Vygotsky's (1987) idea of spontaneous concepts are presented later. In this regard, I view situational and empirical aspects of children's spontaneity as a feature of children's ways of learning and applying mathematics during play and everyday activities and, therefore, EMEs. This feature guided my collection and analysis of data. I will address additional ideas and values related to Vygotsky's (1987) spontaneous concepts later, in the section of theoretical underpinnings.

In addition to Vygotsky's (1987) spontaneous concepts, previous studies noted earlier on pp. 20-21 highlighted children's spontaneity when explaining everyday or informal mathematics. For example, Ginsburg (2006) has referred "everyday mathematics" to what Dewey (1976) called the child's "crude impulses in counting, measuring, and arranging things in rhythmic series" (p. 282). Hannula and Lehtinen (2005) have defined "spontaneous focus on number" as the frequency with which children focus their attention on the exact number of objects in a set, without guidance or prompting. To some extent, these statements show that children are agentive in their mathematics learning. Thus, I argue that children may show an agentive aspect of spontaneity when engaging in their EMEs. However, defining spontaneous from the perspective of involving no guidance or prompting, as noted by these researchers, may be problematic. This is because other studies (e.g., Ramani et al., 2015; Zippert et al., 2019) have found that children's mathematical exploration in free play, and therefore EMES, can be guided and extended when collaborating with more capable peer and adults. In this case, children's spontaneity may not be "natural, universal, and independent of cultural background, home environment, and instructions" (Zuberi, 2002, p. 56) as claimed by Piagetian perspectives. This critique strengthened my decision to not use Piagetian views to frame my study.

The idea of underground proposed in Rogoff et al.'s (2016) theorising of informal learning further assisted me to understand children's agentive aspect of spontaneity in EMEs. Rogoff et al. have explained that:

> Students construct and participate in the unsanctioned underlife in the classroom by employing strategies that are differentiated from teacher-dominated discourse: that is, strategies that undercut the roles students are expected to play. By creating a parallel script to the teachers' formal script, students engage in "counter script," in which students assert unacknowledged cultural references and linguistic practices that are neither recognized nor included in the traditional teacher-led script, as students take stances against roles they are expected to play (p. 377).

Inspired by this idea, I assert that children may show their agentive aspect of spontaneity through creating a parallel script of constructing that differ to the teachers' informal scripts in kindergartens, including teacher-directed learning activities. However, I found it difficult to locate studies to support this argument, thereby providing a rationale for my study. More ideas related to Rogoff et al.'s (2016) idea of informal learning are addressed later, in the section of theoretical underpinnings.

Drawing on the discussion outlined above, I intended to involve *informal and spontaneous* in my definition of EMEs. To further establish the definition of EMEs, I discuss early mathematical concepts and knowledge next.

Early Mathematical Concepts and Knowledge

Review of previous empirical studies regarding children's informal and spontaneous mathematical experiences shows that the mathematical concepts and knowledge applied or explored by children have been consistently noted across these studies. In this regard, I intend to use *early mathematical concepts and knowledge* as a part of the definition of EMEs in my study. In this subsection, I discuss definitions of mathematics and early mathematical concepts and knowledge used in previous studies in connection with children's play and everyday activities in early childhood settings.

Previous studies have defined mathematics in early childhood in various ways; they sometimes used the term mathematics or numeracy in noun form without a clear explanation (as noted earlier on p. 21). In HK, the CDC (2017) does not provide a definition. In the studies reviewed, researchers adopted definitions to serve the purpose of their studies. For example, in Björklund's (2010) study, she used Schoenfeld's (1994) ideas and defined mathematics as "a comprehensive phenomenon including concepts, strategies and principles used in social settings, such as when communicating or problem solving, that describe numerical, spatial or time relationships between objects and events" (p. 75). She highlighted that the purpose of adopting this definition was to investigate toddlers' strategies for learning mathematics in day care settings in Sweden. It is noticeable that most of Schoenfeld's work was undertaken with older or school-aged children and focused more on formal mathematical concepts and knowledge taught through instructions in schools. Thus, it might not be suitable for investigating early mathematical concepts and knowledge that children appropriate through interacting with surroundings during play and everyday activities. However, Schoenfeld's (1994) definition of mathematics has potential for exploring children's mathematical experiences in the early years and, therefore, was used by Björklund (2010) as well as inspiring my study.

In order to explore children's EME in HK kindergartens in my study and drawing on sociocultural theory, I argue that mathematics has a cultural foundation, and include this idea in my definition. My argument is supported by a group of sociocultural researchers who believe that human beings' learning, or appropriation, takes place through participating in cultural practices (Rogoff, 1995). Through cultural activities, human beings "assimilate the experiences of humankind" (Leont'ev, 1981, p. 55). Researchers, such as Brandt and Tiedmann (2009), have indicated that mathematics is a human product inseparable from its cultural context. Ernest (2016) has also posited that mathematical knowledge develops from human activities bound to the world and cultures that people experience.

Building upon a cultural perspective of mathematics, Bishop (1988), which has been widely cited by 628 international articles (according to the statistics of Google Scholar in April 2021), has identified six "universal mathematical activities" (i.e., counting, measuring, locating, designing, playing, and explaining). He has argued that these activities can be identified in the everyday practices of people across varying cultures and contexts. MacDonald et al.'s (2018) Australian study exploring young children's mathematical learning opportunities in family shopping experiences, and therefore EMEs, adopted Bishop's (1988) framework and found all six universal mathematical activities. In HK, the CDC (2017) give "examples of learning expectations" (p. 40) which cover mathematical topics, such as number and quantity, as a reference for mathematics teaching and learning in kindergartens. The mathematical topics noted in the CDC (2017) align with Bishop's (1988) mathematical activities. However, of the 628 international articles that cited Bishop's (1988) mathematical activities, none have investigated HK kindergartens. Thus, my study fills this gap by using Bishop's (1988) six types of mathematical activities to collect and analyse data.

In relation to 3-to 4-year-olds, Ginsburg et al. (1998) summarised previous research and identified a range of children's informal (out-of-school) mathematical concepts and knowledge. These included: enumeration, number relations, simple arithmetic reasoning, subitising and counting, informal addition, subtraction, and division. However, all the studies discussed by Ginsburg et al. (1998) used experimental or quasi-experimental methods. Tudge and Doucet (2004) have critiqued that although these types of methods are excellent at finding out what children can and cannot do at various ages, they do not allow us to understand the types of experiences that children have had that allow them to attain their various mathematical competencies. Tucker (2014) has suggested that for children to understand mathematical concepts and use them in meaningful ways, they need to be engaged in personally meaningful activities (e.g., play) to facilitate the learning process.

Previous studies have found that children demonstrate a good deal of mathematical competence in play that takes place in early childhood settings. For example, Seo and Ginsburg's (2004) North American study examined the nature and frequency of 4- to 5-year-old children's naturally occurring mathematical activities in free play in early childhood settings. Their research results identified the mathematical content of children's activities as classification, magnitude, enumeration, dynamics, pattern and shape, and spatial relations. However, everyday activities, such as play, may reflect a limited range of mathematical concepts naturally and spontaneously engaged with by children (Ginsburg et al., 1999). Studies focused on children's natural and spontaneous mathematical practices in everyday activities in early childhood settings other than play (e.g., having the informal talk with others) are difficult to find.

I located two international studies whose findings indicate a range of mathematical concepts and knowledge used by children aged 13-45 months during play and everyday activities in European early childhood settings. For instance, Reikerås et al. (2012) found that toddlers aged between 30-33 months expressed mathematical competencies in number and counting, geometry, and problem-solving through play and daily life activities in Norwegian kindergartens. In Björklund's (2008) study of children aged between 13-45 months in Sweden day care settings, findings showed that children interact with concepts of dimension or proportion, location, extent, numerosity, and succession (e.g., each number is generated by adding one to the previous number), and use a range of strategies to express their understanding. However, there is still a dearth of research focusing on the range of mathematical concepts and knowledge employed by 3- to 6-year-old children (which is the age of the child participants in my study) during play and everyday activities in early childhood settings other than European countries.

In summary, although previous studies indicate inconsistency and tensions in terms of defining mathematics, the scope of early mathematical concepts and knowledge, and young children's mathematical competence, previous empirical and theoretical studies and HK curriculum documents provide a reference for me to understand the breadth of children's mathematical concepts and knowledge. Furthermore, after reviewing the studies of Björklund's (2008), Reikerås et al. (2012), and Seo and Ginsburg (2004), my study fills the gaps identified in these studies by exploring the content of 3- to 6-year-old children's EMEs during their play and routine activities in HK kindergarten settings. I argue that the content of children's EMEs is evidence of children's mathematical competence during play and everyday activities.

Next, I explain the relationship between EMEs and relevant ideas in HK curriculum documents. As framed by sociocultural perspectives, it is important to understand and analyse ideas related to EMEs addressed in cultural artefacts.

Notions of Children's Mathematical Experiences in HK Curriculum Documents

Some of the ideas expressed in curriculum documents about children's mathematical experiences align with my idea of EMEs. As noted in Chapter 1, the CDC (2017) values children "using mathematical concepts to solve practical problems in everyday life" (p.39) and highlights "[being] aware of the relationship between mathematics and life" as a learning objective for children in early childhood mathematics education (p. 44). Because these ideas echo EMEs, I expanded the CDC's (2017) idea of "using mathematical concepts to solve practical problems in everyday life" in detail using previous literature.

I used Benigno's (2012) North American study to understand and analyse the idea of "using mathematical concepts to solve practical problems in everyday life" noted in the CDC (2017). As mentioned earlier, Benigno (2012) is the only empirical study I located that has used the term EMEs as the research topic. In her doctoral study investigating three, 4-year-old children's EMEs, Benigno (2012) adopted the analytic concepts proposed by Street et al. (2005) to analyse children's EMEs in different settings (i.e., homes, informal day care centres, and other everyday settings, such as supermarkets). In my study, I only focus on children's EMEs in kindergarten settings.

Benigno's (2012) analytic concepts of EMEs helped me understand the notion of children's mathematical experiences addressed in the CDC (2017). She focused on children's application and understanding of mathematical concepts in social and cultural contexts. She analysed children's EMEs as two layers. The first layer was the mathematical content, which was relevant to various mathematical concepts employed and the "mathematical understandings an individual appeared to demonstrate" (p. 16). At the second layer, she defined mathematical practices from social perspectives, which included "purpose and setting" (similar to the idea of "context" in Street et al.'s (2005) model), "values and beliefs", and "social relations" (p. 14). Drawing on these concepts, I understand the CDC's (2017) idea of "using mathematical concepts to solve practical problems in everyday life" from three aspects. These aspects include mathematical concepts and knowledge, the purpose and practice (relating to "solving practical problems"), and the context of using mathematics (relating to "everyday life"). I involve these aspects in my operational definition of EMEs.

After unpacking ideas related to children's EMEs noted in previous literature and HK curriculum documents, I extended and finalised my operational definitions of EMEs as *children's experiences in informally and spontaneously applying and exploring early mathematical concepts* *and knowledge with surrounding objects, peers, and teachers or adults in play and everyday activities* and used it to collect and analyse data. Inspired by the studies noted in this section, diverse ways of investigating children's EMEs are introduced next.

Ways of Investigating Children's EMEs

I investigated EMEs naturally constructed by children in authentic HK kindergartens. This approach aligns with previous empirical studies related to children's "everyday, naturally occurring mathematical activities" (Tudge & Doucet, 2004, p. 34). Although previous studies set the research background outside the kindergarten (e.g., home), the meaning of "natural" noted by Tudge and Doucet (2004) refers to an authentic environment in which research is naturalistic in that it occurs without any interventions. Thus, my study was set in the research background of authentic HK kindergartens.

I used the technique of naturalistic observation to explore children's EMEs in kindergartens, as some previous studies also used naturalistic observation to investigate children's mathematical competencies during play and routine activities. For example, in the study by Reikerås et al. (2012), the play and daily activities of children aged 30-33 months were observed to determine children's competence in relation to number and counting, geometry, and problem-solving. Competencies in all areas were observed, with particularly high levels of competence related to puzzle-making and following spatial instructions. Additionally, Lee (2012) observed the outdoor play of children aged 12-36 months and found evidence of competencies in the areas of space, number, measurement, shape, pattern, classification, and problem-solving.

Furthermore, while observing children's mathematical activities during play in early childhood settings, many researchers have also adopted diverse techniques (e.g., video- and audio-recording) to record their observations. In the Seo and Ginsburg's (2004) study described earlier, the researchers explored children's naturally occurring mathematical activities by video-recording 90 children's verbal and non-verbal mathematical behaviours during free play. They also used inductive methods to develop a coding system to capture the mathematical content of children's behaviours.

Emfinger (2009) similarly used video-recordings to record the mathematical behaviours demonstrated in children's spontaneous pretend play. Hendershot et al. (2016) analysed audio-recordings of children's unstructured play to identify children's mathematics-related conversations. These studies indicate that researchers can perceive the mathematical concepts explored by children through observing their verbal and non-verbal mathematical behaviours, which had implications for my study design.

I explain more details of the methodology and methods that I adopted for my study and how previous studies influenced my decision-making regarding research design in Chapter 3 to avoid repetition. The role of EMEs in children's learning and relevant challenges of promoting EMEs in kindergartens are addressed as follows.

The Role of EMEs in Children's Learning

This section aims to discuss the role of EMEs in children's learning and the difficulties children may encounter when bringing their EMEs from out-of-kindergarten contexts to the kindergarten context (as noted earlier on pp. 37-38). There are four sub-sections in this section, including: the importance of EMEs, the discontinuity between children's EMEs in out-of-school and school settings, the importance of teachers' roles and responsibilities in facilitating children's mathematics learning, and the challenges of promoting EMEs by teachers in HK kindergartens.

The Importance of EMEs

Scholars paying attention to the value of early mathematical concepts and knowledge obtained naturally and informally with guidance from adults (Charlesworth & Leali, 2012) highlight the importance of these concepts and knowledge for children's readiness for future mathematics learning. They believe that children's mathematical experiences in early years, including EMEs, play an enormous role in developing their understanding of mathematics (Tudge & Doucet, 2004) and provide a foundation for later mathematics learning within the framework of formal, statutory education (Alsina & Berciano, 2018). In this sense, I argue that an aspect of the importance of children's EMEs lies in their value in in preparing young children for the next stage in their formal mathematics education.

The notion of children's readiness for learning strengthened by a growing wealth of longitudinal studies that have found that early childhood mathematics is a strong predictor of their future academic achievement and competence in relation to mathematics, reading, and other subjects (e.g., Aunola et al., 2004; Claessens & Engel, 2013; Wolfgang et al., 2001, 2003). Through testing, Aunola et al. (2004) found that children with better mathematical understanding in kindergarten performed better in mathematics in formal education than children with less mathematical understanding. Claessens and Engel (2013) found that early numeracy predicted reading comprehension and science achievement in middle school. Wolfgang et al. (2001, 2003) examined the relationship between children's mathematical competence in block play and future mathematics abilities. Wolfgang et al.'s (2001, 2003) findings showed that children's mathematics-related block building skills demonstrated in block play in early years predicted their mathematics achievement in middle school and high school. Although these longitudinal studies' findings suggest the importance of early childhood mathematical abilities for future mathematics learning, they are generated through testing rather than observing naturally occurring mathematics. As a result, these studies tied with the readiness agenda may imply that more formal teaching is required rather than putting in the hard work and long hours of naturalistic observations of competent children as my study did.

Interestingly, Ginsburg (2006) has argued that an exaggerated focus on the future may devalue the benefits of EMEs on "current activities". Children's EMEs have been found to be beneficial to current activities and play, such as block play, in which children can deal with ideas of shapes, space, and patterns while playing with blocks (Leeb-Lundberg, 1996). Trawick-Smith et al.'s (2017) North American study was undertaken in an early childhood setting and investigated the relationship of block play variables with the mathematical competence children demonstrated while playing with blocks. They video-recorded 41 children playing with blocks in naturalistic free play settings. Block play variables, such as "time spent in playing blocks", "the number of structures built", "levels of social participation", "the frequency of teacher interactions", "the percentage of buildings without replica play toys", and "structure complexity" were coded. Within these variables, research findings indicated that the number and complexity of children's structures aligned with children's growth in mathematical competence. That is, the mathematical competence children demonstrated as they played with blocks enhanced the complexity of their block play. In other words, children's EMEs associated with block play could enhance the complexity of their block play, through which they could practise mathematical concepts and knowledge that are essential for future learning.

Vygotsky's (1987) notion of spontaneous and scientific concepts provides an explanation for the importance of early childhood mathematical abilities for formal mathematics learning as well as children's everyday life. As discussed earlier in the second section of this chapter, Vygotsky (1987) believed that children learn spontaneous concepts from everyday life. Spontaneous concepts lay an essential foundation for developing scientific concepts. That is, they create the potential for the development of scientific concepts within the context of more academic situations (Fleer & Ridgway, 2007). Thus, children's EMEs, constructed through interacting with their surroundings during play and everyday activities, may create potential for the development of formal mathematical concepts. However, this does not mean the development of mathematics-related spontaneous concepts is just for readiness for future formal mathematics learning, as developing spontaneous concepts is also essential for children's living (Fleer & Ridgway, 2007). More thoughts about spontaneous and scientific concepts are addressed later in this chapter.

The importance of EMEs in children's future mathematics learning and current activities and play, as outlined above, provided a rationale for my study to investigate children's EMEs in kindergartens, particularly in the context of HK. As mentioned in Chapter 1, HK kindergarten education focuses mainly on children's learning, and promoting mathematics learning, including EMEs, is a key concern in kindergarten education. However, although EMEs are an important kind of children's mathematics learning, there are difficulties when children bring their EMEs constructed in out-of-kindergarten contexts to the kindergarten. As discussed earlier on pp. 36-37, out-of-kindergarten contexts are a central aspect of children's everyday mathematics. The discontinuity between children's EMEs in out-of-school and school settings is discussed below.

The Discontinuity between Children's EMEs in Out-of-school and School Settings

Some previous studies have highlighted the discontinuity between children's mathematical experiences in out-of-school settings and the mathematical knowledge or skills required in school settings. In Carraher et al.'s (1985) and Saxe's (1988) studies, the researchers observed school-age children who sold candy on the streets of Brazil using informal procedures that they had developed to solve mathematics problems associated with the practice of selling (e.g., buying candy to sell, pricing candy for sale, and calculating sums of purchases). In their research, they found that although the sellers could solve problems within the context of selling, these strategies did not transfer to the formal school context. The sellers were not able to solve similar computational problems when traditionally presented in school mathematics settings. According to these findings, I argue that school settings do not always recognise children's mathematics learning from other contexts.

Previous researchers, such as Gifford (2005), have explained that the discontinuity mentioned above may be because children are not able to use their experience-based knowledge in a new learning context. Gifford's explanation aligns with Vygotsky's (1987) notion of spontaneous concepts. Children's spontaneous concepts originate in their everyday life. These concepts come into being when a child is faced with real things, and the content of these real things are explained by adults. Vygotsky (1987) indicated that, "The weakness of the [spontaneous] concept lies in its incapacity for abstraction, in the child's incapacity to operate on it in a voluntary manner" (p. 168). That is, children cannot operate on spontaneous concepts in a deliberate manner (e.g., a child already has a cookie on the plate. After taking two cookies from the teacher, this child may know that they now have three cookies. However, without giving them any cookies, he might not know the answer to the question: How many cookies will you have if the teacher gives you two more cookies?). Regarding mathematical concepts, I argue that not being able to use their mathematics-related spontaneous concepts may make it difficult for children to understand the mathematical knowledge or skills required by teachers in kindergartens.

Gifford (2005) has suggested that the involvement of an adult is needed to support children's learning, including mathematics learning. Gifford's explanation aligns with Vygotsky's (1987, 2011) notions of scientific concepts and ZPD. Scientific concepts are based on the development of spontaneous concepts and usually require teachers' or adults' instruction in a formal context (Vygotsky, 1987). Vygotsky (2011) viewed interaction with teachers as an efficient way of facilitating children's learning within their ZPD. In this sense, teachers are important for helping children extend their mathematics-related spontaneous concepts to scientific concepts within their ZPD (as will be addressed again shortly in this chapter). With teachers' assistance, Vygotsky (1987) stated,

> [w]hat lies in the zone of proximal development at one stage is realized and moves to the level of actual development at a second. In other words, what the child is able to do in collaboration today [he or she] will be able to do independently tomorrow (p. 211).

Ginsburg et al. (1999) have also suggested that more varied and conceptually rich assistance from teachers is essential for extending children's informal mathematical competencies.

Drawing on the above discussion, I argue that teachers are essential to assist children to cope with the discontinuity caused when using their mathematics-related spontaneous concepts in school settings. Teachers' roles and responsibilities in facilitating children's mathematics learning are discussed next.

Teachers' Roles and Responsibilities in Relation to Children's EMEs

Although appropriate teacher practices enhance children's mathematical understanding, ranging from numbers to simple spatial relationships (Baroody et al., 2008), the appropriateness and effectiveness of teachers' roles and responsibilities may vary in children's play. For example, in

Trawick-Smith et al.'s (2017) research on block play, findings indicated that teacher interaction was not significantly related to block structure complexity or mathematical learning, as measured by assessment tools. Trawick-Smith et al. (2017) have explained that only "good-fit" interactions, which support rather than interrupt children's play (Trawick-Smith et al., 2016), are associated with children's mathematical learning. However, the teacher interaction they investigated might not have aligned with this notion of being a "good-fit". Thus, the appropriateness and effectiveness of teachers' roles and responsibilities may influence children's practices in promoting EMEs.

To achieve teaching appropriateness and effectiveness, making use of moments when children are willing to learn (Ginsburg et al., 2008) and making mathematics fun for children (Seo & Ginsburg, 2004) are two important pedagogical strategies. Limited research has examined the relationship between positive emotions and older children's self-efficacy for mathematics and their mathematics performance (e.g., Bryan & Bryan, 1991). Inspired by this kind of research, I argue that when mathematics learning is fun, it is also meaningful, motivating, and valuable for children. My study is required to support this argument. In contrast, rote learning without understanding leads to children to dislike mathematics and "does little to inculcate the spirit of mathematics—learning to reason, detect patterns, make conjectures, and perceive the beauty in regularities" (Ginsburg et al., 1999, p. 88). The CDC (2017) also notes, "One-way teaching and repetitive drilling on operations of mathematics can cause resentment" (p. 41).

Although making use of favourable teaching moments is appropriate for "scaffolding" (Wood et al., 1976) children's mathematics learning during play, Salomonsen (2019) summarised some challenges involved in this approach. These challenges include: (i) teachers are rarely present when children are playing freely (Seo & Ginsburg, 2004); (ii) teachers often have insufficient mathematical knowledge to recognise the opportunities for, or to make use of, children's interest in participating in mathematical exploration (Stipek, 2008); (iii) teachers find it difficult to facilitate children's mathematics learning at an appropriate mathematical level (Bennett et al., 2012); and (iv) teachers have difficulties in making use of favourable teaching moments when a large group of children is engaged in free play (Ginsburg et al., 2008). With these challenges, previous studies have found that adults, including teachers, rarely manage to observe and make use of such favourable moments (Ginsburg et al., 2008).

In HK, the CDC (2017) also highlights teachers' roles and responsibilities in facilitating children's mathematics learning and, therefore, EMEs. It suggests that teachers adopt different approaches, such as applying cultural tools (e.g., language and mathematics concepts)—as will be discussed later is this chapter—planning everyday activities and play, and arranging the physical environment. These statements in the CDC (2017) align with international studies, such as Öçal and Işık (2017), which have identified teachers' roles in the learning process, learning environment, and with children. Öçal and Işık (2017) have also noted teachers' responsibilities as promoting children's mathematical readiness and interests, planning and implementing mathematical activities, teacher guidance and interaction, arranging environment and materials, and encouraging family participation.

However, as noted in Chapter 1, many HK teachers teach mathematics in a teacher-directed way. As noted earlier, the CDC (2017) highlights the disadvantage of "one-way teaching". Thus, teacher-directed mathematics teaching may constrain children's construction of EMEs in kindergartens. Reasons regarding this phenomenon and the challenges of promoting EMEs by teachers in HK kindergartens are discussed next.

Challenges of Promoting EMEs by Teachers in HK Kindergartens

There are two factors that may challenge teachers' promotion of children's EMEs and, therefore, constrain children's construction of EMEs in HK kindergartens. These are parents' expectations of academic achievement and ambiguous descriptions in curriculum documents. They are described below.

Parents' Expectations of Academic Achievements. As discussed in Chapter 1, HK parents' expectations are a factor that may influence the inconsistency between curriculum documents and classroom practices. Thus, while I did not involve parents in my study, it was likely that teachers would refer to parent expectations as a reason for some of their practices regarding EMEs. I further argue that some HK parents, who hold academic expectations for their children's subject learning and do not value the potential of EMEs for learning through play, may not value the importance and potential benefits of EMEs. Some researchers, who have investigated HK parents' expectations of children's learning in kindergartens, have indicated that parents who are concerned about their children's academic achievements consider kindergarten education merely as a preparation for primary school (Rao et al., 2010). Many parents believe that academic achievements, including mathematical achievements, are more important than play in their children's education (Chang, 2003). Parents in HK tend to encourage their children to learn through direct teaching (Fung & Cheng, 2012) and expect teachers to teach academic concepts, including mathematical concepts, rather than allowing children to play most of the time (Fung, 2009). Such parents' expectations might influence teachers' perceptions of and responses to children's EMEs in HK kindergartens, which deserves to be investigated.

Ambiguous Descriptions in Curriculum Documents. As discussed in Chapter 1, vague, contradictory, or insufficient descriptions of pedagogical approaches noted by the CDC (2017) may cause the inconsistency between curriculum documents and teacher practices. In this subsection, I argue that the CDC (2017) provides unclear and inadequate descriptions regarding teachers' roles and responsibilities regarding mathematics learning and teaching (e.g., teacher interaction, applying mathematical tools, planning activities, and arranging physical environment), which also restrict teachers' understanding of children's EMEs in kindergartens. For example, the CDC (2017) suggests that teachers "assist children to understand basic mathematical language step by step and foster their sensitivity to numbers and space" (p. 46). Without further explaining the meaning of "basic mathematical language", teachers may be confused with the meaning of mathematical language and mathematical concepts. Their understandings of mathematical concepts may be limited to "numbers and space" noted by the CDC (2017).

Additionally, the CDC (2017) has not provided supplementary explanations with more foci on specific pedagogical guidelines for EMEs. To prepare children for formal mathematical education, the CDC (2017) indicates that children have different experiences that shape what they know and bring to kindergarten. It suggests that teachers should recognise and understand what these experiences are and build upon them. However, the CDC (2017) has not provided a precise description of how to build on children's out-of-kindergarten experiences to facilitate children's mathematics learning. Instead, it only describes mathematics learning as a progressive process and suggests that teachers should teach mathematical concepts with increasing levels that align with children's cognitive development. In this regard, children are more likely to be encouraged to develop mathematical concepts that rely on formal and systematic teacher instruction. Teachers might also dismiss or overlook the value of children's EMEs.

In addition to these two factors, traditional Chinese cultural beliefs in education, such as Confucianism, may also influence teachers' promotion of EMEs in HK kindergartens. Next, some theoretical concepts, including Confucian educational beliefs, of my study are discussed.

Theoretical Underpinnings

In this section, I outline seven theoretical concepts based on, or related to, sociocultural theory and explain their relevance to my study. These concepts include informal learning (Rogoff et al., 2016), sociocultural perspective of play and emotions (Rogoff, 1990; Vygotsky, 1978; Vygotsky, 1934/1999), spontaneous and scientific concepts (Vygotsky, 1978), the zone of proximal development (ZPD) (Vygotsky, 2011), cultural tools (Vygotsky, 1981), Confucian views of education, and concepts of affordances theory (Carr, 2000; Gibson, 1977; Norman, 1988, 1993). I introduce these theoretical concepts through three subsections, including (i) learning and teaching from sociocultural perspectives, (ii) cultural and historical aspect of sociocultural perspectives, and (iii) sociocultural views of affordances theory. These concepts shape the theoretical framework for my study, and I used them to analyse the data.

Learning and Teaching from Sociocultural Perspectives

As discussed in this chapter, learning and teaching play an important role in children's construction of EMEs. In this subsection, I use concepts of informal learning, sociocultural perspective of play and emotions, spontaneous and scientific concepts, and ZPD to discuss learning and teaching from sociocultural perspectives.

Informal Learning. Rogoff et al.'s (2016) notion of informal learning, which was discussed in the second section of this chapter, is an important notion that I used to understand children's EMEs and frame my study. In line with sociocultural theory, Rogoff et al. (2016) have stated, "Informal learning is interactive and embedded in meaningful activity" for children and the "involvement builds on individual initiative, interest, and choice" (p. 359). I posit that the features of children's informal learning proposed by Rogoff et al. (2016), are features of children's ways of learning and applying mathematics and, therefore, constitute EMEs. The application of Rogoff et al.'s (2016) informal learning enabled me to (i) unpack the informal nature of EMEs, (ii) understand the value of EMEs for children's learning and development from sociocultural perspectives, and (iii) guide the data collection and analysis process of my study.

Sociocultural Perspective of Play and Emotions. As discussed earlier, play is an essential arena for children to construct EMEs. I interpret the concept of play from a sociocultural theoretical standpoint where social interactions are central to processes of children's play, learning, and development (Rogoff, 1990; Vygotsky, 1978). Sociocultural approaches to play recognise that children create their own peer and play culture (Corsaro, 2009). In this sense, the people with whom children interact and the contexts in which children's EMEs are located shape children's EMEs. Thus, children's EMEs constructed during play, and their perspectives of play, are not just influenced from outside—from contextual factors, such as parents, and social and cultural beliefs and values—they are simultaneously reinterpreted and actively shaped by the children themselves (Huser, 2018).

As also previously mentioned, researchers often associate emotions with children's play. Mesquita (2012) has argued that the notion of emotions is an underexplored and unfinished part of

Vygotsky's work by Vygotskian scholars. Mesquita has further noted that "Vygotsky gives to emotion a character similar to cognitive processes, as constituent of units of the psyche, but the term emotion appears dispersed in his work (Gonzalez-Rey, 2000), not as a theory systematically formulated" (p. 810). Magiolino (2010, as cited in Mesquita, 2012) and Veresov (2009) have indicated that Vygotsky used different terms, such as emotion, affection, passion, and feeling, to describe this phenomenon. They have also expressed the difficulty in establishing a single, inclusive term and definition to represent the meaning of this phenomenon (Mesquita, 2012). In my study, I adopted the term emotion. Vygotsky (1934/1999, as cited in Mesquita, 2012) criticised the dualistic tendency of separating intellect from emotion or affection in psychology. He stated:

> Admitting that thought depends on the affection is not much to do, we need to go further, go from metaphysical study to the historical study of phenomena: it is necessary to examine the relationship between intellect and affection, and the relationship of these with the social signs, and avoiding reductionism dualisms. (p. 121)

Vygotsky believes that emotion or "affection" is a part of mental functions, rather than an isolated cognitive entity (Mesquita, 2012). Drawing on this idea, Williamson et al.'s (2020) study paid attention to children's social and emotional learning and described it as "substantial and cognitively demanding work" (p. 199). However, this kind of research is limited, thus, my study adds value to it by exploring the relationship between children's emotions and their EMEs. Vygotsky's ideas about emotions enabled me to focus on children's emotional expressions when constructing EMEs.

Spontaneous and Scientific Concepts. As mentioned earlier, Vygotsky (1987) characterised young children's concepts as spontaneous concepts developed in collaboration with others through everyday activities. According to Vygotsky (1987), spontaneous concepts are intuitive and usually based on empirical observations of the way things look or feel. At this level, children might not know the scientific meaning behind their actions and consciously apply spontaneous concepts to their practices every day (Fleer & Hedegaard, 2010). For example, children might know how to get a packet of chocolate from the shelf in the supermarket, but they might not understand that food in the supermarket is organised by a classification system. On the other hand, Vygotsky (1987) stated that scientific concepts are usually learned in a formal context and require introduction by another more knowledgeable person. Scientific concepts are closely intertwined with everyday concepts as a dialectical relationship. Moreover, scientific concepts strengthen everyday concepts supporting the structural formation of concepts (Fleer & Ridgway, 2007).

Collaboration with teachers is essential to involve both spontaneous concepts and scientific concepts, and children may learn with the teacher's assistance and participation in this process (Vygotsky, 1987). The most powerful learning contexts are those in which teachers keep in mind the spontaneous and scientific concepts when planning for learning (Hedegaard & Chaiklin, 2005). If scientific concepts are introduced to children away from their everyday experiences, they may be disembedded and hold little meaning for children (Vygotsky, 1987). Moreover, it is only when scientific concepts become integrated with a child's everyday concepts that they transfer into competence in the child's life outside the classroom (Hedegaard, 2007) (e.g., use the knowledge of quantity to take five apples at a time and give them to five people instead of doing it one apple at a time).

Vygotsky's (1978) idea of concept formation enabled me to understand the importance of social interaction in constructing children's spontaneous concepts in EMEs. Social interaction with peers and teachers is an essential concept in sociocultural theory, and understood to be positioned within children's local context that are shaped by their host communities' culture, time, and place. Through interacting with adults and more capable peers, children's spontaneous concepts used in EMEs may be extended to scientific concepts. This idea echoes the operational definition of EMEs for my study (see p. 45), in which I noted the role of social interaction. In this study, I specifically investigated children's EMEs during their interactions with physical objects, peers, and teachers.

Zone of Proximal Development (ZPD). The concept of ZPD serves a primary role in Vygotsky's theory of social interaction (as previously discussed on p. 52). Vygotsky (1978) defined the concept of ZPD as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem-solving under adult guidance or in collaboration with more capable peers” (p. 86). The ZPD can be viewed as the gap between a child’s achievement level without another person’s support and their potential achievement level with the assistance of more competent others (Crain, 2000). Vygotsky (1987) believed that with adult guidance or collaboration with more capable peers, children are able to solve more challenging tasks and do more than they can independently.

Drawing on the idea of ZPD, more competent partners, who can be teachers and peers in kindergartens, play an active role in promoting the individual learning of others through the ZPD in the learning process and context. In relation to my study, I was interested in observing if and how teachers and capable peers extend children’s EMEs through the ZPD. According to Vygotsky (1994), teachers’ assistance through social interaction can support children to extend their learning and ultimately internalise new learning experiences (Vygotsky, 1994). As mentioned before, the CDC (2017) has also suggested that teachers use social interactions and approaches, such as real-life themes, an integrated approach, and play, to extend children’s everyday experience, including EMEs, in HK kindergartens. These suggestions align with Vygotsky’s concept of ZPD.

Cultural and Historical Aspect of Sociocultural Perspectives

Culture plays an important role in sociocultural theory. I unpack the cultural and historical aspect of sociocultural perspectives by discussing two concepts: cultural tools and Confucian views of education.

Cultural Tools. Human actions depend on cultural tools organised in society (Wertsch & Tulviste, 2005). Cultural tools include material and symbolic tools. While material tools facilitate “the process of natural adaptation by determining the form of labour operations” (Vygotsky, 1981, p.137), symbolic tools direct human minds and behaviours (Vygotsky, 1981). Vygotsky (1978) defined culture mainly by symbolic systems. Language, signs, and symbols are important symbolic tools (Kozulin, 2018). Children learn about and apply material and symbolic tools during play and everyday activities (Bruner, 2006). Rogoff (1990) has stated that "children's cognitive development is embedded in the context of social relationships and sociocultural tools and practice" (pp. 7-8). It is important for children to appropriate cultural tool from others to extend their understanding (Rogoff, 1995). Thus, cultural tools have an essential role in children's construction of EMEs during classroom practices.

Vygotsky views language as the most powerful symbolic tool (Vygotsky, 1978). Vygotsky explored language in two ways, as (i) linking language with thoughts and (ii) considering language as a tool to serve specific social practices (Guo, 2010). While elaborating on ideas of language and thoughts, Vygotsky (1978) distinguished private speech from social speech, noting that children process information and think about their actions when speaking to themselves. Private speech plays the role of linking thoughts and speech with actions and is embedded in social context. It can transform social behaviours into "a function of individual adaptation" (Vygotsky & Luria, 1996, p. 168). Moreover, Vygotsky (1978) also considered language as a tool for conducting social practices. He believed that language enables people to carry out their social roles (Minick et al., 1993). Using language means using certain rules and tools to conduct cultural practices in "socially appropriate ways" (van Oers, 2007, p. 300).

Vygotsky also believes that expressing cultural meanings is an important function of cultural tools, particularly symbolic tools. Vygotsky (1981) emphasised the importance of human beings' use of symbolic tools to encode the cultural meanings of these tools. Wartofsky (1979) also described the use of cultural tools as cultural modes of representation and highlighted the importance of understanding knowledge construction in a cultural context. Wartofsky (1979) suggested that scholars should explore cultural modes of representation to understand how people retain and embody cultural meanings within the same cultural group. In which case, people, including children, of the same cultural groups are "making sense of the world, in broadly similar ways" (Hall, 2003, p.2). Thus, to make meanings of a culture, it is important to investigate individuals' actions associated with their use of cultural tools.

People of the same cultural communities, however, may give different meanings to their cultural tools. Interactions between children and their surroundings determine how they interpret the meanings of cultural tools (Guo, 2010). "Any event or situation in a child's environment will have a different effect on him depending on how far the child understands its sense and meaning" (Vygotsky, 1994, p. 343). Characteristics of the child in their development process are a key factor influencing the relationship between the child and their environment (Guo, 2010). In which case, the same cultural environment may influence different children in different ways.

The concept of cultural tools enables me to understand how the abstract notion of culture concretely works on, between, and among children and teachers. Thus, exploring children's EMEs should focus on how children utilise material tools (e.g., toys and resources provided by teachers) and symbolic tools (e.g., language, kindergarten timetables, rules, and approaches to discipline employed by teachers), and on how children's utility influences their actions in relation to EMEs.

Confucian Views of Education. Traditional Chinese cultural values about education are embedded in Confucian doctrines. As mentioned in Chapter 1, I considered Confucianism as a factor that explains the inconsistency between curriculum documents and classroom practices. I now introduce Confucian views of education to offer a useful background to understand a Chinese perspective of learning and teaching. In doing so, important aspirations, such as Chinese ways of child rearing and teaching and learning styles, can be understood.

In this subsection, I begin by briefly outlining Confucian views on human nature and education for human perfection. I then discuss the relationship between Confucian ideology and serious learning, creativity, critical thinking, and problem-solving that could be related to opportunities for children's construction of EMEs and teachers' responses to children's EMEs. As the main influence on Chinese traditional culture, Confucianism has a notable influence on HK culture and education.

Confucian Views on Human Nature and Education for Human Perfection. Confucianism profoundly influences Chinese views of human nature and education for human perfection (Bai, 2007). According to Confucius, the perfection of human nature is a means to creating a perfect society in which individuals influence one another through interactions (Tu, 1994). In this sense, education plays an essential role in pursuing the perfection of human nature (Kim, 2007). Moreover, it is crucial to educate children when they are very young, aiming to build up their capabilities and to, ultimately, become perfect adults (Hong & Howes, 2014). I view these beliefs as the cultural explanation for the emphasis on school readiness promoted in the HK curriculum documents (as discussed in Chapter 1). Drawing on this belief, many HK children, as young as three years of age, are enrolled in classes (e.g., music, dancing, and drawing) to prepare them for the future.

As the main education sector for the early years, parents expect kindergartens to provide children with necessary knowledge and experiences (Kinney, 1995) for transition to primary education and becoming a perfect primary school student. Thus, although the curriculum documents promote the six learning areas, many kindergartens in HK specifically emphasise teaching academic skills, consisting of reading, writing, and arithmetic/the “three Rs” (Fung & Cheng, 2012), that formal schooling requires. This practice aligns with international educational practices that have long viewed the “three Rs” as basic learning skills taught in schools, with which educators believe that children can gain more knowledge in any subject (Chapman-Novakofski, 2017). However, this kind of practice begins with kindergarten education in HK.

According to Confucius, young children are viewed as “little adults”, which is associated with a successful adulthood in the future (Bai, 2005). In this regard, Confucian ways of education expect children to avoid childish behaviours such as being “curious”, “active”, and “loving to play” (Hong & Howes, 2014, p. 40) and be self-cultivating, show calmness and diligence, and dislike play (Fen, 2002). These children are considered as ideal or well-brought up children (Bai, 2005). Hong and Howes (2014) have summarised multiple elements of the Confucian approach to teaching a child to be a perfect member of society:

> getting along with others, which was mainly about how to do good things and treat people well rather than badly; disciplines of behaviour, which was mainly about daily

> behaviour such as how to stand and sit etc. and about how to respond to parents or elders; fundamental knowledge of reading, writing and calculating, among which reading was most addressed; hygiene habits, including forming habits to maintain personal hygiene and help to keep the family environment hygienic; and self-care ability such as how to eat, and how to speak properly according to gender. (p. 41)

The purpose of preparing children to be a perfect member of society reflects Confucian emphasis on collective harmony and needs rather than personal needs. That is, children are not expected to express their views and feelings (Fen, 2002), as doing so may “disrupt group harmony or make one vulnerable to being hurt by others” (Heyman et al., 2008, p. 344). In which case, teachers may pay more attention to keeping all children’s learning at the same pace and pay less attention to children’s individual needs (Stevenson & Stigler, 1992). The pre-determined learning objectives discussed in Chapter 1 align with this idea. However, as noted in Chapter 1, contemporary ECE principles have extended beyond Confucian perspectives, with frequent reference to Western ECE pedagogical approaches (e.g., learning through play) that are in alignment or tension with Confucian views of education. I address Confucian views of play and other aspects of serious learning next.

Confucian Ideology and Serious Learning. Confucianism considers education to be a serious and sober matter (Bai, 2005). I elaborate on the idea of serious learning based on Confucian ideology from two aspects.

First, serious learning emphasises the negative impact of play on children (Hui et al., 2015). Most Confucian educators in ancient China were concerned that “play and a playful environment would divert children’s attention from serious study” (Bai, 2005, p. 15). This belief is in line with an old Chinese saying, 勤有功，戲無益/kan4 jau5 gong1, hei3 mou5 jik1 (success lies ahead of diligence, but nothing is gained by play). Some researchers have used this saying to elaborate on the negative connotation of play within traditional Chinese cultural beliefs of education (e.g., Hui et al., 2015). In the current HK context, teachers who hold this belief value play as a strategy to help children achieve prescribed learning objectives, rather than just doing free play at kindergartens

(Lam, 2018).

Second, serious learning also emphasises teachers' as authority figures. According to Confucian ideas in education, HK teachers are expected to uphold appropriate behaviours and maintain classroom discipline (Opper, 1992; Rao et al., 2010). Features of Confucian pedagogy include focusing on knowledge transmission and being teacher-directed (Ho, 2014). Obedience is one of the most important character qualities that Confucianism expects children to have (Fen, 2002). Moreover, Hue (2007) has found that "the importance of discipline is emphasized more than guidance or pastoral care" (p. 39); that teachers look like "technical managers" in their classrooms and are enthusiastic about telling children what to do (Li, 2006, p. 43).

Drawing on these two aspects of serious learning, traditional Confucian classrooms are authoritarian, in which order, self-discipline, obedience, stability, and hierarchy are promoted (Salili et al., 2001). Unfortunately, this kind of serious classroom atmosphere may constrain children's construction of EMEs through interacting with surroundings during play and everyday activities, as children are expected to be passive and uncritical learners and rely on rote learning (Biggs, 1996), rather than learning through play. Confucian views of creativity, problem-solving, and critical thinking are discussed as follows.

Confucian Ideology and Notions of Creativity, Problem-Solving, and Critical Thinking. The CDC (2006, 2017) has identified creativity, critical thinking, and problem-solving as important goals in kindergarten education. I argue that these skills are important for children to construct EMEs (as will be unpacked in this subsection). However, Confucianism is also keen on educating children to have certain characteristics, such as obedience, conformity, and dislike of play (Fen, 2002). Such characteristics may run counter to a person's creativity (Kim, 2007), critical thinking, and problem-solving skills. This may explain why Kim (2007) has suggested that people in Confucian societies were less creative than people in Western societies. Thus, it is important to discuss how creativity, critical thinking, and problem-solving skills relate to Confucian ideology.

Creativity is a process that involves certain characteristics, such as flexibility, sensitivity, thinking with multidimensional perspectives, interest in surroundings, awareness, being able to think and act comfortably, fluidity, independently, promptly, originality, and being able to draw different conclusions (Özdemir & Çakmak, 2008). Children can "experience creativity only when they feel free to play in their thinking, experiments, exploration, and imagination" (Kim, 2007, p. 30). According to Shen and Edwards (2017), creativity is important for children's mathematics learning because "children can draw on their own inner resources to play with mathematical ideas" (p. 327), and thereby construct EMEs.

However, circumstances, such as being in restricted-choice and competitive environments, strict and demanding routines, and expectations of reward can reduce children's creativity and motivation (Cheung & Leung, 2014). Chien and Hui's (2010) study found that some teachers in HK did not promote a deeper investigation of creative ideas in classroom. This may challenge or constrain children's creativity enactment in early childhood settings (Cheung, 2012), as creativity can be extinguished or weakened if it is not developed (Kilic, 2013). To promote children's creativity, teachers need to be "creative thinkers, flexible and open to innovations" (Kilic, 2013, p. 123).

In addition to creativity, it is crucial to promote critical thinking by teaching children how to critically make inquiries and reason (Heyman, 2008). Critical thinking refers to the "intellectually disciplined process of actively and skilfully conceptualising, applying, analysing, synthesising, and/or evaluating information gathered from, or generated by observation, experience, reflection, reasoning, or communication as a guide to belief and action" (Paul, 1993, p. 22). Critical thinking skills are important because they enable children to effectively deal with problems (Shakirova, 2007) they may encounter in everyday life and, therefore, when constructing EMEs.

Many kindergarten teachers in HK, however, tend to tell children what to do and transmit knowledge to them (Chan & Yuen, 2014). When children are constantly being told what to do, they might only take in what is offered to them without critiquing, and they might not be able to make their own judgments and decisions (Ho, 2015). As previously discussed, according to Confucianism, expressing one's thoughts and feelings is not encouraged. Thus, adults do not encourage children to challenge the authority of teachers and parents.

The CDC (2017) has highlighted the skill of problem-solving in children's mathematics learning. Children develop problem-solving strategies through free play (Pound, 2008), as they could discover mathematics-related problems during play and incorporate these in their EMEs. For example, they could figure out how to use informal measurement during construction play (e.g., Lego) to build a desired building or make a desired object (Charlesworth & Leali, 2012). By using different thinking processes, children can solve problems in everyday life (Sen, 2013). During the problem-solving process, their ability to ask and answer questions is an important component of knowledge construction (Tawfik et al., 2020). However, Confucian classrooms emphasise skill-based practise, drilling, and rote learning that require memory strategies (Li & Cutting, 2011). This may restrict children's opportunities to engage in problem-solving, as children do not engage in the "high-level discussion" unless teachers require them to give reasons for their answers or explain (Gillies & Khan, 2009). Moreover, children are rarely asked challenging questions that encourage them to think about issues and justify their answers, as teachers tend to ask factual or close-ended questions, or make announcements that require no answers (Gillies & Khan, 2009).

Next, I discuss the concept of affordances theory in relation to sociocultural factors that enable and constrain children's construction of EMEs.

Sociocultural Views of Affordances Theory

Affordance refers to "the perceived and actual properties of an object or artefact; those properties that determine just how it could possibly be used" (Norman, 1988, p. 9). The concept of "affordance" has been a useful way to describe the relationship between the learner and aspects of the setting (Norman, 1988, 1993). Affordances theory enabled my study to explore how cultural tools and other sociocultural factors enable and constrain children's EMEs. Affordances theory also enabled me to consider properties of objects or artefacts that afford children's EMEs "as co-developing in a cultural and historical context, and thus largely constructed rather than simply 'predetermined', often 'potential' rather than just 'given'" (Glăveanu, 2012, p. 195).

An environment “provides or furnishes either for good or ill” (Gibson, 1979, p. 127). Therefore, affordances can be positive or negative, depending on the individual’s perception and what they wish to achieve (Clarkin-Phillips, 2018). To distinguish positive and negative affordances, I used the term “affordances” to represent positive affordance and the term “constraints” to represent negative affordances. In relation to children’s EMEs, affordances refer to whatever it is about the environment or context that enables children’s EMEs in kindergartens (e.g., square shaped cookies enable children to talk about the concept of shape during snack time). Constraints refer to whatever it is about the environment or context that restricts or stops children’s EMEs and makes children carry out their mathematical actions in formal ways in kindergartens (e.g., teachers requiring children to keep quiet and sitting properly during teacher-directed activities).

The kindergarten environment can either enhance or prohibit experiences for children, influencing their learning, interactions, and relationships. Many studies have demonstrated the affordance of the physical environment to provide opportunities for children’s exploration, use of the senses, and creativity (e.g., Canning, 2010; Quayle, 2017). However, while a growing number of studies have examined children’s mathematics learning, little research has been directed towards how the physical and social environment of a kindergarten affords opportunities for the construction of children’s EMEs. Thus, my study adds value to the application of concepts of affordances theory.

Additionally, an environment offers affordances through physical objects, social interaction, and atmosphere, which can be physical, social, and cultural. Most previous research has focused on the affordances offered by the physical aspects of an environment, particularly outdoor environment (e.g., Canning, 2010; Quayle, 2017). However, affordances can also be considered in relation to the social and non-physical aspects of an environment, such as social interaction and “classroom atmosphere”. Classroom atmosphere refers to “the quality of the overall emotional climate that is prevalent in a school setting” (Moulton et al., p. 3). Therefore, my study included investigation of the affordances offered by the physical and sociocultural aspects of the kindergarten environment.

In summary, this section has discussed a set of theoretical concepts based on or related to sociocultural theory. These theoretical concepts provide the theoretical underpinnings for my study to understand how sociocultural contexts influence children's EMEs in HK kindergarten settings.

Chapter Summary and Research Questions

I began Chapter 2 by clarifying why I adopted sociocultural perspectives, rather than nativism and constructivism, to guide my study. The sociocultural theoretical framing discussed in this section provided a basis for this literature review chapter.

In the second section, I explored how previous literature and HK curriculum documents describe children's EMEs. This process led to a working definition of children's EMEs for my study, which referred to *children's experiences in informally and spontaneously applying and exploring early mathematical concepts and knowledge with surrounding objects, peers, and teachers or adults in play and everyday activities*.

In the third section, I summarised methods of investigating children's EMEs used in previous empirical studies, which provided a basis for the research design of my study. To avoid repetition, I explain more details of the methodology and methods that I adopted for my study and how previous studies influenced my decision-making regarding research design in the next chapter.

In the fourth section, I explored the role of EMEs in children's learning through four subsections. First, I discussed the importance of EMEs in children's future mathematics learning and current activities and play. Second, I stated the discontinuity between children's mathematical experiences in out-of-school settings and the mathematical knowledge or skills required in the school settings. Third, I outlined teachers' roles and responsibilities in relation to children's EMEs. Fourth, I talked about the challenges of promoting children's EMEs by teachers in HK kindergartens.

In the fifth section, I addressed the theoretical underpinnings of this study. I outlined seven theoretical concepts based on, or related to, sociocultural theory and explained their relevance to children's EMEs. These concepts included: informal learning, sociocultural perspective of play and emotions, spontaneous and scientific concepts, ZPD, cultural tools, Confucian views of education, and affordances theory to discuss (i) learning and teaching from sociocultural perspectives, (ii) cultural and historical aspect of sociocultural perspectives, and (iii) sociocultural views of affordances theory.

Based on the above theoretical concepts, and informed by gaps identified through literature review, the three research questions of my study are:

- What are the nature and content of children's EMEs in HK kindergarten settings?
- What are the affordances and constraints that influence the availability of children's EMEs?
- What are teachers' perceptions and practices with regard to children's EMEs?

The next chapter discusses the methodology and methods used in my study. The study design is inspired by the literature reviewed in this chapter.

Chapter 3: Research Methodology and Methods

This chapter describes the methodology and methods that I employed in my study. Methodology is "the analysis of the intersection (and interaction) between theory and research methods and data" (Sjoberg et al., 1991, p. 29). Framed by sociocultural perspectives, I adopted a qualitative interpretative paradigm and a case study approach drawing on ethnographic techniques (i.e., classroom observation, individual semi-structured interviews, documentation, and reflective journaling) to gain an in-depth understanding of children's everyday mathematical experiences (EMEs) in Hong Kong (HK) kindergarten settings.

The methodology and methods are addressed through six sections in this chapter. The first section describes the study's qualitative approach and interpretative paradigm. The second section outlines the choice of methodology and methods, which involved case study drawing on ethnographic techniques. The third section describes ethical considerations. The fourth section discusses the data collection procedures. The fifth section presents the data analysis procedures. The last section presents details about research rigour.

Qualitative Approach and Interpretative Paradigm

Two factors influenced my decision to employ a qualitative approach. The first factor was the aims and features of the research. This study aimed to investigate what children's EMEs look like and relevant beliefs and practices in HK kindergartens. For me (the main researcher), the most important feature of this study was the nature and content of children's EMEs in the HK kindergarten context and the meanings indicated. Exploring children's experiences, in this instance their EMEs, and the meaning constructed around them is consistent with a qualitative approach, as it offers researchers "valuable insights into how people [including children] construct meaning in various social settings" (Neuman, 2006, p. 308). Regarding meaning, Merriam (1998) has suggested that it is embedded in people's experiences and mediated by the researcher's perceptions. In this study, to understand children's EMEs and their associated meanings, I focused on children's mathematical actions. Actions noted here are "situated in a cultural setting, and in the mutually interacting intentional states of the participants" (Bruner, 1990, p. 19). Through observing children's mathematical actions and interactions, a qualitative approach provided me insight into understanding the EMEs constructed by children and the meaning indicated.

To uncover the meanings constructed, I listened to children and respected their rights to express their ways of understanding mathematics in their worlds (Ernest, 2016). I immersed myself in the kindergarten by observing children's actions and their interactions. I engaged in informal conversations with children about their mathematical actions and the artefacts created by them and participated in their activities and play if they invited me. Moreover, guided by sociocultural perspectives, this study viewed teachers as key informants for understanding children's EMEs in kindergartens. Therefore, I observed teachers' practices, interviewed teachers, and read relevant documents created and provided by teachers.

Second, the methodology and methods adopted by previous studies influenced my choice of a qualitative approach. Previous research has increasingly used a qualitative approach to investigate children's learning experiences (e.g., Benigno, 2012; Corsaro & Molinari, 2000; Papandreou & Konstantinidou, 2020; Papandreou & Tsiouli, 2020; Pramling Samuelsson & Johansson, 2006). Some of these studies have specifically shed light on 3- to 6-year old children's informal and spontaneous mathematical experiences (as discussed in Chapter 2) in early childhood settings (e.g., Papandreou & Konstantinidou, 2020; Papandreou & Tsiouli, 2020). Inspired by previous studies, I also adopted a qualitative approach to investigate children's EMEs in HK kindergarten settings.

This qualitative study is situated within an interpretative paradigm, which conceptualises knowledge as being constituted by the understanding of the meaning of the process or experience (Merriam, 1998). It is also concerned with meaning and understanding persons as actors in the society in which they interpret meanings and actions in line with their viewpoints (Hesse-Biber & Leavy, 2011). I argue that children come to kindergartens with different mathematical knowledge and experiences, characteristics, and backgrounds from their families. They react and interact with surroundings in their unique communicative approaches and negotiate strategies to share experiences and meaning with peers and teachers (Schwandt, 2000). Teachers also have their own social and cultural experiences that influence their perspectives of making sense of children's EMEs. Moreover, as the researcher, I also brought my experience and understanding of children's EMEs into this study, which influenced what I saw and heard and the meaning that I made from the data with children and teachers. In this case, an interpretative paradigm and the flexible nature of qualitative study enabled me to explore and understand children's EMEs from different perspectives (i.e., that of the children and teachers).

Case Study Drawing on Ethnographic Techniques

This qualitative study is a case study drawing on ethnographic techniques. Terminologies such as "mini-ethnographic case study" (Fusch et al., 2017) and "ethno-case study" (Parker-Jenkins, 2018) have been used to label this kind of qualitative research. According to Fusch et al. (2017), this "blended design" means a case study that employs ethnographic techniques (e.g., participant observations and interviews) associated with long-term and intensive ethnography, but "is limited in terms of research time, engagement with the data and the extent of the findings" (p.29). This section explains how and why I adopted case study drawing on ethnographic techniques as the methodology of this study through three subsections, including case study and the case, a blended design of case study drawing on ethnographic techniques, and roles, positionality, and reflexivity of researchers.

Case Study and the Case

Case study was favoured in my study as it is appropriate for studies in complex settings (Anderson et al., 2005). The complex settings mentioned here mean that they "cannot be reduced to single cause and effect relationships" (VanWynsberghe & Khan, 2007, p. 84). In my study, the sociocultural context of HK showed a complex feature which might influence the variability of the research focus of the study or the case investigated (i.e., children's EMEs). Details are addressed below.

As noted in Chapter 1 and 2, mathematics as an academic subject is valued in the HK context. Many parents in HK "are keen to prepare their children to learn academic content from the very beginning of their education (Cheng, 2015). Most kindergartens value children's academic learning, including mathematics; formal mathematics learning and teaching is promoted in kindergartens (Ng & Rao, 2008). In this case, the expectations, aims, and ways for educating children may influence children's EMEs in kindergartens. For instance, through social interactions, children may learn which actions in relation to EMEs are preferred or discouraged in kindergartens. Also, children may have more opportunities to be exposed to mathematical concepts and knowledge, which are "a cultural construct" (Papandreou & Tsiouli, 2020, p. 1) valued by the HK culture.

Within such a complex sociocultural context, I argue that case study is suitable for investigating children's EMEs in HK kindergartens as this methodology may provide "an in-depth, multifaceted investigation, using qualitative research methods, of a single social phenomenon" (Orum et al., 1991, p. 2). I will describe the phenomenon in my study shortly. Additionally, with the case study approach, although the findings cannot be generalised, children's EMEs in HK kindergarten settings could be viewed as a particular and instrumental case for other researchers who are interested in gaining understanding of children's EMEs (Stake, 1995). The findings may also provide a useful comparison for similar research in the future (Atkins & Wallace, 2012).

Merriam (1998) has stated that, "the single most defining characteristic of case study lies in delimiting the object of study, the case" (p. 27). Many researchers have mentioned phenomenon and boundary while describing the case in case study (e.g., Creswell, 2003; Merriam, 1998; Miles & Huberman, 1994; Stake, 1995; VanWynsberghe & Khan, 2007; Yin, 1994). The two terms are discussed below.

Regarding phenomenon, Merriam (1998) has identified "a phenomenon or entity" as the case (p.29). Many researchers have provided the definition of phenomenon. Pearsall (1999) has defined it as "some thing or situation that is of interest to the researcher that gives rise to a process of inquiry about which little is known" (p. 1071); van Manen (1997) has addressed it as perceived events, situations or objects of concern which surface in the everyday world of human beings; and

Marton (2000) has viewed a phenomenon as “the thing as it appears to [people]” (p. 105). In my study, the phenomenon of interest is children’s EMEs, which might occur for children every day. I could observe the experiences (e.g., verbal and physical actions, facial expressions, voices, and children’s interactions with surrounding objects and others) mentioned here through children’s everyday activities and play.

Boundary is another key element to define the case, and the case can be viewed as a phenomenon in a bounded context (Merriam, 1998, 2009). Thus, it is possible to identify a phenomenon with boundaries as a case in a case study. In this study, children’s EMEs were bounded by time (the period of fieldwork) (Stake, 1995), place (two HK kindergartens) (Creswell & Poth, 2018), context (HK context), and the operational definition of EMEs developed from reviewing previous literature (Miles & Huberman, 1994). I, therefore, identified the phenomenon of children’s EMEs in HK kindergarten settings as the case of my case study.

Case study identifies operational links between events over time (Andrade, 2009; Yin, 2014). It is the preferred strategy used by researchers when asking how, what, or why questions (Andrade, 2009; Stake, 1995). However, case study research may not take a long period of time and engage in data immersion, which may be challenging for unearthing the complexities and nuances embedded in the research field or data. Therefore, in addition to case study, my study also used ethnography as a methodology. Fusch et al. (2017) have suggested that one can blend study designs to be able to use the best of each design and mitigate their limitations. This blended design is introduced as follows.

A Blended Design of Case Study Drawing on Ethnographic Techniques

The design of case study drawing on ethnographic techniques used by this study is a blended design. An introduction to ethnography is necessary after presenting the description of case study mentioned above. Ethnography refers to a qualitative research design aimed at exploring the cultural interactions and meanings in the lives of a group of people (Barbour, 2010). An ethnographic study involves learning the beliefs, feelings, and meanings of relationships between people (Fields &

Kafai, 2009). Ethnography was appropriate for my study because I focused on exploring and learning about children's EMEs and how they construct these kinds of experiences in early childhood settings, particularly in HK kindergartens. This methodology also aligned with sociocultural perspectives that underpinned my study, as social interactions and culture are two important aspects of sociocultural perspectives.

Participant observation is an essential technique of ethnographic studies. Previous research has used participant observation to investigate children's informal and spontaneous mathematical experiences (e.g., Benigno, 2012; Papandreou & Konstantinidou, 2020; Papandreou & Tsiouli, 2020). My study adopted both participant observations and non-participant observations to explore children's EMEs. Due to the flow of this section, I explain the approach of non-participant observations later in the section about data collection procedures.

I chose the method of participant observation as appropriate for my study, particularly during "free time", which refers to periods of child-directed activities scheduled in early childhood settings (Papandreou & Tsiouli, 2020). My observations also took place at other times (e.g., during teacher-directed activities) (as will be discussed later in this chapter). During free time, children can exercise agency and express their perspectives by making choices in their play and interacting naturally with each other, without being directed and controlled by teachers. This is key for understanding aspects of their lives (Chesworth, 2018).

Researchers who undertake ethnographic research may immerse themselves by spending sustained periods of time in the field to generate and engage with data sets. Although this may help them to unearth complexities and nuances embedded in the research context or data, an ethnographic study can involve years observing and/or interacting with participants. This may not suitable for a doctoral study, which usually is required to be completed within a limited time. Hence, the use of case study complemented ethnography, as a case study can be bounded by diverse boundaries, such as time (Papandreou & Tsiouli, 2020). Moreover, inspired by ethnography, I immersed myself and spent a prolonged time (four to six weeks) in the field to establish relationships with children and teachers (as will be explained in the sub-section on familiarisation later in this chapter). The following subsections discuss the diverse ethnographic techniques used by me for data collection.

Photography and Video- and Audio-recording. I used photography and video- and audio-recording for classroom observations. The use of digital devices to record observations allows a comprehensive record of uninterrupted sequences of interactions, collaboration, conversations, and actions between teachers and children in kindergartens (Caldwell & Atwal, 2005). Thus, during classroom observations, I photographed and video- or audio-recorded some stances of children's EMEs (e.g., verbal and nonverbal actions, facial expressions, voices, and children's interactions with surroundings) by using a small hand-held digital camera[7] or a digital voice recorder in line with permissions obtained. I also occasionally recorded my own questions and ideas. In addition to classroom observations, I also used audio-recording to record two phases of teacher interviews.

In the fieldwork, I was aware that the use of digital recording devices sometimes might make participants feel uncomfortable. Thus, I valued and was sensitive to the participants' verbal and non-verbal cues to ensure I had sought their assent to continue observing and recording. Regarding some special situations (e.g., children's interactions associated with EMEs in the toilet area), I only used field notes to record observations. More information about photography and recording was addressed in the subsection of ethical consideration.

Field Notes. Field notes, including jotted field notes, mental field notes, and expanded field notes (Tolich & Davidson, 1999), are based on what researchers saw, heard, experienced, and thought (Bogdan & Biklen, 2007). I used this method to complement observations and to capture special incidents that occurred in settings (Ary et al., 2010). Field notes also record potential biases and subjectivities regarding the researcher's thinking, possible ways to organise the data, and ideas to analyse the data collected (Merriam, 1998). During my fieldwork, I used a template (see Table 1)

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[7] In HK kindergartens, it is usual for each teacher to have a small hand-held digital camera for recording children's activities and play as part of their documentation.

to record observations. I used incomplete phrases and words to quickly jot down field notes in a paper notebook during observation, aiming to describe events that I watched and heard and to address my personal feelings and reactions to those events. After each classroom observation, I used my jotted field notes and mental field notes to complete expanded or retrospective field notes. I also transferred the field notes from the paper notebook to a computer for easy storage on the same day.

Table 1

Field Notes Template

Time	Jotted Field Notes		Retrospective Field Notes
	Children	Teacher	

I realised that a researcher's action of taking field notes might create "unwelcome reactive effects and jeopardize trust and goodwill built up between the researcher and the [participants] in the social setting" (Moug, 2007, p. 112). Therefore, when there were times in the field that children showed interest in what I wrote, I shared my notebook with them and explained what I had written. This demonstrated that participation to a certain extent is unavoidable in classroom contexts with children. Moreover, I also took the initiative in showing my field notes to the teachers at the beginning of the classroom observations, explaining that the field notes were about my observations in terms of children's naturally occurring practices and some of their naturally occurring practices in relation to my research topic. After hearing the content of my field notes, teachers behaved more naturally by not watching me or asking me about what I was writing. One teacher even took the initiative by informing me about some children's informal mathematical talk during snack time, which also created an opportunity for me to have an informal conversation with her.

Informal Conversation. Framed by the idea of participant observations, during the course of classroom observations, I engaged in informal conversation to immediately get rich and detailed answers from participants (Khanal, 2016). As a type of interview, informal conversation resembles a chat. Most of the questions asked flow from the immediate context and are not predetermined, which is useful for participant observation fieldwork (Patton, 1987). Moreover, I used excerpts of informal conversations with children and teachers in the second individual semi-structured interview with individual teachers (as will be explained later in this chapter).

In the fieldwork, informal conversations with children and teachers were sometimes initiated by them. For instance, when children came to me and asked me to join their EMEs-related activities or play, I had informal conversations with them. When teachers came to me and discussed EMEs-related situations with me, I also had informal conversations with them. In these cases, the content of the informal conversation was based on the EMEs-related situations initiated by children and teachers. At other times, I initiated the informal conversations with participants, aiming to discuss concepts and questions with them and gain clarification for my observations, field notes, and reflective journaling (Ortlipp, 2008).

During my fieldwork, I was aware that my presence in the field may have had impact on the context and phenomenon of interest. Thus, I was sensitive to the responses of children and teachers when having informal conversations with them. More details about the influences of my presence in the field will be discussed later in this chapter.

Individual Semi-structured Interview. In addition to having informal conversations with teachers during classroom observations, I also employed two phases of semi-structured interviews to explore teachers' perspectives of and responses to children's EMEs. Interviews are a way to know what participants feel and think about their world (Rubin & Rubin, 2012). According to Holstein and Gubrium (2003), "interviewing provides a way of generating empirical data about the social world by asking people to talk about their lives" (p. 3). That is, interviews gather "participants' experiences, views and beliefs concerning a specific research question or phenomenon of interest" (Ryan et al., 2009, p. 309).

In my study, the goal of semi-structured interviews was to "understand the experience of [teachers] and the meaning they make of their experiences by helping them construct and articulate their stories" (Shkedi, 2005, p. 77). Telling stories is a progression of meaning making because when people talk about their experiences in detail, it requires reflection. Vygotsky (1987) has indicated that "every word people use in storytelling constitutes a microcosm of their consciousness" (pp. 236–237). In light of Vygotsky's (1987) idea, Seidman (2006) has further explained that "individual's consciousness gives access to the most complicated social and educational issues" (p. 7). In other words, interviews provided a way for me to explore teachers' experiences and the meaning made by teachers within the stories told by them.

Documentation. Documents indirectly reveal the social world of the people who create them (Merriam, 1998). Public documents (e.g., curriculum documents and kindergarten guidelines) may reveal social and cultural values in relation to children's EMEs. For example, in one of the participating kindergartens, teachers used documentation (e.g., children's learning reports in different areas, including mathematics) in a narrative form to report assessment of children's learning (Helm et al., 2007). These narrative documents show examples of children's activities and experiences in kindergartens (Ho, 2015). I collected four types of documents, including (i) official curriculum documents; (ii) kindergarten guidelines and suggestions for the implementation of mathematics; (iii) textbook and homework in mathematics; and (iv) children's mathematics learning reports and planning documents created by teachers. With the participants' consent, I photocopied some sample pages for analysis. I did not analyse any of the documents I collected except for the teachers' reports about children's mathematical learning. However, reading and re-reading all of these documents helped me to understand the data collected from the field.

Reflective Journaling. Journaling is an essential part of qualitative research (Hayman et al., 2012) and was, therefore, used in my study. Reflective journals refer to notes that address personal statements and feelings about the research (Richardson & Adams St. Pierre, 2008). They are also a way of recording researchers' assumptions. Its employment is especially useful for studies conducted in a short time, as reflective journaling is not limited to the time spent in the field collecting data (Fusch et al., 2017). Regarding my fieldwork, I was aware that my experience and cultural identity would influence the research, from the conceptualisation of the research through to the collection, interpretation, and analysis of data. Hence, writing personal impressions and feelings from the researcher's experiences in the field would help identify personal bias that may impact the researcher's interpretation of the data (Sangasubana, 2011).

I used "the [reflective] journal as a tool for reflection" (Annink, 2017, p. 3), "a written representation of [my] personal lens about the study" (Fusch et al., 2017, p. 930), and as a synthesis of all the data (Fusch, 2013). My reflective journals contained thoughts and reflections on the process of collecting data and the participants (Fusch et al., 2017); contextual information; additional items such as children's artefacts (e.g., drawings) or photographs provided by teachers; the data obtained via observation and interviews; and ideas and plans for subsequent research steps (Altrichter & Holly, 2005). During fieldwork, journaling assisted me to understand the interpretation of the culture through reflecting on my interactions with participants and the field. Once away from the research site, although I did not analyse my reflective journals, reading and re-reading them would help me identify themes and patterns from the data (Ben-Ari & Enosh, 2011).

A limitation of the reflective journaling is that the researcher is the data collection instrument and cannot separate oneself from the research (Marshall & Rossman, 2016). I realised that some personal factors (e.g., my own experiences and cultural identity) influenced the fieldwork. For instance, due to my previous working experience as a kindergarten teacher and my new identify of a novice researcher, I realised and reflected that at the beginning of observation, I sometimes still interpreted children's activities from a teacher's perspective, which influenced my interaction with them. To mitigate the limitation, some researchers have suggested that a researcher should spend time in reflection to ensure that one's interpretation aligns with the culture (e.g., Marshall & Rossman, 2016; Sangasubana, 2011). Thus, it is important to highlight researchers' roles, positionality, and reflexivity as follows.

Roles, Positionality, and Reflexivity of Researchers

In a qualitative study, the researcher is the primary instrument (Creswell, 2007) and plays essential and diverse roles. In my study, I was the research designer and data collector, as well as the person who interviewed participants, reviewed documents, conducted the analysis, and interpreted findings (Creswell, 2007). Among diverse roles, the role of interpreter has been highlighted by previous researchers. For instance, Smith and Osborn (2003) have suggested that qualitative researchers should strive to “make sense of the participants” and “their world” (p.51). Other researchers have argued that ethnographic studies reflect the interpretations of meaning in the lives of communities, as obtained by the researcher (Tajfel, 1981) capturing respondent insights and lived experiences (Cohen et al., 2007). As the researcher of my study, I viewed myself as an important research instrument and shifted across diverse roles to interpret how children constructed the meaning of their EMEs through interacting with surrounding objects, peers, or teachers in kindergartens.

While playing multiple roles addressed above, I positioned myself as an insider as well as an outsider in this study. Being a kindergarten teacher in one HK kindergarten for almost four years, I am familiar with HK kindergarten education. As a Chinese researcher, I am a member of Chinese society and carry my values, beliefs, and understandings shaped by Chinese culture. Moreover, guided by ethnography, I was also a participant observer in the field. In this sense, I considered myself as an insider researcher who shared “common languages, themes and experiences with their participants” (Kim, 2012, p. 264). However, the status of insider is “conditional and unstable” (Subedi, 2006, p. 589), as the participants could treat the researcher as an outsider if they feel they could not share similar perspectives (Chan, 2017). Therefore, I argue that my researcher role was fluid and shifted between insider and outsider depending on different situations throughout the study (Gregory & Ruby, 2011).

In addition to the insider-outsider role, I also positioned myself as a support of sociocultural theory, as I used sociocultural perspectives as the theoretical underpinning of my study (as discussed in Chapter 2). This is essential to undertake an ethical study, which emphasises "positionality, reflexivity, the production of knowledge and the power relations that are inherent in research processes" (Sultana, 2007, p. 382). In the process, researchers could reflexively justify their values, personal background, and biases that explicitly influence their interpretations of a study (Creswell, 2014).

In my study, I have considered the importance of researcher reflexivity, a method to recognise a researcher's positionality. Individual reflexivity is "a necessary prerequisite and an ongoing process for the researcher to be able to clearly identify, construct, critique and articulate their positionality" (Holmes, 2014, p. 3). Bogdan and Biklen (2007) have considered the idea of researcher as the research instrument as an advantage, as "In no other form of research are the processes of doing the study and the people who do it so consciously considered and studied as part of the project" (p. 124). With constant self-evaluation and reflection when engaging in data collection and analysis (Bloomberg & Volpe, 2012), qualitative researchers could develop a deeper understanding of the data and research purposes. As a result, critical reflexivity could increase research rigour and reliability (Savvides et al., 2014). Therefore, I highlighted researcher roles, positionality, and reflexivity in my study.

Ethical Considerations

Investigations in all forms of research should be undertake ethically (Merriam, 1998). It is important that researchers be aware of ethical issues in relation to their research. Ethical issues are particularly salient when conducting research with young children (Flewitt, 2005). Denzin (1989) has indicated:

> our primary obligation is always to the people we study, not to our project or to a larger discipline. The lives and stories that we hear and study are given to us under a promise, that promise being that we protect those who have shared them with us. (p. 83)

Denzin's words inspired me to involve ethical considerations to guide every decision throughout the study (Cullen et al., 2009) and value the protection of the participants.

My study was approved by the University of Auckland Human Participants Ethics Committee in 2018. I also complied with ethical issues and requirements noted by the participating kindergartens (e.g., avoid talking about children's performance in the kindergarten with parents). I followed three main ethical principles to conduct the study: confidentiality, informed consent, and participants' rights to withdraw.

Confidentiality

The participants were informed that all electronic data (i.e., photographs, video footage, and audio files) would be stored on a password-protected computer and backed up on the University of Auckland server. All hard copy data provided by the teachers and principals, signed Consent Forms (CF) for principals (Appendix A), teachers (Appendix B), and parents (Appendix C), and Assent Forms (AF) for children (Appendix D) were securely stored in a locked cabinet at the University of Auckland. The teachers who gave their consent were invited to approve and edit the transcript of their interview. They were informed that they had two weeks to provide edits, from the time they received the interview transcript. The data were only transcribed and translated by me. Only my supervisors and I could access the data. The participants were given assurance that the data were kept confidential and would not be released to any other party without their consent. They were also informed that the data might be used for journal article publications and conference presentations for research purposes.

As this research was a small-scale study, it is possible that the participants could be identified in the final thesis and other publications due to the photographs associated with children's EMEs and interpretation of audio-video recordings presented in the findings. However, every attempt has been made to protect the identity of the participants and kindergartens. Pseudonyms were used in all publications and presentations to represent names of the children and teachers. I assigned each participant a pseudonym with their permission. In doing so, my intention was to manage all the pseudonyms effectively and help readers of this thesis grasp the types of the pseudonym used (e.g., using plant names to name teachers). However, pseudonyms cannot hide a participant's real identity when visual data are generated (Robson, 2011). Therefore, if the information provided by the participants was to be reported or published, I would do this in a way that does not identify the participants as its source. Before being included in the final thesis and any other publications or presentations, I would digitally alter all selected photographs or video footages to a low resolution and blur out all faces so that participants cannot be identified.

Informed Consent

I approached teachers in the kindergarten only after the principal provided me a signed CF (Appendix A). I approached children only after the teacher provided me a signed CF (Appendix B). I only involved the children who provided their assent (Appendix D) and whose parents/guardians had provided their consent (Appendix C) for their children to be participants. Moreover, I acknowledged that consent was an ongoing process. During each visit, and throughout the observation, I was sensitive to both children's and teachers' verbal and non-verbal cues (e.g., turning or walking away) to ensure I had sought their assent to continue observing and recording. When I perceived that they did not wish to be observed, photographed, recorded, or to converse, I stopped immediately. The participants could choose to be observed, but not to be photographed, video- and/or audio-recorded. They indicated their choices on the CF and AF (Appendix B, C, D). I also sought the teachers' assistance whenever necessary to maintain the smooth flow of experiences and interactions in the kindergartens.

Due to the familiar relationship established with them, I found that many child participants in my study were able to give their assent. However, because they could not read the words of the AF (Appendix D), their process of filling the AF had to rely on their parents. For instance, there was a child from a K3 class said, "My mom was busy yesterday, but I will ask her to help me fill [the AF] today." Moreover, they were also capable of offering me their oral consent during my observations. There were times that some children directly expressed, "I don't want to play with you today" or

"Don't take photo/video of me". There were also times children invited me to record their play, e.g., by saying, "I'm going to play in the art corner, you can video-record me."

I did not feature any non-participants in my study. I never asked them to leave an activity or play so that I could start recording. When occasions involved both participants and non-participants, I only photographed participants or used field notes to record the observation. When non-participants entered the frame and might become accidentally photographed or recorded, I stopped immediately. When I found that I had included them by mistake, I only analysed the material from participants and not use the photographs, video footage, or audio files in any publications or presentations.

Participants' Rights to Withdraw

All teacher and child participants, (and parents on behalf of their children) are hereafter referred to as participants. Participation was voluntary. Participants were informed that they could withdraw from the research or withdraw any of their information provided at any time without giving a reason. They were also informed that they had the right to ask me to unconditionally destroy all information that could be identified as their contribution up until data analysis started (I provided the participants an approximate date through the Participant Information Sheet (PIS) (Appendix F & G) and CF (Appendix B & C)). However, they were informed that they were not able to ask for data collected from the classroom observation to be destroyed because of other participants' information on the same recording. Moreover, teachers and parents were given assurance that their decisions to participate or not participate in this research would not affect their relationships with each other and the principle or kindergarten.

Children were informed that they could tell me, or let any adults know, if they did not want to be observed, photographed, video-recorded, audio-recorded, and/or to engage in conversation. They could leave the situation or activity being observed. They also could refuse to answer any questions during the informal conversations. While participating in the individual semi-structured interviews, each teacher was informed that she could suspend the interviews and leave the place where the interview took place or stop recording at any time. They could also refuse to answer any questions during the informal conversations and/or individual semi-structured interviews.

Data Collection Procedures

My data collection procedures reflected ethnographic features. I visited each kindergarten during the first semester[8] in the academic year 2018/2019. This section describes the data collection procedures of my study through three subsections: research sites and participants, familiarisation, and data collection after familiarisation.

Research Sites and Participants

The Process of Recruiting Kindergartens and Participants. My study selected two kindergartens as the research sites. I employed purposive sampling (Patton, 1990) to recruit kindergartens. According to Merriam (2009), researchers need to decide what selection conditions are crucial in selecting the sites to be studied as the criteria established for purposeful sampling. I recruited kindergartens that (i) followed the Curriculum Development Council (CDC) (2017) to design the curriculum; and (ii) were located in two specific districts with different socio-economic characteristics. The intention was to see if differing availability of kindergarten resources and parents' expectations might influence children's EMEs. As a result, I selected one kindergarten from the Central & Western district and one from the Tai Po district[9].

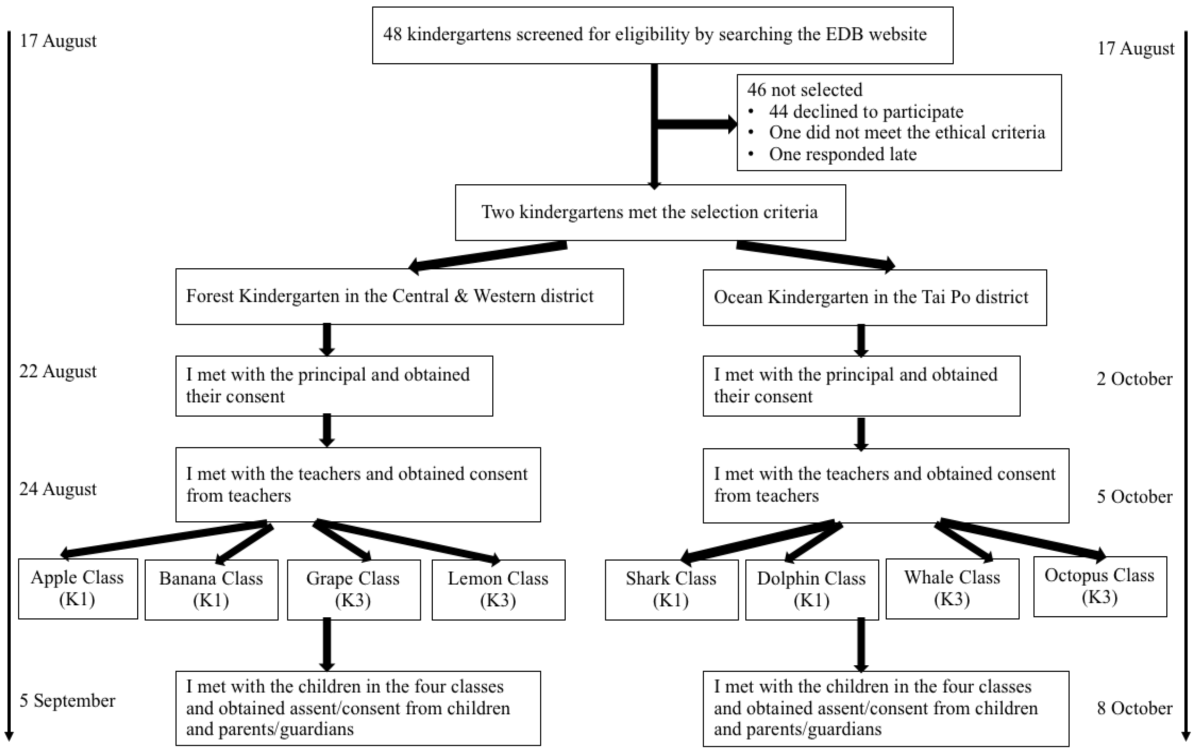
I successfully recruited two kindergartens as the research sites. Recruiting two kindergartens was appropriate for this study for two reasons. Firstly, my study did not intend to generalise from a sample to a population (Patton, 1999). As a small-scale study, a small sample size was appropriate to provide an in-depth exploration of children's EMEs. Secondly, I was the only researcher who conducted the data collection procedure, so two kindergartens (a total of eight half-day classes) were manageable for me to visit in parallel each week. I present a detailed description of data

[8] HK kindergartens operate on a system based on two semesters per year.
[9] The median monthly household income of domestic household in the Central & Western district was ranked as the highest and the Tai Po district was ranked as the 15th among 18 districts in HK in 2017 (Hong Kong Census and Statistics Department, 2017).

collection procedure and arrangement later in this section. Figure 1 shows the procedure of recruiting kindergartens and participants for my study. I spent around eight weeks on recruiting kindergartens and attaining the participants' assent or consent.

Figure 1

The Procedure of Recruiting Kindergartens and Participants



Regarding the procedure of recruiting kindergartens, I first searched on the EDB website for suitable kindergartens located in the two districts. I then sent an email invitation letter (Appendix H), PIS, and CF written in Chinese to 48 kindergartens that fulfilled the two criteria listed above. I selected the first kindergarten from each district that voluntarily responded to my email. I also met the kindergarten principals in person to explain my research purposes and procedures, and to request their participation. I stopped the recruitment once I got the consent from one kindergarten of each district. At the beginning of the process, only the principals of Forest Kindergarten (FK) from the Central & Western district and another kindergarten from the Tai Po district showed interest in participating in my study. However, due to ethical issues[10], I later chose Ocean Kindergarten (OK) from the Tai Po district as the second research site after the principal contacted me to express interest in participating in my study. As a result, I obtained consent from the principal of FK approximately six weeks before I obtained consent from OK.

Both principals provided me a signed CF before I approached teachers and children in their kindergartens. I applied purposive sampling to recruit participants. In kindergartens, teachers have different expectations of younger versus older children (Saft & Pianta, 2001). Children of different ages might show differences in terms of the nature and content of EMEs. Thus, I recruited children and teachers from K1 and K3 classes as participants. The head teachers helped me schedule a convenient time to meet the teachers from K1 and K3 classes in their kindergartens.

During my visit to each kindergarten, I introduced the research procedures, duration, method, and data collection techniques to teachers. I also explained my role during the observations and how I would handle the information with confidentiality. I explained that they would have the right to withdraw from the study at any time if they desired. I explained that the purpose of this study was not to judge either the children or teachers, and that I would not discuss them with the principal. After this visit, I requested the principals' permission to leave some copies of the PIS and CF for the teachers (Appendix B & F) so that individual teachers who wished to participate in my study could contact me directly. All teacher participants in the two kindergarten provided a signed CF on the day of my visit.

I recruited the children in the teacher participants' classes as child participants. With the principal's permission, I discussed with the teachers the best way to distribute and collect the AF for the children (Appendix D) and the CF for the parents/guardians (Appendix C) in their class. The teachers arranged the opportunity for me to introduce myself to children and answer their

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[10] Prior to gaining consent from OK, I was contacted by the principal of another kindergarten in the Tai Po district. However, during my initial visits, I noticed that some of the teachers had not volunteered to participate in my study. Therefore, I withdrew from this kindergarten and started the recruitment process again.

questions when I visited each class for the first or second time. I prepared an information booklet (Appendix I) to talk about the research aims, duration, data collection techniques, and my role as a researcher during the observations. With the teachers' permission, I laminated the booklet and left one copy in each classroom for those children who showed interest in me and my study to read. In the fieldwork, I answered questions proposed by some children in terms of the pictures presented in the booklet.

With the suggestion and permission of the principals and teachers, I left each child a copy of the AF together with the information booklet for children (Appendix D & I), a copy of the PIS and CF for parents/guardians (Appendix C & G), and an empty envelope in the notice bag that children brought home every day. In doing so, the parents/guardians who wished or did not wish their children to participate in my study could confidentially return the AF that they helped their children fill in along with the CF. In the PIS, I noted that if any questions came up, they could contact me in person or through my contact details listed in the PIS and information booklet (Appendix G & I). I only involved the children who provided their assent and whose parents/guardians provided their consent as participants. A summary of the participants is listed in Table 2.

Table 2

A Summary of the Participants

Kindergartens	Grades	Classes	Participants	Pseudonyms of teachers
Forest Kindergarten (FK)	K1	Apple Class	1 teacher; 17 children	Violet
		Banana Class	1 teacher; 17 children	Cherry
	K3	Grape Class	1 teacher; 19 children	Olive
		Lemon Class	1 teacher; 20 children	Amber
Ocean Kindergarten (OK)	K1	Shark Class	1 teacher; 20 children	Rose
		Dolphin Class	1 teacher; 18 children	Iris
	K3	Whale Class	2 teachers; 23 children	Ivy; Lily
		Octopus Class	1 teacher; 11 children	Daisy

Two research sites. The learning environments in the two kindergartens were set up differently. At FK, interest corners played an important role to encourage children's free choice activities and play in the classroom. This feature was influenced by the CDC (2017) which guides teachers to set up diverse interest corners and prepare learning resources to encourage children's "self-directed learning" (p. 26) (as noted in Chapter 1, p. 7). Teachers allowed children to choose whichever corner they liked when they had finished their required work or activities. Each interest corner usually included toys and resources (hereafter referred to collectively in this study as resources) which were displayed on the shelves, walls, ceilings, floors, and/or tables. At FK, teachers took the main role in designing the classroom settings. Therefore, the design of interest corners showed individual teachers' ideas and considerations and varied across different classrooms. Figure 2 (left) shows the setting of a dramatic play corner. Moreover, as a result of "free play" having been actively promoted in HK since the 2018/2019 academic year, FK had constructed a "free play area" (see Figure 10 (right)) in the common area. The principal and head teachers set a time schedule for each class to use the free play area once weekly. The setting of the free play area was similar to the interest corner in the classroom.

At FK, the environment was set up to reflect thematic learning, and the principal and head teachers pre-determined seven main themes for exploration over the year as part of the curriculum. The kindergarten policy required all teachers to set up an environment that aligned with the monthly theme. During my period of observing (around seven months), the settings of all classes and common area in FK changed monthly according to the monthly theme explored by children and guided by teachers (the theme and its exploration will be discussed in Chapter 5).

Figure 2

A Dramatic Play Corner (Left) and a "Free Play Area" (Right)

Compared with the teachers at FK, teachers from OK had less control in deciding the classroom settings and resources, as the principal and head teachers took the main responsibility for designing classrooms and resources. In HK, many kindergartens which were termed as "traditional kindergartens" by early childhood practitioners showed the same situation as FK. In these kindergartens, the setting of all classrooms showed a similar design. According to Figure 3 that I took in the Shark/Dolphin Class and Whale Class, the teachers used children's artwork as wall or ceiling decorations, arranged furniture of similar design in similar ways, displayed worksheets or pictures related to the themes investigated (as will be discussed later) on the wall, and kept all resources in similar baskets and storage on shelves. During my period of observing, the environment rarely changed.

Figure 3

The Classroom Setting of Shark/Dolphin Class (Left) and Whale Class (Right)

Resources in the two kindergartens were distributed in a top-down manner. That is, the principals and head teachers of both kindergartens bought and prepared resources for teachers to use to teach the six key learning areas (Chapter 1, p. 2). However, there were some differences between the two kindergartens. At FK, teachers selected resources they desired from the "storage room of resources"[11] in accordance with the monthly themes and the teachers' ideas for designing

[11] This term was used by the teachers from FK, which was a place to keep all resources for teaching and learning in FK.

the classroom. At OK, the head teachers prepared resources for each class at the beginning of the semester. Resources in the classes of each grade were limited and similar. According to my observation and informal conversations with the teachers, some teachers bought or created resources for children when they thought it was needed. During my period of observing, resources of the classes that I observed rarely changed. Findings in terms of the set up of the learning environment will be presented and discussed in Chapter 5.

In HK, children who attend half-day classes normally attend for three hours per day, for either a morning or afternoon session. The two kindergartens in this study offered both morning and afternoon sessions, but children only attended either a morning or afternoon session for three hours a day. The principals and the head teachers of the two kindergartens pre-arranged timetables in all classes. All children and teachers had to follow daily timetable which involved set activities at set times. As the timetables and activities from Monday to Friday were very similar in all classes, after discussion with the teachers and head teachers or principals, I observed each half-day class for a half-day weekly (approximately 3 hours). Tables 3 to 6 show my observation arrangement and the timetables for the eight half-day classes that I observed.

Table 3

Ocean Kindergarten K1 Half-day Class Timetable (Monday)

Shark Class/Dolphin Class		
Time	Activity	Duration
9:00am-12:10am/ 1:00pm-4:10pm	Whole-class activity: Morning assembly	10 minutes
	Half-class activity: English activity and music activity	50 minutes
	Whole-class activity: Toilet time and snack time	20 minutes
	Group activity (three group tasks): Thematic activity, homework, art activity, operating realia, sharing activity, or playing toys	65 minutes
	Whole-class activity: Free play	40 minutes
	Whole-class activity: Getting ready to go home	5 minutes

Table 4

Ocean Kindergarten K3 Half-day Class Timetable (Tuesday)

Whale Class/Octopus Class		
Time	Activity	Duration
9:00am-12:00am/ 1:00pm-4:00pm	Whole-class activity: Morning assembly	10 minutes
	Whole-class activity: Free play	40 minutes
	Whole-class activity: Toilet time and snack time	20 minutes
	Group activity (three group tasks): Thematic activity, homework, art activity, operating teaching aids and realia, sharing activity, or playing toys	65 minutes
	Half-class activity: Mandarin activity and physical activity	50 minutes
	Whole-class activity: Getting ready to go home	5 minutes

Table 5

Forest Kindergarten K3 Half-day Class Timetable (Wednesday)

Lemon Class (Group A)/Grape Class (Group B)		
Time	Activity	Duration
9:00am-12:00pm/ 1:30pm-4:00pm	Whole-class activities: Morning assembly and free choice activity	15 minutes
	Group/half-class activity: English activity	25 minutes
	Group/half-class activity: Music activity	25 minutes
	Group/half-class activity: Physical activity (indoor)	25 minutes
	Group/half-class activity: Toilet time, snack time, and free choice activity	35 minutes
	Group/half-class activity: Circle time (including homework)	40 minutes
	Group/half-class activity: Toilet time and changing shoes	10 minutes
	Whole-class activities: Getting ready to go home	5 minutes

Table 6

Forest Kindergarten K1 half-day class timetable (Thursday)

Apple Class (Group A)/Banana Class (Group B)		
Time	Activity	Duration
9:00am-12:00pm/ 1:30pm-4:00pm	Whole-class activity: Morning assembly and free choice corner activity	15 minutes
	Sub-group activity: Art activity and/or circle time	40 minutes
	Group/Half-class activity: Toilet time, snack time, and free choice activity	35 minutes
	Group/Half-class activity: Physical activity and outdoor play	25 minutes
	Group/Half-class activity: English activity	25 minutes
	Group/Half-class activity: Free play (common area)	25 minutes
	Group/Half-class activity: Toilet time and changing shoes	10 minutes
	Whole-class activities: Getting ready to go home	5 minutes

These four tables summarise the time allocation of half-day activities in different classrooms. They indicate an emphasis on structured aspects of learning, as all classrooms allocated fixed time for all routine activities, including child-directed activities and play (i.e., “free play” and free choice activities). Routine activities in the two kindergartens typically included morning or afternoon assembly[12], learning activities (e.g., thematic activities), “free play”, free choice activities, physical/music/art activities[13], toilet time, snack time, tidying-up, and getting ready to go home. The time allocation for “free play” and free choice activities were limited in all classrooms, ranging from 25 to 40 minutes.

The structured aspect of learning was also reflected through different forms of organising routine activities. Throughout the half-day, the teachers from both kindergartens followed the timetable and used a variety of grouping techniques for organising different activities for children. As can be seen from the four tables, at OK, all activities were organised according to whole-class, half-class, and smaller groups each day. At FK, the activities were organised according to whole-class, groups/half-class, and sub-groups (applicable to K1 classes) each day. Although the teachers at FK referred to half-class activities as group activities, these actually involved half the class. The teachers usually assembled a name for each group (e.g., Group A or B in some classes) to help children remember and identify their small community within the larger community of the whole class. Most of the time, whole-class/group activities were found to be the predominant pedagogical mode in HK kindergarten classrooms (Li et al., 2012), and most learning activities were conducted with all children in the class or group.

The timetable did not indicate all activities children participated in each day. For instance, although the timetable did not explicitly highlight teacher-directed mathematics activity, the kindergartens encouraged teachers to use the group activity time for formal mathematics teaching.

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[12] Teachers would guide children to sing and dance with music, do show and tell, and have casual conversations with children during the assembly.

[13] An assistant teacher assigned by the head teacher would help the teacher conduct an art activity during this time.

In specific, the teachers in some classes provided mathematics instruction through introducing the content of mathematics homework at the end of assembly or before the group learning activity started. In other classes, teachers' responses to children during routine activities (e.g., morning or afternoon assembly) would highlight mathematics if it was important to the teachers. In this case, mathematics activities were temporally embedded in many routine activities.

Mathematics activities were also physically located in difference spaces of the classroom. For example, teachers set up mathematics corners/areas (Figure 4), mathematics applications were accessible to children through the desktop computer (Figure 5), and mathematics resources were available for children to use during free play and free choice activities in some classrooms (Figure 6). More details about the temporal and physical influence of timetables on children's EMEs are further discussed in Chapter 5.

Figure 4

A Photo of Mathematics Corner/Area

Figure 5

A Photo of a Mathematics Application for Children to Learn Composing and Decomposing Number Seven on a Desktop Computer

Figure 6

A Photo of Mathematics-focused Toys/Materials/Resources

Additionally, teachers' responsibilities in caring for children was an issue related to class size in the two kindergartens. For instance, during my period of fieldwork, at FK, there were 23-28 children in each class. There were two qualified teachers who worked together. Except for whole-class activities (i.e., morning/afternoon assembly, getting ready to go home, and other special kindergarten events), each teacher was in charge of a group/half the class of children. Both teachers were main teachers[14]. In this case, the main teacher to child ratio was 1:11-14. At OK, the number of children in each class was 23-28. There were also two qualified teachers who worked together. One was the main teacher, and the other one was the assistant teacher[15] during school hours. Except for some group activities (i.e., English/Putonghua activity, music/physical activity, and thematic activities), the main teacher was usually the person who guided all group and whole-class activities. In this case, the main teacher to child ration was 1:23-28. Thus, although the CDC (2017) advocates a teacher-child ratio of 1:11, the main teacher-child ratio in the two kindergartens in this study showed a higher number. This could lead to issues related to the class size Crouch et al. (2007) have indicated that a large class size means that the teacher knows fewer children well (as mentioned in Chapter 1).

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[14] Main duties of main teachers were planning and conducting activities for all children in the class.

[15] Main duties of assistant teachers were assisting the main teacher to conduct the activities prescribed by the main teacher and supervising children to do homework in some K3 classes.

This subsection outlined the process of recruiting kindergartens and participants and provided background information about the two research sites. Next, the familiarisation process of my fieldwork is introduced.

Familiarisation

When commencing fieldwork, undertaking a familiarisation process allows the researcher to set up a study and build researcher-participant relationships (Frankham & Howes, 2006). In my study, I conducted familiarisation over a period of 4-6 weeks in each kindergarten, depending on the situation in each classroom. For the participants, the phase of familiarisation would make them familiar with not only the research process, but also my presence in the classroom and kindergarten. In my fieldwork, when we met in the kindergarten, some children had informal chats with me, saying things like, “Which class do you stay today?” or “I know today is Wednesday, and you will be in my class on Thursday.” For me, the major aims of the familiarisation were related to building relationships with participants, negotiating and establishing a researcher role, and learning the language of the kindergarten (Barley & Bath, 2014). These aims are addressed below.

Building Relationships. Learning how to locate and build relationships in the initial period of fieldwork is crucial to ensure that preliminary and ongoing access is granted (Burgess, 1991). Hatch (2002) has stressed the importance of developing a positive relationship between the researcher and participants, placing the responsibility of developing the relationship in the hands of the researcher. In my study, I had recruited the teachers as participants at the start of the familiarisation process. Becoming familiar with participants is essential “to grasp what [participants] experience as meaningful and important” in research sites (Emerson et al., 1995, p. 2). To become familiar with the teachers and the kindergartens and establish a relationship with the teachers in a short time, I actively sought opportunities to get along with them. Thus, I participated as a helper after school hours when needed during familiarisation and throughout the fieldwork. For example, I attended kindergarten events as a volunteer, and I supported teachers to prepare teaching materials and help them decorate the classroom.

The benefits of establishing familiar relationships became evident during the interview process. Many teachers were very supportive of my study. Although they were informed that the interview would take around one hour, some of them asked me not to worry about the time and expressed their support by saying, "I hope all information [provided by me] is valuable to your study." Therefore, some of the interviews were carried out for more than one hour, as the teachers had additional information and experiences to share with me.

Children were not participants until I attained their parents'/guardians' consent and their assent during the familiarisation phase. Recruiting children as participants and establishing a relationship with them were two main purposes of the familiarisation phase in my study. A process of familiarisation is important for children as they often need time and space to observe adults in a setting before they are ready to interact and confide with them (Punch, 2002). I distributed the AF and CF (Appendix C & D) to children and their parents/guardians two weeks after the familiarisation stage had begun. Allowing participants to have sufficient time to get to know my study and me was intended to help build trust, so that they would feel comfortable giving their full consent to participate in the study and allow me to access their world (Barley & Bath, 2014). Some children, particularly those in some K3 classes, self-selected to become participants by saying that they would like to "help me with my study"; they also allowed me to be an onlooker of their play and sometimes invited me to play with them.

I was also aware that being too familiar with participants could be a dilemma in qualitative research. Hence, negotiating a researcher role becomes important in my study. This is addressed below.

Negotiating and Establishing a Researcher Role. Familiarisation allows a researcher to establish their role as a researcher within the research site (Barley & Bath, 2014). According to my prior working experience as a kindergarten teacher, it is common to have different people in HK kindergartens, such as pre-service teachers doing practicums and parent volunteers telling stories to children. However, all teachers in my study reported that they, and the children in their classes, had never worked with a student researcher conducting qualitative research before. Thus, negotiating a researcher role with both teachers and children within the classroom was particularly important for my study. As a researcher in the classroom who was not a member of staff or any of the roles outlined above, I believed that negotiating the boundaries of my role as researcher was an ongoing process and a part of my own familiarisation within the kindergarten. Schensul et al. (1999) have suggested that it is important for a researcher to learn how to be a "self-reflective tool of enquiry" and engage in ongoing reflection of one's own experiences (p. 72). Thus, during my fieldwork, I recorded my reflections through reflective journaling after visiting each class.

By establishing a researcher role, the researcher also establishes rules of working that allow participation in activities and the effective collection and recording of data, which can be viewed as learning the practicalities of doing fieldwork (Barley & Bath, 2014). The phase of familiarisation enabled me to learn how to effectively collect and record data within each classroom. Breglia (2009) has adopted the term "work-break game" to describe the process through which the researcher and participant co-construct the rules of collecting and recording data. This was my first time conducting a qualitative study as a novice researcher working on my own in kindergartens with which I was not familiar. As noted above, the participants (particularly children) had not encountered a researcher before. In this case, when introducing myself to children, I explained to them that I was doing a project at university and related this to their thematic learning activities in kindergartens. I positioned myself as an adult who lacked knowledge about children's worlds and needed them to teach me (Mayall, 2008).

During the process of learning how to collect and record data effectively, the concepts of "least-adult role" (Mandell, 1988) and "horizontal relationships" (Milstein, 2010) enabled me to join in children's activities, play, and conversations as a participant observer when invited. Moreover, as the spaces at the two kindergartens hosted a range of activities and were governed by different rules (e.g., some spaces were quiet places where children were encouraged to work independently while others were social areas), I adopted different techniques listed earlier to collect and record data depending on which space children were located in. Last, learning with the children how to record their actions and words in a culturally sensitive way, not only taught me how to collect and record my data effectively, but also to negotiate ongoing informed consent (Cocks, 2006).

Learning the Language of the Kindergarten. In my study, learning the language of the kindergarten referred to becoming familiar with the routines, rules, beliefs, norms, and the jargon or slang of each kindergarten (Barley & Bath, 2014). These aspects are key to gaining a foundational understanding of the research site and the social rules it abides by (Schensul et al., 1999). In my study, I worked within the norms and beliefs of a culture with which I was previously familiar. This enabled me to have a better understanding of participants' ways of exploring and expressing mathematics within the cultural context (e.g., in Chinese culture, numbers six to ten are commonly represented by finger symbols using one hand).

Framed by sociocultural perspectives, my study explored children's EMEs through investigating children's lived experiences in kindergarten settings. Thus, becoming familiar with the rules and routines of a kindergarten day was an important aspect of the initial period which helped to structure my initial observations. Learning the classroom rules, especially those unspoken or explicit rules, was an important first step in understanding the field location (Barley & Bath, 2014). Some explicit rules were those that appeared to be "openly stated by the teacher" (Barnikis, 2015, p. 299). For example, some teachers required children to keep quiet during snack time by stating, "Quiet". Other rules could be implicit and understood as "hidden curriculum", which is a "set of implicit messages relating to knowledge, values, norms of behaviour and attitudes that learners experience in and through educational processes" (Skelton, 1997, p. 188). These implicit rules were not at first apparent so needed to be elicited through observations and conversations with participants. For example, at FK, children in some classes could only start free-choice activities after completing tasks prescribed by teachers.

Becoming acquainted with the jargon or slang used in kindergartens was important when I visited the two kindergartens, as some terms spoken by the teachers were only understandable to the people in the kindergarten. For example, the teachers at OK adopted the term "生活數學/sang1 wut6 sou2 hok6 (mathematics in life)" to represent teacher-directed mathematics activity. They adapted this term from a mathematics textbook used by children and teachers at the kindergarten called 幼兒生活數學/jau4 ji4 sang1 wut6 sou2 hok6 (children's mathematics in life). Knowing the kindergarten jargon or slang enabled me to have a better understanding of teachers' and children's language and, therefore, the experiences and meanings created by them.

Following the phase of familiarisation, the data collection process included conducting classroom observations and individual semi-structured interviews. They are introduced as follows.

Data Collection after Familiarisation

Classroom Observations. Observation is one of the main sources of data in studies that take place in natural settings (Mertens, 2005). Observations enable researchers to collect data on a variety of areas of interest (e.g., children's EMEs constructed by interacting with surroundings in kindergartens in this study) (Patton, 2002). In educational research, classroom observation refers to a "non-judgmental description of classroom events that can be analysed and given interpretation" (Gebhard & Oprandy, 1999, p. 286). It also presents a clear picture of classroom activities and allows researchers to collect information about types of language, activities, interactions, instruction and other important moments in the classroom (Mackey & Gass, 2015). In my study, classroom observation commenced after the familiarisation phase. This arrangement aimed to make the classroom observations smoother, as the familiarisation process enabled the teachers and children in each classroom to become familiar with my presence in the classroom and the kindergarten; it also enabled me to be familiar with the participants and the field.

As mentioned earlier, in addition to participant observation (Yin, 1994), I also included non-participant observation (Robson, 2011) in my data collection process. Non-participant observation means that the researcher interferes as little with teachers and children as possible to avoid affecting their behaviours (Robson, 2011). It allows identifying what is happening in a classroom situation, for example, who is there and when and how things happen in a given context. It allows the researcher to focus on collecting data and at the same time not affect the research processes (Moug, 2007). According to these ideas, I could observe behaviours or processes of children and teachers, as they would naturally occur in classrooms. However, participation to a certain extent is unavoidable in classroom contexts with children (Ho, 2015). For example, although researchers may be silent when they were taking notes, this action might actually attract children's attention (Moug, 2007).

I was also aware that due to my presence or "observer effect", "participants may alter their behaviours as a result of being observed" (Casey, 2006, p. 77). I therefore involved participant observation in the fieldwork to encourage the participants to be more comfortable with me in general, and more at ease with having me in the class for a prolonged period (Ho, 2015). I hoped that by doing so, participants might carry on natural behaviours and ways of interacting with one another, including maintaining their daily routines and common activities (Benigno, 2012). In other words, they might tend to forget my presence (Patton, 2002).

The observations were undertaken in each classroom for three hours per week for approximately 16 weeks in my study. This "prolonged engagement in the field" enhanced the authenticity of the results (Nicholls, 2009, p. 642). In spending time in the classroom, I aimed to achieve a kind of "saturation" (Hesse-Biber & Leavy, 2011) or arrive at a point where no new matters were rising from the data (Casey, 2006). During the classroom observations, the techniques of informal conversation, photography, video- or audio-recording, and making field notes were employed to generate observation data. Moreover, I used the technique of collecting documents throughout the fieldwork, as teachers' writing of learning reports about children's mathematics learning and planning for mathematics learning was ongoing.

Individual Semi-structured Interviews. Nine teachers agreed to attend two, up to one-hour individual semi-structured interviews during non-contact hours. These two phases of interviews took place after the familiarisation period and after my classroom observation respectively. The first interview aimed at understanding teachers' perceptions of children's lived experience in the kindergarten. In the second interview, I used selected photographs and video footage of children's EMEs, with and without the teacher's participation, and the excerpts of informal conversations with children and teachers to encourage discussion of children's EMEs and to "explore the meanings that lie behind observed behaviours" (Edwards, 2001, p. 131). Video-recording teachers' practices is useful to foster professional dialogue and reflection (Cherrington & Loveridge, 2014), as it might stimulate teachers' memories of interacting with children. Through this process, teachers may "perceive discontinuities between their intentions and actions" (Wood & Bennett, 2000, p. 639). However, teachers might feel uncomfortable viewing episodes of their own practices (Zhang et al., 2011). To help make teachers feel comfortable, I used two strategies: (i) sending the episodes to each teacher to view prior to the interview; and (ii) describing this process in the PIS for teachers. Santagata and Guarino (2011) have explained that providing clear guidance on what aspects to attend to is an important element of using video footage successfully.

I scheduled the semi-structured interviews and conducted them with the teachers at mutually convenient times, as the environment can influence the quality of the interview (Yow, 2005). The teachers decided where the interviews took place. They chose their workplaces or quiet cafes as venues because such arrangement saved their travel time. Each interview averaged 60 minutes, but the length of the interviews varied due to the complexity of the information provided by the teachers and other factors, such as my probing. I conducted the interviews in Cantonese as it is the dialect used by the teachers, and they felt comfortable expressing themselves using Cantonese. I transcribed the interviews in verbatim accounts to make sure that each transcript would reflect the actual meanings intended by the teachers throughout the interviews.

The creation of an interview schedule is crucial to identify the sequence and process to be used with participants (Ryan et al., 2009). After considering issues such as the aim, objectives, and nature of the study, I used open-ended questions (Appendix J) to support flexible dialogue and to enhance the natural flow of thoughts of teachers in the two interviews (Mukherji & Albon, 2010). I carefully developed and organised the interview questions linked to the research questions with the supervisors' suggestions. I put demographic questions, which were easy to answer, at the beginning of the interview. Questions that were related to the focus of the study followed, while potentially sensitive questions were withheld until trust had been developed and the interviewees were at ease. The general sequencing scheme was to move from easy to difficult or sensitive. Moreover, because questioning is a means to find out the meaning of a phenomenon when interviewing teachers, I "put questions in a straightforward, clear, and non-threatening way" (Robson, 2011, p. 273). I also employed audio-recordings to enhance the data quality or accuracy and reduce bias by replicating the precise contents of interviews (Wellington, 2000). The use of audio-recording contributed to a continuous flow of conversation, which enabled me to concentrate on listening and probing the teachers' views when necessary.

Data Analysis Procedures

Data analysis was undertaken to look for answers to the three research questions in my study. To reach this goal, I explored children's EMEs in each class and synthesised areas of congruence and difference across all the classes to create a holistic description of the phenomenon. I followed a four-step inductive analysis based on "thematic analysis" (Braun & Clarke, 2006) to address the three research questions. Thematic analysis is a method for "identifying, analysing and reporting patterns (themes) within data" (Braun & Clarke, 2006, p. 79). It is "a process that can be used with most, if not all, qualitative methods" (Boyatzis, 1998, p.4). It can accommodate rich and comprehensive data as it is applicable to different theoretical and epistemological approaches (Braun & Clarke, 2006).

First Step: Preparing, Transcribing and Translating the Data

During the fieldwork phase, I started preparing the data by creating eight documents on my computer to systematically organise the data collected from the eight classes. I sorted the data collected from each class according to the type of data collection method used (i.e. field notes, audio-recordings, video-recordings, photographs, first teacher interview transcripts, second teacher interview transcripts, documentation, and reflective journaling). I filed them in chronological order for further management and reference.

Data transcription is a key phase of data analysis within the interpretative qualitative methodology, which influences the understanding and interpretation of data and considers how meanings are created (Braun & Clarke, 2006). I transcribed the interviewed data and audio-/video-recorded observation data verbatim. Moreover, translation was particularly important for data analysis in this study. This study was designed in English but conducted in Chinese then reported in English. Thus, I used English to analyse all transcribed Chinese data first, and only translated the excerpt of transcripts that would appear in the final thesis from Chinese into English (Cormier, 2018). In this way, "late translation" (Cormier, 2018) could enhance the research validity, as it retained the participants' words in their language for as long as possible (Nurjannah et al., 2014; Sutrisno et al., 2014). Temple and Young (2004) have explained that the late translation may indicate a "recognition of the ontological importance for people of their first language" (p. 174). That is, late translation shows the importance and respect of participants' cultures.

Researchers need to have "intimate knowledge of their data" (Howitt, 2010, p.164). I therefore read and re-read the transcribed data, made preliminary jottings to become increasingly familiar with the data, and got a sense of EMEs and the meaning of EMEs for children. I also listened to audio-recordings of the interviews with the teachers and watched episodes of video-recordings repeatedly to become familiar with all the data. Some documents provided by the teachers and kindergartens and reflective journals were not part of the case description so were not analysed. Still, they helped me to analyse the data and reminded me of how I understood something when it happened. When I was satisfied with the process of preparation and familiarisation, I then moved to the second step of the analysis.

Second Step: Conducting the Initial Coding

I coded the "data corpus" (Braun & Clarke, 2006) at an individual class level as the second step of the analysis to make the data analysis manageable. This step aimed to conduct the initial coding regarding three research questions: *What are the nature and content of children's EMEs in HK kindergarten settings? What are the affordances and constraints that influence the availability of children's EMEs?* And *what are teachers' perceptions and practices with regard to children's EMEs?* I identified evidence that indicated why and how children employed mathematical concepts and knowledge in mathematical events[16] (nature) and the mathematical concepts and knowledge employed by children (content). I identified evidence that indicated factors enabling (affordances) and constraining (constraints) the availability of children's mathematical events. I identified evidence that indicated what teachers said (teachers' perceptions) and did (teachers' practices) regarding children's EMEs. I used nature, content, affordances, constraints, teachers' perceptions, and teachers' practices as six main categories to guide this coding process, as they were six key ideas addressed in the research questions.

I made some decisions in relation to conducting the initial coding at an individual class level. Firstly, the data for this step of analysis were (i) all children's mathematical events, which were from the field notes, transcriptions of audio- and video-recordings, and some documents provided by four teacher participants[17]; and (ii) transcripts of all teachers' first and second interviews. Secondly, all events were processed in the original Chinese version but coded in English. Thirdly, I performed manual coding by using the functions of track changes and comments on Microsoft Word. Although some data (e.g., reflective journals) that did not show close relevance to children's EMEs were not analysed, they helped me to analyse and understand the data in this step.

I scrutinised the data to develop preliminary codes related to the six main categories. I used a line-by-line coding strategy to extract the notes and transcripts that directly pertained to understanding the six categories outlined above. The preliminary codes created in this process were based on (i) the collection of mathematical concepts and knowledge prepared prior to data

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[16] Mathematical events are units of observed occasions, in which children's mathematical action is "integral to the nature of the participants' interactions and their interpretive processes" (Street et al., 2005, p. 12).
[17] At FK, the teachers were required by management to create observation records in different learning areas, including mathematics, as a part of the assessment of children's learning.

collection; (ii) insights gained from observing children's everyday kindergarten life; and (iii) patterns that emerged through the process of coding the data. I put aside the data that did not show close relevance to the research phenomenon. I also wrote "analytic memos" (see Table 7) to document and reflect on "coding processes and code choices; how the process of inquiry is taking shape; and the emergent patterns, categories and subcategories, themes, and concepts in the data—all possibly leading toward theory" (Saldana, 2013, p.41). Table 7 illustrates the analytic memos created by me and the line-by-line coding of a fieldnote extract of Gary's counting process.

Table 7

A Line-by-line Coding of a Field Note Extract

Analytic memos	Fieldnote extract	Coding
It was very important to be well-behaved and concentrate in this classroom. At the beginning of this episode, the whole class had a relaxed atmosphere, and children chatted with each other. However, after seeing that one boy did not wear his schoolbag properly and one girl did not sit properly, the teacher started talking about discipline. After then, the classroom became quiet. Later, the teacher started reviewing what she had taught today. This aligns with the CDC's (2017) suggestion about the "conclusion and sharing of the day's activities" (p. 58) while engaging in tidying-up and getting ready to go home.	Before going back home, all children are sitting in their seats and are ready for the end of school, and most of them are talking.	A non-academic classroom event
	At this time, several children in the Watermelon Group are laughing, pointing their fingers at the others to count the number of children in the class.	Peers' counting actions
	However, the teacher Iris asked them to stop, so they put their hands down.	The presence of the teacher
	Andy in the Apple Group points to the children in the distant Watermelon Group and counts, "1, 2, 3, 4, 5."	Counting the number of children in the class
It seemed like Iris wanted to stop Andy's counting process. She interpreted that the purpose of Gary's counting was to know the number of people in the classroom, so she directly told him that there were 29 people here.	At this time, Iris is passing Gary's seat to get the New Year's decorations and says, "No need to count, there are 29 people here, oops, no, including teachers the number is not 29."	The presence of the teacher
Gary tried to explain, but Iris didn't hear and know.	Gary takes back his hand, looking at the opposite side and says in a low voice that "Grace (who usually sits opposite to Gary) is absent."	Explaining the intention of counting by the child
	At this time, Iris continues to tell the children about how to make New Year's decorations tomorrow, and Gary is sitting quietly.	The teacher is unresponsive

After developing a set of preliminary codes, I sorted them to search for potential themes within the six main categories for each class. I worked on the six main categories one by one in the following order: nature, content, affordances, constraints, teachers' perceptions, and teachers' practices. Braun and Clarke (2006) have described a theme as an idea that captures something important about the data in relation to the research question that represents a pattern in responses. Building on this definition, within each main category, I created themes and sub-themes based on preliminary codes that emerged in this initial sorting. Other codes were discarded or kept as outliers. I used mind maps to help sort the different codes into potential themes.

Reviewing themes was important after identifying potential themes. To achieve this goal, I re-read all "data extracts" (Braun & Clarke, 2006) that fit into each theme to ensure that the data formed a coherent pattern. In this process, reflection was necessary to consider if the data fit into the themes. If the data fit into each theme coherently, I considered each theme in relation to the data corpus. I used a thematic map to help me visualise the relationship between the themes. Developing themes was not a static process in my data analysis. As part of the process, I asked myself: Do the relationships between the themes reflect the meaning of the data as a whole? If not, I returned to searching for themes and keep reviewing themes. I also generated a thematic map of the data and extracted all themes and codes to new Microsoft Word and Microsoft Excel documents for the third step of the analysis.

Third Step: Conducting the Second Coding

Building on the themes and codes created at an individual class level in the second step of the analysis, the third step of the analysis aimed to not only keep addressing the research questions, but also identify the nature, content, affordances, constraints, teachers' perceptions, and teachers' practices regarding children's EMEs across all eight classes. In doing so, I was able to analyse data in a holistic way. In other words, this step of analysis helped me understand children's EMEs in the two kindergartens as a whole.

I created six new documents and tables for the four main categories. Each document included all relevant potential themes, codes, and data extracts identified from the raw data in all eight classes. I worked on the six categories one by one. In this process, I also included analytic memos to analyse the themes and codes, aiming to examine the robustness of the themes and codes identified and clarify my understanding of children's EMEs. I re-read all data extracts that fit into each theme to ensure that all of the data formed a coherent pattern. I also generated a thematic map of the data at this holistic level.

Defining and naming themes are essential qualitative data analysis (Braun & Clarke, 2006). In my study, this process was based on my interpretations of children's EMEs and understandings about the children, their everyday experiences in kindergartens, and the culture of the classes and the kindergartens. I also used existing literature to inform and facilitate my interpretations. Through this process, I captured the essence of what each theme was about and what aspect of the data each theme captured. I also created an overall narrative with all the data. I analysed each theme and its individual narrative by asking myself: Does it fit into my overall narrative? At the end of the third step of the analysis, the names that I used to label the themes identified were thought to be concise and informative in giving readers a sense of the themes.

Fourth Step: Synthesising Categories and Themes

Subsequent data analysis entailed a synthesis of all categories and themes in a coherent description of children's EMEs in each class, grade, and kindergarten. The purpose of this step was to draw together similar ideas to create an integrated description and explanation of children's EMEs in the two kindergartens. In the process, to develop a consistent understanding of children's EMEs, I examined and reviewed all the codes, categories, and themes built in the previous three steps of analysis. I also searched and created connections among relevant data. Additionally, I stepped out of studying the parts and considered the whole picture and how each part contributed to it. The back-and-forth movement of data and text created provided me with evidence for the final holistic analysis (Van Manen, 1990).

Research Rigour

The quality of research is an essential issue, particularly if findings are utilised to inform educational practices and policies. A qualitative study of high quality must be rigorous (Tracy, 2010). The notion of rigor in qualitative research has been regarded as equating to the notions of reliability and validity and all are necessary components of quality (Cypress, 2017). Rigor is defined as the strength of the research design and the appropriateness of the method to answer the questions (Morse et al., 2002). The following paragraphs describe the strategies that I used to ensure the rigor of this study.

First, to validate findings, I adopted the strategy of "participant validation" (Bryman, 2004), which means that researchers return to the research participants with their tentative results and refine them according to the feedback of the participants. Accordingly, I returned the interview transcripts to the teachers to verify that the transcripts reflected their knowledge and experience. All transcripts were presented in Chinese. The teachers' feedback led to revisions and further points from them were incorporated into the data analysis. Seven out of nine teachers did this. One teacher added some comments to the transcripts, the other teachers did not make any changes. Moreover, due to the complexity of observation data (e.g., mixed multiple children and teachers) and that the children were not able to read written words, I did not return the transcripts of observation data and my field notes to the children and teachers to verify the observation data. This might be a limitation in relation to using participant validation in my study.

Second, I sought to triangulate the data through the use of several data gathering methods to gain insights into the research questions. In qualitative research, "triangulation" (Patton, 1999) has been viewed as a strategy to ensure validity of the data collected. Data triangulation refers to the use of multiple methods or data sources in qualitative research to develop a comprehensive understanding of phenomena (Patton, 1999). Qualitative studies often used participant observations alongside other methods. In my study, I adopted diverse techniques to collect data and for the purpose of method triangulation, including some techniques for classroom observations (i.e., photography, video- or audio-recording, field notes, and individual informal conversations), two phases of semi-structured interview for teachers, documentation, and reflective journaling. Method triangulation involves the use of multiple data collection methods to investigate the same phenomenon (Denzin, 1989). Moreover, I also collected data from different sources (i.e., children and teachers). This echoed data source triangulation, which involves the collection of data from different types of people, aiming to gain multiple perspectives and validation of data (Carter et al., 2014).

Third, I adopted the strategy of “thick description” (Tracy, 2010) to obtain credibility in my study. As Merriam (1998) has contended, “The qualitative study provides the reader with a depiction in enough detail to show that the author’s conclusion ‘makes sense’” (p. 199), thereby increasing the credence of their interpretation. According to Ponterotto (2006), thick description means “the researcher’s task of both describing and interpreting observed social action (or behaviour) within its particular context” (p. 543). Thus, in this thesis, I described instances or vignettes of children’s EMEs in a narrative form, aiming to disclose details of the phenomenon and tell a story about children’s EMEs. I adopted Ponterotto’s (2006) five components of thick description to guide the narrative. These included: describing and interpreting in the context; capturing the "thoughts, emotions, and web of social interaction"; assigning "motivations and intentions", using rich details to provide readers with "a sense of verisimilitude", and meaningful descriptions and interpretations that "resonate with readers" (pp. 542-543).

Chapter Summary

This chapter has presented a thorough account of the research methodology. I employed a qualitative case study drawing on ethnographic techniques to explore children’s EMEs in HK kindergartens. I recruited eight classes in two kindergartens. I also adopted four ethnographic techniques (i.e., classroom observation, individual semi-structured interviews, documentation, and reflective journaling) to gain an in-depth understanding of the phenomenon of children’s EMEs in HK kindergarten settings. A range of strategies, including data triangulation, enhanced the study’s trustworthiness. I viewed ethical considerations as essential to guide every decision throughout the study. During data analysis procedures, I identified key themes from the findings through thematic analysis. Chapter 4, the first of two findings and discussions chapters, presents and discusses the findings regarding the nature and content of children's EMEs.

Chapter 4: The Nature and Content of Children's Everyday Mathematical Experiences (EMEs)

This chapter is the first of two findings and discussion chapters that report on and discuss the study's findings about the nature and content of everyday mathematical experiences (EMEs) constructed by children in two Hong Kong (HK) kindergartens. Chapters 4 answers the first research question, *What are the nature and content of children's EMEs?* To achieve this aim, I sought evidence about children's engagement with mathematical language, including symbols, because Vygotsky (1987) viewed language as the most important symbolic tool. I also interpreted the findings in relation to the literature and theory from Chapter 2. The findings comprise data collected from classroom observations, interviews, field notes, documents (i.e., teachers' observation records focused on mathematics and mathematics learning objectives provided by the head teachers), and my reflective journaling. In this chapter, I present my findings mainly in narrative form in that examples of children's EMEs are presented as vignettes or descriptive excerpts to illustrate relevant findings in this chapter.

Regarding the nature of children's EMEs, I analysed four types of children's EMEs: exploratory, playful, informal, and spontaneous. Indicators or features for each type of EME are presented and discussed. My thoughts in relation to these four types of EMEs were inspired by Gray (2013), although he focused on diverse types of children's play rather than EMEs. He noted, "The types of play […] are not mutually exclusive categories. They are functional types, meaning that they refer to the different functions that play can serve. Any given instance of play may serve more than one function" (p. 125). Aligned with these ideas, I also view the types of EMEs I identified as different features of EMEs in that they are not mutually exclusive categories. At any given instance, an EMEs may indicate more than one feature. Regarding the content of children's EMEs, I summarise a range of mathematical concepts and knowledge explored and applied by children, such as numbers, quantity, size, patterns, and shapes.

The chapter is divided into five sections. The first section focuses on exploratory EMEs and their two indicators. The second section focuses on playful EMEs and their two indicators. The third section addresses informal EMEs and their indictors, and the fourth section discusses spontaneous EMEs and their indictor.

To identify the sources of data, I outlined the names of the teacher participants (pseudonyms) and their classes in Table 2 (Chapter 3, p. 89). Table 8 summarises the annotations used in the two findings chapters. Aligned with being a case study where the case is the phenomenon of children's EMEs, I present the data from the two research sites together rather than separately.

Table 8

Data Source Annotations

Data Source	Alpha Code
Teacher interviews 1	(Name/Kindergarten/Class/TI1/ddmmyy) e.g., (Rose/Ocean Kindergarten (OK)/Apple Class/TI1/ddmmyy)
Teacher interviews 2	(Name/Kindergarten/Class/TI2/ddmmyy) e.g., (Rose/OK/Apple Class/TI2/ddmmyy)
Kindergarten documentation	(KD/Kindergarten/year) e.g., (KD/OK/2018)
Field notes	(FN/ddmmyy)
Journaling	(RJ/ddmmyy)

Exploratory EMEs

I identified exploratory as a salient feature of some instances of children's EMEs constructed in kindergartens. I termed this type of EMEs as exploratory EMEs. Two aspects indicated the exploratory feature, including (i) mathematical curiosity and sensitivity and (ii) self-directed mathematical inquiries and discoveries. By constructing exploratory EMEs during classroom experiences, children could satisfy their curiosity, discover mathematical discoveries, find and solve practical problems independently, and understand their surroundings.

Mathematical Curiosity and Sensitivity as an Indicator of Exploratory EMEs

My observations showed that children often displayed a desire and interest to know and discover mathematical aspects in their immediate context. I termed this as mathematical curiosity and sensitivity and identified it as an indicator of children's exploratory EMEs. Three excerpts of children's mathematical curiosity and sensitivity are shown below:

> Excerpt 1[18]: After the group activity, the teacher asked all children to line up. While lining up, Daisy (6 years, 1 month) counted the children's photos displayed on the wall one by one in a low voice. After counting, she looked at me and said in an excited voice, "Ye, Ye, do you know if we have 23 children in the class?" As she initiated the conversation with me, I asked her the reason for her counting. She explained, "Because I don't know whether we have 23 or 24 [children in our class]. I want to know". (FN/260219)

> Excerpt 2: After the Mandarin teacher walked into the classroom, there were three adults in the classroom, including the Mandarin teacher, the assistant teacher, and me as the researcher. At this time, Lucas (4 years, 1 month), who was looking at the teachers and sitting on the chair, said in a low voice, "one, two, three, three teachers". Then the Mandarin teacher started the activity. (FN/250219)

> Excerpt 3: The teacher asked children to walk from the first floor to the ground floor to engage in an English activity. I was walking with Sam (3 years, 4 months) at the end of the queue. When we were at the entrance of the stairs, he stopped and looked at upstairs. Then he looked at me and asked, "Which floor is above the first floor?" I said, "What do you think?" Sam thought for a few seconds and said, "The second floor, because of one, two". (FN/280219)

As shown in the above instances of EMEs, the children actively engaged with the environment and made connections to mathematics. They were specifically interested in the

[18] To better engage in the discussion of findings in this thesis, I marked all excerpts with sequential numbers.

numbers embedded in quantitative aspects of their surroundings. Daisy and Lucas were particularly curious about and sensitive to counting while Sam was interested in ordinal numbers and their application. Ponticorvo et al. (2018) have explained that with development, children keep refining their senses and coordinate them with their actions, so that they can obtain relevant information from the environment. In my study, mathematical information was one type of information that children were keen to obtain from the environment.

All the mathematical information that children were curious about and sensitive to were embedded in their immediate social and physical context. For instance, Daisy was particularly interested in the number of children in the classroom; Lucas was focused on the number of adults in the room; and Sam wondered about the different levels of the building. The children encountered all these activities or situations in the present moment, rather than drawing on the past or future. Therefore, I argue that the immediate social and physical context provides resources for children to construct exploratory EMEs.

The display of children's mathematical curiosity and sensitivity required observation and the use of their senses. Observation is important in children's informal learning (Paradise & Rogoff, 2009) because it often leads to discovery. The Curriculum Development Council (CDC) (2017) has noted that observation is an important strategy used by children to "Explore the properties of quantity, shapes and space (e.g. front, back, left, and right)" (p. 40) and that observation could be carried out "by using the senses of sight, hearing, taste, smell and touch" (p. 43). In my study, children used their observations as channels to experience and explore their surroundings with mathematical lenses. As can be seen from the examples outlined above, they used their senses to see mathematical information embedded in their environment.

Children were keen to observe, paying attention to what was happening in the context in which they located. According to the examples of EMEs outlined earlier, Daisy observed her peers in the classroom while they were lining up; Lucas was aware of adults in the classroom; and Sam's curiosity was piqued by his observation of the stairs. These children all demonstrated curiosity in relation to mathematics in their environment.

Children were also curious about and sensitive to what they heard in the environment. For example, at the two kindergartens, assembly followed the prescribed timetable and started at the same time every day. During assemblies, teachers usually played the same piece of music to signify the start of assembly. In most the classes, teachers led the children to do actions in time with the music. One day, the teacher who was responsible for playing the music accidentally played the piece twice. I observed a child in the Apple Class asking, “Why did they play the same music twice today?” (FN/251018). Using his sense of hearing, this child noticed this difference and was curious about what he had heard.

Children sometimes showed mathematical curiosity and sensitivity to the physical objects they touched. Another day, I observed an art activity in the Apple Class as the teacher, Violet, introduced and demonstrated to children how to apply glue. She specifically reminded children to use the proper amount of glue. During this activity, Leo (2 years, 11 months) was curious about the glue and this motivated him to explore. He scooped up some glue with his finger and said happily, “It’s a lot” (FN/111018). His smile and speech indicated that he enjoyed exploring the concept of quantity by touching the glue.

Children’s curious natures, questions, and interests underpinned their mathematical curiosity and sensitivity and led to inquiry-based exploratory EMEs. “Children are curious in nature” (CDC, 2017, p. 41). Their curiosity is essential to maintaining “a positive disposition toward mathematics” (Baroody & Coslick, 1998, vii). When Daisy expressed her desire to know how many children were in the class, her questioning was motivated by curiosity. Senthamarai (2019) has explained that children’s curiosity leads them to know mathematical concepts and knowledge and explore it. In Daisy’s case, her counting was a way to satisfy her curiosity and a form of investigating designed to resolve uncertainty about the number of children in the class.

Children's mathematical curiosity and sensitivity often emerged from a social context. During my fieldwork, I observed that children from all classes were keen to talk to each other during snack time. Food was an important topic of conversation. For example, when eating bananas or other food, some that they had taken a big bite. They held the food in their mouths without swallowing and said to their peers through puffed cheeks, "Look, how big my bite is". Sometimes, their peers would also take a bite of banana happily saying, "my bite is also big" or "mine is bigger than yours". In the process, they used the concepts of quantity and comparing and turned their conversation into playful EMEs (as will be discussed later in this chapter). Children's curiosity and sensitivity to how much food they could put in their mouths was influenced by playful interactions with their peers.

Children's mathematical curiosity and sensitivity was also influenced by cultural beliefs and values. As noted in Chapter 2, the goal of education framed by Confucianism is to help children develop perfect characters (Kim, 2007). According to my observation, taking big bites while eating was a good character valued by teachers. Some teachers, particularly from some K1 classes, often took the initiate in praising children by highlighting they took big bites while eating. As a previous kindergarten teacher in HK and an adult grown up in a Chinese family, I know that many Chinese teachers and parents believe that children's big bites mean that they have a good appetite and a strong body. In this case, it is understandable that children sometimes actively showed their puffy cheeks with food to teachers. After swallowing, some children even exclaimed to teachers in a proud voice, "It was really a big bite". This indicates that they might be sensitive to the big bites valued by teachers. I therefore argue that children's mathematical curiosity and sensitivity is influenced by cultural values and beliefs.

The findings presented and discussed in this sub-section showed children's mathematical curiosity and sensitivity to their environment led to exploratory EMEs. Moreover, all instances of children's exploratory EMEs described in this subsection occurred without direct guidance from teachers. While independently engaging in exploratory EMEs, children's interest in mathematics was directly influenced by the physical environment and their social and cultural context. In the process, children's observations, use of senses, curious natures, questions, and interests underpinned their mathematical curiosity and sensitivity. This curiosity and sensitivity was a strength that supported children's engagement in exploratory EMEs. From the observation data, I interpreted another indicator for children's exploratory EMEs, which is presented next.

Self-directed Mathematical Inquiries as an Indicator of Exploratory EMEs

Drawing on their curiosity and sensitivity to mathematical properties in their world, children often made mathematical inquiries when engaging in exploratory EMEs. They applied mathematical concepts and knowledge in their exploration or investigation of their environments. I therefore identified the mathematical inquiries made by children as the second indicator of their exploratory EMEs. The children's behaviours described on p. 115 illustrated their use of mathematical concepts and knowledge to solve to solve their independent inquiries and make sense of their worlds.

In this study, children's inquiries, which are "an act of seeking or building knowledge" (Marin, 2014, p. 21), were often indicated by the questions they asked. For example, when Violet told the children that they "can use just a little glue", Leo asked "What is too little?" And when she used the concept "much", Leo asked "What is too much?" (FN/111018). In this example, Leo's two questions were both explicit questions in relation to the concept of quantity.

In addition to asking explicit questions, children's mathematical questions were sometimes implicit. For example, one day in the Dolphin Class, children were waiting for their parents to pick them up. After seeing his peers leaving, Gary (3 years, 11 months) raised his finger and pointed at other children in the classroom. He counted, "One, two, three, four, five…" (FN/070119). Although Gary did not explicitly ask the question "how many children are in the classroom?", his thinking appeared to be what Mead (1934) referred to as an "internalised conversation with oneself" (p. 47). In such conversations, the other person's responses can be imagined or anticipated (Carpendale et al., 2009). In this instance, it is likely that Gary was thinking about "how many children were left in the classroom". I defined all direct questions and explorations as mathematical inquiries in this study.

While making mathematical inquiries, children often asked "information-seeking questions" (Lau & Lee, 2018). According to the examples outlined above, the children's explicit or implicit questions made their inquiries understandable to me. For instance, Leo's inquiry referred to exploring possible outcomes of using too little or too much glue and Gary's inquiry referred to seeking information about the number of children in the classroom. All the inquiries revealed important information that they would like to know. Through making mathematical inquiries, the children aimed to learn about mathematical information or knowledge related to their everyday experiences.

Children's mathematical inquiries sometimes led to the discovering of mathematical knowledge, which I termed as mathematical discoveries in my study. Discovery refers to finding concepts through a series of data or information obtained through observation or experimentation (Gallenstein, 2005). For example, it was evident that Gary was curious about the number of children in the classroom, rather than other information, as he used his finger to point to other children in the room. Furthermore, he looked at his peers and counted the number of children one by one, which showed that he was observing them. His mathematical speech indicated that he employed the counting strategy in the form of "private speech" (Vygotsky, 1987) to directly obtain the mathematical knowledge that he was interested in knowing.

The reason why children made mathematical inquiries and gained the information they desired was connected to their mathematical curiosity and sensitivity. As mentioned earlier, children are naturally curious. Their interest in exploring the world around them is the hallmark of their curiosity (Chak, 2010). The purpose of satisfying her curiosity was evident in Daisy's exploratory EMEs (see p. 115). Her words "I want to know" indicated that she made the mathematical inquiry for her own sake and the inquiry derived from her own curiosity. She took the initiative in making her mathematical inquiry. In this case, her inquiry could be viewed as a "self-directed inquiry" (Stone, 2016), which is a series of questions that belong to the individual (Eick et al., 2005). Drawing on these discussions, being self-directed could be viewed as a feature that characterised the mathematical inquiries children made in their exploratory EMEs.

Self-directed mathematical inquiries are also closely connected to children's agency. As discussed in Chapter 2, with agency, children are able to show possibilities and willingness to control their own actions (Tertoolen et al., 2017). For example, both Daisy and Gary were eager to investigate and solve the inquiry by themselves rather than asking for help from others. Stone (2016) has explained that because children are "the owner and stakeholder of the inquiry process", their self-directed inquiry allows them "the freedom to explore, create, adapt, modify, and play with ideas" (p. 2). Therefore, I argue that the mathematical inquiry addressed in Daisy's and Gary's exploratory EMEs indicated agency in their own learning and development.

Self-directed mathematical inquires made and solved by children led to the achievement of self-directed mathematics learning. Self-directed learning refers to "learning projects" undertaken by either a child or some children without the assistance of teachers, but it can include the presence of a "resource person" who does not regard themselves as an educator (Schugurensky, 2000). For instance, in Leo's case, motivated by his mathematical curiosity and sensitivity, he viewed Violet as a fellow inquirer (as will be discussed in Chapter 5) to ask and solve questions that were important to him. Dann (2013) has explained that curiosity becomes a positive, intrinsic strength when children are able to struggle with their own questions and problems and are aware of what can help them to answer their questions or solve those problems.

Further analysis of the data indicated that children independently used the cultural tools (Vygotsky, 1981) discussed in Chapter 2 to solve mathematical inquires. I identified two main types of cultural tools used by children to construct exploratory EMEs, which will now be discussed: First, mathematical concepts and knowledge were an essential cultural tool that enabled children to solve mathematical inquires independently. In the excerpts outlined on p. 115, the children were keen to make sense of their surroundings from a mathematical perspective and satisfy their curiosity, which led to their mathematical inquiry. To satisfy their curiosity, Daisy and Lucas used their knowledge of numbers, counting, and quantity; and Sam used his knowledge of ascending order numbers. According to Engeström (1987), cultural tools are the results of human beings' collective life activity in practice, which enable collective activity related to other individuals. Drawing on this idea, Forsström and Afdal (2020) have viewed mathematics as a cultural tool, which has been "created over time by human beings" (p. 35). Thus, the mathematical knowledge used by these children might have been appropriated through their interactions with their teachers, parents, or peers. The children were then able to use their mathematical knowledge to make and solve mathematical inquiries independently.

Second, the use of private speech enabled children to solve mathematical inquiries and express mathematical discoveries, and further extend their mathematics learning. As discussed in Chapter 2, Vygotsky (1987) viewed language, including private speech, as an important symbolic tool. According to Vygotsky (1987), private speech is a product of a child's social environment. In the process of counting, the strategies of one-to-one correspondence and counting numbers in ascending order used by Lucas probably appropriated through his interactions with other people, such as his teachers or parents. Moreover, Vygotsky (1987) proposed that private speech is located in social context, functioning to connect thoughts and words with actions. He further indicated that children could use private speech as a tool to facilitate their cognitive processes (e.g., solving problems). In Lucas's case, his questioning about the social context (i.e., the number of teachers in the room), led him to count the teachers in the room. With private speech, Lucas was able to seek the answer to his mathematical inquiry, which led to mathematics learning.

Children in this study sometimes indicated their inquiries and discoveries by using private speech to describe the mathematical attributes of physical objects or mathematical information they had noticed. For example, one day, I observed that Max (3 years) had taken his temperature sheet (a card that recorded his daily temperature) from his schoolbag and unfolded it. He stared at the unfolded sheet in his hands. Without looking at others in the classroom, he said slowly and happily, “The temperature sheet is so big” (FN/031218). Because his oral language was overtly for personal communication but not for communication with others, his speech could be perceived as private speech (Gholami et al., 2016). The content of his private speech was an expression of his discovery, which indicated that he was interested in, and enjoyed discovering, the mathematical concept of size in relation to his temperature sheet.

As can be seen from the examples of exploratory EMEs outlined in this section, the self-directed mathematical inquires made and solved by children were linked to them making sense of their worlds. Through using mathematical concepts and knowledge during classroom experiences (e.g., waiting to go home in Gary’s case), gained mathematical understandings of the context in which the investigations took place (e.g., knowing how many children were left in the classroom).

In summary, this section discussed two indicators for children’s exploratory EMEs: (i) mathematical curiosity and sensitivity and (ii) self-directed mathematical inquiries. Findings in terms of playful EMEs are presented and discussed below.

Playful EMEs

That the EMEs constructed by children in kindergartens were sometimes playful was the second feature I identified. In this study, I viewed playful as being fond of play and the emotions that play and playfulness engender (e.g., feeling amused and happy). I analysed two indicators for children’s playful EMEs, including (i) the use of different languages and resources to express mathematics and (ii) expressions of emotions. Through constructing playful EMEs in kindergartens, I found that children created and practiced diverse ways of using mathematics and gained diverse emotional experiences through engaging with mathematics.

The Use of Different Languages and Objects to Express Mathematics as an Indicator of Playful EMEs

I identified children’s use of different languages and objects to engage with mathematical concepts in diverse and creative ways as an indicator for playful EMEs. I observed that instances of playful EMEs often occurred when children were directly playing with mathematical concepts, particularly with number-based concepts (e.g., counting, date, and age). The diverse and creative ways noted here refer to the use of different cultural tools, such as language (i.e., oral language, written language, body language, and music) and the objects children used to express mathematics playfully.

As discussed in the last section, children sometimes used oral language as a tool to make mathematical inquiries. They also sometimes played with mathematical concepts through oral language. There was evidence in the findings that some children viewed mathematical terms in the form of speech as an object to play with. The following two excerpts show how Tracy (3 years, 8 months) played with numbers and how Zita (5 years, 2 months) played with the days of the week through their oral language:

> Excerpt 4: While others were tidying up before going home, Tracy stood in front of the desk in the language corner. She looked at the coloured paper with numbers that teachers prepared for this corner and said with a smile, “one, two, three, four, five, six, seven, eight, nine, ten, ha-one, ha-two, ha-three, ha-four, 15, 16”. Then she turned around and left. (FN/140319)

> Excerpt 5: During morning assembly, the teacher introduced the date in both Cantonese and English to children. After the teacher announced that today was Tuesday in English, Zita showed a big smile on her face and said in an excited but low voice, “one-sday, two-sday, three-sday, four-sday (raising fingers accordingly)”. (FN/012219)

In these two excerpts, Tracy’s playful mathematical action involved carrying out rote counting with self-created numbers. Zita’s playful mathematical action involved her inventing terms to represent the days of the week. Although Tracy and Zita engaged with different key mathematical concepts (i.e., numbers and the days of the week respectively) in their speech, they both mimicked the patterning of numbers in a playful way. I identified this process as playful EMEs according to the content of their speech, their playful manner of speaking, and facial expressions. The smiles on their face indicated that they both enjoyed playing with mathematics.

Two circumstantial factors seemed to have contributed to the playful EMEs demonstrated by Tracy and Zita: (i) although Cantonese was the main teaching language, English and Mandarin were also formally taught at the two kindergartens as required by the CDC (2017); and (ii) some children's home language was Mandarin and/or English. Within these contexts, many children in this study used these three languages interchangeably among themselves, with their peers, teachers, and when speaking to me. Drawing on these two factors, the processes through which Tracy and Zita engaged in playful EMEs indicated that their playful EMEs were influenced by social and cultural factors, particularly the use of language in the kindergarten and family.

Further analysis of the data showed that children's mathematical speech was created using a pattern of numbers that mixed the same sound or suffix. In Tracy's case, she showed interest in saying numbers from 1-16 in Cantonese. After counting to ten, she said the numbers in an unconventional way, referring to "ha1, ha2, ha3, ha4". Later, she continued saying the numbers 15 and 16. According to the number sequence she expressed, it could be understood that she used "ha-one, ha-two, ha-three, ha-four" to represent the conventional numbers 11-14. This indicated her understanding of the Chinese number system as she used the same sound "ha" to represent the ten in numbers 11-14. Regarding Zita, instead of saying Monday to Thursday in English in a conventional way, she used the combination of numbers 1-4 and the suffix "-sday" to represent four days of the week, which could be understood as Monday, Tuesday, Wednesday, and Thursday.

Tracy and Zita's knowledge of the Chinese number system and expressions in English and Chinese contributed to their playful EMEs. Knowledge of the Chinese number system enabled Tracy to recreate the names. Ng and Rao (2010) have explained that in Chinese, the number names from 1-9 are single-syllable words. Number words above ten are generated by consistent rules. The spoken Chinese number names from 11-19 systematically follow the base-ten rule. For example, the literal translation of 11 from Chinese into English is "ten-one". According to this rule, 12 is read as "ten-two" and 13 as "ten-three". Therefore, the spoken Chinese number names and the base-ten rule could enable Tracy to recreate "ha1, ha2, ha3, ha4" to represent numbers 11-14.

The pattern indicated in Zita's playful EMEs related to the days of the week in English. In English, the seven days of the week all end with the same suffix "-day"; Tuesday, Wednesday, and Thursday end with "-sday". Because the teacher announced that the day was Tuesday on that day, which ends with "-sady", Zita used "sday" instead of "day" in her playful EMEs. Furthermore, the Chinese way of expressing the days of the week might also have influenced Zita's thinking and language. In Chinese, Monday is considered to be the first day of the week. Thus, it is expressed as "week-one/星期一/sing1 kei4 yat1" if translated into English directly. Also, Thursday is expressed as "week-four/sing1 kei4 sei3/星期四". Therefore, my finding was that language is a flexible cultural tool that enables children's playful EMEs.

Observations also showed that the children sometimes involved music in their playful EMEs. Music is cited as a characteristically human form of communication (Zatorre & Baum, 2012). Some children enjoyed improvising mathematical lyrics through song. I therefore identified singing songs or nursery rhymes with improvised mathematical lyrics as a type of playful mathematical action carried out by children. The following excerpt shows an example of Ross's (3 years, 3 months) singing process:

> Excerpt 6: When walking down the corridor, the teacher introduced the children to the art display outside the music room. While the teacher was talking, Ross used the melody of *Twinkle Twinkle Little Star* but replaced the words with "one two, one two, one two five; one two, one two, one two five …". (FN/140319)

Based on the melody, I identified that Ross was singing the tune to *Twinkle Twinkle Little Star*. However, he used numbers (i.e., one, two, and five) to improvise the lyrics, which I identified as a type of playful mathematical action.

Children showed their knowledge and understanding of patterns (e.g., the "one two, one two, one two five" sung by Ross) in their playful EMEs involving music. Virtually all mathematics is based on pattern and structure, and a mathematical pattern may be described as any predictable regularity, usually involving numerical, spatial, or logical relationships (Mulligan & Mitchelmore, 2009). Through music, children can "find patterns in the repeated melodies, refrains, or rhythms of a song" (Edelson & Johnson, 2003, p. 66). In Ross's case, he repeatedly used the combinations of "one two" and "one, two, five" in time with the melody. He used the combinations of "one two" and "one, two, five" to create patterns, which also functioned as the content of the song. In addition to using numbers, I observed that some children made patterns in their improvised singing by repeating individual syllables (e.g., using two syllables "ah" and "ha" to sing the song *Twinkle Twinkle Little Star*). This finding aligned with Smith (2009), who has indicated that some of the earliest experiences with patterns are found in the repetitive phrases or rhythms of children's songs and nursery rhymes. I therefore argue that children's playful EMEs involving music can be viewed as a demonstration of their knowledge and understanding of patterns.

Features of songs contributed to children's engaging in music during their playful EMEs. *Twinkle Twinkle Little Star* is a common children's song. The melody of the song is from a French song *Ah! vous dirai-je, maman* and has been used in many other famous songs (e.g., *The Alphabet Song* and *Baa Baa Black Sheep*). Some music fans have analysed this song to understand why the melody has such broad appeal:

> Musically, it's simple, memorable, and easy for children and adults to sing and play because of its short-range (6 notes, less than an octave). It is rhythmically repetitive and pretty much as simple as you can get, […] which makes it very easy to put words to, and very easy to remember (Some_Guy, 2017, para. 6).

All the above information indicates that the existing melody of *Twinkle Twinkle Little Star* contributes to the creation of many new songs via replacing the lyrics. Ross's song was an example of a new song based on a familiar melody that included knowledge of number and patterning.

Private speech also enabled children's engagement with mathematics via oral language in a playful way. Gholami et al. (2016) described private speech as the language used by the child overtly for intrapersonal communication. Thus, talking to oneself is an essential feature of private speech and can be used to identify children's private mathematical speech. Tracy, Zita, and Ross all expressed themselves through private speech not intended for others. Private speech enabled them to creatively explore making sounds with the same patterns in ways that were meaningful for them rather than functioning as verbal communication with others.

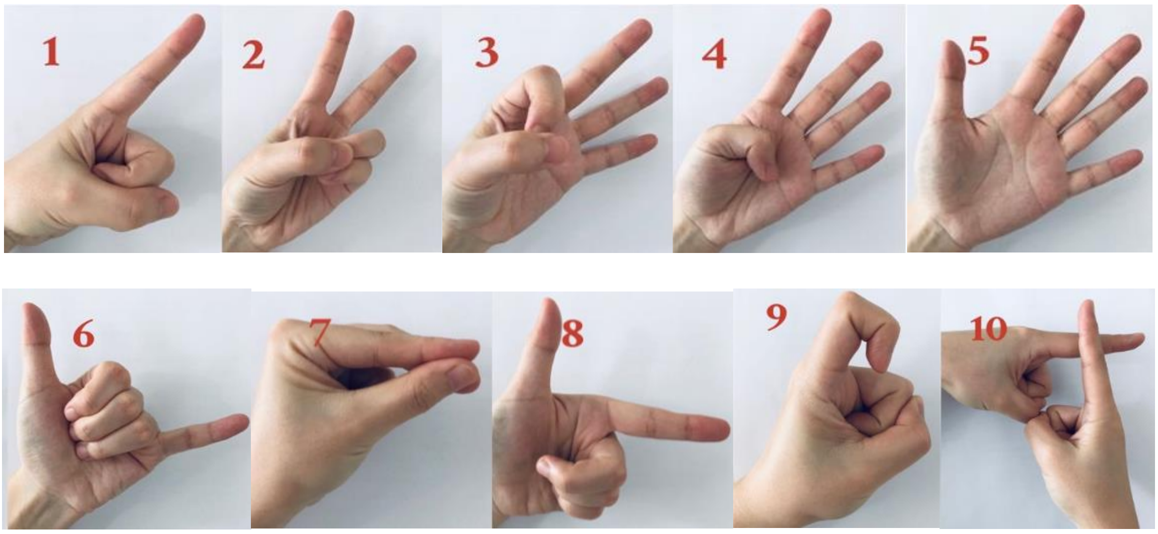
Although children's private speech is not always understandable to others, the mathematical information embedded in their private speech could be used as clues to demonstrate their mathematical understandings. Most teachers in this study noted that they often recognised children's EMEs through the mathematical information conveyed through their speech, including private mathematical speech. Moreover, Vygotsky (1987) has also considered private speech as the transition to social speech, which is communication to others. It is noted that although private speech is not intended to communicate with others, children in this study often shared their playfulness with mathematical words (e.g., numbers) with others, aiming to have fun, and this will be discussed later.

In addition to oral language and music, children also used body language as a tool to express mathematical concepts while engaging in playful EMEs. Children mainly used three types of gestures and movements to represent mathematical concepts and ideas. (i) I observed that children sometimes represented cardinal numbers by using "finger-number gestures" (Morrissey et al., 2020), such as raising two fingers to represent the number two. These gestures were also used by teachers. (ii) Children sometimes used finger gestures to create geometric shapes (e.g., triangles and circles) and non-geometric shapes (e.g., hearts). And (iii) children also used their hands and feet to represent mathematical concepts. For example, they used the distance between two hands to represent the concept of size (e.g., big/small). Details of these three types of gestures and movements are addressed below.

First, children's use of finger-number gestures was a socially and culturally embedded practice. In Chinese culture, the cardinal finger-number gestures include a one-to-one correspondence up to number 5, and numbers 6-10 can be represented by finger symbols using one or two hands (see Figure 7). There are some differences in relation to the finger symbols used for zero and number 10 across different regions in China.

Figure 7

Chinese Finger-number Gestures



Second, children sometimes used hand gestures to make mathematical concepts concrete and visible. For example, the following excerpt related to Sophia's (3 years, 1 month, an English-speaking child) hand gestures that took place on her way to the toilet.

> Excerpt 7: Sophia put two index fingers and two thumbs together and enclosed them into a hollow shape. She looked at me and showed her finger-shape, they said with a smile, "Triangle". Later, she extended the shape horizontally. She looked at me and showed her finger-shape, then said with a smile again, "Big triangle".
>
> (FN/131218)

In this excerpt, Sophia's playful mathematical actions referred to (i) using thumbs and index fingers to create an enclosed space and naming the enclosed space as a "triangle"; and (ii) exploring the concept “big” with her fingers. I observed that her triangle had three sides. That is, she used two index fingers to construct two sides of the triangle, and she used two overlapping thumbs to construct the base of the triangle. The enclosed shape she made indicated that she understood that a triangle has three sides. Her smile indicated that she enjoyed creating a “triangle” with her fingers. Thus, she continued her interest by creating another shape by extending her fingers horizontally. She believed that the second triangle she made was bigger than the previous one, so she described it as a “big triangle”. If a child’s gestures reflect the state of the child’s knowledge, these gestures could serve as a signal that the child is ready for certain kinds of learning (Goldin-Meadow, 2009). Through representing her exploration and understanding using hand gestures, Sophia demonstrated that she was an active mathematical learner.

Third, in addition to hand gestures, children also used other types of body language to support their communication e.g., stretching out their arms horizontally while talking about the concept of length. The following excerpt shows how Joni (3 years, 11 months), Flora (3 years, 5 months), and Jack (3 years, 11 months) supported their mathematical conversation using body language.

> Excerpt 8: While waiting for the end of the morning session, Joni, Flora, and Jack were sitting together. They were chatting about their older brothers and sisters. Joni initiated the conversation by stating that she had an older brother. Then Flora shared the same experience, she smiled and added, “My brother is very big”. While speaking, she closed her hands but left a gap between two hands. Joni responded, “My brother is ve…ry big”. She also nearly closed her hands, but the gap between her hands was wider than Flora’s. At this time, Jack joined the conversation and noted that he only had an older sister. He asked Flora, “Is your brother very big?” Flora nodded her head and answered, “He is very, very big”. She laughed and fully extended her hands. (FN/171218)

In this excerpt, Flora and Joni used an arm gesture to support their conversation about size with their peers. Also, their speech and hand movements showed them making comparisons. They used the words “big”, “very big”, “ve..ry big” and “very, very big” to describe the size of their older brothers. They expressed mathematical meanings through playing with language and actions. While using verbal descriptions, their body movements also reflected their understanding of size and comparison. Previous research has shown that human’s body language often conveys semantic information relevant to that in speech (e.g., Kita & Özyürek, 2003). The children used physical gestures to complement their verbal statements and convey the comparisons they were making.

Sometimes, children also used written language to engage with mathematical concepts during playful EMEs. For example, children showed their geometric knowledge and thinking when they drew shapes and created patterns. Moreover, sometimes they created symbols that were meaningful to them to address mathematical information. For example, drawing symbols (i.e., short lines) to represent different numbers. Children also wrote conventional numbers using Arabic and Chinese numerals, e.g., 1, 2, 3 in Arabic numerals is represented as 一, 二, 三 in Chinese, to represent ideas in their drawings. Hedges (2007) has noted that children are interested in writing from an early age. Figure 8 was a drawing created by Tim (5 years, 4 months) during a music activity. While playing a piece of music about outer space, the teacher asked all the children to create drawings based on what they felt listening to the music (FN/241018).

Figure 8

A Child’s Drawing with Numbers

In Tim's drawing, he drew a planet, a sun, a moon, four stars, a rocket and wrote down three numbers using conventional Arabic numerals (i.e., 1, 2, 3) next to the rocket. After Tim had finished, he showed me his drawing. He told me that the numbers "3, 2, 1" referred to the countdown for the rocket launch in his drawing. This demonstrated that he understood about counting numbers backwards and that a countdown needed to happen before the launching of a rocket.

The instruction and practice of reading and writing numbers promoted in kindergartens had an impact on children incorporating written language into their mathematics play. As discussed in Chapter 2, many kindergartens in HK focus on learning academic skills, such as reading and writing (Wu, 2014). Teaching children to read and write numbers played an important role in the two kindergartens in this study. In K3 classes, the teachers intentionally and formally guided children to read and write Arabic and Chinese numerals. This phenomenon can be explained as Confucian way of teaching children, which highlights writing as fundamental knowledge (Hong & Howes, 2014). Although the CDC (2017) highlights that, "[kindergartens] should not ask K1 children to hold a pencil and write" (p. 36), I observed that children in the K1 classes in OK had to do homework, such as colouring pictures or numbers on a daily basis. Furthermore, as mentioned in Chapter 3, mathematics activities were physically situated in some classes. For example, K1 teachers at FK prepared worksheets for use in the art corner for children to colour in numbers (see Figure 9).

Figure 9

Worksheet of Number Colouring

In addition to language, children also played with mathematics using physical objects. I observed that they sometimes endowed mathematical meanings to the things they created, which helped to make mathematical concepts more concrete. In the teacher interview with Cherry, she described an excerpt she had observed, and thought was an example of children's playful use of mathematics and therefore EMEs:

> Excerpt 9: Once, I saw them folding face cloths after snack time. […] When a child has finished folding his face cloth, he told me about the shape of the face cloth he had folded. […] I praised him and said, "It turned into a square". Another child saw it, so he also started folding his face cloth. […] They both told me [the shapes they folded] (laughing). (Cherry/FK/Banana Class/TI2/010319)

In this instance, the children's playful mathematical actions referred to folding the face cloths into shapes. Enabling by the interpersonal space involving peers and the teacher Cherry, they were motivated to engage in playful EMEs. They used mathematical terminology to describe the shapes they had created, which made the final product understandable to Cherry. Moreover, using the face cloths enabled the children to translate the abstract concept of shape into a physical form, and what they had created matched with what they said.

Self-directed play contributed to children's representation of mathematical concepts through their manipulation of physical objects. Children from all classes, regardless of age, were situated in physical environments that contained a multitude of physical objects. They sometimes employed mathematics concepts when manipulating physical objects (e.g., blocks, Lego, and other toys). For example, during block play, they would consider the lengths of the blocks, the positioning of where they placed the blocks, and the geometrical nature of the blocks (e.g., using cylinders for some purposes, and using rectangular prisms for others). The outcomes of children's Lego play indicated in Figure 10 and Figure 11 showed how two children from two K1 classes at FK applied their mathematical understanding in relation to patterns, symmetry, spatial awareness, number, shape, and classification of colour while playing with Lego.

Figure 10

A Lego Cave with Symmetrical Features

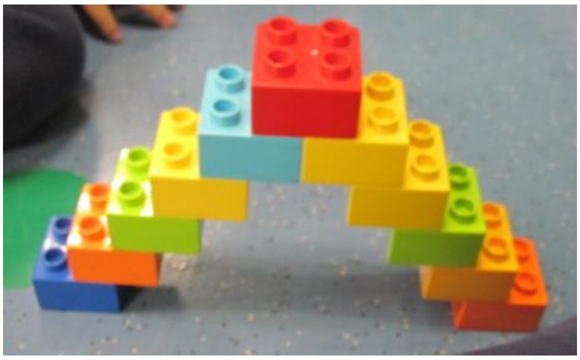
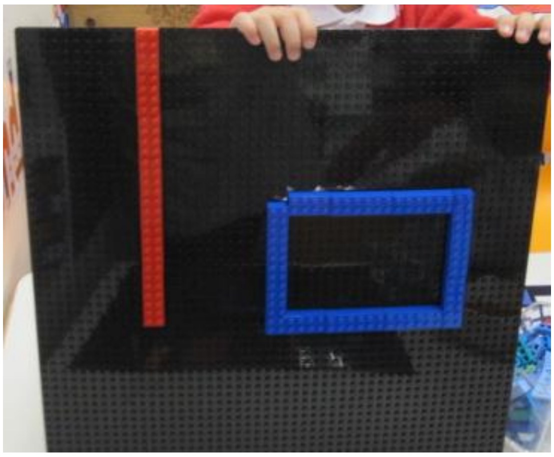


Figure 11

A Lego Arabic Numeral (left) and A Lego Rectangle (right)



These two examples indicated that children were agentive in using physical objects (e.g., Lego) within their play to represent objects with mathematical attributes and mathematical concepts directly. Regarding Figure 10, the child claimed that he had constructed a cave to allow trains to pass through, although he did not construct a Lego train. Although the child did not describe what the cave looked like in words, according to Figure 10, the cave he created showed his understanding of symmetry. Regarding Figure 11, the child reported that he had used some red and blue Lego pieces to construct the Arabic numeral 1 and rectangle respectively.

In addition to self-directed play, private speech also contributed to children's representations of mathematical concepts through physical objects that sometimes guided their play. Vygotsky (1978) indicated that private speech helps children connect thoughts with words.

Children in my study often engaged in EMEs during play. They would employ mathematics via private speech to guide their play and construct playful EMEs. For example, in the following excerpt, Adam (3 years, 10 months) used private speech to guide his construction play:

> Excerpt 10: In a group activity, the teacher asked one group of children to play with Lego. Adam used Lego to construct a train. After connecting six pieces of Lego together, he pointed to the carriages of the train one by one and counted, “One, two, three, four, five, six, six carriages…need one more”. Then he took a piece of Lego from the basket and connected it to the last carriage of the train and said, “Got one. Seven”. (FN/110319)

A Vygotskian perspective views private speech as a cognitive tool that allows the child to plan, guide, and regulate behaviour (Winsler et al., 1997). Adam’s counting process (i.e., “One, two, three, four, five, six, six carriages”) indicated that he wanted to know how many carriages he had constructed. The words “need one more” functioned as “self-guidance” (Krafft & Berk, 1998), which refers to describing what a child is doing as they are thinking out loud. Accordingly, he got another piece of Lego and constructed a train with seven carriages. The process demonstrated his understanding of number-based concepts (i.e., counting, one-to-one correspondence, and quantity). Using mathematical concepts and knowledge guided his play with construction and led to a playful EMEs.

Findings presented and discussed in this sub-section showed that children were capable of engaging with mathematical concepts in diverse and creative ways. In socially and culturally embedded practices, they constructed playful EMEs through playful use of oral language, music, body language, written language, and objects to express mathematics.

Children’s diverse ways of expressing mathematics sometimes made their playful EMEs visible to teachers. During the second teacher interviews, regarding the evidence that enabled teachers to identify children’s mathematical actions in EMEs, I asked each teacher the question “How did you know that the child/children was/were using mathematics?” They provided their answers:

Violet (FK/Apple Class/TI2/280219): During free choice time, they operated those Russian nesting dolls [prepared by teachers as toys for them].

Cherry (FK/Banana Class/TI2/010319): Once, I saw them fold face cloths after snack time. […] He told me that he created a square. I saw the shape of face cloths folded by him. […] They were happy in the process.

Olive (FK/Grape Class/TI2/160319): I often saw them playing puzzles and Ludo (a board game). […] [I knew they were using mathematics through their] movements and language.

Amber (FK/Lemon Class/TI2/050319): Movements, as he did not talk in this process [of playing wooden blocks].

Rose (OK/Shark Class/TI2/260219): During “free play” time. […] Mostly language, because she talked about the number of children in the small group. Sometimes I also observed children’s movement…but less than language.

Iris (OK/Dolphin Class/TI2/110319): Once in a group learning activity, we were learning the features of fresh strawberries. […] They talked about big and small, long and short [without my guidance].

Ivy (OK/Whale Class/TI2/050319): During physical education, […] they counted and got the corresponding quantity of the equipment.

Lily (OK/Whale Class/TI2/120319): During snack time, […] they talked to me and asked for more cookies.

Daisy (OK/Octopus Class/TI2/040319): Mostly during "free play" time, […] the mathematical concepts (terminology) talked about by them and their movement [made me know they were using mathematics].

It was evident that oral language and physical movement were the main evidence that the teachers drew on to identify the mathematics used by children during their EMEs. Moreover, almost all teachers observed or noticed diverse instances of children's playful EMEs during "free play" and free choice activities. Among all teachers, Cherry took the initiative in describing the children's emotional state by saying, "They looked very happy in the process" (the instance of playful EMEs described by her was presented on p. 133). This showed that she paid attention to children's emotions while they were engaging in playful EMEs. Additionally, Cherry reported that she did not engage with the children's playful EMEs but provided the children with brief praise and feedback. Thus, even though she was approving of the children's playful EMEs, she chose not to join in. More discussions in relation to teachers' perceptions and practices regarding EMEs will be addressed in Chapter 5. In most situations, children's playful EMEs were accompanied by expressions of emotions, particularly positive emotions (e.g., smiles and laughter), which I identified as a sign of playful EMEs and discuss below.

Expressions of Emotions as an Indicator of Playful EMEs

Although not all playful EMEs involved smiling and laughter, children's emotional expressions offered me insights into their playful EMEs. As mentioned earlier, I understood playful as being fond of play and the emotions that play and playfulness engender in this study, e.g., feeling amused and happy. Drawing on this understanding, and after discussing how children use different language and objects to play with mathematics, I further argue that children's emotional expressions indicate playful EMEs. Also, having fun is children's main purpose when being playful with mathematical concepts, even though there were times that children had playful EMEs without expressing positive emotions. This indicator aligns with Vygotskian ideas of emotions discussed in Chapter 2. It also aligns with Williamson et al. (2020), who have described children's social and emotional learning as "substantial and cognitively demanding" processes (p. 199). The findings presented and discussed in this subsection derived from instances of children's playful EMEs that I noticed during classroom observations.

Emotional expressions refer to observable changes in facial expressions, body movements, and vocalisations in response to a particular stimulus (Losonczy-Marshall, 2007). Five typical excerpts of children's emotional expressions while engaging in playful EMEs are shown below:

> Excerpt 11: While six boys (5-6 years old) were lining up outside the toilet, all of them did rote counting starting from one together. Their faces were full of smiles while counting. […] Their voices grew louder and louder. (FN/230119)

> Excerpt 12: Samuel (3 years, 4 months) and Betty (3 years, 3 months) looked at each other and giggled together, "Ha-ha…We three play together". (FN/140319)

> Excerpt 13: Zita (5 years, 2 months) said in an excited voice and with a big smile on her face, "one-sday, two-sday, three-sday, four-sday (raising fingers accordingly)". (FN/220119)

> Excerpt 14: Tracy (3 years, 8 months) said in a low voice and with a smile on her face, "One, two, three, four, five, six, seven, eight, nine, ten, ha1, ha2, ha3, ha4, 15, 16". (FN/140319)

> Excerpt 15: Jack (3 years, 2 months) grinned and said in an amusing and silly voice, “Four raisins, five raisins (raising five fingers)”. Frank (3 years, 1 month) looked at Jack, and they laughed out loud together. (FN/131218)

Children could feel emotions, particularly positive emotions, through constructing playful EMEs. As can be seen from these examples, there were two main types of children’s emotional behaviours related to their playful EMEs. (i) I often noticed and recorded smiling (including grinning) and laughing (included giggling) during children’s playful EMEs. (ii) children sometimes made amusing and silly sounds while talking about mathematics. This was usually accompanied by smiles and laughter. Therefore, I argue that children’s positive emotions, particularly smiling, laughing, and making amusing and silly sounds, could be the manifestation of joy and playfulness during playful EMEs. As discussed in Chapter 2, many previous studies have also indicated the connection between play or playfulness and positive emotional expressions.

Many teachers in the study, including Cherry (as mentioned earlier on p. 136), noticed children’s positive emotions, including joy, as they explored mathematics. For example, in the individual teacher interviews, Iris (OK/Dolphin Class/TI2/110319) described the children’s counting actions before going home as “seeking joy for themselves when they felt bored”; Ivy (OK/Whale Class/TI2/050319) and Lily (OK/Whale Class/TI2/120319) perceived the children’s actions of comparing the length of pencils while doing homework as “being fond of play and having fun”; Daisy (OK/Octopus Class/TI2/040319) indicated that the children in her class “loved comparing and competitions no matter when and where, and the only purpose was just for fun”; and Olive (FK/Grape Class/TI2/160319) perceived children’s counting during snack time as their play and described them as showing “great interest in playing with mathematics”. However, the teachers did not always value the positive emotions children demonstrated while exploring mathematics in a playful manner. Teachers sometimes put a stop to children’s playful EMEs, as will be discussed in Chapter 5.

Humour and playfulness were an impetus that motivated children's playful EMEs associated with positive emotions. Many children from different classes in my study often counted numbers using an amusing and silly voice. According to Excerpt 15 (p. 139), with a sense of humour and playfulness, Jack demonstrated to Frank how to express mathematical language (i.e., "Four raisins, five raisins") in an amusing and silly voice. McGhee (2002) has explained that human beings can develop a sense of humour, which is a form of play and based on playfulness, from playing with language and concepts. Also, humour enables human beings to "understand, produce, and appreciate amusement" for joyfulness and laughter (Pande, 2014, p. 386) and, usually, is associated with a positive emotional state and "overall psychological well-being" (Pande, 2014, p. 386). The instances of EMEs listed on pp. 138-139 indicated that children had fun while they played with mathematical language (e.g., in Jack's case, he and Frank laughed out loud together after making amusing and silly sounds). Therefore, I argue that humour and playfulness are part of children's playful EMEs, with which children can see mathematics as enjoyable and fun.

I identified three factors that contributed to children's emotional expressions associated with playful EMEs. First, private speech contributed children's expressions of emotions while engaging in playful EMEs. Although children sometimes showed neutral feelings, they often smiled or laughed without making obvious sounds during or after engaging in private speech related to mathematics. Previous research has explained that "People often smile when they are […] having private thoughts" (Gholami et al., 2016, p. 185). Private speech is considered to be an expression of personal thoughts. Children's smiling when they are alone suggests that they may enjoy the process of exploring and playing with mathematics by themselves.

Second, peer interactions provided an arena for children to feel emotions while engaging in playful EMEs. According to Excerpt 15 (p. 139), peer interaction created an opportunity for Jack and Frank to be playful with mathematics together. They expressed numbers in an amusing and playful way. Their laughter indicated that they enjoyed playing with numbers together. Human beings are social in nature (Schmitt, 2018). Data analysis also suggested that when children engaged in playful

EMEs with peers, they showed more diverse emotional expressions than when they were alone. I observed that when children played with peers they often smiled, grinned, laughed loudly, giggled, talked in silly/dramatic/exaggerated voices, and showed expressions of pride (i.e., a small smile and head titled slightly back). All these emotional expressions are an expression of joy and playfulness. Furthermore, children sometimes also felt upset if they showed disagreement with peers when engaging with playful EMEs together. Thus, it can be said that children felt diverse emotional experiences in the process of engaging in playful EMEs with peers.

Third, imagination, creativity, and opportunities for play had a positive influence on children's play and, meanwhile, their emotions during playful EMEs. Imagination is the broadening of a child's experience (Fleer, 2011). Children in this study often involved imagination in their playful EMEs and generated a sense of joy from their imaginative play. The following excerpt shows how John (3 years, 1 month) practised and linked his imagination and creativity with mathematics through Lego play. Vygotsky (2004) theorised the relationship between creativity and imagination. He denoted that imagination serves as an imperative impetus of all human creative activity. Play provides an arena for creative activities and processes (Saracho, 2002). The excerpt of John's Lego play is presented as follows.

> Excerpt 16: John vertically stacked around twenty pieces of Lego of the same size and shape but different colours to construct a Lego building. In the constructing process, he looked calm and sometimes showed a slight smile on his face. Then he came to me and said with a smile, "YeYe, he is 18 years old and very tall". After confirming I understood that what he was talking about was his Lego building, he said, "When he is 18, he will collapse soon". Later, he also happily showed his Lego building to his teacher and employed the same words to describe it. (FN/181018)

I viewed John's Lego construction as a process of practising the concepts of size and shape, as he intentionally and repeatedly selected around twenty pieces of Lego of the same size and shape from a pile of different shaped Lego. Additionally, his calmness, smiles, and the state of being happy were evidence of his emotional experience. He had a pleasant and enjoyable Lego play time.

John's pleasant and enjoyable Lego construction process indicated his creativity in using mathematical knowledge. Robson (2014) has defined creativity as the production of original content and evidence of diverse forms of thinking. Both are often presented in children's play. As discussed in Chapter 2, children could "draw on their own inner resources" and creatively play with mathematics (Shen & Edwards, 2017, p. 327). In John's case, he selected Lego of the same size and shape to design a Lego building that was different from other children's Lego buildings in terms of appearance. Thus, I argue that playful EMEs could be evidence of children's creative nature.

While talking about John's Lego building, "tall" was an important attribute of his Lego building. In his conversations with adults (i.e., the teacher and me), he labelled the Lego building as an 18-year-old and mentioned the concept of "tall" at the same time. He might believe that age and height were related. In Vygotsky's (2004) view, the principle that formulates the operation of the imagination is contingent on the richness and wideness of an individual's experience because "imagination always builds using materials supplied by reality" (p. 14). John might have imagined the Lego building as a human being and believed that in real life a person would be tall at 18 years of age. Møller (2015) has indicated that play offers an opportunity for a child to become a creator of rules. In this sense, John seemed to create his own rule of using mathematical concepts in play (e.g., using the concept of age to represent the concept of height).

The connection John made between age and height might have a cultural basis. In HK and many other cultures, turning 18 is a sign of maturity and the official start of adulthood. Thus, it was possible that John had learnt this concept from his family members or other out-of-kindergarten contexts. Furthermore, according to my observations, the teachers in this study often linked age with children's height. For instance, one day I was observing children's snack time in the Apple class. Violet said to a boy, "You are really a three-year-old now. You can […] eat more [than before]. If you keep eating more, you will be as tall as your father soon" (FN/070319). In the process, Violet linked the child's age and height with the amount of food he could eat, explicitly making a connection between these factors. In my experience, I know that many Chinese parents and teachers place emphasis on children's eating. They value eating habits such as taking big bites of food, positively thinking that children who are good eaters will grow to be tall. Height is regarded as a sign of being healthy and good looking. Thus, it is understandable that John was interested in the concepts of age and height while playing Lego.

While discussing children's emotions in relation to mathematics, most previous studies simply noted that children took pleasure in engaging with informal mathematics without providing further elaboration (e.g., Ginsburg, 2006). Additionally, there is a large body of literature focused on older children's and adults' apprehensiveness of mathematics or "math anxiety" (e.g., Hart & Ganley, 2019; Sorvo et al., 2017). However, I did not observe any evidence of children's anxiety in relation to mathematics while constructing EMEs, particularly playful EMEs. Therefore, my study adds value to the body of research around young children's emotional states related to mathematics by discussing children's expressions of emotions during EMEs.

In summary, I focused on presenting and discussing findings about the two indicators of children's playful EMEs in this section. These include the use of different languages and resources to express mathematics and the expression of emotions while engaging mathematics. Next, I present and discuss findings in terms of the informal aspect of children's EMEs.

Informal EMEs

EMEs constructed by children in kindergartens are all informal. According to the discussion regarding informal learning addressed in Chapter 2 and the operational definition of EMEs developed for this study, I adopted the features of informal learning proposed by Rogoff et al. (2016) (Chapter 2, p. 38) to define the features of children's informal EMEs. Rogoff et al. (2016) have highlighted that informal learning is "interactive" and that it involves "meaningful activity" and "conversational talk" (p. 359). Aa a result of analysing the data, I noticed that children often talked about what they had encountered or were experiencing in everyday life with peers and adults as meaningful informal learning experiences. I identified two indicators for informal EMEs, including informal mathematical talk and out-of-kindergarten mathematical experiences. Through constructing informal EMEs, children could engage in interpersonal interactions, develop self-esteem, and build on out-of-kindergarten mathematical experiences to extend EMEs or facilitate informal learning in kindergartens.

Informal Mathematical Talk is an Indicator of Informal EMEs

Children often involved mathematical information in their language interactions with peers and adults. In doing so, they constructed informal EMEs in kindergartens based on their informal talk. Informal talk refers to “the type of language used in casual everyday conversations, a way of speaking which is supposed to convey familiarity, closeness and equality” (Decock et al., 2021, p. 5). I termed this kind of talk as informal mathematical talk and identified it as an indicator of children’s informal EMEs. Four typical excerpts of children’s informal mathematical talk are shown below:

> Excerpt 17: Shortly after physical education, children were lining up to get ready to go back to the classroom. Mason (4 years, 10 months) looked at me with an excited expression on his face, he pointed at Owen (5 years) and himself and said, "You know, we got 44 and 48”. I asked him: "What are 44 and 48?" Owen happily explained, "We threw and caught balls [during physical education], we got 44 points and 48 points”. Mason further noted with a proud look, "I got 48 points and he got 44 points. I got more”. Owen argued, "[It’s because] he caught more”. (FN/101018)

> Excerpt 18: Children were lining up to get ready to go to the toilet. Because It was nearly Christmas time, the kindergarten would soon have a feast for the children. Ava (5 years) said to me with a happy look, "We will eat turkey tomorrow!" I responded, "It sounds great that you will eating turkey”. At this time, another girl Mia (5 years, 2 months) said, "Yes, Tomorrow is Thursday”. Ava looked confused and said, "No, today is Tuesday. Two, three. Tomorrow is Wednesday”. When she

mentioned two and three, she used two and three fingers respectively to count at the same time. (FN/111218)

Excerpt 19: Children were lining up to get ready to go back to the classroom. Levi (5 years, 2 months) and Isaac (5 years, 1 month) stood face to face. Levi used his hand to measure his height and Isaac's height. He said, "I'm taller than you, how tall are you?" Isaac replied, "64 [cm]". Then Levi said, "You are 64 [cm]. I am also 64 [cm]. We are the same height". At this time, the teacher heard them and said, "Who is playing?" Then Levi and Isaac stopped their conversation. (FN/181218)

Excerpt 20: The teacher distributed the children's homework back to them. After receiving it, Andrew (5 years) opened his homework and said excitedly, "I got five stamps". Eli (5 years, 2 months) said with a smile, "I have six. Six plus six equals twelve". Nora (4 years, 11 months) said, "One hundred plus 200 equals 300". At the same time, Hunter (5 years, 2 months) showed three fingers on each hand and said in a low voice, "Three plus three equals six". Then Nora said, "Two plus five equals seven". When the teacher started talking about homework the four children stopped. (FN/181218)

As shown in these examples, the children embedded mathematical information or utterances in their informal talk with others, and the topics were often related to their everyday activities. For instance, Mason and Owen talked about the points they scored in during physical education; Ava and Mia talked about the day they would eat turkey; Levi and Isaac compared their heights; and Andrew, Eli, Nora, and Hunter talked about the stamps on the homework they got from the teacher. I therefore argue that what children experience and encounter every day may become resources for them to construct informal EMEs.

Children's informal mathematical talk sometimes was intertwined with their mathematical discoveries. Earlier, in the section focused on exploratory EMEs, I argued that through making mathematical inquiries, children could find out mathematical information. This process was evident in children's informal mathematical talk with peers. For instance, Levi was curious about Isaac's height, so he measured Isaac to see how tall he was; and Andrew was interested in the number of stamps he got on his homework, so he counted to discover the number of stamps. These behaviours indicated that children were constructing exploratory EMEs, which became the basis of their informal EMEs. For example, after measuring Isaac's height, Levi discovered that they were actually the same height; his discovery triggered mathematical talk among the four children. The children actively involved the mathematical information they had discovered in their informal talk and in doing so, constructed informal EMEs. This finding suggested a connection between children's exploratory EMEs and their informal EMEs.

Mathematics was sometimes a topic of children's informal mathematical talk with others. Children in this study often took the initiative in talking about what they knew about mathematics in front of their peers. For example, during physical education, Dylan (5 years, 10 months) talked to Peter (6 years) in a proud tone and facial expression, "Two, four, six, eight, ten. There are almost ten of them. I know to count [the number of children in the other team] in even numbers. It's very easy" (FN/120319). Furthermore, they often talked about their mathematical abilities with adults. For example, one day, Kim (5 years, 6 months) initiated a conversation with me. He smiled and told me proudly, "I know eight plus eight equals 18" (FN/181218). Although Kim did not correctly work out the outcome of "eight plus eight", his words "I know eight plus eight equals 18" and the smile on his face indicated that he was willing and confident to show me what he knew about addition.

Through initiating informal mathematical talk, children could demonstrate their mathematical competencies and gain self-esteem. Kim's proud tone and smiling face indicated that he felt proud of knowing how to add. According to Hosogi et al. (2012), self-esteem is a "feeling of self-appreciation" (p. 1), which is an essential emotion for people to adapt to society and live their lives. Thus, I interpreted this sense of feeling proud as self-esteem.

Children's informal mathematical talk also related their parents' aspirations (Briley et al., 2014). After telling me what he knew about addition, Kim also added with a smile, "If I don't know, my dad will be angry with me". As discussed in Chapter 2, many parents in HK have high expectations of their children's academic achievements, including mathematics (Chang, 2003). However, Costigan et al. (2010) have asserted that high expectations might do more harm than good for some Asian children who worry about falling short of parental expectations. On the other hand, Kim smiled when he mentioned his father which indicated that he thought his father would be happy with his knowledge of addition. During the period of my observation, Kim often initiated conversations about the topic of addition with his peers. His continuous interest in addition indicated that he was confident and happy to share his mathematical ability to others.

In addition to obtaining self-esteem, engaging in informal mathematical talk with others, particularly teachers, sometimes helped children to develop a sense of fairness. I observed that children often used speech with mathematical information to express their needs and negotiate with teachers. Many teachers also noted this purpose of using mathematics by children. In the interview with Lily, she described her experience in terms of children engaged in mathematics talk to express their needs in her class. She stated:

> Excerpt 21: During snack time, children usually looked at their plates and other children's plates, and then they would say, "Why has he got more cookies than me?" Or "Why has he got four cookies and I only have three?" …They liked to compare. Because his cookies were fewer than others, he asked me to give him one more. I think…they did this to express their needs and fight for their rights (laughing).
>
> (Lily/OK/Whale Class/TI2/120319)

Children's use of mathematical information to express their needs and negotiate with teachers indicated their desire for obtaining fairness (i.e., an equal quantity of cookies). Four decades ago, Deforges and Deforges (1980) found that children as young as three and a half show a number-based concept of fairness in sharing tasks. In the above instance, the child compared the number of cookies he had with his peers. When he discovered that he had one less than the others, he requested that the teacher give him another cookie. The process demonstrated the child's understanding of the concepts of comparing, quantity, and fairness. Furthermore, he was able to use informal mathematics talk to negotiate with adults.

During language negotiation and communication, children's mathematical talk was different from the language they used during playful EMEs. Communication is a reciprocal process (Velentzas & Broni, 2014). It is important to express oneself so that others understand what you mean. In the example described by Lily, the children's language used number and other mathematical concepts designed to describe a mathematics-related problem that they needed her to understand and address.

Children's informal mathematical talk was beneficial in their interpersonal interactions with peers and adults/teachers in kindergartens. I observed that informal mathematical talk often occurred in a dyad or among a small group of children. Sometimes this talk involved adults/teachers, with the talk being initiated by the children or adults/teachers. This informal mathematical talk was interactive and included children talking to each other, children talking to adults/teachers, and adults/teachers talking to children. For example, the informal mathematical talk between Levi and Isaac was reciprocal and showed a back-and-forth flow (see Excerpt 19). As discussed in Chapter 2, social interactions with teachers and peers play an important role in facilitating children's mathematics learning. Thus, children's informal mathematical talk might be evidence of how mathematics learning is facilitated through social interaction. Through social interaction with peers and teachers/adults, children could share mathematics-related information.

I observed that children's informal mathematical talk often took place during snack time. As discussed in Chapter 2, previous studies have found that snack time provides rich opportunities for social referencing, attention, and the use of rich mathematical language (Chen et al., 2017). Snack time also promotes the building of shared meaning through communication (Johnston & Degotardi, 2020). However, there were times that these kinds of conversations were not valued or allowed by teachers in the kindergartens. This will be explained in Chapter 5.

In addition to snack time, children's informal mathematical talk also often took place during transitions. Two possible reasons are addressed below:

First, transitions enabled children to explore their immediate context and make mathematical inquiries, which often led to informal mathematical talk with others. At the two kindergartens, transition activities included tidying up, lining up, walking from one venue to another, and preparing to go home. Although none of the class timetables mentioned transitions, they played an important role in connecting all routine activities in the kindergartens. Transitions signalled the immediate change of surroundings (e.g., activities and venue). As noted in the section focused on exploratory EMEs, children were keen to observe by watching what was happening in their immediate context. The change of surroundings experienced by children during transitions enabled them to observe their immediate context and construct informal EMEs.

Second, during transitions, children had opportunities to engage in child-initiated social interactions. As described in Chapter 3, at both kindergartens, all routine activities and play were structured to promote a serious learning environment. Due to the large number of group activities, waiting during transitions provided opportunities for children and teachers to have a break from formal learning. During transitions, many teachers in my study allowed children to initiate informal conversations with others. More details about the serious nature of the learning environments will be discussed in Chapter 5.

Findings presented and discussed in this sub-section showed that children often engaged in informal mathematical talk with peers and adults, thereby constructing informal EMEs in kindergartens. The second indicator of informal EMEs is discussed below.

Out-of-kindergarten Mathematical Experiences as an Indicator of Informal EMEs

As discussed in Chapter 2, Rogoff et al. (2016) have indicated that informal learning takes place in diverse settings, including kindergartens and out-of-kindergarten settings. Many studies, such as Resnick (1989, 1991), have viewed out-of-school settings (e.g., home) as an arena for building up children's informal mathematical knowledge. In my study, I observed that children often brought their out-of-kindergarten mathematical experiences to kindergarten. I therefore identified this kind of children's out-of-kindergarten mathematical experiences as another salient indicator of informal EMEs.

As mentioned in the previous subsection, children were keen to engage in informal mathematical talk with peers and adults in kindergartens. They sometimes mentioned their out-of-kindergarten mathematical experiences in the informal mathematical talk. For example, one day, Kale (5 years, 11 months), Tony (5 years, 5 months), and Lara (5 years, 11 months) were eating chunky cheese bread during snack time. They sat at the same table. While eating, Kale, who thought the cheese was egg, announced his discovery, "I found a piece of egg in my bread, the yellow one, see". Kale's comment promoted Tony to talk about the sandwiches his mother makes. He said, "You know, my mom knows how to make sandwiches. Different shaped sandwiches". Lara asked, "What kinds of shapes?" Tony answered, "sometimes rectangles, and sometimes squares, like this bread (he showed his bread to Lara and Kale). She also knows how to make triangle ones". Kale grinned and said, "My mom also knows [how to make sandwiches]. She can make the rocket shape and schoolbag shape". Tony laughed and replied in a funny voice, "My mom also knows. Shoe shapes, homework shapes, pencil shapes". Then all of them laughed (FN/041218).

Social interaction, particularly peer interaction, provided opportunities for children to share their out-of-kindergarten mathematical experiences. This conversation was initiated by Kale, as he just shared his discovery about the bread with Tony and Lara. Tony's response turned the conversation into a playful conversation about the shapes of sandwiches his mom made. Through this kind of reciprocal peer interaction, the children were able to bring mathematical information related to their out-of-kindergarten life to kindergartens.

Children's out-of-kindergarten mathematical experiences were also beneficial for their peer interaction. The informal talk of Kale, Tony, and Lara was woven with mathematical language. Each child played a role in this mathematical interaction. Kale initiated the conversation by noting the number of chunky cheeses, thereby contributing to the theme of food; Tony developed Kale's theme and used it to explore the mathematical concept of shape; and Lara's involvement encouraged them to further explore shape. The involvement of mathematical ideas extended their language interaction.

In addition to informal mathematical talk, children sometimes used mathematical skills appropriated from home contexts in their kindergarten activities. For example, during free choice activity, Benjamin (3 years, 2 months) chose to play in the stamping area. He took a blank paper and a stamp that had been prepared by the teacher. After opening the cap of the stamp, he stamped on the paper. He used two hands to press the stamp and said, "One, two, three, four, five, six, seven, eight, nine, ten" at a fast speed. After counting to 10, he lifted the stamp off the paper. Then he showed me his work with an excited look, "YeYe, look". I was curious about his counting and asked, "That's a nice one, but why did you count 1-10?" Benjamin explained, "My older sister also counts at home". I asked again, "When stamping?" He explained that he sometimes played stamping with his sister at home (FN/291118).

"Funds of knowledge" (Moll et al., 2005), as mentioned in Chapter 2, contributed to Benjamin's counting process and, therefore, can be said to be present in children's informal EMEs. Funds of knowledge may include information, ways of thinking and learning, approaches to learning, and practical skills (González et al., 1993). In Benjamin's case, he had knowledge of his sister counting from 1-10 at home and brought this mathematical experience to kindergarten. Counting while stamping became a resource for his informal EMEs in the kindergarten. However, as this study did not involve parent interviews or observations at children's homes, it was unknown what experience of counting and stamping Benjamin engaged in at home.

In addition to supporting activities prescribed by teachers, children sometimes used mathematical skills appropriated from home to construct other types of EMEs, such as playful EMEs. As discussed in the section on playful EMEs, I identified written language as an important way that children expressed mathematics and constructed playful EMEs. In playful drawing activities during free choice time, some children from K1 classes showed me the Arabic numbers they had drawn or written. They reported that their parents or grandparents had taught them to write numbers at home. In this case, the numbers that they learned from home became a resource for their playful EMEs in kindergartens, and their playful EMEs were also informal EMEs. This echoes what I noted in the introduction of this chapter (p. 113), that the four types of EMEs identified are not mutually exclusive categories, and any given instance of EMEs may show more than one type.

Children's formal learning experiences in tutoring centres in the out-of-kindergarten context also contributed to their informal EMEs in kindergartens. As discussed in Chapter 2, in HK, tutoring services are available for young children (Bray, 2015; Eng, 2019). The formal mathematics learning and teaching in tutoring centres play a role in preparing children for interviews when applying to primary school and for formal schooling. Children in my study, particularly in the K3 classes, often talked about the tutoring centres they attended.

According to the children in my study, their parents sent them to different tutoring centres to learn academic subjects and other skills (e.g., mathematics, English, Mandarin, musical instruments, and sports). In the following conversation, Winston (6 years, 1 month) told me what he had learned about mathematics at the tutoring centre.

> Excerpt 22: Before the afternoon assembly started, Winston looked at me and said (smiling), "YeYe, you know, I know even numbers, 2, 4, 6, 8, 10, 12, 14, 16, 18." I said, "Right, these are some even numbers." Winston smiled and said, "I went to the tutoring centre yesterday, and the teacher taught me". (FN/181218)

As indicated in this conversation, Winston correctly recited even numbers up to 18, that he had learnt from his teacher at the tutoring centre. Thus, his learning experience at the tutoring centre provided resources (i.e., knowledge of even numbers) for his mathematics-related conversation with me at kindergarten. Of note, was that Winston had learnt to recite even numbers, rather than apply this knowledge in practical ways (e.g. to solve an inquiry). However, he felt confident about and proud of himself that he knew some even numbers and enjoyed showing this ability to me (Hasan 2012).

In summary, this section focused on presenting and discussing findings of the two indicators of children's informal EMEs, informal mathematical talk and out-of-kindergarten mathematical experiences. Next, findings in terms of the children's spontaneous EMEs are presented.

Spontaneous EMEs

I identified spontaneity as the fourth feature of the EMEs constructed by children in kindergartens. As discussed in Chapter 2, although previous researchers showed diverse understandings of the concept of spontaneous to some extent, the findings from my study indicated connection between spontaneity and children's agency in mathematics learning. In line with Rogoff et al. (2016), children in this study showed agency by creating a parallel script of constructing EMEs to prescribed activities by teachers. Thus, I argue that EMEs constructed through children's self-directed play and activity (discussed in the section on exploratory EMEs) can be viewed as spontaneous.

In this section I specifically focus on EMEs constructed during activities and play prescribed by teachers, but where children's purpose and intentions differ from those prescribed by teachers (e.g., the excerpt of EMEs described in the abstract). According to Jones (2015), children are agentive in constructing cultures that are different from those of teachers. Thus, I viewed these kinds of EMEs as spontaneous EMEs. I identified one indicator of spontaneous EMEs, which linked children's mathematical spontaneity and mathematics-related underground culture. This indicator aligned with Rogoff et al.'s (2016) concept of underground informal learning. Details are addressed below.

Mathematical Spontaneity in Constructing Underground Culture as an Indicator of Informal EMEs

Data analysis showed that when participating in activities and play prescribed in kindergartens, children often showed spontaneity in initiating mathematics-related actions with independent intentions and purposes. I termed this kind of spontaneity as mathematical spontaneity and identified it as an indicator of children's spontaneous EMEs. Three typical examples of children's mathematical spontaneity are listed below:

> Excerpt 23: During group learning activity, Tony (5 years, 11 months) was asked to do homework (teachers in this study required children to complete some of their homework at kindergartens. More details will be explained about this in the next chapter). Just as he was taking the pencils from the pencil basket, he found two pencils of different lengths. He took out the two pencils and held them in his hand. Then he asked Kitty (5 years, 8 months), sitting opposite him, which one was longer. Then they started to guess and compare the lengths of different pencils. (FN/141218)

> Excerpt 24: During an indoor group learning activity, the teachers asked some children to practise stacking lids of different sizes. Jerry held a disposable coffee lid and said to me, "YeYe, coffee, 51 dollars. No, 59 dollars". He intended to sell "coffee" to me for 59 dollars. The teacher asked him to focus on his task. Later, when playing in the outdoor sandbox, he held a bottle of sand and said to me, "YeYe, 51 dollars, coffee", intending to sell me coffee again. (FN/240119)

> Excerpt 25: During snack time, Julia (3 years, 10 months) and Tiffany (3 years, 11 months) were eating rectangle shaped cookies. After one bite, they found that both of their cookies turned into a similar shape. They talked about the shape and put their cookies together to compare. Julia said in a delighted voice, "Same." At this

> time, the teacher saw them. Due to the outbreak of influenza in HK at the time, she reminded the children not to touch each other's food to avoid spreading germs. However, they did not listen and continued their exploration of shapes. This time, they compared cookies without talking and left a gap between their cookies so that they did not touch. However, the teacher stopped them again. After a while, they resumed comparing their cookies just by holding them in their hands, without saying a word. (FN/210119)

In HK, kindergarten activities have been described as having two dimensions: academic and non-academic (Opper, 1992). Academic activities in HK kindergartens are designed to prepare children for primary school and are, therefore, focused on the teaching and learning of academic skills (Wu, 2014). This is essential in HK, where teaching strategies tend to be didactic and teacher-centred (Li & Lim, 2009). Non-academic activities, conversely, are more focused on enhancing other skills such as independence, co-operation, and self-regulation (Opper, 1992). As can be seen from the above excerpts, children's spontaneous EMEs were embedded in academic activities prescribed by teachers (e.g., doing homework and practising the skill of stacking lids with different sizes) and non-academic activities, such as snack time, in which teachers demonstrated a serious demeanour.

In the context of kindergarten, children's spontaneous EMEs showed "underground" (Rogoff et al., 2016) features. Inspired by Rogoff et al.'s (2016) concept of underground informal learning, which was discussed in Chapter 2, I argue that children in my study created a parallel script of carrying out informal and playful EMEs to the teachers' formal script in kindergartens. In most situations, these EMEs were not allowed or appreciated by their teachers. However, children were keen to create opportunities for EMEs. In this case, children sometimes played the role of the guard when their peers were carrying out EMEs (as will be discussed again in Chapter 5). Underground EMEs constructed by the children were particularly evident during activities designed for formal instruction.

As can be seen from Excerpt 23, the children explored the different length of pencils in a playful way. Kitty responded to Tony's playful intention by joining in. It was important to note that Fiona played the role of the guard and was on alert for teachers approaching. Rogoff et al. (2016) have explained that "in the 'underground' informal learning in schools, students' creative ways of addressing constraints are especially appreciated by their classmates, if not by their teachers" (p. 389). Although the teacher did not approach the children during this episode, the children's reactions (i.e., putting down their pencils and focusing on their homework) indicated that they knew what they did was not allowed or valued by the teacher (as will be discussed in Chapter 5). Therefore, before the teacher noticed them, they took the initiative in stopping and cleaning up the traces of their playful mathematical actions, making the scene look like nothing happened.

The intention of obtaining joy and reducing boredom might be a factor that motivated children to construct spontaneous EMEs. Boredom is described as "an unpleasant deactivating emotion" (Putwain et al., 2018, p. 74). Due to group routines and associated expectations of conformity or quietness; when things in kindergartens are not fun and playful, children can feel bored. I wrote in my reflective journals, "I have a feeling that children play with mathematics is when they feel bored with kindergarten life" (RJ/111219). According to Excerpt 4, the boys overcame boredom and had fun by engaging in playful rote counting during toilet time. In most situations at the two kindergartens, teachers expected children to stand still and keep quiet while lining up. Westgate and Wilson (2018) have indicated that an activity or situation perceived as meaningless leads to boredom. Being bored is a signal to change behaviour and affects behaviour (Martarelli & Wolff, 2020). Therefore, meaningless sheer queuing and waiting might lead to children's boredom, which inspired them to start their playful rote counting.

In addition to the waiting time involved during routines, the content of teacher-directed activities might also cause children's boredom and therefore inspire them to construct spontaneous EMEs. I found that in some group/whole-class learning activities, teachers taught children mathematical concepts and knowledge that they already knew. For instance, once during a group learning activity in Shark Class, the teacher revisited the concept of number five with all the children by asking them to count one by one till five. She intended to teach them number six later. Without knowing the learning objectives prescribed by the teacher, Simon (4 years, 1 month) showed interest in the textbook the teacher was holding, as there were six bears on the page. While others were counting, he raised his finger and counted the bears one by one. Later, he smiled and said to himself, "Six bears" (FN/110319).

Lack of interest and challenge might underline children's boredom. Simon's spontaneous EME indicated that he already knew the concept of six. Thus, the teacher's learning objectives may not have been of interests to him. According to Van Tilburg and Igou (2012), disengagement and lack of interest are features of boredom. Moreover, in Excerpt 24, I observed that Jerry started playing with me after practicing the skill of stacking lids many times. He might have found mathematics-related task prescribed by the teacher not challenging and interesting. Simon's and Jerry's boredom further inspired them to construct spontaneous EMEs.

Teachers played an essential role in driving EMEs underground. As noted in Excerpt 25, Julia and Tiffany took their spontaneous exploration of cookie shapes underground because the teacher did not value or allow this play due to health and hygiene reasons. In this case, they gradually turned their spontaneous mathematical action underground (i.e., from putting cookies together and talking about the discovery to holding cookies in their hands without saying a word) (see Figure 12).

Figure 12

Two Children Compared the Shape of their Cookies

In summary, the findings presented and discussed in this section showed that when engaging in activities and play prescribed by teachers, children often constructed spontaneous EMEs by spontaneously initiating mathematics-related actions with independent intentions and purpose. The next section presents and discusses findings regarding the content of children’s EMEs in kindergartens.

The Range of Mathematical Concepts and Knowledge Employed and Explored in Children’s EMEs

The previous four sections have discussed four salient types of children’s EMEs and relevant indicators or features in kindergartens. I bring together now the range of mathematical concepts and knowledge explored and employed by the children in this study (see Table 9). The mathematics terminologies noted in Table 9 are based on Bishop’s (1988) mathematical activity framework. This framework provides a broad mathematical lens for understanding children’s exploratory, playful, and spontaneous mathematical activities. The data were from teacher interviews, classroom observations, and teachers’ documented observations of children’s mathematics learning.

Table 9

A Summary of Mathematics Concepts and Knowledge Applied by the Children

<table>
<tr><th colspan="3">Mathematical Concepts</th><th>Instances of EMEs</th></tr>
<tr><td rowspan="4">Number: Identifying and employing verbal (i.e., number words), symbolic (i.e., numerals) representations of elements, or body gestures (i.e., number-finger gestures) from the set of natural numbers (i.e., zero, one, two, three …) to signify nominal, ordinal, and/or numeric meanings</td><td colspan="2">Nominal meanings: Number to label</td><td>Sam thought for a few seconds and said, “The second floor, because of one, two”. (FN/280219)</td></tr>
<tr><td colspan="2">Ordinal meanings: Number to indicate</td><td>At this time, Ada said, “Yes, three. We three came first. I am the first, she (Betty) is the second, and he (Samuel) is the third”. (FN/140319)</td></tr>
<tr><td rowspan="2">Numeric meanings</td><td>Counting: Expressing number words orally in a count sequence without reference to specific objects</td><td>During scheduled toilet time (gender segregated), the boys were asked to wait in line at the door after going to the toilet. At this point, a child began to count from one, then several children joined him. Later, almost all children counted together. (FN/230119)</td></tr>
<tr><td>Demonstrating concepts of enumeration</td><td>Daisy counted the children’s photos displayed by the teacher on the wall one by one in a low voice. After counting, she looked at me and said in an excited voice, “YeYe, do you know if we have 23</td></tr>
</table>

			children in the class?" (FN/260219)
		Employing number words or numerals to describe attributes of physical quantities or entities	Raymond raised five fingers and said, "The teacher said five raisins". (FN/131218)
		Numeric part-total relationships	After connecting six pieces of Lego together, Adam pointed at the carriage of the train one by one and counted, "One, two, three, four, five, six, six carriages…need one more". Then he took a piece of Lego from the basket and connected it to the last carriage of the train and said, "Got one. Seven". (FN/110319)
		Number operation (e.g., adding and subtracting)	While waiting to go home, two girls decided to play the *rock, scissors, paper* game. Sophie said, "Let me help you track the score". During the game, Sophie only used finger-number gestures to track the score without saying any words. Her finger-number gestures changed when either of the two players won another round. At the end of the game, she pointed at the girl who got three points said, "Three to two, you win". (FN/051218)
		Engaging numbers as the lyrics in singing	While the teacher was talking, Ross sang (smiling) in the melody of the song *Twinkle Twinkle Little Star*, "One two, one two, one two five, one two five…". (FN/140319)
		Written numerals	In Tim's drawing, he drew a planet, a sun, a moon, four stars, a rocket and wrote down the numbers 1, 2, 3 next to the rocket. (FN/241018)
Size and quantity			Max said slowly and with a smile, "The temperature sheet is so big". (FN/031218)
Pattern, shape, design, and geometric thinking			John vertically stacked around twenty pieces of Lego of the same size and shape but different colours to construct a Lego building. (FN/181018)

Spatial sense	At this point, Ada said, "The driver should sit in the front". Betty moved forward and said, "I'm sitting in the front". Samuel also moved forward but sat next to Betty. (FN/140319)
Classification, sorting, and matching	Kaden sorted the Chinese chess into two categories according to the colour of the chess. (FN/270219)
Comparing and measuring	Tony found two pencils of different lengths. He took out the two pencils and held them in his hand. Then he asked Kitty, sitting opposite him, which one was longer. (FN/141218)
Money	Karen said, "This green one is for the daughter…But you have to pay". Then she asked Zoe and Sophie for five dollars. (FN/090119)
Time and age	John, "YeYe, he is 18 years old and very tall." (FN/181018)
Logical thinking and explaining	Karen said, "Excuse me, as you bought so many things, this ring is a reward for you two". Then she took a jade ring from the shelf and showed it to Zoe and Sophie. (FN/090119)
Playing and creating games	While waiting to go home, two girls decided to play the *rock, scissors, paper* game. (FN/051218)

Table 9 indicated that the mathematics concepts and knowledge explored and applied by the children in their EMEs were broad, ranging from number to playing and creating games.

Additionally, I analysed the mathematics learning objectives required by the CDC (2017) and the two kindergartens in relation to formal mathematics learning and teaching. Table 10 is a summary of the pre-determined mathematical concepts and knowledge that teachers were responsible for teaching.

Table 10

A Summary of Pre-determined Mathematics Concepts and Knowledge

Mathematical Concepts K1	Mathematical Concepts K3
Number (i.e., recognising numbers from 1-10, understanding the quantity of numbers from one to ten and rote counting from one to ten)	Number (i.e., knowing and learning Chinese numbers from 1-20, counting numbers backwards from 20-1, classification statistics by chart, composition and decomposition within the number 10, comparison of values up to 50, the order of 51 to 100, even numbers from 1 to 100, the concept of "one pair", recognising symbols “+” and “=”, the concepts of singular and even numbers, adjacent numbers within 50 and subtraction within 5)
Size and quantity (i.e., knowing the meaning of many, big/small, tall/short, fat/slim, long/short, more/less, fast/slow, more/less and far/near and how to compare these relative concepts)	Size and quantity (i.e., unit for measuring food, capacity unit, classification statistics, measurement methods, the concept of half, dividing the number of objects into half, width, conservation of quantity, comparative capacity, counting in groups of 5 and 10, combining the currency value of coins, and customising measuring tools to compare distance)
Logic and relation (i.e., one-to-one correspondence, same and different, part and whole, classification, matching, and ordering)	Logic and relation (i.e., learning to arrange four figures according to rules, arranging by height, classifying by features, reasoning according to rules, cycle of the week, using comparative reasoning, and the relationship between three-dimensional and plane graphics)
Shapes (i.e., circle, square, rectangle, and triangle) and space (e.g., up/down, inside/outside, far/close and front/back)	Shapes (i.e., arrangement position: front/middle/last, solid figure: cube/sphere/triangular cylinder, and left/right,)
Time (i.e., day and night, the order of a day)	Time (i.e., arranging by time or the order of events, clock: hour and half hour, year/month/day and their order)
Colour (i.e., red, yellow, blue, green, orange, black and white)	

Table 10 indicates that the mathematics formally taught to children in both K1 and K3 classes covered five main categories including, number, magnitude and quantity, logic and relation, shapes, and time. Moreover, knowing colour was listed as a learning objective for children at K1 age.

After comparing Table 9 and Table 10, I found that the pre-determined mathematics learning objectives, particularly in K3 classes, focused more on formal mathematics (e.g., recognising symbols “+” and “=”). Additionally, the mathematical concepts and knowledge explored by children in their EMEs both covered, and were broader than, these five categories (e.g., children used number knowledge in EMEs, and the number knowledge was also listed as a key mathematical concept in both K1 and K3 classes. However, children used mathematical concepts and knowledge to play and create games, which was not mentioned in the pre-determined mathematics concepts and knowledge of both K1 and K3 classes). In which case, children might sometimes find the mathematics-related tasks or learning objectives prescribed by teachers not challenging or interesting, which could lead to their boredom (as discussed on pp. 156-157).

Chapter Summary

This chapter has presented and discussed the nature and content of children's EMEs in the context of two kindergartens in HK. The findings were presented and discussed in four sections.

In the first section, I identified exploratory EMEs as the first type of children's EMEs, which were indicated by children's (i) mathematical curiosity and sensitivity and (ii) self-directed mathematical inquiries. Motivated by mathematical curiosity and sensitivity, children could make mathematical inquiries by asking mathematics-related questions that interested them and led to mathematical discoveries. Children's self-directed mathematical inquiries were connected to their agency, and self-directedness characterised their mathematical inquiries. Drawing on self-directedness, children would use mathematical concepts and knowledge and private speech as cultural tools to solve mathematical inquiries, which further led to them achieving self-directed mathematics learning and making sense of the world.

In the second section, I described children's playful EMEs, which were indicated by the use of different languages and objects to express mathematics. The findings showed that, children were capable of engaging with mathematical concepts in diverse and creative ways. In socially and culturally embedded practices, they constructed playful EMEs through playful use of oral language, music, body language, written language, and objects to express mathematics. Furthermore, expressions of emotions while engaging in mathematics were the second indicator of playful EMEs. Children could feel positive emotions through constructing playful EMEs. With a sense of humour and playfulness, children actively constructed playful EMEs associated with positive emotions by playing with mathematics. Three salient factors contributed to children's expressions of emotions during playful EMEs, including (i) private speech, (ii) social interactions with peers and other people, and (iii) imagination, creativity, and opportunities for play.

In the third section, I discussed children's informal EMEs, which were indicated by (i) informal mathematical talk and (ii) out-of-kindergarten mathematical experiences. Findings showed that children were keen to engage in informal mathematical talk with peers and adults and used it in their construction of informal EMEs in kindergartens. Topics of children's informal mathematical talk were related to their everyday activities in kindergartens and their prior mathematical knowledge. Children's informal mathematical talk was sometimes intertwined with their mathematical discoveries. Opportunities, such as snack time and transition time, enabled children to explore and engage in informal mathematical talk. Through constructing informal EMEs, children could develop their self-esteem and a sense of fairness. Moreover, children's mathematical experiences gained from out-of-kindergarten contexts (e.g., home and tutoring centres) sometimes became resources for their informal EMEs.

In the fourth section, I talked about children's spontaneous EMEs, which were indicated by children's spontaneous mathematics activity and linked to the construction of underground culture. The findings showed that when participating in activities prescribed by teachers, children often spontaneously initiated mathematics-related actions with independent intentions and purposes. I referred to these as spontaneous EMEs. Children's spontaneous EMEs were embedded in academic activities prescribed by teachers and non-academic activities, such as snack time, when teachers demonstrated serious demeanours. In kindergartens, children's spontaneous EMEs showed underground features. The intention of having fun and reducing boredom and teachers were two main factors that drove children's EMEs underground.

In the fifth section, I brought together the range of mathematical concepts and knowledge explored and employed by the children in this study. I also analysed the mathematics learning objectives required by CDC (2017) and the two kindergartens in relation to formal mathematics learning and teaching. The findings showed that the mathematical concepts and knowledge explored by children in their EMEs both covered, and were broader than, the mathematics learning objectives pre-determined by teachers. Moreover, the pre-determined mathematics learning objectives, particularly in K3 classes, focused more on formal mathematics.

Later in Chapter 6, I will present more discussions about the answer to the first research question, *What are the nature and content of children's EMEs?* Key findings and arguments presented in this chapter will be addressed again in Chapter 6.

Chapter 5: Affordances and Constraints for Children's Everyday Mathematical Experiences (EMEs) in Kindergartens

This chapter aims to answer the second research question, *What are the affordances and constraints that influenced the availability of children's (everyday mathematical experiences) EMEs?* And the third research question, *What are teachers' perceptions and practices with regard to children's EMEs?* To achieve these aims, I explored factors that enabled and constrained children in my study to construct their EMEs in kindergartens. I also examined teachers' perceptions and practices regarding children's EMEs which could be affordances and constraints for children's EMEs. Additionally, I investigated cultural factors that might influence children's EMEs and teachers' perceptions and practices regarding children's EMEs.

Based on the data analysis, I present and discuss relevant findings on three themes: (i) physical environment; (ii) social environment; and (iii) classroom atmosphere. The findings comprise details from classroom observations, interviews, and field notes compiled during data collection. I present instances of children's EMEs, relevant photos taken by me during the fieldwork, and excerpts of teacher interviews to support discussion of the findings. Before addressing the findings in this chapter, it is necessary to revisit key ideas in relation to the concept of affordance described in Chapter 2.

The notion of affordance is a useful way to describe the relationship between the individual and features of the environment (Norman, 1988, 1993). It refers to the perceived and actual properties of an object or artefact which determine how it could possibly be used by the individual (Norman, 1988). The use of perceived and actual properties of an object or artefact depends on the individual's perception and what they wish to achieve (Clarkin-Phillips, 2018). According to Kyttä (2004), affordances exist on the ecological level, regardless of whether the individual perceives them or not. Kyttä has also noted that constraints in the physical and sociocultural environment determine which and when the perceived affordances are utilised. Adopting the concept of affordance enabled me to explore objects or aspects within kindergarten settings that facilitated children's engagement in EMEs that I was interested in.

In my study, I view affordances as features of the physical and sociocultural environment that enable children's EMEs in kindergartens. I refer to constraints as features of the physical and sociocultural environment that restrict children's EMEs or cause them to stop, and/or make children behave in particular ways related to formal mathematics education in their construction of EMEs. As mentioned in Chapter 2, an environment offers affordances through features, objects, or artefacts, which could be physical, social, and cultural. Although many previous studies have focused on the affordances offered by physical aspects of an environment, my study also focused on the affordances and constraints provided by the sociocultural environment. Moreover, classroom atmosphere—underpinned by the physical and sociocultural environment—is an essential aspect of classroom environment that provides affordances and constraints for children's EMEs in kindergartens. In Hong Kong (HK) kindergartens, the combination of rules and disciplines, teachers' beliefs and practices about play, teaching, and learning, and the timetable and routines contribute to the classroom atmosphere.

Knowing more about the mathematical possibilities afforded by kindergarten environments is important for noticing what children explore in their EMEs. Conversely, noticing what children explore in their EMEs also provides insight into the mathematical possibilities afforded by physical and sociocultural aspects of the kindergarten environments. Details are addressed as follows.

Physical Environment

The term physical environment in my study includes all indoor and outdoor spaces in the kindergartens, from the ceiling and different interest corners in classrooms to the sandpit outside the classroom. Children in my study were situated in a structured kindergarten environment every day and attuned to mathematical ideas embedded in the context. "The mathematical affordances of the objects make it more likely that children attend to and engage with mathematical concepts" (Jasien et al., 2018, p. 1339). Many teachers reported that their kindergartens promoted the value and belief of practising, doing, and “playing” mathematics in a structured physical environment. For example, Violet (Forest Kindergarten (FK)/Apple Class/Teacher Interview 1 (TI1)/091118) said, “We believe that the children acquire knowledge [including mathematical knowledge] by touching and experiencing. Through the design of the [kindergarten] environment, they can learn from everywhere, no matter from the walls, stairs, or the corners in the classroom.” This value and belief aligned with the Curriculum Development Council (CDC) (2017), which the teachers and kindergartens were seeking to follow. This document highlights the importance of the physical environment in facilitating children’s learning, including mathematics learning. It notes that providing opportunities for children to operate with physical objects can help reinforce children’s mathematical learning.

Although all teachers had some control in constructing classroom environments (as noted in Chapter 3), their ideas for setting up the environment were influenced by curriculum documents. For example, Amber explained the idea of designing the classroom at her kindergarten:

> Our curriculum is planned according to Education Bureau’s (EDB) requirements, so it contains six learning areas. […] We hope to include six learning areas in the environment setting. […] However, some spaces are not only designed for one area. For example, […] some activities in the art corner also contain scientific elements. In doing so, all six learning areas can be combined, and they will not be too clear-cut. (Amber/FK/Lemon Class/TI1/081118)

The other three teachers from FK shared the same idea as Amber when explaining the classroom design in the individual interviews. They were aware of the integration of all six learning areas in creating the environment. This aligned with the CDC (2017), which highlights the “integrated approach” in kindergarten education (p. 18). Mathematics is one of the six learning areas, which was integrated with other learning areas in the physical environment by teachers. In this case, the teachers’ approach of integrating all six learning areas might enable the mathematical affordances to be available in different corners of the classroom (as will be discussed later in this chapter).

Some teachers reported that decisions they made about the design of the environment were further guided by the kindergarten policy, which was an elaboration of the curriculum document. In this case, in addition to considering the six learning areas, they also designed and decorated the classroom according to the themes being investigated. Each theme lasted around five weeks. When the theme changed, the environment would be altered. The following excerpt describes how policies at the kindergarten level influenced Amber's classroom design:

> We also design the corners based on the [pre-determined] learning objectives under each theme investigated [and decided by the principle and head teachers]. The kindergarten provides a reference for every teacher during each theme. […] It is a booklet, in which the learning objectives for children are clearly listed. In this way, the teacher will know what kinds of knowledge children need to learn within each theme. We then design the classroom environment to meet the [prescribed] learning objectives. (Amber/FK/Lemon Class/TI1/081118)

This excerpt indicated the importance of making connections between the physical environment and themes and learning objectives pre-determined by the kindergarten. In this case, the teachers used pre-determined learning objectives, including learning objectives related to mathematics, as a guideline for preparing resources and designing the classroom environment for children.

In this section, I argue that the physical environment is central to the enabling and constraining of children's construction of EMEs in kindergartens. I present and discuss the findings related to affordances and constraints provided by the physical environment through three subsections. Firstly, I investigate children's opportunities to engage with the toys and resources (as discussed in Chapter 3, hereafter I use the term resources to represent) prepared by teachers for constructing EMEs. Secondly, I explore how the classroom environment designed by teachers provided opportunities for children to explore mathematics and construct EMEs. Thirdly, I look at the influence of resources and settings that were constantly available to children on their EMEs in kindergartens.

Toys and Resources Prepared by Teachers

As described in Chapter 3, teachers, particularly those from FK, played an essential role in purposively preparing resources for children's learning in kindergarten. Some instances of children's EMEs discussed in Chapter 4 were associated with resources prepared by teachers (e.g., Excerpt 4 on p. 124, Excerpt 10 on p. 135, and Excerpt 16 on p. 141). Through interacting with resources prepared by teachers, children could construct resource-related EMEs in kindergartens. However, this kind of opportunities was sometimes limited and bound by diverse factors (as will be discussed shortly). In this subsection, I argue that it is how children used resources prepared by teachers, rather than the resources themselves, that enabled their EMEs. Evidence is presented as follows.

Observation data showed that children from different classes utilised different resources prepared by teachers in their EMEs. For example, during my fieldwork, only children from Apple Class and Banana Class[19] used lids of different sizes to construct EMEs in (Excerpt 24, p. 154). This phenomenon could have been influenced by teachers' intentions as I did not observe teachers in other rooms providing caps resources for children to play with. Thus, teachers' intentions in relation to the resources they prepared for children varied. This influenced the kinds of resources that children engaged with during EMEs. I argue that, to some extent, teachers are the people who determine the affordances of classroom resources for children's EMEs.

I identified three main purposes considered by the teachers when preparing the resources for children. The three purposes included supporting children's play and games (e.g., constructive toys, puzzles, and board games), academic learning (e.g., number matching cards, books, and number games on computers or tablets), and hand strengthening or fine motor skills (e.g., playdough and beads). With specific intentions, teachers expected, and sometimes required, children to practice the knowledge and skills they thought those resources would provide children. These three purposes aligned with the CDC (2017), which promotes "learning through play", pre-

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[19] The teachers of these two classes were partner teachers. The children from these two classes used the same classroom.

academic knowledge (e.g., literacy/language and mathematics) and skills (e.g., writing skills). Therefore, I argue that the resources prepared by teachers in kindergartens aligned with the practices outlined in the CDC (2017). Table 11 lists the three main purposes of preparing resources considered by teachers and relevant examples of resources.

Table 11

The Purposes of Resources Prepared by Teachers and Examples of Resources

Purposes of resources	Examples of resources prepared by teachers
Play and games	
Academic learning	
Hand strengthening or fine motor skills	

Drawing on these three purposes, different teachers demonstrated different foci in relation to resource preparation. For instance, some teachers, such as Violet and her co-teacher Cherry, prepared many resources with explicit academic learning-oriented functions, including mathematics learning (e.g., number matching cards), in the classroom. In doing so, they believed that children could practice pre-determined academic learning objectives. However, it was rare to find resources with explicit academic learning-oriented functions in Olive's and Amber's classroom.

I identified three main factors that influenced teachers' decisions in regard to the types of resources they preferred children to use more often. The first factor, pre-determined learning objectives, was briefly described in the introduction to this section. Violet stated that:

> The booklet distributed by the school lists some knowledge, skills, and attitudes that children are expected to acquire. […] And the teacher needs to cooperate with the requirements [of the prescribed learning objectives] to select toys for children's play. For example, if the booklet requires children to learn how to count, then I may put some [mathematics learning-oriented] items for them to count. (Violet/FK/Apple Class/TI1/091118)

I observed that the existing kindergarten resources could not fulfil the learning objectives, Violet and her co-teacher would often create resources to facilitate children's learning in the six learning areas required by the CDC (2017). Many teachers in this study used the same strategy as Violet. Some instances of children's EMEs observed in these teachers' classes related to the academic learning - oriented resources that teachers had made (see Excerpt 4 on p. 124 and Excerpt 24 on p. 154). Moreover, I observed that teacher interactions mainly occurred when children interacted with resources that were designed for children to practise pre-determined learning objectives. As this section focuses on the physical environment, I will discuss this later in the chapter.

The second factor was teachers' expectations of children depending on their age. At the two kindergartens, pre-determined learning objectives for learning and teaching were categorised by children's age level. While preparing resources, teachers showed different expectations in relation to children's age. For example, teachers from K1 classes often prepared resources to facilitate children's hand strengthening and fine motor skills, which sometimes enabled children's EMEs. The preparation for writing practice started from K2 grades might lead into this phenomenon. More discussion regarding children's age is provided in the last section of this chapter.

The third factor was children's interests. Some teachers, such as Olive and Amber, liked to create an environment based on children's interests and motivation. They believed that children's interests were related to play. They also mentioned that children, particularly K3 children, learned best with playful and fun resources or games. Thus, resources, such as puzzles, board games, and games created by them (e.g., I spy games) were frequently played by the children in their classroom. I observed that many instances of EMEs took place when children engaged with resources such as puzzles and board games. Olive's and Amber's thoughts aligned with Rogoff (2013), who has indicated that teachers need to create a learning environment that is meaningful, challenging, and welcoming, in which children can explore and construct knowledge actively.

Olive and Amber noted that children's interests contrasted with academic learning-oriented resources. In contrast to Violet, they preferred not to select academic learning-oriented resources. Olive explained:

> Actually, my partner [teacher] (Amber) and I have screened [the toys and resources]. For example, we really don't want to put the toys that are too academic in the [interest] corner. For example, those cards used to practise addition and pairing. We observed that in fact, if children have some choices, most of them rarely choose [academic learning-oriented resources]. (Olive/FK/Grape Class/TI1/091118)

With such a preference, it was understandable that children in her class rarely had the chance to interact with resources that had explicit academic learning-oriented functions.

Although teachers purposively prepared resources, in most situations, children sometimes used the resources for other purposes than those prescribed by teachers. For instance, children sometimes used the resources that teachers prepared for academic learning purposes and hand strengthening or fine motor skills as resources for playful EMEs (e.g., pretending a book or playdough is a biscuit and describing its size and shape to peers). They also used the resources that teachers designed for play as resources for exploratory EMEs (e.g., classifying Lego according to colour) which involved mathematics learning (e.g., the concept of classification). Therefore, I argue that the purposes of encouraging or facilitating play, academic learning, and hand strengthening or fine motor skills prescribed by teachers were not obvious or important to children when interacting with teacher prepared resources.

Children's creativity, play, and social interactions with others were key factors that enabled children to use teacher-prepared resources in ways they desired and to construct EMEs. According to the notion of creativity discussed in Chapter 2, using teacher-prepared resources in flexible ways is evidence of "children's creativity" (Kupers et al., 2019). Moreover, as discussed in Chapter 4, opportunities for children's play and social interactions with others played an important role in transforming teachers' expectations of children's mathematics learning into exploratory, playful, and/or informal EMEs. For example, in the Excerpt 24 on p. 154, after practising the skill of stacking lids prescribed by the teacher, Jerry started playing with the lids with me. In doing so, he constructed a playful and informal EMEs.

Although resources prepared by teachers could enable children to explore mathematics and construct EMEs, these opportunities were not available all the time in classrooms. This phenomenon constrained children's EMEs associated with teacher prepared resources. I identified three factors for explaining this phenomenon. These are presented below.

Firstly, time for "free play" or free choice activities that enabled children to play with resources prepared by teachers was limited and fixed by timetable. As discussed in Chapter 2, play was pre-arranged as an activity that occurred at a fixed time according to the timetable in all classes at both kindergartens. The teachers from Ocean Kindergarten (OK) reported that children had fixed "free play" time for about 40 minutes every day in the classroom or the physical education room in place of the free choice activity time they used to have. Moreover, the principal and head teachers of FK arranged free choice activities for all classes before assembly or after snack time each day, and they also scheduled "free play" time for each class once a week. During "free play" and free choice activity time, "children can choose their own tools, ways of playing, playmates, and activity area" (CDC, 2017, p. 119). Therefore, although children's EMEs in relation to teacher-prepared resources were afforded by opportunities for play and “free play”, it was limited and bound by time and activities.

Secondly, within the limited time for children’s play, teacher authority was another factor that enabled and constrained children’s resources related EMEs. Authority means a person having the right or power to make decisions and have these acted on. Teacher authority refers to how teachers make decisions and prescribe action, and how students follow through on their’ direction (Hargreaves, 2017). In my study, teachers were the people who decided what kinds of resources children were allowed to play with and when. Although all teachers claimed that they were aware of the benefits of providing a wide range of resources for the children to explore and play with, some teachers only provided limited choices for children. I also observed that during group learning activities in some classes, teachers sometimes only allowed those children who were waiting for teacher-directed learning activities to play with certain types of resources (mostly constructive toys and playdough). Over the seven months of fieldwork, these toys or resources never changed. In this case, children’s EMEs were sometimes repetitive and monotonous (e.g., only constructing Lego buildings). Thus, I argue that children’s EMEs are afforded by the resources made available by teachers and constrained within the activities and play prescribed by teachers.

Thirdly, a lack of funding to buy resources was a factor that might influence this phenomenon. Due to limited funding, the choice of resources was limited in some classrooms, particularly at OK. As discussed in Chapter 1, a lack of funding could lead to less opportunities for children to engage in outdoor activities and play (Robson & Smedley, 1996), as some kindergartens could not afford an outdoor playground (Ho, 2015). My study found a lack of funding could lead to less classroom resources and, therefore, less choices for children to construct EMEs during indoor activities and play. I argue that lack of adequate resources constraining children to construct EMEs related to resources.

Interest Corners

In addition to those resources prepared by teachers, as discussed in Chapter 3, interest corners also played an important role in classrooms in HK kindergartens designed to encourage children's "self-directed learning" (CDC, 2017, p. 26). In this subsection, I argue that through the platform of the interest corners created by teachers, children had opportunities to incorporate their out-of-kindergarten experiences into their play and EMEs. Findings and discussions that support this argument are presented as follows.

The influence of interest corners on children's EMEs was particularly evident at FK. As noted earlier in this section, the design of interest corners was influenced by themes being investigated. At FK, the principal and headteachers pre-determined seven themes for exploration over the year in the curriculum. The teachers played a central role in preparing the physical environment and learning resources for children, and the kindergarten policy required all teachers to set up an environment that aligned with the monthly theme. Through observation it was evident that all the teachers at FK spent much time creating interest corners with different functions and various learning resources. These were aimed at encouraging children's self-directed learning in different learning areas, including mathematics. Through presenting three examples of children's dramatics play that took place in the dramatic play corner of the Grape Class at FK, this section discusses the finding that the physical environment constructed by teachers afforded children's EMEs in a play context.

In the first example, Olive (FK/Grape Class) and her partner teacher Amber (FK/Lemon Class) created a Chinese antique store in the dramatic play corner, which was named "The Treasure House" (see Figure 13). In the store, the teachers had selected a range of resources from a range of kindergarten resources[20] and displayed different kinds of traditional Chinese jewellery (e.g., jade bangles and rings) and other artefacts (e.g., dolls) as the goods for sale. They also set up a place as

[20] The kindergarten resources were distributed according to the theme of exploration (e.g., in the theme "China and me", all resources were related to Chinese culture).

the checkout counter with a cash register and a Chinese abacus[21] for the cashier, using a recycled paper box with six compartments as a register. Olive explained that, as the kindergarten resources did not have pretend HK currency, they had no choice but to prepare copper coins and silver ingots[22] for children to use as the currency in the store.

Figure 13

A Dramatic Play Corner: "The Treasure House"

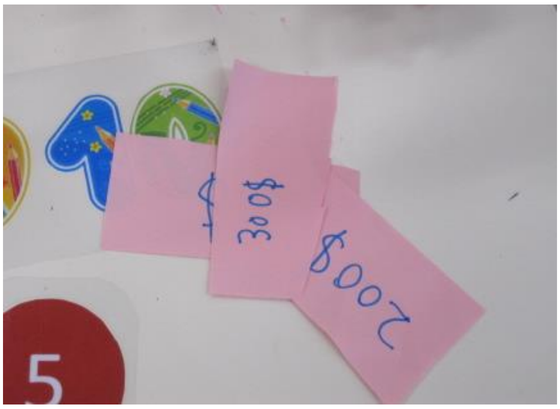
The two teachers stated that the idea of constructing a Chinese antique store was based on a number of considerations. First, due to the monthly theme "China and me", the main objective required teachers to construct an environment that could help children "know more about traditional Chinese culture and custom and learn to respect and accept different cultures" (Kindergarten documentation (KD)/FK/2019). Second, within each theme, the principal and head teachers followed the curriculum documents to pre-determine a set of learning objectives related to the six learning areas. Regarding the learning area of mathematics, constructing a Chinese antique store aligned with some of the objectives, including recognising numbers written in Chinese, addition within ten, and distinguishing modern objects from ancient ones. Third, the teachers' noticing and understanding children's interest in the concept of money. For example, Olive stated that during this period of time, she noticed children bringing toy money they had made at home to

[21] A Chinese abacus is a calculating apparatus used in ancient China.
[22] Two types of currency used in imperial China.

the kindergarten (see Figure 14). Last, the teachers noticed that the children in their classes were fond of dramatic play, therefore they wanted to encourage this.

Figure 14

Money Created by Children



A number of children responded actively in this dramatic play corner. There were three 5- to 6-year-old children (i.e., Karen, Zoe, and Sophie) play in “The Treasure House” during free-choice time. Picking up on the theme of buying and selling set up by the teacher, Karen played the role of storekeeper and Zoe and Sophie played the role of customers. In pretend play, the imaginary situation stimulates children to try on social roles that they do not yet have a mastery of (Vygotsky, 1977). Zoe and Sophie took the initiative in establishing a mother-daughter relationship in the play. While the children were playing, Olive was supervising other children doing homework, so she did not have a chance to witness their play. I recorded the whole process and discussed it with her during the interview (as will be addressed later).

In the children’s play, I observed two main types of mathematical actions: (i) number-based concepts in the interactions between mother and daughter and (ii) money-related concepts in the interactions between the customers and storekeeper.

Number-based Concepts Observed in the Mother-daughter Interactions. In the play, Sophie pretended to be the daughter and relied on Zoe, who pretended to be the mother. Their interactions are illustrated below:

__Interaction 1__: Sophie selected a pink hair accessory and gave it to Zoe. She said, "Mommy, […] I want one, this one." Zoe took over the hair accessory and said, "Ok. It must be very expensive." Then she walked to Karen.

__Interaction 2__: Sophie selected a silver hair accessory and gave it to Zoe. She said, "Mommy, I also want to buy this one. I want to buy two [items]." Zoe took over the hair accessory and said, "Ok. [You] want to buy one more, right?" Sophie replied, "Yes."

__Interaction 3__: Sophie took a gold hair accessory and gave it to Zoe. She said, "Mommy, I want this one." Zoe took over the hair accessory and talked to Karen.

The interactions between Sophie and Zoe showed a pattern, including two steps: (i) Sophie expressed her wants and asked Zoe to buy the hair accessories for her; and (ii) Zoe responded to Sophie and took the items to the storekeeper to pay for.

These interactions also illustrated how Sophie and Zoe imagined the way a daughter and mother incorporate talk related to number-based concepts while playing artefacts that encouraged pretend play. In Sophie's role as daughter, it was essential for her to describe the merchandise she wanted to Zoe. Conversations related to mathematics, especially number-based concepts (e.g., "two items" and "one more"), occurred naturally throughout their interactions. As illustrated in the first two interactions above, Sophie used numbers (i.e., one and two) to address the quantity of the merchandise she wanted to buy. Through these interactions, she demonstrated an ability to employ cardinal numbers to express her needs in a shopping context.

Zoe's responses in her role as mother were essential for Sophie to continue shopping. Zoe took the merchandise Sophie wanted and paid for it. In Zoe's second response, she unfolded the mathematical meanings expressed by Sophie. Zoe prompted a question to confirm her understanding of Sophie's request and addressed the cardinal number "two" expressed by Sophie as "one more". Moreover, in the first interaction, without checking the price tag, Zoe defined the hair accessory Sophie wanted as "very expensive". This judgement indicated her understanding of the value of jewellery and the concept of comparing, as she might perceive other merchandise as less expensive.

The children's interactions illustrate the concept of affordances, as an "affording environment" provides opportunities for developing a range of learning attributes (Claxton & Carr, 2004). The objects in the environment are not themselves mathematics, but they afford mathematical thinking (Ginsburg, 2006). In this case, the countable merchandise prepared by the teachers afforded the two children's application of number-based concepts during conversation. Thus, while addressing what they wanted to buy in both the daughter-mother interactions and customer-storekeeper interactions, quantity became an essential attribute of the merchandise. The mathematical attributes of the objects perceived by the children (e.g., quantity) afforded mathematical actions in their interactions (e.g., describing the quantity and value of merchandise).

Money-related Concepts Observed in the Customer-storekeeper Interactions. After making the decision about what to buy, Zoe went to the checkout and initiated customer-storekeeper interactions with Karen. In this process, Karen took the initiative in involving the concept of discount into the play.

> Karen took two silver ingots from the register and gave them to Zoe. She said, "[As] you bought so many pieces of jewellery, so I can give you a discount." Zoe said thanks to Karen, and then she took over the money and put them into the purse.

The reason for providing a discount (i.e., "you bought so many pieces of jewellery") explained by Karen indicated her understanding of multiple purchases and total value. In HK, some business sellers use multiple purchases as a strategy, encouraging customers to buy more by offering hem a discount. In this context the word discount referred to "a reduction made from the regular price" (Herbert, 2017, p. 692). Karen perceived the concept of discount as charging less. Thus, after Zoe paid, she returned two silver ingots to her as the cash discount:

> Later, while Zoe and Sophie were looking at the merchandise, Karen used the Chinese abacus to pretend to calculate. She said, "Excuse me, I need to calculate …

> Excuse me, because you bought so many things, this ring is a reward for you two."
>
> Then she took a jade ring from the shelf and showed it to Zoe and Sophie. Sophie asked, "Do we need to pay?" Karen waved her left hand and said, "No, it's free."

Karen's calculating action was mediated by the abacus, which was a cultural tool provided by the teachers. She randomly moved the beads up and down to pretend to "calculate". This process demonstrated her early concept of calculating, as she knew an abacus could be used as a tool for calculating. This also aligned with the expectation to "know about some Chinese artefacts" as noted by Olive. This example illustrated how children know what cultural artefacts are used for even if they do not know exactly how to use them.

After totalling Zoe's purchases, Karen involved a concept of reward in their play, in the form of a jade ring. Similar to the concept of discount, her words "you bought so many things" indicated her understanding of reward was also based on the concept of multiple purchases or spending a large sum of money in a transaction. She selected a jade ring as a free gift, which demonstrated that she understood that gifts are given with no monetary cost. In HK, the use of free gifts is a common practice to entice people to buy more. While Karen proposed this idea, Zoe and Sophie were looking at the merchandise but expressed no intention in buying the jade ring. Thus, Karen might use a free gift to tempt Zoe and Sophie to buy more.

> Karen gave the free jade ring to Sophie. However, Zoe took the ring first and wore it. She found the ring was too big for her finger, so she said, "Because this ring is too big, you gave it to her [for free]." Then Karen took another ring from the shelf and gave it to Sophie. She said, "This green one for the daughter…But you have to pay".
>
> Then she asked Zoe and Sophie for five dollars.

This vignette provides the rationale for the free gift Karen provided earlier. The act of selling the second ring for five dollars indicated that her intention of providing the free gift was to sell more rings. Zoe interpreted Karen's sales strategy after realising the ring was "too big". This demonstrated her mathematical understanding of size.

In the interview, when talking about this instance, Olive took the initiative[23] in addressing how Karen's home experience influenced her inclusion of the concepts of discount and reward in the play. She stated:

> She has rich experience in dramatic play. Her mother told me that she loved doing the dramatic play with her older sister at home…Her mother often took her sister and her to do shopping, so I think she also has rich shopping experience…I am not surprised that she could role-play shopping so well. (Olive/FK/Grape Class/Teacher Interview 2 (TI2)/160319)

The antique store created by teachers could have reminded Karen about her out-of-kindergarten experiences, which supported her pretend play in the kindergarten.

In the process of their play, the three children, particularly Karen, showed mathematical knowledge of value and the sociocultural understanding of commodities trading. Their sociocultural understandings sat alongside their mathematical concepts and gave meaning to each other. I therefore argue that through the platform of the antique store created by teachers, children were provided the opportunity to incorporate their everyday experiences from out-of-kindergarten contexts into their play.

Daily Kindergarten Resources and Settings

In addition to the resources and interest corners intentionally set up by teachers, children's EMEs also related to resources and settings in kindergartens that were available all the time and not directly connected to teachers' planning. I termed them as daily kindergarten resources and settings. In this subsection, I argue that daily kindergarten resources and settings provided many opportunities for children to explore mathematics, thereby enabling their EMEs. Table 12 summarises all relevant daily kindergarten resources and settings, the potential for mathematical

[23] In the interview, I did not specifically ask Olive to talk about Karen.

concepts used by children, and relevant instances of children's EMEs evident in the analysis of the data.

Table 12

Daily Kindergarten Resources and Settings

Daily kindergarten resources and settings	Mathematical concepts	Instances of EMEs
Recourses for daily kindergarten life		
Stationery • Pencils • Coloured pencils • Paper • Scissors Watercolour paints	Number; size and quantity; classification; comparing and measuring; logical thinking and explaining	Excerpt 23 described on p. 154, which is also the example of EMEs described in the abstract of the thesis
Food and drinks • Cookies with diverse shapes, bread, and cake • Eggs • Fruit (e.g., mandarins and grapes) • Traditional food (e.g., steamed vermicelli roll) • Water	Number; size and quantity; classification; patterns and shapes; designing; geometric thinking; spatial sense; locating; comparing and measuring; logical thinking and explaining; playing and creating games	The informal mathematical talk among Kale, Tony, and Lara on p. 150
Household appliances/Information and communication technology (ICT) • Fans/air conditioners • Radios/speakers • Desktop computers/tablets	Number; size and quantity; comparing	One day, children from the Banana Class were walking across the building to participate in an English activity. Luke (3 years, 4 months) was walking at the end of the queue. While walking, he stopped and looked at the fans on the ceiling. Then he said loudly, "We had three fans. One is not working." (FN/2102112)
Other resources • Cups, bowls, and plates • Spoons • Plastic water jugs • Face cloths • Tongs • Tissue paper	Classification; patterns and shapes; geometric thinking; comparing and measuring	Excerpt 9 on p. 133
Floor/Wall/Ceiling decorations		
Holiday decorations • Chinese New Year (e.g., firecracker ornaments,	Number; size and quantity; classification; patterns and shapes; comparing and measuring; logical thinking	Before Chinese New Year, the teachers decorated the classroom environment with some Chinese ornaments (e.g., lanterns). While

Spring Festival couplets, and signs) • Christmas (e.g., Christmas trees, Christmas garlands, and Christmas ornaments) • Regular decorations (e.g., children's work and photos)	and explaining; playing and creating games	lining up and waiting to go home, Ava (3 years, 7 months) looked at the hanging fish ornament (see the photo below) and asked me, "Why does the fish have two tails?" (FN/240119) 
Indoor areas other than the classroom		
The internal structure of the building • Stairs • Lifts • Corridors	Number; magnitude and quantity; classification; patterns and shapes; geometric thinking; spatial sense; locating; comparing and measuring; logical thinking and explaining	Excerpt 3 on p. 115
Bathroom • Taps • Toilet paper • Sinks • Toilets • Taps, knives, fridges	Number; size and quantity; patterns and shapes; geometric thinking; spatial sense; comparing and measuring	Liam (3 years, 6 month) was washing hands in the toilet. He showed me how to control the tap (see the photo below) and said, "YeYe, look. If I turn [the tap a little way] back [towards the closed position], the water is very small." (FN/030119) 
Outdoor areas		
Outdoor playground • Bicycles, tricycles, and scooters for children • Sand pit/bean pit • Slide	Number; size and quantity; classification; patterns and shapes; designing; geometric thinking; spatial sense; locating; comparing and measuring; logical thinking and explaining; playing and creating games	In the outdoor playground, Jerry (3 years, 7 months) pointed to the sign (i.e., 1⟷2) on the tricycle that he just rode and said to me, "YeYe, this [tricycle] is No. 12." (FN/030119)

As shown in Table 12, children's EMEs were relevant to what they encountered in kindergartens every day. The daily kindergarten resources that enabled children's EMEs were diverse and rich and included all the kindergartens' everyday resources (e.g., stationery, food and drinks, and household appliances/information and communications technology (ICT)). The daily kindergarten settings that enabled children's EMEs included the internal structure of the building, toilets, and outdoor areas (e.g., the outdoor playground). Mediated by these daily kindergarten resources and settings, children constructed their EMEs with a range of mathematical concepts, from numbers to playing and creating games.

The richness of daily kindergarten resources and settings was beneficial to children's mathematical curiosity and sensitivity and, therefore, their EMEs. As discussed in Chapter 4, children were often curious and sensitive to their immediate context. Ginsburg and Seo (1999) have explained that children in all cultures develop in physical environments containing a multitude of objects and events that can support mathematics learning in everyday life. In this sense, the objects are not themselves mathematics but afford mathematical thinking (Ginsburg, 2006). Thus, I argue that children's construction of EMEs can happen everywhere in kindergartens. However, due to the formal and serious classroom atmosphere in HK kindergartens, teachers sometimes did not allow these kinds of opportunities to happen (as will be discussed later).

In this section, I discussed how the physical environment enabled and constrained children's EMEs in kindergartens. Moreover, as discussed in Chapter 2, children's mathematical competence with physical objects can be determined by positive teacher interactions and participation in creative activities (Howes & Smith, 1995). Thus, while engaging with physical environments, social interactions with teachers and peers also play an important role in influencing children's EMEs. This is discussed as follows.

Social Environment

In addition to the physical environment, the social environment in kindergartens also provides affordances and constraints to influence children's EMEs. Social relationships are central to sociocultural theory. The best way for people to learn is through interacting with people who are more capable than the learners themselves (Feden & Vogel, 2003). From a sociocultural perspective, this means that I should pay attention to the relationships that children develop with others when investigating their learning and development.

In my study, children often constructed their EMEs through interacting with peers and sometimes interacting with teachers. Data analysis showed that both teachers and peers functioned as different roles in children's EMEs. Some roles enabled children's EMEs, and other roles constrained children's EMEs. In this section, I unpack social affordances by presenting and discussing the relevant findings under two themes: (i) the five roles of teachers in children's EMEs and (ii) the two salient roles of peers in EMEs.

Roles of Teachers in Children's EMEs

This subsection focuses on the diverse roles teachers had in relation to children's EMEs and the reasons underpinning these roles. It answers the third research question: *What are teachers' perceptions and practices with regard to children's EMEs?* I identified five salient roles of teachers for enabling and constraining children's EMEs, including: a fellow mathematical inquirer, an outsider, a playmate in mathematics-related play, a social partner in mathematical conversations, and a facilitator to extend children's EMEs. To elaborate on these roles, I focus on what teachers did during the process of enabling and constraining children's EMEs. I argue that although teachers are important for enabling and extending children's EMEs by playing diverse roles, they also constrain children's EMEs by not always encouraging and valuing EMEs.

A Fellow Mathematical Inquirer. As discussed in Chapter 4, children in this study were keen to make mathematical inquiries, and teachers were an important informant or a fellow inquirer for them to ask mathematics-related questions. According to Leo's instance of EMEs described on p. 117 and p. 119, motivated by mathematical curiosity and sensitivity, Leo viewed his teacher Violet as a fellow inquirer to ask relevant questions and expected to gain answers from her. Violet seemed to value and respect Leo's questions, as she stopped the instruction process to answer Leo's questions.

As an informant, Violet provided Leo with answers from her perspective. Violet's responses aligned with the CDC (2017), which has stated that, "Teachers are recommended to use mathematical concepts more often to discuss with children about the things they encounter in their everyday life" (p. 41).

Violet's timely responses were beneficial to Leo's mathematical inquiries from two aspects. First, she provided Leo with relevant information in terms of his questions. This might help Leo solve his mathematical inquiries, as he did not ask more questions later. Second, Violet's responses seemed to extend and continue Leo's interest in exploring volume. When children began to practise the skill of applying glue, Leo did not follow Violet's instruction (i.e., use a little glue), rather he dipped his whole finger into the glue bottle. While experimenting, he stared at the glue on his finger and said with a smile, "A lot." The experience of putting glue on his finger could have deepened his understanding of the concepts of "a lot" and "too much".

Teachers who acted as fellow inquirers in children's EMEs held the belief that it was important to establish children's life skills in mathematics. All teachers in my study believed that mathematical knowledge and skills were important life skills in that they help children solve problems in everyday life. For example, Amber (FK/Lemon Class/TI2/160319) believed, "With mathematical knowledge, children can solve some problems by themselves without asking the teacher." This belief aligned with the CDC (2017), which has stated, "Children should be encouraged to put into practice what they have learnt by using mathematical concepts to solve practical problems in everyday life" (p. 39) (as discussed in Chapter 1). Olive (FK/Grape Class/TI2/160319) viewed mathematical concepts and skills learned in kindergartens as "basic skills that make children's life more convenient". According to teachers, children's early mathematical knowledge and skills seemed to be important for functioning in everyday life.

The teachers' focus on establishing life skills also aligned with Confucian beliefs and values in relation to education. As discussed in Chapter 2, Confucianism believes that education can lead a person to perfect their character (Kim, 2007). Teachers in my study viewed life skills in mathematics as important characteristics that children should have. In which case, teachers may think it crucial to establish children's life skills in mathematics, aiming to build up their capabilities and, ultimately, become perfect adults (Hong & Howes, 2014).

In addition to becoming perfect adults, helping children to become perfect primary school student and prepare them for primary schooling was also a priority for teachers. They believed that children's early mathematical knowledge and skills were important for preparing children for primary education. According to Ho (2015), in HK, when transitioning to primary school, teachers expect children to possess prescribed mathematical concepts and knowledge (e.g., being able to identify odd and even numbers). In my study, teachers also expected children to possess certain mathematics-related life skills that might enable them to be independent. For example, "knowing about time; […] distinguishing left and right; […] understanding teachers' mathematics-related instructions" (Amber/FK/Lemon Class/TI1/081118), and distinguishing "little" from "much" (as indicated in the discussion between Leo and Violet). Daisy (FK/Octopus Class/TI2/040319) viewed mathematical concepts and skills learned in kindergartens (e.g., counting from 1-100) as "the skills before moving to the primary school". Some teachers, including Olive, Lily, Cherry, Rose, and Iris shared the same belief as Amber and Daisy. This indicated that in addition to prescribed mathematical concepts and knowledge, many teachers also viewed mathematics-related life skills and, therefore EMEs, as essential for preparing children for the transition to primary school.

Teachers sometimes used the term "common sense" (e.g., knowing that two cookies are more than one cookie) to describe children's mathematics-related life skills which were explored by children in their EMEs. They viewed kindergarten mathematics as the mathematics that everyone should have. Ivy noted that she sometimes paid no attention to common sense in mathematics as she believed that it was not worth acknowledging. She explained:

> In fact, [I think this demonstrates that] they just know how to apply [the mathematical concepts that they learned before]. We just don't care. […] Teachers will not pay attention to the details of what concepts they applied. In fact, they have

> learned these concepts before, and I won't consciously notice that they used it. It's not surprising to see them used these concepts in class or in other activities. (Ivy/OK/Whale Class/TI2/050319)

Ivy's thoughts about common sense aligned with Rogoff et al.'s (2016) ideas of informal learning that imply that the concepts used in informal learning are inaccurate.

Another teacher Amber shared a similar idea to Ivy. While addressing her thoughts about how children employed mathematics in block play, she explained:

> Maybe when they play, they themselves don't even know that they are actually applying mathematical concepts. That is to say, they use those mathematical concepts invisibly and naturally. […] In fact, it's the same with adults. You won't emphasise and reflect on what kinds of mathematical concepts you're using right now. (Amber/FK/Lemon Class/TI2/160319)

Amber used the term "invisible mathematics" to describe the mathematics one can do but which one does not recognise as mathematics (e.g., provide five plates for five people on the table). Teachers knew children were capable of applying and exploring mathematics but sometimes did not recognise this as academically valuable or directly linked to pre-determined mathematics learning objectives. Therefore, I argue that once teachers consider children's mathematics-related life skills mentioned earlier as mathematical common sense, the mathematical common sense acknowledged by teachers is not important to them.

An Outsider. Teachers sometimes were an outsider in children's EMEs. That is, children sometimes excluded teachers from their EMEs, particularly spontaneous EMEs (e.g., Excerpt 23 described on p. 154 and the abstract). During my observation, this never happened to children's peers. It seemed that teachers were sometimes unwelcome in the children's world of EMEs. This finding aligned with the one addressed in Chapter 4, which referred to children's construct of spontaneous EMEs being linked to their underground culture.

My interpretation of data indicated that there were four main reasons for the phenomenon of teacher as outsider. First, some teachers demonstrated unsupportive and discouraging attitude towards children's EMEs, particularly during formal and serious situations (e.g., group learning activities). They sometimes used responses, such as "stop…" or "no need to…" as indicators for children to discontinue their off-task EMEs. In the following excerpt, Iris intended to use the time before children went home to introduce the New Year decorations to be made the next day.

> Gary (3 years, 11 months) raised his finger and pointed at other children in the classroom. He counted, "One, two, three, four, five…" At this time, the teacher Iris passed Gary's seat to get the New Year decoration. She said to Gary, "No need to count. There are 29 people here…No, including the teachers, more than 29." Then, Iris continued introducing the details of creating the New Year decoration. Gary said in a low voice, "But Grace didn't come back [to the kindergarten] today".
>
> (FN/070119)

The teacher-child interaction was initiated by Iris, and Gary counted in the form of private speech. Iris's words, "No need to count" indicated that she recognised Gary's counting action. However, she responded in a way intended to stop him counting in this context.

Gary's words (i.e., "But Grace didn't come back [to the kindergarten] today.") indicated that he had doubts about Iris's words (i.e., "There are 29 people here…No, including the teachers, more than 29."), which could be evidence of his "critical thinking" (Scriven & Paul, 2007). As discussed in Chapter 2, critical thinking skills enable children to solve problems they may meet when constructing EMEs. In the interview, after reading the transcript of this instance, Iris viewed Gary's words as the expression of disapproval of her. According to her tone, it seemed that she did not appreciate Gary's criticism. As noted in Chapter 3, according to Confucianism, teachers do not encourage children to challenge the authority of adults, including teachers. Thus, Confucian ideology might influence Iris's unsupportive and discouraging attitude in this situation.

Second, some teachers showed unsupportive and discouraging attitude to the joy children obtained through EMEs (as discussed in the section of playful EMEs, Chapter 4), particularly in task-oriented activities (e.g., doing homework). In the individual interview with Ivy and Lily, I showed them a transcript of the pencil measuring excerpt outlined in Excerpt 23 (Chapter 4, p. 154). In the interview, both teachers identified the purpose of the children's EMEs as play, as other children were doing worksheets. They described the children's actions with the term "貪玩/taam1 waan4 (enjoying oneself by behaving in a playful way or being fond of play)". Lily's unsupportive and discouraging attitude, which was indicated by her tone, in regard to the children's enjoyment during play in this example was due to the lack of focus they showed towards completing the worksheet. Moreover, the teachers' unsupportive and discouraging attitude regarding play aligned with Confucian view of serious learning and play. As noted in Chapter 2, to Confucius, childish behaviours such as "loving to play" should be avoided (Hong & Howes, 2014, p. 40).

Third, the idea of age-appropriate mathematics, children's mathematics learning progression, and teacher expectations led to teachers' unsupportive and discouraging attitudes. Ivy indicated:

> I don't think there's any excuse that they don't know how to compare. This is basically what K1 children would do. […] They knew it from K1 and K2. By the K3, it's just "貪玩/taam1 waan4". (Ivy/OK/Whale Class/TI2/050319)

She judged the mathematics employed by the children as under their age level. Her judgement was influenced by the pre-determined mathematics learning objectives for children in different grades. She expected K3 children to use the mathematics that was appropriate to or beyond their age level (as will be discussed again in the last section of the chapter).

Fourth, teacher authority underlying their unsupportive and discouraging attitudes might lead to teachers' unwelcome role in children's EMEs. Teachers are seen as authority figures within Confucian ideology (Ho & Ho, 2008). When talking about the reason why Iris wanted to stop Gary's counting process described earlier, she explained:

> I think [Gary] should sit properly and be ready for home. […] Because at that moment I was introducing…what they were going to do tomorrow. I was talking about important information, so he should listen to me carefully first…and then do other things later. (Iris/OK/Dolphin Class/TI2/110319)

She took an authoritative position in deciding that the activity of introducing the New Year decorations directed by her was the only important thing in this instance. Gary's counting action was perceived as not important by her.

Children knew that their EMEs, particularly their playful EMEs were not valued and could be discontinued by teachers. During the individual interview, Ivy and Lily reported that they had noticed similar instances of pencil comparing many times in their class. Lily stated, "[If they] measure pencils, […] I will ask them to stop playing and focus on doing homework". Ivy shared the same attitude as Lily. They did not approve of this kind of behaviour or ignored children's EMEs when children were supposed to be engaging in teacher-directed tasks and redirected children back to what they thought the children should be doing. This indicates that teachers' unwelcome role in children's EMEs might be caused by the unequal power relation between children and teachers.

In addition to being excluded by children, data analysis also indicated that in most situations, teachers also chose to take an outsider role in children's EMEs. The outsider role taken by teachers mentioned here has two meanings: non-responder and non-participant observer.

Regarding the role of non-responder, some teachers sometimes did not value and, therefore, ignored children's EMEs. For example, one day in the Grape Class, a group of children were making a game of adding up numbers out loud (e.g., saying 100 plus 100 equals 200) together during snack time. The teacher Olive noticed this and initiated a conversation with me. She reported, "I won't join [or respond to] them because it's snack time. They can talk freely to each other". Olive's non-responder role was influenced by the time allocation and how she viewed her role in snack time. She allowed children to talk and engage in peer interaction during snack time.

While addressing her understanding of the children's intention, Olive (FK/Grape Class/TI2/160319) said, "They always played in this way. […] I think they just want to show off to others that they know how to do addition, but actually, the result was often incorrect." She viewed children's playful mental arithmetic on addition and informal mathematical talk are problematic and showing-off. However, as discussed in Chapter 4, children in this study often took the initiative in talking about what they knew or created with mathematics with others to develop self-esteem. Chairilsyah (2019) has argued that children often feel proud of themselves when they do something that they think is great or spectacular. Therefore, although Olive allowed the children's informal mathematical talk, she did not value or understand the benefits of children's informal mathematical talk for their self-esteem. In this case, she chose to be a non-responder.

Regarding the role of non-participant observer, as noted earlier, teachers sometimes noticed children's EMEs through observing children's "free play" and free choice activity. In the process, they often adopted a non-participant observer role physically distant from children's play. This finding aligned with Devi et al. (2018). The teachers in their study also positioned them as outsiders to children's play. They explained that this reflected teachers' beliefs that "adults do not become directly involved in children's play" (p. 306). In my study, even though children's play involving mathematics is valued by Chinese culture (as discussed in Chapter 1), teachers did not attempt to become involved in children's play. The paragraph below lists the reasons Violet gave for this phenomenon.

Violet indicated that doing non-participant observation was important for learning about children's EMEs through their play. Observation means more than watching and listening. The CDC (2017) highlights observation as the assessment of learning conducted by teachers, referred to as "continuous observation" (p. 27), documenting, and providing feedback. Moreover, it is "a process by which educators can understand and give meaning to what they see and hear, drawing on their own knowledge and experience as well as on the evidence of their senses" (Drummond et al., 1992, p. 24). Violet explained:

> If I observe that she can count from 1-10, then I will know what kind of learning resources I can prepare for her to use next and facilitate her self-directed learning in mathematics. (Violet/FK/Apple Class/TI2/280219)

It seemed like non-participant observation helped her collect data about children's mathematical development status. With these data, she might extend children's EMEs in the form of self-directed learning in mathematics through creating an environment that afforded opportunities for children. Moreover, encouraging children's self-directed learning in mathematics, which is a feature of EMEs, was the purpose of Violet's observation. As previously noted, the CDC (2017) has suggested teachers set up interest corners and prepare learning resources (e.g., toys) to encourage children's self-directed learning. In the interview, Violet highlighted that she preferred not to provide direct guidance to children but "provide opportunities for children to explore" in play. These opportunities referred to preparing learning resources for children to interact with. She indicated that the decision of how to prepare learning resources was "based on the teacher's sensitivity to each child in a specific context [during observation]".

In summary, drawing on teachers' unsupportive and discouraging attitudes to EMEs, teacher authority, and the power imbalance between children and teachers, it is understandable that teachers were sometimes not welcome in children's EMEs. Children might think that their EMEs were not allowed or welcome by teachers. In which case, they would hide their engagement in EMEs from teachers. Additionally, due to teachers' initiative in playing a non-responder and non-participant observer role in EMEs, children might think that teachers were not interested in their EMEs. Thus, they sometimes ignored teachers while constructing EMEs.

It was noticeable that some teachers also expressed that they wanted to play with children rather than taking on an outsider role. However, due to class size and the focus on academic learning activities, they sometimes had to be an outsider. As mentioned earlier, in many situations, teachers were more focused on managing large groups and the academic imperatives they valued (e.g., supervising children doing homework). In this case, being an outsider meant that they did not just ignore or devalue children's play, they had other priorities and objectives. Despite this, sometimes children looked to teachers as playmates.

A Playmate in Mathematics-related Play. Children in my study sometimes invited teachers to be playmates in their playful EMEs. I observed that in most situations, this happened when teachers took the initiative to approach children during their play. I identified two main factors underlying teachers' initiative in being children's playmate in EMEs.

First, as required by curriculum documents and kindergartens, teachers had a responsibility to observe and record children's learning cross a range of activities and play. Thus, teachers at FK sometimes took the initiative in being the playmate in children's playful EMEs. The purpose of observation records was to "record important information of children at various stages as evidence of children's growth" (CDC, 2017, p. 73). The teachers at FK reported that after the principal and the head teachers checked the observation records created by them, parents were the main readers of those records. This aligned with the CDC (2017), which notes that teachers should "let parents obtain and understand the information on assessment and the performance of their children" (p. 70). The CDC (2017) has also indicated that provision of this information would encourage parents' "support for children's learning" (p. 70).

According to the observation records I collected, while being children's playmates, teachers at FK often employed the strategy of making inquiries as part of observing children's play and, subsequently, sometimes their EMEs. In doing so, some teachers reported that they could assess the mathematical concepts and knowledge children employed or explored in play and to what extent they had engaged with these. However, some teachers only made inquiries regarding instances of play that could be used as evidence of children's mathematics learning. For example, Amber (FK/Lemon Class/TI2/160319) noted, "For those instances of children's mathematical exploration in play [and, therefore, EMEs] that I really need to report and document, then I will ask children's questions about what you are doing and why you are doing that". Thus, creating observation records was a factor that motivated some teachers to be children's playmates and make inquiries as part of their observations.

Second, those teachers who valued the benefits of the mathematics employed by children in their play would like to extend the mathematical experiences that they thought were under children's age level in play. For example, one day in the free choice activity in the Lemon Class, Kaden was engaging in imaginative play. He employed some mathematical concepts (e.g., classification and colour) in the process of selling a set of Chinese chess as "mooncakes" to his peer without mentioning money and price (see Figure 15).

Figure 15

Kaden's Dramatic Play: "Selling Mooncakes"

Kaden was playing, the teacher Amber was taking care of other children, so she did not have a chance to observe him using the mathematical concepts above to initiate imaginative play with his peer. In the interview, after watching the video footage of Kaden's play, and in response to my query about how she would have responded had she witnessed this interaction, Amber stated that she would have provided Kaden with play money and introduced using this money via a set of questions (e.g., "How much is each mooncake?" and "How much for five mooncakes?"). She believed that the concept of money and the use of play money would mediate and extend the spontaneous mathematical actions in Kaden's play. Amber explained:

> For children at K3 level…I think their play should be more 營養/jing4 joeng5 (nutritious) (laughing). [The concept of] selling is not just giving [customers] everything that [the shop owner] has. […] If I knew (laughing), I would have provided

> them with the play money in the classroom. […] In doing so, they may know…you have to use the money to buy things. If you pay more money, you have to give change. With these props (i.e., play money), I hope they will know the concepts of… [that is,] you can pay dollar by dollar, and how much change you need to give. I think they will know more about the process of buying and selling rather than just giving everything to others without payment. (Amber/FK/Lemon Class/TI2/160319)

As discussed in the previous section, teachers played an important and authoritative role in providing resources for children in their play. Without Amber's support, Kaden and his peer did not have chance to access to the resources Amber mentioned.

While involving teachers as playmates, children sometimes ignored teachers' efforts to extend children's mathematics learning experiences through play. For example, in an instance of children's EMEs (FN/240119) I collected in Banana Class, Julie was interested in buying and selling. Through her interaction with teacher Cherry, Julie explored the notions of expensive and cheap (the descriptive terms for the value of merchandise) and discount. Cherry showed interest in knowing what children were doing, so she took the initiative to join the children's play. In the process, she tried to guide Julie to focus on the concept of price. However, Julie was not interested in the concept of price and continued her play. Thus, although Cherry asked her twice, she did not reply, choosing to give Cherry flowers instead. It appeared that Julie was avoiding the learning focus of the concept of price proposed by Cherry. She was more intent on engaging with the teacher as a playmate rather than in her usual teaching role. Therefore, Cherry's intention of extending Julie's experience in the concept of price was not successful in this instance of EMEs, which I identified as a dilemma. Bäckman (2016) has explained that "it can be problematic for the teacher to direct the child's attention to a mathematical content when the child's attention is directed towards a play content" (p. 225). Thus, teachable and learnable moments sometimes pose a dilemma depending on the children's play intentions.

A Social Partner in Mathematical Conversations. Teachers were an important adult social partner for children in kindergartens. As discussed in Chapter 4, children were interested in discovering aspects related to mathematics in their immediate context and sharing their discoveries with others, including teachers, through informal mathematical talk. Children also sometimes appropriated the mathematical information mentioned by teachers to construct their EMEs. In this subsection, I argue that as an adult social partner of children's mathematical conversations, teachers' words were sometimes resources for children's EMEs.

Observation data indicated that teachers often involved mathematical information in their verbal and non-verbal language to communicate with children and other adults in kindergartens. As noted in Chapter 2, previous research has noted teachers' "math talk" (i.e., mathematical language input) is a specific aspect related to stimulating children's mathematics learning (Boonen et al., 2011). The CDC (2017) has also recommended that teachers "use mathematical concepts more often to discuss with children the things they encounter in their everyday life (p. 41)". I found that the mathematical information provided by teachers was embedded not only in their speech (e.g., using ordinal numbers such as first and second to introduce a procedure of creating an art craft), but also in their gestures (e.g., using Chinese number gestures, as discussed in Chapter 4, to indicate numbers), enabled children's mathematics-related speech and gestures.

Teachers sometimes intentionally involved mathematical language in non-mathematics learning activities (e.g., art). At the two kindergartens, teacher-directed activities were a place where teacher-child interactions mainly took place. During these activities, teachers took the main role in talking. Sometimes their instructions explicitly included mathematical information. For instance, one day during an art activity in the Banana Class, Cherry was teaching the skill of stacking lids. In the process, she used number words, "Get the lids and stack them one by one [in this way], one, two, three". Without any teacher direction, Vicky (3 years, 2 months) took the initiative in appropriating the key mathematical information Cherry expressed in her instructions. While Cherry was talking, she said, "Three [lids]. One, two, three." (FN/030119). In the informal conversation between Cherry and me, she explained that "I [counted] on purpose. I want to show them I used three lids and stacked one by one. That is the procedure of this [stacking] skill. Also, they are learning numbers and counting [as a prescribed learning objective] now. In doing so, I can also teach them to count one, two, three".

Additionally, teachers often involved mathematical language in their informal conversations with children or adults at kindergartens. In this case, their mathematical language was not specifically aimed at talking to children. However, it sometimes enabled children to appropriate the mathematical information embedded in teachers' conversations into their EMEs. For example, one day in the Apple Class, children had corn kernels and raisins as their snack. Two teachers were talking about the shortage of raisins. One said, "Later, I can ask each child of my class to get five [raisins], and they can practise counting (laughing) (the teachers were teaching number five and its quantity this week)". The two teachers' private conversation was not aimed at the children. However, when Raymond (3 years, 2 months) finished his snack and was tidying up, his peer reached out to take some more raisins. Raymond said to him, "The teacher said five raisins" (raising five fingers). (FN/131218). Raymond had appropriated the mathematical information "five [raisins]" embedded in the teachers' informal conversation to inform his peer not to take too many raisins. This vignette indicated that children paid attention to mathematical concepts in teachers' private conversations.

As an adult social partner, teachers sometimes showed children what kinds of mathematics they valued by praising and encouraging children's EMEs. Teachers sometimes responded positively to EMEs through timely verbal compliments and affirmation. For instance, in the interview with Cherry, I played a video footage of two children playing with playdough. In response to my query about how she would have responded had she witnessed this interaction Cherry stated:

> If they talk about [big and small], I will give them more affirmation…That is, if [I] see them creating a big ball [with playdough], then I will say, oh, right, you made a big ball. If he can create a triangle that is really a triangle, [I will say,] oh, right, you really

made a triangle. I will give this kind of affirmation to them […] to praise them when they are able to create [these shapes] (laughing). (Cherry/FK/Banana Class/TI2/010319)

Cherry would provide the children with praise or affirmation regarding the mathematical concepts used by children to construct their EMEs. Many other teachers reported that they would also say statements such as "叻/lak6 (impressive and smart)", "precious", and "deeds accord with words" to give compliments and affirmation.

Teachers who said they praised and encouraged EMEs valued children's proactiveness and independence. In the interview with Amber, she described an instance of children's EMEs she had observed. In her example, she said the child spontaneously explored the concept of symmetry by creating a seesaw device with wooden blocks. While addressing her understanding of the process, she noted that "what the child did was really 'without any guidance' (Amber said these words in English). I feel it was very precious". The words "without any guidance" and "it was very precious" indicated that she valued the child's agency and independence that she could perceive in this spontaneous and unprompted EMEs.

Cherry shared a similar thought to that of Amber. In the interview, when explaining the reason why she believed that the instance of children's exploratory and playful EMEs observed by her deserved to be praised, she said:

> They…discovered [the shapes of the towel] by themselves. That is, through their own exploration, they found out something they had learned before. They could employ what they had learned by themselves. (Cherry/FK/Banana Class/TI2/010319)

Cherry explained that the concept of shape was "something [children] had learned before", as she has previously taught this concept to children. It seemed like she particularly valued the children's proactiveness in exploring and applying the mathematics they had learned.

Teachers who said they praised and encouraged children's construction of EMEs believed that their praise would enhance children's self-confidence. Once, I observed an EME in Iris's class which a child counted before going home. In the second interview, Iris addressed her thoughts about the child's EME, she noted:

> I think if I knew, I would…praise him for this "叻/lak6 (impressive)" competence [of counting], to make him realise his [mathematical] competence. […] Making him more confident. That is, [I can] "幫/ bong1 (help)" him via this method. (Iris/OK/Dolphin Class/TI2/110319)

Although she had not observed this child counting, she viewed his ability as an "impressive competence" and envisioned the above response during the interview. The term "幫/ bong1 (help)" employed by Iris indicated that she might perceive the child needed assistance in self-confidence. Thus, after perceiving the child as being less confident, she assumed that praising his competence would build his confidence. Cherry shared a similar thought to Iris. In the interview, she stated:

> I think [the praise and affirmation] will help [children] discover more or apply more […] mathematics [in their daily life] […] and "幫/ bong1 (help)" them affirm themselves, that is, they will be more confident. (Cherry/FK/Banana Class/TI2/010319)

Like Iris, Cherry used the term "幫/ bong1 (help)" to describe the relationship between her praise and children's self-affirmation, confidence, and ongoing interest in exploring or applying mathematics.

Observation data indicated that teachers' praise and encouragement sometimes extended children's development of EMEs. For example, in the interview, Cherry described an example of children's exploratory and playful EMEs (Excerpt 9, p. 133), in which the children used face cloths to explore shapes. In the process, Cherry verbally praised the children on what they created by using mathematical ideas. She believed that teachers' praise would motivate and extend children's interest in spontaneously exploring mathematics. This might explain why the second child started folding face cloth, as he heard the compliment that the teacher gave to his peer and wanted to show what he could do as well.

According to my observation, in some classes where children obtained affirmation and praise from their teachers in relation to their EMEs, children would repetitively engage in similar EMEs valued by the teacher (e.g., comparing the similarity or difference of the colour of spoons during snack time in Cherry's class). There is a dearth of studies that explore the impact of teacher affirmation and praise on children's mathematics learning (Floress & Jenkins, 2015), particularly that of pre-school aged children. Wang et al. (2017) have investigated the relationship between older Chinese children's mathematics-related beliefs and achievement and teacher praise. These researchers found that children were more motivated by teachers praise. In my study, examples (e.g., Excerpt 9, p. 133) demonstrated that teacher affirmation and praise related to children's mathematical exploration process and product could motivate children to constructed EMEs. However, during my fieldwork across the eight classes, I only collected a few relevant examples from Cherry's class. This suggests that most teachers might not have responded to children's EMEs with an encouraging attitude in most situations. The formal and serious classroom atmosphere in HK kindergartens contribute to this phenomenon, which will be discussed later in this chapter.

The belief of timely and positive responses to some of the children's EMEs aligned with the CDC (2017). It describes one of the teacher's role as "[providing] timely and positive feedback to children" (p. 54). It suggests that teachers should encourage children "to reflect, so as to foster their learning and growth" (p. 54). Öçal and Işık (2017) have also highlighted the importance of teachers promoting children's mathematical readiness and interest by providing teacher guidance and interaction. However, most teachers in my study just provided brief compliments and recognition by saying "well done" or "good job". It seemed like they praised children's EMEs without real interest and care, which I termed as perfunctory praise and discuss as follows.

In an activity of introducing homework, Ivy gave perfunctory praise to a child's EMEs. The following instance shows the interaction:

> Ivy was teaching children how to finish the Chinese homework today, she said, "Now we learn two words "白雲/baak6 wan4 (white cloud)". Ok, be careful about the order of the strokes." While she was talking, George (5 years, 3 months) used his finger to point at the poster describing stroke order Ivy has displayed on the white board (see Figure 16). He said loudly, "It has 12 strokes, 12 strokes." Ivy looked at George and said, "[You mean] the character "雲/wan4 (cloud)?" George nodded his head and smiled. "You calculated [the sum of the strokes] as 12. Really? Fine, that's 叻/lak6 (impressive and smart)." (FN/080119)

Figure 16

Explanation of Stroke Order

When discussing this instance in the interview, Ivy noted that "I think the praise I gave him was "敷衍/ fu1 jin2 (perfunctory)" at that time…to make him stop talking." She admitted that she had provided praise in a perfunctory manner. That is, she provided a compliment without really commending the carefulness that she perceived from George's exploratory EMEs (i.e., being curious and sensitive to discovering the number of strokes of the Chinese character that Ivy was teaching). Ivy explained:

> He kept saying that. It was very annoying. He is the child who would disturb the classroom [discipline], so if you don't praise him, he won't stop talking.
>
> (Ivy/OK/Whale Class/TI2/050319)

The terms "annoying" and "disturb" employed by Ivy indicated that George's action had irritated her. She perceived talkativeness as a personal trait of his along with a constant need for praise. Ivy made it clear that his talking was a key factor that caused her to feel annoyed. In this case, she viewed his spontaneous mathematical action as a "misbehaviour" which upset the classroom order, hampered her teaching process (Sun, 2015) and violated the implicit norms or expectations of classroom behaviour (Sun & Shek, 2012). However, she praised the child by saying his response was "叻/lak6 (impressive and smart)".

It was clear that the real purpose of using praise in this situation was to end the child's EME which Ivy perceived as disruptive. Her responsibility to manage the class and maintain discipline might have led Ivy to respond this way to George. At the same she had to guide the whole-class activity (as will be discussed later). By praising George, Ivy could continue introducing the stroke order of Chinese characters. However, a perfunctory attitude was potentially confusing for children. After Ivy praised George, another two children carried out a similar spontaneous mathematical action, which could have been motivated by Ivy's compliment. However, Ivy responded differently by ignoring the children, as she believed they were just showing off. Based on these findings, I assert that teachers should not respond to children's EMEs in a perfunctory way, as such response might be confusing for children.

A Facilitator to Extend Children's EMEs. All teachers in my study viewed facilitating children's mathematics learning as one of their responsibilities. However, during my period of fieldwork, I observed very few instances of teachers extending children's EMEs. The following excerpt shows one of them, in which Olive responded to some children's counting and extended it via a physical activity.

> Olive used the form of a "bowling game" to make children practise ball skills. The aim of the bowling game was for children to knock down Lego pieces with the ball. When all the Lego pieces had been knocked down, the game was over. While she was randomly placing 12 pieces of Lego on the ground as the target, some children were also counting the number of Lego pieces on the floor. Later, she came up with a mathematical activity built upon the children's interests in counting. After each child had played once, she used instructional questions (e.g., "How many Lego pieces are on the floor?", "How many Lego pieces did he/she knock down?", and "How many Lego pieces are left now?") to inspire children to make calculations using the number of Lego pieces. The children were very happy in the process. They actively engaged in calculating how many Lego pieces were knocked down and left and responding to Olive's questions. (FN/230119)

In the process, I observed that the children smiled, laughed, and were very happy. They actively engaged in making calculations with the Lego and responding to Olive's questions. This indicated that they enjoyed the learning moment Olive had created.

There were four main factors that enabled the children to enjoy their EMEs being extended by Olive. First, Olive built on children's interest in counting to extend children's EMEs. In the interview, Olive believed that mathematics learning could be based on children's interest in mathematics. Her highlighting of children's interests indicated that she recognised and valued the children's interests in carrying out spontaneous counting during the bowling game. Hyun and Marshall (2003) have explained that teachable moments occur when teachers observe, discern, and interpret children's spontaneous play interests. Olive's values in relation to authentic context and children's interests in mathematics drove her to capture this "good opportunity". Olive's beliefs also aligned with the CDC (2017), which indicate that "[mathematics learning] activities should match children's interests. Teachers can create [authentic] learning contexts and select suitable objects and teaching aids to facilitate children's learning" (p. 40).

Second, the process was playful and fun. Olive noted that with the involvement of mathematics, she hoped the bowling game would also become playful. In this case, she responded to children's EMEs during the facilitation of a formal learning activity (i.e., practising ball skills through the bowling game). Amber (FK/Lemon Class/TI2/160319) shared a similar idea to Olive. She also liked using children's EMEs to make physical education more exciting and thereby facilitate children's engagement. Thus, I argue that teachers' extension of EMEs provided potential for children's mathematics learning and facilitated children's learning in other areas (e.g., physical skills) as well as extending teachers' teaching.

Third, children's mathematics learning in this excerpt took place in an authentic context (i.e., the bowling game took place during physical education). Olive was also aware of this factor. She distinguished the authentic context from the artificial context addressed in children's homework. She believed that an authentic context was better for children to integrate mathematical concepts than the written tasks assigned in the homework. Olive's belief aligned with the CDC (2017), which indicates that "contextualised learning can help children become aware of the relationship between mathematics and their life" (p. 39). Furthermore, "teachers should make use of daily activities and play to introduce and consolidate mathematical concepts as appropriate" (p. 40).

Fourth, children's EMEs were extended by Olive within the ZPD. When extending children's EMEs, age-appropriate mathematical concepts appeared to be a factor Olive considered. Vygotskian research is underpinned by the belief that the most powerful forms of learning take place when children are working within a ZPD (Thompson, 2013). In this case, Olive's actions could be viewed as extending children's EMEs within their ZPD. According to the pre-determined mathematics learning objectives for K3 children, at the time this instance took place, children were practising subtraction in their homework. Thus, Olive might have considered the concept of subtraction as an age-appropriate concept and believed that introducing it to children aligned with the pre-determined mathematics learning objective. The CDC (2017) highlights that "learning of mathematical concepts should be in line with children's cognitive development" (p. 39).

Moreover, Olive extended the children's EMEs through scaffolding when she asked mathematics-related questions during the bowling game. She explained her questioning process:

> Each time after a child played, I asked them, "How many pieces of Lego did he knock down?" and "How many pieces of Lego are left?" [For example,] when children said that they had pushed down four, of course, they didn't know they could obtain the answer through calculating 12 minus 4 equals 8, but they would know to count the rest of the Lego pieces on the floor. I think in the process, I want them to experience the idea of subtraction. That is, although they didn't know the subtraction algorithm, at least they knew that every time Lego pieces were knocked down, the number of Lego pieces will decrease. (Olive/FK/Grape Class/TI2/091118)

Olive unpacked the idea of subtraction as two relevant questions: "How many Lego pieces did he knock down?" and "How many Lego pieces are left?". These two questions reflected the processes involved in subtraction. Olive used the bowling game to contextualise subtraction.

As discussed in Chapter 2, from the perspective of Vygotsky's (1987) spontaneous and scientific concepts, teacher guidance is necessary to build a structure for formal mathematics on the foundation of children's everyday mathematics. Olive explained why she intentionally responded to children's EMEs.

> I came up with that idea at that moment because I think it was a good opportunity. In most situations, children do the subtraction in their homework through writing or drawing, and it is rare to take place in the authentic context…It was a good opportunity…I saw them counting, and as they were interested [in mathematics], so I made use of it. (Olive/FK/Grape Class/TI2/091118)

She used the phrase "good opportunity" to describe the children's spontaneous counting during the bowling game. That she noticed and recognised this counting made it possible for her to facilitate children's mathematics learning through extending children's experience of subtraction. Her description of "I came up with that idea at that moment" and "it was a good opportunity" aligned with the concept of "teachable and learnable moments" proposed by Bäckman (2016). In Bäckman's (2016) study, she found that "'here-and-now' situations provide teachable and learnable moments" for children's learning (p. 223).

Drawing on the instance of extending EMEs co-constructed by Olive and the children, I argue that teachers can create learnable moments and take advantage of the teachable moments that arise (Cheeseman, 2015). However, this study found that only a few teachers caught the "good opportunity" or teachable moment to extend children's EMEs. Reasons for this relate to the kindergartens' structured and tight timetables (as will be discussed later).

Roles of Peers in EMEs

In line with the importance of peer interaction as discussed in Chapter 2, it is not surprising that peers played an essential and diverse role in children's construction of EMEs. In this subsection, I identify two salient roles played by peers in enabling and constraining EMEs, including (i) a cooperative social partner in mathematics-related situations and (ii) a playmate and play facilitator in mathematics-related play. I argue that peers are essential in providing a more affording social environment than one that involves teachers for enabling, extending, or constraining children's construction of EMEs in kindergartens.

A Cooperative Social Partner in Mathematics-Related Situations. Peers are essential social partners for children in kindergartens. Many instances discussed in Chapter 3 showed that children often had informal mathematical talk with peers and cooperated with peers to cope with mathematics-related situations. Data analysis indicated that as social partners, children's responses to their peers' EMEs were more sincere and honest than those teachers who provided perfunctory responses to children's EMES (as discussed on pp. 201-203). Rogoff (2006) has explained that playing with peers may empower opportunities to explore ideas in a more equal relationship than with adults, where adult-child power relations may disrupt reciprocity. In my study, if children were not interested in the mathematics-related talk and activities initiated by their peers, they did not make any response to or join them. Moreover, they rarely praised other children's EMEs like teachers did. Thus, I argue that when constructing EMEs with peers, children are exposed to a more authentic social environment in which they could obtain more sincere and honest responses than those provided by teachers.

As social partners, children sometimes used mathematics to complete a cooperative activity or task with their peers. Cooperation forms the foundation of human culture (Rogoff, 1990). The beginning of cooperation with peers is representative of children entering peer culture (Brownell et al., 2006). As discussed in Chapter 2, peer culture refers to activities that children create and share with their peers (Corsaro, 2009). In my study, children sometimes cooperated to complete an activity or task (e.g., toilet time). For example, in my fieldwork, during children's timetabled toilet time in the Whale Class and Octopus Class, children had to queue outside the toilet and wait for their turn. At this time, I often observed that some children who had finished using the toilets/urinals started counting unoccupied toilets/urinals while washing their hands, perhaps to explain the delay between them leaving and others coming in. They used finger-number gestures and verbal mathematical speech as a sign to show how many of the children who were waiting outside could come in. The children who were waiting outside also started counting. Their EMEs referred to using numbers to exchange mathematical information (i.e., the number of unoccupied toilets/urinals and children waiting outside). In doing so, a precise number of children could use the toilets/urinals at a time.

This instance was an example of showing how children collectively organised their EMEs. The teachers reported that they had never asked the children to do this. Almost all children in the two classes actively involved themselves in this EME. They developed and shared the same objective, which was to use the toilets/urinals effectively and efficiently. The process indicated that they understood that the use of number knowledge would help them reach this collective objective.

With mathematics-related peer cooperation, children sometimes made their games competitive and fun. Children from the Grape Class often played the rock, scissors, paper game when they had free time. The excerpt below records how Sophie (5 year, 2 months) used finger-number gestures to track scores in a rock, scissors, paper game.

> While waiting to go home, two girls decided to play the rock, scissors, paper game. Sophie said, “Let me help you track the score”. During the game, Sophie only used finger-number gestures to track the score without saying any words. Her finger-number gestures (see Figure 17) changed when either of the two players won another round. At the end of the game, she said, “three to two, you win (pointing at the girl who got three points)”. (FN/051218)

Figure 17

Using Finger-Number Gestures to Track the Score of a Rock, Scissors, Paper Game

Sophie’s EME involved using the concept of number to help her peers track the score of their game. She used two hands to represent each player and lifted a finger every time a player scored a point. The process of adding a finger each time a player scored a point demonstrated her understanding of the concept of addition. She understood that the final finger-gestures on each hand showed the final score for each player. She translated the finger-gestures into numbers and accurately announced the final scores. She also employed the concept of bigger and smaller numbers to compare the final score and announced that the player who got the most points won the game.

Before Sophie proposed her idea, her peers were playing by themselves. Sophie joined in the game by suggesting that she could take the role of scorekeeper. In this situation, taking on a mathematics-related role in the game helped her to join in with her peers and her action of tracking the score made the game competitive and playful. I observed that both players were happy after Sophie announced the result and continued to play for several rounds. As noted in Chapter 2, playful peer interactions can lead children to develop new understandings and thinking, and more complex play and sense making about concepts, including mathematical concepts (Williamson et al., 2020). In Sophie's case, all the children involved practised number-based concepts through playing the role of either scorekeeper or player.

A Playmate and Play Facilitator in Mathematics-related Play. As noted in Chapter 2, play was an important part of children's everyday life, including at kindergarten. Play created opportunities for children co-construct EMEs through their mathematical explorations. In play, peers are the main playmates and play facilitators for children in kindergartens.

The excerpt below shows how Ada played the role of playmate and play facilitator by providing mathematical ideas to support her imaginative play with Samuel and Betty. I described three brief versions of this excerpt with different focuses (see p. 138, 158, & 160) in Chapter 4.

> Ada, Samuel, and Betty were playing in the library corner. Samuel and Betty each held a book in their hands, pretending to be a steering wheel, and played the role of driver. They sat side by side against the wall, turning the "steering wheel" and talked while they were driving. At this point, Ada said, "The driver should sit in the front." Betty moved forward and said, "I'm sitting in the front." Samuel also moved forward but sat next to Betty. Then Betty moved forward again to show that she was in the front. Because of the limited space, Samuel couldn't move any more. At this point, Ada also held a book without moving to anywhere, she said, "Here is the front. I'm also sitting in the front." Samuel looked at her and said, "Here is the front too." Then the three of them played the role of the driver in their own positions. (FN/140319)

This vignette demonstrated Ada's understanding and experience of driving a vehicle in everyday life. Vygotsky (1978) explained, "Whenever there is an imaginary situation, there are rules" (p. 95). Ada understood that a rule of driving is that drivers sit in the front of a vehicle so she proposed that "The driver should sit in the front".

Ada's suggestion motivated Samuel's and Betty's exploration of the concept of space and location (i.e., front). In response to Ada's suggestion, Samuel and Betty adjusted their positions and moved forward. This indicated that they understood that that the word "front" refers to a location in space. Moving forward was connected with the state of being in front. However, due to the limited space of the library corner, Samuel could not move forward any further and yet he still claimed to be at the front in his space. Vygotsky (1977) has asserted that the pretend play allows children freedom from the constraints of the real world that surrounds them, and children recreate real-life events regardless of the fact that they take place in an imaginary situation of the play. In this case, Ada defined the place where she sat could count as the front. Samuel accepted Ada's definition and also viewed the place where he sat as the front. This excerpt indicated that peers, and their ability to engage together in pretend play, played an important role in providing a mathematical focus for children to explore in play. Therefore, children's play is an affordance for their EMEs.

In their play with other children, children sometimes used their mathematical speech to guide and facilitate their peers' play behaviours. One day, I observed Adam (3 years, 8 months) engaging in construction play. Through his play he demonstrated his knowledge of classification and the shape of a car, as he used interlocking stickle bricks construct a red car (see Figure 18). While Adam was constructing, his peer Chad (3 years, 10 months) took an interest in what Adam was making. Without asking for permission, he joined Adam's play by taking a blue stickle brick from the basket and adding it to Adam's car.

Figure 18

A Car Constructed by Two Children

Adam allowed Chad's participation, as he did not ask Chad to leave. However, Chad's participation had disrupted his design. Thus, Adam said, "No, I just want red one." After explaining his plan to build a red car, Adam said to Chad "You gave the red ones to me. [I need] three, no, four [pieces]." Chad followed Adam's verbal instructions by selecting four red stickle bricks from the basket and passing them to Adam. Later, Chad was keen to add more to the car, so while Adam was attaching green wheels, he selected two red wooden blocks from another basket and put them on the top of car (see Figure 18). However, this disrupted Adam's design again. Adam took them away, pointing at the car and said, "No, just this one. It's done now" (FN/210119).

This instance of informal EMEs was based on Adam's playful EMEs, as he used interlocking stickle bricks to express the mathematical concepts of classification, shape, and design. In addition to demonstrating his mathematical knowledge, Adam also demonstrated his social abilities. He used "social speech" (Vygotsky, 1987), which is external communication used to talk to others, that involved mathematical information as a strategy to play with and guide Chad's behaviour in the play he had initiated. With Chad's cooperation, they built a red car desired by Adam.

However, classroom observations showed that children's ideas about formal mathematics learning sometimes constrained their peers' playful EMEs, particularly in some K3 classes. In my study, formal mathematical ideas referred to highlighting conventions and correctness in using mathematics and emphasising the use of mathematics in ways taught by teachers. For example, in the following excerpt, Leon (5 years, 10 months) used formal mathematical ideas to restrain Joshua's (5 years, 4 months) playful mathematical actions.

> During the free-choice activity before the morning assembly, Joshua and Leon chose to solve three mathematical problems prepared by teachers on a whiteboard in the mathematics corner. This required making calculations and writing the answers in the answer section (see Figure 19). Joshua wrote a "5" in the answer section as the answer to the first problem. He smiled and said to Leon, "It's a snake." However, Leon wiped his answer, and said, "It's an ugly '5', just write slowly". Joshua did not say anything but wrote the 5 in the answer area again, which looked better than before. Then Joshua wrote a small "4" in the upper-left corner of the answer section as the answer. Leon wiped off his answer again. Leon said, "It's too ugly. Why [is this 4] so small? There are so many spaces here." While answering, "I like it." Joshua wrote a bigger "4" than before in the centre of the answer area. After Joshua finished the last problem, he passed the pen to Leon and left. (FN/181218)

Figure 19

Two Children in the Mathematics Corner

In the process of completing the calculation problems, Joshua tried to make the process playful and creative. For example, after writing the answer to the first problem, he described the "5" written by him as a snake. His smile indicated that he was happy with his work. Rubin et al. (1983)

have explained that children's playful behaviour is internally motivated and involves self-imposed goals, with a tendency for children to attribute their own meanings to objects and behaviours. In contrast to Joshua, Leon seemed to take on the role of teacher during this interaction. He used writing conventions to correct Joshua's work (i.e., writing numbers in a formal and standard way and writing numbers in the centre of the answer area). This process indicated that Leon's ideas about formal mathematics constrained Joshua's playful mathematical actions.

When I discussed this instance with Lily during the interview, she explained Leon's behaviour:

> For homework, we often ask children to practice writing numbers. There are grids in the homework books, and we require children to write the numbers in the grid. That is, they can't write too large or too small, so that the handwriting will be neat. I think because Leon had this kind of experience in writing numbers according to norms, he required Joshua to do so…just like what we did to them. However, Joshua is a child who normally won't pay much attention to these norms. (Lily/OK/Whale Class/TI2/120319)

This excerpt indicated that children sometimes used ideas related to formal mathematical learning to restrain their peers' playfulness. In this sense, children's understanding of formal mathematics can act as an external constraint on children's playful EMEs.

Children in my study, particularly those K3 children, viewed being on alert for teachers approaching while their peers were playing with mathematics as an important responsibility of playmate. As noted earlier, teachers sometimes did value or permit children's EMEs. In this case, they would stop children and ask them to focus on teacher prescribed tasks or activities. However, as discussed in Chapter 4, these responses did not discourage children. Instead, children would turn some of the EMEs that were not allowed and valued by teachers into underground experiences invisible to adults. During such times, playmates took on the role of guard to keep an eye out for teachers. I argue that teachers' unsupportive and discouraging attitudes create a classroom atmosphere that leads to this phenomenon. This is discussed below.

Classroom Atmospheres

The kindergarten classroom is a place where children interact with peers and teachers to carry out activities and play. The classroom atmosphere is the emotional climate of the learning environment created by teachers (Ma, 2016). According to Confucianism, teachers are authority figures who have an essential role in creating the classroom atmosphere in HK kindergartens. Thus, teachers' interactions with children are essential for identifying the classroom atmosphere. In this study, I argue that the combination of rules and discipline, teachers' beliefs and practices about play, teaching, and learning, and timetable and routines create classroom atmospheres that either constrain or enable children's EMEs. The predominant underlying atmosphere in the two kindergartens was formal and serious.

Formal and Serious Atmospheres

I identified having a formal and serious atmosphere as a key factor that influenced teachers' unsupportive and discouraging perceptions and practices regarding children's EMEs and constrained the availability of children's EMEs in kindergartens. I used the term formal to refer to teaching and learning in kindergartens that was structured and prescribed by teachers and curriculum documents. Additionally, I used the adjective serious to describe the formal atmospheres of the kindergartens in HK. Some teachers in my study also used the word serious to describe their approach to teaching mathematics, which aligned with Confucian ideology. As discussed in Chapter 2, Confucianism considers education to be a serious and sober matter (Bai, 2005). In my study, a formal and serious atmosphere was usually less flexible and playful.

My interpretation of data showed that teachers constructed formal and serious classroom atmospheres by providing instructions for children and guiding routine activities in accordance with timetables. I generated three relevant themes for unpacking features of formal and serious classroom atmospheres: rules and discipline, pre-determined mathematics learning objectives, and timetables. They are discussed as follows.

Rules and Discipline. I identified obeying rules and discipline as the first feature of formal and serious classroom atmospheres. According to Charles (2001), rules and discipline refer to children's misbehaviour and teachers' actions to enhance children's appropriate behaviour during class. As noted in Chapter 2, in HK kindergartens, it is important for children to listen to teachers, obey rules, and comply with teachers' discipline. In this subsection, I argue that teachers' focus on classroom rules and discipline may constrain children's EMEs.

Almost all teachers in my study stated the importance of discipline and children following the rules, particularly during teacher-directed or task-oriented activities. To maintain classroom discipline, many teachers expected children to concentrate on prescribed tasks, sit quietly, and listen carefully. Many instances of EMEs outlined earlier showed that if teachers perceived children's EMEs as off-task and inappropriate, they usually redirected those EMEs. Chan (2012) has explained that obeying the rules and discipline are necessary skills for preparation for primary schooling. In this study, most K3 teachers' pedagogy aligned with Chan (2012). Furthermore, many K1 teachers reported that learning to obey rules and rules and discipline were important, not only to prepare children for primary education, but also their learning at K2 and K3 grades.

Large class size and the limited number of teachers in classrooms were key factors that influenced teachers' emphasis on rules and discipline. As discussed in Chapter 3, although the CDC (2017) advocates a teacher-child ratio of 1:11, the main teacher-child ratio in the two kindergartens in this study showed higher ratios. According to Amber and Olive, there were no extra teachers allocated for their classes. I observed that they worked alone to guide all activities within three hours at the kindergarten. The reason for not allocating extra teachers could be because Amber and Olive taught K3 children, and there were only 14 children in both classrooms. Thus, the principal may have assumed that they did not need or qualify for further assistance. However, this could lead to issues related to the class size, as a large class size means that teacher knows fewer about individual child (Crouch et al., 2007). Furthermore, this could also lead to issues related to teachers' wellbeing. I noticed that both Amber and Olive did not even have time to go to the toilet during school hours.

Due to the emphasis on rules and discipline, and having to manage a large group of children, teachers sometimes did not allow children to construct EMEs by talking about and playing with mathematics. For example, in the EME described on p. 189, the teacher, Iris, sought to end Gary's playful counting. In the interview, Iris explained that she expected Gary to sit quietly and listen to her carefully, as paying attention to what the teacher says is an important discipline that she expected all children to obey, especially when she was introducing something new. However, as noted in Chapter 2, Vygotsky (1978) has indicated the advantages of encouraging conversations among children in classrooms and the learning that can occur with more knowledgeable others being able to help children enhance their understandings. Interactive conversations may enhance children's communication skills, build confidence, and above all, improve children's critical thinking skills (Rose & Rogers, 2012). Moreover, as a result of teachers' expectations in relation to discipline and following the rules, children might think that talking about and playing with mathematics are harmful for their learning.

Pre-determined Mathematics Learning Objectives. I identified pre-determined mathematics learning objectives as the second feature of a kindergarten's formal and serious atmosphere. As discussed in Chapter 1, the CDC (2017) notes some developmental characteristics or learning objectives of children at different ages, including learning related to mathematics. The findings indicated that these developmental characteristics or pre-determined mathematics learning objectives influenced teaching and learning in kindergartens in two main ways, as noted earlier in this chapter. Firstly, teachers considered pre-determined mathematics learning objectives when designing the classroom environment. Secondly, pre-determined learning objectives sometimes led to teachers' unsupportive and discouraging attitudes towards EMEs. I view these factors as evidence of the manifestation of formal and serious kindergarten atmospheres. This subsection elaborates on teachers' unsupportive and discouraging attitudes towards EME as evident in their verbal communication with children. I argue that pre-determined mathematics learning objectives constrain children's EMEs.

When observing children's play and activities, teachers reported that they often categorised children's concepts and knowledge in relation to the six learning areas (including mathematics) into under, appropriate to, and beyond children's age levels. These three categories aligned with the CDC (2017), which categorised pre-determined learning objectives by children's age. Accordingly, teachers demonstrated expectations of children in relation to these three age levels which underpinned their practice in relation to the way they perceived and responded to children's EMEs.

As discussed earlier on pp. 199-201, through praising and encouraging children's EMEs, teachers showed children what kinds of mathematics were valued by them. Further data analysis indicated that most teachers valued and paid attention to the mathematics that was appropriate to children's age level. They viewed those children who were able to apply age-appropriate mathematics as "叻/lak6 (impressive and smart)". This view aligned with Coben (1998), who indicated that being "mathematically knowledgeable" is often treated as an indicator of general intelligence.

In contrast, teachers thought that the mathematics children used in their play was usually under the children's age level. As a result, most teachers, particularly K3 teachers, did not value children's mathematical play. Ivy expressed her attitude:

> I look down on this kind of mathematics. […] They already know how to apply [them]. I don't care, or I think I have taught you, […] so there was nothing special that they were able to apply […] I don't think there was any reason not knowing [how to] compare at their [K3] age, and it should be basically done at K1 age. (Ivy/OK/Whale Class/TI2/050319)

The terms "look down upon", "don't care", and "nothing special" indicated Ivy's uncaring and discouraging attitude towards the mathematics applied by children that she viewed as being under their age level. Ivy indicated that she perceived children's mathematical play as being of low value and less important than formal mathematics learning. Ivy's thoughts aligned with Seo and Ginsburg's (2004) study as noted in Chapter 2. They found that teachers rarely joined children's play where their mathematical experiences were constructed. Thus, considering children's EMEs as under or appropriate to children's age level might be a factor that influences teachers' decisions to not join children's play and extend children's EMEs.

Regarding the mathematics beyond children's age level or pre-determined mathematical learning objectives, some K1 teachers showed approving but less caring attitudes. In the interview with Iris (OK/Dolphin Class/TI2/110319), I played a video-recording of a child's counting. In the video, a child from her class was counting the 15 children waiting to go home. After watching the video, Iris stated that "the [mathematical learning objective in the] curriculum only requires children to count from one to seven. He makes me think that he is 叻/lak6 (impressive and smart)." Her comment indicated that her judgement of the level of children's mathematics was based on pre-determined mathematics learning objectives. She was impressed by the child's action of employing mathematics that was beyond the requirement of the curriculum. However, even so, she preferred not to respond to the child, as she thought it was unnecessary to pay attention to this kind of mathematics.

Some K3 teachers also did not pay attention to the mathematics employed by children beyond their age level. In the interview with Olive, while talking about several children's calculating actions, which showed understanding beyond their age level, during snack time, she indicated:

> I think there is no need to care much about addition after 20. […] They already know that [the sum of] addition was becoming more and more, so I think it's enough. (Olive/FK/Grape Class/TI2/091118)

She used the word "no need to care much" and "enough" to address her attitude to the mathematics applied by the children. This indicated that her judgement of the appropriate level of mathematics that children at this age should employ was based on that indicated by the pre-determined mathematics learning objectives.

According to the findings discussed in this subsection, I argue that within a formal and serious classroom atmosphere, pre-determined mathematics learning objectives may lead to teachers' unsupportive and discouraging attitudes, perceptions, and practices regarding EMEs. Most teachers only valued and paid attention to the mathematics they thought was appropriate to children's age level. In this case, they might not value and overlook some aspects of children's mathematical competencies, which constrain children's EMEs. In addition to pre-determined mathematics learning objectives, the influence of routine activities prescribed in timetables is addressed next.

Timetables. Within the formal and serious classroom atmosphere, everything adhered to timetables. As a cultural tool used in HK kindergartens, I viewed timetables as an indicator of a formal and serious atmosphere. In this subsection, I argue that the focus of academic activities in kindergarten timetables is characteristic of a formal and serious atmosphere and constrains children's EMEs.

The timetables of the two kindergartens placed much time on teacher-directed learning activities. Moreover, to reach pre-determined learning objectives, time allocation for teacher-directed learning activities were usually fixed and not flexible. For example, as described in Chapter 3, some classes had 25 minutes for physical education, music, or art activities, 25 minutes for English or Mandarin activities; and 40-65 minutes for circle time or group learning activities. Moreover, children from all K3 classes had to do homework, which played an important role in kindergarten life. Although the time for homework was not highlighted in the timetables, according to my observation, all the K3 teachers arranged a special time (around 20-30 minutes) as a whole class activity each day for children to do their homework. As noted in Chapter 1, in HK, children from half-day classes usually attended kindergartens for three hours each day. Of these 180 minutes, children had to attend teacher-directed learning activities for 110-145 minutes in total.

A further dimension to the importance of teacher-directed learning activities is evident in the degree of support it receives from kindergarten principals. At the kindergartens in this study, during circle time or group learning activities, extra teachers were allocated to the two K1 classes at FK and all four classes at OK. The extra teachers were to make sure children did their homework properly and to speed up the transition time to activities. As introduced in Chapter 2, teachers required children to complete some parts of their homework while at kindergarten. Children could not proceed to the next activity until they finished their homework. Thus, assistance was an important part of ensuring that this routine was enacted within the time allocated.

Highlighting the importance of teacher-directed learning activities led to little and limited time placed on children's play and, therefore, constrained children's EMEs in the two kindergartens. My observations showed that due to the restriction of timetables, time for "free play" and free choice activities that enabled children to play and thereby construct EMEs was limited. At FK and OK, some classes only allocated either 25 or 40 minutes for children to play in interest corners or common area each day. Other classes' timetables did not schedule free choice activity as an independent activity but combined it with teacher-directed learning activities or other routine activities (e.g., assembly and snack time). If the objectives of learning activities could not be achieved within the timeframe, teachers sometimes used the time for free choice activities. In this case, "free play" and free choice activities seemed to function as transition activities between different teacher-directed learning activities at the two kindergartens. Time for children's play, and the construction of EMEs through play, both depended on and was restricted by the progress of teacher-directed learning activities.

Child Friendly and Less Serious Atmosphere

Occasionally the timetable allowed for children's play. As discussed in Chapter 4, many instances of children's EMEs took place during "free play" or free choice activities and diverse routine activities (e.g., snack time and transitions) that allowed children to play, talk, and interact with other people. Although these occasions happened less often than activities directed by teachers, they were endowed with an atmosphere that was pleasant and conducive to child-initiated activities and play. The CDC (2017) noted that children are allowed to guide their own actions within "a pleasant and joyful learning atmosphere" (p. 63). In my study, I termed this kind of classroom atmosphere as child friendly and less serious. I used the term less serious not only as a complement to child friendly, but also to contrast with the serious atmosphere discussed earlier. Rusdinal and Afriansyah (2017) have indicated that a "conducive" (or child friendly and less serious in this study) classroom atmosphere could "make children become more open and flexible" (p. 289). In contrast to the formal and serious atmosphere discussed earlier, I argue that a child friendly and less serious classroom atmosphere enables children to construct EMEs. I identified minimal teacher interference as the main characteristic of this atmosphere and elaborate below.

Most teachers, particularly those from OK, reported that whether or not children constructed their mathematical experiences, they preferred not to join in children's "free play" and free choice activities. For example, Rose (OK/Shark Class/TI2/260219) explained, "Actually, unless an accident occurs, […] we should not participate in children's 'free play'". However, as discussed in Chapter 2, sociocultural theory has stated that children's development of mathematical thinking is influenced by teachers' guidance and assistance (Van Oers, 2010). Vygotsky (1987) stated that scientific concepts require introduction by another more knowledgeable person, including teachers. "Good-fit" (Trawick-Smith et al., 2016) teacher interactions were a factor that influenced children's mathematics learning during play.

One factor that influenced teachers' views about not joining in children's play, was a government project that included workshops on "free play". In support of the launch of the latest curriculum document, the OK kindergarten joined the government's *On-site Professional Support Service-Promoting Learning through Free Play* (EDB, 2017) in the 2017/2018 academic year. Many teachers from OK had attended workshops organised by the government. According to teachers, the speakers at the workshops suggested that unless an accident occurred, teachers should not participate in children's "free play". According to the CDC (2017), this promotes "children's autonomy and free participation" (p. 199). Although decisions not to get involved in children's play provided some opportunity for children to play and construct EMEs without adult interference, Lee and Ginsburg (2009) have asserted that teachers should actively assist children to advance beyond their everyday mathematics rather than "step back, and let the children play" (p. 40).

Some teachers believed that decreasing teachers' involvement in "free play" was one way to promote "children's autonomy and free participation" (CDC, 2017, p. 199). In the interview with Rose, she highlighted:

> Sometimes if teachers intervene, it can negatively impact children making their own rules for play. They will wait for you. [For example, they may say,] well, how should we play? […] Then, it will turn into the play directed by me rather than the spontaneous play. (Rose/OK/Shark Class/TI2/260219)

Rose perceived teacher response as direct involvement or intervention. She believed that teachers' direct responses would have a negative effect on children's spontaneity in using mathematics to make rules for play.

Daisy shared the same idea as Rose. In the interview, she highlighted that "as a teacher, I will not provide any suggestions for children's 'free play'". She explained:

> The "free play" is not for me to direct them to play, but for them to explore by themselves. […] They can apply existing knowledge to discover and explore, […] including mathematics. For example, if [they] create different shapes with Popsicle sticks, [they] are exploring mathematical concepts. [They may explore] how to keep the shape stable…these are all…mathematical concepts. […] If I teach him/her to play, it will turn out to be my ideas. (Daisy/OK/Octopus Class/TI2/040319)

Daisy's words "I will not provide any suggestions for children's 'free play'" and "if I teach him/her to play, it will turn out to be my ideas" indicated that she might perceive teacher response as a teacher-directed response. Moreover, she highlighted the importance of providing opportunities for children to discover and explore mathematics by themselves through play. She expected children to take more ownership of applying their knowledge, including mathematical knowledge, through play rather than requiring teacher direction. Daisy's thoughts aligned with Landreth (2002) who has indicated that self-guided play allows children to pursue play interests that hold personal meaning.

In addition to "free play" and free choice activities, as noted in Chapter 2, although much of children's time in HK kindergartens is spent following prescribed timetables and teacher-directed activities, I argue that children nevertheless found opportunities to play and engage in EMEs. I observed that children's play and playful moments, including instances of children's EMEs, took place in almost all routine activities in kindergartens, particularly during activities that were non/less teacher-directed (e.g., snack time, transitions, and toilet time). In non/less teacher-directed activities, teachers rarely participated in children's play and playful moments, which provided children with opportunities to guide their own play and construct EMEs with little interference from teachers.

The value of "child-centredness" highlighted by the CDC (2017) influenced teachers' decision to not participate in children's play and playful moments, thereby creating opportunities for children to construct EMEs. For example, Violet preferred not to disturb children's playful EMEs during free-choice activity. She explained:

> I always respect and follow child-centeredness. When they are playing, I will not say or do anything. […] Setting up learning resources is not just about learning. For example, the skill of ordering is just a concept that the teacher wants you to learn, but the way children play can be various. They can [use the resource] to tell a story or tell [other people that] They like eating chicken. I think the range [of how children manipulate the learning resources] is very wide. (Violet/FK/Apple Class/TI2/280219)

She believed that adopting child-centredness referred to no involvement in children's play and respecting the variety children created in their play. However, her ideas did not align with the CDC (2017), which indicates that guided by the core value of child-centredness, teachers should "give [children] timely guidance, so as to stimulate children's interest and confidence in learning" anytime in kindergartens (p. 62). Because the CDC (2017) has not detailed the meaning of child-centredness, this could lead to teachers' limited understanding of the concept and appropriate pedagogy.

Iris believed that teacher responses sometimes had a negative influence on children's emotional states in play. After watching a video of a child counting, she explained why she preferred not to make any response to the child:

> I am afraid that I would scare him… [he might think that] shouldn't I count at this time? [I am afraid that] it would affect his interest [in counting] …He might feel stressed (laughing). […] He is a child who always looks like a frightened bird (laughing). […] Thus, I would rather do something that will not affect him, that is… "Do no harm" (Iris said these words in English). (Iris/OK/Dolphin Class/TI2/110319)

Iris compared teacher responses to harming a child. Not wanting to cause harm, stopped her from responding to the child.

Some teachers believed that decreasing teacher involvement would encourage peer interaction. Iris believed that children would gain more benefits from communication with peers than with teachers. She explained:

> I think children will have more responses [when interacting with peers than with teachers]. Because my role [in the classroom] is a teacher, once I talk, they will just listen…even I ask them questions. They will talk more when communicating with peers. […] Sometimes I prefer not to disturb [them]. I would like them to talk more with their friends. (Iris/OK/Dolphin Class/TI2/110319)

As can be seen from this subsection, teachers' decisions to not participate in children's play provided opportunities for children to play without teachers' interference, which enabled children to construct their EMEs without teachers' involvement.

Chapter Summary

This chapter has presented and discussed the affordances and constraints of children's EMEs in kindergartens and teachers' perceptions and practices regarding children's EMEs. The findings were presented and discussed in three sections.

In the first section, I analysed affordances and constraints related to the physical environment through three subsections. Firstly, I investigated children's opportunities to engage with resources prepared by teachers and construct related EMEs. The findings showed that how children used teacher prepared resources enabled their EMEs. Secondly, I used interest corners as an example to discuss influences of classroom settings on children's EMEs. I found that through the platform of the interest corners created by teachers, children had opportunities to bring their out-of-kindergarten experiences into their play at kindergarten, thereby constructing EMEs. Thirdly, I explored opportunities for EMEs provided by daily resources and settings in kindergartens that were available all the time and not directly connected to teachers' planning. The findings indicated that daily kindergarten resources and settings provided many opportunities for children to construct EMEs.

In addition to the physical environment, the social environment in kindergartens also provided affordances and constraints that influenced children's EMEs. Therefore, in the second section, I discussed the diverse roles of teachers and peers that enabled and constrained children's EMEs in kindergartens. Firstly, I identified five roles of teachers in children's EMEs: a fellow mathematical inquirer, an outsider, a playmate in mathematics-related play, a social partner in mathematical conversations, and a facilitator to extend children's EMEs. I found that although teachers were important for enabling and extending children's EMEs by playing diverse roles, they also constrained children's EMEs by not always encouraging and valuing EMEs. Secondly, I identified two peers' roles: (i) a cooperative social partner in mathematics-related situations and (ii) a playmate and play facilitator in mathematics-related play. The findings showed that peers were essential in providing a more affording social environment for children's EMEs than the one that involved teachers.

In the last section, I explored the impact of classroom atmosphere on children's EMEs. I unpacked features of classroom atmosphere through addressing two relevant themes: (i) formal and serious atmosphere and (ii) child friendly and less serious atmosphere. I identified formal and serious atmosphere as a key factor that influenced teachers' unsupportive and discouraging perceptions and practices regarding children's EMEs and constrained children's EMEs in kindergartens. In contrast, child friendly and less serious classroom atmosphere enabled children to construct their EMEs in kindergartens.

Later in the conclusion chapter, I will address more details about the key findings and arguments presented in this chapter with the intention of answering the second research question, *What are the affordances and constraints that influenced the availability of children's EMEs?* And the third research question, *What are teachers' perceptions and practices with regard to children's EMEs?*

Chapter 6: Conclusion

This study investigated and theorised children's everyday mathematical experiences (EMEs) within the context of kindergartens in Hong Kong (HK), where formal mathematics teaching was dominant. Play, where children prominently constructed their EMEs, was scheduled as an activity within limited time. As a result, children in my study sometimes used transitions (e.g., lining up) as opportunities to engage in playful EMEs with their surroundings. Within this context, the investigation and theorisation of EMEs by my study offers a response to the problem of the vague and insufficient explanations of the terminology "everyday" used in the HK Curriculum Development Council (CDC) (2006, 2017) (Chapter 1, p. 9). This problem leads to issues around teachers' misunderstanding of how to facilitate children's learning in relation to everyday experiences and apply the pedagogical approaches referred to in curriculum documents, which further contributes to inconsistencies between curriculum documents and classroom practices (Chapter 1, p. 10). These problems and issues were the catalysts for my study.

Sociocultural theoretical concepts shaped and framed my study. I defined EMEs as *children's experiences in informally and spontaneously applying and exploring early mathematical concepts and knowledge with surrounding objects, peers, and teachers or adults in play and everyday activities*. The three research questions in relation to EMEs of my study were:

i) What are the nature and content of children's EMEs in HK kindergarten settings?

ii) What are the affordances and constraints that influence the availability of children's EMEs?

iii) What are the affordances and constraints that influence the availability of children's EMEs?

Answering these questions involved the process of exploring the nature and content of children's EMEs, factors that enabled and constrained children's EMEs, and teachers' perceptions and practices regarding children's EMEs. Through this exploration, I:

i) investigated features indicated in children's EMEs;

ii) explored the mathematical concepts and knowledge used by children to construct their EMEs;

iii) identified factors that afford children to construct their EMEs;

iv) identified factors that constrain children to construct their EMEs;

v) explored teachers' perceptions and practices regarding children's EMEs;

vi) investigated factors that shape teachers' perceptions and practices regarding children's EMEs.

Drawing on my investigation and interpretation, this thesis presented findings about EMEs of children from two HK kindergartens. I involved children's and teachers' perspectives in illustrating children's EMEs within the sociocultural context of HK.

This chapter provides a summary of the study along with discussion of its conclusions, contributions, and implications. I begin the chapter with a brief overview of the study encapsulating key ideas of each chapter, including the three research questions that guided my study. Following this overview, I summarise key findings and arguments in response to each research question. I then focus on the study's contributions and implications before stating the limitations of my study and suggestions for future research. I conclude this chapter with my personal reflections.

Study Overview

In Chapter 1, I introduced the background information of kindergarten education and early childhood mathematics education in HK. I identified that the inconsistency between curriculum documents and practices that aligned with the CDC (2017) might constitute tensions for children's construction of their learning experiences through everyday activities and play in HK kindergartens. I also provided the rationale for the study by highlighting personal interests and experiences and the importance of investigating children's EMEs within the context of HK.

In Chapter 2, I began by clarifying why I adopted sociocultural perspectives, rather than nativism and constructivism, to guide my study. I explored what previous literature and the HK curriculum documents contributed to understanding children's mathematical learning in formal and informal situations, leading to an operational definition of children's EMEs for my study. I summarised methods of investigating children's informal and spontaneous mathematical experiences used in previous empirical studies, which provided a basis for the research design of my study. I also illustrated the role of EMEs in children's learning from four aspects: (i) I discussed the importance of EMEs in children's current activities and play and in HK as readiness for future mathematics learning; (ii) I stated the discontinuity between children's EMEs in out-of-school and school settings, which called for; (iii) the importance of teachers' roles and responsibilities in relation to children's EMEs; and (iv) I identified the challenges of promoting children's EMEs by teachers in HK kindergartens.

I also addressed the theoretical underpinnings of my study in Chapter 2. I outlined seven theoretical concepts based on, or related to, sociocultural theory and explained their relevance to children's EMEs. These included informal learning, sociocultural perspective of play and emotions, spontaneous and scientific concepts, ZPD, cultural tools, Confucian views of education, and concepts of affordances theory to discuss (i) learning and teaching from sociocultural perspectives, (ii) cultural and historical aspect of sociocultural perspectives, and (iii) sociocultural views of affordances theory. Based on the above theoretical concepts, and informed by gaps identified through literature review, the three research questions noted earlier were developed.

In Chapter 3, I presented the methodology and methods. I employed a qualitative case study drawing on ethnographic techniques to investigate children's EMEs. The fieldwork took place for seven months in eight half-day classes in two HK kindergartens. There were 79 children and 9 teachers who voluntarily participated in my study. A range of strategies, including data triangulation, enhanced the study's trustworthiness. I adopted ethnographic techniques (i.e., classroom observation, individual semi-structured interviews, documentation, and reflective journaling) to gain in-depth understanding of EMEs. I viewed ethical considerations as essential to guide every decision throughout my study. Regarding the data analysis procedure, I identified key themes from the findings through thematic analysis (Braun & Clarke, 2006).

Chapters 4 and 5 were two findings and discussion chapters. Chapter 4 presented and discussed the findings regarding the nature and content of children's EMEs. I identified exploratory, playful, informal, and spontaneous as four types of EMEs. I unpacked indicators or features for each type of EMEs. I concluded Chapter 4 by bringing together the range of mathematical concepts and knowledge explored and employed by the children in my study. Chapter 5 presented and discussed the affordances and constraints of children's EMEs in kindergartens and teachers' perceptions and practices regarding children's EMEs. I analysed affordances and constraints related to the physical and social environment of kindergartens. I also explored the impact of classroom atmosphere on children's EMEs.

Key Findings and Arguments

This chapter shed light on the importance of my investigation of EMEs within the kindergarten context. I revisited and addressed the research questions in turn by drawing upon the key findings and arguments noted in Chapters 4 and 5.

Relating to the Nature and Content of Children's EMEs

The key findings regarding the first research question *What are the nature and content of children's EMEs in HK kindergarten settings?* included two parts. First, regarding the nature of children's EMEs, the findings pointed out that there were four types of EMEs in kindergartens, which I termed as exploratory, playful, informal, and spontaneous EMEs. Children's exploratory EMEs (Chapter 4, p. 114) reflected their mathematical curiosity and sensitivity, which motivated them to make and solve self-directed mathematical inquiries. I argued that by constructing exploratory EMEs during classroom experiences, children could satisfy their curiosity, discover mathematical discoveries, find and solve practical problems independently, and understand their surroundings.

While constructing playful EMEs (Chapter 4, p. 123), children often used different languages and resources to express mathematics. Their expressions of diverse emotions, particularly positive emotions, also characterised playful EMEs. I argued that through constructing playful EMEs in kindergartens, children could create and practice diverse ways of using mathematics and gain diverse emotional experiences through engaging with mathematics.

The EMEs constructed by children in kindergartens were all informal (Chapter 4, p. 143). There were two indicators or features of informal EMEs, including informal mathematical talk and out-of-kindergarten mathematical experiences. I argued that through constructing informal EMEs, children could engage in interpersonal interactions, develop self-esteem, and build on out-of-kindergarten mathematical experiences to extend EMEs or facilitate informal mathematics learning in kindergartens.

Spontaneous EMEs (Chapter 4, p. 153) were the fourth type of EMEs, which linked with children's mathematical spontaneity and mathematics-related underground culture. I argued that children's construction of spontaneous EMEs indicated that they were agentive in constructing cultures that are different from those of teachers.

Second, regarding the content of children's EMEs, children of this study used a range of mathematical concepts and knowledge to construct EMEs in kindergartens (Table 9, pp. 158-160). Table 9 contrasts with the summary of pre-determined mathematical concepts and knowledge that teachers were responsible for teaching (Table 10, p. 161). The mathematics concepts and knowledge explored by children in EMEs covered, and were broader than, the mathematics learning objectives pre-determined by teachers and kindergartens. I argued that the range of pre-determined mathematics learning objectives did not align with the range of mathematical concepts and knowledge that children used to construct their EMEs.

Through addressing and discussing findings in response to the first research question *What are the nature and content of children's EMEs in HK kindergarten settings?* I argue that children's construction of different types of EMEs is important and valuable for them to learn and practise a range of mathematical concepts and knowledge.

Relating to Affordances and Constraints

The key findings regarding the second research question *What are the affordances and constraints that influence the availability of children's EMEs?* are related to the physical environment, social environment, and classroom atmosphere.

Regarding the affordances related to the physical environment, the resources made available by teachers afforded children's EMEs, as children had opportunities to engage with resources prepared by teachers and construct related EMEs in kindergartens. Additionally, classroom settings, such as interest corners, also played an important role in enabling children's EMEs. An excerpt of three children's dramatic play described on pp. 175-181 demonstrated that while playing in interest corners, children demonstrated mathematical knowledge and sociocultural understanding of mathematical concepts appropriated from out-of-kindergarten contexts. I argued that through the platform of the interest corners created by teachers, children had opportunities to bring their out-of-kindergarten experiences into their play at kindergarten, thereby constructing EMEs. Furthermore, children's EMEs were also afforded by daily kindergarten resources and settings (Table 12, pp. 182-183), as their mathematical curiosity and sensitivity motivated them to explore surroundings from mathematical lenses. I argued that the richness of daily kindergarten resources and settings enabled children's mathematical curiosity and sensitivity and, therefore, their EMEs.

Regarding the constraints related to the physical environment. When preparing the resources for children, teachers expected, and sometimes required, children to practise the knowledge and skills they thought those resources would provide children (Table 11, p. 170). However, in most situations, the purposes prescribed by teachers were not obvious or important to children when interacting with teacher prepared resources. Thus, I argued that it was how children used resources prepared by teachers, rather than the resources themselves, that enabled EMEs. Furthermore, due to a lack of funding to buy resources and teacher authority, children's choices of and opportunities for engaging with teacher prepared resources were limited and not always available, which sometimes constrained children's EMEs.

Regarding the affordances related to the social environment, peer and teacher interactions that created a child-friendly and less serious classroom atmosphere influenced children's EMEs. Peer interactions enabled children to construct EMEs through the role of cooperative social partner in mathematics-related situations. Children could use mathematics to engage in cooperative activities and games with peers. Also, as the cooperative social partner, children's responses to their peers' EMEs were sincere and honest. That is, if children were not interested in the mathematics-related talk and activities initiated by their peers, they did not make any response to or join them. Second, peers were also a playmate and play facilitator in children's EMEs, as children could use their mathematical speech to guide and facilitate their peers' play behaviours. In addition, teachers could also enable children's EMEs by playing diverse roles (Chapter 5, pp. 185-207) in kindergartens. However, these kinds of moments usually took place during "free play" or free choice activities and diverse routine activities that teachers allowed children to play, talk, and interact with other people. I argued that underlying teacher interaction, a child friendly and less serious classroom atmosphere (Chapter 5, pp. 221-225) enabled children to construct EMEs.

Through addressing and discussing findings in response to the second research question *What are the affordances and constraints that influence the availability of children's EMEs?* I argue that although children's EMEs can be enabled by the kindergarten context, they sometimes are constrained and not permitted in kindergartens.

Relating to Teachers' Perceptions and Practices

The key findings regarding the third research question *What are teachers' perceptions and practices with regard to children's EMEs?* were that to allow or support children's constructions of EMEs, teachers who said that they valued and encouraged children's EMEs: (i) provided timely responses; (ii) chose to be an outsider by allowing children to construct EMEs without adults' interference; (iii) provided resources for children to extend their EMEs during play; (iv) showed children what kinds of mathematics they valued by praising and encouraging children's EMEs; and (v) caught the "good opportunity" or teachable moment to extend children's EMEs. In response to teachers' support and encouragement, children in my study sometimes invited teachers to be their playmate during EMEs-related play, repetitively engaged in similar EMEs valued by their teachers, and enjoyed the learning moments that their teachers had created.

Teachers who valued and encouraged less children's EMEs showed unsupportive and discouraging attitudes to children's EMEs by providing brief compliments and recognition without real interest and care, which I termed as perfunctory praise (Chapter 5, pp. 201-202) in my study. They also redirected children's construction of EMEs by asking them to focus on teacher prescribed tasks or activities. However, teachers' unsupportive and discouraging responses did not discourage children. Instead, children in my study would turn some of the EMEs that were not allowed and valued by teachers into underground experiences invisible to teachers (Chapter 4, pp. 153-158).

Through addressing and discussing findings in response to the third research question *What are teachers' perceptions and practices with regard to children's EMEs?* I argued that a deeper understanding of teachers and policy makers on teachers' roles, responsibilities, and knowledge regarding children's EMEs is needed to improve teachers' corresponding responses. Furthermore, drawing on key findings and arguments regarding the three research questions, as discussed in Chapter 4 and 5, the contributions my study makes are addressed next.

Contributions to Theories and Knowledge

My study was the first in-depth qualitative case study investigation into children's EMEs in HK kindergarten settings, and as such, contributes to current theories and knowledge in relation to children's EMEs. As noted in Chapter 2, although previous studies have investigated children's informal and spontaneous mathematical experiences, they have not clearly defined and justified these kinds of experiences. My study adds to prior research by adopting seven theoretical concepts based on, or related to, Vygotskian sociocultural theory (Chapter 2, p. 55) to critically explore and present a theorised concept of EMEs. Thus, I viewed the theorisation and definition of EMEs as the most significant contributions of my study. Below I synthesise the notion of children EMEs into four key points to align with the key findings of the four types of children's EMEs addressed earlier.

First, exploratory EMEs. The idea of spontaneous concepts (Vygotsky, 1978) enabled me to understand children's construction of exploratory EMEs as their formation of mathematics-related spontaneous concepts within the sociocultural context. According to Vygotsky (1987), children learn spontaneous concepts from everyday life, which are usually based on empirical observations of the way things look or feel. Children in my study actively obtained mathematical information that interested them from their immediate social and physical context by using their senses. Motivated by their mathematical curiosity and sensitivity, children's formation of spontaneous concepts or exploratory EMEs showed a sociocultural basis.

The concept of cultural tools (Vygotsky, 1981) also enabled me to understand children's exploratory EMEs as they independently used cultural tools to solve mathematical inquires. Firstly, children used their mathematical knowledge as a cultural tool, a cultural tool that has been "created over time by human beings" (Forsström & Afdal, 2020, p. 35), to construct exploratory EMEs. They appropriated mathematical knowledge through interactions with their peers, teachers, and family members. Secondly, private speech also enabled children to construct exploratory EMEs. Vygotsky (1987) viewed language, including private speech, as an important symbolic tool, which he considered is a product of a child's social environment. Instances of EMEs described in the first section of Chapter 4 demonstrated that through using private speech, children could extend their mathematics learning by practising the mathematical knowledge they learned from their social environment.

Second, playful EMEs. Drawing on sociocultural perspective of play (Rogoff, 1990; Vygotsky, 1978) and the notion of cultural tools (Vygotsky, 1981), I understood children's construction of playful EMEs as socially and culturally embedded practices and play. Children in my study demonstrated their competence in playfully using oral language, music, body language, written language, and objects as cultural tools to express mathematics. Tracy and Zita's instances of EMEs (Chapter 4, p. 125) demonstrated that their knowledge of the Chinese number system and expressions in English and Chinese contributed to their playful EMEs. In the process, private speech enabled their engagement with mathematics via oral language in a playful way.

Additionally, framed by sociocultural perspective of emotions (Vygotsky, 1934/1999), children's expressions of diverse emotions, particularly positive emotions, while engaging in mathematics provided insights for their playful EMEs. As noted in Chapter 2, Williamson et al. (2020) have argued that children's social and emotional learning "is substantial and cognitively demanding work" (p. 199). The findings related to children's playful EMEs showed that with a sense of humour and playfulness, children actively constructed playful EMEs associated with positive emotions by playing with mathematics. This can be evidence for Williamson et al.'s (2020) argument and adds value to Vygotskian views of emotions.

Play created opportunities for children to co-construct playful EMEs through their mathematical explorations. The two excerpts described on pp. 210-212 show how children used their mathematical speech to guide and facilitate peers' play behaviours. These two excerpts are evidence of the zone of proximal development (ZPD) (Vygotsky, 2011) in which children learn best from more knowledgeable others, including more advanced peers, who can scaffold or guide their exploration and learning.

Third, informal EMEs. Drawing on the concept of informal learning (Rogoff et al., 2016), I understood children's construction of informal EMEs as their informal learning in mathematics. Rogoff et al. (2016) have stated, "Informal learning is interactive and embedded in meaningful activity" for children and the "involvement builds on individual initiative, interest, and choice"(p. 359). In my study, children's informal mathematical talk often took place during snack time and transition time, which was intertwined with their mathematical discoveries, everyday activities in kindergartens, prior mathematical knowledge, and out-of-kindergarten experiences.

Drawing on the notion of spontaneous concepts (Vygotsky, 1978), I also viewed children's construction of informal EMEs as their formation of everyday concepts. Children in my study were keen to engage in informal mathematical talk with peers and adults and used it in their construction of informal EMEs in kindergartens. Vygotsky (1987) characterised young children's concepts as spontaneous concepts developed in collaboration with others through everyday activities.

Fourth, spontaneous EMEs. In academic activities prescribed by teachers and non-academic activities (e.g., snack time) when teachers demonstrated serious demeanours, children in my study sometimes spontaneously initiated mathematics-related actions with independent intentions and purposes. With the intention of having fun and reducing boredom, they constructed underground EMEs and excluded teachers from their EMEs (Chapter 4, pp. 153-158). As noted in Chapter 2, Rogoff et al. (2016) have explained that "in the 'underground' informal learning in schools, students' creative ways of addressing constraints are especially appreciated by their classmates, if not by their teachers" (p. 389).

In discussion the four types of EMEs, it became clear that some Confucian beliefs and values afford children's construction of EMEs. For example, as noted in Chapter 2, the goal of education framed by Confucianism is to help children develop perfect characters (Kim, 2007). In my experience, I know that many Chinese parents and teachers place emphasis on children's perfect characters regarding eating and height. They value eating habits such as taking big bites of food, positively thinking that children who are good eaters will grow to be tall. Taking big bites of food leads to being healthy, and height is regarded as a sign of being good looking. Thus, it was understandable that children sometimes actively showed their puffy cheeks with food to teachers during snack time and talked about their "big bites" with others (Chapter 4, p. 118), and in John's case (Chapter 4, p. 141), he was interested in the concepts of age and height while playing Lego. These two examples indicated that children might be sensitive to the big bites and height valued by teachers. Therefore, Confucian beliefs, such as developing perfect characters, sometimes enabled children's EMEs.

On the other hand, Confucian beliefs and values can also constrain children's construction of EMEs. Many instances of EMEs outlined in chapter 5 showed that if teachers perceived children's EMEs as off-task and inappropriate, they usually redirected those EMEs and expected children to concentrate on prescribed tasks, sit quietly, and listen carefully. These indicated that Confucian values regarding rules and disciplines can constrain children's construction of EMEs.

The discussion of four types of EMEs also provided insights into understanding the value of children's EMEs or informal mathematics learning. As noted in Chapter 2, the value of informal learning is often overlooked "because of its assumed inferior opposition to formal learning" (Hedges, 2021, p. 4). My study found that EMEs were valuable and important for children. Through constructing EMEs in kindergartens, children could explore mathematical ideas, satisfy mathematical curiosity and sensitivity, make sense of surroundings, experience joy from playing with mathematics in ways desired and created by them, communicate with peers and adults, bring out-of-kindergarten mathematical experiences to kindergartens, demonstrate mathematical spontaneity, and solve practical problems. All these benefits showed the power and value of children's informal learning in the aspect of mathematics, which can be viewed as a contribution to theorising values of children's informal learning (Rogoff et al., 2016).

Implications for Teachers and Policy Makers

This section draws out the educational implications of this study for teachers and policy makers, including teacher professional learning, teacher practices, and policy making.

Teacher Professional Learning

Teachers' perceptions and practices regarding children's EMEs that I summarised guided me to propose that teacher professional learning is an implication of my study. To increase teachers' sensitivity and recognition regarding EMEs, it is important to provide teacher professional learning opportunities for teachers to understand and learn what children EMEs look like in kindergartens. My study provides an interpretation of the nature and content of children's EMEs, affordances and constraints that influence children's EMEs, and teachers' own perceptions and practices regarding EMEs. These ideas provide a reference for teachers to develop a better understanding of children's EMEs and empower teachers to recognise the value of EMEs and their potential for children's mathematics learning.

Teacher Practices

To create a child friendly and less serious classroom atmosphere that enables children to construct EMEs, teachers can benefit from reflecting on their attitudes and responses to children's EMEs. As discussed earlier, teachers' unsupportive and discouraging attitudes drove children to turn some of the EMEs that were not allowed and valued by teachers into underground experiences invisible to teachers. In which case, children's construction of EMEs was both constrained to certain contexts, such as queuing for the bathroom, and not accessible to teachers. My study explicitly outlines the roles of teachers in enabling and constraining children's EMEs (Chapter 5, pp. 185-207), which provides a research-based tool for teachers to examine or reflect on their classroom practices.

By catching the "good opportunity" or teachable moment, there were times that teachers in this study could create learnable moments for children to extend their EMEs (Chapter 5, p. 204). Thus, it is possible for teachers to develop the sensitivity to teachable moments and extend children's EMEs. Examples noted in Chapter 5 (p. 204) demonstrated that through building on children's EMEs, engaging in a playful and fun process, facilitating children's mathematics learning in an authentic context, and adopting the pedagogical approach of scaffolding within children's ZPD, children would enjoy their EMEs being extended by teachers and achieve the same outcomes as teachers.

Policy Implications

As discussed in Chapter 2, the CDC (2017) values children "using mathematical concepts to solve practical problems in everyday life" (p.39). This idea echoed the informal mathematics learning, or EMEs, explored by my study. However, the CDC (2017) has not provided a precise description of how to facilitate children's informal mathematics learning or extend children's EMEs. Instead, it only describes mathematics learning as a progressive process and suggests that teachers should teach mathematical concepts with increasing levels that align with children's cognitive development. In this regard, children are more encouraged to develop mathematical concepts that rely on formal and systematic teacher instruction and teachers may dismiss the value of children's

EMEs.

Drawing on the issues noted above, my study provides evidence that can be drawn on for curriculum policy making in HK. The theorisation of EMEs has made explicit four types of EMEs constructed by children in kindergartens and the indicators or features of each type of EMEs. I have also discussed physical and sociocultural affordances and constraints that influence the availability of children's EMEs. These ideas provide a research-based tool for the CDC in HK, which has responsibility for creating and promoting curriculum documents, to make supplementary explanations with more foci on specific pedagogical guidelines for EMEs and to promote the understanding of children's EMEs among early childhood teachers.

Limitations of the Study

No matter how carefully a study is designed and conducted, there will always be some limitations that need to be acknowledged (Bloomberg & Volpe, 2012). However, limitations may also open up space and opportunities for future research. Despite the study's potential limitations, my study provides an in-depth investigation of children's EMEs in two HK kindergartens. I identify three limitations of my study and elaborate on them as follows.

The first limitation is the researcher's subjectivity. In my study, I was the only person who handled the data collection, recording, and interpretation of the data. As my cultural background and past experiences influenced the selection, understanding, and presentation of the data, there was a potential for bias. Thus, self-reflection and ongoing reading and learning was essential to ensure the understanding of the data collected. I also discussed the research plan, coding schemes, and data analysis procedures with supervisors before proceeding. In doing so, I may have minimised this limitation.

The second limitation refers to the nature of my study. This qualitative study was an exploratory study in kindergartens. Hence, the findings cannot be interpreted for every aspect of children's EMEs. For example, because my study did not involve parent interviews or observations at children's homes, it was unknown what mathematical experiences children engaged in at home that they brought to kindergartens (as noted in Chapter 4, pp. 151-152). However, this exploratory study will enable me to set a strong foundation for exploring ideas about children's EMEs in the future.

The third limitation of my study was the relatively small sample. As a qualitative case study, I only conducted my study in two kindergartens. This limitation restricted the possibility of generalising (Stake, 2005) findings to other kindergartens and settings. However, the aim my study was to address the issue of transferability (Lincoln & Guba, 1985) rather than generalisability. As a result, Lincoln and Guba (1985) suggested that "thick and rich" description and comprehensive information concerning the context and background of the study contribute to its practical application in other contexts.

Future Research

Drawing on the limitations outlined above, there are two main suggestions for future research. First, it is anticipated that more EMEs-related studies will be conducted to investigate EMEs (i) with children at other age levels; (ii) in other non-profit making and private kindergartens from other districts of HK; (iii) at family or other out-of-kindergarten settings; and (iv) in other Asian and non-Asian cultural contexts. This is because my study only theorised and examined EMEs of 3- to 4-year-old and 5- to 6-year-old children from two non-profit-making kindergartens located in two out of 18 districts of HK. In doing so, future research could (i) provide deeper insights into understanding children's EMEs, (ii) show that a qualitative study (e.g., my study) can be transferred to other contexts, and (ii) gain holistic understanding of children's experiences that inform their EMEs.

Second, subsequent research in HK could be undertaken using different types of qualitative studies, as this study was an exploratory case study which mainly focused on exploring the phenomenon of children's EMEs in HK kindergartens. Relevant findings indicated that children's EMEs were important and valuable for children to explore mathematical elements embedded in their surroundings, support social interactions, facilitate play and informal learning, and practise agency. Thus, future research could use the indicators for the four types of EMEs, affordances and constraints of EMEs, and teachers' perceptions and practices regarding EMEs that I identified to undertake more in-depth exploration about the effects of these factors on children's EMEs or informal mathematics learning. Outcomes of qualitative studies might add a teacher professional learning dimension to support teachers to understand the nature, content, and value of EMEs and facilitate their perceptions and practices.

Personal Reflections

I chose to study children's EMEs because of my fascination with children's interest in using their own ways to learn and engage with mathematics in kindergartens. This inspired me to explore what was happening underlying these kinds of children's mathematical actions. However, my adult role and previous professional experiences as a kindergarten teacher have challenged me as a researcher to explore children EMEs from the perspective of respecting and valuing children's informal learning. For instance, I found myself in situations where I regretted what I did and had wanted to avoid, such as acting like a teacher or adult who played an authoritative role. For example, during classroom observations, I aimed to approach children and learn their process of constructing EMEs as an interested adult, rather than as a teacher. At first, I assumed that children's construction of EMEs linked to their play and agency, which might be against the rules and restrictions set by teachers (e.g., sit properly and keep quiet). Hence, I tried to be different from their teachers. However, there were several times that I noticed that children immediately stop talking when I approached them. I listened back to audio-recordings and checked my reflective journals to reflect on this phenomenon. I noticed that I did not set up any rules for them. However, I realised that without the children's permission, I sometimes talked about their EMEs, which I identified as spontaneous EMEs later, with their teachers in front of them. Thus, those children who hid their talk from me might have viewed me as a traitor, or as a teacher-like adult.

I viewed this instance as an opportunity through which children taught me how to respect and value their informal learning in mathematics. I realised that by only focusing on avoiding rules but using an authoritative way to share children's EMEs with others, I reacted similarly to the role of teacher or adult which I had aimed to avoid. As a result, I might have lost the trust of some children and missed opportunities to access their spontaneous EMEs. After this realisation, I avoided discussing children’s underground EMEs with their teachers in front of them and noting children’s names.

Looking back to where I started and where I stand now, I am excited about the journey children allowed me take with them in their construction of EMEs and will keep reflecting on my views of education. As a former kindergarten teacher in HK and an adult who grew up in a Chinese family, the educational beliefs that I held before I started my PhD Journey were similar to many Chinese teachers and parents in that I thought children learnt best through formal education. However, through my PhD journey, I came to realise that there should be more than one way to facilitate children’s learning. Children in my study were confident and competent learners in their informal mathematics learning in ways that are currently unrealised in HK kindergartens. Nevertheless, with the development of teachers’ professional learning, I believe that teachers and other early childhood practitioners can see potential and value in children’s informal learning.

Appendices

Appendix A: Consent Form for Principals

Consent Form (Principal)

(This form will be held for a period of six years)

Research Title: Children's everyday mathematical experiences (EMEs) in Hong Kong kindergarten settings

Name of Principal Investigator/Supervisor: Dr Angel Chan

Name of Student Researcher: Ye Zhang

I have read the Participant Information Sheet, and I have understood the nature of the research and why the kindergarten, teachers, and children have been invited. I have had the opportunity to ask questions and have the researcher answered to my satisfaction. In giving my consent:

- I agree the kindergarten to participate in this research.
- I understand that this kindergarten's participation is voluntary.
- I understand that the researcher will invite the teachers with their consent and the children with their parents'/guardians' consent and their assent to participate in this research.
- I agree to give my assurance that whether or not teachers, children, and parents/guardians are willing to participate in this research will not affect their employment status (the teachers) or their relationship with myself as the principal (the children and parents/guardians) and the families' access to the kindergarten services.
- I agree / do not agree (please choose one) to provide access for the researcher to read the kindergarten's documents relating to this research, and photocopy approved sample pages at the researcher's expense.
- I understand that the researcher will not form part of the teaching ratio and not teach the children.
- I understand that the process of classroom observation will include photographing, video-recording, and/or audio-recording in line with permissions obtained.

- I understand that research activities will be suspended if any safety issues become apparent.
- I understand that the content of two individual semi-structured interviews will not be disclosed to me.
- I understand that the researcher and her University supervisors can access all data.
- I understand that the data will be transcribed and translated by the researcher.
- I understand that anonymity cannot be guaranteed. However, the researcher will make every effort to protect the identity of the participants and kindergarten.
- I understand that all electronic data, hard copy data, and signed consent forms and assent forms will be securely stored for a period of six years, after which they will be securely destroyed.
- I understand that the selected data will be used in the researcher's PhD thesis, and may be used to support future academic publications as well as conference presentations.
- I wish to choose this pseudonym ________________________ to represent the kindergarten's name.
- I wish / do not wish (please choose one) to receive a summary of findings, which can be provided to me at this email address: ________________________.

Name: ____________________________ Position: ____________________________

Name of kindergarten: ____

Signature: __________________________ Date: _______________________________

Approved by the University of Auckland Human Participants Ethics Committee on 06-July-2018 for three years. Reference Number 021583

Appendix B: Consent Form for Teachers

Consent Form (Teacher)

(This form will be held for a period of six years)

Research Title: Children's everyday mathematical experiences (EMEs) in Hong Kong kindergarten settings

Name of Principal Investigator/Supervisor: Dr Angel Chan

Name of Student Researcher: Ye Zhang

I have read the Participant Information Sheet, and I have understood the nature of the research and why I have been invited. I have had the opportunity to ask questions and have the researcher answered to my satisfaction. In giving my consent,

- I agree to participate in this research.
- I understand that my participation in this research is voluntary.
- I understand that I am free to withdraw myself at any time without giving a reason, and I have the right to ask the researcher to unconditionally destroy all the information that could be identified as my contribution until (date).
- I understand that I will not be able to ask for data collected from the classroom observation to be destroyed.
- I agree / do not agree (please choose one) to be photographed by the researcher during classroom observation.
- I agree / do not agree (please choose one) to be video-recorded by the researcher during classroom observation.
- I agree / do not agree (please choose one) to be audio-recorded by the researcher during classroom observation.
- I agree / do not agree (please choose one) to provide access for the researcher to read my planning documents and other documents relating to this research, and photocopy approved sample pages at the researcher's expense.

- I wish / do not wish (please choose one) to receive a copy of my interview transcript and the translation of interview excerpts and review, edit, and make additional comments within 2 weeks from the time I receive the materials.
- I understand that the researcher will not form part of the teaching ratio and not teach the children.
- I understand that the kindergarten (principal) has given the assurance that whether to participate or not will not affect my employment status.
- I understand that I can ask the researcher to stop photographing, video-recording, audio-recording, and/or conversing my practice at any time during classroom observations.
- I understand that selected photographs and/or video footage of children's EMEs with and without my participation will be used in the second individual semi-structured interview.
- I understand that two individual semi-structured interviews will take place at a time and venue that suit me.
- I understand that two individual semi-structured interviews will be audio-recorded by a digital voice recorder.
- I understand that I can ask the researcher to suspend the individual semi-structured interviews and/or audio-recording at any time.
- I understand that the content of two individual semi-structured interviews will not be disclosed to anyone without my consent.
- I understand that research activities will be suspended if any safety issues become apparent.
- I understand that the researcher and her University supervisors can assess to all data.
- I understand that the data will be transcribed and translated by the researcher.
- I understand that anonymity cannot be guaranteed. However, the researcher will make every effort to protect the identity of the kindergarten and participants.
- I understand that all electronic data, hard copy data, and signed consent forms and assent forms will be securely stored for a period of six years, after which they will be securely destroyed.

- I understand that the selected data will be used in the researcher's PhD thesis, and may be used to support future academic publications as well as conference presentations.
- I wish to choose this pseudonym ________________________ to represent my name in the publication or presentation of the research.
- I wish / do not wish to (please choose one) receive a summary of findings, which can be provided to me at this email address: ________________________.

Name: ____________________________ Class: _______________________________

Name of kindergarten: ____

Signature: _________________________ Date: ________________________________

Approved by the University of Auckland Human Participants Ethics Committee on 06-July-2018 for three years. Reference Number 021583

Appendix C: Consent Form for Parents or Guardians

Consent Form (Parent/Guardian)

(This form will be held for a period of six years)

Research Title: Children's everyday mathematical experiences (EMEs) in Hong Kong kindergarten settings

Name of Principal Investigator/Supervisor: Dr Angel Chan

Name of Student Researcher: Ye Zhang

I have read the Participant Information Sheet, and I have understood the nature of the research and why my child have been selected. I have had the opportunity to ask questions and have the researcher answered to my satisfaction. In giving my consent,

- I agree my child to participate in this research.
- I understand that my child's participation in this research is voluntary.
- I understand that I free to withdraw my child at any time without giving a reason, and I have the right to ask the researcher to unconditionally destroy all the information that could be identified as my child's contribution until the point when data analysis starts (date).
- I understand that I will not be able to ask for data collected from the classroom observation to be destroyed.
- I agree to help my child complete the Assent Form at home.
- I agree / do not agree (please choose one) my child to be photographed by the researcher during classroom observation.
- I agree / do not agree (please choose one) my child to be video-recorded by the researcher during classroom observation.
- I agree / do not agree (please choose one) my child to be audio-recorded by the researcher during classroom observation.

- I agree / do not agree (please choose one) to provide access for the researcher to read my child's mathematical learning reports and other research-related documents, and photocopy approved sample pages at the researcher's expense.
- I understand that the kindergarten (principal) has given the assurance that my decision and my child's participation or not will not impact on my child's and my relationship with the principal and on my family's access to the education and care services of the kindergarten.
- I understand that my child can ask the researcher to stop photographing, video-recording, and/or conversing at any time during classroom observations.
- I agree that selected photographs and/or video footage of children's EMEs that may involve my child will be used in the individual semi-structured interview with my child's teacher.
- I understand that research activity will be suspended if any safety issues become apparent.
- I understand that the researcher and her University supervisors can assess to all data.
- I understand that the data will be transcribed and translated by the researcher.
- I understand that my child's name will not be used in any occasions.
- I understand that anonymity cannot be guaranteed. However, the researcher will make every effort to protect the identity of the kindergarten and participants.
- I understand that all electronic data, hard copy data, and signed consent forms and assent forms will be securely stored for a period of six years, after which they will be securely destroyed.
- I understand that the selected data will be used in the researcher's PhD thesis, and may be used to support future academic publications as well as conference presentations.
- I wish / do not wish (please choose one) to receive a summary of findings, which can be provided to me at this email address: ________________________.

Child's Name: ________________________ Class: ____________________________

Name of kindergarten: ____

Parent's/ Guardian's Name: ____

Parent's/ Guardian's Signature: ___________________ Date: ____________________

Approved by the University of Auckland Human Participants Ethics Committee on …… for three years. Reference Number 021583

Appendix D: Assent Form for Children

Assent Form (Child)

(This form will be held for a period of six years)

Research Title: Children's everyday mathematical experiences (EMEs) in Hong Kong kindergarten settings

Name of Principal Investigator/Supervisor: Dr Angel Chan

Name of Student Researcher: Ye Zhang

I have been told about this research and understand what it is about. All my questions have been answered.

- I know that I do not have to participate if I don't want to and nothing will happen to me.
- I can stop Ye watching or recording me at any time, and I do not have to give a reason.
- I know that I can leave the activity or situation being observed by Ye.
- If I have any worries or if I have any other questions, then I can talk about these with Ye, my teachers, my parents/guardians, and other adults.
- I am happy / not happy to take part in the research. Please tick one:

Happy Not happy

- I am happy / not happy that Ye will photograph me. Please tick one:

Happy Not happy

- I am happy / not happy that Ye will video-record me. Please tick one:

Happy Not happy

- I am happy / not happy that Ye will audio-record me. Please tick one:

Happy Not happy

Child's name: **Parent/Guardian's name:**

Parent/Guardian's signature: **Date:**

Approved by the University of Auckland Human Participants Ethics Committee on 06-July-2018 for three years. Reference Number 021583

Appendix E: Participant Information Sheet for Principals

Participant Information Sheet (Principal)

Research Title: Children's everyday mathematical experiences (EMEs) in Hong Kong kindergarten settings

Name of Principal Investigator/Supervisor: Dr Angel Chan

Name of Student Researcher: Ye Zhang

Researcher Introduction

My name is Ye Zhang. I am a PhD candidate at the Faculty of Education and Social Work, the University of Auckland. My supervisors are Dr Angel Chan, Dr Deborah Widdowson, and Professor Helen Hedges.

Research Description

This research is being undertaken as part of my PhD project, which is about children's everyday mathematical experiences (EMEs). The concept of children's EMEs in this research refers to children's experiences in exploring early mathematical concepts (e.g., number, shape, size, weight, and pattern) with surrounding objects, peers, or adults in day-to-day activities and play. This research aims to explore children's EMEs, including teachers' perceptions of and practices associated with children's EMEs in Hong Kong kindergarten settings. I will be visiting the kindergarten for a total of 21 weeks from September 2018 to early February 2019. I will visit each class for a half-day weekly (approximately 3 hours).

Invitation to Participate I would like to invite one or two teachers from K1 and K3 half-day classes respectively and the children from these teachers' classes to participate in this research. If this research is of interest to your kindergarten, I would like to meet you at a convenient time to explain this research and the proposed nature of the participants' involvement. For your information, I have attached the Participant Information Sheets for teachers and parents/guardians, the Consent Forms for them to sign and return, an information booklet about the research for children, and an Assent Form to be completed by children at home with the assistance of their parents/guardians.

If you decide to participate, I would request your permission to leave some copies of the Participants Information Sheet and Consent Form in the secretary's office for the teachers to consider participating in this research. I would also like to seek your help to enable me to introduce this research to the teachers at a convenient time, such as at a staff meeting. If the teachers have any questions, they can contact me through the email address and phone number listed in the information sheet for them. If none of your K1 and K3 teachers decide to participate, I will not continue with the research in your kindergarten.

Once the teachers give their consent, then I will discuss with the teachers and you the best way to recruit children. In regard to children's participation, the parents'/guardians' consent will be obtained first, and then the children's assent will be requested. If the parents/guardians have any questions, they can contact me through the email address and phone number listed in the information sheet for them. I will only involve the children whose parents'/guardians' have given their consent and who have also given their assent as participants. I would also like to discuss with the teachers and you the best way and time for me to introduce myself to and share information about the research with the children through the information booklet. If less than 50% of the children in all classes decide to participate, I will not continue with the research in your kindergarten. I would also request your permission to place a sealed collection box in the secretary's office for all participants to return their Consent Forms and Assent Forms within two weeks of the receipt of their forms.

Research Procedures

I would like to invite the children and teachers to involve a section of familiarisation and classroom observation during kindergarten hours. I would also like to invite the teachers to attend two individual semi-structured interviews during non-contact hours. I am aware that having a researcher in the kindergarten may be seen as inconvenient, but I hope to minimise the influence of my presence. I will discuss my role in detail with the teachers and you. However, I hope you understand that I will not form part of the teaching ratio and not teach the children. The research procedures are described below.

Familiarisation. I would like to visit each class for a half-day weekly (approximately 3 hours) over about 4 weeks. I wish to be familiar with the regular events and experiences in each class. I will respect the routines and policies of each class and the kindergarten. I will make every effort not to interrupt the teaching and learning interactions, and the teachers' relationships with the children. As a further aspect of research, I request your approval to use the kindergarten documents relating to my research, such as the kindergarten's guidelines and suggestions for the implementation of mathematics and other research-related documents, to support the findings. I also request your approval to photocopy some sample pages at my expense for analysis later. I will check with you before I do so. I want to emphasise that my interest in the documents is to gain more information about children's EMEs, I am not in any way evaluating the kindergarten.

Classroom Observation. I would like to do the formal classroom observation in each class for a half-day weekly (approximately 3 hours) over about 16 weeks. I will not ask the teachers to set up or introduce any new or special activities, nor will I interrupt children's everyday activities/play or interactions with their peers and teachers. I will suspend research activities if any safety issues become apparent. I will use a small hand-held digital camera and/or a digital voice recorder to photograph, video-record, and/or audio-record episodes of children's EMEs with and without teachers' participation in line with permissions obtained. The participants may choose to be observed, but not to be photographed, video-recorded, and/or audio-recorded. They can indicate their choices on the Consent Forms and Assent Form. They can ask me to stop photographing, video-recorded, and/or audio-recorded at any time. Moreover, I may have informal conversations with the participants during the classroom observation. The informal conversations will be based on the content of EMEs-related activities or situations initiated by them. If I notice that they do not wish to be observed, photographed, audio-recorded, video-recorded, and/or conversed, I will cease immediately.

Non-participants (children and adults) will not be featured in this research. They will never be asked to leave an activity/play so that I can start recording. If any occasions and activities involve both the participants and non-participants, I will record my observations through photographing the participants only or writing in my field notes. If any non-participants enter the sense in which photographing, video-recording, and/or audio-recording is taking place, I will cease immediately. If I find that I have included any non-participants by mistake in any photographs, video footage, and/or audio files, I will delete the photographs on the same day and not use the video footage and/or audio files in any publications or presentations.

Individual semi-structured interview. I will invite the teachers to attend two up to 1-hour individual semi-structured interviews during non-contact hours within the last week of familiarisation (the 4th week) and the last week of data collection (the 21st week) respectively. Interviews will take place at a time and venue that suit the teachers, and they will be audio-recorded by a digital voice recorder. The teachers can stop the interview and ask me to turn off the recorder at any time during the interview. I will transcribe the interview data verbatim and analyse it in Chinese. Excerpts of the interview transcript will be translated from Chinese into English. Once the interviews have been transcribed and the interview excerpts have been translated, I will invite the teachers to review, edit, and make additional comments within 2 weeks from the time they receive the interview transcript and the translation of interview excerpts. I will make changes accordingly. I hope you understand that the content of two interviews will not be disclosed to you.

Data Storage, Retention, Destruction and Future Use

All electronic data (photographs, video footage, and audio files) will be stored on a password-protected computer and backed up on the University of Auckland server. All hard copy data will be stored in a locked cabinet at the University of Auckland. All signed consent forms and assent forms will be stored in a locked cabinet in the principal investigator’s office at the University of Auckland. All electronic data, hard copy data, and signed consent forms and assent forms will be stored for six years. After six years, they will be destroyed through the secure destruction service at the University of Auckland. All data will be transcribed and translated by me and only accessible to my supervisors and me. The selected data will be used in my PhD thesis and may be used for academic publications and conference presentations, and will not be used for any other purpose or released to any other without the participants' consent. If you would like a copy of the summarised findings of this research, please state this on your consent form.

Right to Withdraw from Participation

I wish to give you the following assurances: The participation of the children and teachers in this research is voluntary. They are free to withdraw themselves at any time without giving a reason, and they have the right to ask me to unconditionally destroy all the information that could be identified as their contributions until the point when data analysis starts (date). However, I hope you understand that they will not be able to ask for data collected from the classroom observation to be destroyed because of other participants' information on the same recording. The participants and you will have the right to check and approve the photocopies of selected relevant documents at a convenient time before the end of the data collection.

Benefits and risks of research

Through this research, the teachers may gain benefits from viewing and reflecting on their pedagogical practices through photographs and/or video footage that are relevant to their practices, and from engaging in reflective discussions about their practices with the researcher. These processes may contribute to the teachers' understandings of children's EMEs. Children may also benefit from their teachers' understandings of EMEs. As this research is a small-scale study where the researcher is present for some time, it is possible that the participants may be identified in the thesis and other publications, due to the findings presented. However, the efforts I will make to try and prevent that from happening are explained next.

Anonymity and Confidentiality

The identity of the kindergarten and participants will be treated with utmost care and respect. However, if the photographs and video footage are used, anonymity cannot be guaranteed, but every attempt will be made to preserve the confidentiality of the kindergarten and participants. The children's names will not be used in any occasions. Each teacher will be invited to choose a pseudonym to represent his/her name. I will also invite you to choose a pseudonym to represent the kindergarten's name. The photographs and video footage will be digitally altered before using them in the thesis, academic publications, and/or conference presentations. I assure that the data collected will be kept confidential and will not be released to any other without the participants' and your consent.

If you do decide to participate, I would like to request your assurance that the teachers are under no obligation to participate and that any of their decisions will not affect their employment status. I would also like your assurance that the children are free to give their assent. The parents/guardians are free to decide whether or not to allow their children to participate, and that any of their decisions will not impact on the relationship with you as the principal and on their families' access to the education and care services of the kindergarten.

If you have any further queries, please contact my supervisors or me. I do hope you will agree your kindergarten to participate in this research and provide access to the participants. If so, I would appreciate you signing the Consent Form to confirm your agreement and returning it to me within two weeks of the receipt of the form.

Contact Details and Approval

Student Researcher	Principal Investigator	Head of Department
Ye Zhang PhD Candidate University of Auckland ye.zhang@auckland.ac.nz +852 5429 3917 (HK)	Dr Angel Chan School of Curriculum and Pedagogy University of Auckland angel.chan@auckland.ac.nz +64 9 373 7999 ext 48884	Professor Helen Hedges School of Curriculum and Pedagogy University of Auckland h.hedges@auckland.ac.nz +64 9 373 7999 ext 48606

For any queries regarding ethical concerns, you may contact the Chair, the University of Auckland Human Participants Ethics Committee, the University of Auckland, Research Office, Private Bag 92019, Auckland 1142. Telephone 09 373-7599 ext. 83711. Email: ro-ethics@auckland.ac.nz.

Approved by the University of Auckland Human Participants Ethics Committee on 06-July-2018 for three years. Reference Number 021583

Appendix F: Participant Information Sheet for Teachers

Participant Information Sheet (Teacher)

Research Title: Children's everyday mathematical experiences (EMEs) in Hong Kong kindergarten settings

Name of Principal Investigator/Supervisor: Dr Angel Chan

Name of Student Researcher: Ye Zhang

Researcher Introduction

My name is Ye Zhang. I am a PhD candidate at the Faculty of Education and Social Work, the University of Auckland. My supervisors are Dr Angel Chan, Dr Deborah Widdowson, and Professor Helen Hedges.

Research Description

This research is being undertaken as part of my PhD project, which is about children's everyday mathematical experiences (EMEs). The concept of children's EMEs in this research refers to children's experiences in exploring early mathematical concepts (e.g., number, shape, size, weight, and pattern) with surrounding objects, peers, or adults in day-to-day activities and play. This research aims to explore children's EMEs, including teachers' perceptions of and practices associated with children's EMEs in Hong Kong kindergarten settings. I will be visiting the kindergarten for a total of 21 weeks from September 2018 to early February 2019. I will visit your class for a half-day weekly (approximately 3 hours).

Invitation to Participate

I would like to invite you to participate in this research. If you want to know more about this research and have any questions, you can contact me through the email address and phone number listed in this information sheet. If you agree to participate, I would like to discuss with the principal and you the best way to recruit the children in your class, and the best way and time to enable me to introduce myself to and share information about this research with the children through an information booklet. If less than 50% of the children in your class decide to participate, I will not continue with the research in your class.

Research Procedures

I would like to invite the children and you to involve a section of familiarisation and classroom observation during kindergarten hours. I would also like to invite you to attend two individual semi-structured interviews during non-contact hours. I am aware that having a researcher in your class may be seen as inconvenient, but I hope to minimise the influence of my presence. I will discuss my role in detail with the principal and you. However, I hope you understand that I will not form part of the teaching ratio and not teach the children. The research procedures are described below.

Familiarisation. I would like to visit your class for a half-day weekly (approximately 3 hours) over about 4 weeks. I wish to be familiar with the regular events and experiences in your class. I will respect the routines and policies of your class and the kindergarten. I will make every effort not to interrupt the teaching and learning interactions, and your relationships with children. As a further aspect of research, I request your approval to use your planning documents and other research-related documents, to support the findings. I also request your approval to photocopy some sample pages at my expense for analysis later. I will check with you before I do so. I want to emphasise that my interest in the documents is to gain more information about children's EMEs. I am not in any way evaluating you and your teaching.

Classrooms observation. I would like to do the formal classroom observation in your class for a half-day weekly (approximately 3 hours) over about 16 weeks. I will not ask you to set up or introduce any new or special activities, nor will I interrupt children's everyday activities/play or interactions with their peers and you at any time. I will suspend research activities if any safety issues become apparent. I will use a small hand-held digital camera and/or a digital voice recorder to photograph, video-record, and/or audio-record episodes of children's EMEs with and without your participation in line with permissions obtained. You may choose to be observed, but not to be photographed, video-recorded, and/or audio-recorded. You can indicate your choice on the Consent Form. You can ask me to stop photographing, video-recording, and/or audio-recording at any time. You can discuss with me some EMEs-related situations during the classroom observation, in this case, I may have informal conversations with you. The informal conversations will be based on the content of EMEs-related situations initiated by you. If you do not wish to be observed, photographed, audio-recorded, video-recorded, and/or conversed at any given time, I will cease immediately.

Non-participants (children and adults) will not be featured in this research. They will never be asked to leave an activity/play so that I can start recording. If any occasions and activities involve both the participants and non-participants, I will record my observations through photographing the participants only or writing in my field notes. If any non-participants enter the sense in which photographing, video-recording, and/or audio-recording is taking place, I will cease immediately. If I find that I have included any non-participants by mistake in any photographs, video footage, and/or audio files, I will delete the photographs on the same day and not use the video footage and/or audio files in any publications or presentations.

Individual semi-structured interview. I would also like to invite you to attend two up to 1-hour individual semi-structured interviews during non-contact hours within the last week of familiarisation (the 4th week) and the last week of data collection (the 21st week) respectively. Interviews will take place at a time and venue that suit you. I will prepare some open-ended questions for two interviews. The first interview will provide an opportunity for us to discuss your experiences and perceptions regarding children's EMEs. In the second interview, selected photographs and/or video footage of children's EMEs with and without your participation and the excerpts from the classroom observation and informal conversation will also be used to stimulate our discussion.

Each interview will be audio-recorded by a digital voice recorder. You can suspend the interview and ask me to turn off the recorder at any time during the interview. I will transcribe the interview data verbatim and analyse it in Chinese. Excerpts of the interview transcript will be translated from Chinese into English. Once the interviews have been transcribed and the interview excerpts have been translated, I would like to invite you to review, edit, and make additional comments within 2 weeks from the time you receive the interview transcript and the translation of interview excerpts. This process aims to check if they correctly and accurately convey your meaning in the interviews. I will make changes accordingly.

Data Storage, Retention, Destruction and Future Use

All electronic data (photographs, video footage, and audio files) will be stored on a password-protected computer and backed up on the University of Auckland server. All hard copy data will be stored in a locked cabinet at the University of Auckland. All signed consent forms and assent forms will be stored in a locked cabinet in the principal investigator's office at the University of Auckland. All electronic data, hard copy data, and signed consent forms and assent forms will be stored for six years. After six years, they will be destroyed through the secure destruction service at the University of Auckland. All data will be transcribed and translated by me and only accessible to my supervisors and me. The selected data will be used in my PhD thesis and may be used for academic publications and conference presentations, and will not be used for any other purpose or released to any other without the participants' consent. If you would like a copy of the summarised findings of this research, please state this on your consent form.

Right to Withdraw from Participation

I wish to give you the following assurances: Your participation in this research is voluntary. You are free to withdraw yourself at any time without giving a reason, and you have the right to ask the researcher to unconditionally destroy all the information that could be identified as your contribution until the point of data analysis (date). However, I hope you understand that you will not be able to ask for data collected from the classroom observation to be destroyed because of other participants' information on the same recording. The principal has given assurance that your decisions to participate or not to participate in this research will not affect your employment status.

You will have the right to check and approve the photocopies of the documents provided by you at a convenient time before the end of the data collection.

Benefits and risks of research

Through this research, you may gain benefits from viewing and reflecting on your pedagogical practices through photographs and/or video footage that are relevant to your practices, and from engaging in reflective discussions about your practices with the researcher. These processes may contribute to your understanding of children's EMEs. Children may also benefit from your understanding of EMEs. As this research is a small-scale study where the researcher is present for some time, it is possible that the participants may be identified in the thesis and other publications, due to the findings presented. However, the efforts I will make to try and prevent that from happening are explained next.

Anonymity and Confidentiality

The identity of you will be treated with utmost care and respect. However, if the photographs and video footage are used, anonymity cannot be guaranteed, but every attempt will be made to preserve the confidentiality of you. I will invite the principal to choose a pseudonym to represent the kindergarten's name. I will also invite you to choose a pseudonym to represent your name. The photographs and video footage will be digitally altered before using them in the thesis, academic publications, and/or conference presentations. I assure that the relevant data will be kept confidential and will not be released to any other without your consent.

If you have any further queries, please contact my supervisors or me. I do hope you will agree to participate in this research. If so, I would appreciate you signing the Consent Form to confirm your agreement and returning it to the sealed collection box placed in the secretary's office within two weeks of the receipt of the form.

Contact Details and Approval

Student Researcher	Principal Investigator	Head of Department
Ye Zhang PhD Candidate University of Auckland ye.zhang@auckland.ac.nz +852 5429 3917 (HK)	Dr Angel Chan School of Curriculum and Pedagogy University of Auckland angel.chan@auckland.ac.nz +64 9 373 7999 ext 48884	Professor Helen Hedges School of Curriculum and Pedagogy University of Auckland h.hedges@auckland.ac.nz +64 9 373 7999 ext 48606

For any queries regarding ethical concerns, you may contact the Chair, the University of Auckland Human Participants Ethics Committee, the University of Auckland, Research Office, Private Bag 92019, Auckland 1142. Telephone 09 373-7599 ext. 83711. Email: ro-ethics@auckland.ac.nz.

Approved by the University of Auckland Human Participants Ethics Committee on 06-July-2018 for three years. Reference Number 021583

Appendix G: Participant Information Sheet for Parents or Guardians

Participant Information Sheet (Parent/ Guardian)

Research Title: Children's everyday mathematical experiences (EMEs) in Hong Kong kindergarten settings

Name of Principal Investigator/Supervisor: Dr Angel Chan

Name of Student Researcher: Ye Zhang

Researcher Introduction

My name is Ye Zhang. I am a PhD candidate at the Faculty of Education and Social Work, the University of Auckland. My supervisors are Dr Angel Chan, Dr Deborah Widdowson, and Professor Helen Hedges.

Research Description

This research is being undertaken as part of my PhD project, which is about children's everyday mathematical experiences (EMEs). The concept of children's EMEs in this research refers to children's experiences in exploring early mathematical concepts (e.g., number, shape, size, weight, and pattern) with surrounding objects, peers, or adults in day-to-day activities and play. This research aims to explore children's EMEs, including teachers' perceptions of and practices associated with children's EMEs in Hong Kong kindergarten settings. I will be visiting the kindergarten for a total of 21 weeks from September 2018 to early February 2019. I will visit each class for a half-day weekly (approximately 3 hours).

Invitation to Participate

I would like you to invite your child to participate in this research. I will discuss with the teachers and the principal the best way and time for me to introduce myself to the children and explain why I am in their class through an information booklet. If you agree your child to participate in this research, an Assent Form will be provided for your child to be completed at home with your assistance. I will only involve your child with your consent and his/her assent. If you want to know more about this research and have any questions, you can contact me through the email address and phone number listed in this information sheet.

Research Procedures

I would like to invite your child to involve a section of familiarisation and classroom observation during kindergarten hours. In order not to influence the teaching and learning process, I will not form part of the teaching ratio and participate in any teaching role. The research procedures are described below.

Familiarisation. I will visit your child's class for a half-day weekly (approximately 3 hours) over about 4 weeks. I wish to be familiar with the regular events and experiences in your child's class. I will respect the routines and policies of the class and kindergarten. I will make every effort not to interrupt the teaching and learning interactions, and the teacher's relationship with children. As a further aspect of research, I request your approval to use your child's mathematical learning reports and other research-related documents, to support the findings. I also request your permission to photocopy some sample pages at my expense for analysis later. I will check with you before I do so. I want to emphasise that my interest in the documents is to gain more information about children's EMEs. I am not in any way evaluating your child or his/her learning.

Classroom Observation. I would like to do the formal classroom observation in your child's class for a half-day weekly (approximately 3 hours) over about 16 weeks. I will not ask the teacher to set up or introduce any new or special activities, nor will I interrupt children's everyday activities/play or interactions with their peers and teachers at any time. I will suspend research activities if any safety issues become apparent. Children's EMEs could give this research valuable information and insight. I would therefore like to include some photographs and video footage of children's EMEs in my research. I will use a small hand-held digital camera and/or a digital voice recorder to photograph, video-record, and/or audio-record episodes of children's EMEs in line with permissions obtained. I would like to request your permission to use the selected photographs and/or video footage of children's EMEs that may involve your child in the individual semi-structured interview with your child's teacher.

You may represent your child choose to be observed, but not to be photographed, video-recorded, and/or audio-recorded. You can indicate your choice on the consent form. During classroom observations, children can ask me to stop photographing, video-recorded, and/or audio-recording at any time. If children come to me and ask me to join their EMEs-related activities/play, I may have informal conversations with them. The informal conversations will be based on the content of EMEs-related activities/play initiated by them. If I notice that they do not wish to be observed, photographed, audio-recorded, video-recorded, and/or conversed at any given time, I will cease immediately.

Non-participants (children and adults) will not be featured in this research. They will never be asked to leave an activity/play so that I can start recording. If any occasions and activities involve both the participants and non-participants, I will record my observations through photographing the participants only or writing in my field notes. If any non-participants enter the sense in which photographing, video-recording, and/or audio-recording is taking place, I will cease immediately. If I find that I have included any non-participants by mistake in any photographs, video footage, and/or audio files, I will delete the photographs on the same day and not use the video footage and/or audio files in any publications or presentations.

Data Storage, Retention, Destruction and Future Use

All electronic data (photographs, video footage, and audio files) will be stored on a password-protected computer and backed up on the University of Auckland server. All hard copy data will be stored in a locked cabinet at the University of Auckland. All signed consent forms and assent forms will be stored in a locked cabinet in the principal investigator's office at the University of Auckland. All electronic data, hard copy data, and signed consent forms and assent forms will be stored for six years. After six years, they will be destroyed through the secure destruction service at the University of Auckland. All data will be transcribed and translated by me and only accessible to my supervisors and me. The selected data will be used in my PhD thesis and may be used for academic publications and conference presentations, and will not be used for any other purpose or released to any other without the participants' consent. If you would like a copy of the summarised findings of this research, please state this on your consent form.

Right to Withdraw from Participation

I wish to give you the following assurances: Your child's participation in this research is voluntary. You are free to withdraw your child at any time without giving a reason, and you have the right to ask me to unconditionally destroy all the information that could be identified as your child's contribution until the point when data analysis starts (date). However, I hope you understand that you will not be able to ask for data collected from the classroom observation to be destroyed because of other participants' information on the same recording. The principal has given assurance that your decision and your child's participation or not will not affect your child's and your relationship with the principal and your family's access to the education and care services of the kindergarten. You will have the right to check and approve the photocopies of your child's selected documents at a convenient time before the end of the data collection.

Benefits and risks of research

Through this research, the teachers may gain benefits from viewing and reflecting on their pedagogical practices through photographs and/or video footage that are relevant to their practices, and from engaging in reflective discussions about their practices with the researcher. These processes may contribute to the teachers' understandings of children's EMEs. Children may also benefit from their teachers' understandings of EMEs. As this research is a small-scale study where the researcher is present for some time, it is possible that the participants may be identified in the thesis and other publications, due to the findings presented. However, the efforts I will make to try and prevent that from happening are explained next.

Anonymity and Confidentiality

The identity of your child will be treated with utmost care and respect. However, if the photographs and video footage are used, anonymity cannot be guaranteed, but every attempt will be made to preserve the confidentiality of your child and the kindergarten. Your child's name will not be used in any occasion. Pseudonyms of the teacher and kindergarten will be used in all publications and presentations. The photographs and video footage will be digitally altered before using them in the thesis, academic publications, and/or conference presentations. I assure that the relevant data will be kept confidential and will not be released to any other without your consent.

If you have any further queries, please contact my supervisors or me. I do hope you will agree your child to participate in this research. If so, I would appreciate you signing the Consent Form to confirm your agreement and then helping your child complete the Assent Form at home to confirm his/her agreement, and returning them to the sealed collection box placed in the secretary's office within two weeks of the receipt of the form.

Contact Details and Approval

Student Researcher	Principal Investigator	Head of Department
Ye Zhang PhD Candidate University of Auckland ye.zhang@auckland.ac.nz +852 5429 3917 (HK)	Dr Angel Chan School of Curriculum and Pedagogy University of Auckland angel.chan@auckland.ac.nz +64 9 373 7999 ext 48884	Professor Helen Hedges School of Curriculum and Pedagogy University of Auckland h.hedges@auckland.ac.nz +64 9 373 7999 ext 48606

For any queries regarding ethical concerns, you may contact the Chair, the University of Auckland Human Participants Ethics Committee, the University of Auckland, Research Office, Private Bag 92019, Auckland 1142. Telephone 09 373-7599 ext. 83711. Email: ro-ethics@auckland.ac.nz.

Approved by the University of Auckland Human Participants Ethics Committee on 06-July-2018 for three years. Reference Number 021583

Appendix H: Email Invitation Letter

Email Invitation Letter

Dear Principal [name],

My name is Ye Zhang. I am a PhD candidate at the School of Curriculum and Pedagogy, Faculty of Education, University of Auckland. I am searching for two kindergartens to undertake a doctoral research project. This research is part of my study towards a Doctor of Philosophy degree at the University of Auckland under the supervision of Dr Angel Chan, Dr Deborah Widdowson, and Prof. Helen Hedges.

This research is about children's everyday mathematical experiences (EMEs). Although studies on children's EMEs have expanded over the past decades, such research is still limited, especially in the context of Hong Kong. Thus, this research aims to explore the nature and content of children's EMEs, including teachers' perceptions of and practices associated with children's EMEs in Hong Kong kindergarten settings. I hope to find the factors that influence the availability of children's EMEs, and how teachers understand, recognise, encourage, or extend children's EMEs in Hong Kong kindergarten settings.

I hope to visit the kindergarten weekly for a total of 21 weeks from September 2018 to early February 2019. This research will invite the teachers from one or two K1 and K3 half-day classes respectively and all children from these teachers' classes as the participants. The children's and teachers' daily routine will be observed. Episodes of children's EMEs with or without teachers' participation will be photographed, audio-recorded, and/or video-recorded in line with permissions obtained. The teachers will be invited to attend two individual semi-structured interviews to share their perceptions regarding children's EMEs. This research will also involve analysing the official curriculum documents, documents of kindergarten's guidelines and suggestions for the implementation of mathematics, children's mathematical learning reports, teachers' planning documents, and other research-related documents.

Through this research, the teachers may gain benefits from viewing and reflecting on their pedagogical practices through photographs and/or video footage that are relevant to their practices, and from engaging in reflective discussions about their practices with the researcher. These processes may contribute to the teachers' understandings of children's EMEs in Hong Kong kindergarten settings. Children may also benefit from their teachers' understandings of EMEs.

If this research study is of interest to your kindergarten, please contact me.

I have attached the Participant Information Sheets for teachers and parents/guardians, the Consent Forms for them to sign, an information booklet about the research for children, and an Assent Form to be completed by children with the assistance of their parents/guardians at home for you to carefully read to decide your kindergarten's participation in this research. I would be grateful for your kind support to conduct this research at your kindergarten. I am looking forward to hearing you soon. Thank you very much!

Approved by the University of Auckland Human Participants Ethics Committee on 06-July-2018 for three years. Reference Number 021583

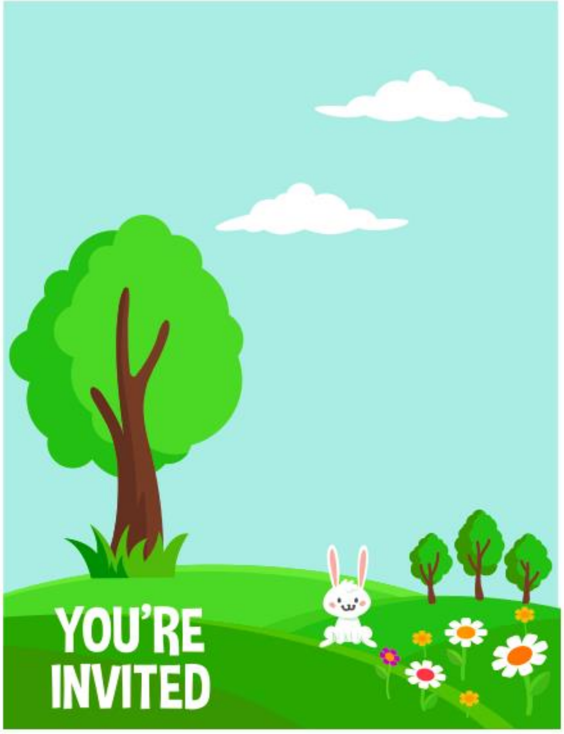
Yours sincerely,

Ye Zhang

Email: ye.zhang@auckland.ac.nz

THE UNIVERSITY OF AUCKLAND
NEW ZEALAND
Te Whare Wānanga o Tāmaki Makaurau

Appendix I: Information Booklet for Children



Hello, my name is Ye.
I am a researcher.

I would like to invite you to take part in a project!

I will stay in your class for about 20 weeks.

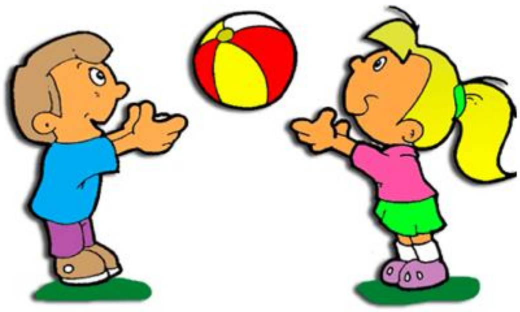
I like you to help me do some research.

I will be coming to your kindergarten to find out:

- How you find out about things like number, shape, size, and weight by yourself or with friends and adults when you are in the kindergarten.

- How your teachers think about this exploration.

Sometimes I will watch you play with your friends.



Sometimes I will watch you play by yourself.

Sometimes i will watch you have snack.

Sometimes I will watch you participate in activities with your teacher and friends.

Sometimes I will video-record you doing things.

Sometimes I may talk with your teachers.

Sometimes I will record your voice.

Sometimes I will take photographs of you.

Sometimes I will take notes.

Sometimes I may talk with you and your friends.

You can stop me watching or recording you at any time. Just tell me or any adults.

If you or your family have any questions, you can contact me:
ye.zhang@auckland.ac.nz

Approved by the University of Auckland Human Participants Ethics Committee on 06-July-2018 for three years. Reference Number 021583

Appendix J: First and Second Teacher Interview Questions

First Teacher Interview Questions

Date ________________________________ Time ________________________________

Kindergarten _________________________ Teacher _____________________________

Venue ______________________________ Interview length ______________________

Teacher's professional background and experience

1. Thank you for volunteering to participate in my research. I would like to know about you, your role as a kindergarten teacher, and your teaching and educational experiences. Please tell me something about yourself.
2. I would like to know about in what occasions you will use mathematical concepts to solve problems in everyday life. Could you please describe some occasions when you think mathematical concepts are useful in your everyday life?
3. Please tell me something about how you teach early childhood mathematics in your class.

Teacher's perception of Children's EMEs

4. How would you describe early childhood mathematics?
5. What does the term "everyday experiences" mean to you?
6. How would you describe the relationship between everyday experiences and early childhood mathematics?
7. How would you describe the term "everyday mathematical experiences (EMEs)" when you hear it?
8. What kinds of expectations do the parents in your class have regarding early childhood mathematics?
9. Why do you think the parents in your class have such expectations?
10. Thank you very much for your participation in this interview. Are there any other questions that you would like to talk about?

Second Teacher Interview Questions

Date ________________________________ Time ________________________________

Kindergarten _________________________ Teacher _____________________________

Venue ______________________________ Interview length _______________________

Teacher's practices of children's EMEs

1. At the beginning of my fieldwork you told me that you understood children's everyday mathematical experiences (EMEs) to be …… I wonder if you want to add anything more to those ideas now.
2. Please describe one example that the children in your class are exploring early mathematical concepts (e.g., number, shape, size, weight, and pattern) in their day-to-day activities and play?
3. Let's see one photograph/video footage of children's activity/play in your class. What do you think they are doing there? (Show one photograph/video footage of children's EMEs without the teacher's participation)
4. What elements do you think are most important if you see children are exploring early mathematical concepts (e.g., number, shape, size, weight, and pattern) in their day-to-day activities and play?
5. Let's see another photograph/video footage of children's activity/play in your class. Can you describe what you were doing at that time in this photograph/video footage? (Show another photograph/video footage of children's EMEs with the teacher's participation)
6. What elements do you think are most important if you intend to join children's exploration of early mathematical concepts (e.g., number, shape, size, weight, and pattern) in their day-to-day activities and play?
7. Thank you very much for your participation in this interview. Are there any other questions that you would like to talk about?

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